Economics and Computation @ Ashoka University

Lecture 2

Computational Barriers to Manipulation

Rohit Vaish (IIT Delhi)


Bucklin
Count
Voting
Random
Plurality
Minimax
Kemeny-Young
Winner
Copeland
Pairs
Ranked
Borda
Majority-Judgement
Range
Approval
Schulze

VOTING RULE

A mapping from preference profiles to candidates.

$$f: \begin{array}{cccc} v_1 & v_2 & & v_n \\ a & b & & c \\ b & c & \cdots & a \\ c & a & & b \end{array} \longrightarrow c$$

(also known as a *social choice function*)

One Rule to Rule Them All?

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Axiomatic Approach

- Formulate a set of "reasonable" properties
- Check if there is a voting rule that satisfies them

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Unique voting rule with
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Axiomatic Approach

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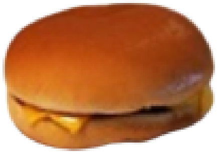
Expectation

Unique voting rule with
all the desirable properties

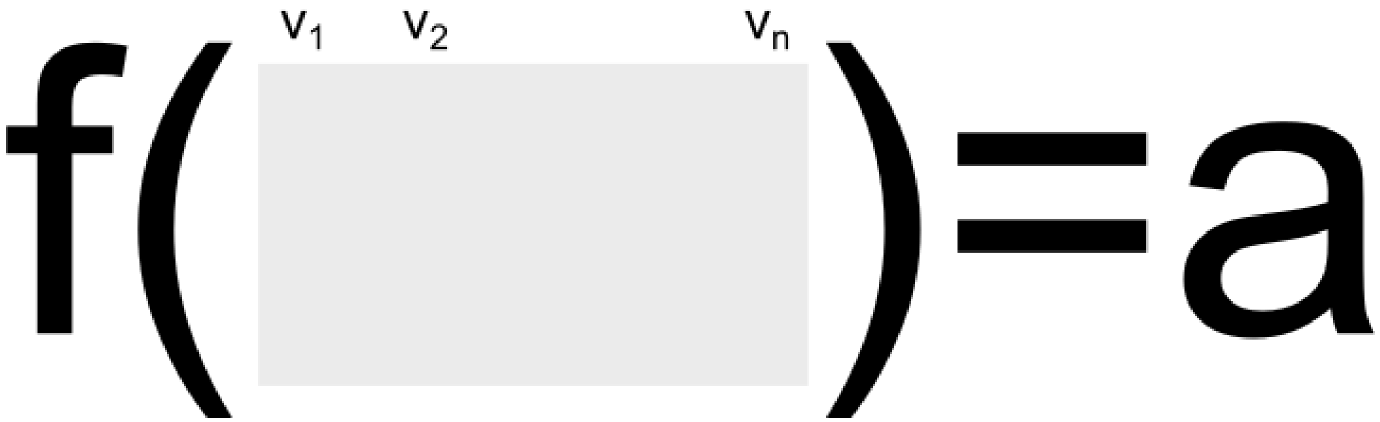
Reality

No such voting rule



ONTO

For any candidate "a", there exists a profile where "a" wins.



STRATEGYPROOF

No voter can improve by misreporting its preferences.

For any profile , any voter v_i, and any misreport , it must be that

An Obviously Bad Voting Rule

A voting rule is called a dictatorship if there exists a voter v_i such that for any preferences of the other voters, the voting outcome is the favorite candidate of voter v_i.

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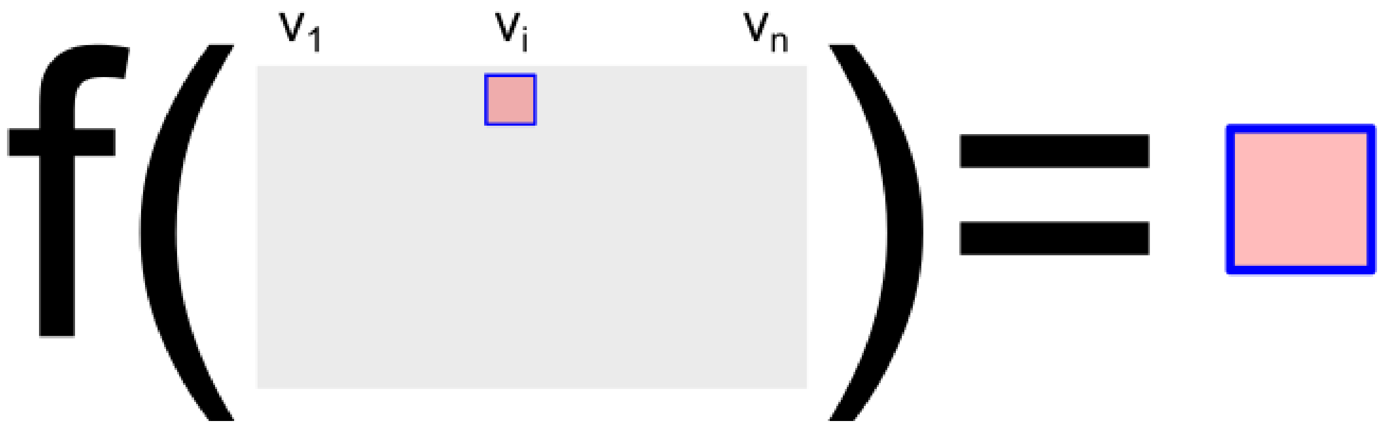

