COL 8184 : ALGORITHMS FOR FAIR REPRESENTATION

LECTURE 11

THE CAREFUL SLIDING METHOD

FEB 19, 2026 | ROHIT VAISH

FEEDBACK ON MID-SEMESTER FEEDBACK

FEEDBACK ON MID-SEMESTER FEEDBACK

7 responses (out of 15)

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗

* Grading feedback

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗

* Grading feedback ✓

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗
* Grading feedback ✓
* Allow in-person regrading

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗
* Grading feedback ✓
* Allow in-person regrading ✗ [Use office hours]

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗

* Grading feedback ✓

* Allow in-person regrading ✗ [Use office hours]

* Proof writing

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗

* Grading feedback ✓

* Allow in-person regrading ✗ [Use office hours]

* Proof writing

* "Remind us to read instructions" 😐

FEEDBACK ON MID-SEMESTER FEEDBACK

* Allow audits ✗
* Grading feedback ✓
* Allow in-person regrading ✗ [Use office hours]
* Proof writing
* "Remind us to read instructions" 😐
* Format of mid-term presentation

MID-TERM PROJECT PRESENTATIONS

* March 12 (Thursday), in person during class hours

* 7 mins per group : 5 mins presentation + 2 mins Q & A

* Update finalized project titles and references

* Project report due by March 10 (Tuesday)

MID-TERM PROJECT PRESENTATIONS

* Clearly convey
  - the main question(s) studied by the project
  - why these questions are interesting / significant
  - brief overview of related literature
  - project milestones (for each group member)

GRIMMETT'S METHOD

* Consider a unif. random permutation of the states (wolog, identity).

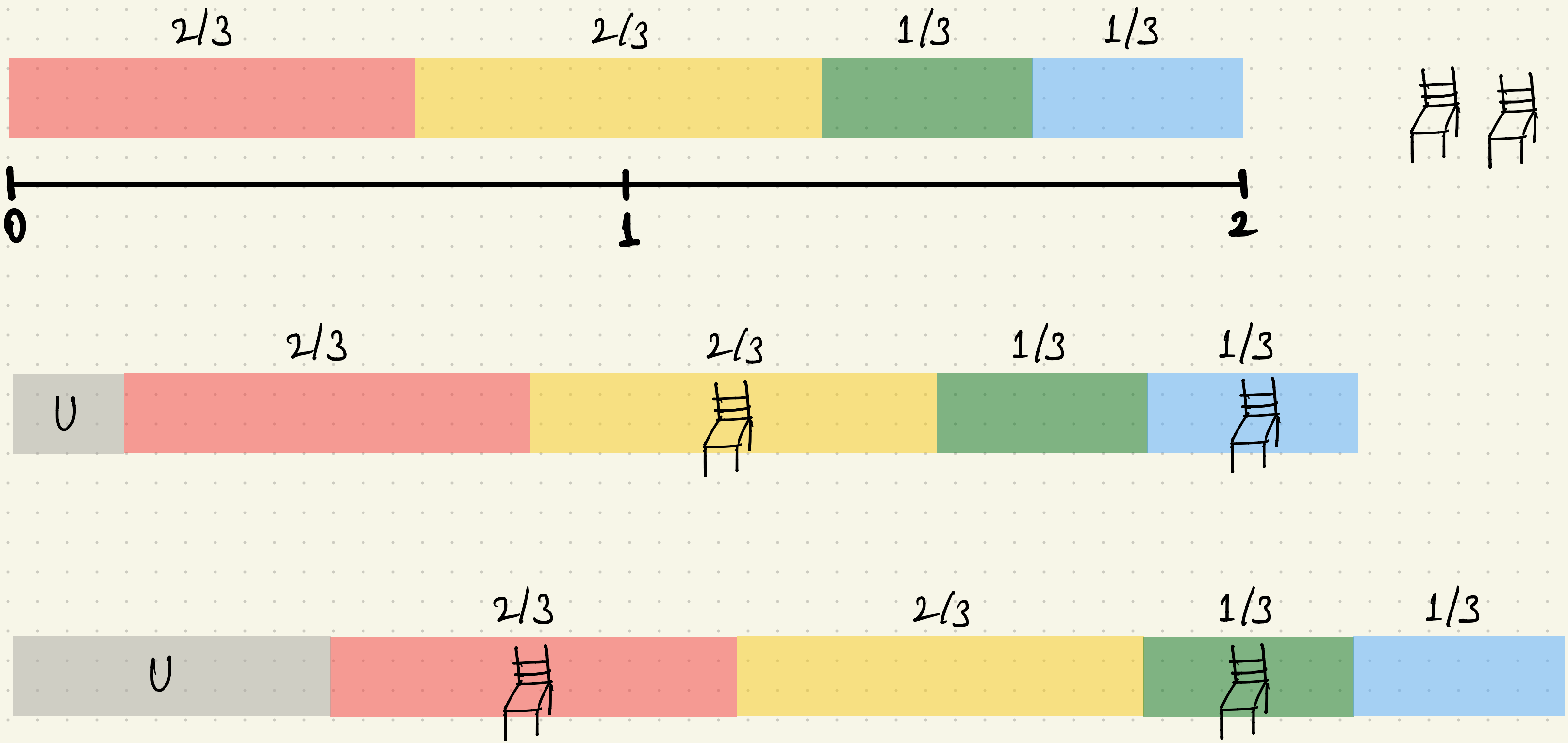
* Provisionally assign $\lfloor q_i \rfloor$ seats to state i. Let $r_i := q_i - \lfloor q_i \rfloor$.

