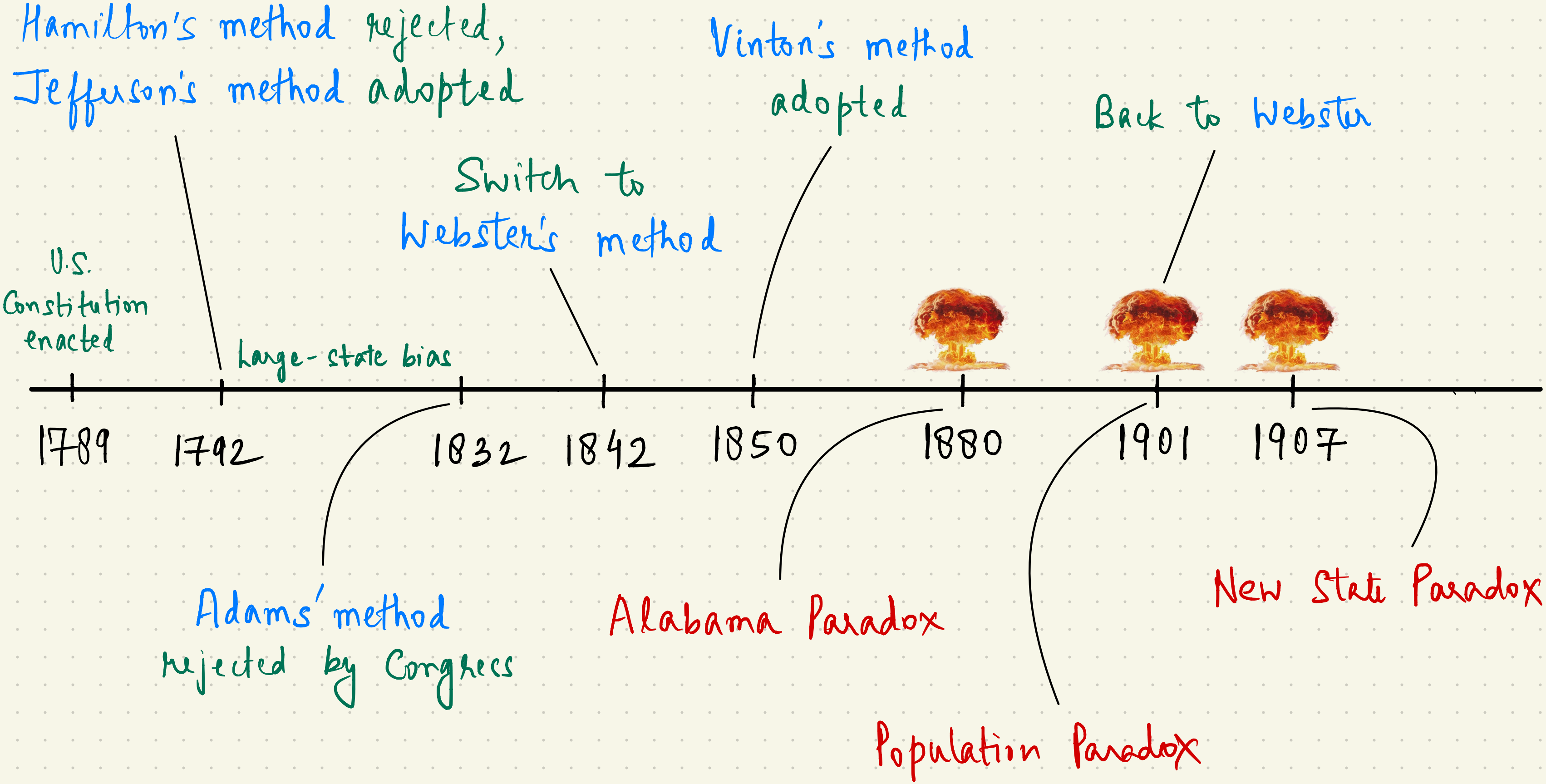
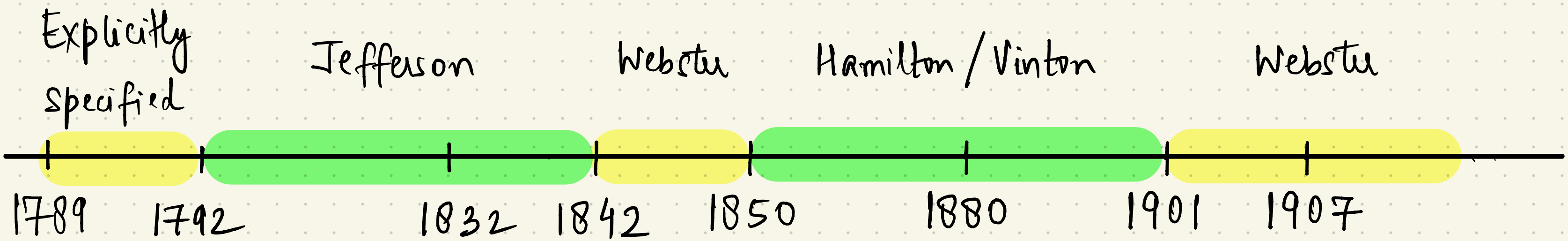
COL 8184 : Algorithms for Fair Representation

