

# COL 8184 : ALGORITHMS FOR FAIR REPRESENTATION

## LECTURE 1

INTRODUCTION TO THE COURSE  
& THE APPORTIONMENT PROBLEM

JAN 05, 2026

|

ROHIT VAISH

# WHAT IS THIS COURSE ABOUT ?



Procedures for collective decision-making



Voting



## Voting

voter 1 : A > B > C

voter 2 : C > B > A

voter 3 : A > B > C

voter 4 : B > A > C



## Voting

voter 1 : A > B > C

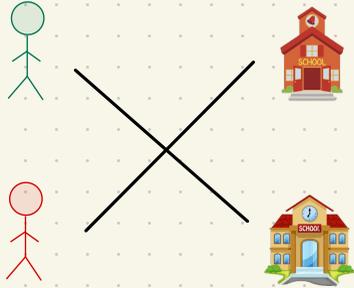
voter 2 : C > B > A

voter 3 : A > B > C

voter 4 : B > A > C



## Matching



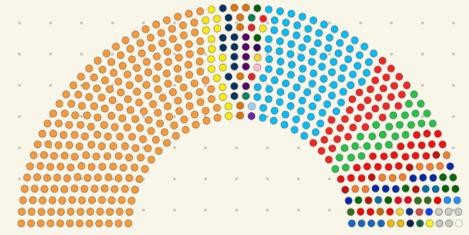
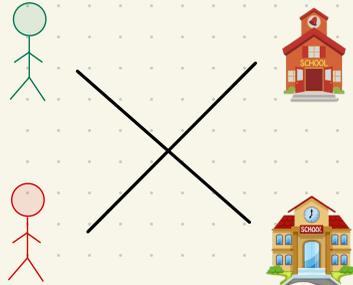


## Voting

Voter 1 : A > B > C  
Voter 2 : C > B > A  
Voter 3 : A > B > C  
Voter 4 : B > A > C



## Matching



## Apportionment

543 Lok Sabha seats

UP : 80

Maharashtra : 48

West Bengal : 42

Delhi : 7

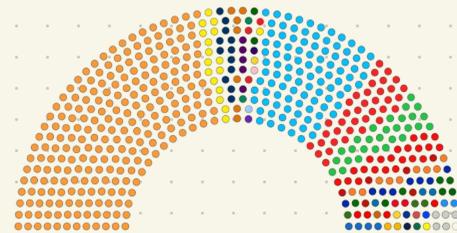
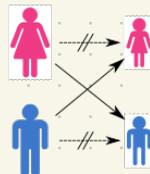


Voting

amazon



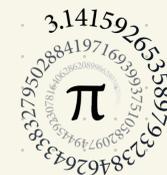
Matching



Apportionment



European Parliament

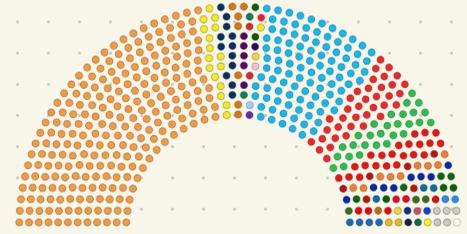




Voting



Matching



Apportionment

Fairness is paramount!