  i.e., the residue

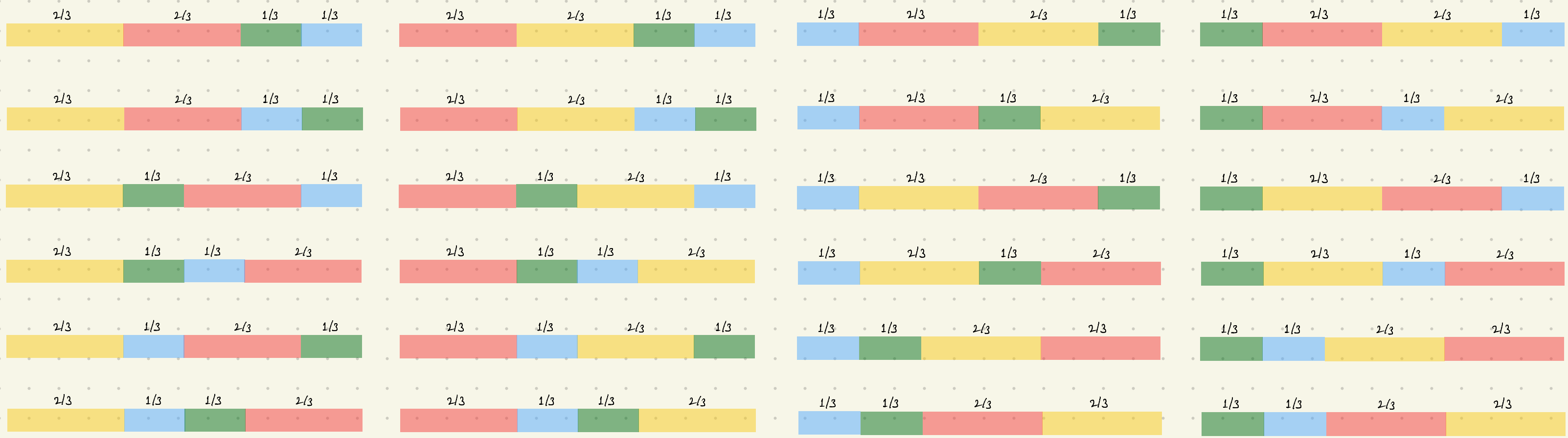
* Draw $U \sim \text{Unif}[0,1]$. Let $Q_i := U + \sum_{j=1}^{i-1} r_j$.

* For each state i, allocate an extra seat to state i if $[Q_i, Q_{i+1})$ contains an integer.

GRIMMETT'S METHOD

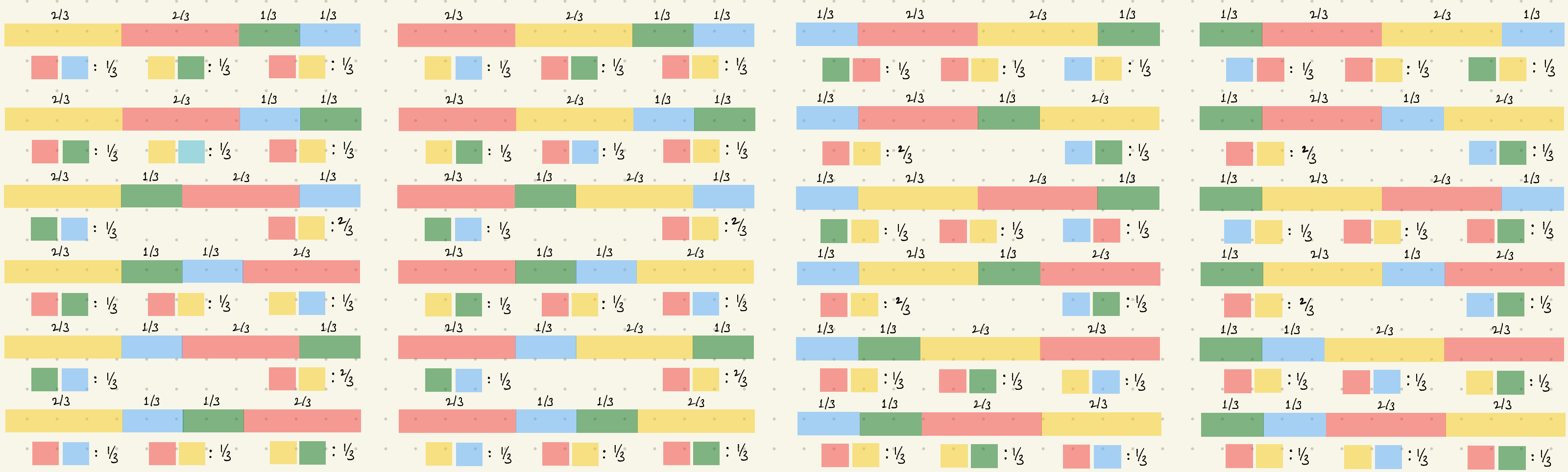


GRIMMETT'S METHOD



4! permutations

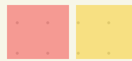
GRIMMETT'S METHOD

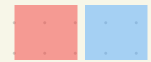


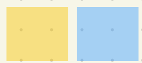
4! permutations and the resulting distributions

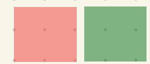
GRIMMETT'S METHOD

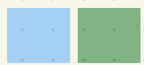
Overall distribution :

