

Lecture 17

Manipulation in Stable Matching Problem

Stable Matching Problem

$w_1 > w_2 > w_3$



$m_3 > m_2 > m_1$

$w_2 > w_1 > w_3$



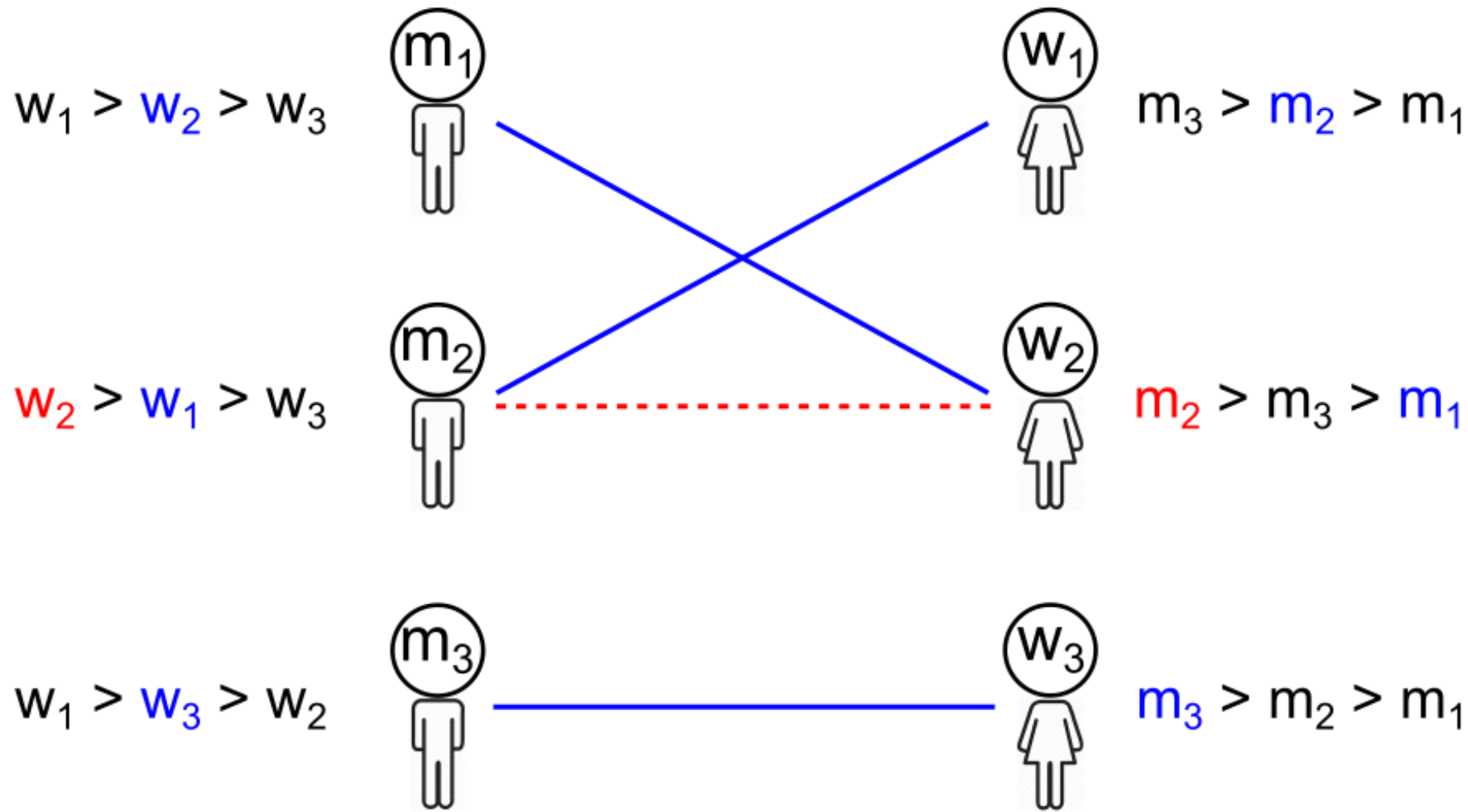
$m_2 > m_3 > m_1$

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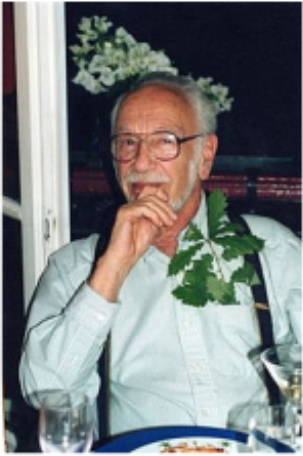


$m_3 > m_2 > m_1$

Stable Matching Problem



A matching is **stable** if there is no **blocking pair**.



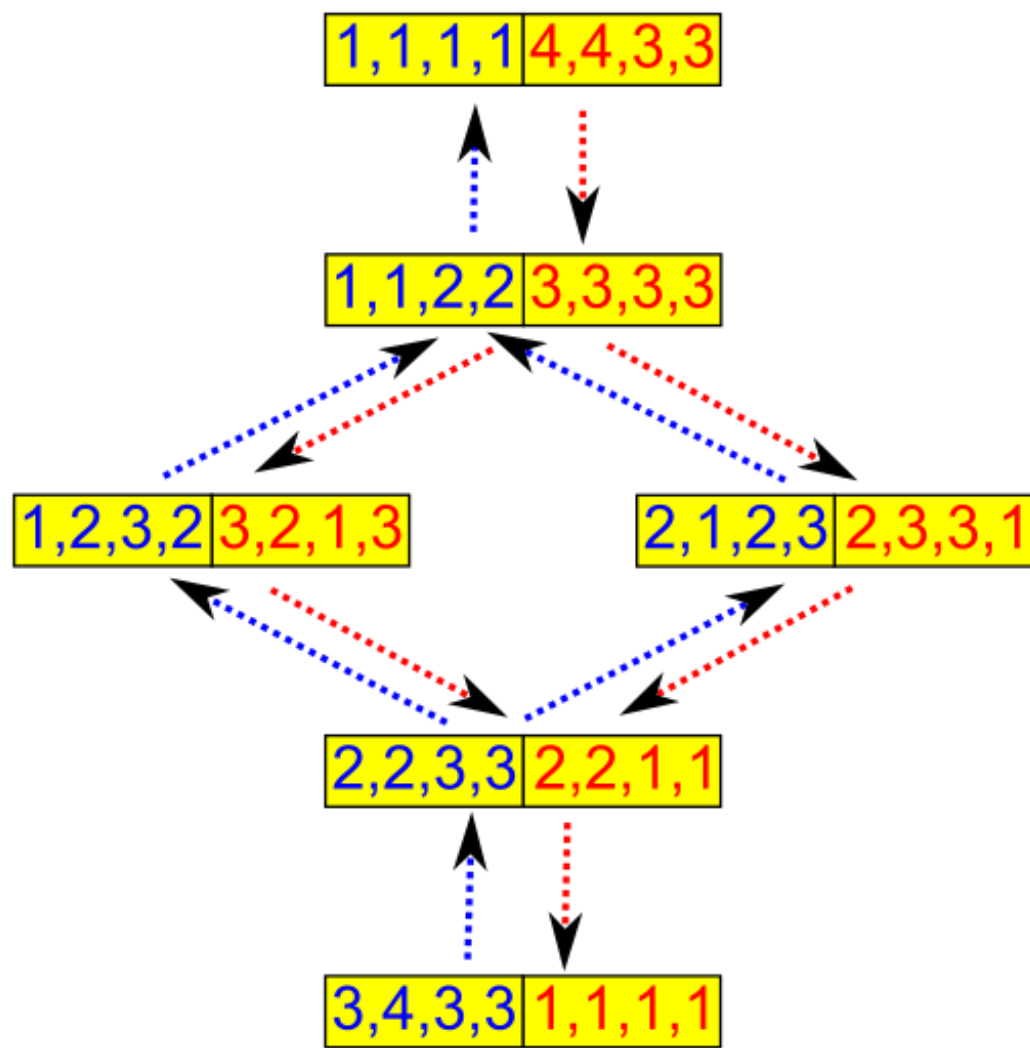
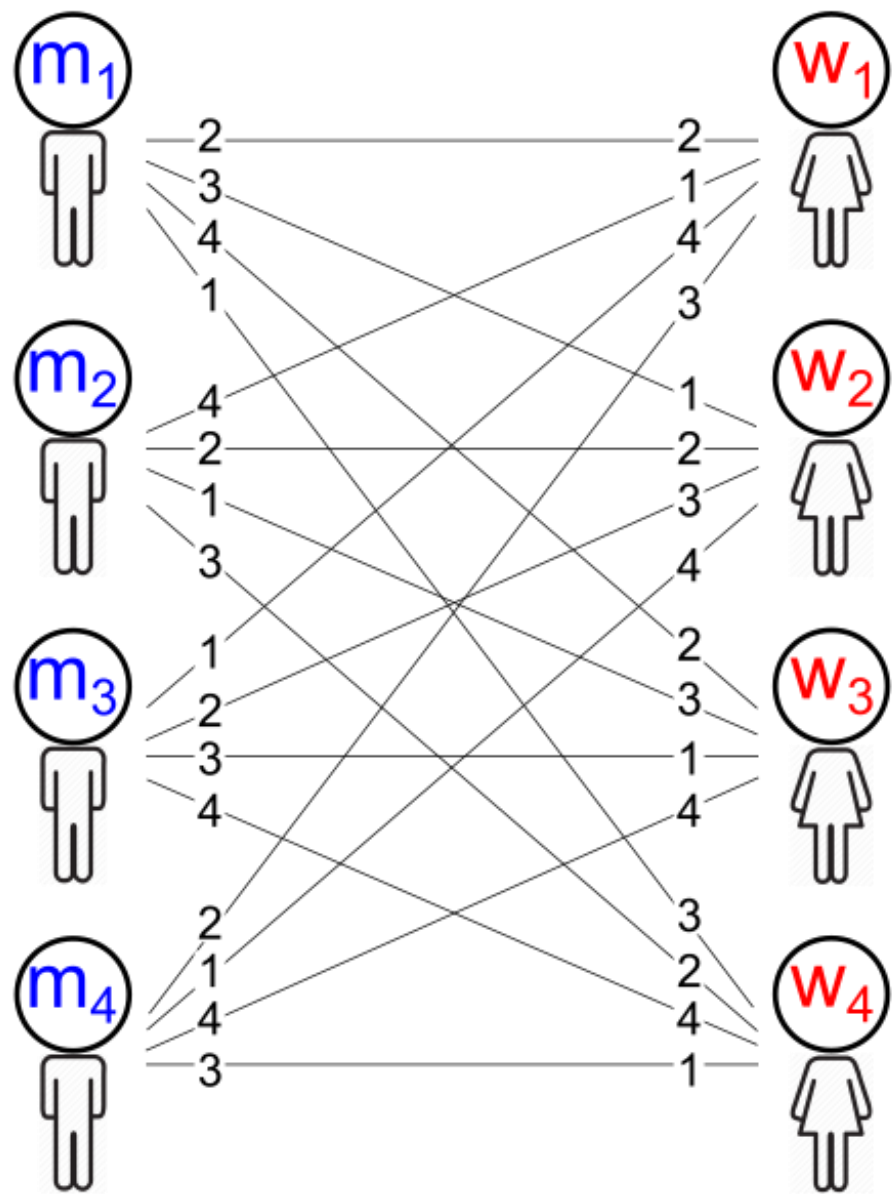
COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

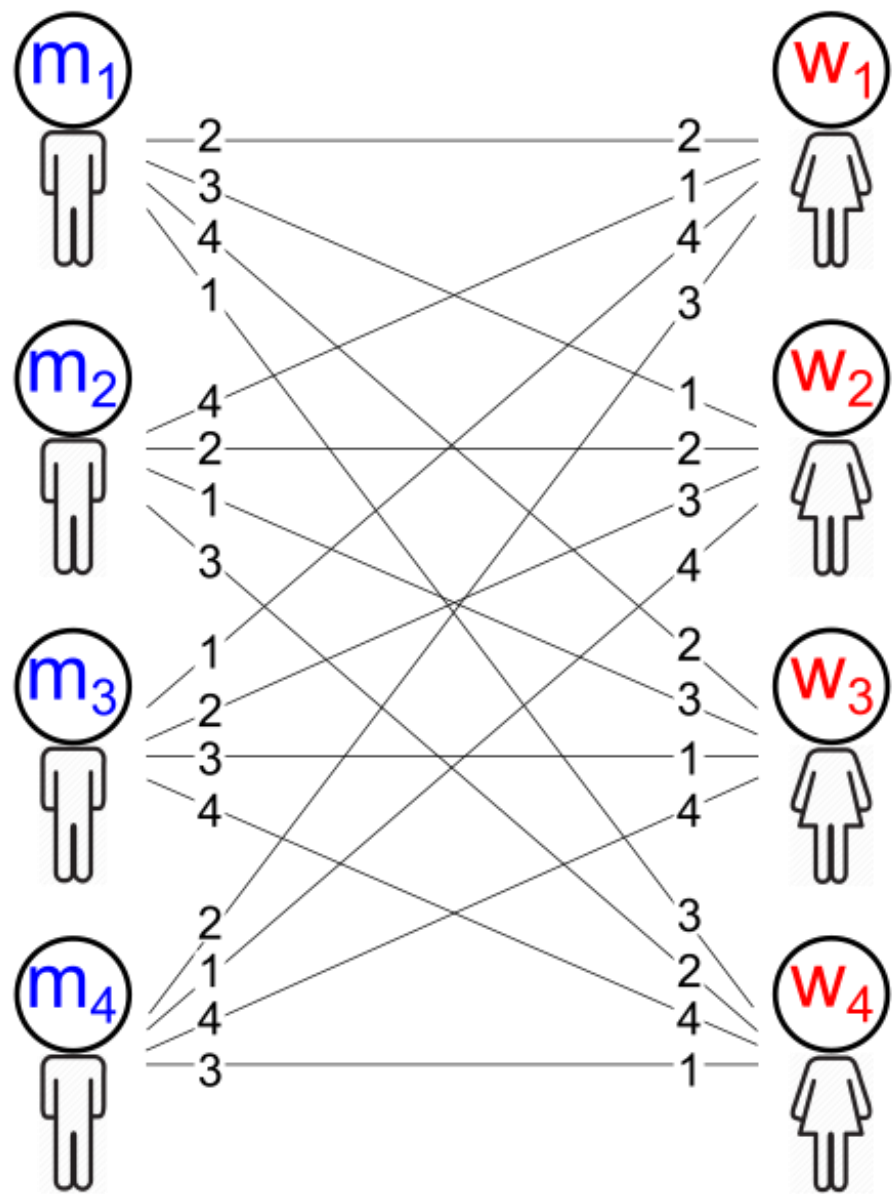
D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation



Source: *The American Mathematical Monthly*, Jan., 1962, Vol. 69, No. 1 (Jan., 1962), pp. 9-15

Given any preference profile, a stable matching for that profile always exists and can be computed in polynomial time.

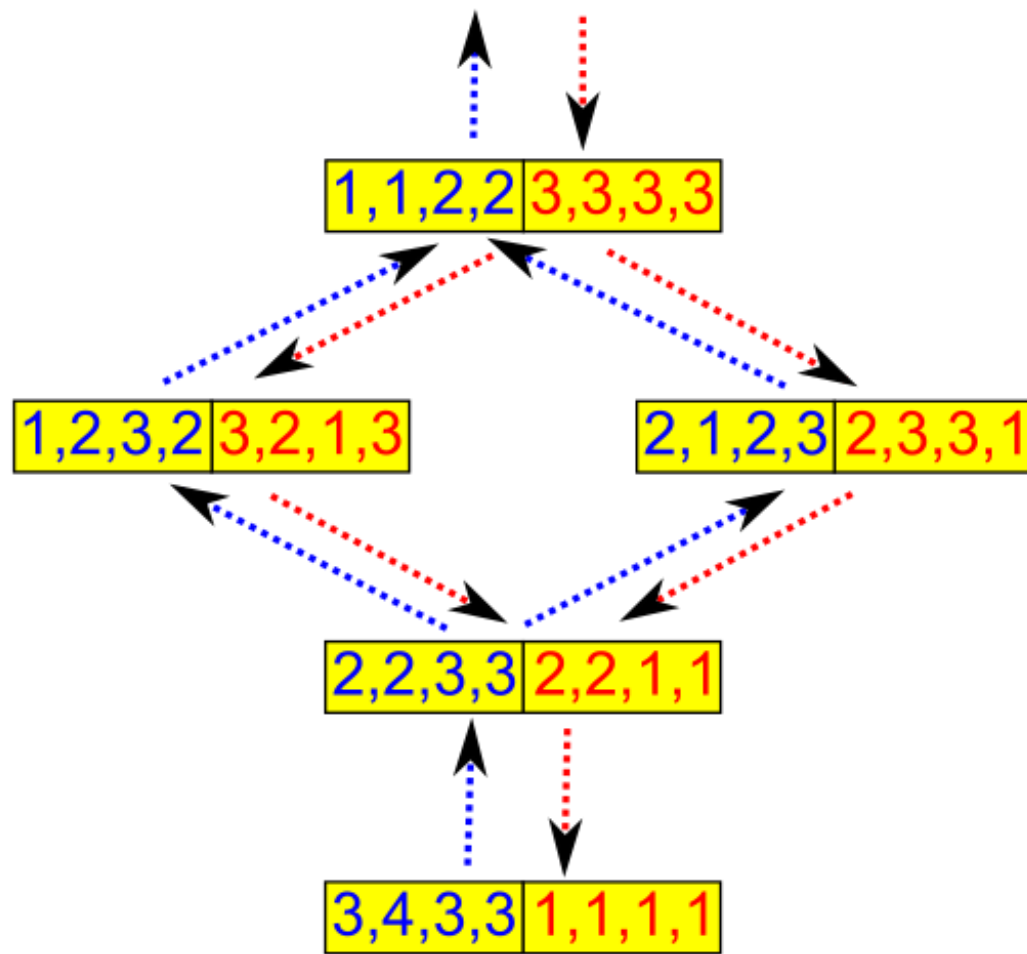


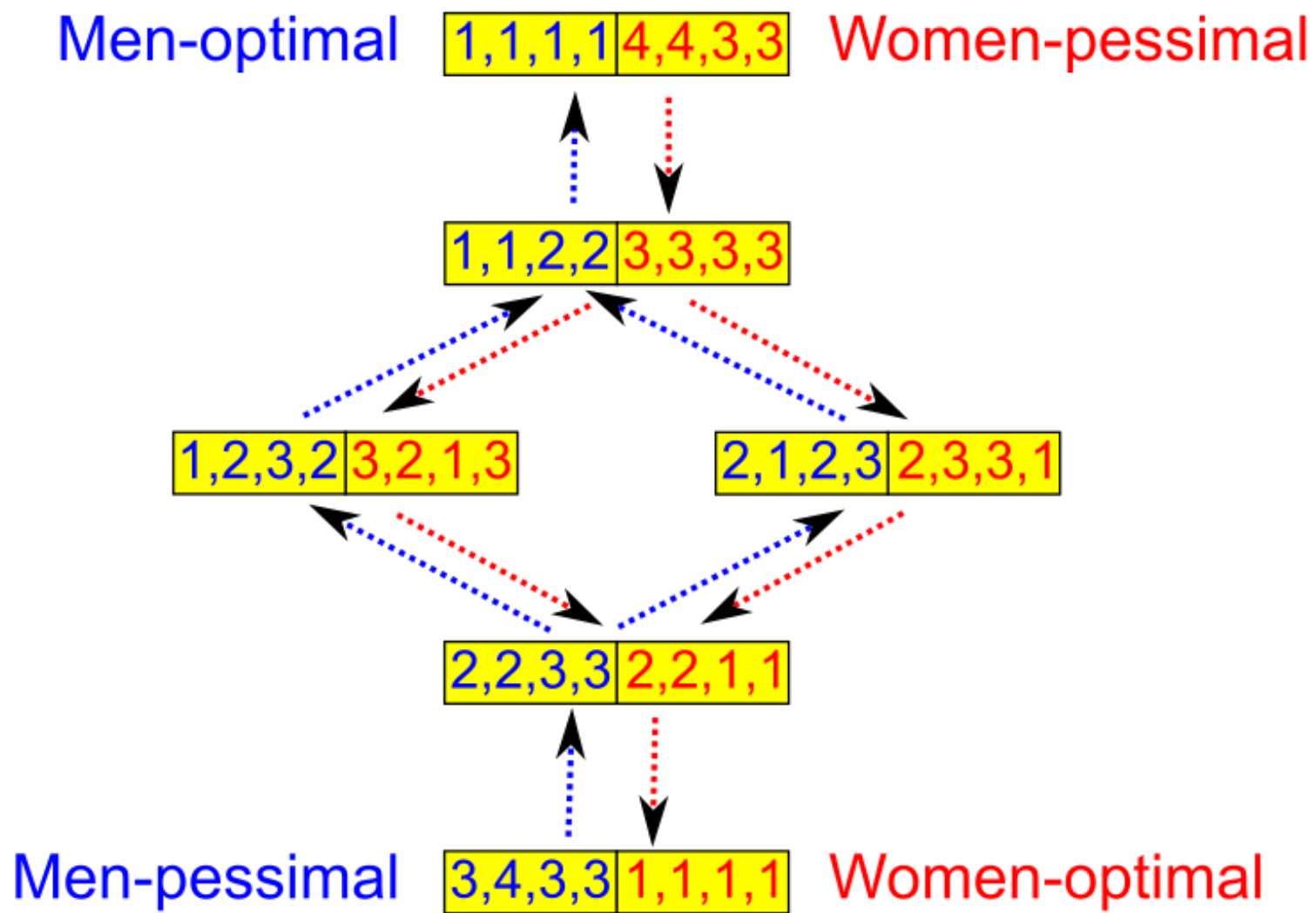
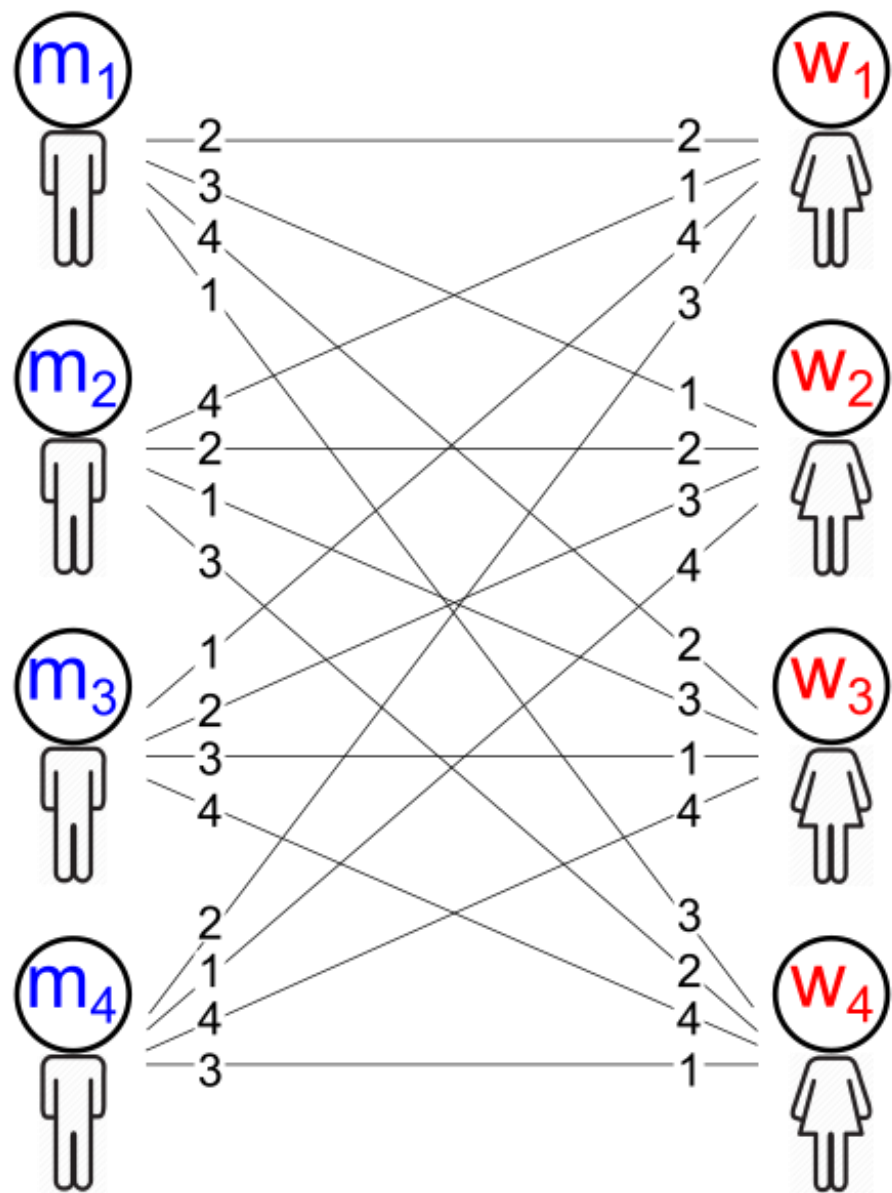