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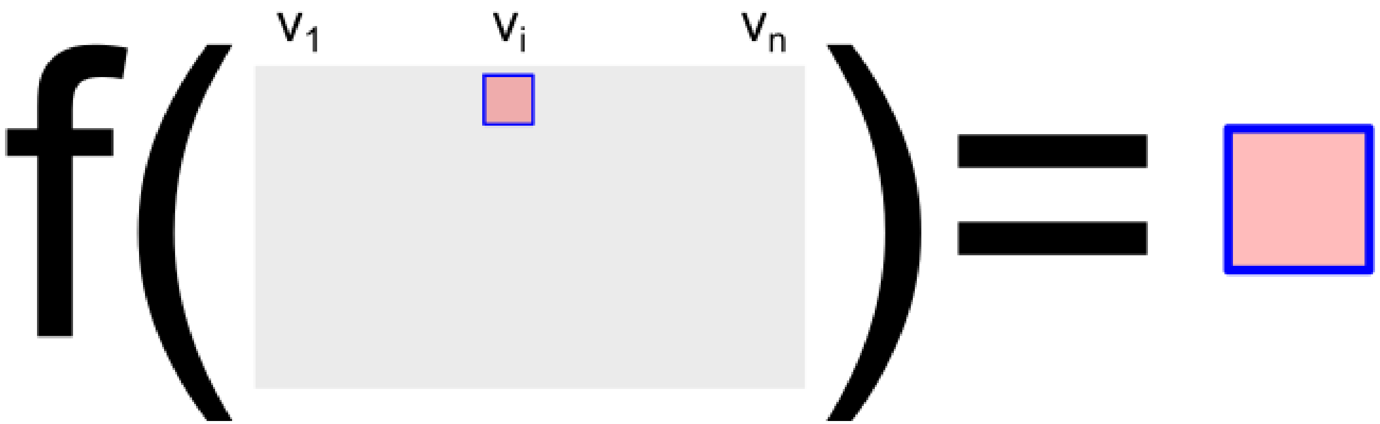

A dictatorship is onto and strategyproof.

[Gibbard'73; Satterthwaite'75]

With three or more candidates,
a voting rule is onto and strategyproof
if and only if it is a dictatorship.

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Quiz!

[Gibbard'73; Satterthwaite'75]

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Onto + strategyproof but two candidates:

[Gibbard'73; Satterthwaite'75]

With three or more candidates, a voting rule is onto and strategyproof if and only if it is a dictatorship.

Onto + strategyproof but two candidates:

≥3 candidates + strategyproof but not onto:

[Gibbard'73; Satterthwaite'75]

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With three or more candidates,
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Onto + strategyproof but two candidates: Majority

≥3 candidates + strategyproof but not onto:

≥3 candidates + onto but not strategyproof:

[Gibbard'73; Satterthwaite'75]

With three or more candidates,
a voting rule is onto and strategyproof
if and only if it is a dictatorship.

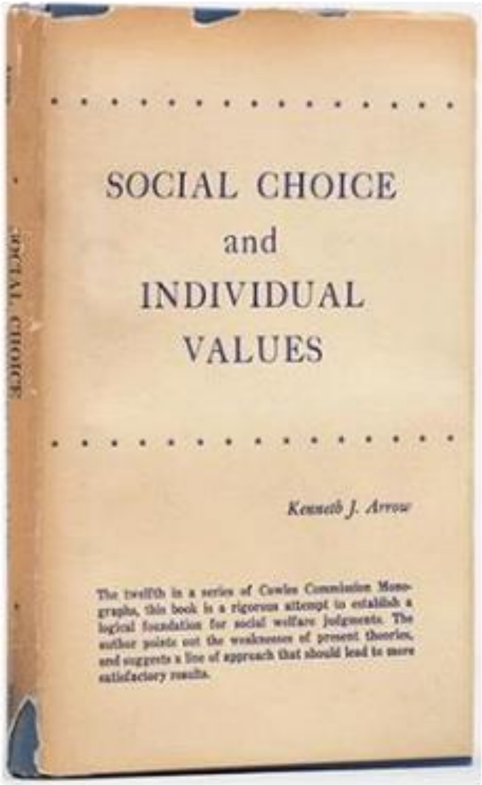
Onto + strategyproof but two candidates:	Majority
≥3 candidates + strategyproof but not onto:	Constant/Restricted Majority
≥3 candidates + onto but not strategyproof:	

[Gibbard'73; Satterthwaite'75]

With three or more candidates,
a voting rule is onto and strategyproof
if and only if it is a dictatorship.

Onto + strategyproof but two candidates:	Majority
≥3 candidates + strategyproof but not onto:	Constant/Restricted Majority
≥3 candidates + onto but not strategyproof:	Plurality/Borda/...

The use of axiomatic approach in voting goes back much further.



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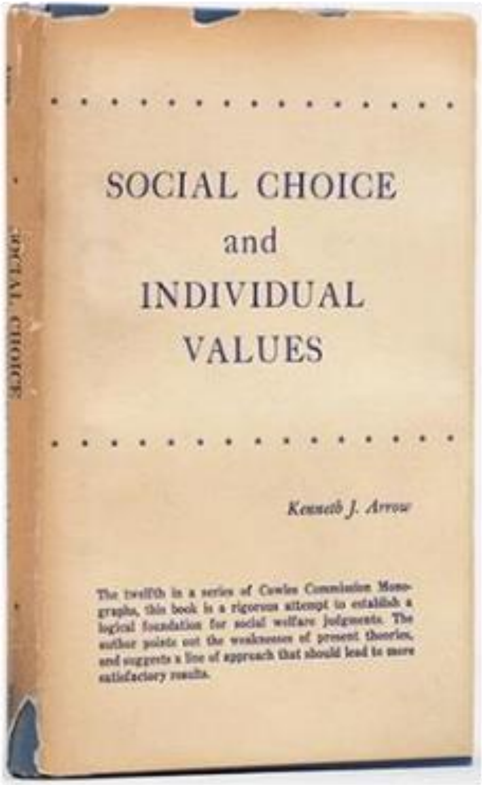