# UNFAIRNESS IN VOTING

# UNFAIRNESS IN VOTING

voter 1 : A > B > C

voter 2 : A > C > B

voter 3 : A > B > C

voter 4 : B > A > C

voter 5 : C > B > A

# UNFAIRNESS IN VOTING

voter 1 : A > B > C

voter 2 : A > C > B

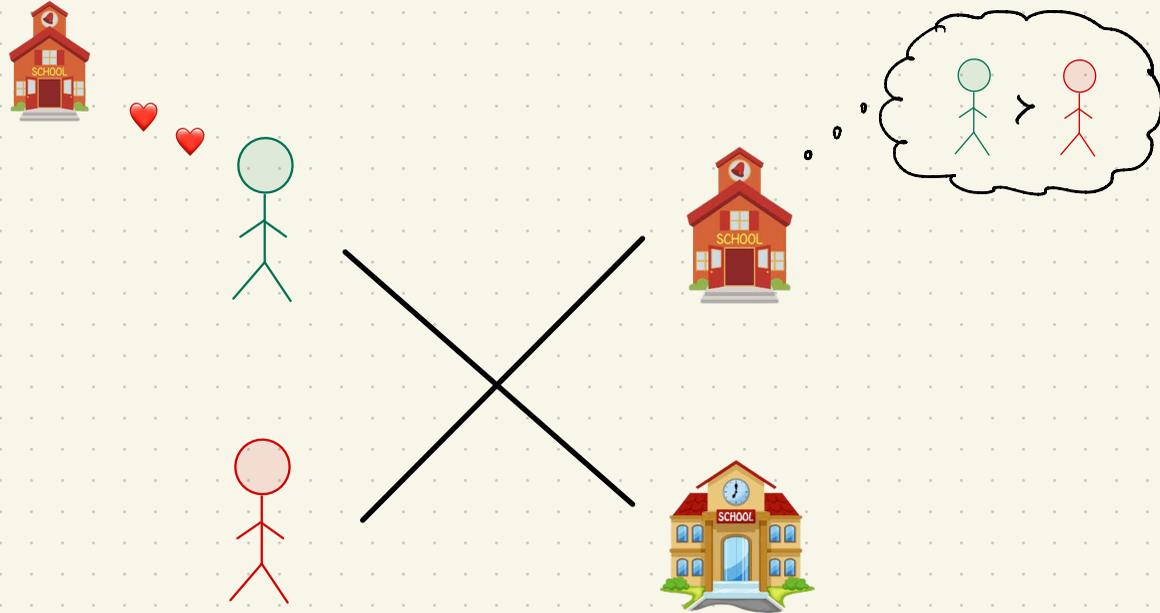
voter 3 : A > B > C

voter 4 : B > A > C

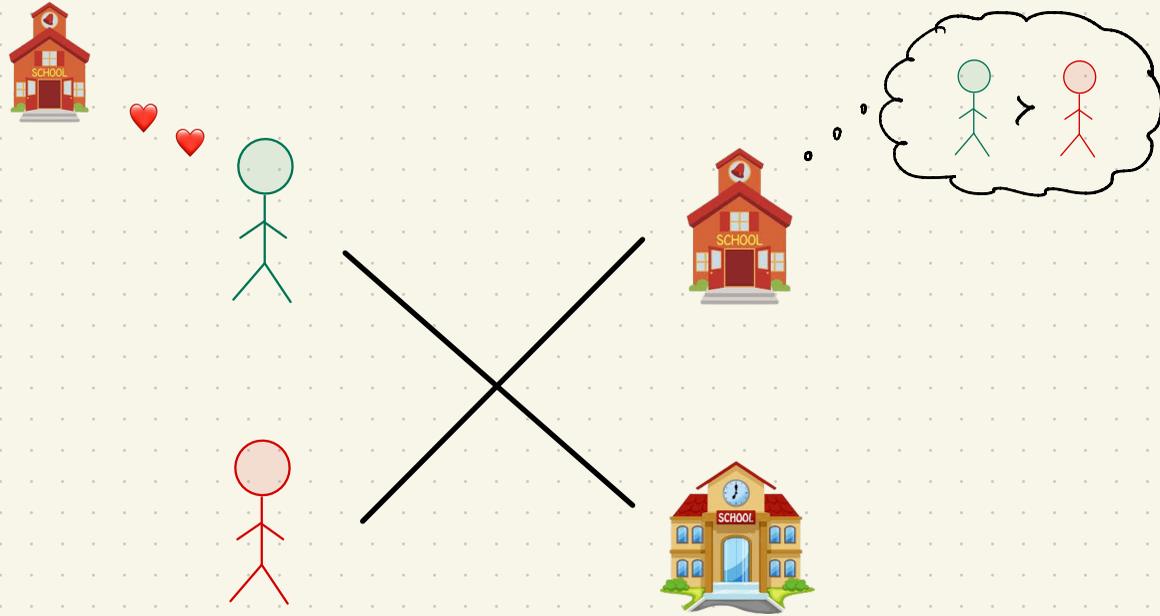
voter 5 : C > B > A

Choosing B as election winner seems unfair  
as majority prefer A over B

# UNFAIRNESS IN MATCHING



# UNFAIRNESS IN MATCHING



has justified envy towards



# UNFAIRNESS IN RESOURCE ALLOCATION

Suppose house size = 2.

State	Population	Seats in parliament
A	2000	2
B	1000	0 
C	1000	0 
	<hr/> 4000 <hr/>	

# GOALS OF THIS COURSE

- \* A mathematical view of societal problems
- \* Provable and rigorous demonstration of fairness
- \* Efficient algorithmic solutions
- \* Learn about historical development of these topics

# WHY SHOULD YOU DO THIS COURSE?

- \* Gain awareness of applications of computer science to problems in economics and social sciences
- \* Research experience
- \* Challenging and practically-relevant problems

# LOGISTICS

# LOGISTICS

INSTRUCTOR : ROHIT VAISH (Please call me Rohit)

TA : PAYAS KHURANA

LECTURES : Mon, Thurs 2-3:20 PM LH 614

OFFICE HOUR: Thurs 4-5 PM Bharti 428

# COURSE WEBPAGE



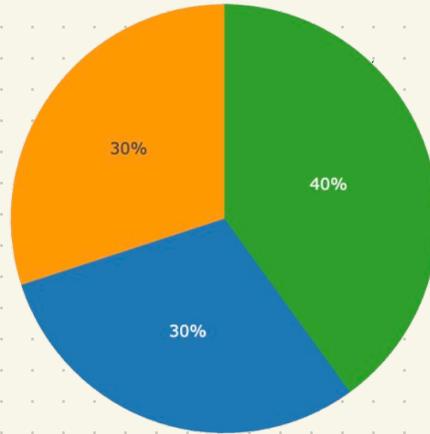
# TEAMS CHANNEL



# EVALUATION

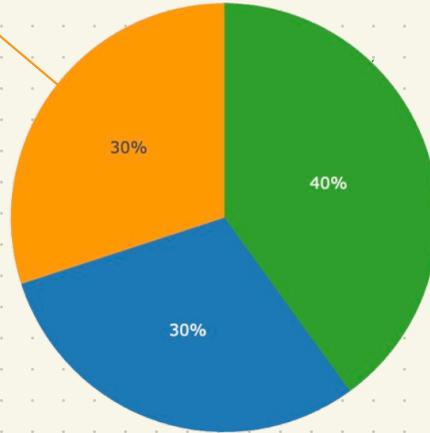
No minor , No major

# EVALUATION



# EVALUATION

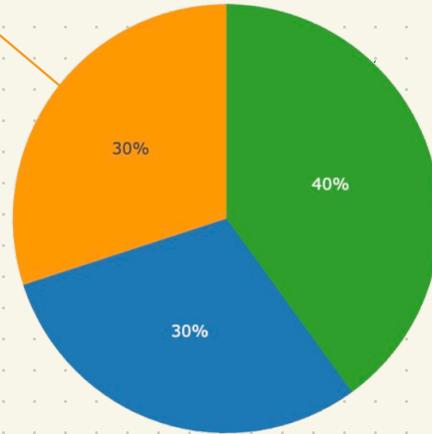
In-class quizzes



# EVALUATION

## In-class quizzes

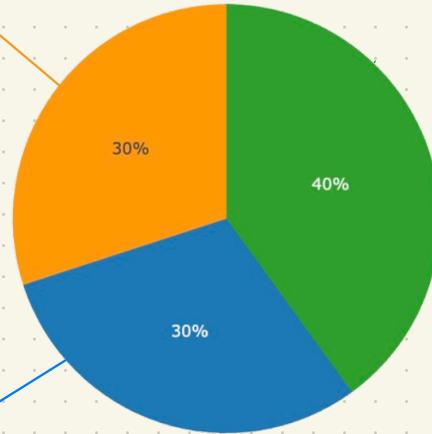
- \* ~ 10 mins / lecture
- \* worst three scores dropped