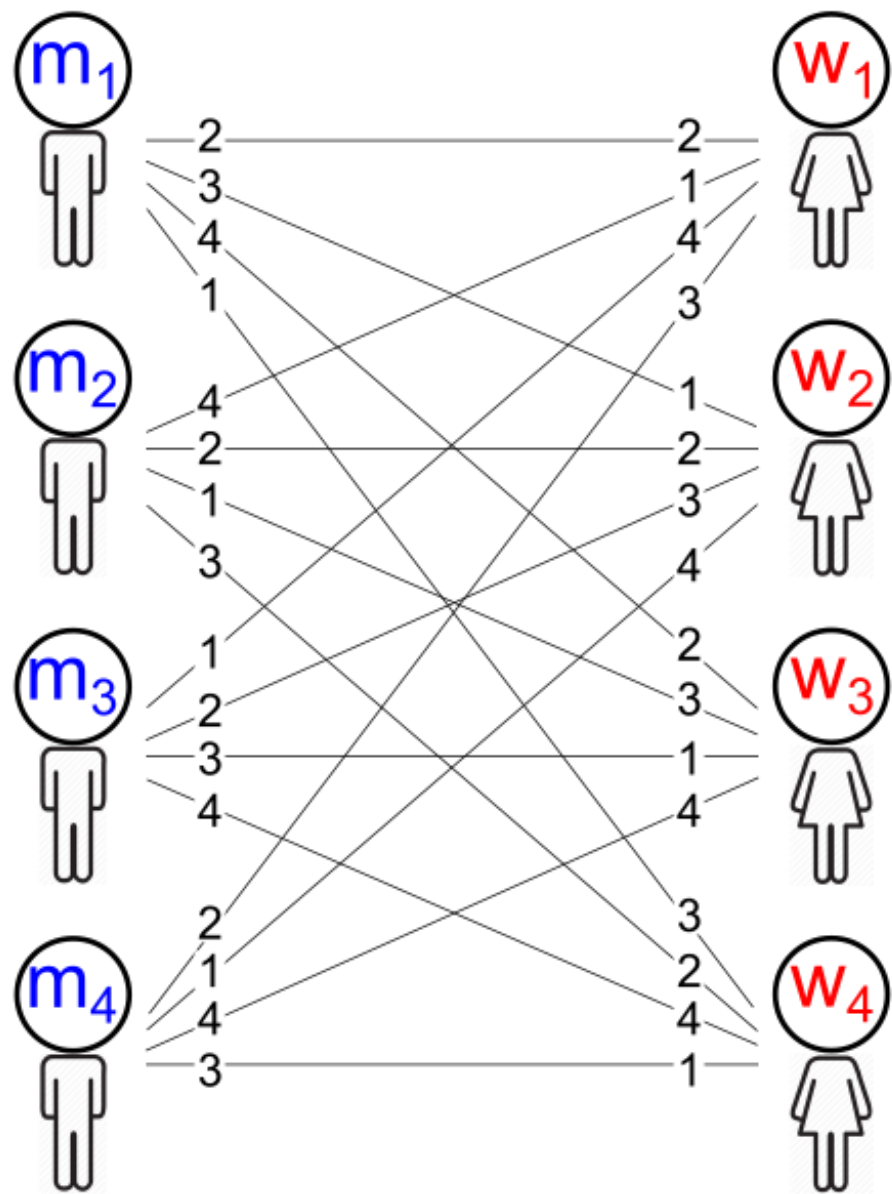
Men-optimal

1,1,1,1 | 4,4,3,3

Women-pessimal

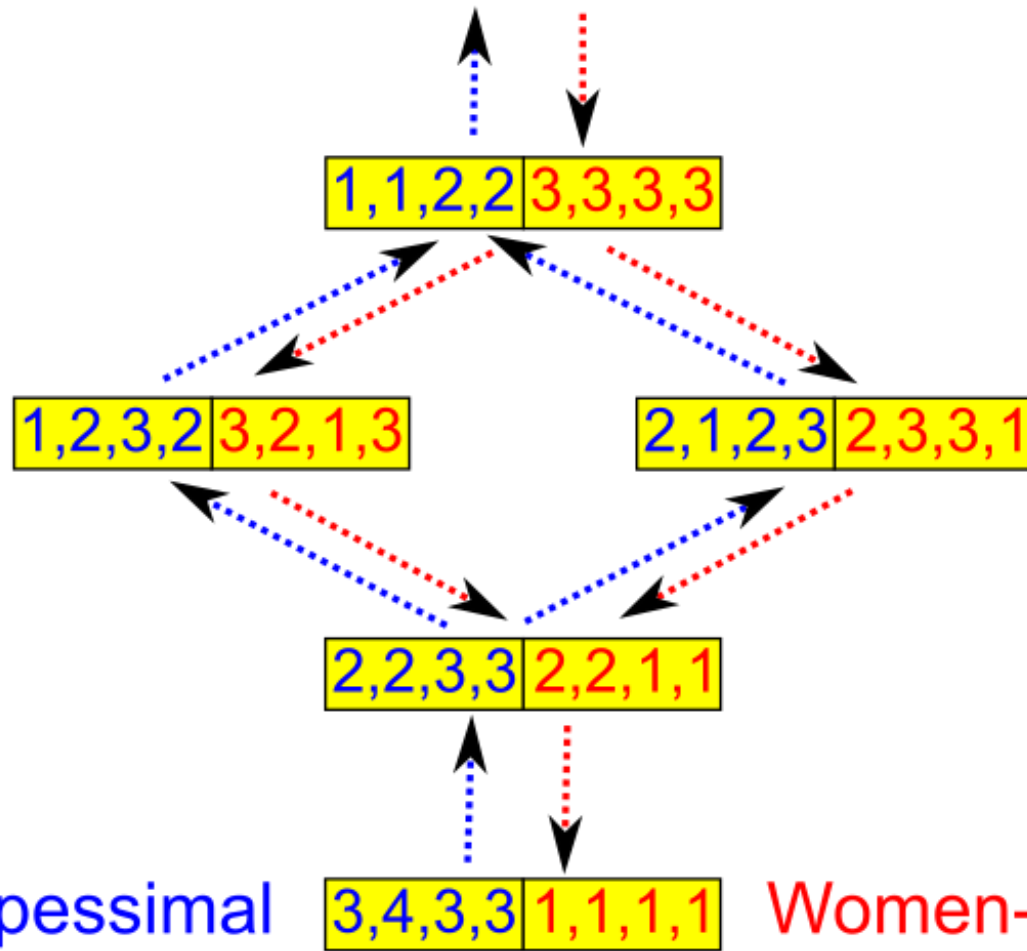




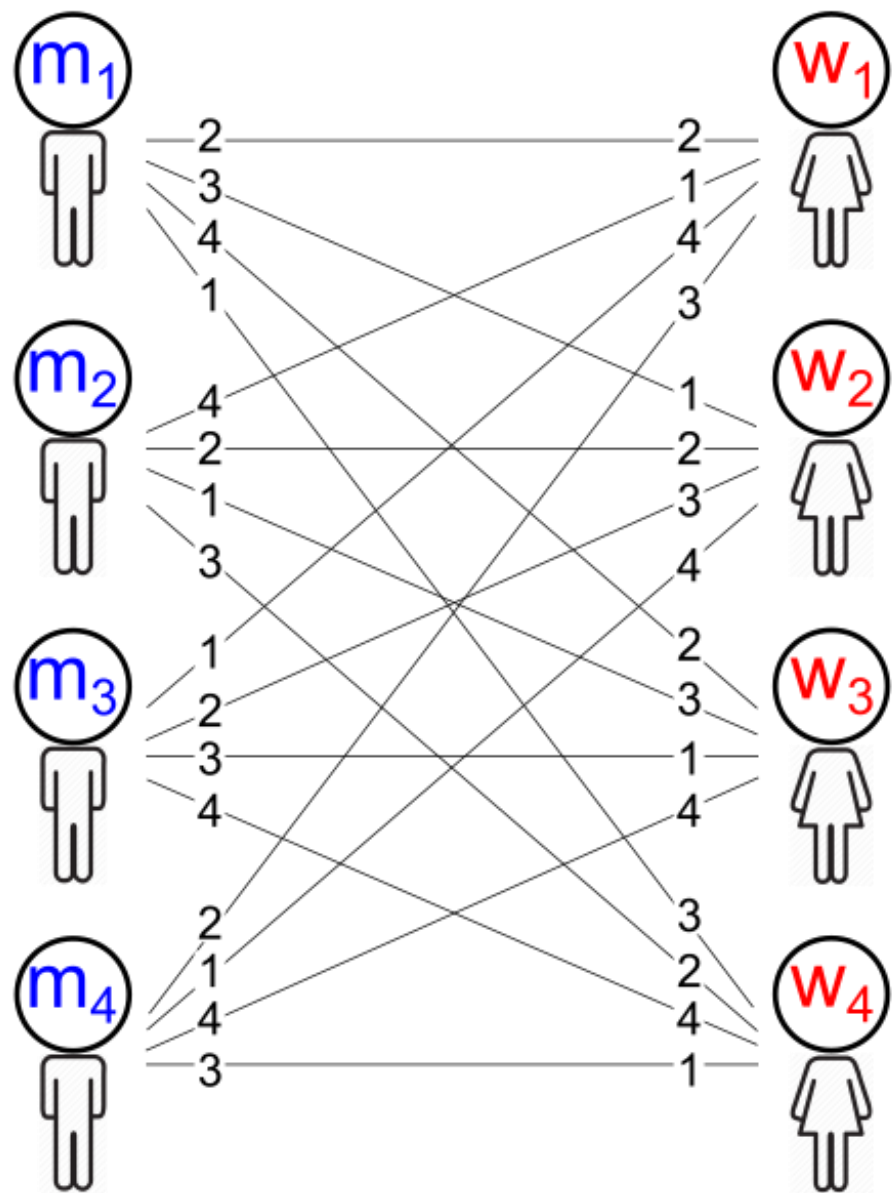


Men-proposing DA algorithm computes this

Men-optimal $1,1,1,1 \mid 4,4,3,3$ Women-pessimal

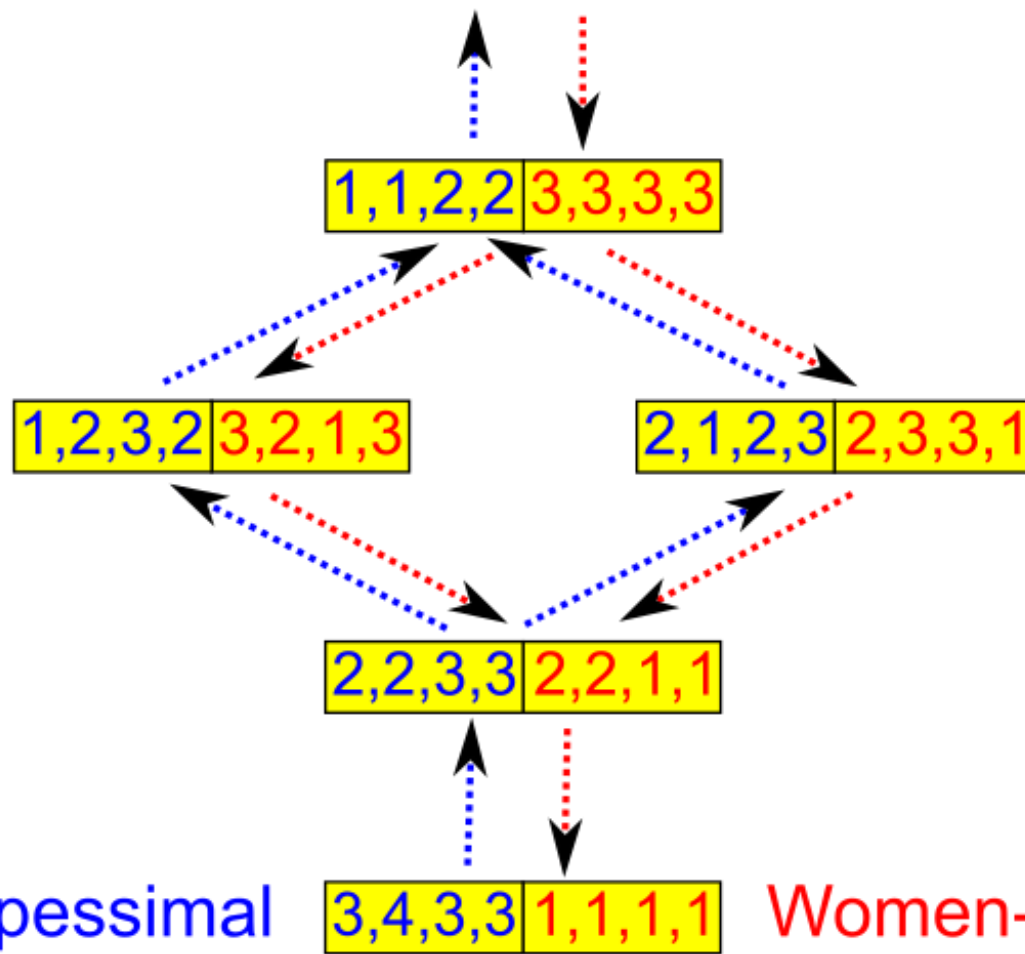


Men-pessimal $3,4,3,3 \mid 1,1,1,1$ Women-optimal



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Men-optimal $1, 1, 1, 1$ | $4, 4, 3, 3$ Women-pessimal



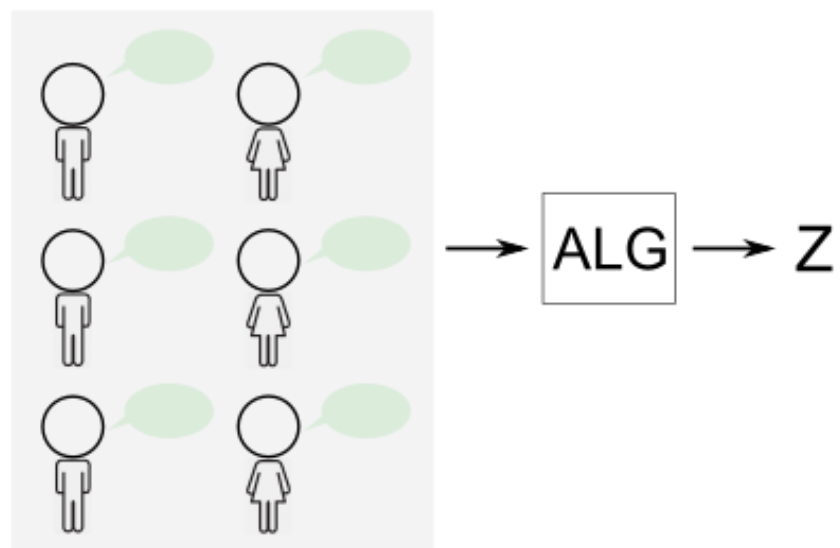
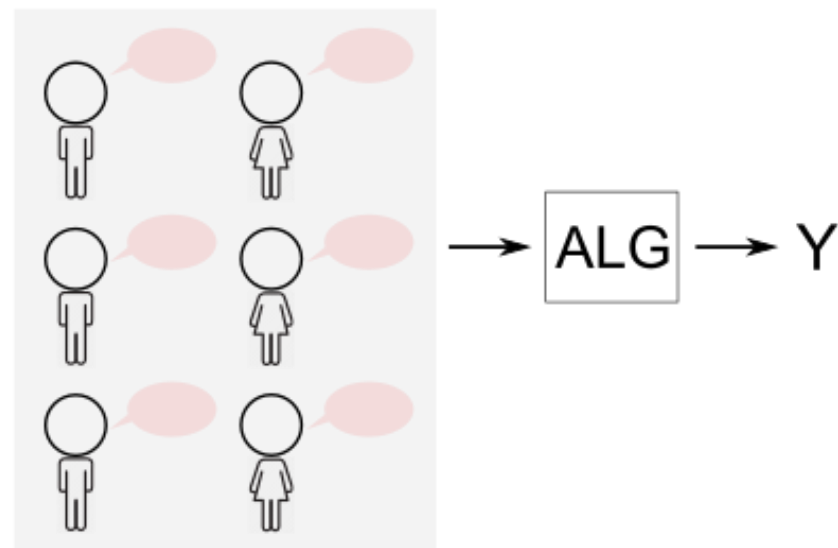
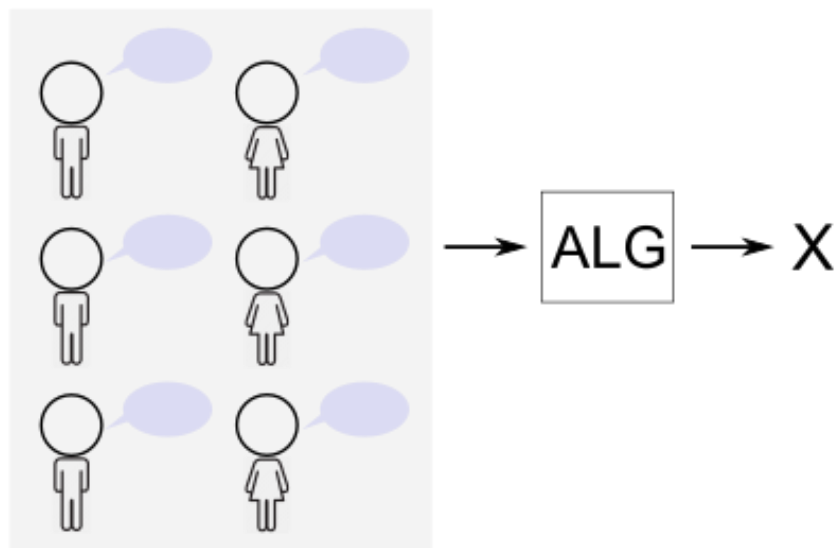
Men-pessimal $3, 4, 3, 3$ | $1, 1, 1, 1$ Women-optimal

Women-proposing DA algorithm computes this

Can an agent get a better partner under DA algorithm
by misreporting his/her preferences?

Strategyproofness

Strategyproofness



...

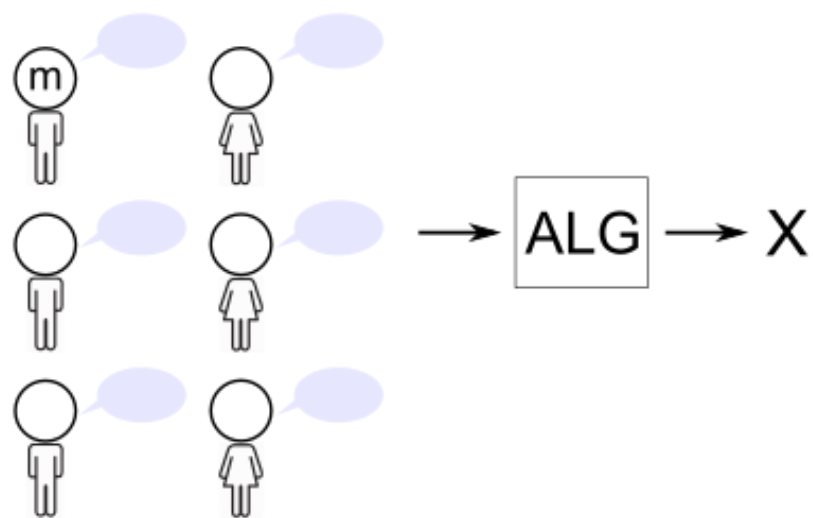
Strategyproofness

An algorithm fails strategyproofness if, for some set of input preferences, some agent can unilaterally misreport and improve (w.r.t. true preference).

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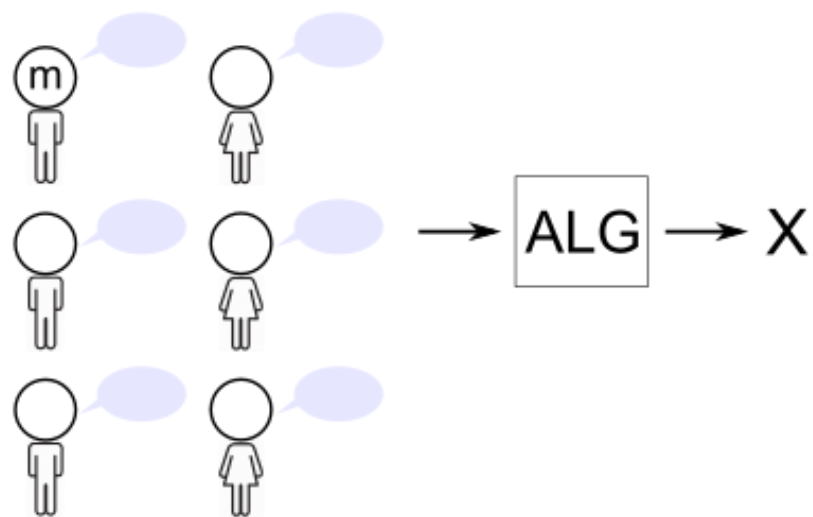
True profile $P = (P_{-m}, P_m)$



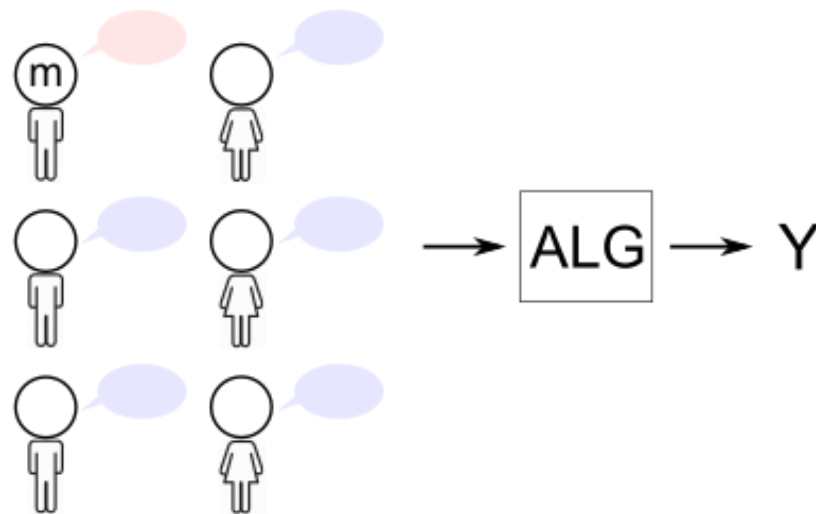
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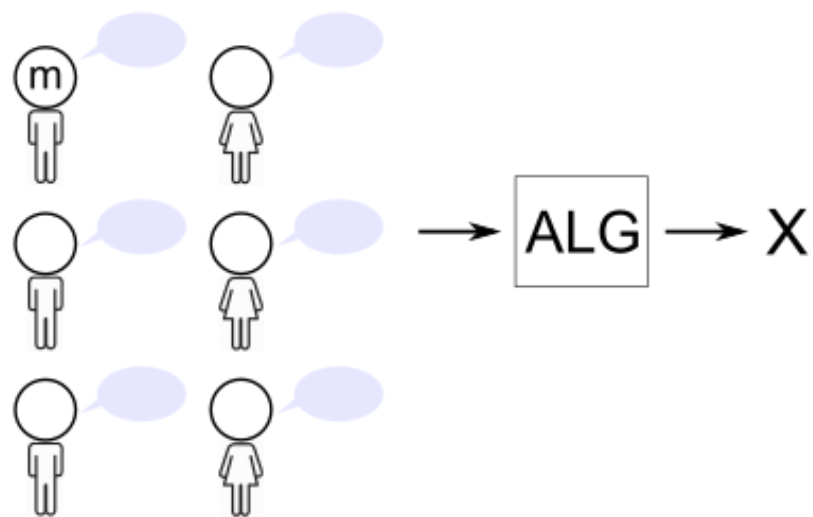
Manipulated profile $P' = (P_{-m}, P'_m)$



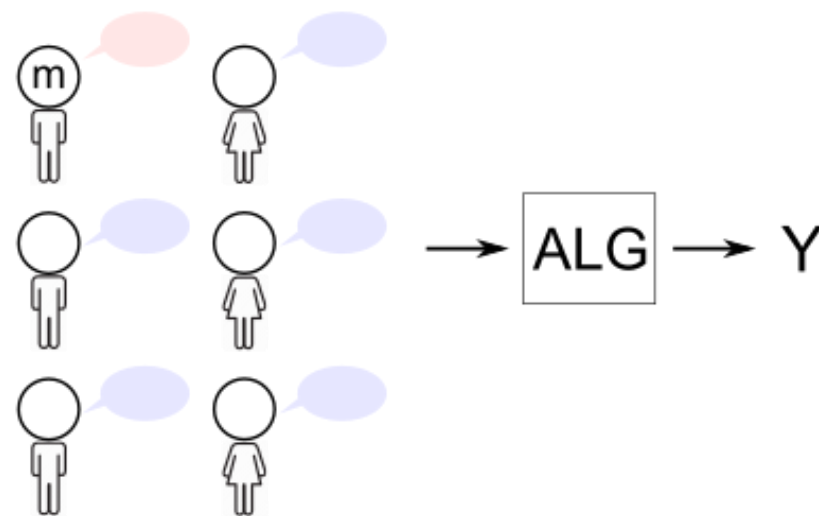
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True preference list P_m ... $> Y(m) > \dots > X(m) > \dots$

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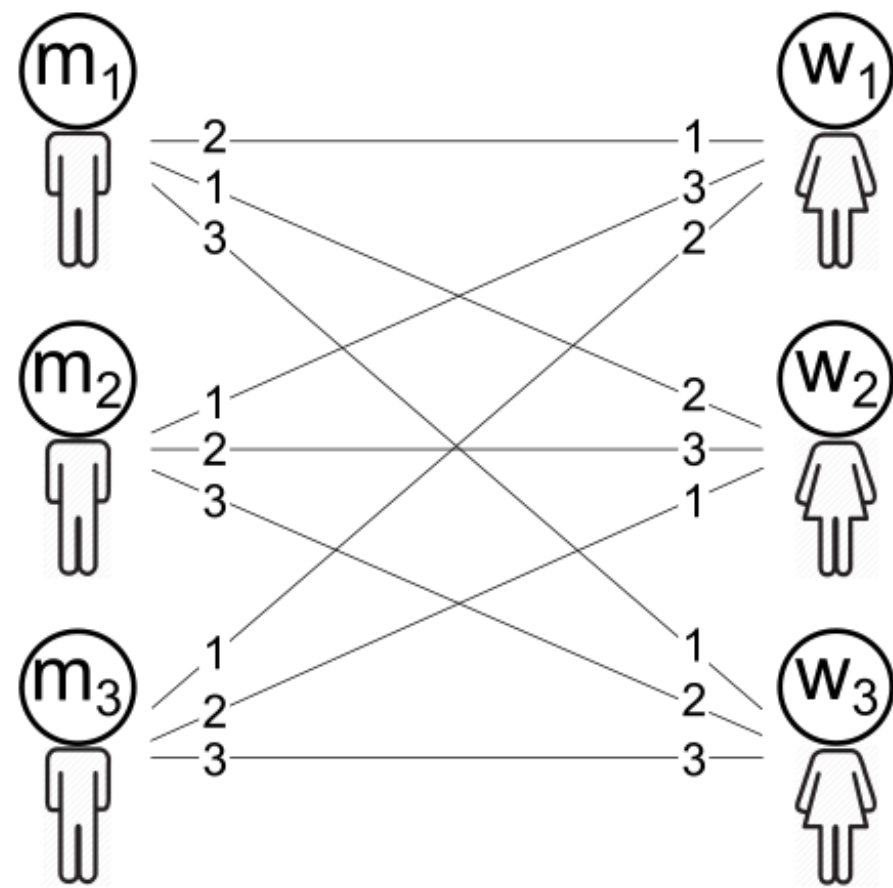
An algorithm is **strategyproof** if it does not fail strategyproofness on any input.

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is not strategyproof.

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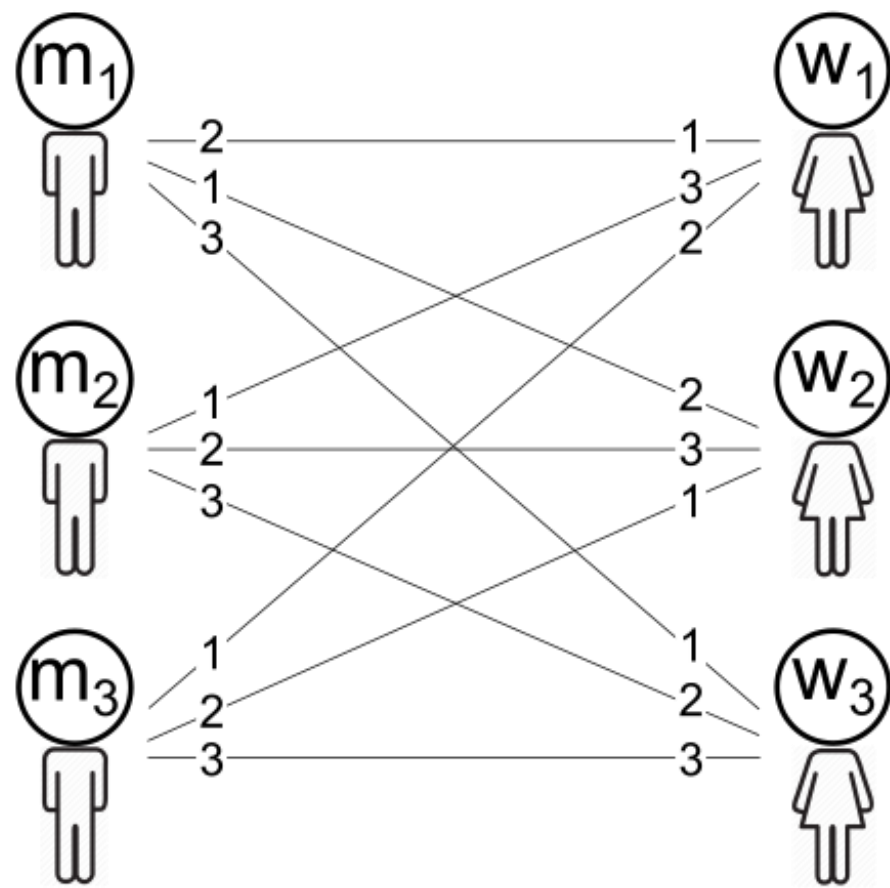
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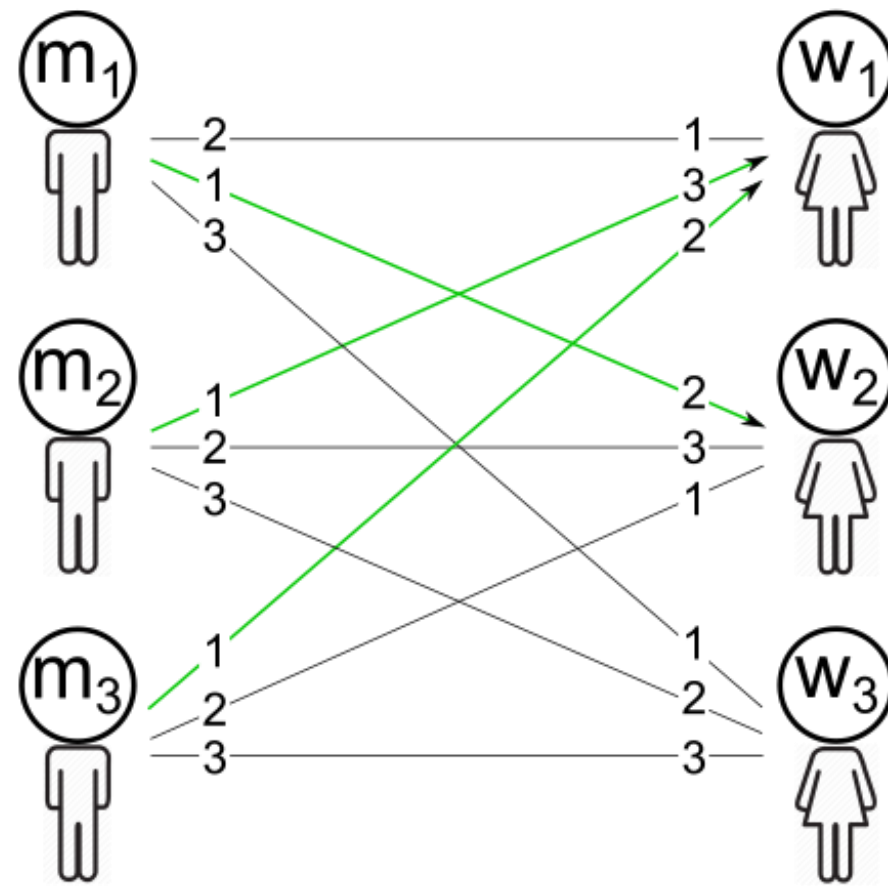
Round 1



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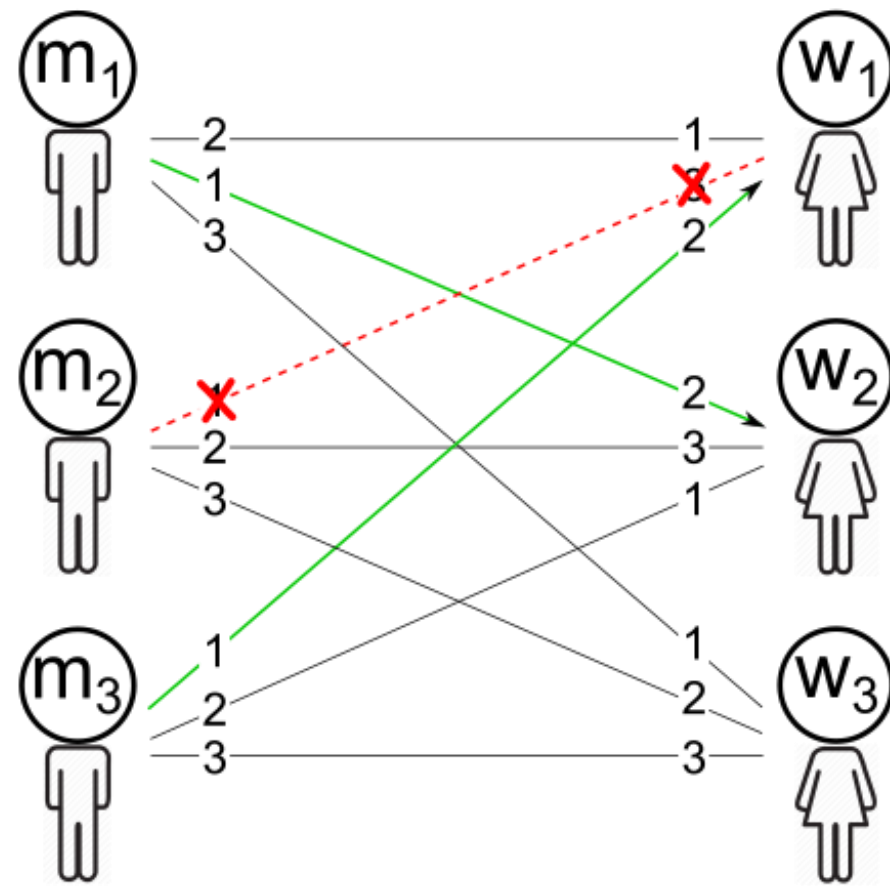
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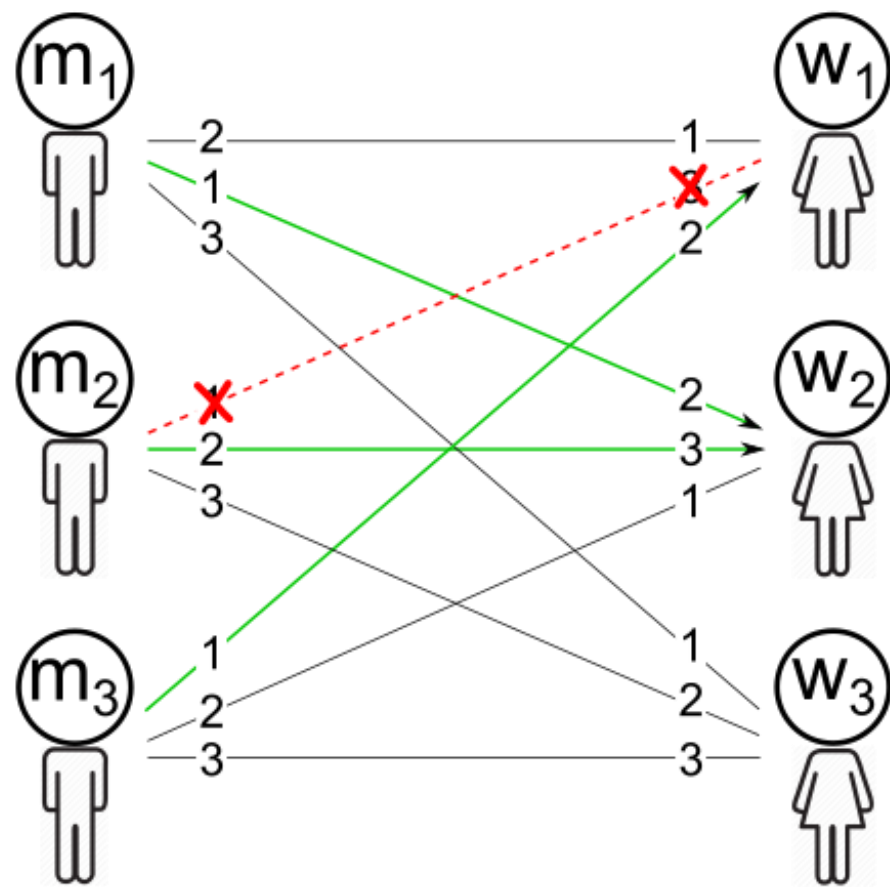
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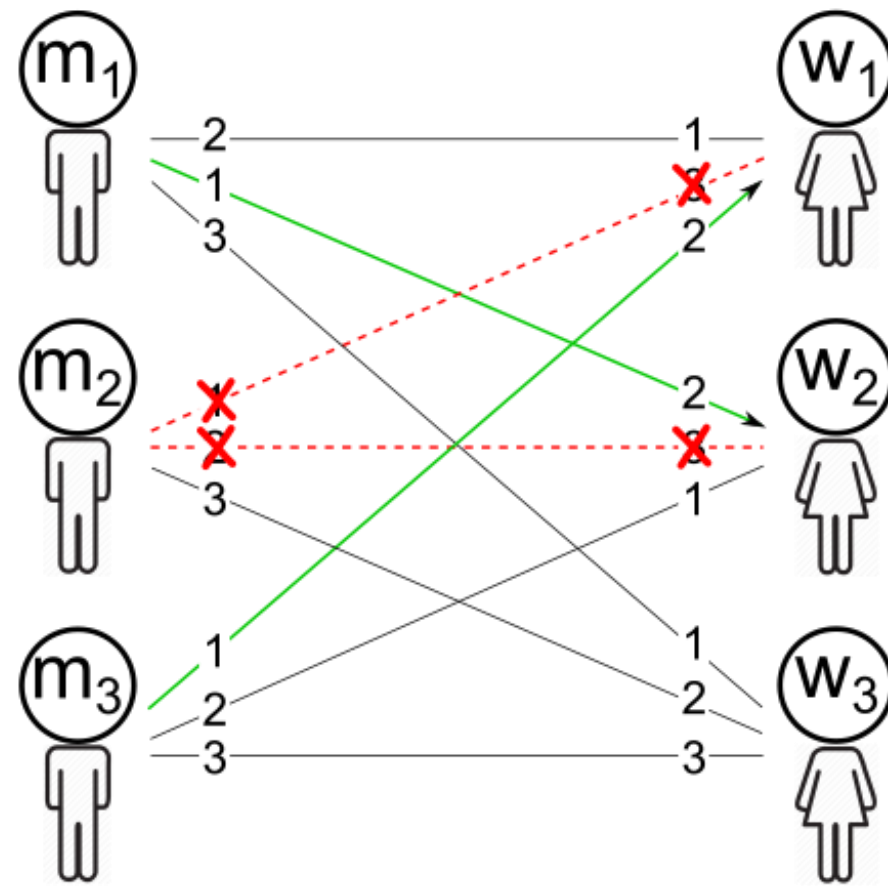
Round 2