Lecture 4

Divisor Methods

Jan 19, 2026 | Rohit Vaish


Hamilton's method rejected,
Jefferson's method adopted
Vinton's method adopted
Back to Webster
Switch to Webster's method
U.S. Constitution enacted
Large-state bias
1789
1792
1832
1842
1850
1880
1901
1907
Adams' method rejected by Congress
Alabama Paradox
New State Paradox
Population Paradox



Explicitly specified
Jefferson
Webster
Hamilton / Vinton
Webster
1789
1792
1832
1842
1850
1880
1901
1907

* Let D be a divisor such that

Jefferson : $\left\lfloor \frac{p_1}{D} \right\rfloor + \left\lfloor \frac{p_2}{D} \right\rfloor + \cdots + \left\lfloor \frac{p_n}{D} \right\rfloor = h.$

Adams : $\left\lceil \frac{p_1}{D} \right\rceil + \left\lceil \frac{p_2}{D} \right\rceil + \cdots + \left\lceil \frac{p_n}{D} \right\rceil = h.$

Webster : $\left[\frac{p_1}{D} \right] + \left[\frac{p_2}{D} \right] + \cdots + \left[\frac{p_n}{D} \right] = h.$

* Assign $s_i = \left\lfloor \frac{p_i}{D} \right\rfloor$, $\left\lceil \frac{p_1}{D} \right\rceil$, or $\left[\frac{p_1}{D} \right]$ to state i.

Jefferson Adams Webster

DIVISOR METHODS (TODAY!)

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APPORTIONMENT PARADOXES

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Alabama Paradox : A state loses a seat despite growing house size.

Population Paradox : A growing state loses a seat to a shrinking state.

New State Paradox : A state loses a seat when a new state joins.

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HOUSE MONOTONICITY

An apportionment method is house monotone if, for any two problem instances $I = (h; p_1, p_2, \dots p_n)$ and $I' = (h+1; p_1, p_2, \dots, p_n)$ with seat assignments $I \longmapsto (s_1, s_2, \dots, s_n)$ and $I' \longmapsto (s_1', s_2', \dots, s_n')$, we have

$$s_i' \geq s_i \quad \text{for every state } i.$$

HOUSE MONOTONICITY

Theorem: Hamilton's method fails house monotonicity.

Theorem: Jefferson's method is house monotone.

Population Monotonicity

An apportionment method is population monotone if, for any two problem instances $I = (h; p_1, \cdots, p_n)$ and $I' = (h'; p_1', \cdots, p_n')$ with seat assignments $I \longmapsto (s_1, s_2, \cdots, s_n)$ and $I' \longmapsto (s_1', s_2', \cdots, s_n')$,

$$\underbrace{s_i < s_i'}_{\substack{i \text{ gets} \\ \text{more seats}}} \text{ and } \underbrace{s_j > s_j'}_{\substack{j \text{ gets} \\ \text{fewer seats}}} \Rightarrow \underbrace{p_i < p_i'}_{i \text{ grows}} \text{ or } \underbrace{p_j > p_j'}_{j \text{ shrinks}}.$$

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Suppose, for contradiction, that $s_j > s_j'$ for some state j.