# EVALUATION

In-class quizzes

- \* ~ 10 mins / lecture
- \* worst three scores dropped

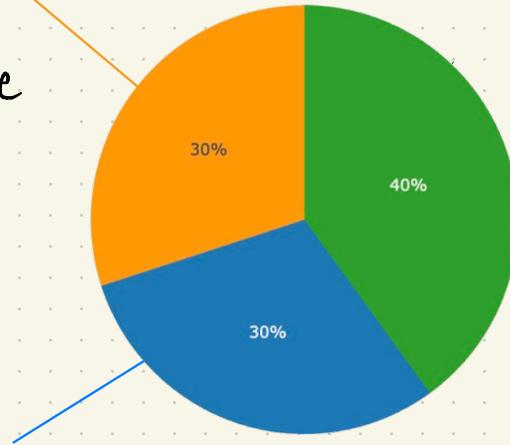


Assignments [3 x 10%]

# EVALUATION

## In-class quizzes

- \* ~ 10 mins / lecture
- \* worst three scores dropped



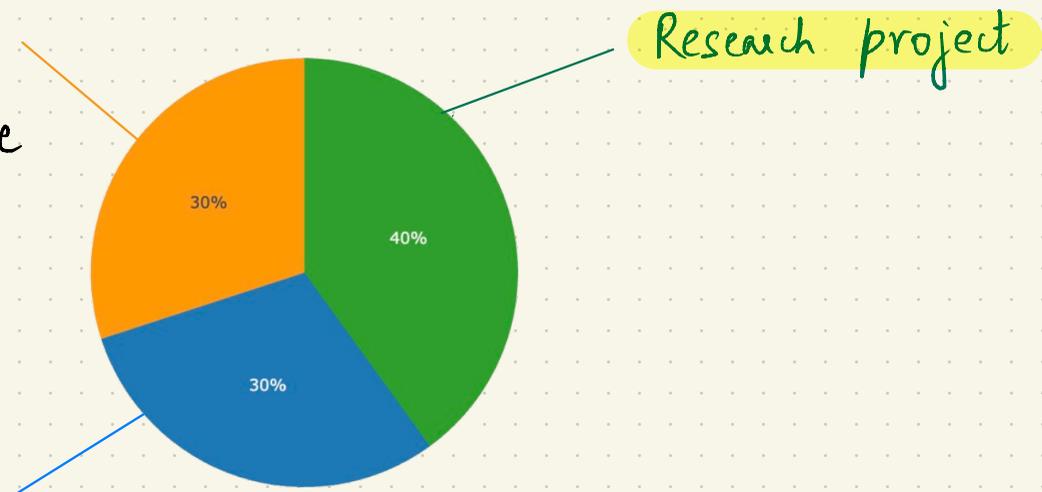
## Assignments [3 x 10%]

- \* ~ 4 hrs on three Sundays

# EVALUATION

In-class quizzes

- \* ~ 10 mins / lecture
- \* worst three scores dropped



Research project

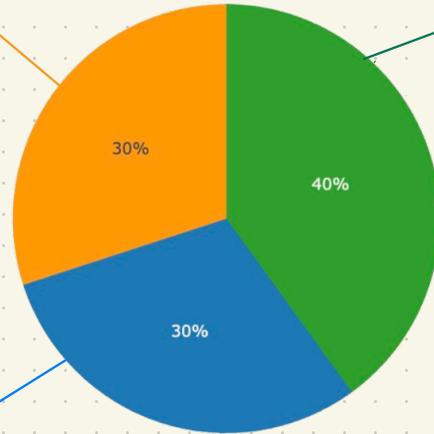
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# EVALUATION

## In-class quizzes

- \* ~ 10 mins / lecture
- \* worst three scores dropped



## Research project

- \* Read papers
- \* Develop your own research questions / open problems
- \* Contribute new insights

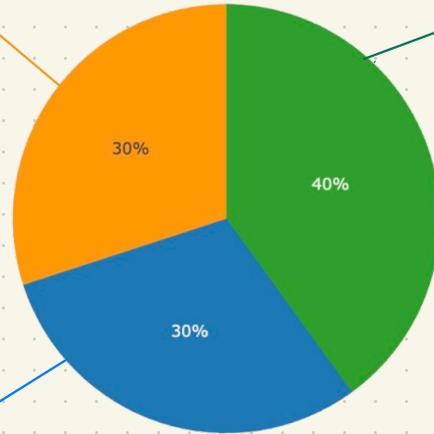
## Assignments [3 x 10%]

- \* ~ 4 hrs on three Sundays

# EVALUATION

## In-class quizzes

- \* ~ 10 mins / lecture
- \* worst three scores dropped



## Research project

- \* Read papers
- \* Develop your own research questions / open problems
- mid-term presentation
- \* Contribute new insights
- end-term presentation + report

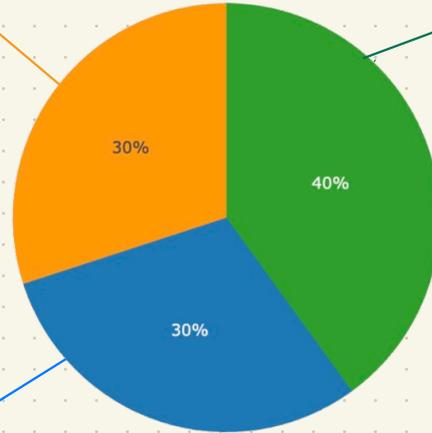
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# EVALUATION