[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

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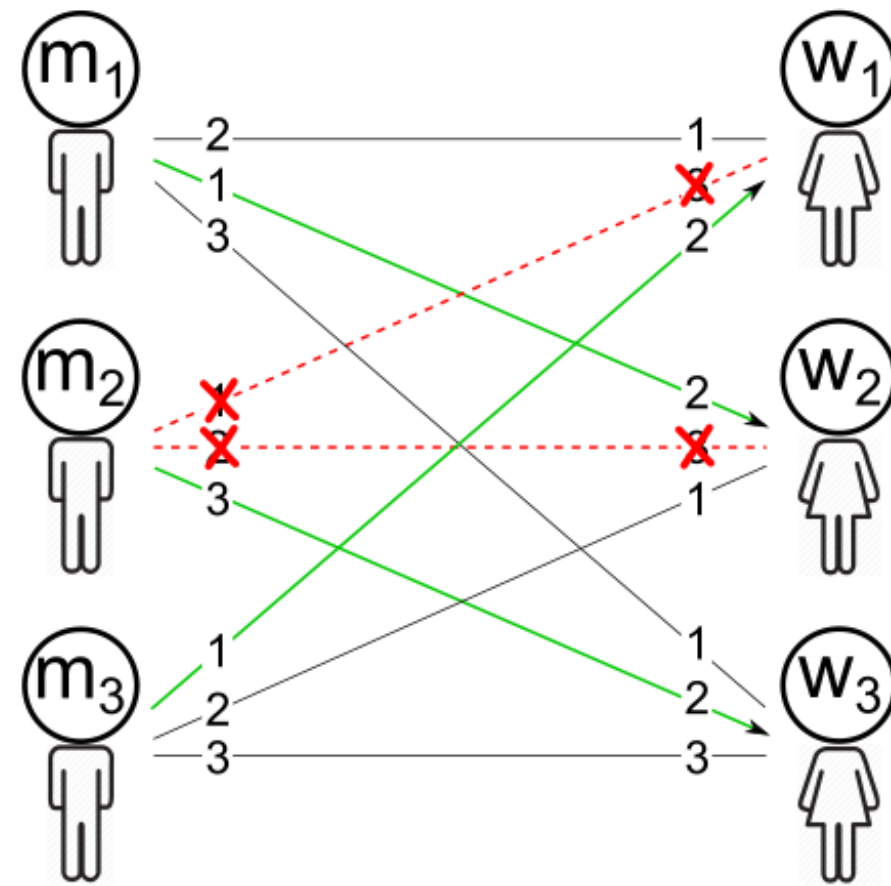
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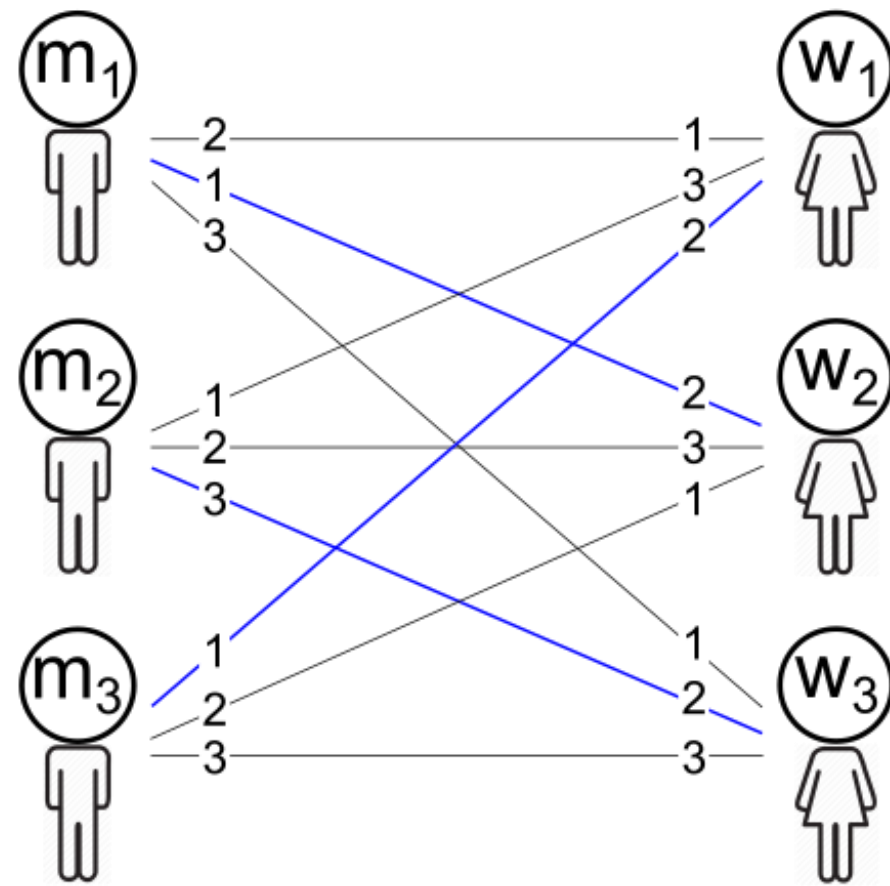
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Round 3



[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

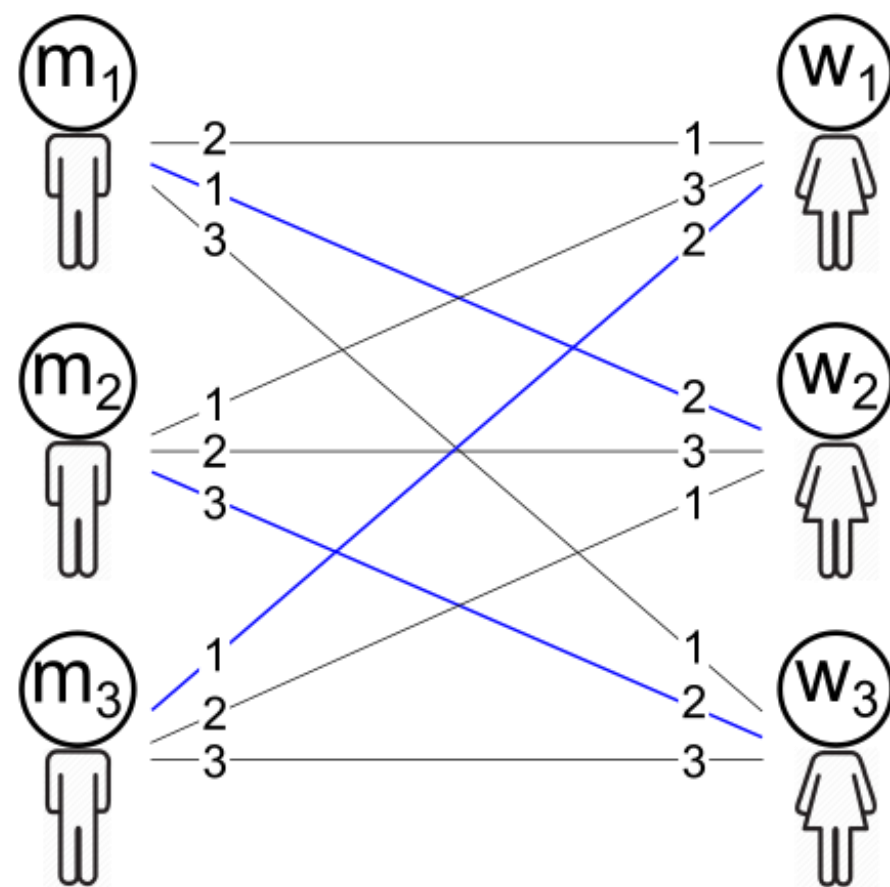
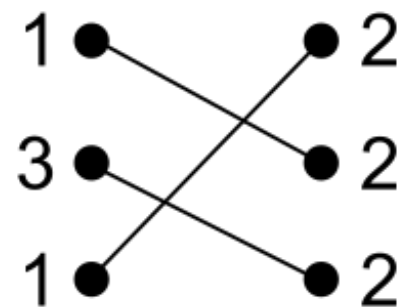
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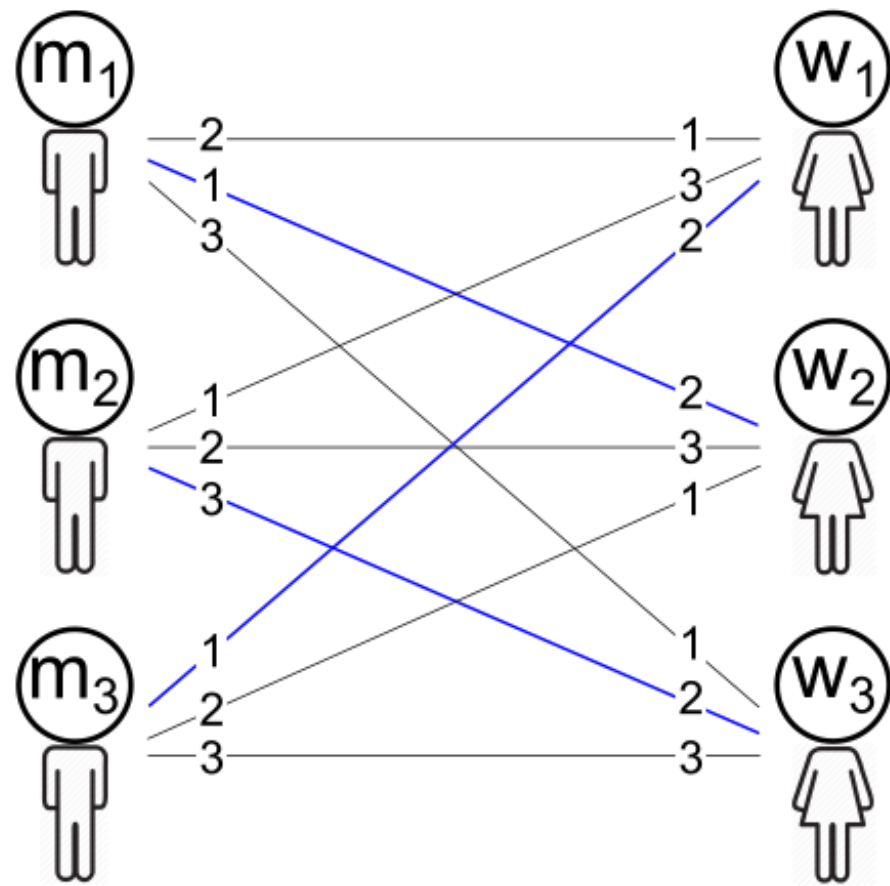
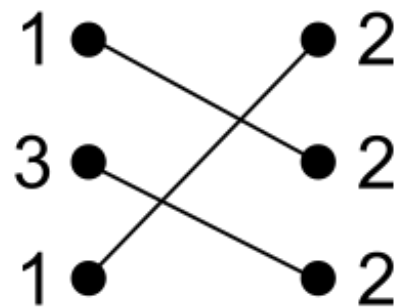
Outcome for true prefs



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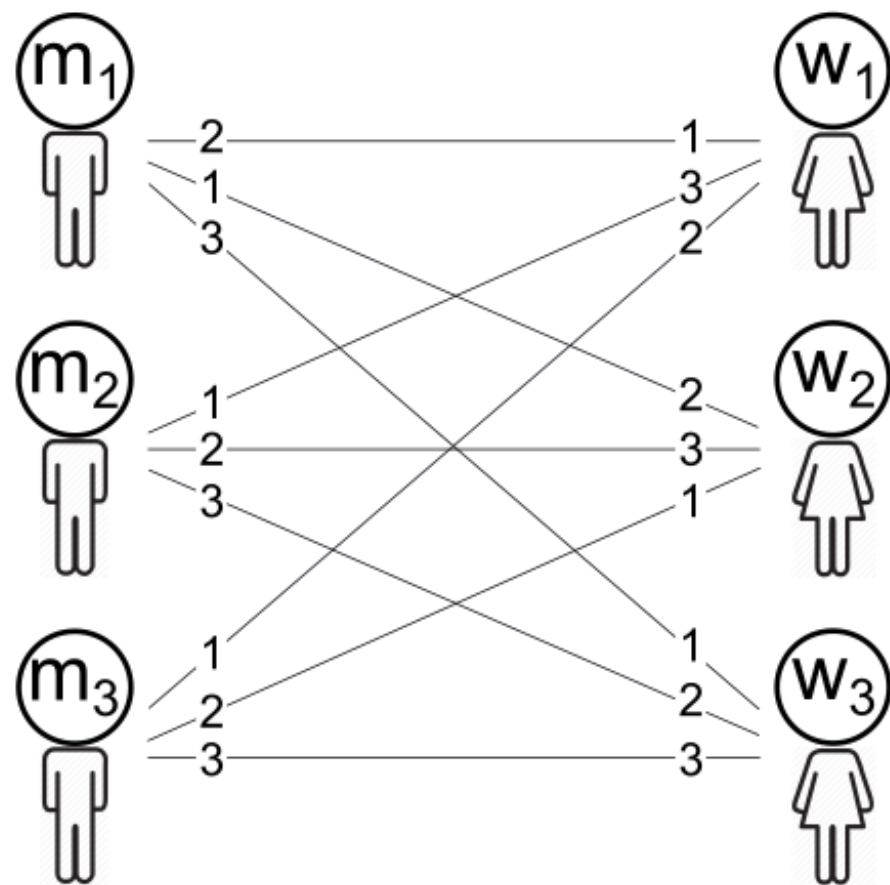
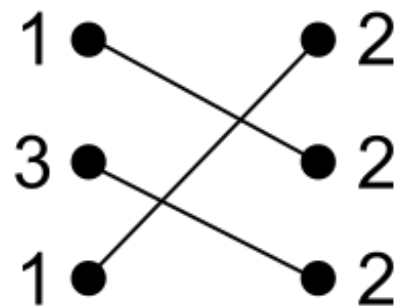


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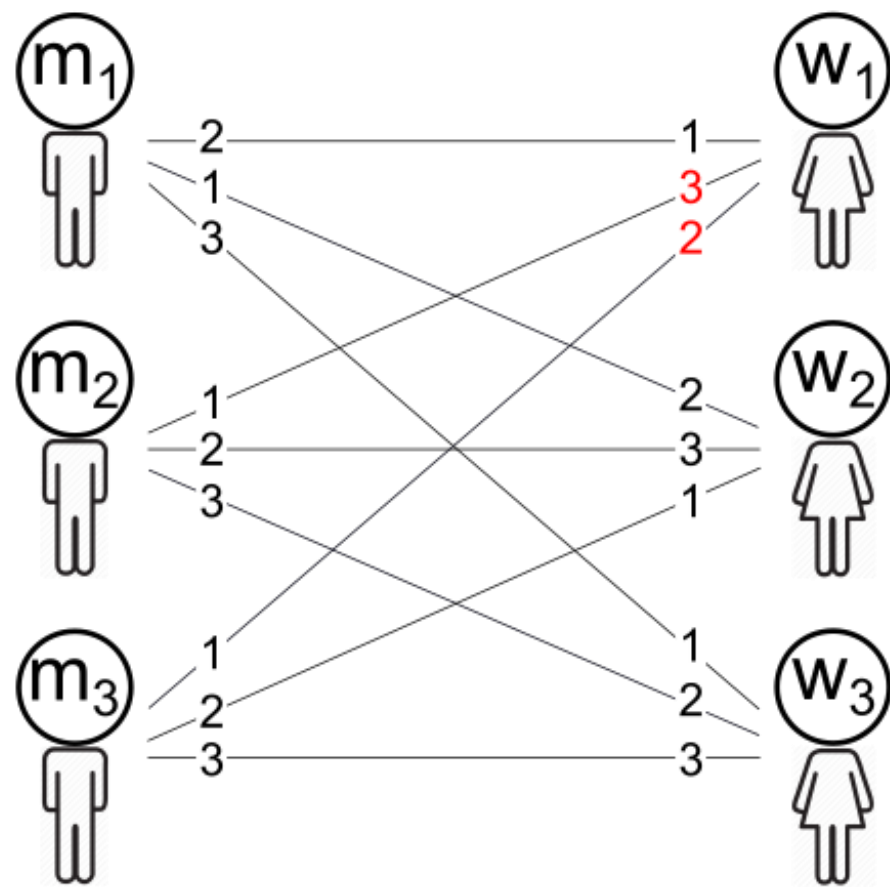
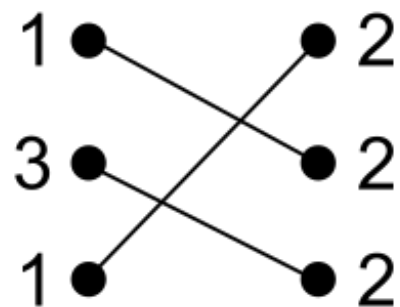


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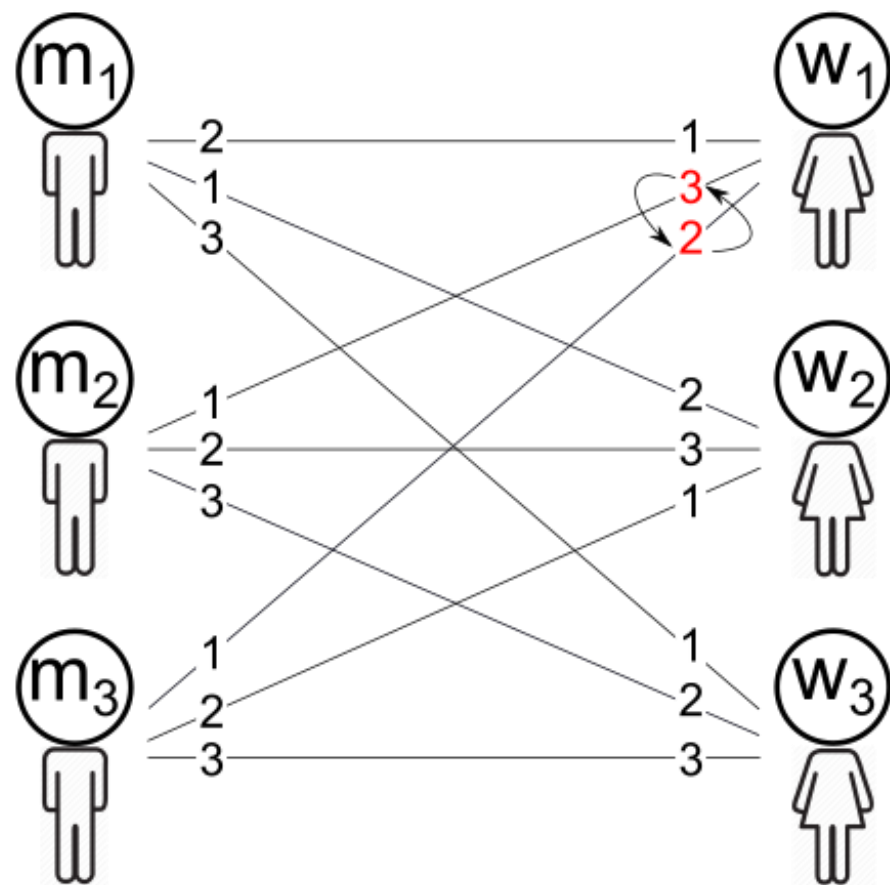
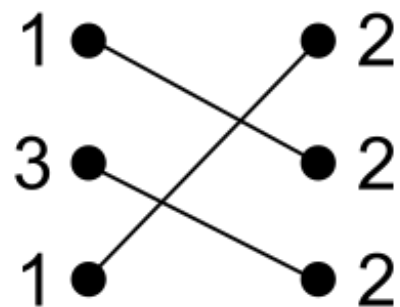


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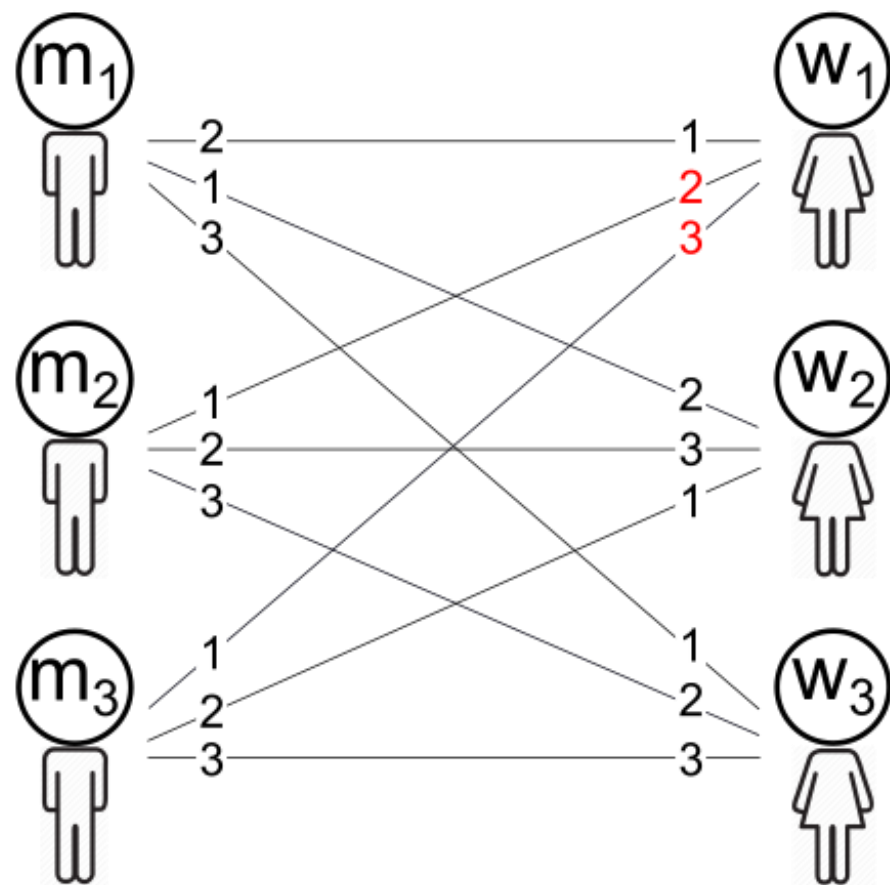
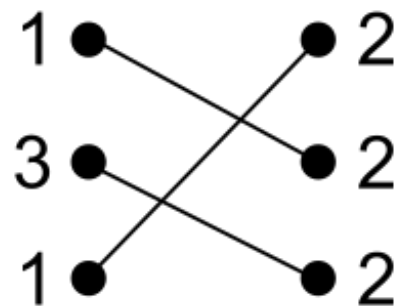


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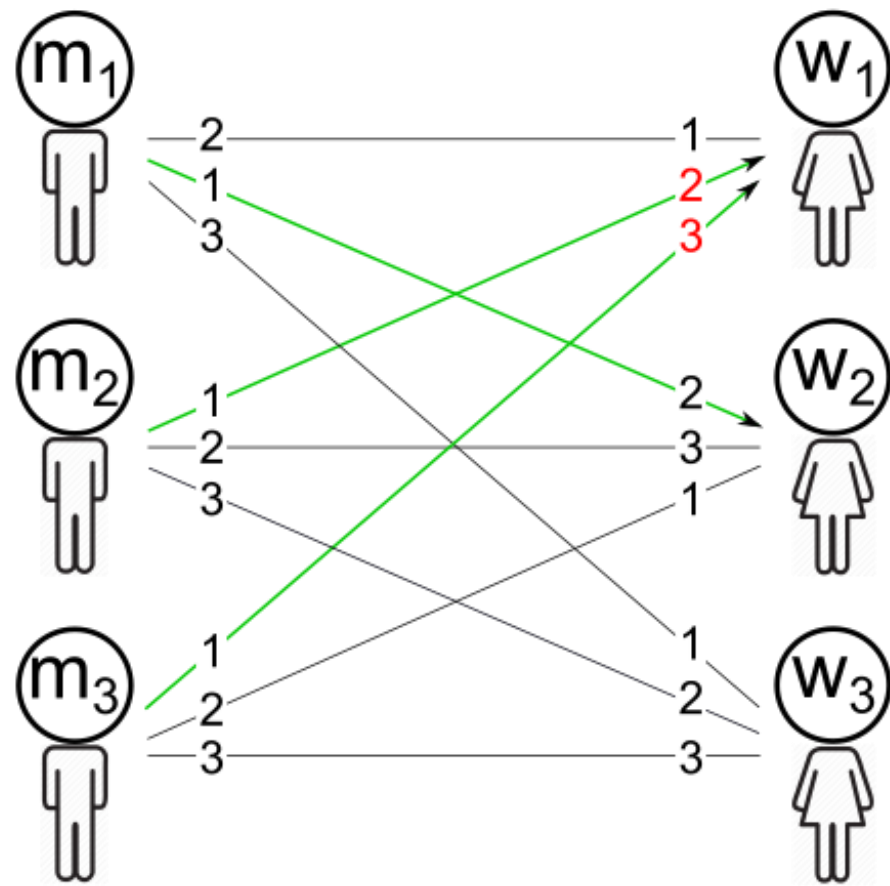
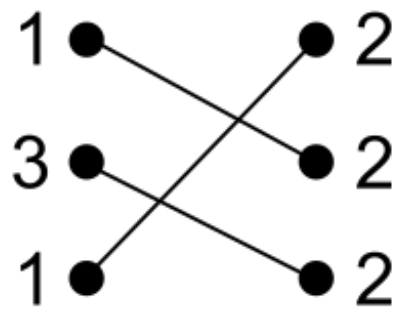


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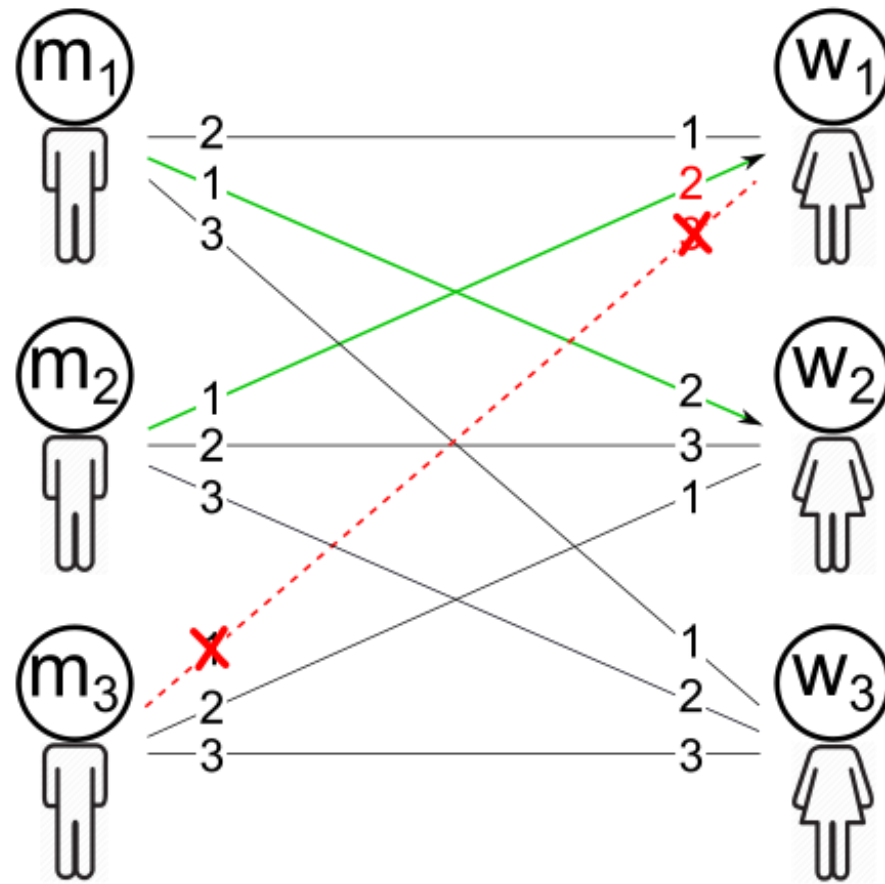
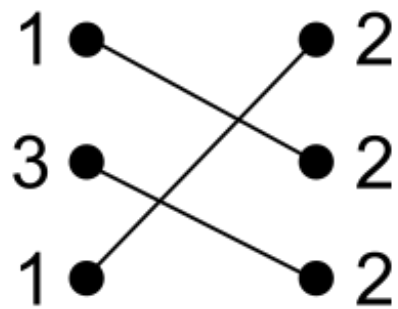