1951 PhD thesis of Kenneth Arrow

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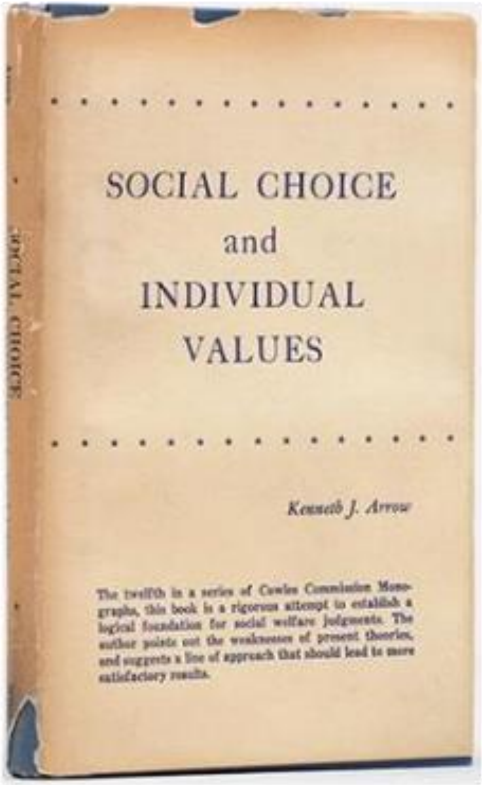

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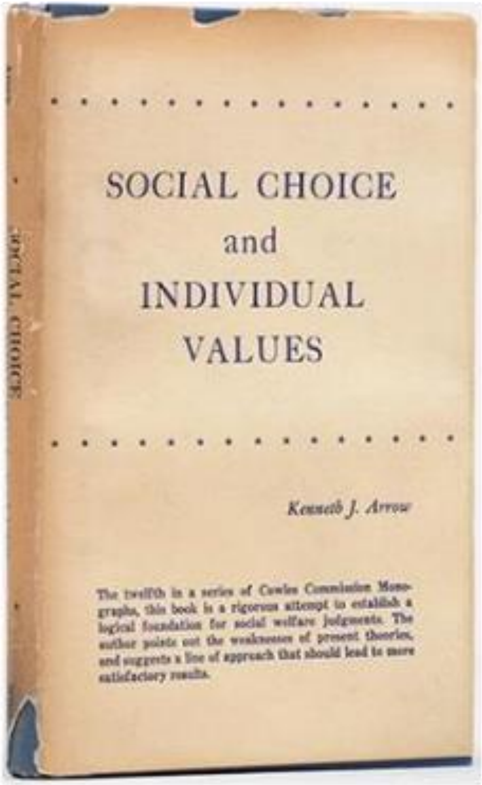

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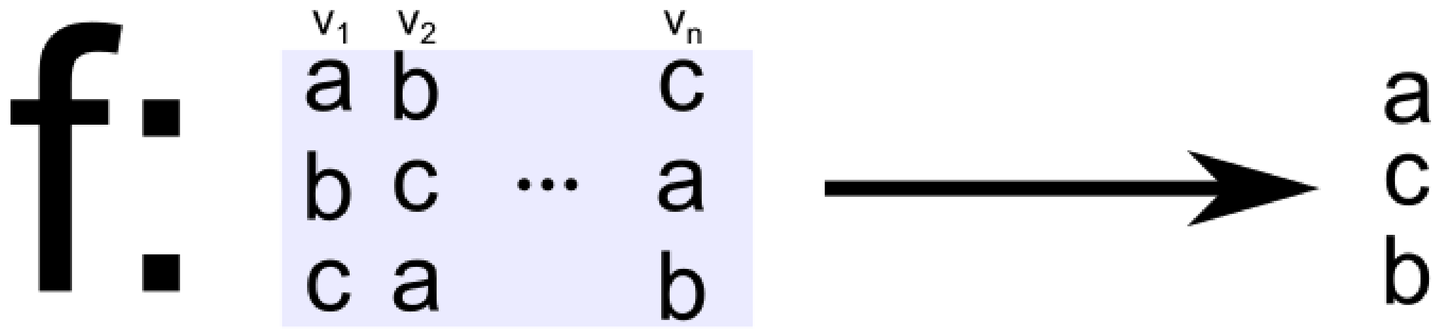
Voting Rules that Output Rankings

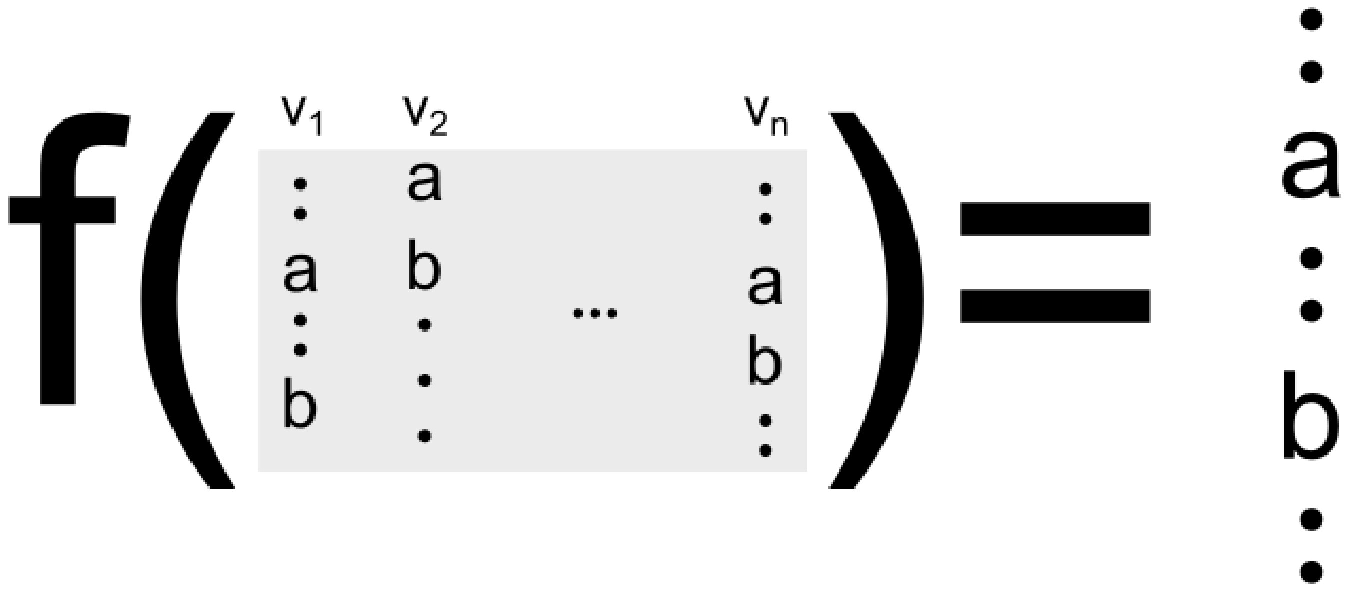
A mapping from preference profiles to rankings over candidates.