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However, both conditions are FALSE for I and I'. $\blacksquare$

DIVISOR METHODS

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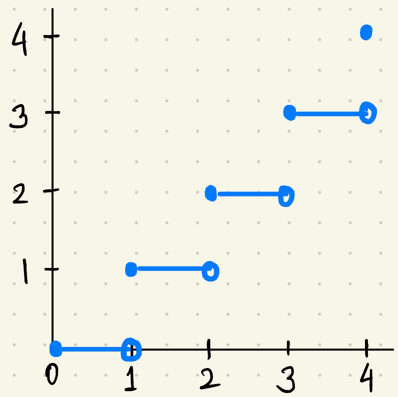

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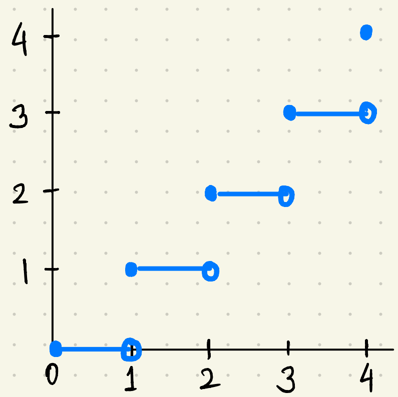

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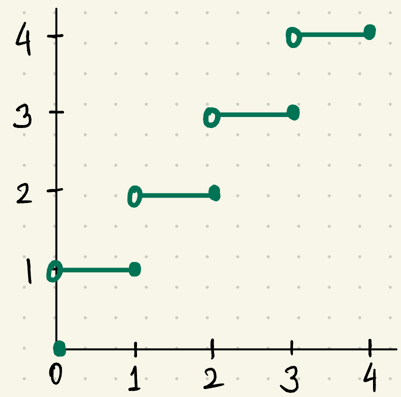

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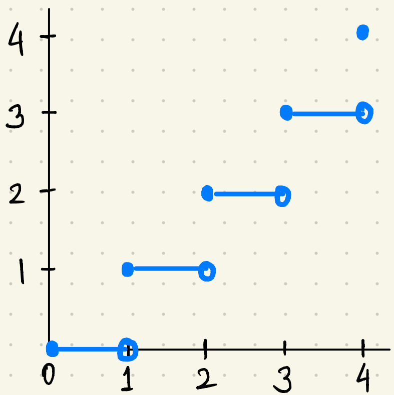

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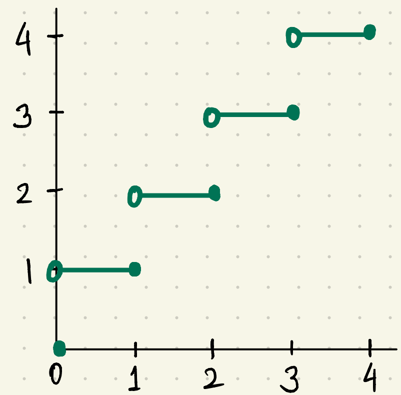

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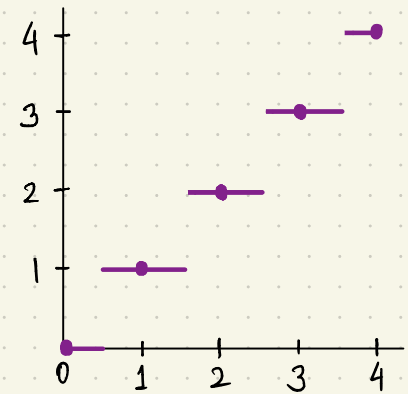

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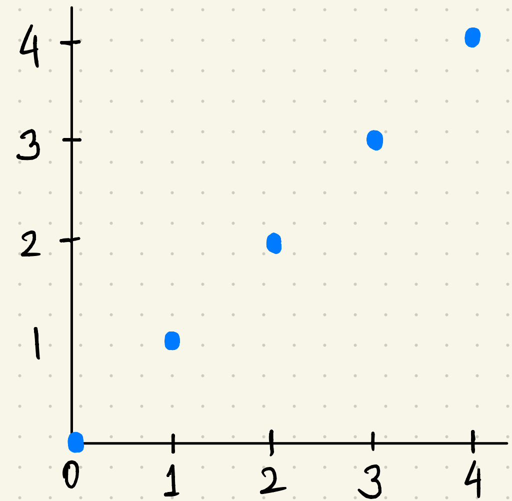

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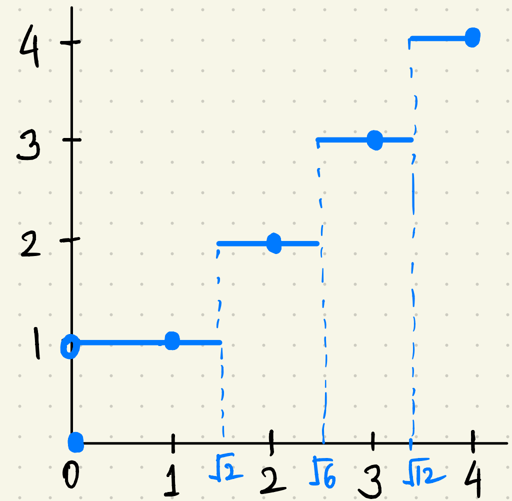