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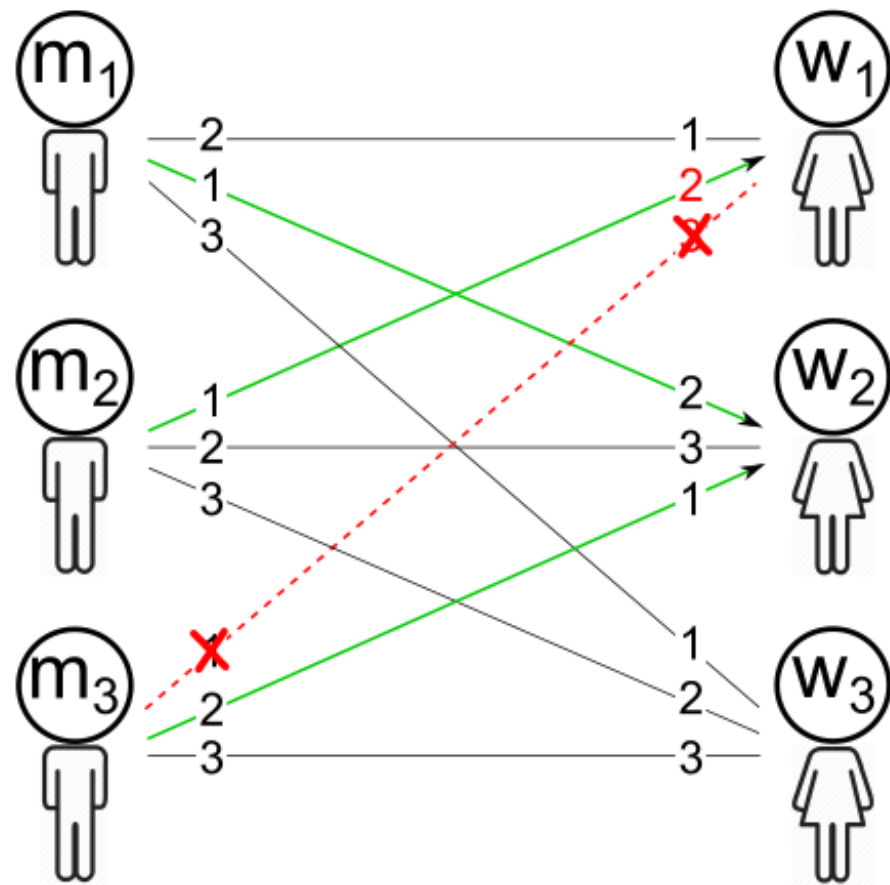
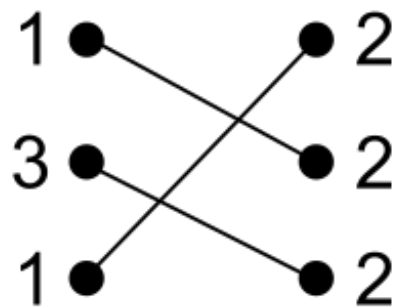


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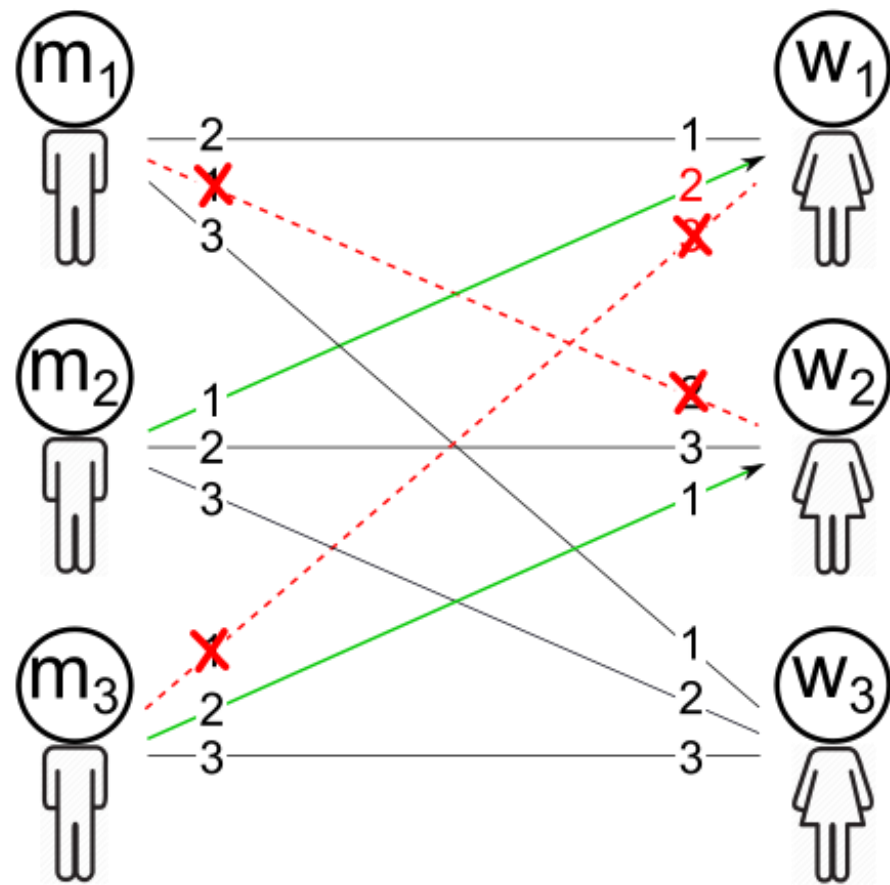
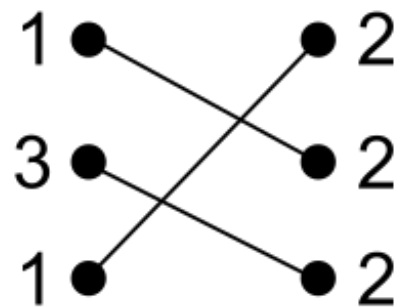


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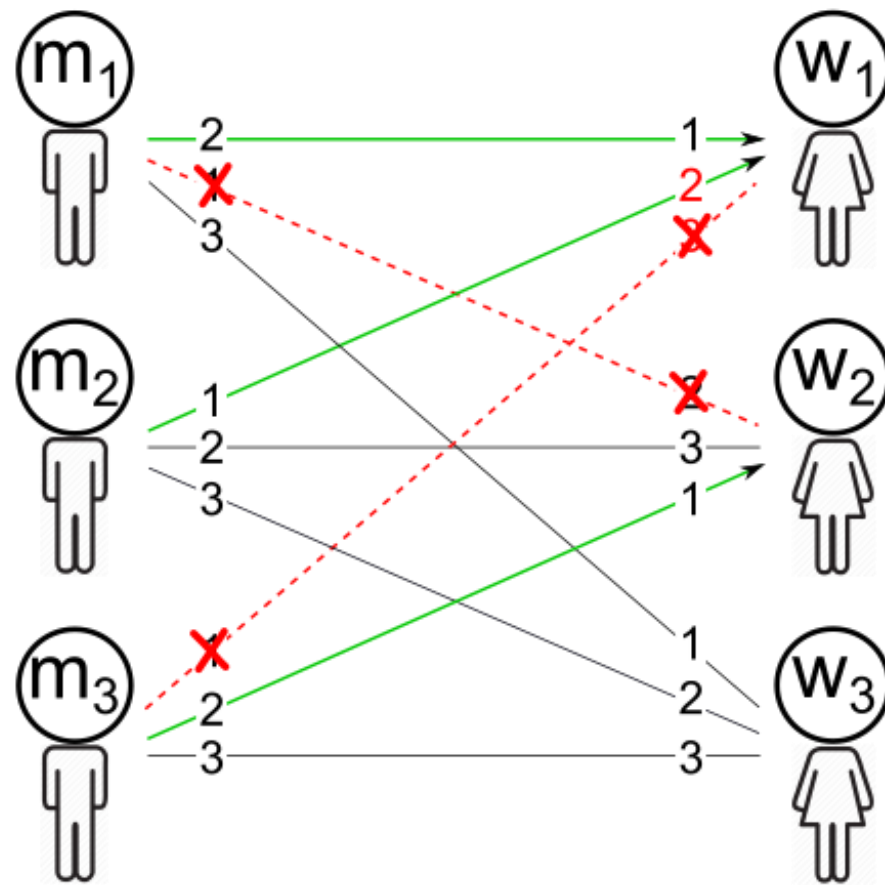
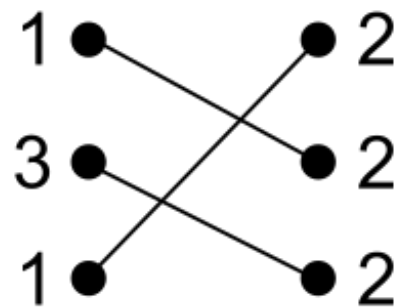


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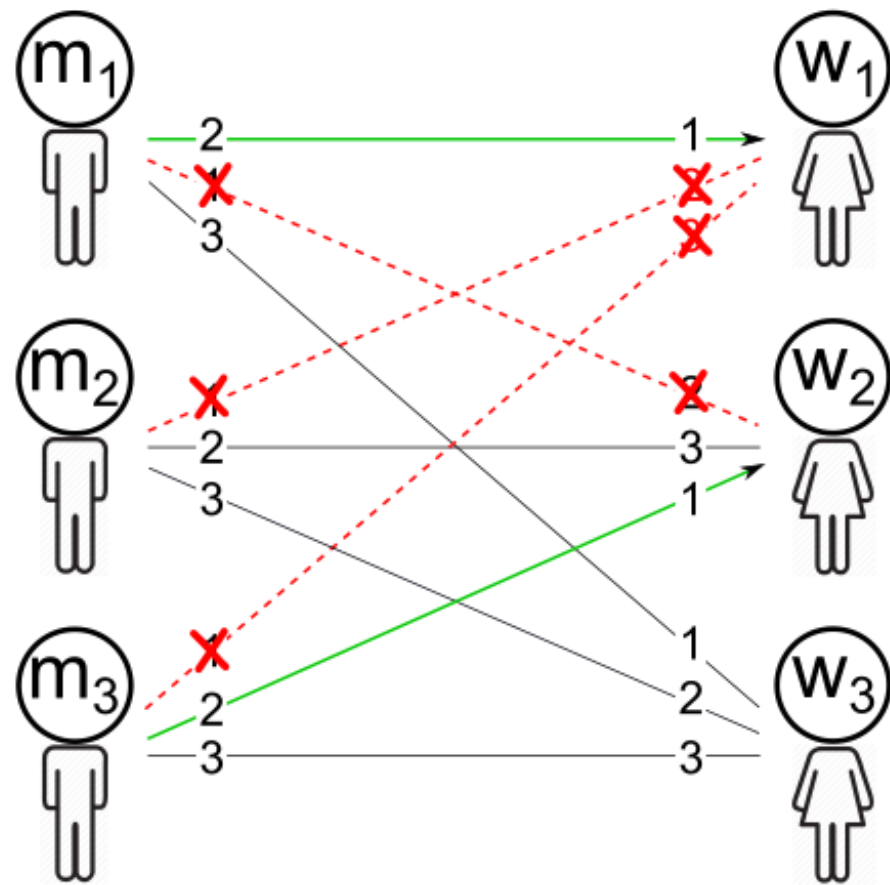
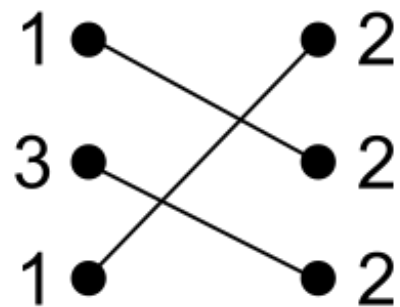


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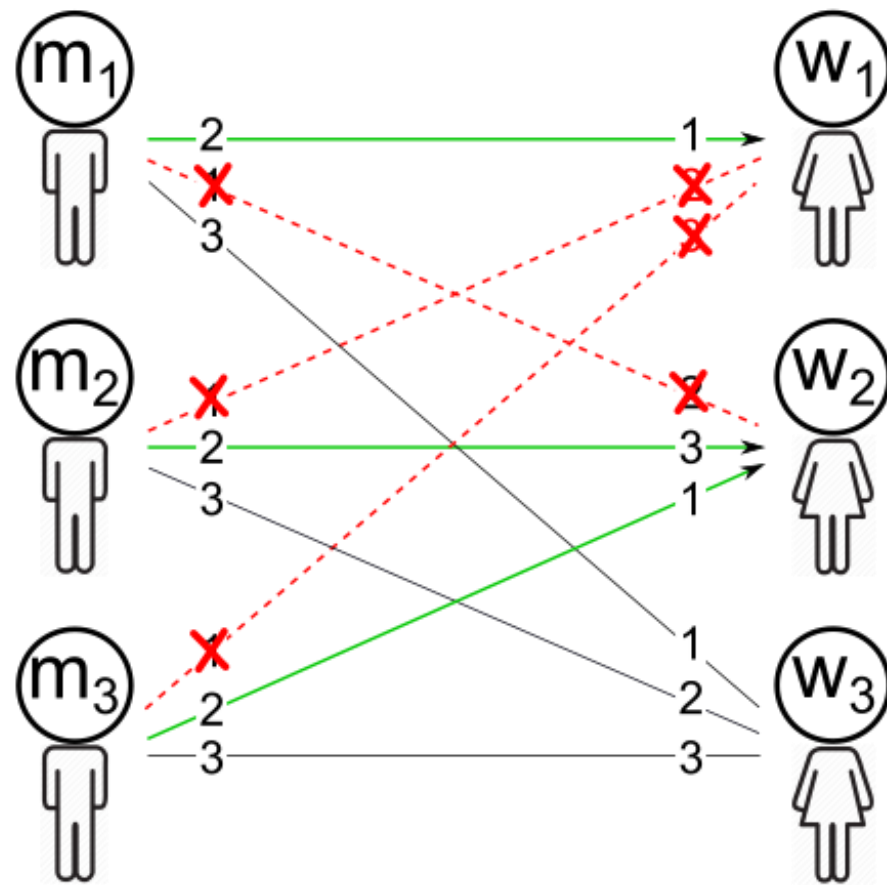
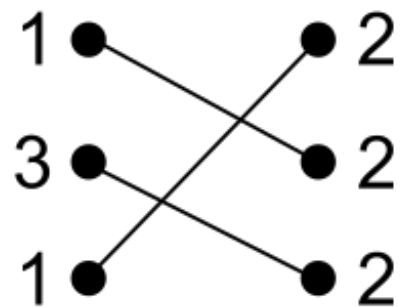


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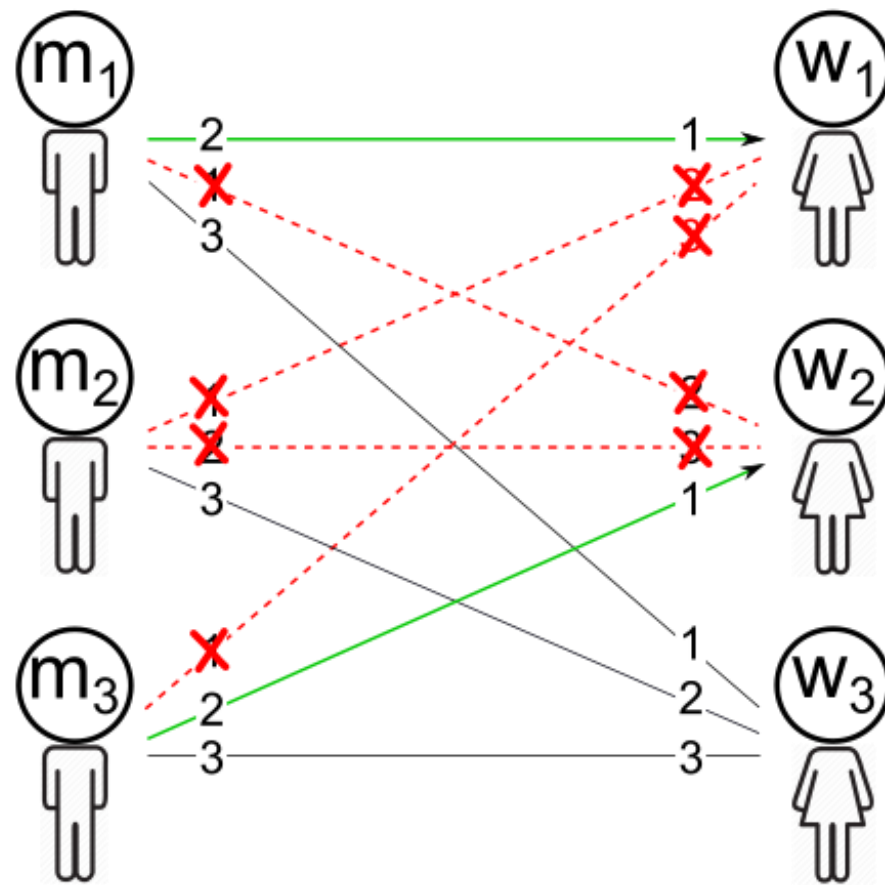
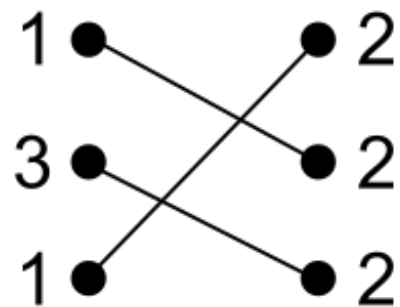


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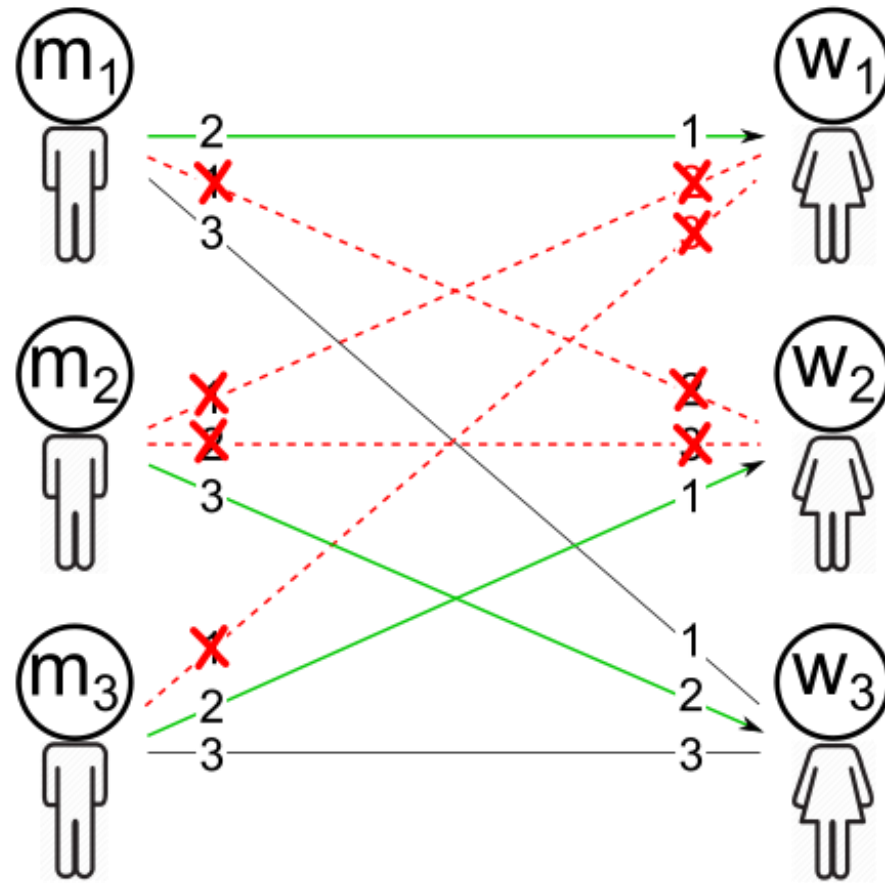
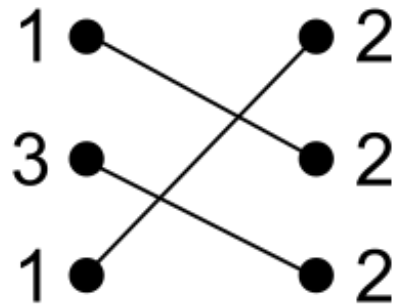


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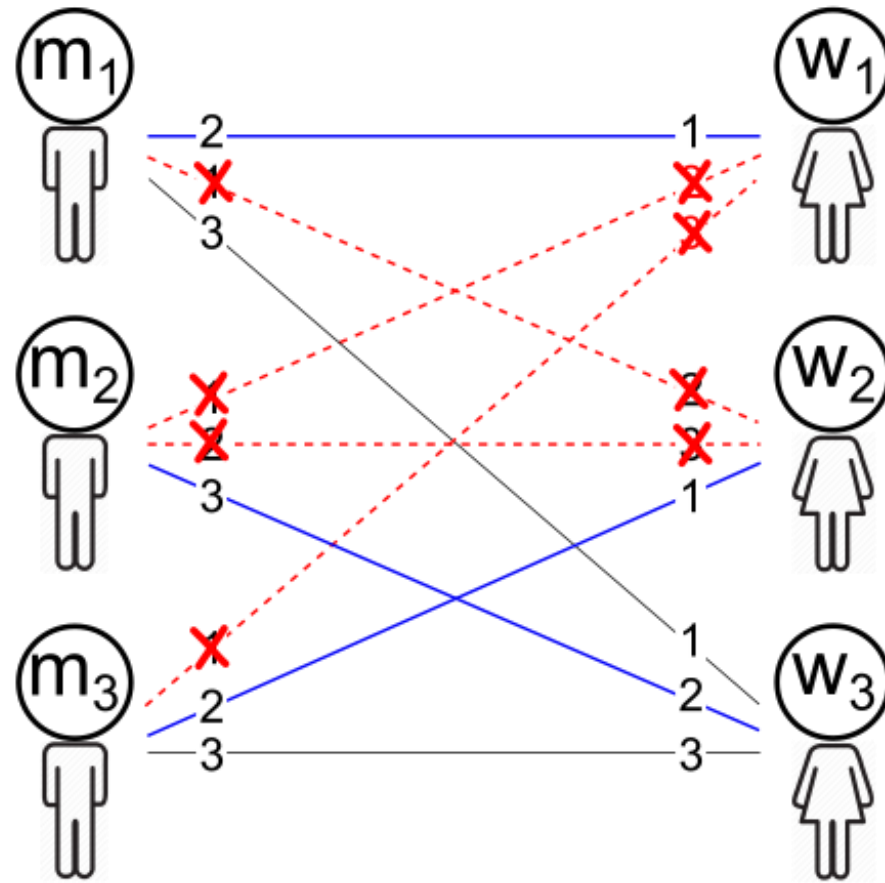
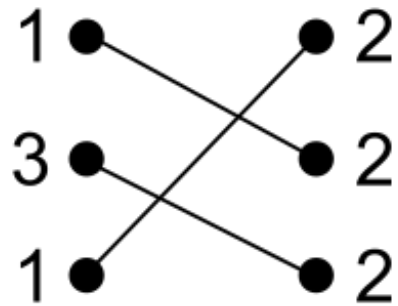


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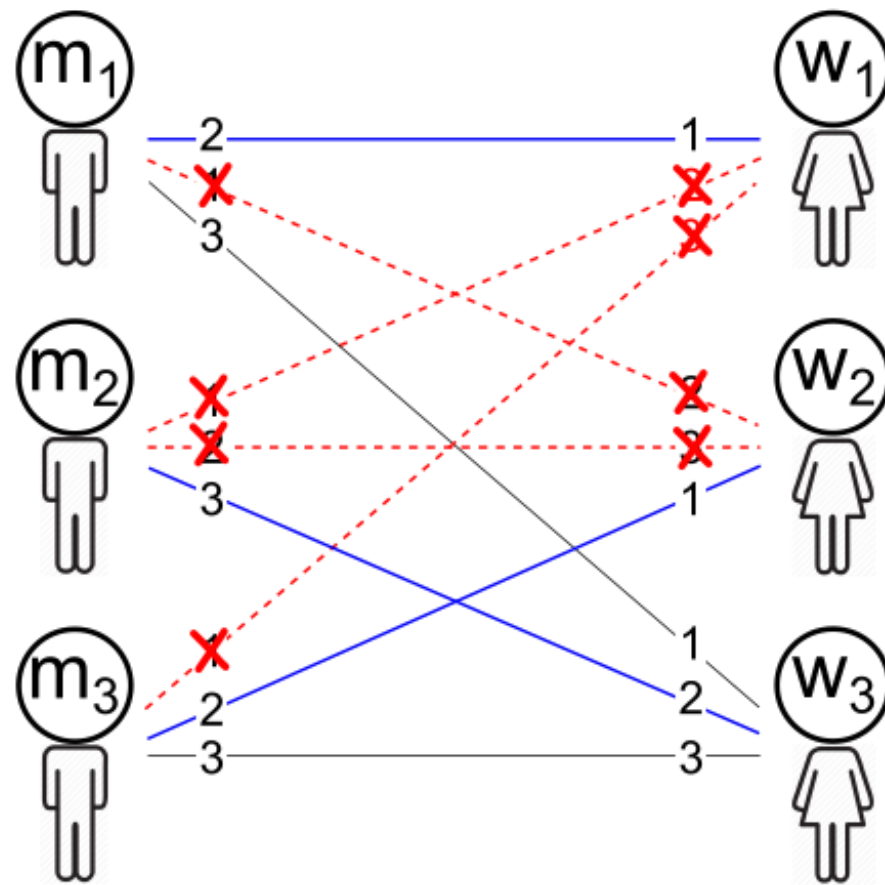
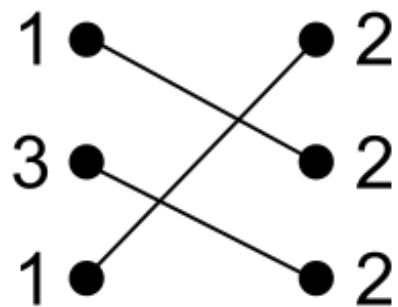