(also known as a *social welfare function* or SWF)

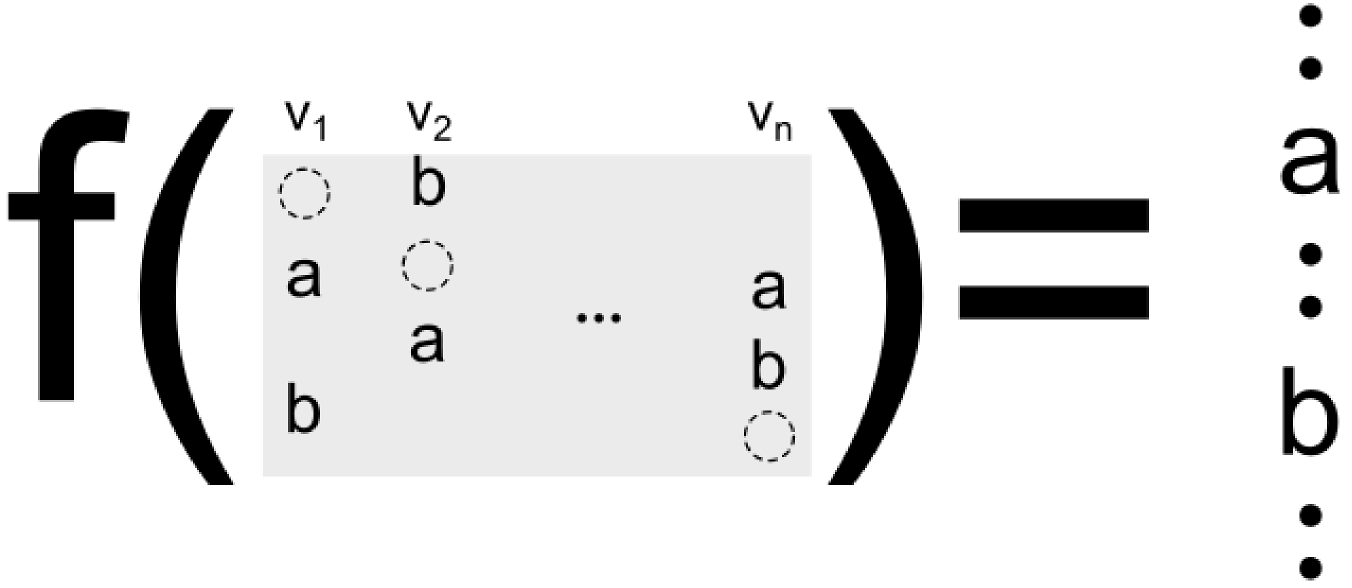
UNANIMOUS

If all voters prefer "a" over "b", then so does the SWF.



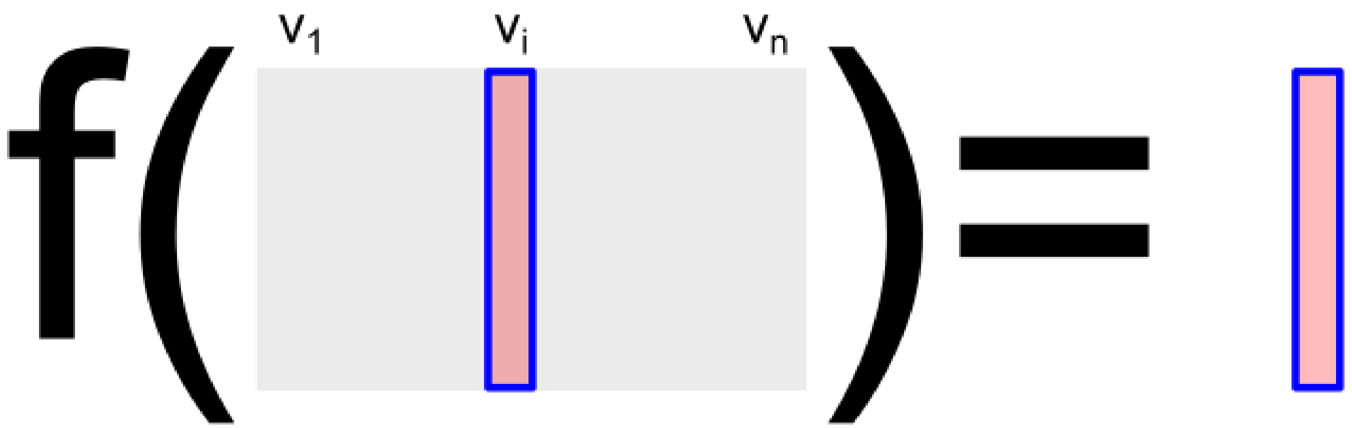
INDEPENDENCE OF IRRELEVANT ALTERNATIVES

If the relative ranking of "a" and "b" in each vote is unchanged,
then their relative ranking in the SWF outcome is also unchanged.



DICTATORSHIP

An SWF that mimics the preferences of a fixed voter on all inputs.