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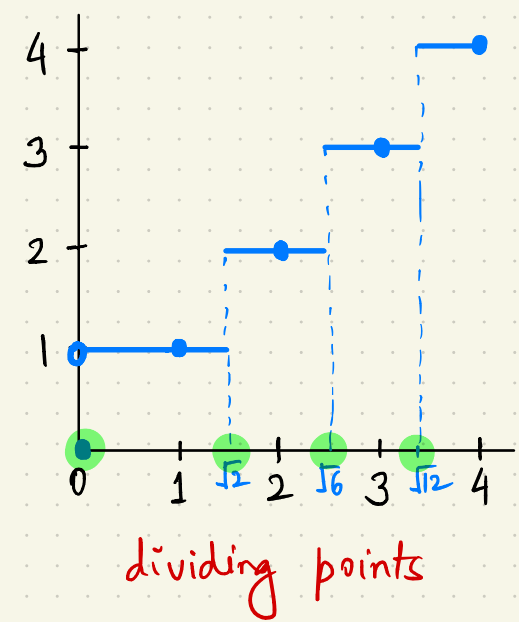

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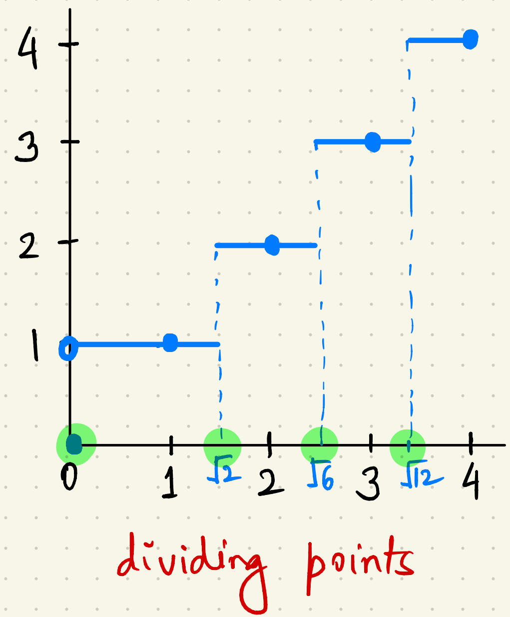

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$f(x) = \lceil x \rceil$: $0, 1, 2, \cdots, k-1$

$f(x) = [x]$: $0.5, 1.5, 2.5, \cdots, k - \frac{1}{2}$

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$$f\left(\frac{p_1}{D}\right) + f\left(\frac{p_2}{D}\right) + \cdots + f\left(\frac{p_n}{D}\right) = h.$$

and assigns $s_i = f\left(\frac{p_i}{D}\right)$ seats for every state i.

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Webster's $\quad f(x) = [x] \quad \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots, k - \frac{1}{2}$

arithmetic mean

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arithmetic mean

Huntington-Hill's $\quad f(x) = \begin{cases} \lceil x \rceil & \text{if } x \geq \sqrt{\lfloor x \rfloor \lceil x \rceil} \\ \lfloor x \rfloor & \text{o/w} \end{cases} \quad 0, \sqrt{2}, \sqrt{6}, \cdots, \sqrt{(k-1) \cdot k}$

geometric mean

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DIVISOR METHODS

Jefferson's	$f(x) = \lfloor x \rfloor$	$1, 2, 3, \cdots, k$
Webster's	$f(x) = [x]$ arithmetic mean	$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots, k - \frac{1}{2}$
Huntington-Hill's	$f(x) = \begin{cases} \lceil x \rceil & \text{if } x \geq \sqrt{\lfloor x \rfloor \lceil x \rceil} \\ \lfloor x \rfloor & \text{o/w} \end{cases}$ geometric mean	$0, \sqrt{2}, \sqrt{6}, \cdots, \sqrt{(k-1) \cdot k}$
Dean's	$f(x) = \begin{cases} \lceil x \rceil & \text{if } x \geq \dfrac{2}{\frac{1}{\lfloor x \rfloor} + \frac{1}{\lceil x \rceil}} \\ \lfloor x \rfloor & \text{o/w} \end{cases}$ harmonic mean	$0, \frac{4}{3}, \frac{12}{5}, \cdots, \frac{2k(k-1)}{2k-1}$
Adams'	$f(x) = \lceil x \rceil$	$0, 1, 2, \cdots, k-1$

DIVISOR METHODS

In the order of the value of the k^{th} dividing point d_k :