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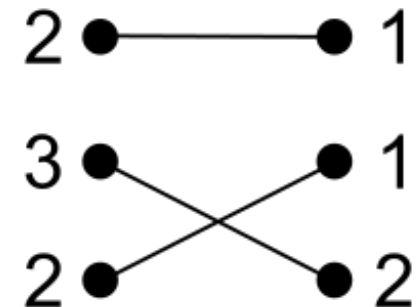
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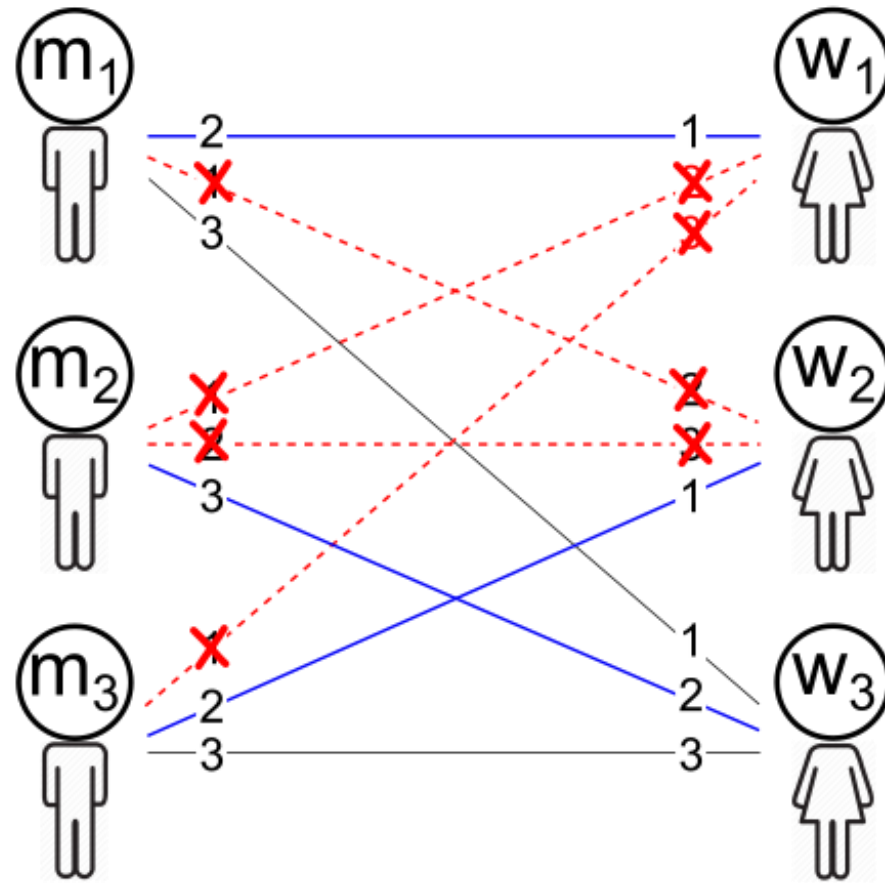
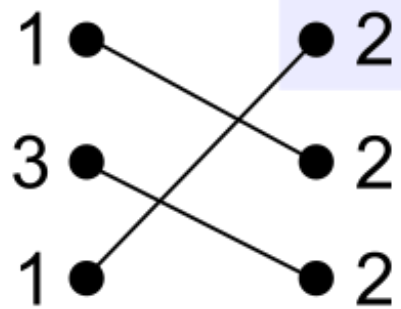
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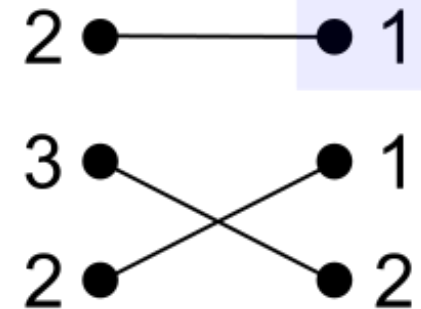
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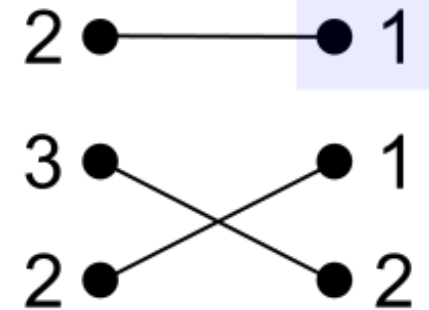
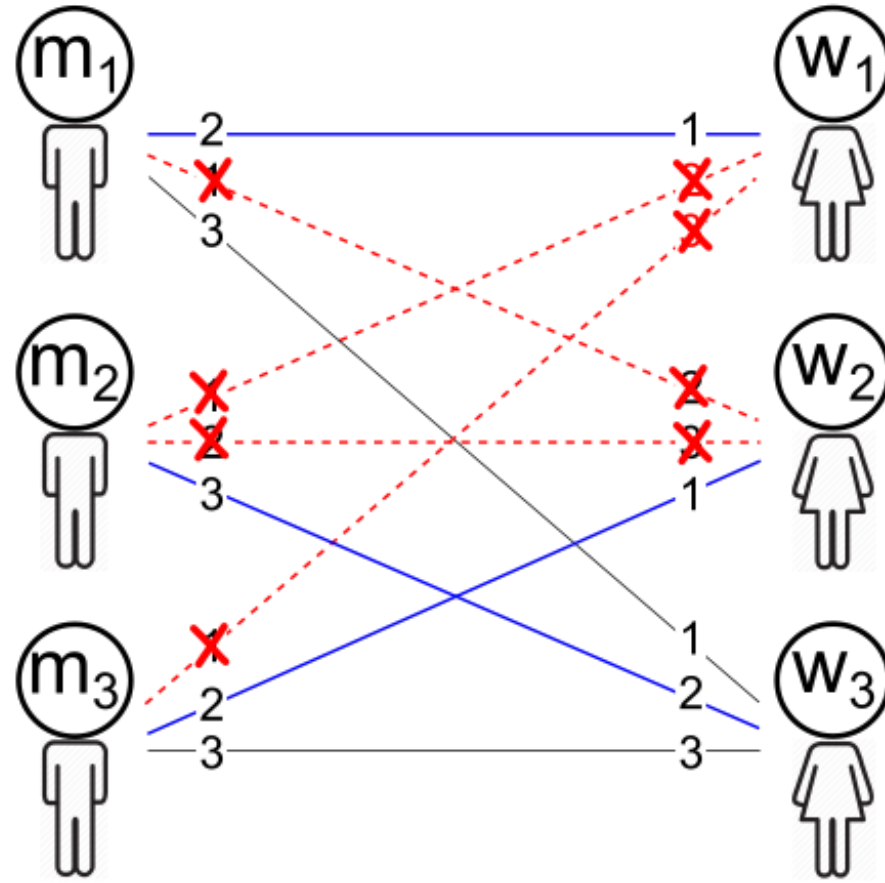
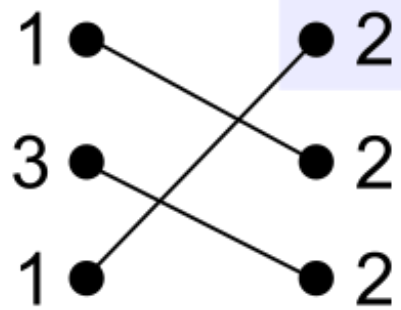
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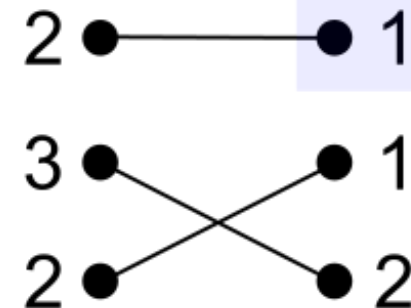
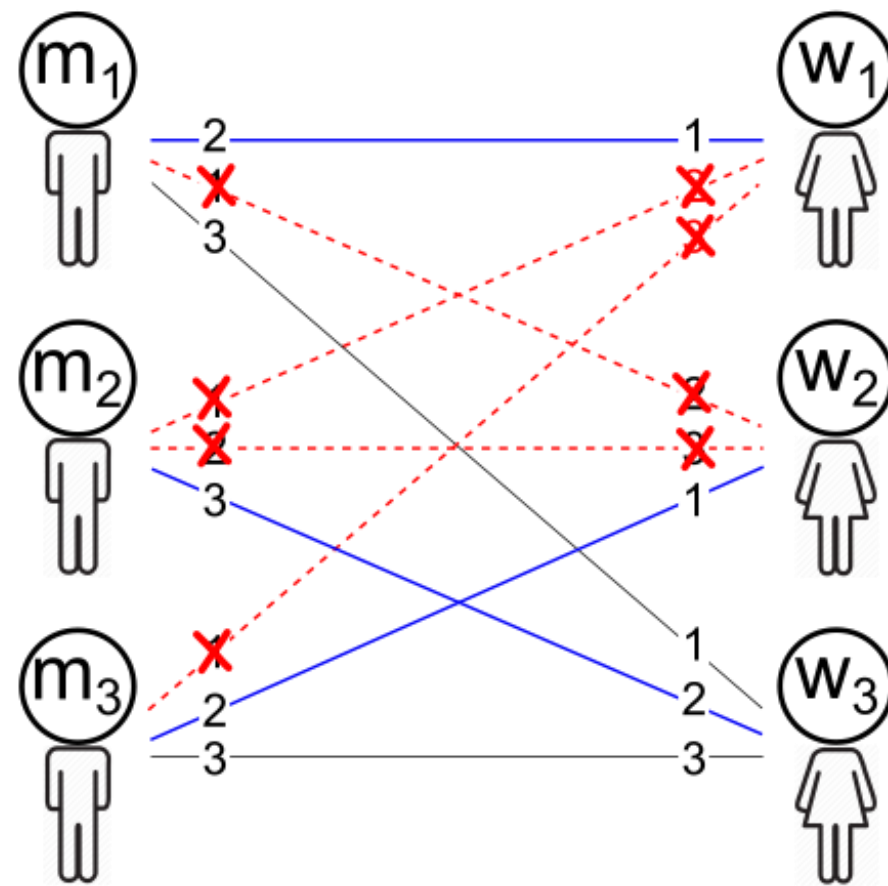
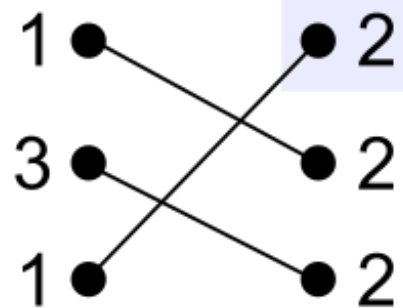
Outcome for true prefs



[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is not strategyproof.

Outcome for true prefs



Any luck for the men?

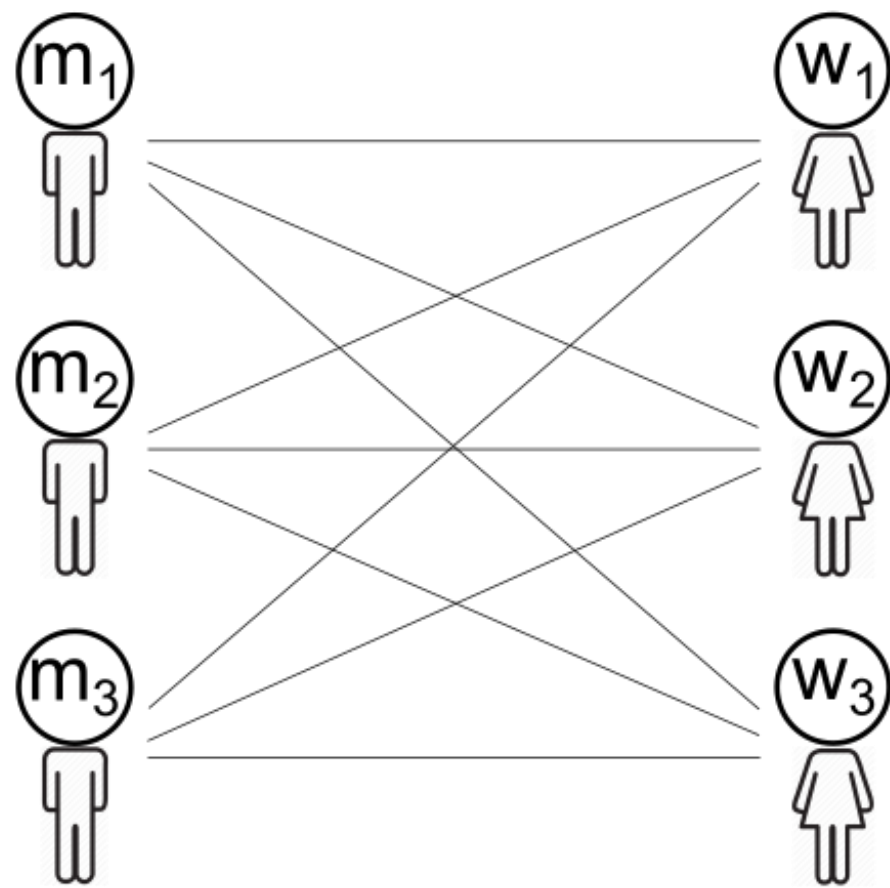
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Can I get a better partner by misreporting my preferences?



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Nope.

Nope.

Nope.

Nope.

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m_1

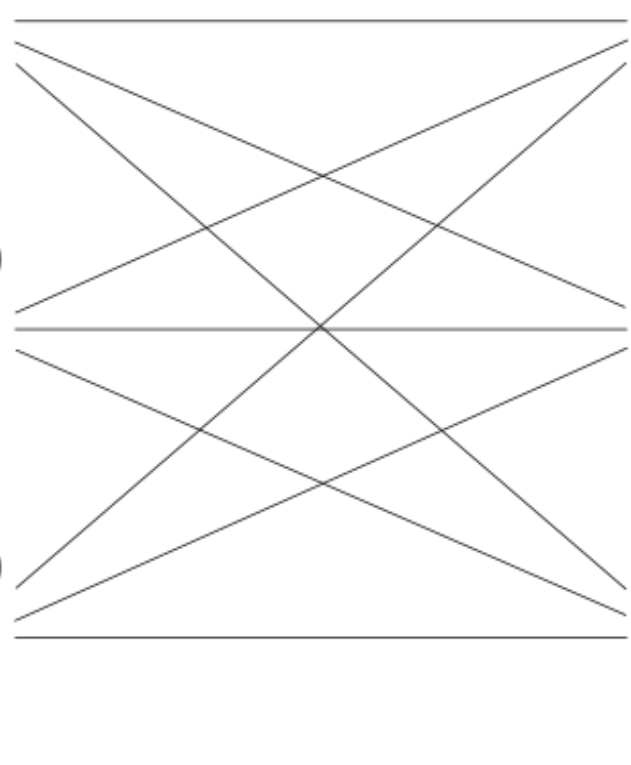
m_2

m_3

w_1

w_2

w_3



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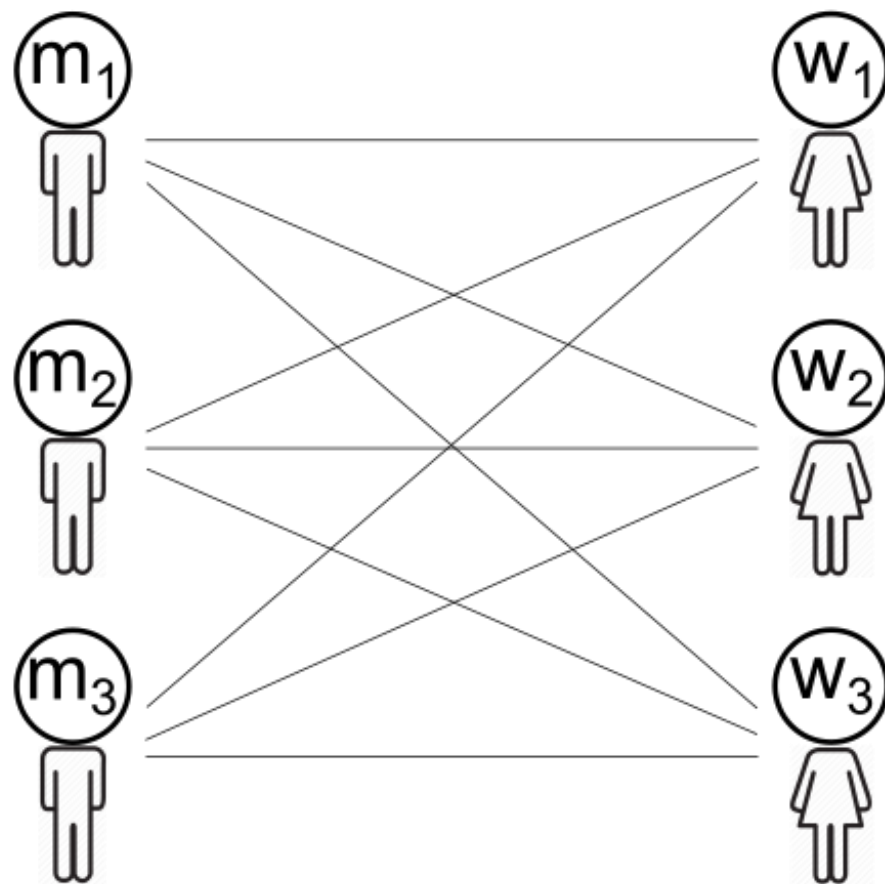
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Proof ~~later in today's lecture~~ in the lecture slides.

So, men can't cheat in the men-proposing DA algorithm but there **exists** an opportunity for a woman to manipulate.

So, men can't cheat in the men-proposing DA algorithm but there **exists** an opportunity for a woman to manipulate.

Is it possible to efficiently **compute** a beneficial misreport whenever one exists?

[Teo, Sethuraman and Tan, *Manag. Sci.* 2001; Vaish and Garg, *IJCAI* 2017]

An optimal manipulation for a woman can be computed
in polynomial time.

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An optimal manipulation for a woman can be computed
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We will use a structural result about optimal manipulation strategies.

Any optimal manipulation for a woman can also be achieved by an "inconspicuous" misreport.

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True list of woman w : $m_1 > m_2 > m_3 > m_4 > \boxed{m_5} > m_6 > m_7 > m_8$

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An optimal manipulation for a woman can be computed in polynomial time.

But what about stability (w.r.t. true preferences)?

The DA matching after optimal manipulation by a woman is stable with respect to the true preferences.

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We will use the following observation:

Suppose a woman promotes a man m in her list and no other changes are made.

If m proposed to her during DA on the old profile, then he proposes to her during DA on the new profile.

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Idea: Any deviation between old and new runs of the DA must involve tentative acceptance/rejection of m , but that can happen only after m proposes.

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If $w' \neq w$, then m and w' are both truthful and will block X' w.r.t. P' ---contradicting the stability of DA.

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P_m	P_w
•	•
•	•
w	m
•	•
•	•
$X'(m)$	$X'(w)$
•	•
•	•

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P_m	P_w	P'_m	P'_w
•	•	•	•
•	•	•	•
w	m	w	$X'(w)$
•	•	•	•
•	•	•	•
$X'(m)$	$X'(w)$	$X'(m)$	m
•	•	•	•
•	•	•	•

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•	•	•	•
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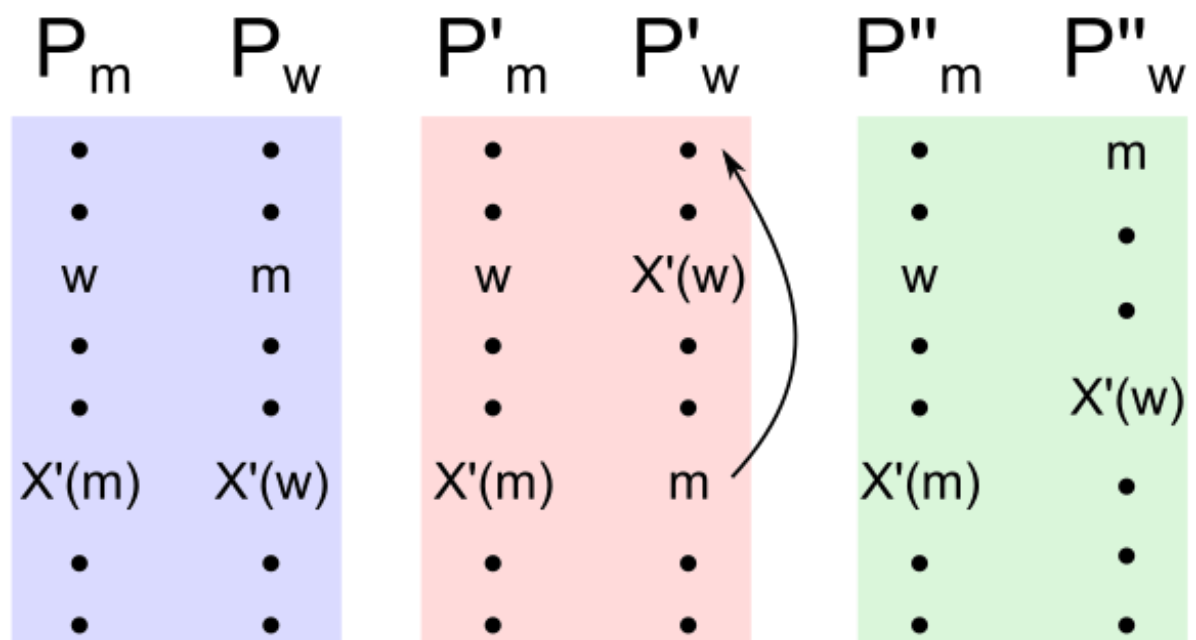
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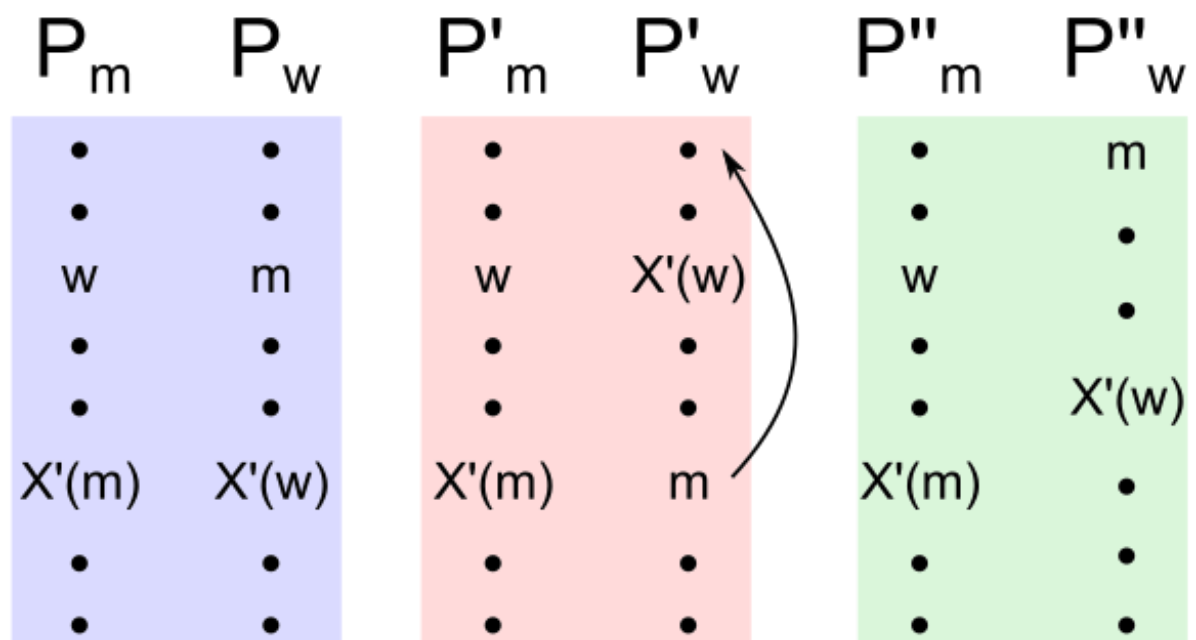
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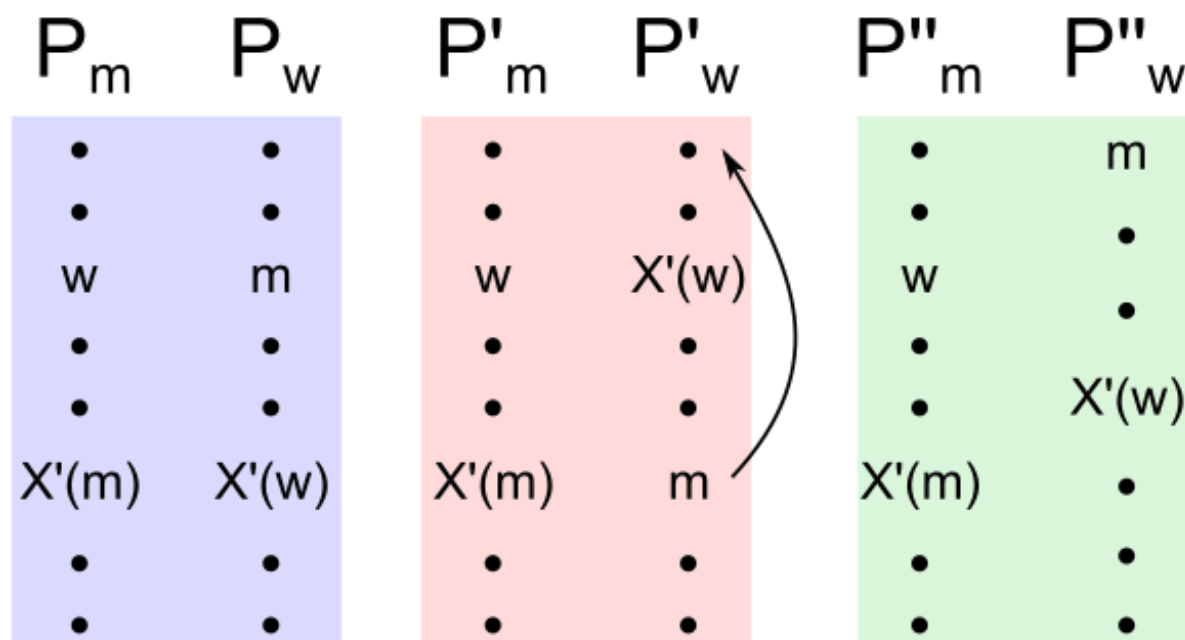
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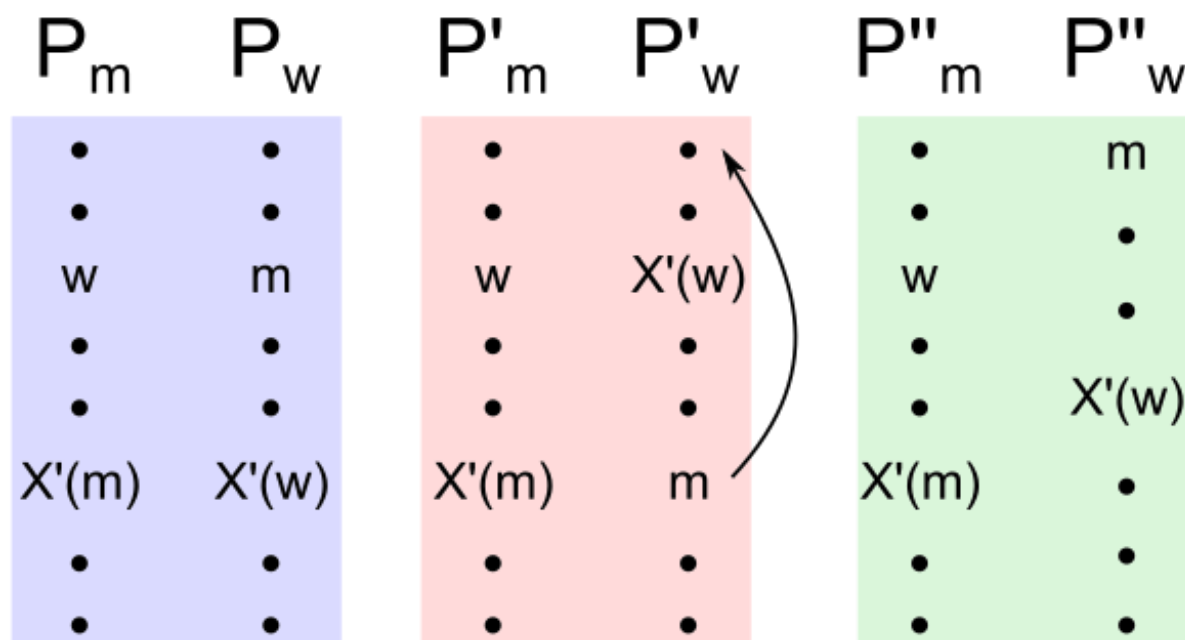
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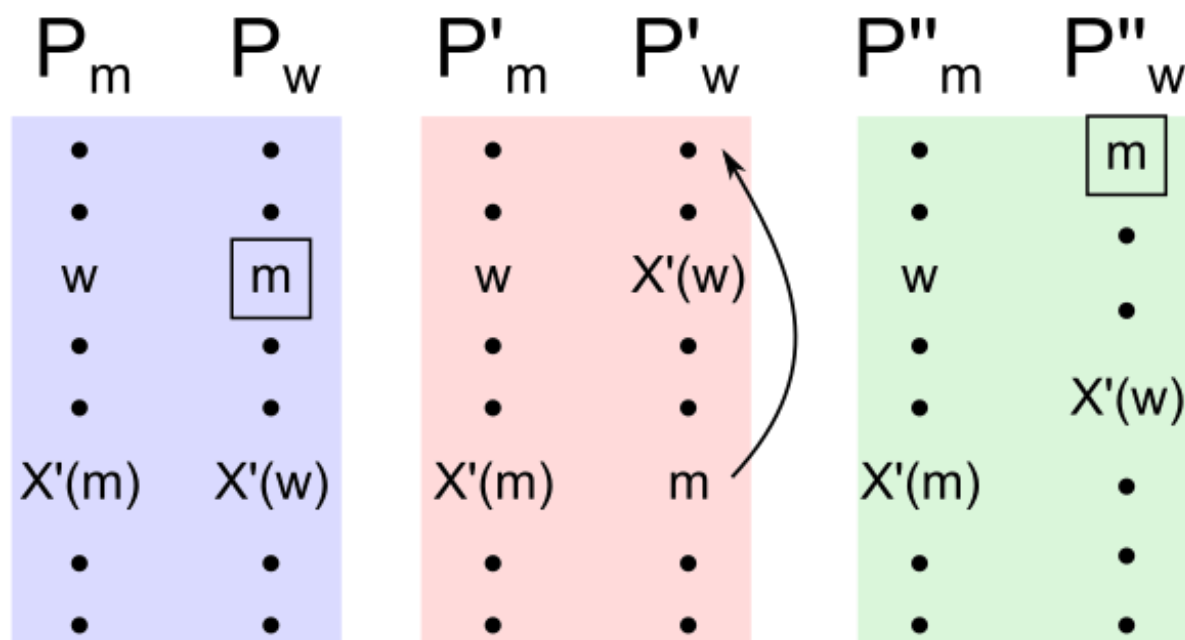
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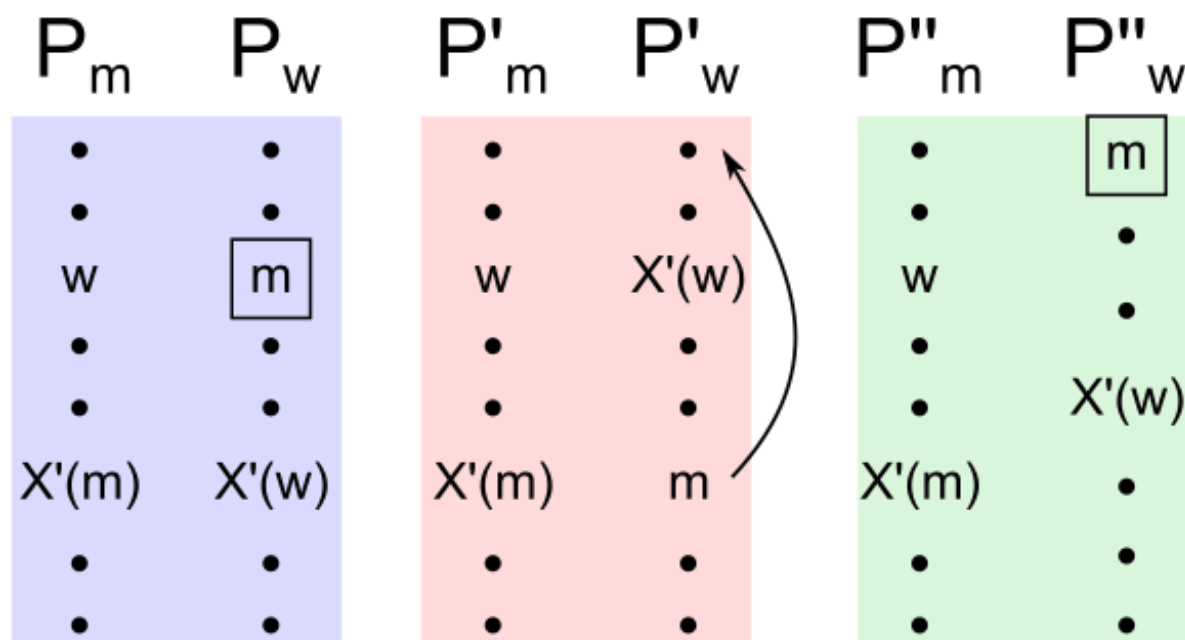
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 But then, P''_w gives w a better partner than under her optimal strategy!