In-class quizzes

Research project

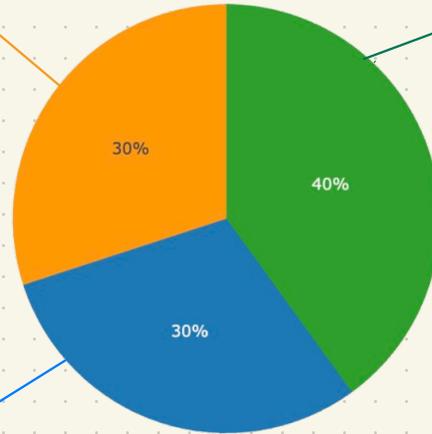


Assignments [3 x 10%]

# EVALUATION

In-class quizzes

Research project 



 Assignments [3 x 10%]

---

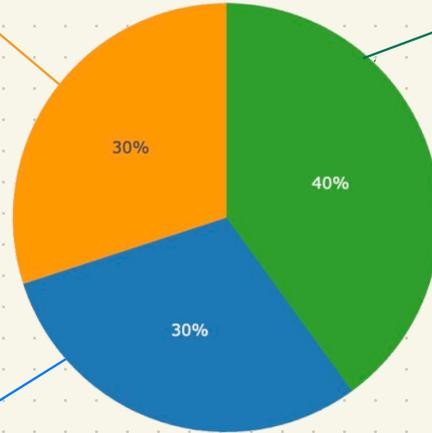
Audits : Not allowed

Attendance : Not recorded [ will use quizzes + assignments  
if required by the institute ]

# RE-EVALUATION

In-class quizzes

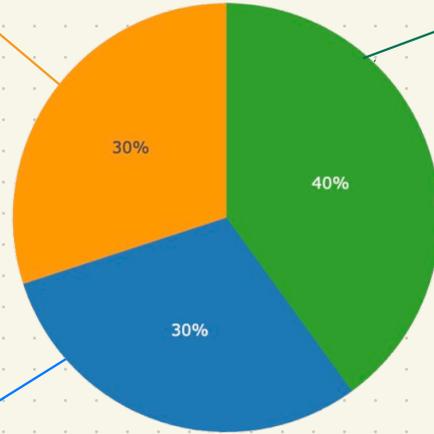
Research project 



Assignments  [3 x 10%]

# RE-EVALUATION

In-class quizzes  
no make-up quizzes  
(must 3 scores dropped)



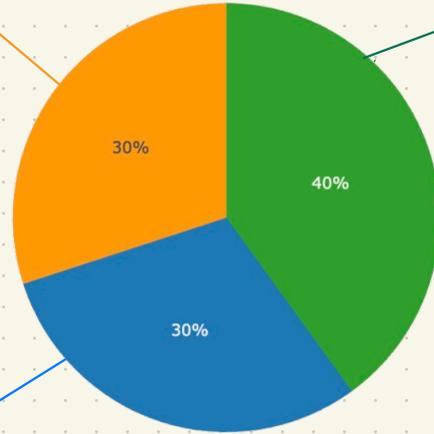
 Research project  
no make-up presentations

 Assignments [3 x 10%]  
no make-up assignments

No minor / major  
⇒ No reminor / re-major

# RE-EVALUATION

In-class quizzes  
no make-up quizzes  
(hurst 3 scores dropped)



 Research project  
no make-up presentations

 Assignments [3 x 10%]  
no make-up assignments

No minor / major  
⇒ No reminor / remajor

If you anticipate missing an evaluation, inform me in advance.

# REGRADING

\* Gradescope : Sign up **only** with @ dept.iitd, **not** @ iitd

\* "Frivolous" requests : **-2%** in overall score for first one  
Subsequently **-4%**, **-8%**, **-16%**, ...

If in doubt, talk to me.

# PLAGIARISM

Any kind, at any stage : F grade

If in doubt, talk to me.

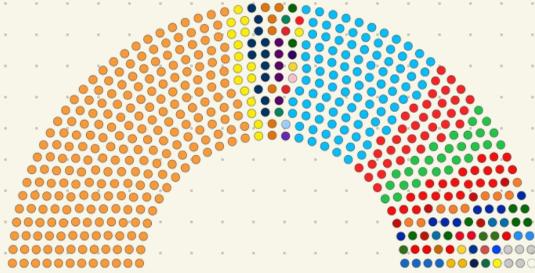
# COMMUNICATION

\* Most important!