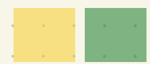
 : 4/9

 : 1/9

 : 1/9

 : 1/9

 : 1/9

 : 1/9

PROPERTIES OF GRIMMETT'S METHOD

Theorem : Grimmett's method satisfies :

* ex-ante proportionality
* ex-ante population monotonicity
* ex-post quota

BIZARRE CORRELATIONS

BIZARRE CORRELATIONS

$h = 11$

State 1	110
State 2	270
State 3	210
State 4	160
State 5	70
State 6	280

BIZARRE CORRELATIONS

$h = 11$

State 1	1.1
State 2	2.7
State 3	2.1
State 4	1.6
State 5	0.7
State 6	2.8

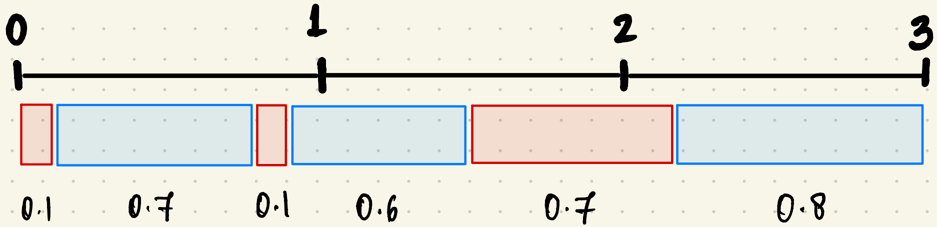
BIZARRE CORRELATIONS

First assign lower quotas.

$h = 11$

(1)	State	1	1.1
(2)	State	2	2.7
(2)	State	3	2.1
(1)	State	4	1.6
(0)	State	5	0.7
(2)	State	6	2.8