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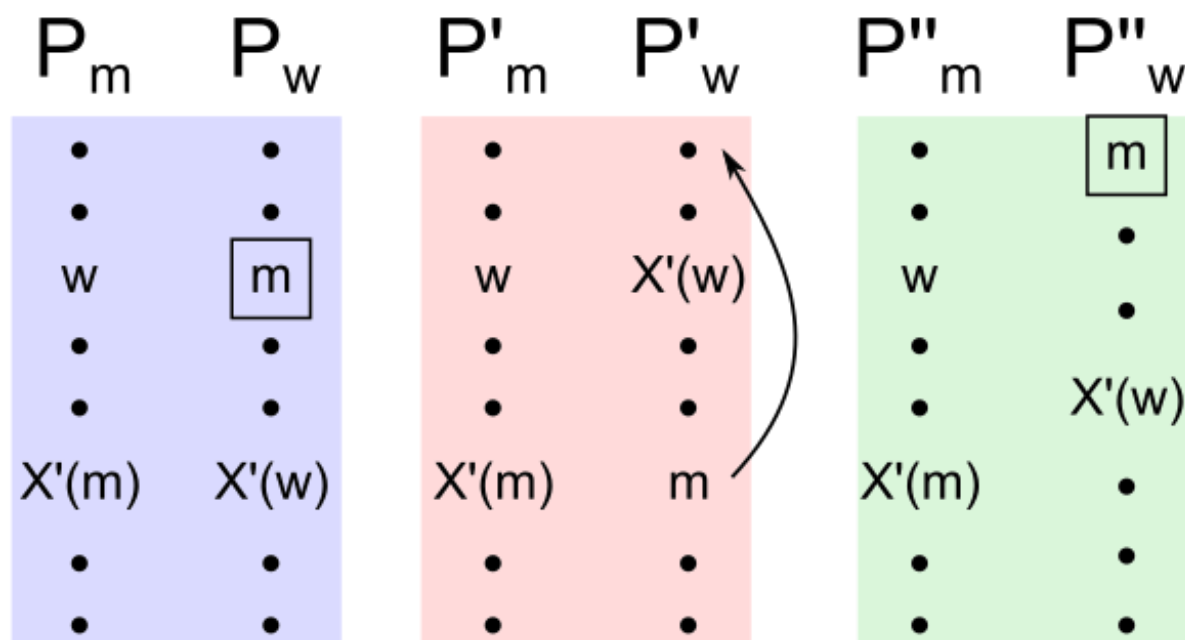
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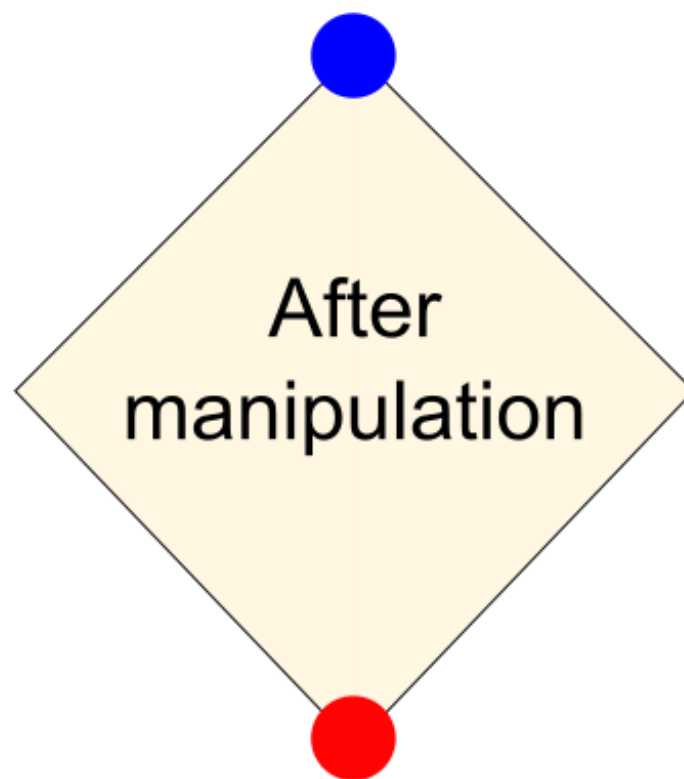
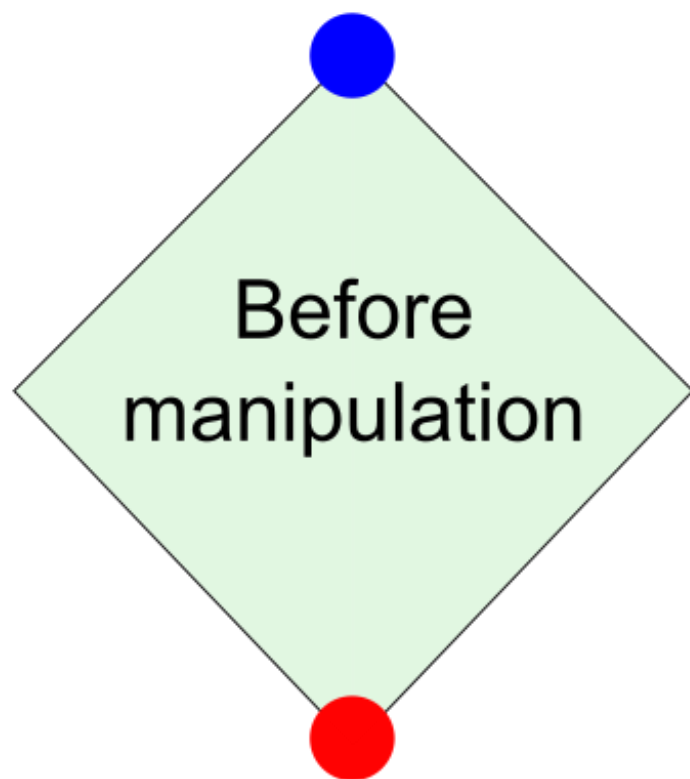
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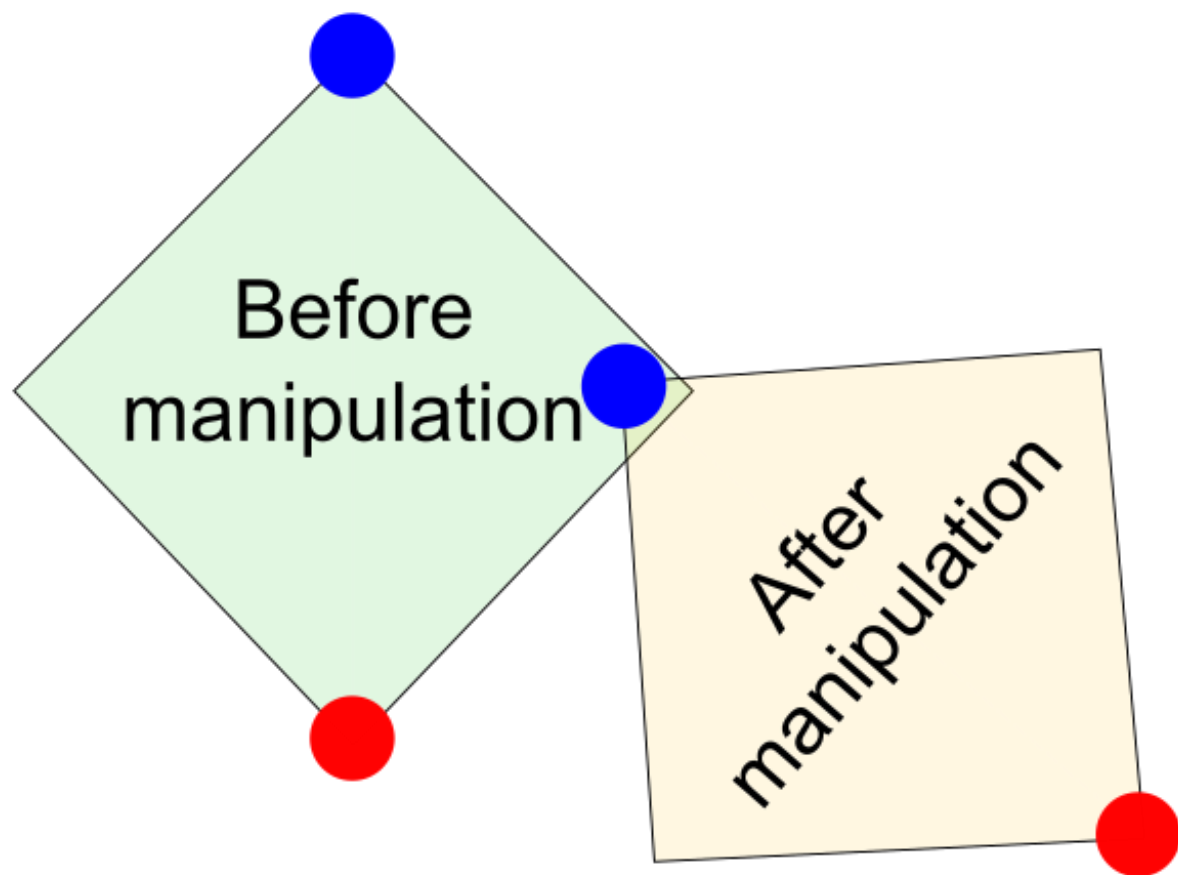
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*Stable marriages are manipulable,
but optimally manipulated marriages are stable.*

References

- DA algorithm fails to be strategyproof.

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"Machiavelli and the Gale-Shapley Algorithm"

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- No stable matching procedure is strategyproof.

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Management Science, 47(9), 2001 pg 252–1267

- Optimally manipulated marriages are stable.

Rohit Vaish and Dinesh Garg

“Manipulating Gale-Shapley Algorithm: Preserving Stability and Remaining Inconspicuous”

IJCAI 2017, pg 437-443

[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

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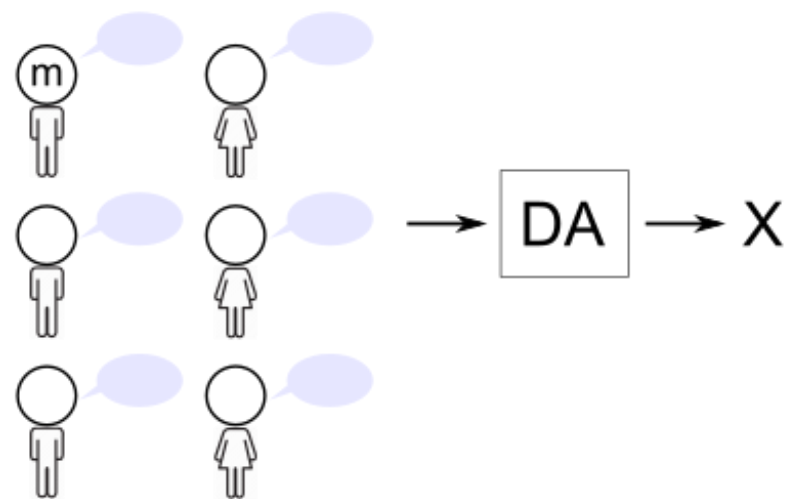
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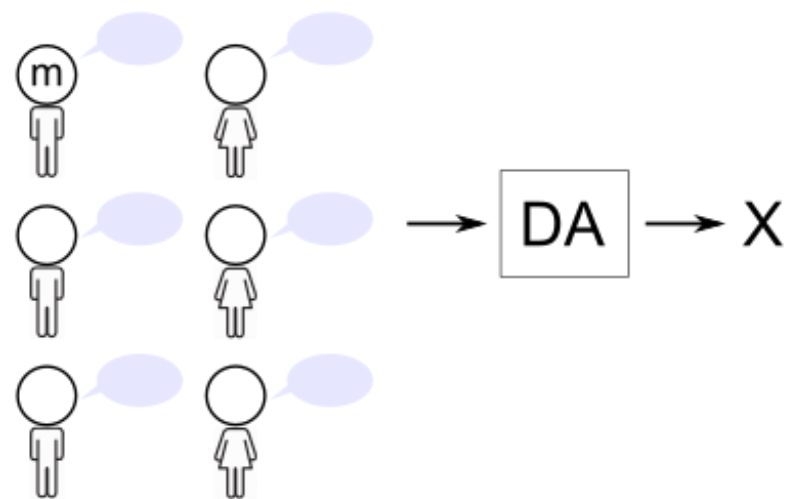


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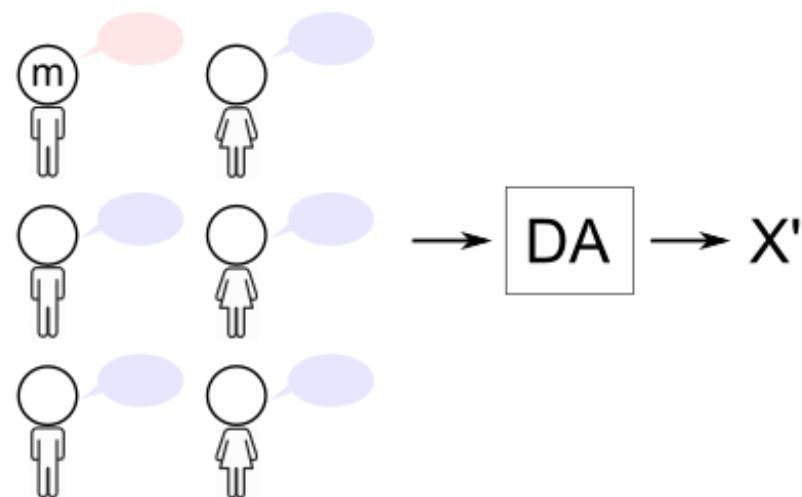
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Manipulated profile $P' = (P_{-m}, P'_m)$

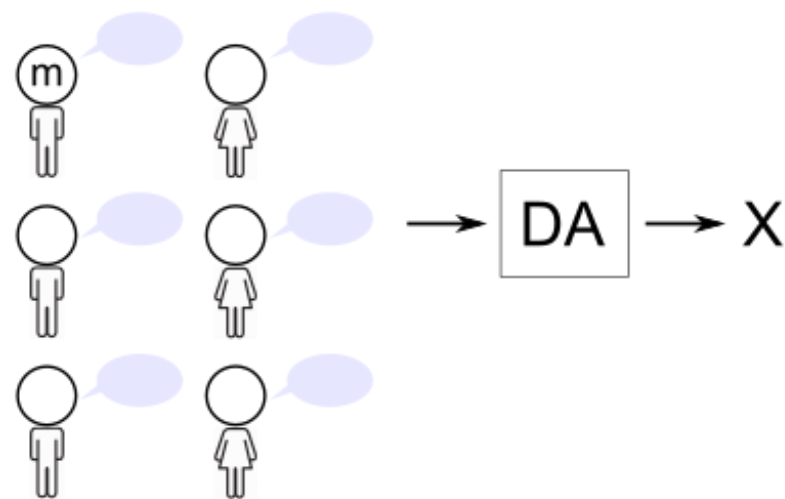


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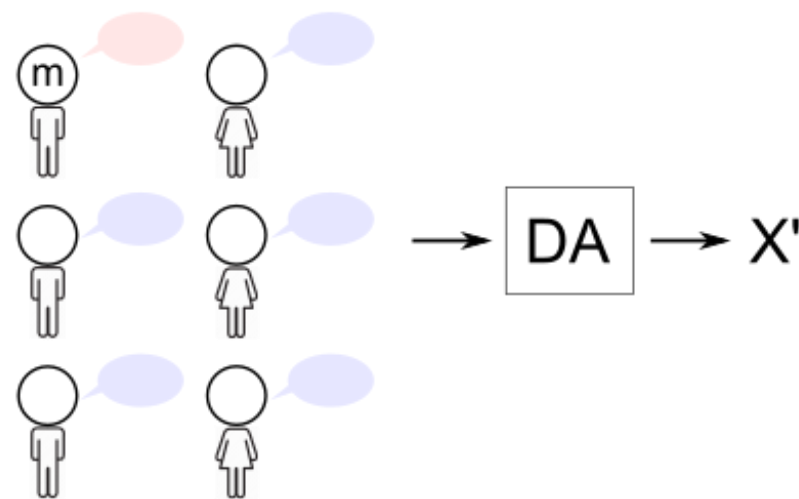
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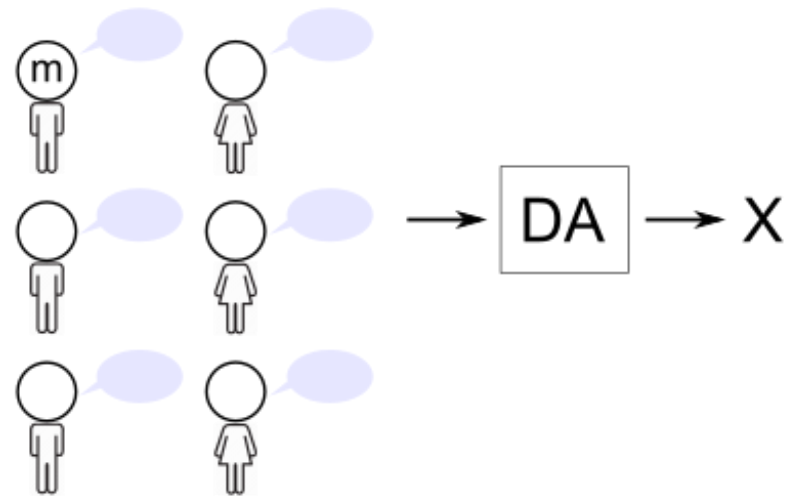
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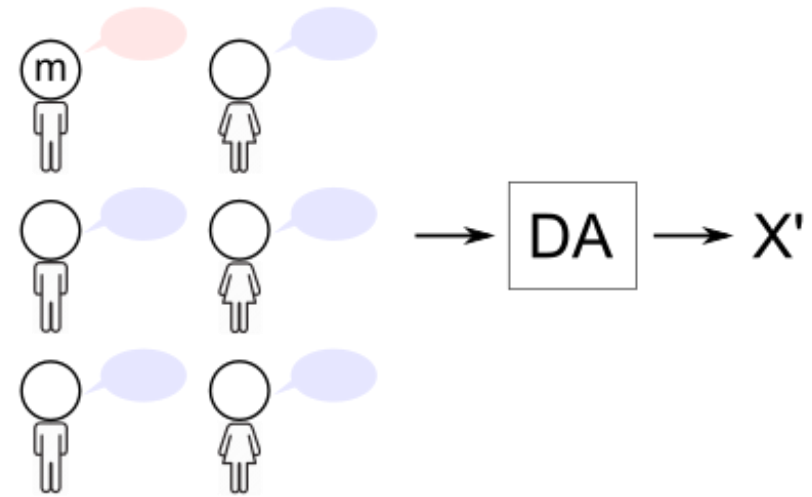
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We will use three lemmas to derive a contradiction.

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Simplicity

If there is a feasible strategy for manipulation, then there is a "simple" feasible strategy that achieves the same outcome.

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Brotherhood

All men are weakly better off under P' (compared to P).

Simplicity

1

If there is a feasible strategy for manipulation, then there is a "simple" feasible strategy that achieves the same outcome.

Brotherhood

2

All men are weakly better off under P' (compared to P).

No new proposal

3

If a man proposes to a woman during DA on the profile P' , then he must also propose to her during DA on the profile P .

1

Simplicity

Let $X' = DA(P')$. Then, m is matched to $X'(m)$ under DA on the profile $P'' = (P_{-m}, P''_m)$ obtained from his true list P_m by promoting $X'(m)$ to the top.

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Then, X' must also be stable w.r.t. the profile P'' .

(The manipulator m gets his top choice in P'' and therefore doesn't block X' . Any other blocking pair must also block X' w.r.t. P' , but that would contradict stability of DA algorithm.)

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When DA is run on P'' , we get the men-optimal stable matching, say X'' , w.r.t. P'' .

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When DA is run on P'' , we get the men-optimal stable matching, say X'' , w.r.t. P'' .

Man m must weakly prefer $X''(m)$ over $X'(m)$ according to P''_m .

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The matching X' is stable w.r.t. the profile P' .