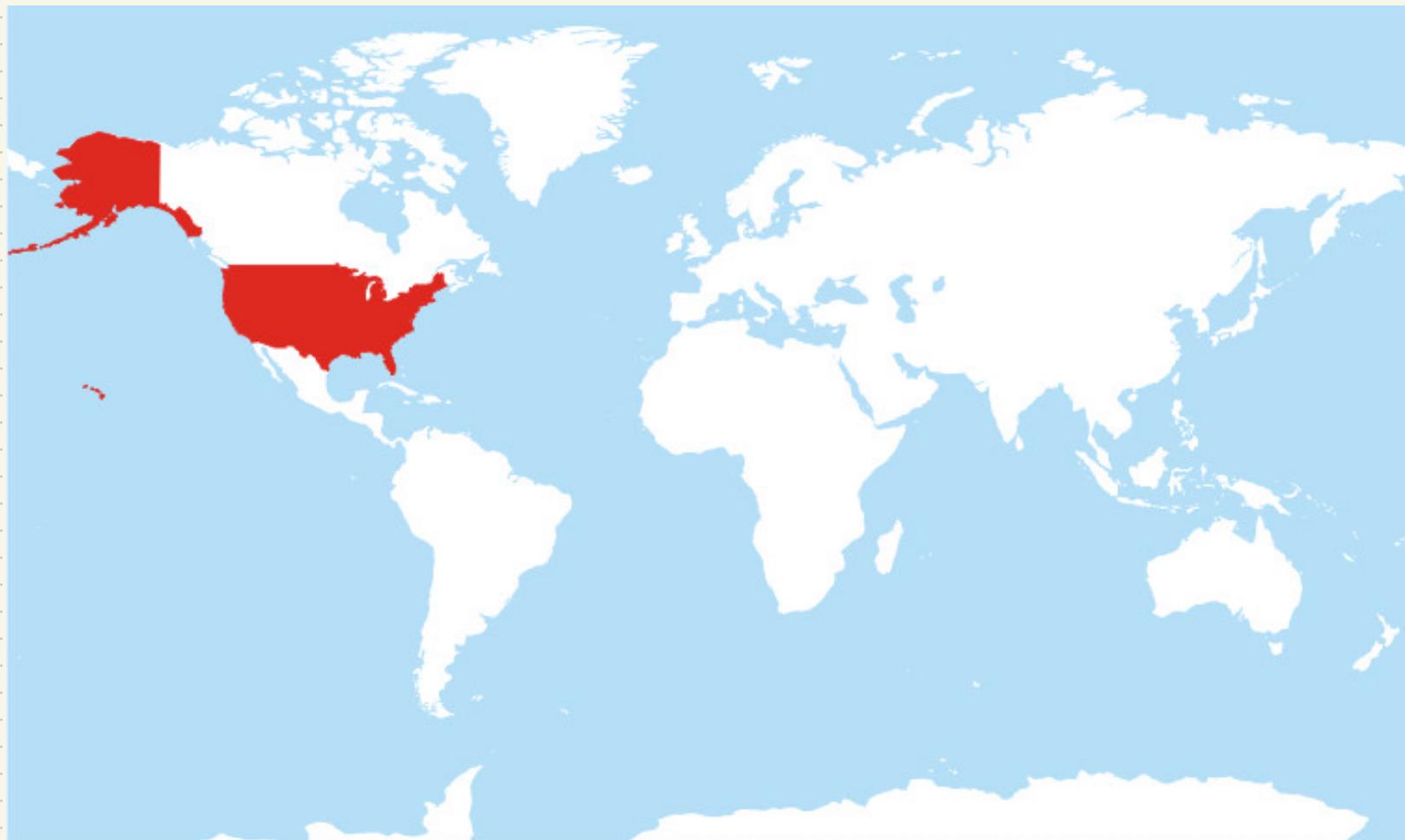
\* Please reach out to me about ANY issues anytime.

\* I hope you will enjoy learning about fair algorithms.

# PART I



# APPORTIONMENT



# We the People

of the United States, in order to form a more perfect Union, establish Justice, insure domestic Tranquillity, provide for the common Defence, promote the general Welfare, and secure the Blessings of Liberty to ourselves and our Posterity, do ordain and establish this Constitution for the United States of America.

## Article 1.

Section 1. All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

Section 2. The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature.

No Person shall be a Representative who shall not have attained to the Age of twenty five Years, and been seven Years a Citizen of the United States, and who shall not, when elected, be an Inhabitant of that State in which he shall be chosen.

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons. The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. The Number of Representatives shall not exceed one for every thirty thousand, but each State shall have at Least one Representative; and until such Enumeration shall be made, the State of New Hampshire shall be entitled to chuse three, Massachusetts eight, Rhode Island and Providence Plantations one, Connecticut five, New York six, New Jersey four, Pennsylvania eight, Delaware one, Maryland six, Virginia ten, North Carolina five, South Carolina five, and Georgia three.

US Constitution , Article 1 , Section 2 , Clause 3

Came into force on March 4<sup>th</sup>, 1789

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① Proportionality is the ideal

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② Basis of proportionality:  
Representative population

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③ Periodic revisions (census) to account for shifts in population

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③ Periodic revisions (census) to account for shifts in population

④ Upper and lower bounds on no. of representatives

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③ Periodic revisions (census) to account for shifts in population

④ Upper and lower bounds on no. of representatives

Missing: Exact no. of representatives & Method of apportionment  
(the "how many") (the "how")

# THE MODEL

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\*  $n$  states with populations  $p_1, p_2, \dots, p_n$  ( $p_i > 0$ )

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- \* House size  $h \geq 0$

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Goal: Assign seats  $s_1, s_2, \dots, s_n \in \mathbb{N}_{\geq 0}$  such that

$$s_1 + s_2 + \dots + s_n = h.$$

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$$s_1 + s_2 + \dots + s_n = h.$$

Apportionment method:  $(h, p_1, p_2, \dots, p_n) \mapsto (s_1, s_2, \dots, s_n)$

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 $s_1 + s_2 + \dots + s_n = h$ . (apportionment method)

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$$s_1 + s_2 + \dots + s_n = h. \quad (\text{apportionment method})$$

\* Total population  $P = p_1 + p_2 + \dots + p_n$

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\* Total population  $P = p_1 + p_2 + \dots + p_n$

\* Standard quota of state  $i$ :  $q_i = \frac{p_i}{P} \times h$  (entitlement)

# THE MODEL

\*  $n$  states with populations  $p_1, p_2, \dots, p_n$  ( $p_i > 0$ )

\* House size  $h \geq 0$

Goal: Assign seats  $s_1, s_2, \dots, s_n \in \mathbb{N}_{\geq 0}$  such that

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\* Total population  $P = p_1 + p_2 + \dots + p_n$

\* Standard quota of state  $i$ :  $q_i = \frac{p_i}{P} \times h$  (entitlement)

\* Upper quota:  $\lceil q_i \rceil$ , lower quota =  $\lfloor q_i \rfloor$

# NAIVE ROUNDING FAILS

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House size  $h = 10$

State	Population $p_i$
-------	------------------

A	36
---	----

B	37
---	----

C	27
---	----

---

$$P = 100$$

---

# NAIVE ROUNDING FAILS

House size  $h = 10$

State	Population $p_i$	Quota $q_i$
-------	------------------	-------------

A	36	
---	----	--

B	37	?
---	----	---

C	27	
---	----	--

---

$$P = 100$$

---

# NAIVE ROUNDING FAILS

House size  $h = 10$

State	Population $p_i$	Quota $q_i$
A	36	3.6
B	37	3.7
C	27	2.7
	<hr/>	<hr/>
	$P = 100$	10
	<hr/>	<hr/>