BIZARRE CORRELATIONS



$h = 11$

(1) State 1 1.1

(2) State 2 2.7

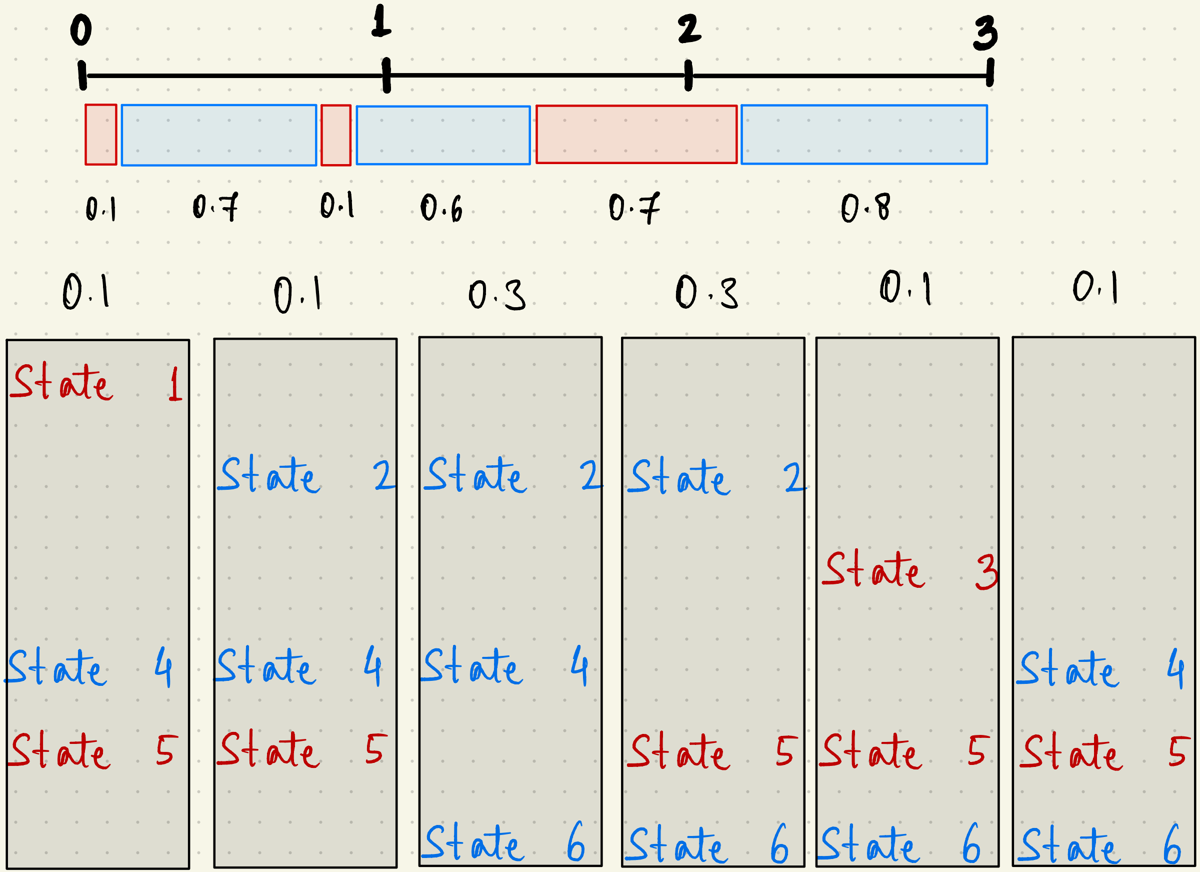
(2) State 3 2.1

(1) State 4 1.6

(0) State 5 0.7

(2) State 6 2.8

BIZARRE CORRELATIONS



$h = 11$

(1) State 1 1.1

(2) State 2 2.7

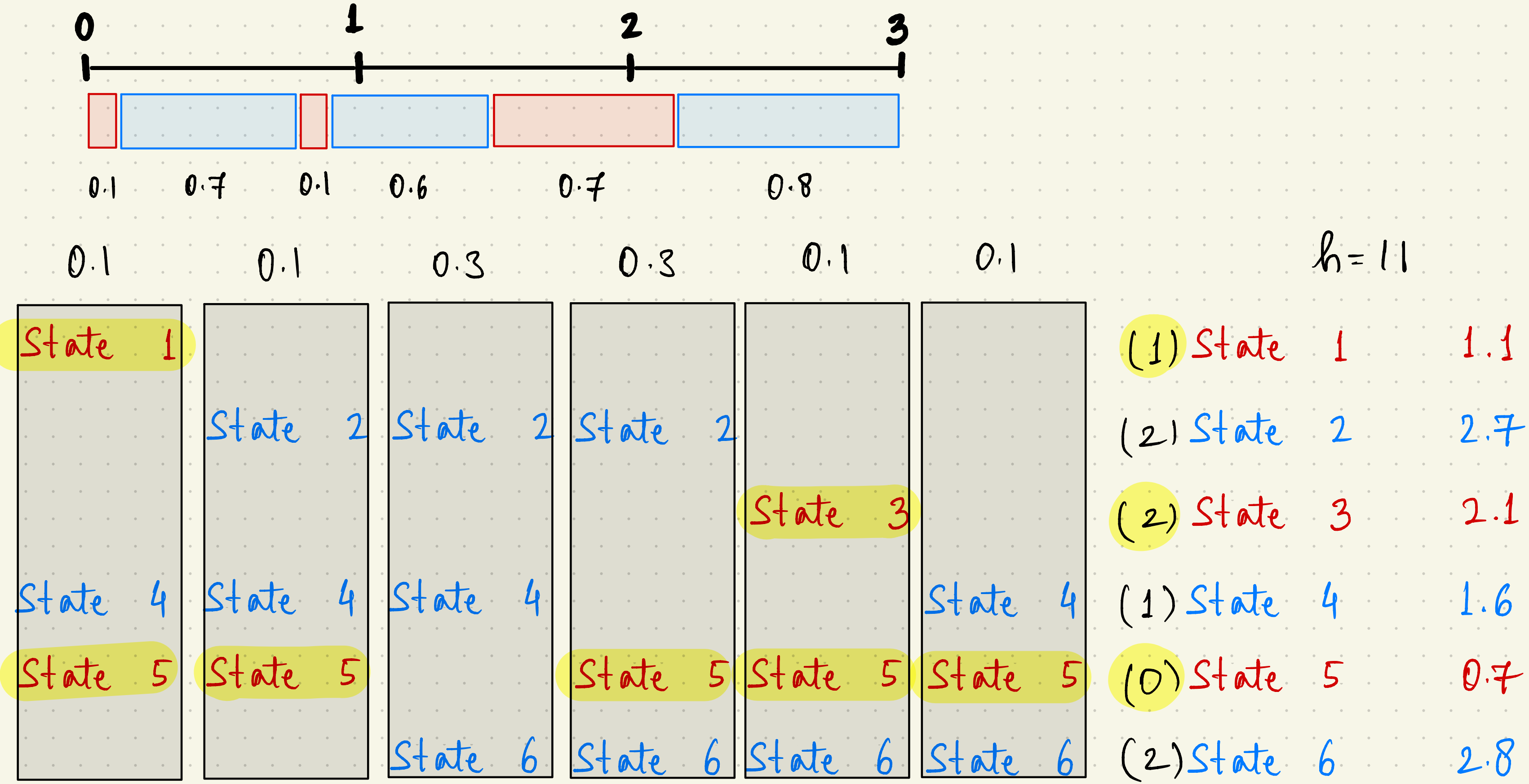
(2) State 3 2.1

(1) State 4 1.6

(0) State 5 0.7

(2) State 6 2.8

BIZARRE CORRELATIONS



RED states never receive more than 5 states ex-post.