Adams' < Dean's < Huntington-Hill's < Webster's < Jefferson's

Adams'	Dean's	Huntington-Hill's	Webster's	Jefferson's
$k-1$	$\left(\frac{\frac{1}{k-1}+\frac{1}{k}}{2}\right)^{-1}$	$\sqrt{(k-1)\cdot k}$	$k-\frac{1}{2}$	k

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In the order of the value of the k^{th} dividing point d_k :

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$k-1$ $\quad$ $\left(\frac{\frac{1}{k-1}+\frac{1}{k}}{2}\right)^{-1}$ $\quad$ $\sqrt{(k-1)\cdot k}$ $\quad$ $k-1/2$ $\quad$ k

Why? For any $0 \leq x < y$

$$x \leq HM(x,y) \leq GM(x,y) \leq AM(x,y) \leq y$$

AWESOME EXAMPLE

House size $h = 36$

States	Populations	Adams	Dean	H-H	Webster	Jeff.
A	27,744	10	10	10	10	11
B	25,178	9	9	9	9	9
C	19,951	7	7	7	8	7
D	14,610	5	5	6	5	5
E	9,225	3	4	3	3	3
F	3,292	2	1	1	1	1

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🤔 Is there a smaller awesome example?

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PROPERTIES OF DIVISOR METHODS

Every divisor method has an equivalent "Table definition".

State i gets at least one seat $\iff f\left(\frac{p_i}{D}\right) \geq 1$

$$\iff \frac{p_i}{D} \geq d_1$$

$$\iff D \leq \frac{p_i}{d_1}$$

PROPERTIES OF DIVISOR METHODS

Every divisor method has an equivalent "Table definition".

State i gets at least k seats $\iff f\left(\frac{p_i}{D}\right) \geq k$

$$\iff \frac{p_i}{D} \geq d_k$$

$$\iff D \leq \frac{p_i}{d_k}$$

PROPERTIES OF DIVISOR METHODS

Every divisor method has an equivalent "Table definition".

	$\frac{p_i}{d_{10}}$	$\frac{p_i}{d_9}$	$\frac{p_i}{d_8}$	$\frac{p_i}{d_7}$	$\frac{p_i}{d_6}$	$\frac{p_i}{d_5}$	$\frac{p_i}{d_4}$	$\frac{p_i}{d_3}$	$\frac{p_i}{d_2}$	$\frac{p_i}{d_1}$
State A	·									·
State B	·			- - -						·
State C	·									·

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Corollary: Every divisor method satisfies house monotonicity.

Quota Methods

QUOTA METHODS

An apportionment method satisfies lower quota if, for any instance $I = (h; p_1, p_2, \ldots, p_n)$ and seat assignment $(s_1\ s_2 \cdots s_n)$, it holds that $s_i \geq \lfloor q_i \rfloor$ where $q_i = h \times p_i / \sum p_j$.

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A quota method is one that satisfies both upper and lower quota.

PROPERTIES OF DIVISOR METHODS

Theorem : Every divisor method fails the quota criterion.

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We will prove a stronger result later.

PROPERTIES OF DIVISOR METHODS

Theorem : Every divisor method fails the quota criterion.

Among divisor methods :

* Only Jefferson's method always satisfies lower quota.
* " Adams' " " " upper " .

COHERENCE

(aka uniformity)

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Idea : Every part of a fair division should be fair.

COHERENCE

(aka uniformity)

Idea : Every part of a fair division should be fair.

An apportionment method satisfies coherence if, for any instance $I = (h; p_1, p_2, \dots, p_n)$ with seat assignment $(s_1, s_2, \dots, s_n)$ and any subset $T \subseteq \{1, 2, \dots, n\}$ of states, the seat assignment for the restricted instance $I|_T = \left(\sum_{i \in T} s_i ; (p_t)_{t \in T} \right)$ is $(s_t)_{t \in T}$.

PROPERTIES OF DIVISOR METHODS

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Therefore, D is also the divisor for the restricted instance $I|_T$, resulting in seat assignment $(s_t)_{t \in T}$. ∎

STORY SO FAR

Divisor methods

+ avoid Alabama paradox (house monotonicity)
+ avoid population paradox (population monotonicity)
+ avoid new state paradox (coherence)

− fail quota criterion

QUIZ

QUIZ

House size $h = 10$

States	Populations	Adams	Jefferson
A	?	2	2
B	40	?	4
C	27	3	?
D	?	2	1
	100		

Fill out the missing entries.

NEXT LECTURE

An optimization view of Apportionment