# NAIVE ROUNDING FAILS

House size  $h = 10$

State	Population $p_i$	Quota $q_i$	Naive Rounding
A	36	3.6	
B	37	3.7	?
C	27	2.7	
	<hr/> $P = 100$ <hr/>	<hr/> 10 <hr/>	

# NAIVE ROUNDING FAILS

House size  $h = 10$

State	Population $p_i$	Quota $q_i$	Naive Rounding
A	36	3.6 ↑	4
B	37	3.7 ↑	4
C	27	2.7 ↑	3
	<hr/> $P = 100$ <hr/>	<hr/> 10 <hr/>	<hr/> 11 <hr/>

---

US Constitution enacted



A horizontal black line represents a timeline. A vertical tick mark is positioned on the left side of the line. A thin black line extends upwards from this tick mark to the text 'US Constitution enacted'. Below the tick mark, the date 'March 04' is written in black text on a yellow highlight.

March 04

1789

US Constitution enacted

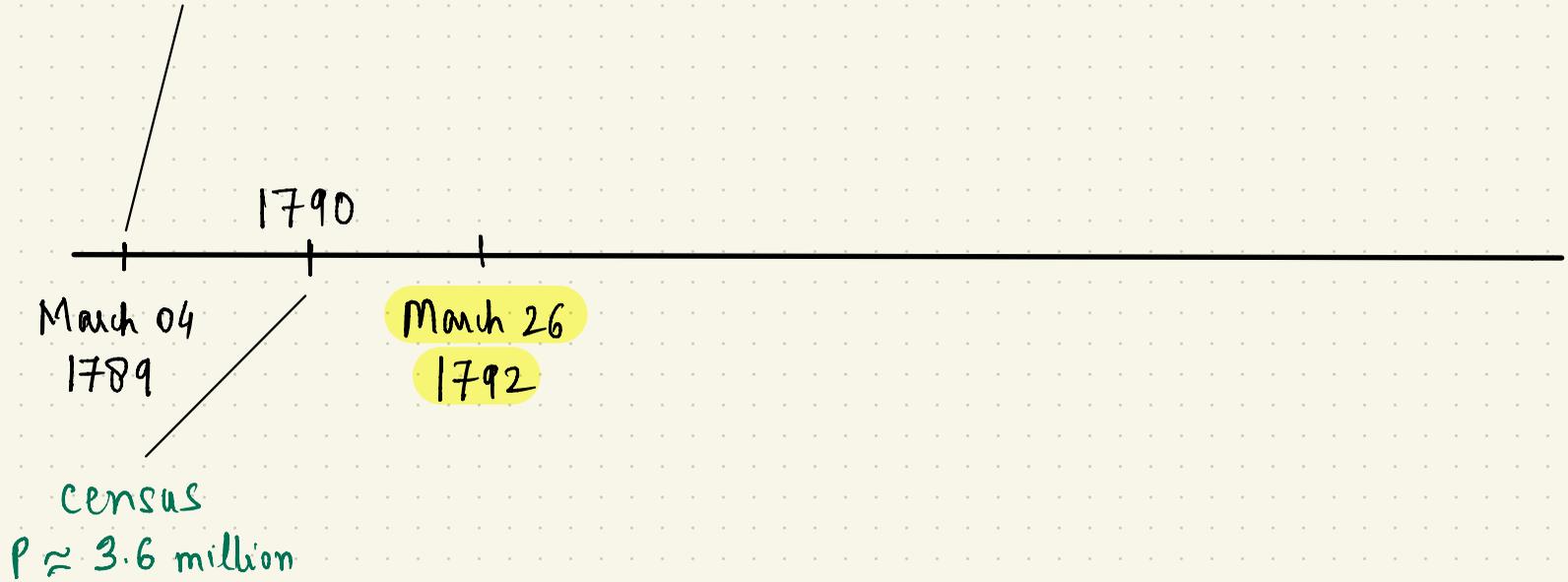
1790

March 04  
1789

census  
 $P \approx 3.6$  million



US Constitution enacted



US Constitution enacted

Congress passes a bill proposing  
the use of Hamilton's method

1790

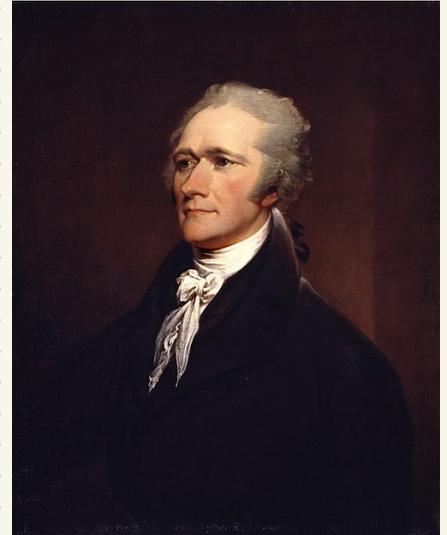
March 04  
1789

March 26  
1792

census  
 $P \approx 3.6$  million

# ALEXANDER HAMILTON

- \* One of the Founding Fathers of US
- \* First US Secretary of Treasury
- \* Founded US's first political party
- \* Subject of a renowned musical



1755 - 1804

# HAMILTON'S METHOD

(aka method of largest remainder)

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\* Assign each state  $i$  its lower quota  $\lfloor q_i \rfloor$ .

# HAMILTON'S METHOD

(aka method of largest remainder)

- \* Assign each state  $i$  its lower quota  $\lfloor q_i \rfloor$ .
- \* Assign remaining seats one at a time to the state with the largest remainder  $r_i = q_i - \lfloor q_i \rfloor$ .