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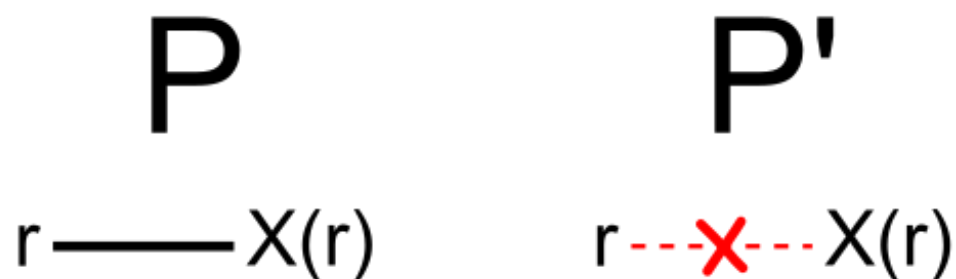
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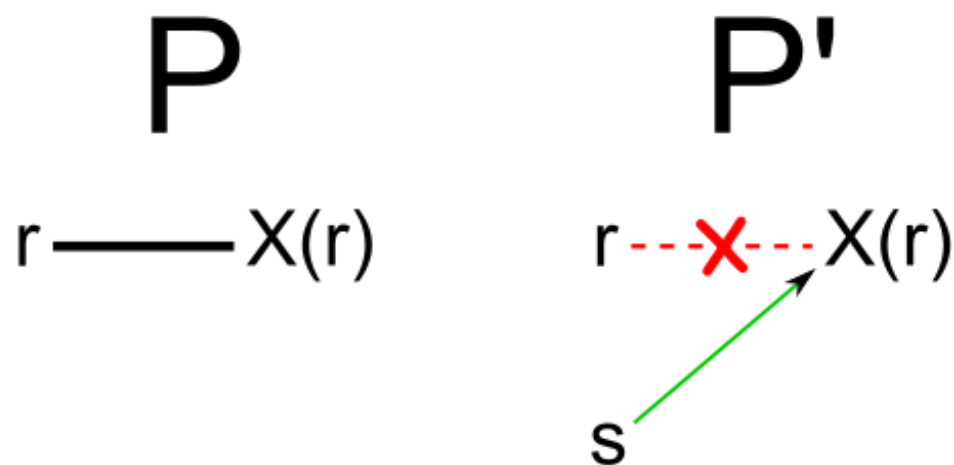
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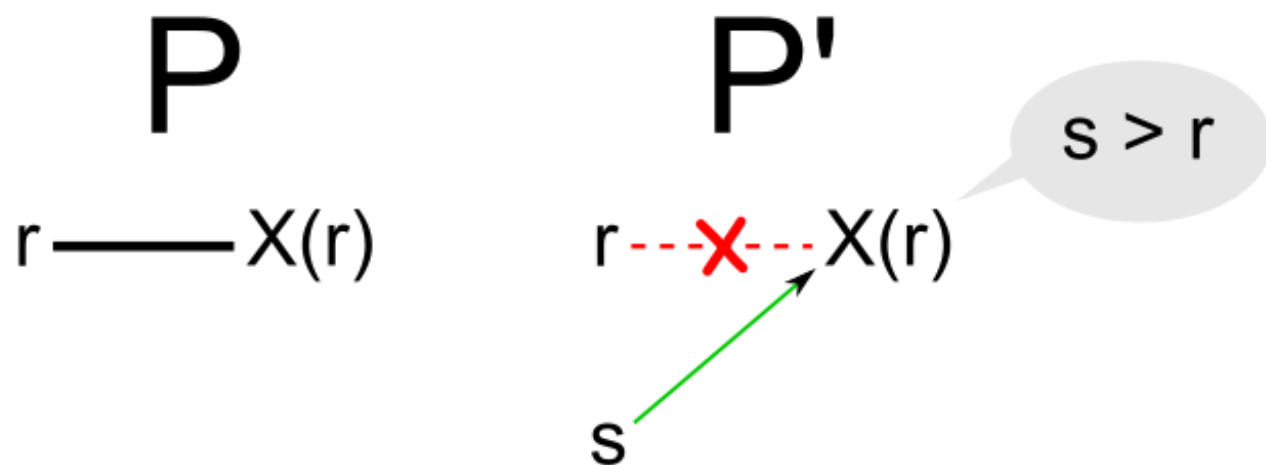
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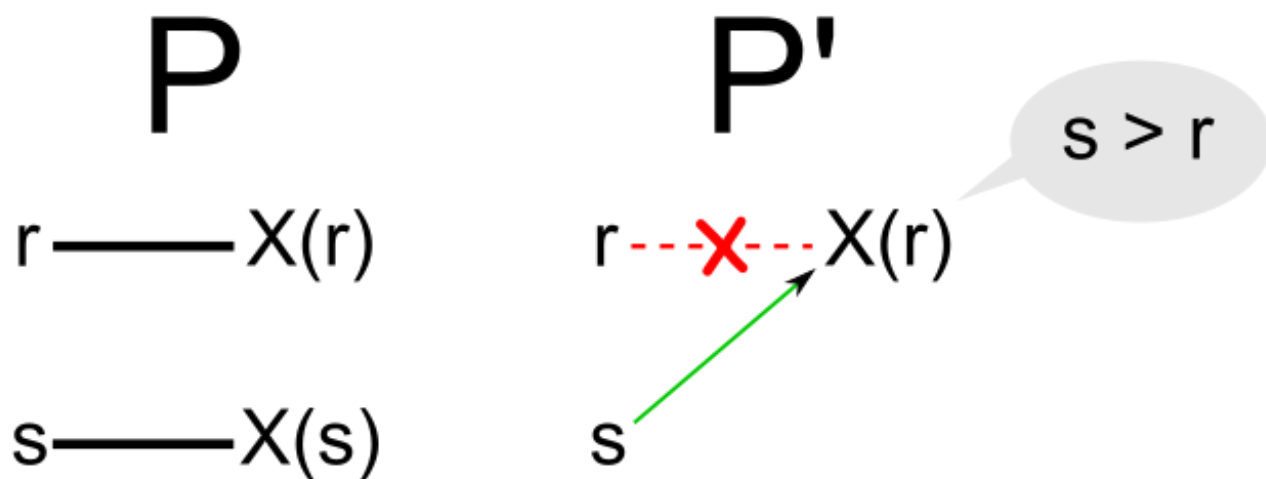
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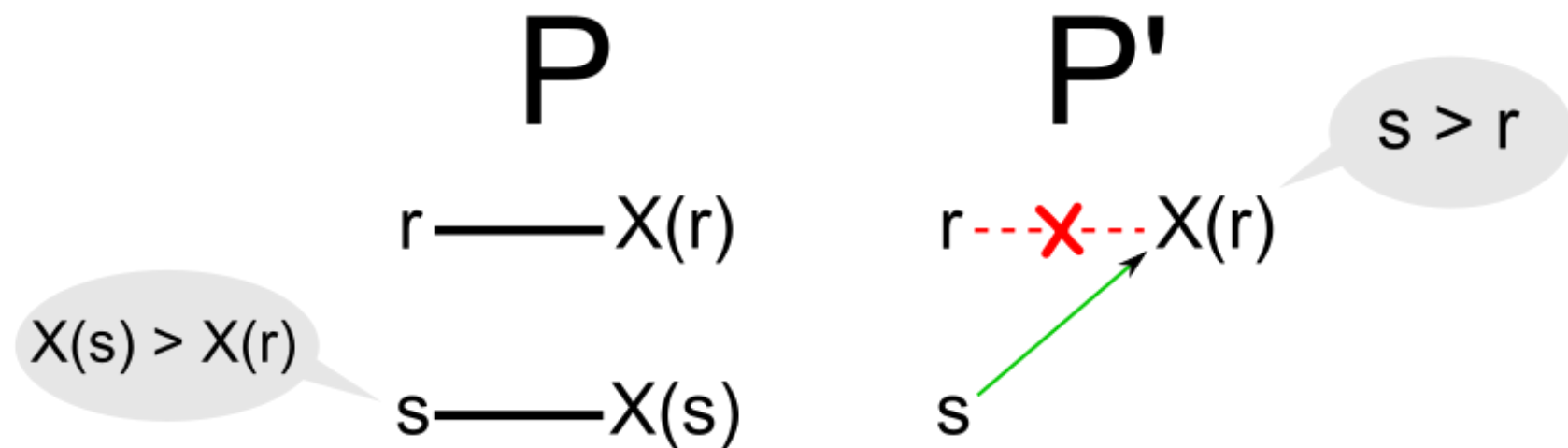
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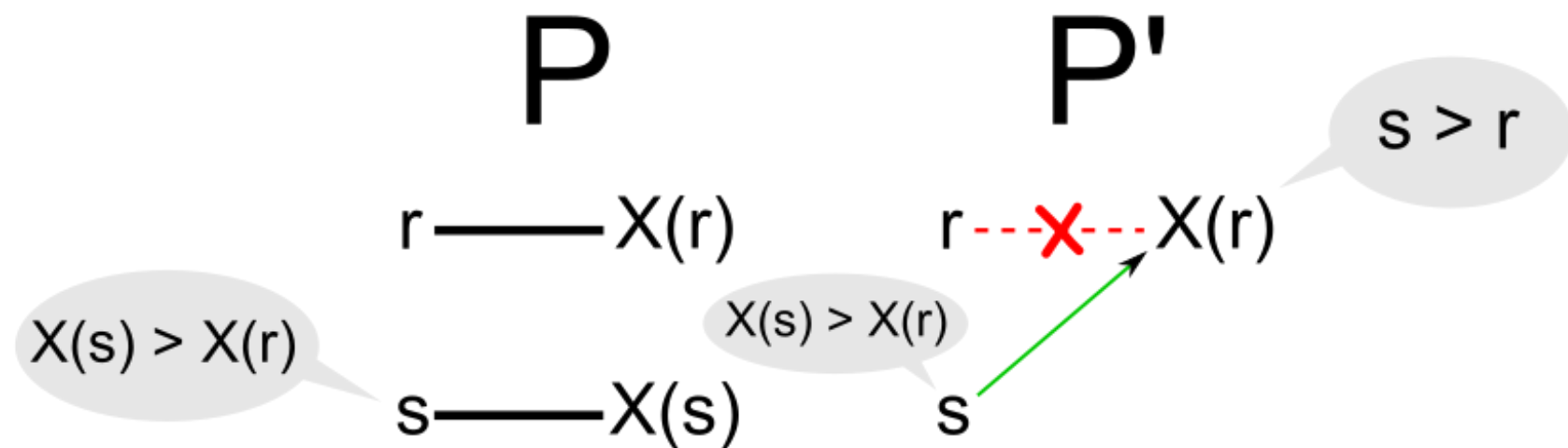
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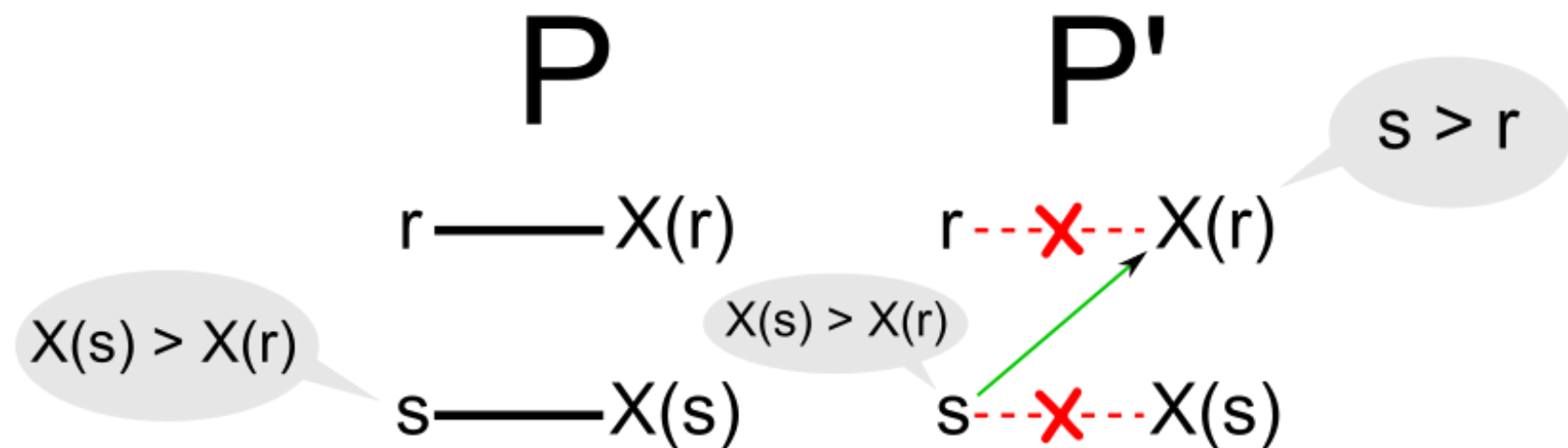
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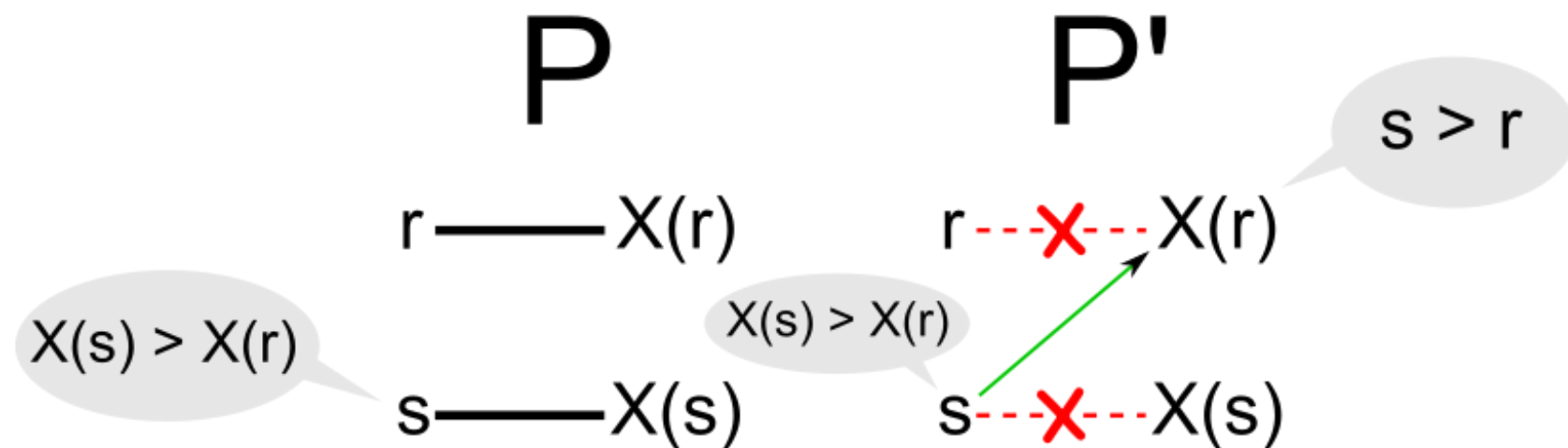
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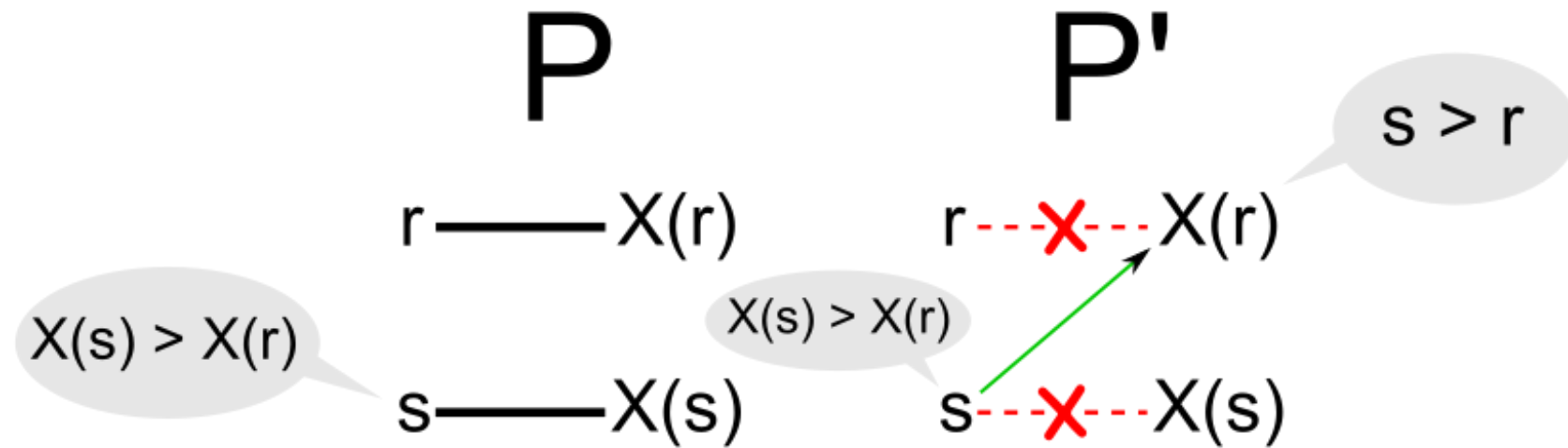
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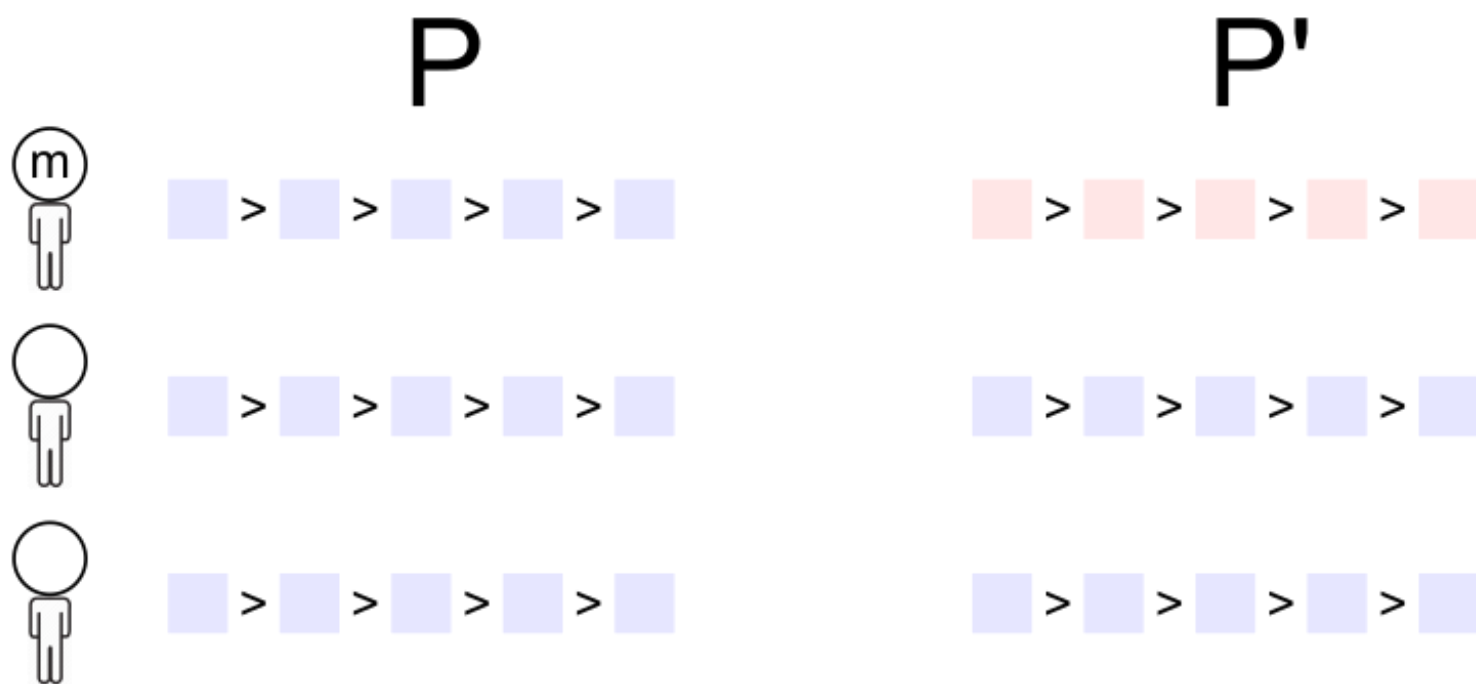
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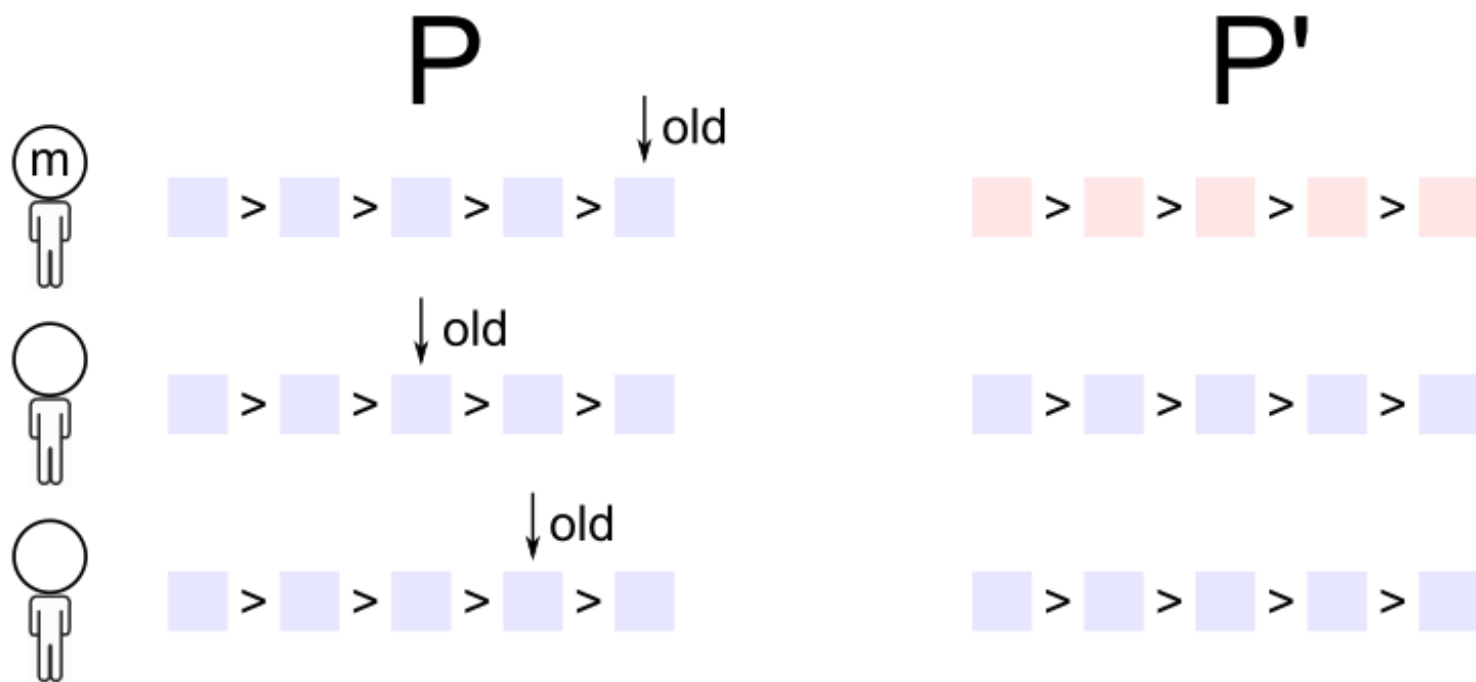
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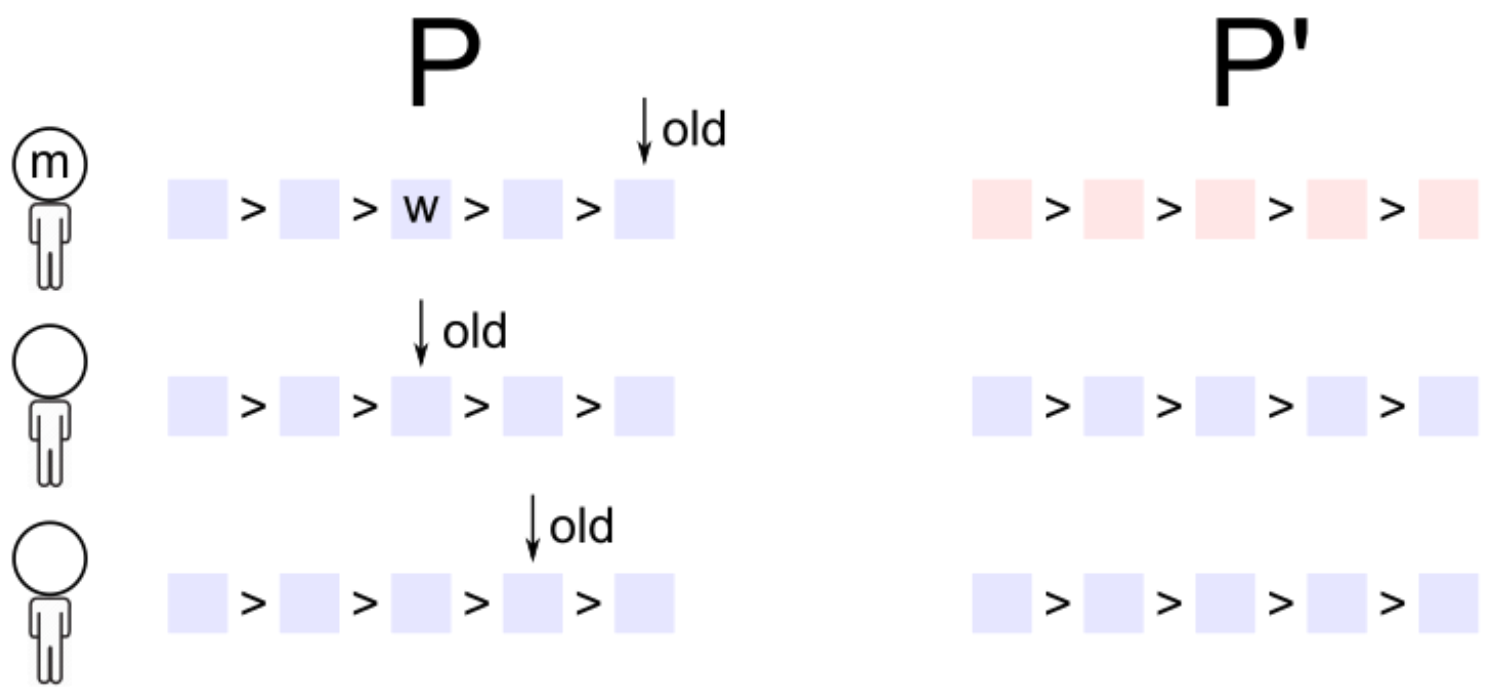
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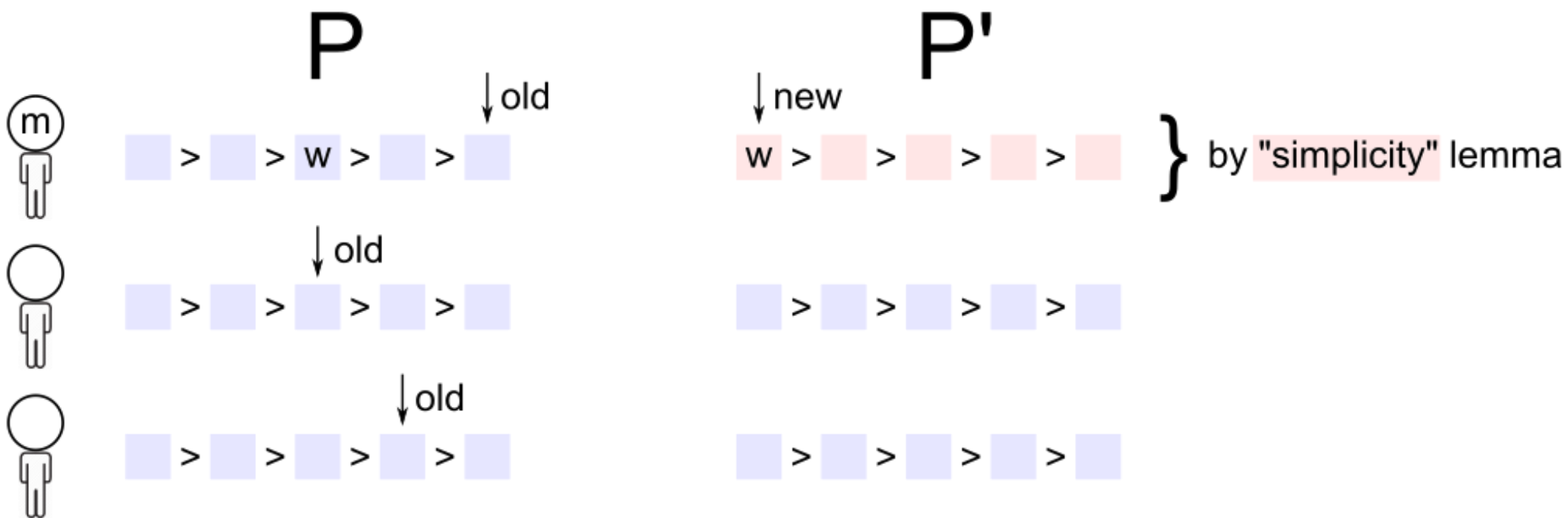
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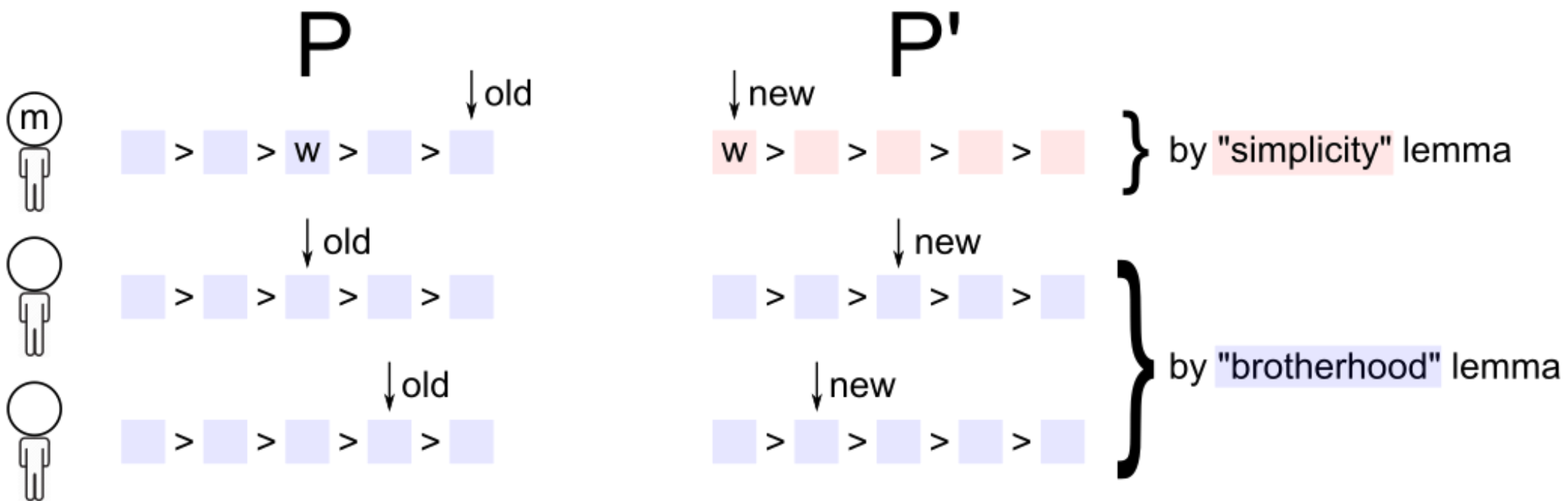
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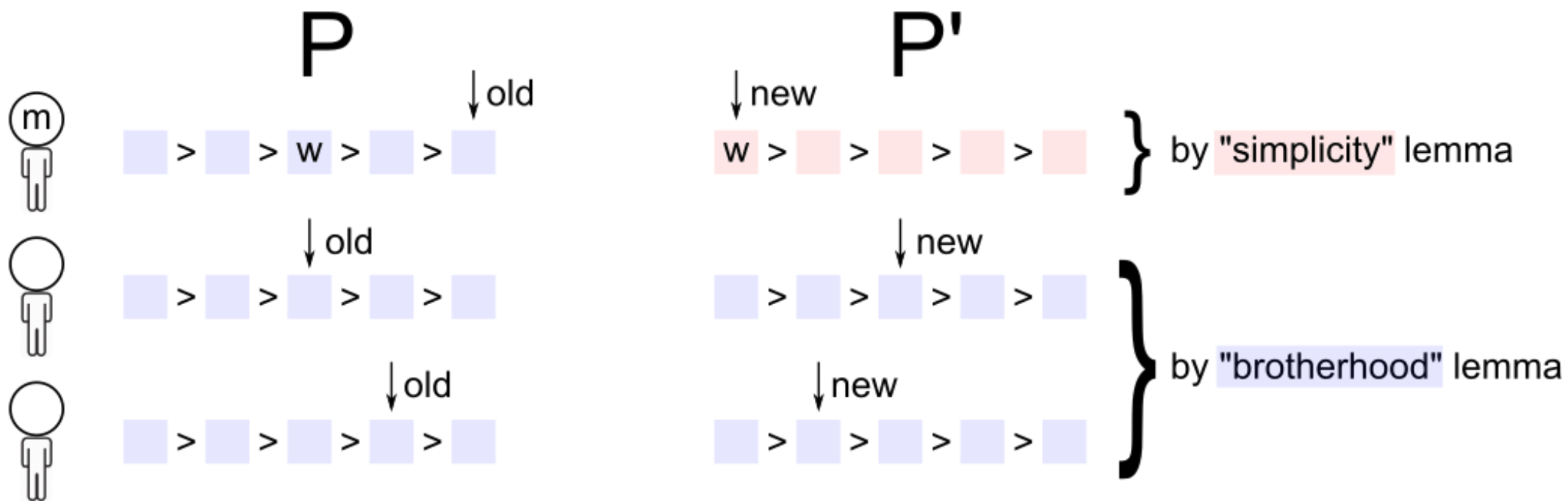
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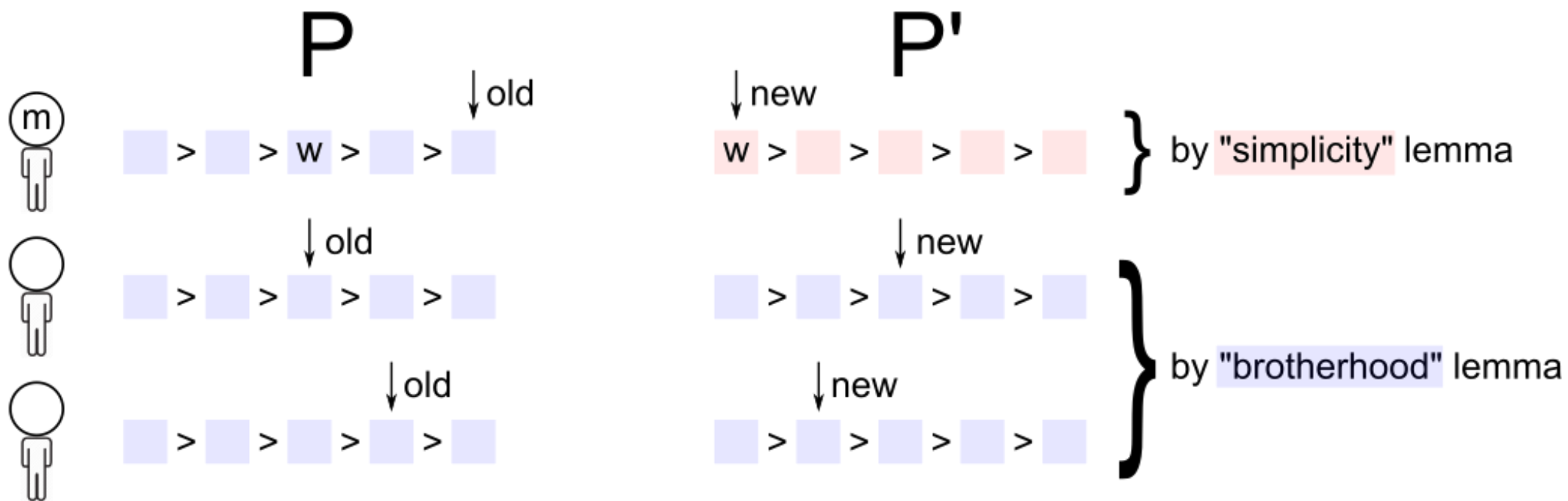
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Any man who proposes to his X -partner in round k or later is matched to his X -partner under X' as well.

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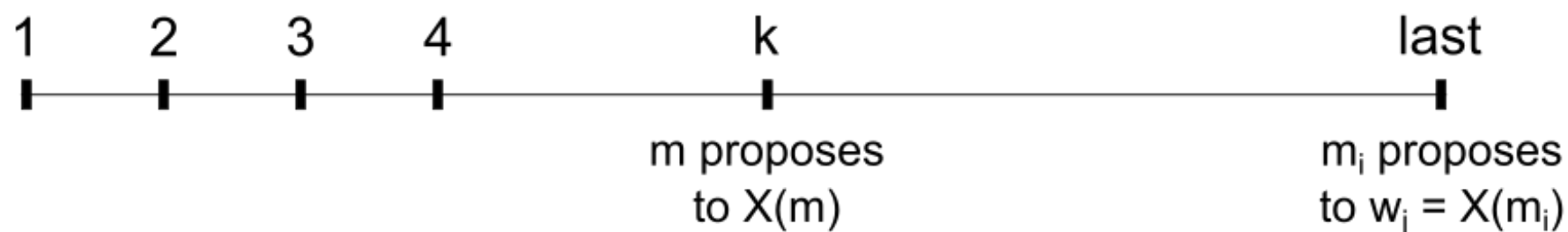
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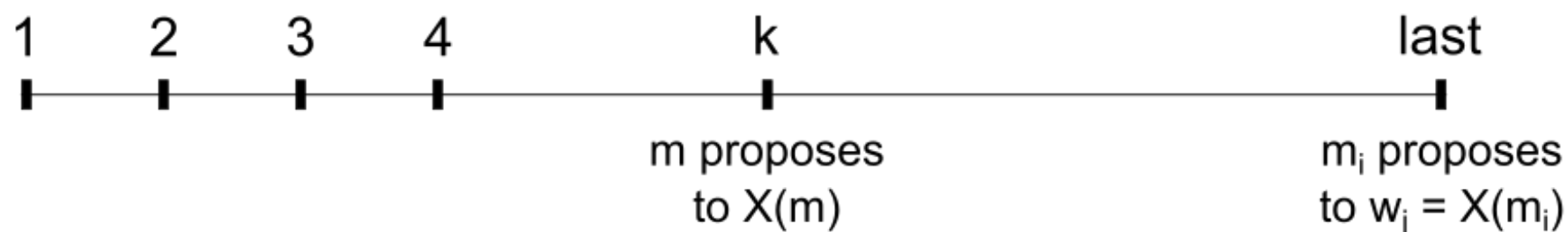


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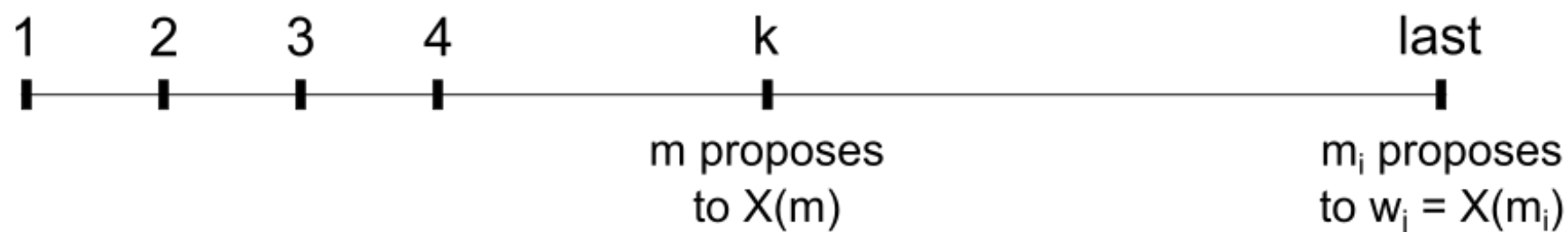
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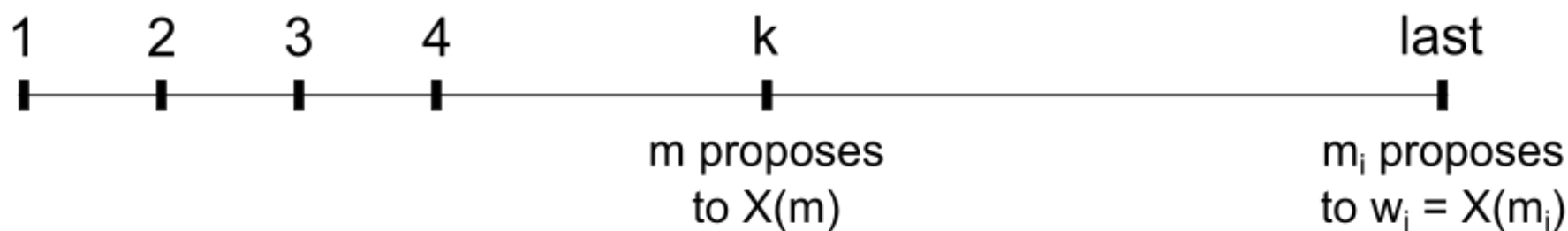
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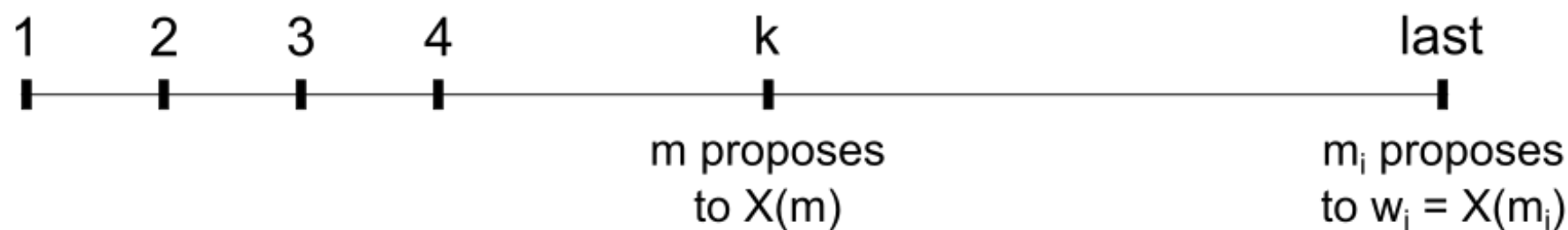
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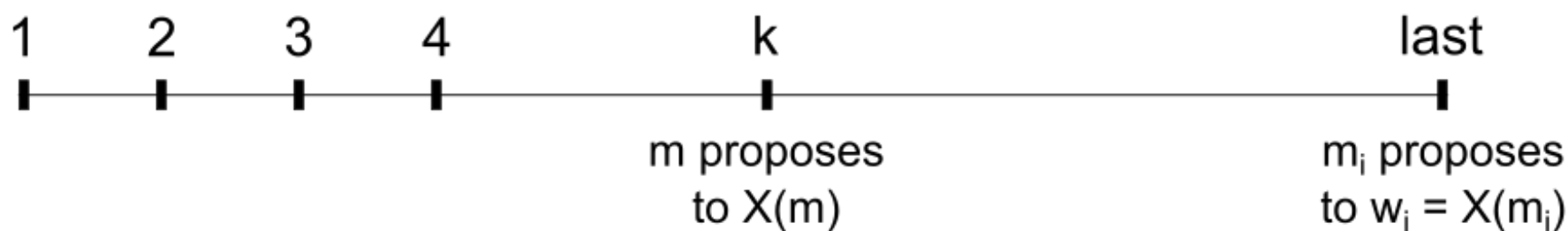
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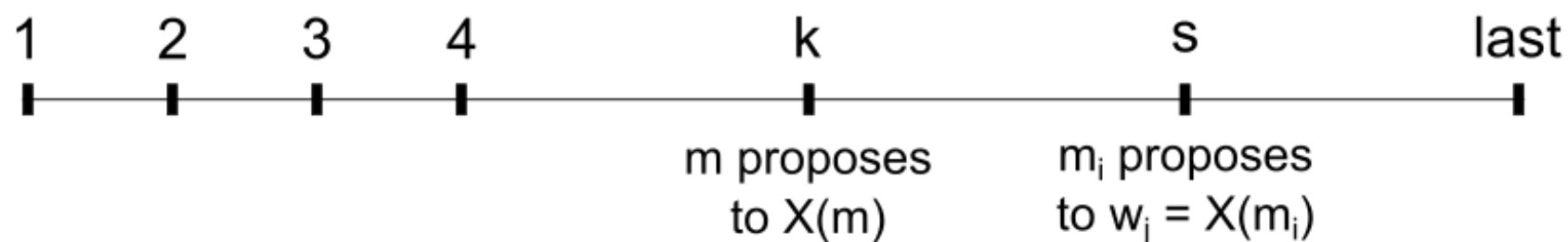
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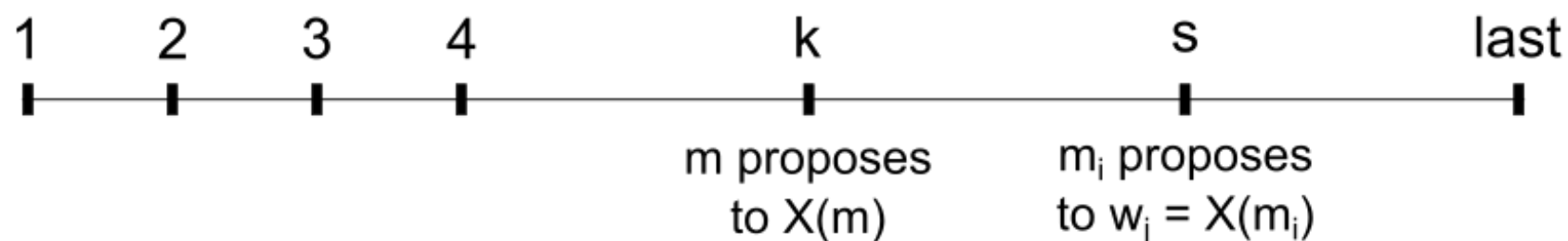