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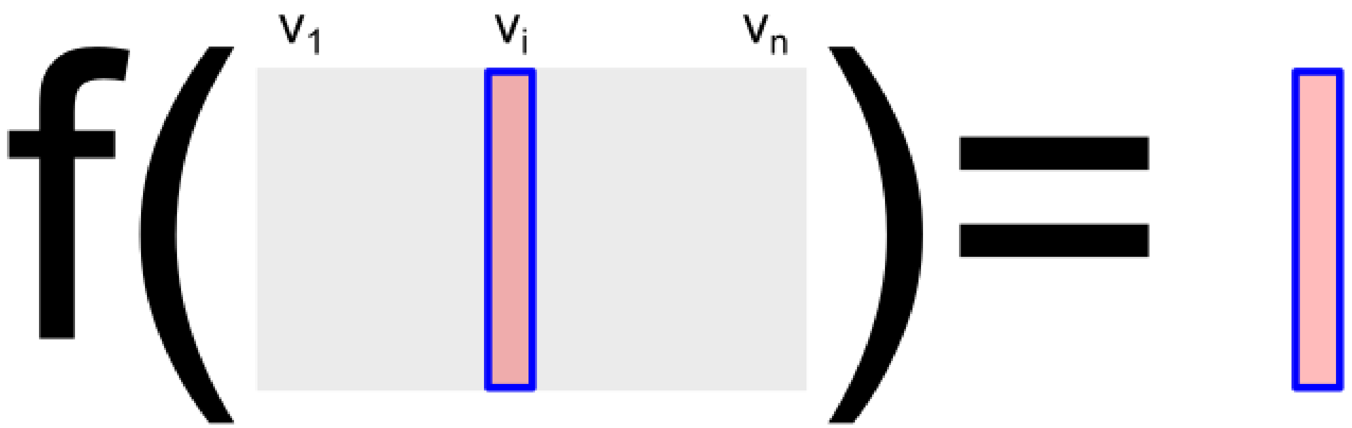

A dictatorship is unanimous and IIA.

[Arrow'51]

With three or more candidates,
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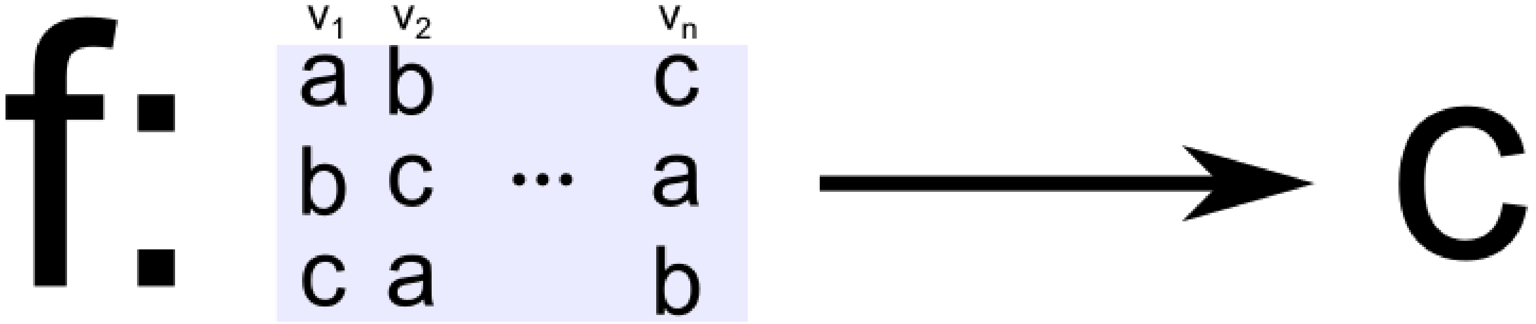
if and only if it is a dictatorship.

Method \ Criterion	Majority	Maj. loser	Mutual maj.	Condorcet	Cond. loser	Smith/ ISDA	LIIA	IIA	Cloneproof	Monotone	Consistency	Participation	Reversal symmetry	Polytime/ resolvable		Summable	Later-no-		No favorite betrayal	Ballot type	Ranks	
																	Harm	Help			=	>2
Approval	Rated[a]	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes[e]	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes[f]	Yes	Approvals	Yes	No
Borda count	No	Yes	No	No[b]	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	Yes	Yes
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Teams, crowds	Yes	No[b]	No[b]	Yes	$O(N^2)$	No	$O(N^2)$	No[b]	No	No[b]	Ranking	Yes	Yes
IRV (AV)	Yes	Yes	Yes	No[b]	Yes	No[b]	No	No	Yes	No	No	No	No	$O(N^2)$	Yes[g]	O(N!)[h]	Yes	Yes	No	Ranking	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Spoilers	Yes	No[b] [i]	No[b]	Yes	O(N!)	Yes	$O(N^2)$[j]	No[b]	No	No[b]	Ranking	Yes	Yes
Highest median/Majority judgment[k]	Rated[l]	Yes[m]	No[n]	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	No[o]	No[p]	Depends[q]	O(N)	Yes	O(N)[r]	No[s]	Yes	Yes	Scores[t]	Yes	Yes
Minimax	Yes	No	No	Yes[u]	No	No	No	No[b]	Spoilers	Yes	No[b]	No[b]	No	$O(N^2)$	Yes	$O(N^2)$	No[b][u]	No	No[b]	Ranking	Yes	Yes
Plurality/FPTP	Yes	No	No	No[b]	No	No[b]	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A[v]	N/A[v]	No	Single mark	N/A	No
Score voting	No	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
Runoff voting	Yes	Yes	No	No[b]	Yes	No[b]	No	No	Spoilers	No	No	No	No	O(N)[w]	Yes	O(N)[w]	Yes	Yes[x]	No	Single mark	N/A	No[y]
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
STAR voting	No[z]	Yes	No[aa]	No[b][c]	Yes	No[b]	No	No	No	Yes	No	No	Depends[ab]	O(N)	Yes	$O(N^2)$	No	No	No[ac]	Scores	Yes	Yes
Sortition, arbitrary winner[ad]	No	No	No	No[b]	No	No[b]	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A
Random ballot[ae]	No	No	No	No[b]	No	No[b]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No

"Most systems are not going to work badly all of the time. All I proved is that all can work badly at times."