BIZARRE CORRELATIONS

$n = 11$

State	1	1.1
State	2	2.7 ↑
State	3	2.1
State	4	1.6 ↑
State	5	0.7 ↓
State	6	2.8 ↑

BLUE states gain population

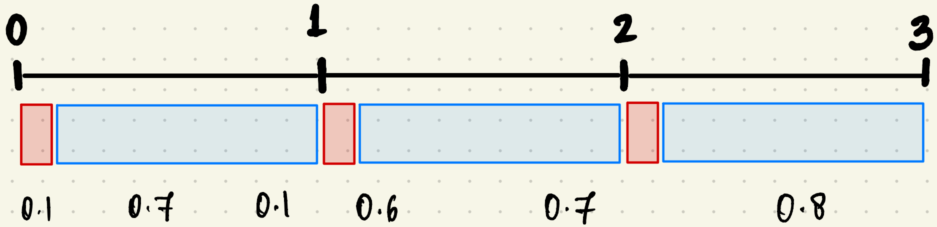
RED states lose population

BIZARRE CORRELATIONS

$n = 11$

State	1	1.1
State	2	2.9
State	3	2.1
State	4	1.9
State	5	0.1
State	6	2.9

BIZARRE CORRELATIONS



$n = 11$

(1) State 1 1.1

(2) State 2 2.9

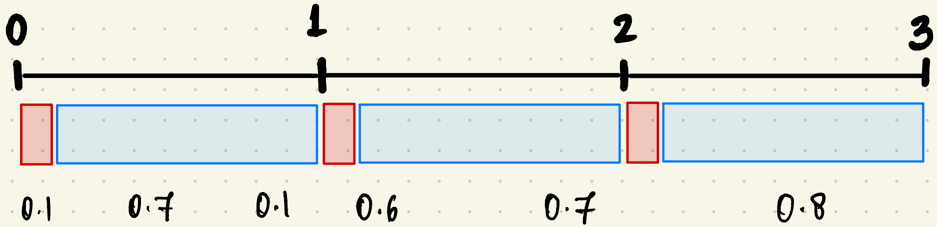
(2) State 3 2.1

(1) State 4 1.9

(0) State 5 0.1

(2) State 6 2.9

BIZARRE CORRELATIONS



0.1	0.9
State 1	
	State 2
State 3	
	State 4
State 5	
	State 6

$h = 11$

(1) State 1 1.1

(2) State 2 2.9

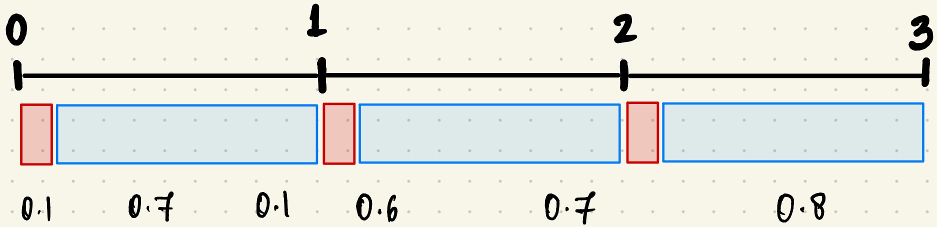
(2) State 3 2.1

(1) State 4 1.9

(0) State 5 0.1

(2) State 6 2.9

BIZARRE CORRELATIONS



0.1	0.9
State 1	State 2
State 3	State 4
State 5	State 6

h = 11

(1)	State 1	1.1
(2)	State 2	2.9
(2)	State 3	2.1
(1)	State 4	1.9
(0)	State 5	0.1
(2)	State 6	2.9

RED states achieve majority with probability 0.1

BIZARRE CORRELATIONS

RED states gain ex-post majority by losing population to BLUE states!

Selection Monotonicity

[Correa, Gölz, Schmidt-Kraepelin, Tucker-Foltz, and Verdugo ; EC 2024]

Selection Monotonicity

[Correa, Gölz, Schmidt-Kraepelin, Tucker-Foltz, and Verdugo ; EC 2024]

Let $\vec{r}, \vec{r}' \in [0,1)^n$ be two residue vectors summing up to the same integer k. Let T be a set of k states such that

$$r_i' \geq r_i \quad \forall\ i \in T \qquad \text{and} \qquad r_i' \leq r_i \quad \forall\ i \notin T.$$

A randomized apportionment method f satisfies selection monotonicity if

$$\Pr_{S \sim f(\vec{r}')}[S = T] \geq \Pr_{S \sim f(\vec{r})}[S = T].$$

Selection Monotonicity

[Correa, Gölz, Schmidt-Kraepelin, Tucker-Foltz, and Verdugo ; EC 2024]

Which randomized apportionment method satisfies selection monotonicity, ex-post quota, and ex-ante proportionality?

🤔

LANDSCAPE

$$\begin{tikzcd}
 & \text{Ex-post proportionality} \arrow[dl] \arrow[dr] & \\
\text{Ex-ante proportionality} \arrow[dr] & & \text{Ex-post quota} \arrow[dl] \\
 & \text{Ex-ante quota} &
\end{tikzcd}$$

$$\begin{tikzcd}
 & \text{Ex-post pop. mon.} \arrow[dl] \arrow[dr] & \\
\text{Ex-post house mon.} & & \text{Ex-ante pop. mon.}
\end{tikzcd}$$

LANDSCAPE

$$\begin{tikzcd}
& \text{Ex-post proportionality} \arrow[dl] \arrow[dr] & \\
\text{Ex-ante proportionality} \arrow[dr] & & \text{Ex-post quota} \arrow[dl] \\
& \text{Ex-ante quota} &
\end{tikzcd}$$

$$\begin{tikzcd}
& \text{Ex-post pop. mon.} \arrow[dl] \arrow[dr] & \\
\text{Ex-post house mon.} & & \text{Ex-ante pop. mon.}
\end{tikzcd}$$

LANDSCAPE

Ex-post proportionality

Ex-post pop. mon.