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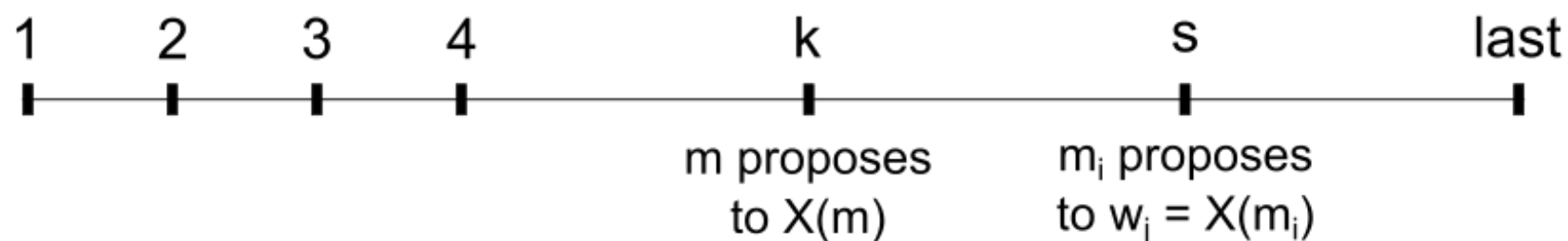
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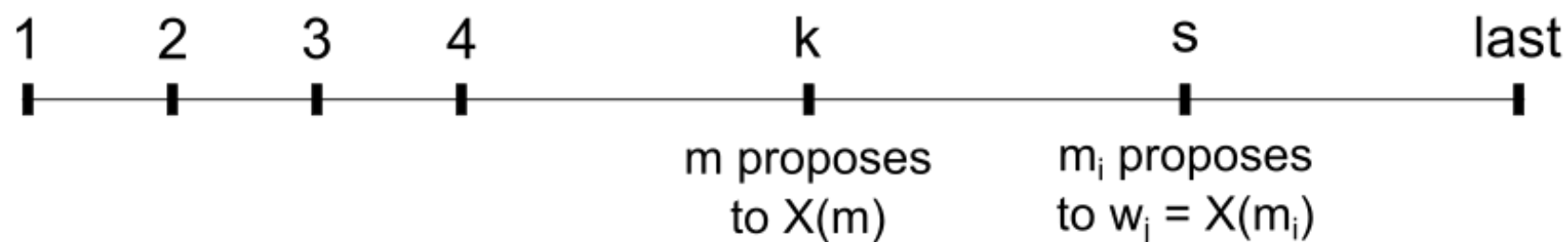
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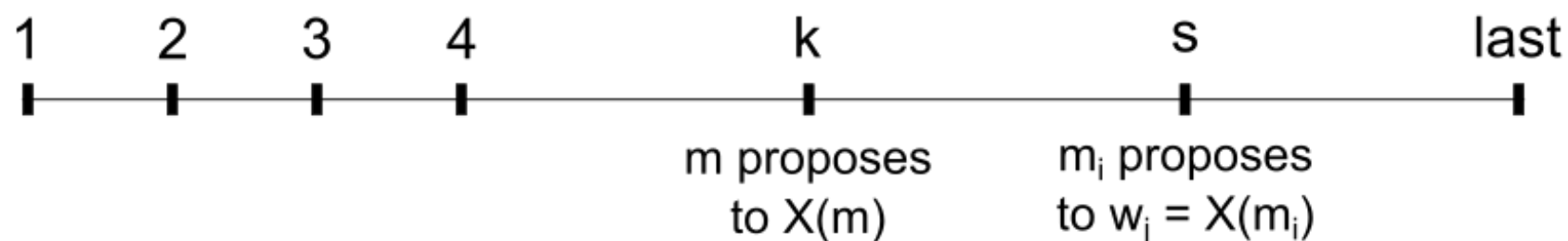


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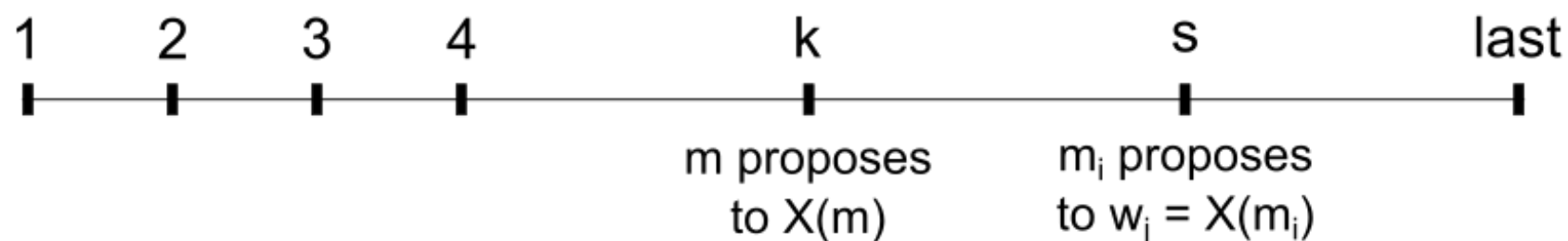
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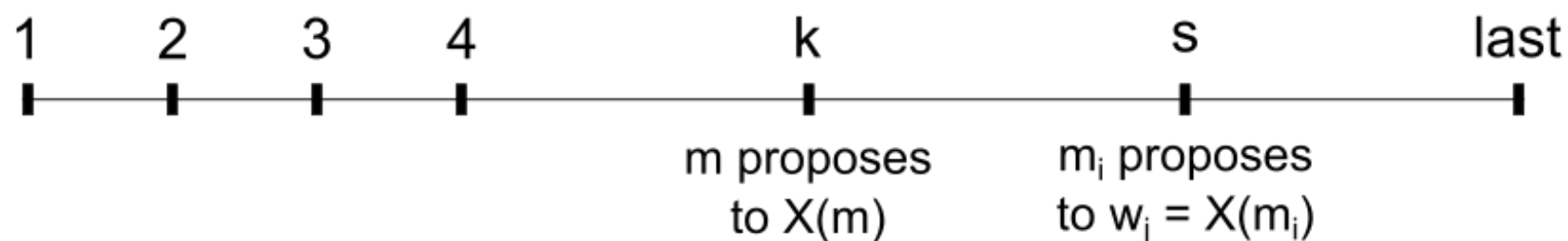
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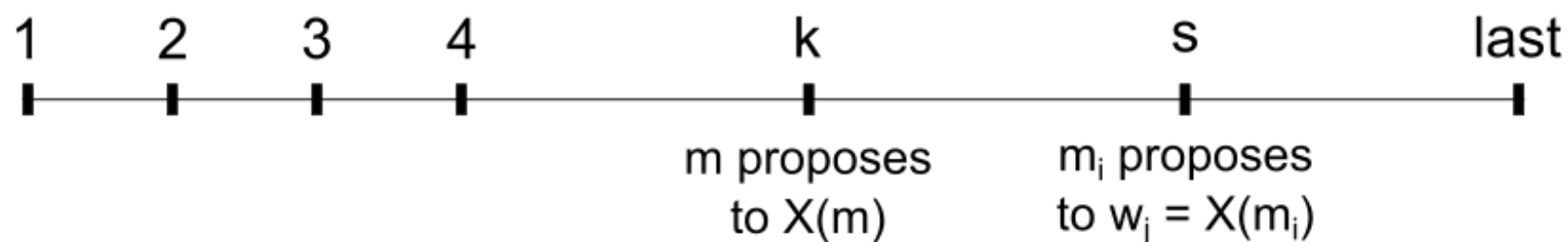
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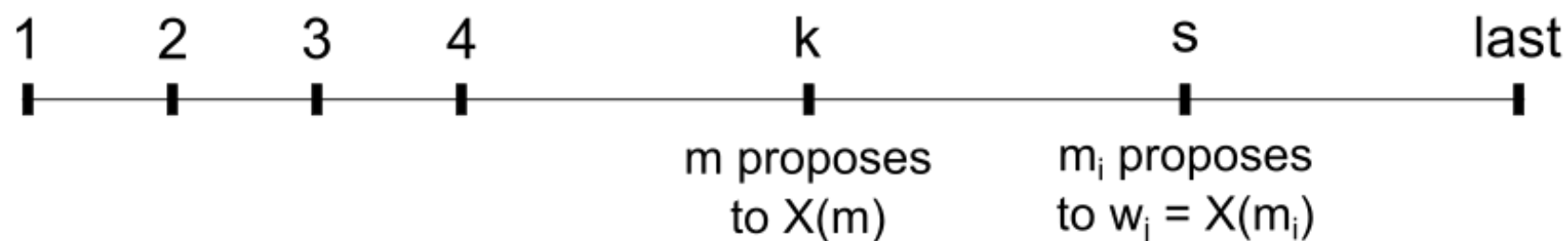
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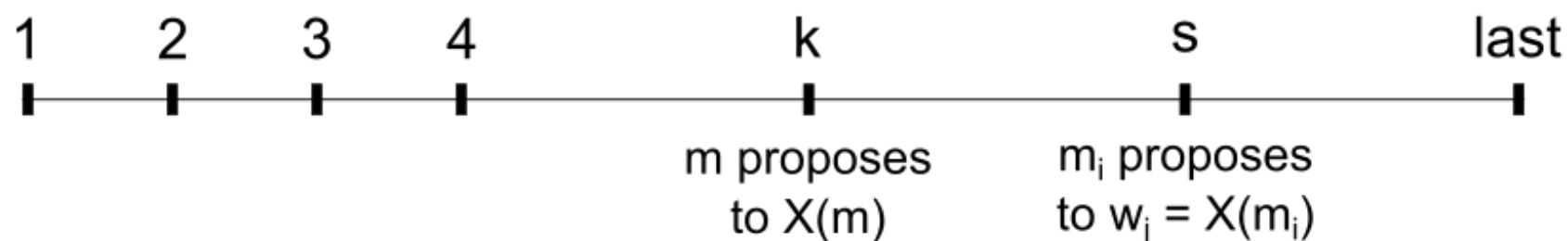
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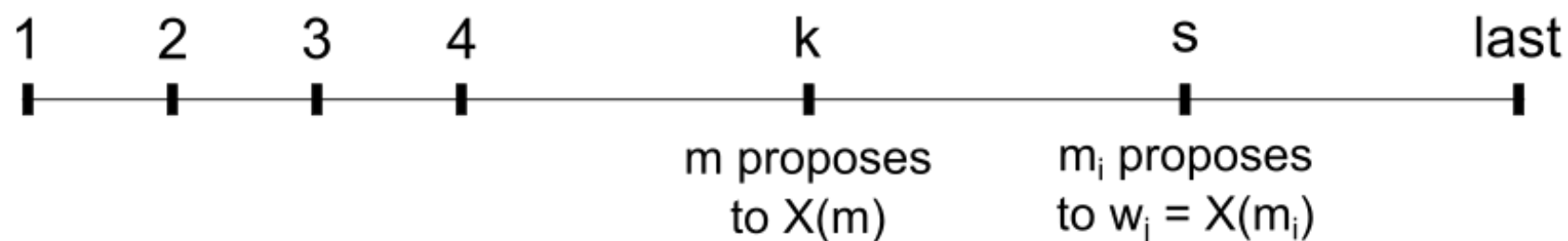
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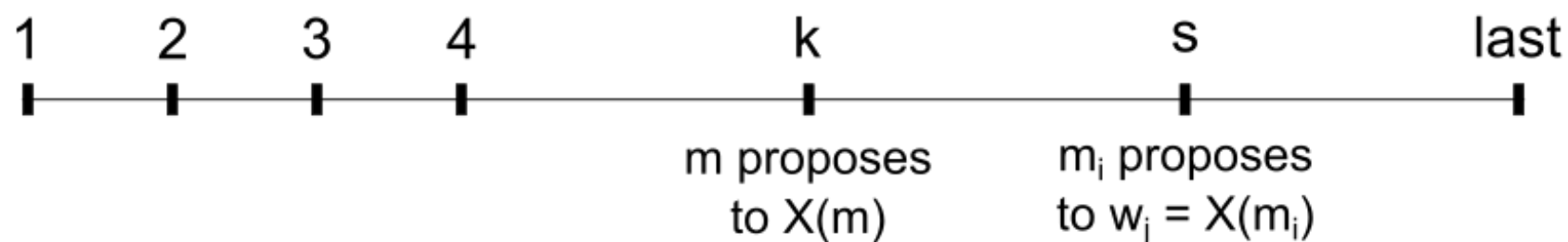
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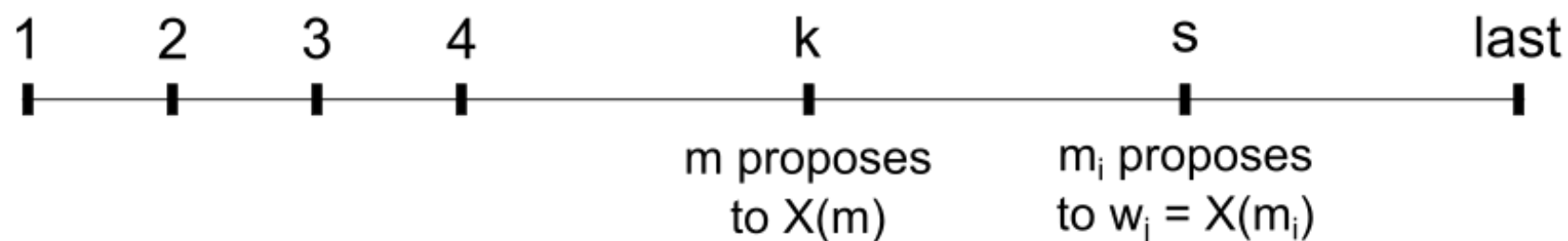


Any man who proposes to his X-partner in round k or later is matched to his X-partner under X' as well.

Proof by induction.

Induction step:

Suppose the claim holds for all rounds $s+1$ or later, and we want to prove it for round s .



Let R = set of men rejected by w_j during DA on profile P (across ALL rounds).

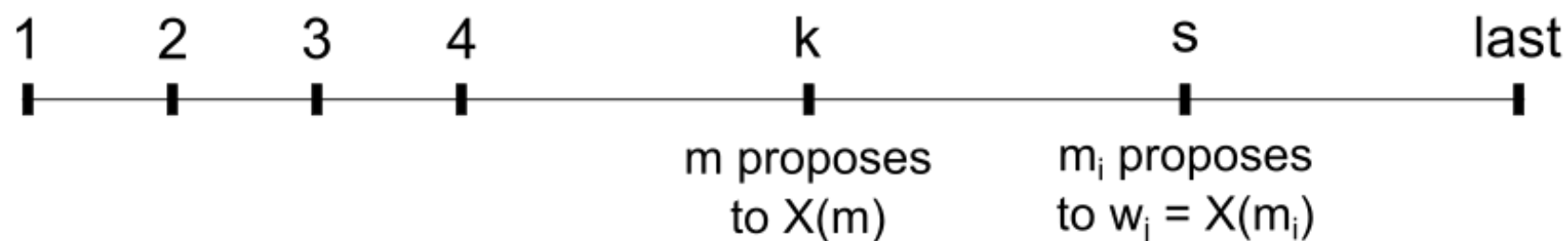
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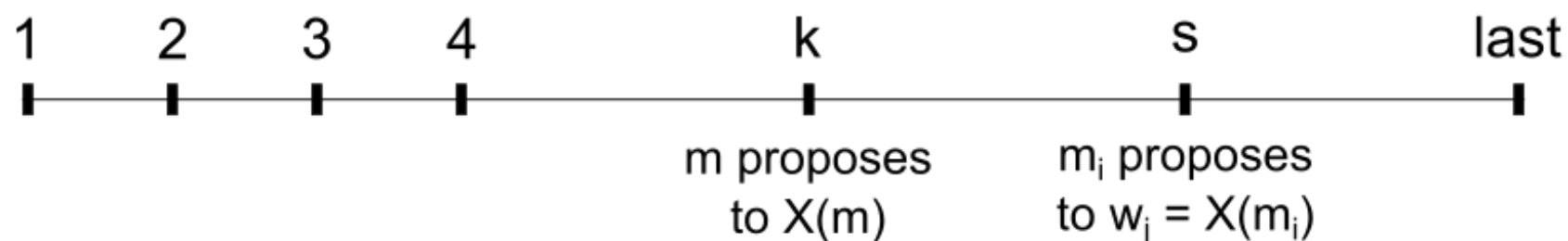
So, m_F must propose to w_j during DA on P' , and is again rejected since $X(m_F) = X'(m_F)$.

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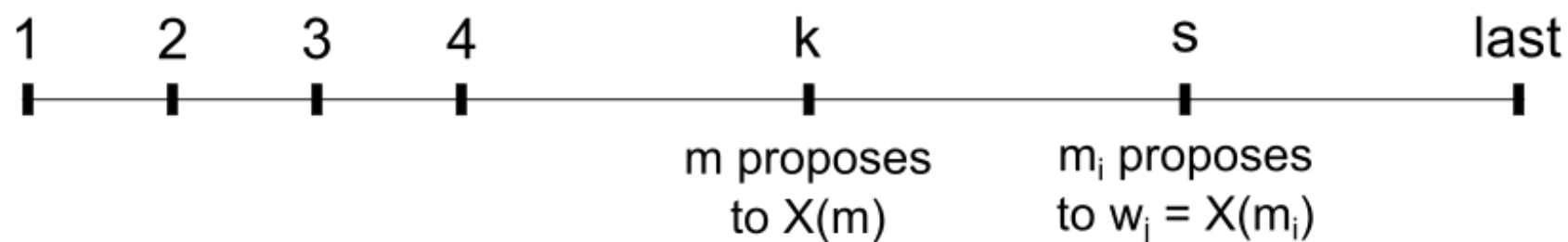
Thus, w_j receives at least one more proposal (besides m_F) during DA(P').

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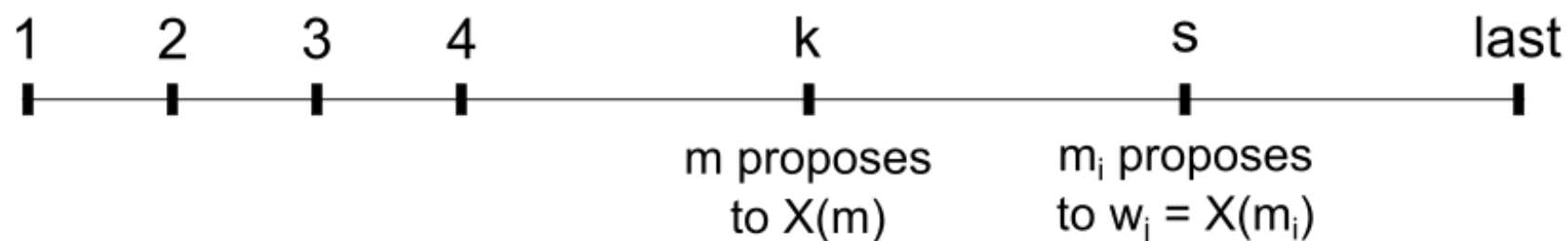


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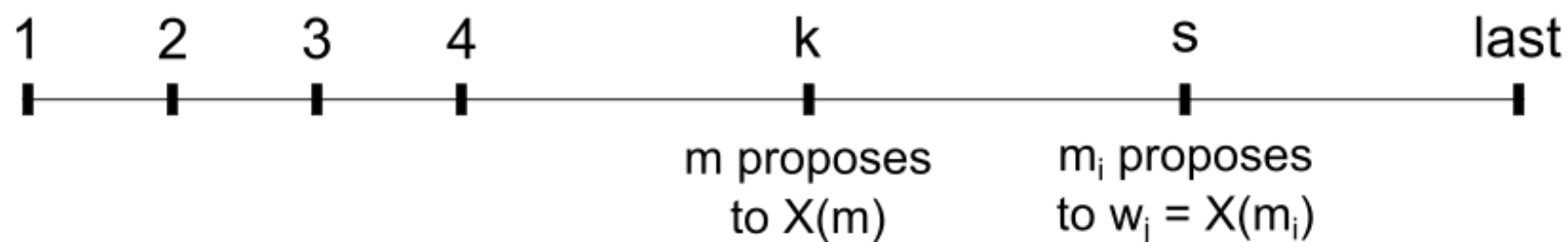
By "no new proposal" lemma, the man $m' = X'(w_j)$ proposes to w_j during $DA(P)$.

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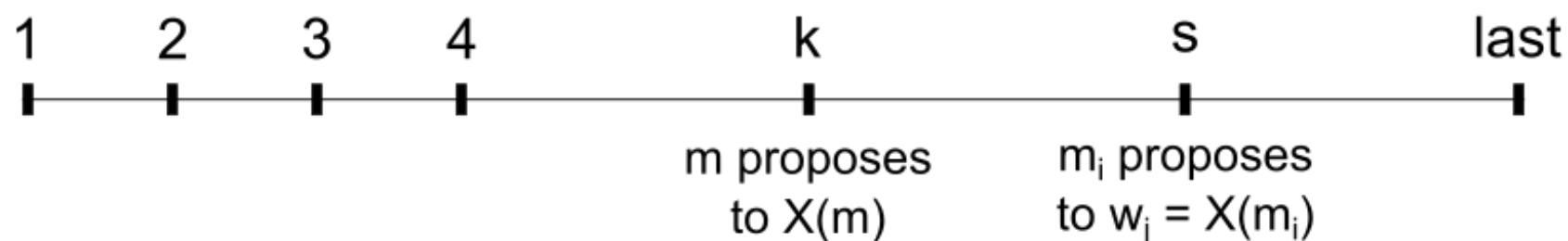
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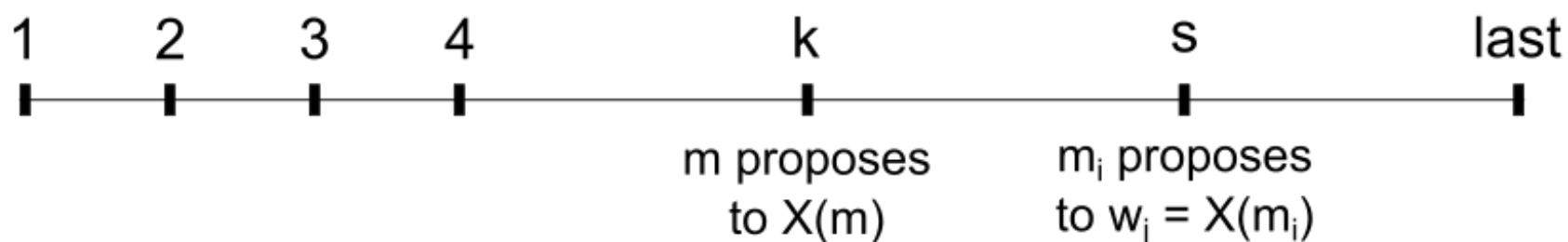
- If $m' = X(w_j) = m_i$, then we are done.
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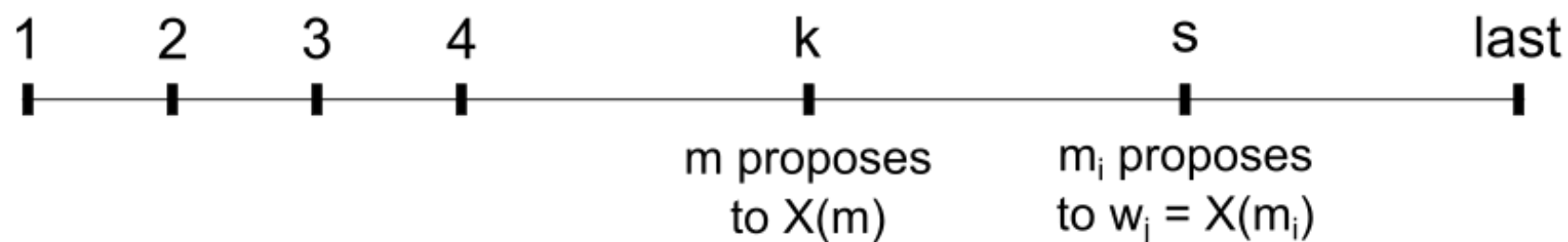
- If $m' = X(w_j) = m_i$, then we are done.
- If $m' >_w m_i$, then w_j would have rejected her X-partner during $DA(P)$ ---a contradiction.
- If $m_i >_w m'$, then w_j weakly prefers m_i over m' , and would have rejected m' during $DA(P')$, again a contradiction.

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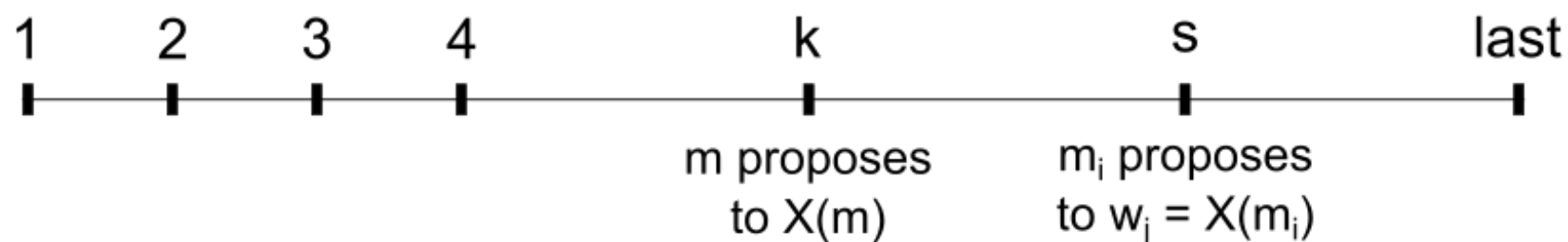


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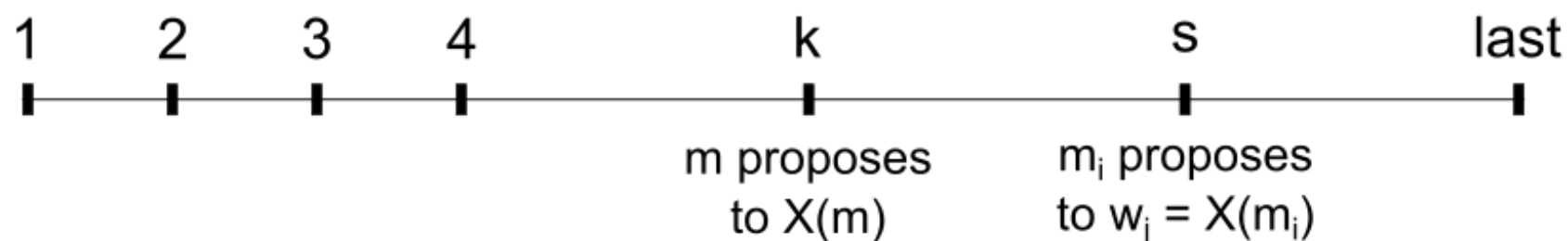
Therefore, $X(m_j) = w_j = X'(m_j)$.

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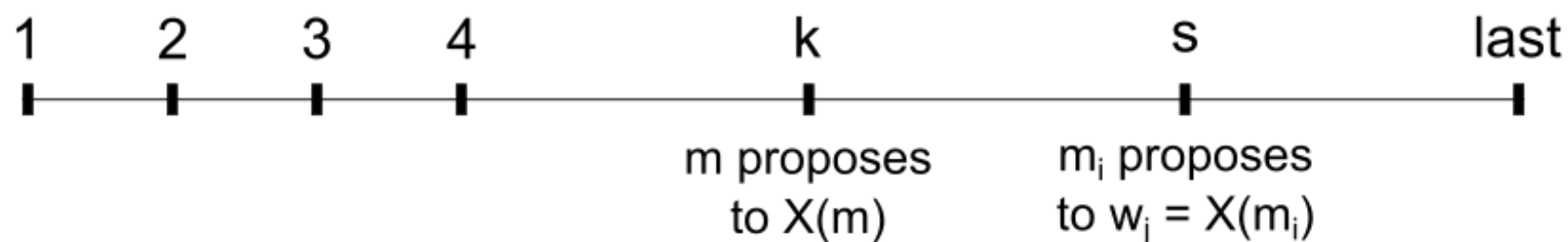


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[Dubins and Freedman, *Amer. Math. Mon.* 1981; Roth, *MOR* 1982]

DA algorithm is strategyproof for the men.

Suppose, for contradiction, that DA can be manipulated by a man m on the profile P .

Consider the execution of DA on the true profile P . Recall that $X = DA(P)$.

Suppose man m proposes to his X -partner (i.e., $X(m)$) in round k of the algorithm.

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Then, $X(m) = X'(m)$, which means the manipulator does not improve.

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