VOTING RULE

A mapping from preference profiles to candidates.



f-Manipulation

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Input:

- A set of candidates and a set of voters $v_1, v_2, \ldots, v_n$

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- Manipulator v_1's favorite candidate c

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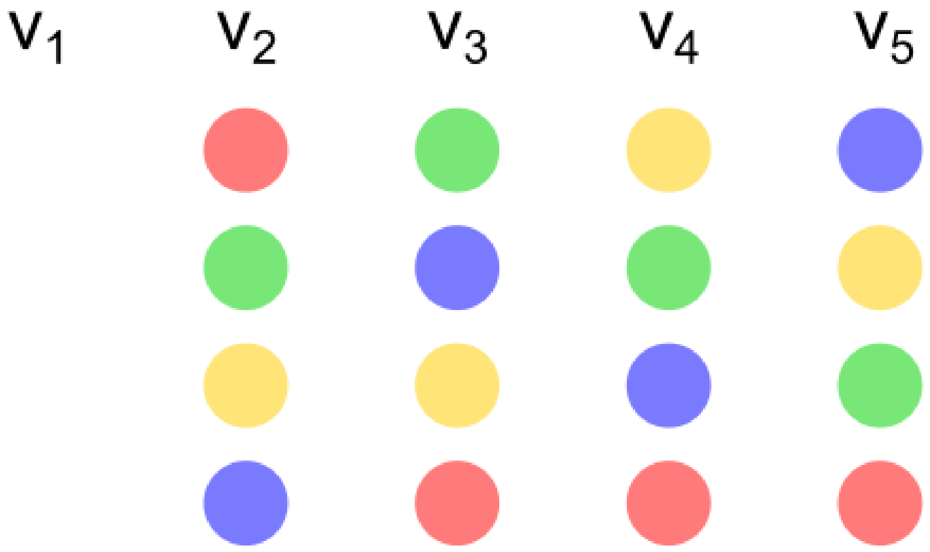
- Manipulator v_1's favorite candidate c

Question:

Does there exist a vote P_1 of the manipulator v_1 such that $f(P_1, P_2, \ldots, P_n) = c$?

Manipulation under Borda Count



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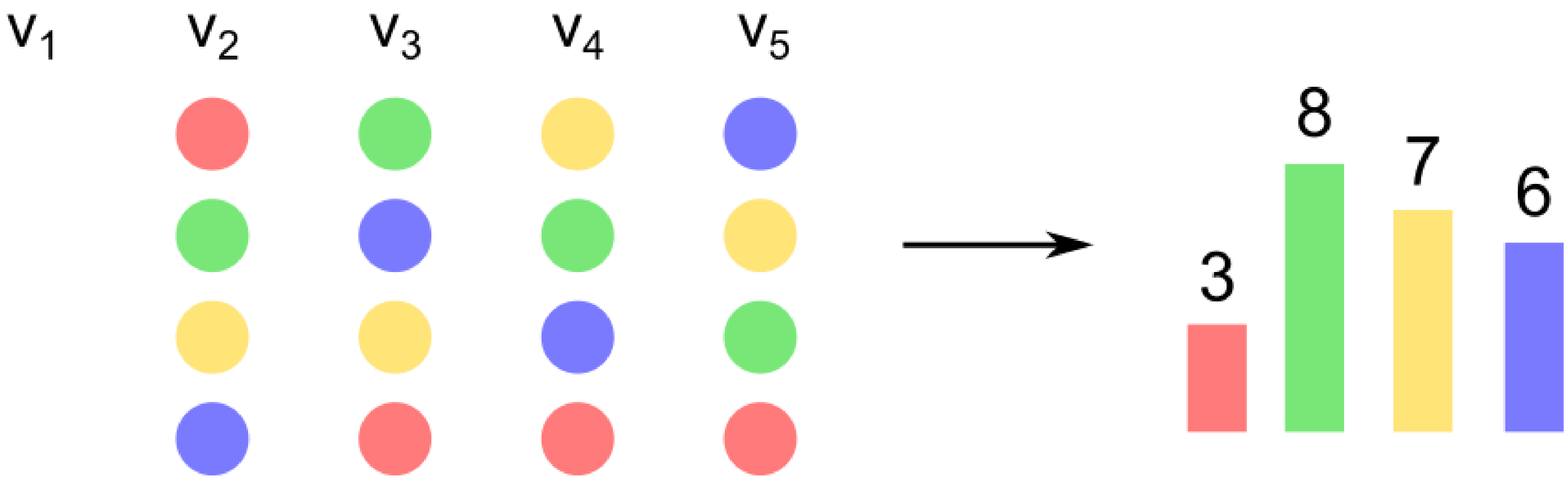