Ex-ante proportionality

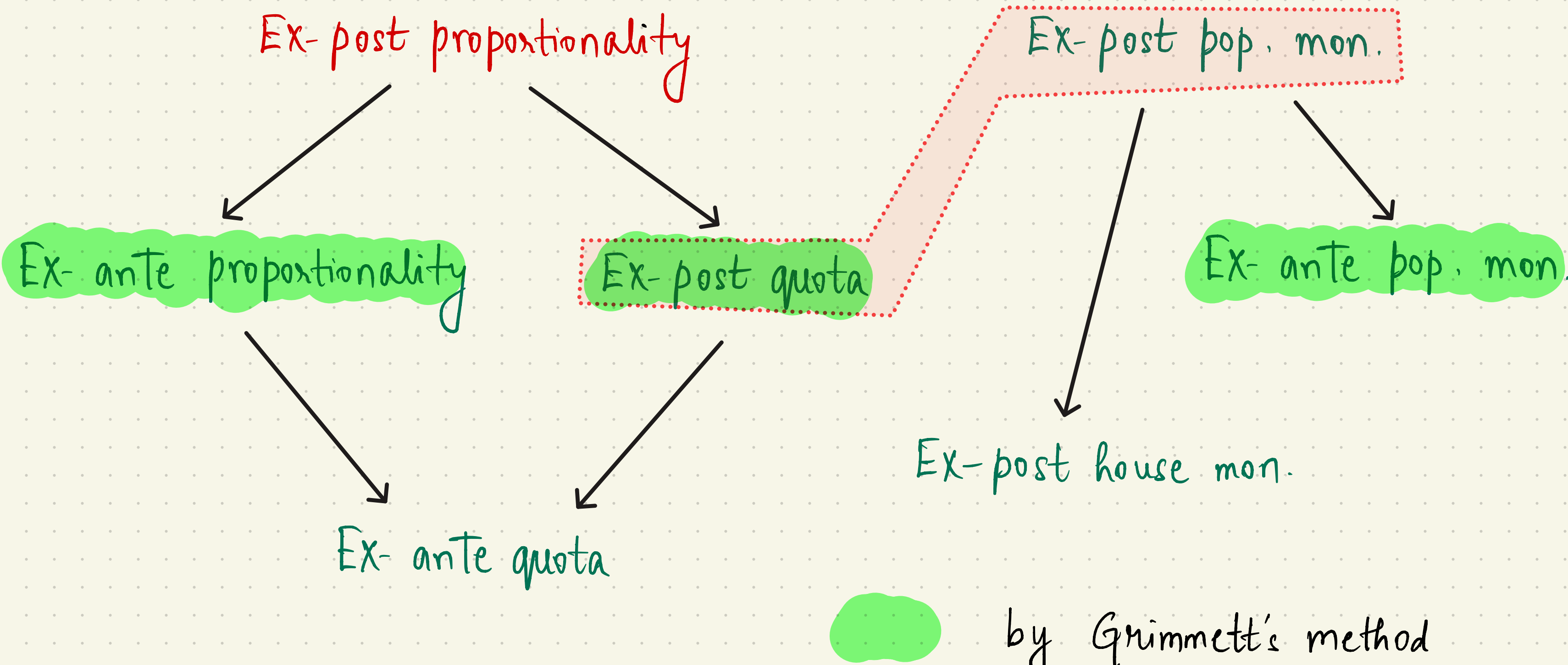
Ex-post quota

Ex-ante pop. mon.

Ex-post house mon.

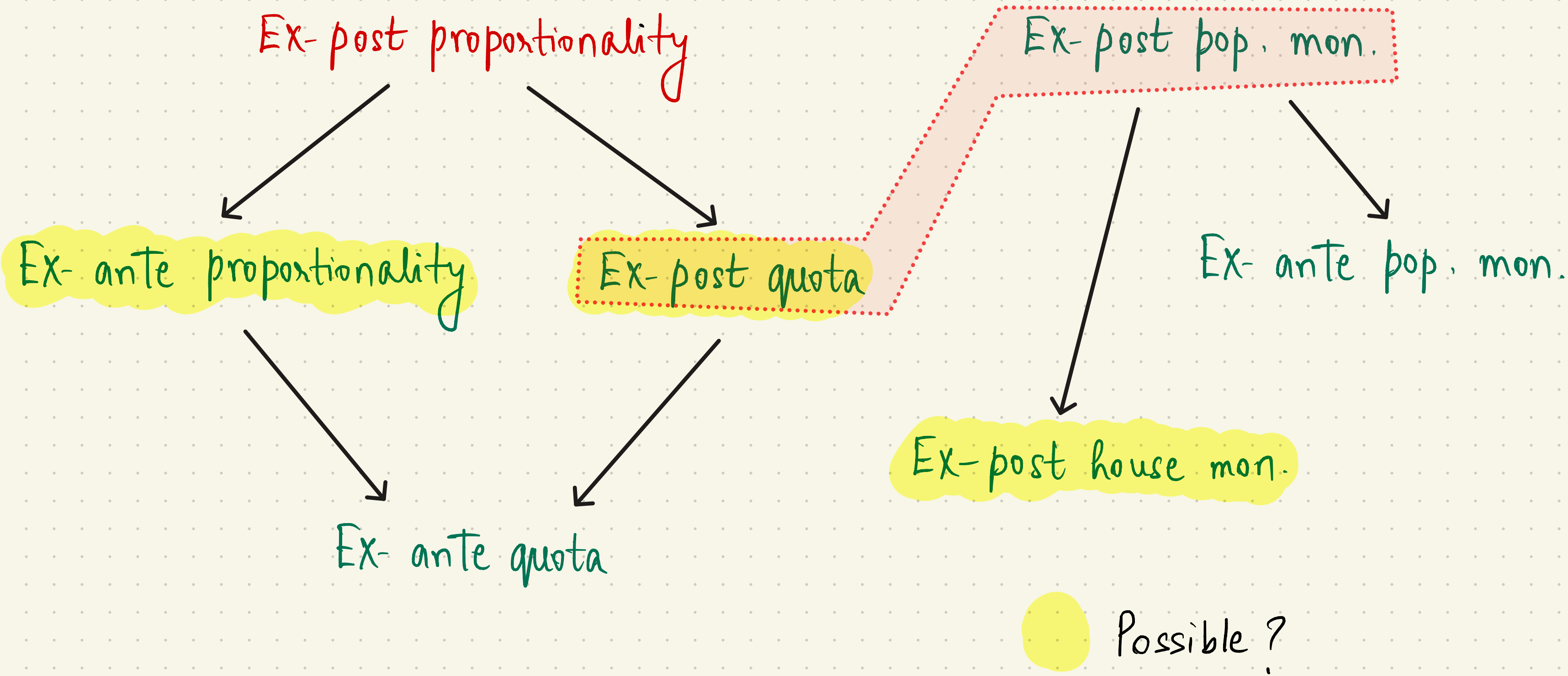
Ex-ante quota

LANDSCAPE



Ex-post proportionality
Ex-post pop. mon.
Ex-ante proportionality
Ex-post quota
Ex-ante pop. mon.
Ex-post house mon.
Ex-ante quota
by Grimmett's method