# HAMILTON'S METHOD EXAMPLE

# HAMILTON'S METHOD EXAMPLE

House size  $h = 10$

State	Population $p_i$
-------	------------------

A	198
---	-----

B	399
---	-----

C	403
---	-----

---

$$P = 1000$$

---

# HAMILTON'S METHOD EXAMPLE

House size  $h = 10$

State	Population $p_i$	Quota $q_i$
A	198	1.98
B	399	3.99
C	403	4.03
	<hr/>	<hr/>
	$P = 1000$	10
	<hr/>	<hr/>

# HAMILTON'S METHOD EXAMPLE

House size  $h = 10$

State	Population $p_i$	Quota $q_i$	Hamilton
A	198	1.98	
B	399	3.99	?
C	403	4.03	
	<hr/> $P = 1000$ <hr/>	<hr/> 10 <hr/>	<hr/> <hr/>

# HAMILTON'S METHOD EXAMPLE

House size  $h = 10$

State	Population $p_i$	Quota $q_i$	Hamilton
A	198	1.98	1
B	399	3.99	3
C	403	4.03	4
	<hr/> $P = 1000$ <hr/>	<hr/> 10 <hr/>	<hr/> 8 <hr/>

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# HAMILTON'S METHOD EXAMPLE

House size  $h = 10$

State	Population $p_i$	Quota $q_i$	Hamilton
A	198	1.98 ←	<del>X</del> 2
B	399	3.99	<del>X</del> 4
C	403	4.03	4
	<hr/> $P = 1000$ <hr/>	<hr/> 10 <hr/>	<hr/> <del>X</del> <del>X</del> 10 <hr/>

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Done!

Census of 1790. 120 seats to be allocated, 30,000 citizens per representative

---

<i>State</i>	<i>Population</i>	<i>Seats allocated</i>		
		<i>“Raw”</i>	<i>Initial</i>	<i>Final</i>
Connecticut*	236,841	7.895	7	8
Delaware*	55,540	1.851	1	2
Georgia	70,835	2.361	2	2
Kentucky	68,705	2.290	2	2
Maryland	278,514	9.284	9	9
Massachusetts*	475,327	15.844	15	16
New Hampshire*	141,822	4.727	4	5
New Jersey*	179,570	5.986	5	6
New York	331,589	11.053	11	11
North Carolina*	353,523	11.784	11	12
Pennsylvania	432,879	14.419	14	14
Rhode Island	68,446	2.282	2	2
South Carolina*	206,236	6.875	6	7
Vermont*	85,533	2.851	2	3
Virginia	630,560	21.019	21	21
Total	3,615,920	120.531	112	120

\*States receiving an additional seat after the initial allocation.

US Constitution enacted

Congress passes a bill proposing  
the use of Hamilton's method

1790

March 04  
1789

March 26  
1792

census  
 $P \approx 3.6$  million

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Bill becomes a law

1790

March 04  
1789

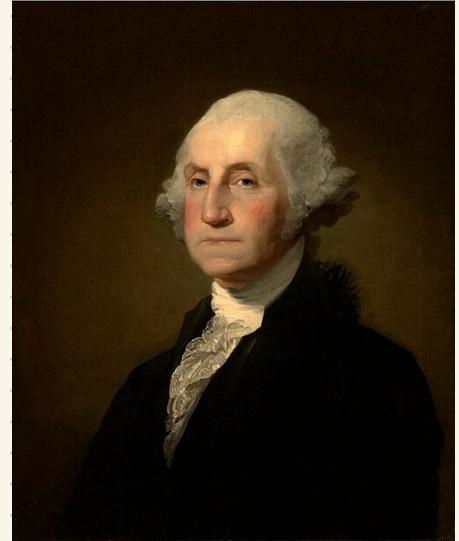
March 26  
1792

April 06  
1792

census  
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# GEORGE WASHINGTON

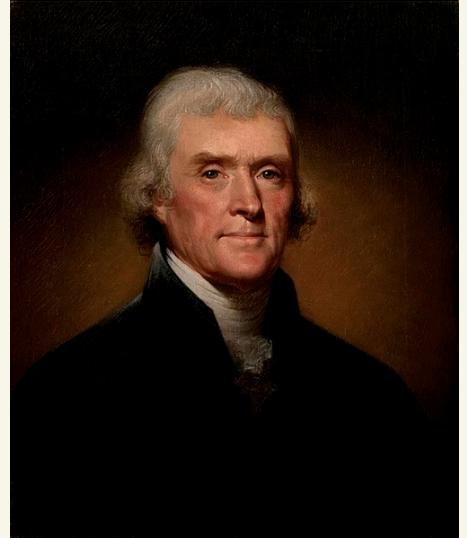
- \* First US President
- \* Known as "Father of the Nation"  
in US



1732 - 1799

# THOMAS JEFFERSON

- \* First Secretary of State  
(under George Washington)
- \* Second Vice - President  
(under John Adams)
- \* Third President



1743 - 1826

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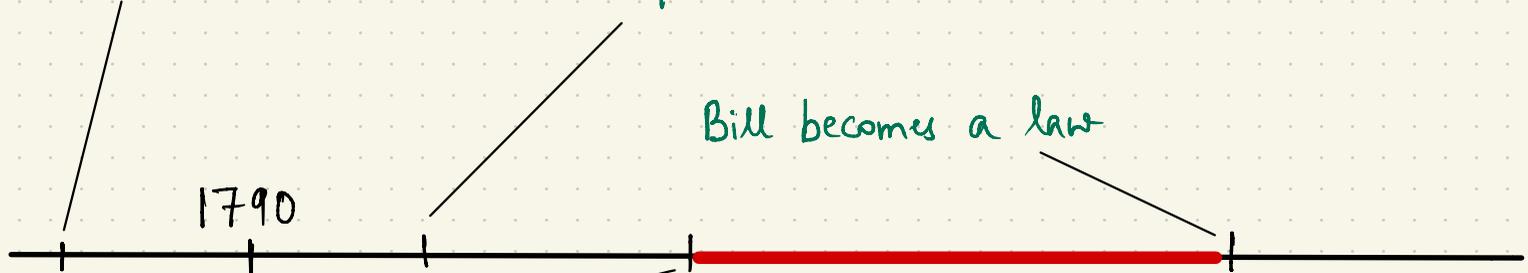
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GW consults Jefferson  
and Randolph  
(both reject Hamilton)