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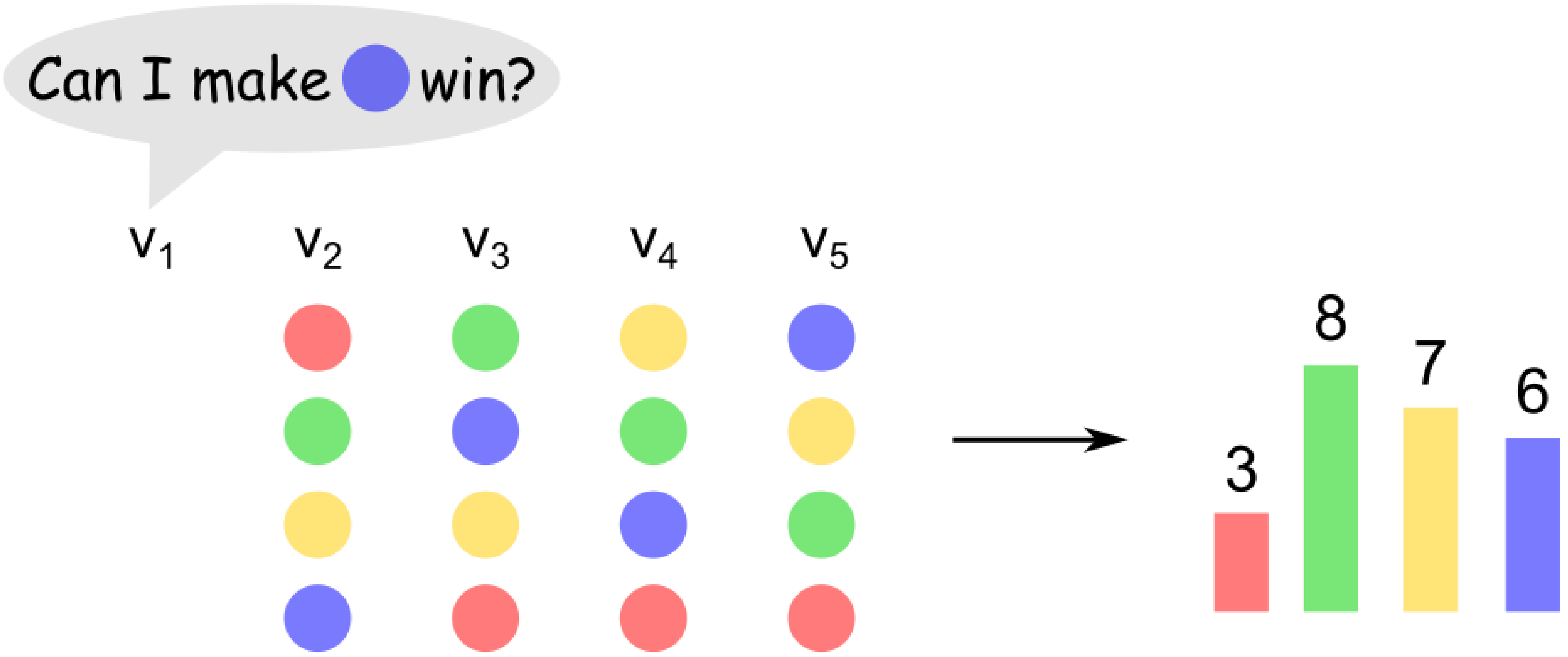

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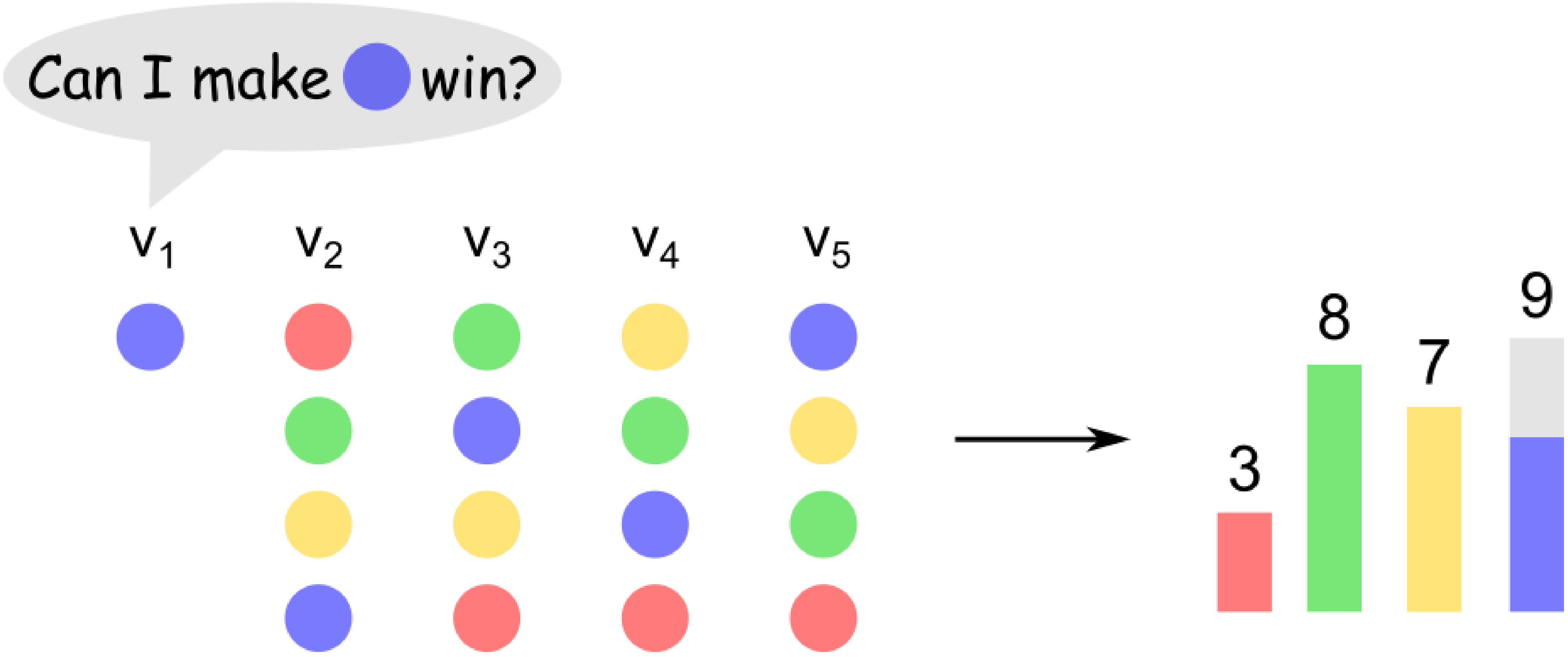