LANDSCAPE


Ex-post proportionality
Ex-post pop. mon.
Ex-ante proportionality
Ex-post quota
Ex-ante pop. mon.
Ex-post house mon.
Ex-ante quota
Possible ?

GRIMMETT'S METHOD FAILS HOUSE MONOTONICITY

GRIMMETT'S METHOD FAILS HOUSE MONOTONICITY

$h = 2$ Four states with populations $(1, 2, 1, 2)$.

Identity permutation and $U > \frac{2}{3}$ $\Rightarrow$ "toxic" assignment

Grimmett returns the seat allocation $(1, 0, 1, 0)$.

Ex-post house monotonicity requires that for $h = 3$,

at least one out of state 2 or state 4 still receives 0 seats.

But quota compliance requires both states to receive exactly

one seat when $h = 3$.

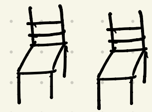
Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



Careful Sliding Method

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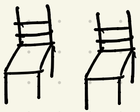
0.1 0.1 0.3 0.7 0.8



sorted residues

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

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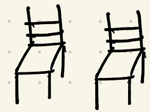


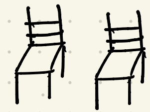
sorted residues

Suppose lower quotas $\lfloor q_i \rfloor$ are already assigned.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8

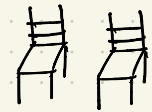


Consider assigning a seat each to #seats-many states with the lowest quotas.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



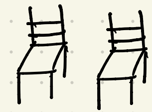
Consider assigning a seat each to #seats-many states with the lowest quotas.

$\Rightarrow$ selecting the seat assignment $(1, 1, 0, 0, 0)$ with prob. 0.1 meets ex-ante proportionality for state 1.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8

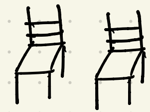


Proposed seat assignment	Probability needed	Probability available	Decision

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

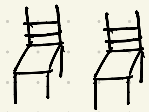


Proposed seat assignment	Probability needed	Probability available	Decision
1 1 0 0 0	?		

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8

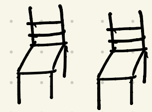


Proposed seat assignment	Probability needed	Probability available	Decision
1 1 0 0 0	0·1		

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

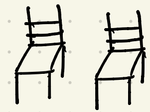


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	?	

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



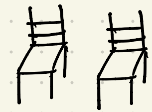
Proposed seat assignment	Probability needed	Probability available	Decision
1 1 0 0 0	0.1	0.2	

Total probability of all seat assignments where state 5 is rounded down $= 1 - 0.8 = 0.2$

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

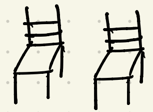


Proposed seat assignment	Probability needed		Probability available		Decision
1 1 0 0 0	0.1	$<$	0.2	$\Rightarrow$	0.1

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8

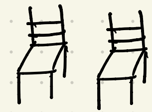


Proposed seat assignment	Probability needed	Probability available	Decision
1 1 0 0 0	0·1	0·2	0·1

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



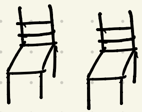
saturated!

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



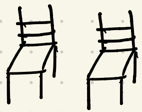
Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1

Resume from the next lowest-residue unsaturated state.

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

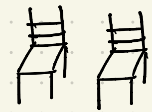


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0			

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

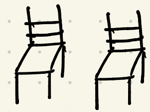


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



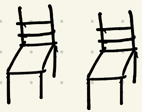
Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1   0.1   0.3   0.7   0.8

Saturated

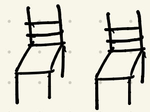


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

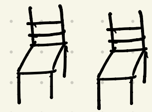


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	?		

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8



Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	?	

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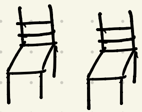

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	

Total probability of all seat assignments where state 5 is rounded down $= 1 - 0.8 = 0.2$

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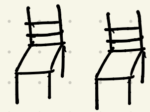

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	?