Census of 1790. 120 seats to be allocated, 30,000 citizens per representative

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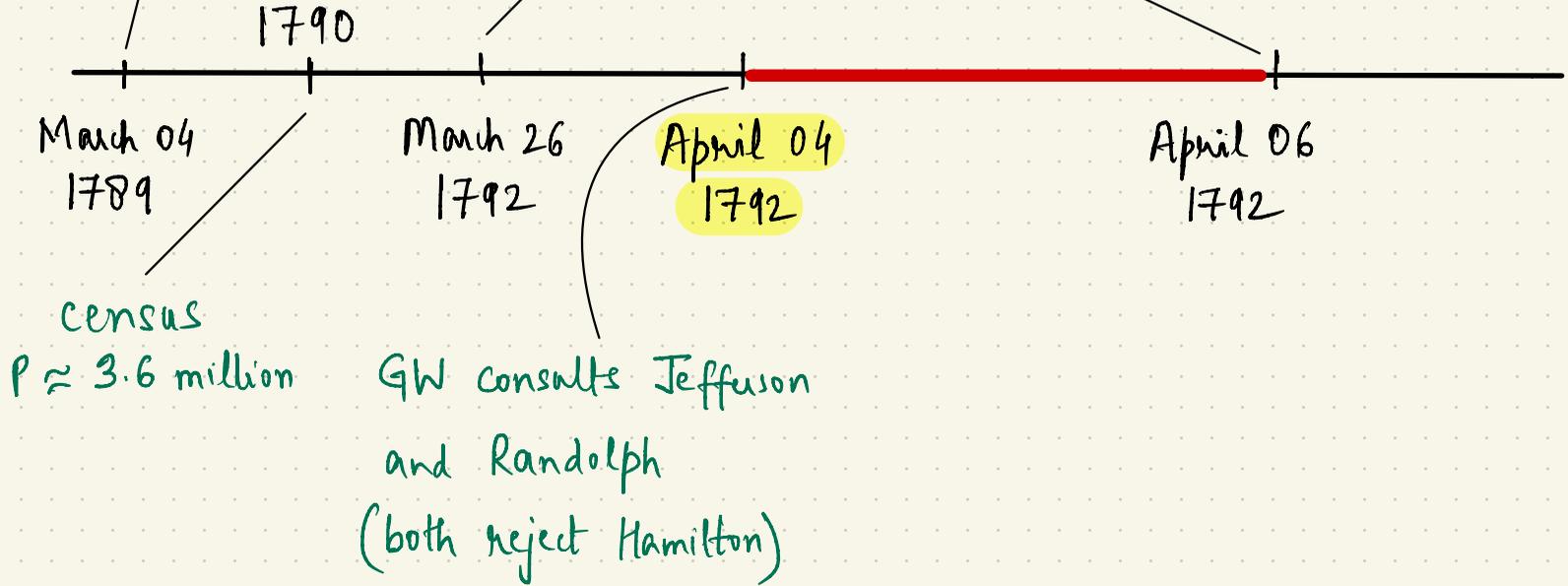
States gaining a seat (\*) have fewer than 30,000 people / representative

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons. The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. **The Number of Representatives shall not exceed one for every thirty Thousand**, but each State shall have at Least one Representative; and until such enumeration shall be made, the State of New Hampshire shall be entitled to chuse three, Massachusetts eight, Rhode Island and Providence Plantations one, Connecticut five, New-York six, New Jersey four, Pennsylvania eight, Delaware one, Maryland six, Virginia ten, North Carolina five, South Carolina five, and Georgia three.

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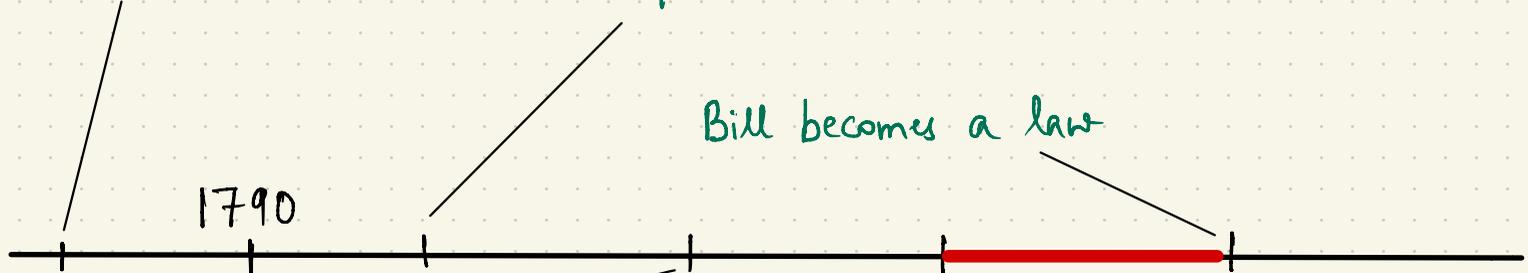
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GW vetoes Hamilton  
(first ever veto in US history)

*Gentlemen of the House of Representatives*

*I have maturely considered the Act passed by the two Houses, intituled, "An Act for an apportionment of Representatives among the several States according to the first enumeration," and I return it to your House, wherein it originated, with the following objections.*

*First—The Constitution has prescribed that representatives shall be apportioned among the several States according to their respective numbers: and there is no one proportion or divisor which, applied to the respective numbers of the States will yield the number and allotment of representatives proposed by the Bill.*

*Second—The Constitution has also provided that the number of Representatives shall not exceed one for every thirty thousand; which restriction is, by the context, and by fair and obvious construction, to be applied to the separate and respective numbers of the States: and the bill has allotted to eight of the States, more than one for thirty thousand.*

*George Washington.*