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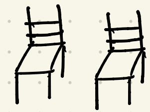


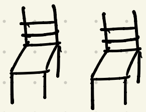
NOT saturated

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8



NOT saturated

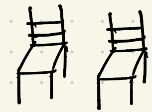
Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0·1	0·2	0·1
0	1	1	0	0	0		0
0	0	1	1	0	0·3	0·1	0·1

State 5 must be rounded up in ALL subsequent seat assignments

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1   0.1   0.3   0.7   0.8



NOT saturated

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1

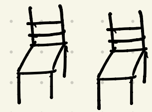
State 5 must be rounded up in ALL subsequent seat assignments

$\Longrightarrow$ Reduced problem with one less seat.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8



Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0·1	0·2	0·1
0	1	1	0	0	0		0
0	0	1	1	0	0·3	0·1	0·1
0	0	1	0	1			

Careful Sliding Method

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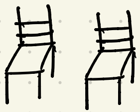

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1
0	0	1	0	1	0.2		

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

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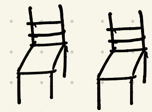
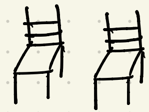


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1
0	0	1	0	1	0.2	0.2	

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0.1 0.1 0.3 0.7 0.8

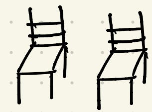


Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1
0	0	1	0	1	0.2	0.2	0.2

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

0·1 0·1 0·3 0·7 0·8



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1	1	0	0	0	0·1	0·2	0·1
0	1	1	0	0	0		0
0	0	1	1	0	0·3	0·1	0·1
0	0	1	0	1	0·2	0·2	0·2
0	0	0	1	1			

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

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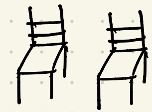

Proposed seat assignment					Probability needed	Probability available	Decision
1	1	0	0	0	0.1	0.2	0.1
0	1	1	0	0	0		0
0	0	1	1	0	0.3	0.1	0.1
0	0	1	0	1	0.2	0.2	0.2
0	0	0	1	1	0.6	0.6	0.6

Careful Sliding Method

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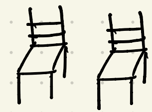

Resulting distribution is :

1	1	0	0	0	with probability 0.1
0	0	1	1	0	with probability 0.1
0	0	1	0	1	with probability 0.2
0	0	0	1	1	with probability 0.6

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

input : populations $p_1, p_2, \dots, p_n$ and house size h

output : a probability distribution over seat assignments

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[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

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output: a probability distribution over seat assignments

Assign each state i its lower quota $\lfloor q_i \rfloor$.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

input : populations $p_1, p_2, \ldots, p_n$ and house size h

output : a probability distribution over seat assignments

Assign each state i its lower quota $\lfloor q_i \rfloor$.

Let state i's residue be $r_i := q_i - \lfloor q_i \rfloor$, and let $k := \sum_i r_i$

denote the number of unassigned seats.

// WOLOG, $r_1 \leq r_2 \leq \cdots \leq r_n$.

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

Initialize all states as unsaturated.

Set unallocated probability $\alpha = 1$.

Set support size $\ell = 0$. // Initially, support is empty.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

// Consider the highest-residue unsaturated state, say j.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

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* If ex-ante proportionality for state j requires rounding up state j in all subsequent allocations, then :

Careful Sliding Method

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// Consider the highest-residue unsaturated state, say j.

* If ex-ante proportionality for state j requires rounding up state j in all subsequent allocations, then:

  - decrease k by 1
  - round up s_j in $(\ell+1)^{th}$, $(\ell+2)^{th}$, ... seat assignments.

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

// Consider the highest-residue unsaturated state, say j.

* If ex-ante proportionality for state j requires rounding up state j in all subsequent allocations, then:

  - decrease k by 1
  - round up s_j in $(\ell+1)^{th}$, $(\ell+2)^{th}$, ... seat assignments.

* Repeat until $\alpha = 0$ or the condition doesn't hold.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

* Round up the k lowest-residue unsaturated states. Call the seat assignment $\vec{s}_\ell$.

CAREFUL SLIDING METHOD

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* Round up the k lowest-residue unsaturated states. Call the seat assignment $\vec{s}_\ell$.

* Assign probability α_ℓ to $\vec{s}_\ell$. // α_ℓ is the maximum feasible probability such that $\alpha - \alpha_\ell$ is sufficient to meet ex-ante prop. for the highest-residue state.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

* Round up the k lowest-residue unsaturated states.
  Call the seat assignment $\vec{s}_\ell$.

* Assign probability α_ℓ to $\vec{s}_\ell$. // α_ℓ is the maximum feasible probability such that $\alpha - \alpha_\ell$ is sufficient to meet ex-ante prop. for the highest-residue state.

* Update $\ell \leftarrow \ell + 1$ and $\alpha \leftarrow \alpha - \alpha_\ell$.

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

* Round up the k lowest-residue unsaturated states. Call the seat assignment $\vec{s}_\ell$.

* Assign probability α_ℓ to $\vec{s}_\ell$. // α_ℓ is the maximum feasible probability such that $\alpha - \alpha_\ell$ is sufficient to meet ex-ante prop. for the highest-residue state.

* Update $\ell \leftarrow \ell + 1$ and $\alpha \leftarrow \alpha - \alpha_\ell$.

* If some state's ex-ante prop. is achieved, mark it as saturated and round it down in all subsequent seat assignments.

Careful Sliding Method

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

Repeat the checks for highest and lowest residue states until $\alpha = 0$

CAREFUL SLIDING METHOD

[Aziz, Lev, Mattei, Rosenschein, and Walsh, AIJ 2019]

Repeat the checks for highest and lowest residue states until $\alpha = 0$

Return the prob. distribution $(\vec{S_0}, \alpha_0)$, $(\vec{S_1}, \alpha_1)$, $(\vec{S_2}, \alpha_2)$, $\ldots$

Quiz

QUIZ

Under the Careful Sliding Method, can the RED states improve their chances of ex-post majority by losing population to BLUE states?

$h = 11$

	BEFORE	AFTER
State 1	110	110
State 2	270	270 (+20)
State 3	210	210
State 4	160	160 (+30)
State 5	70	70 (−60)
State 6	280	280 (+10)

NEXT LECTURE

Randomized apportionment via network flows