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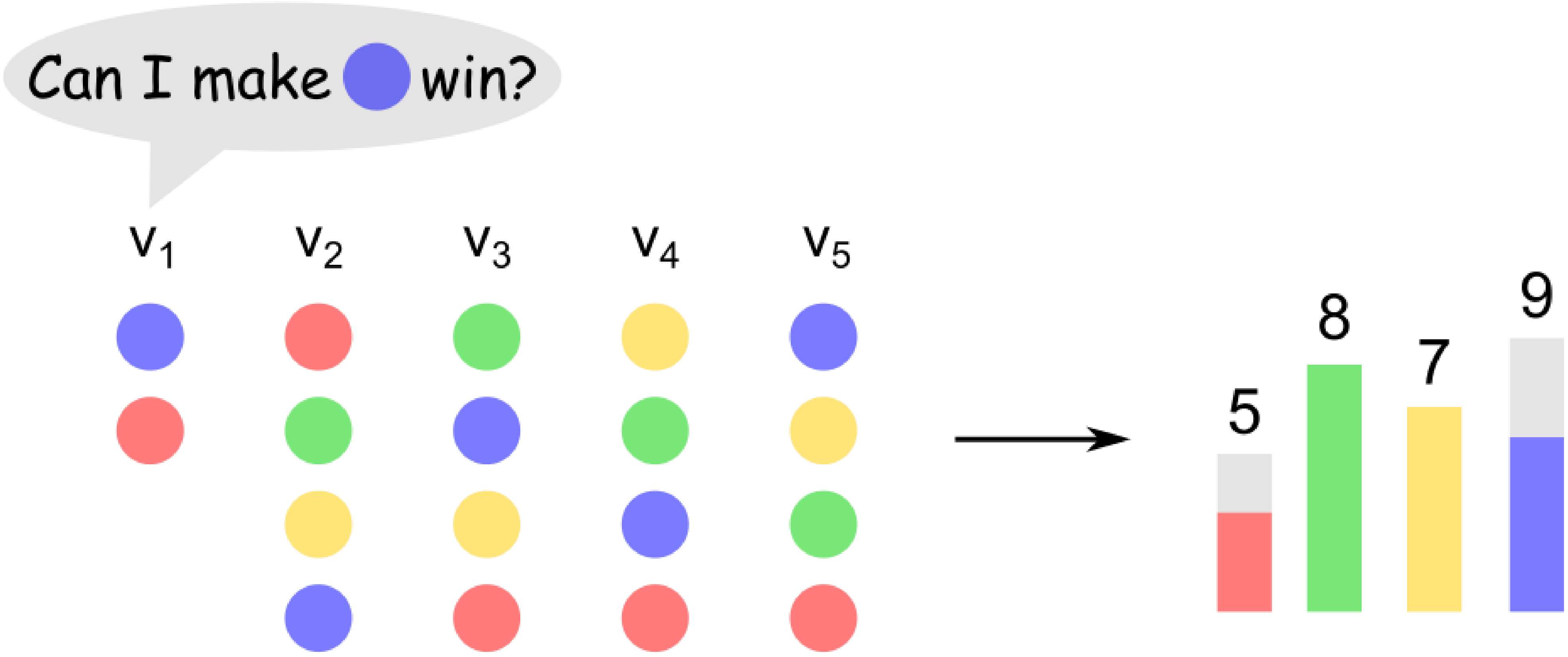

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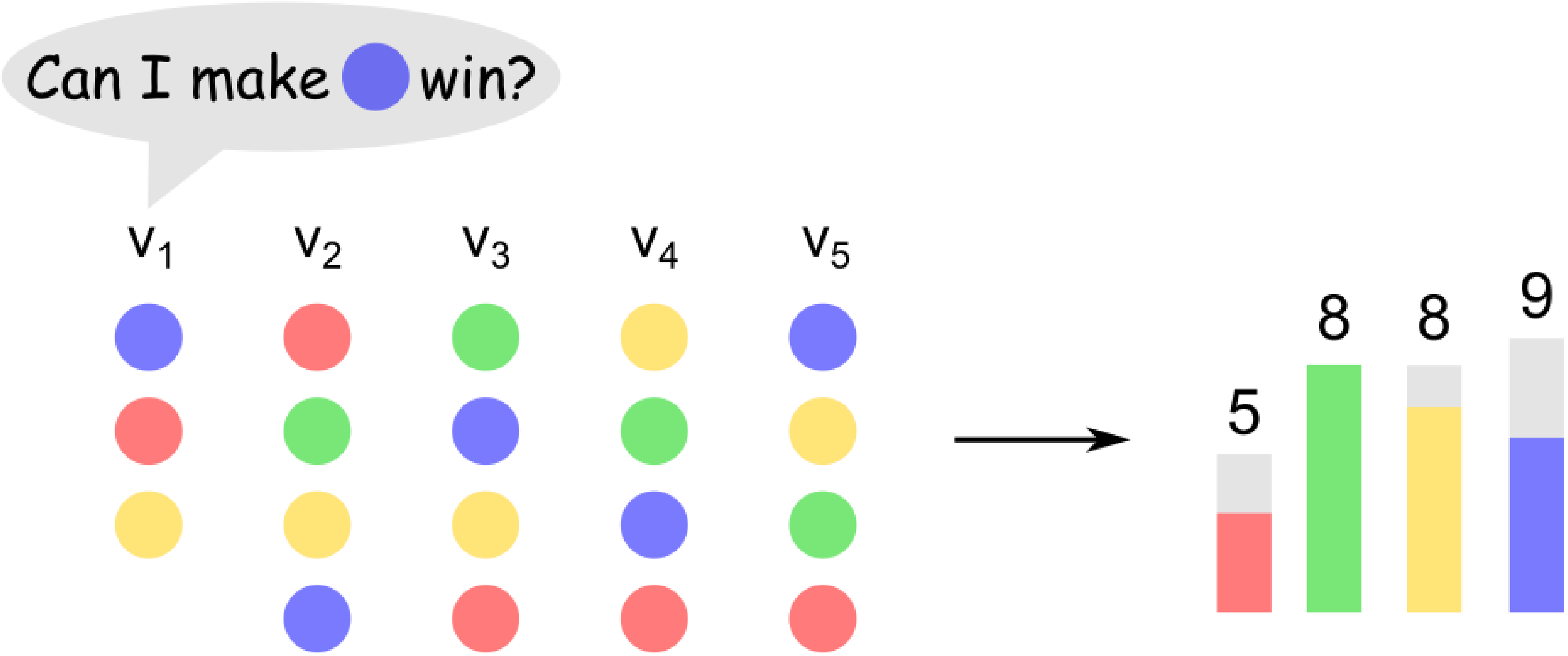

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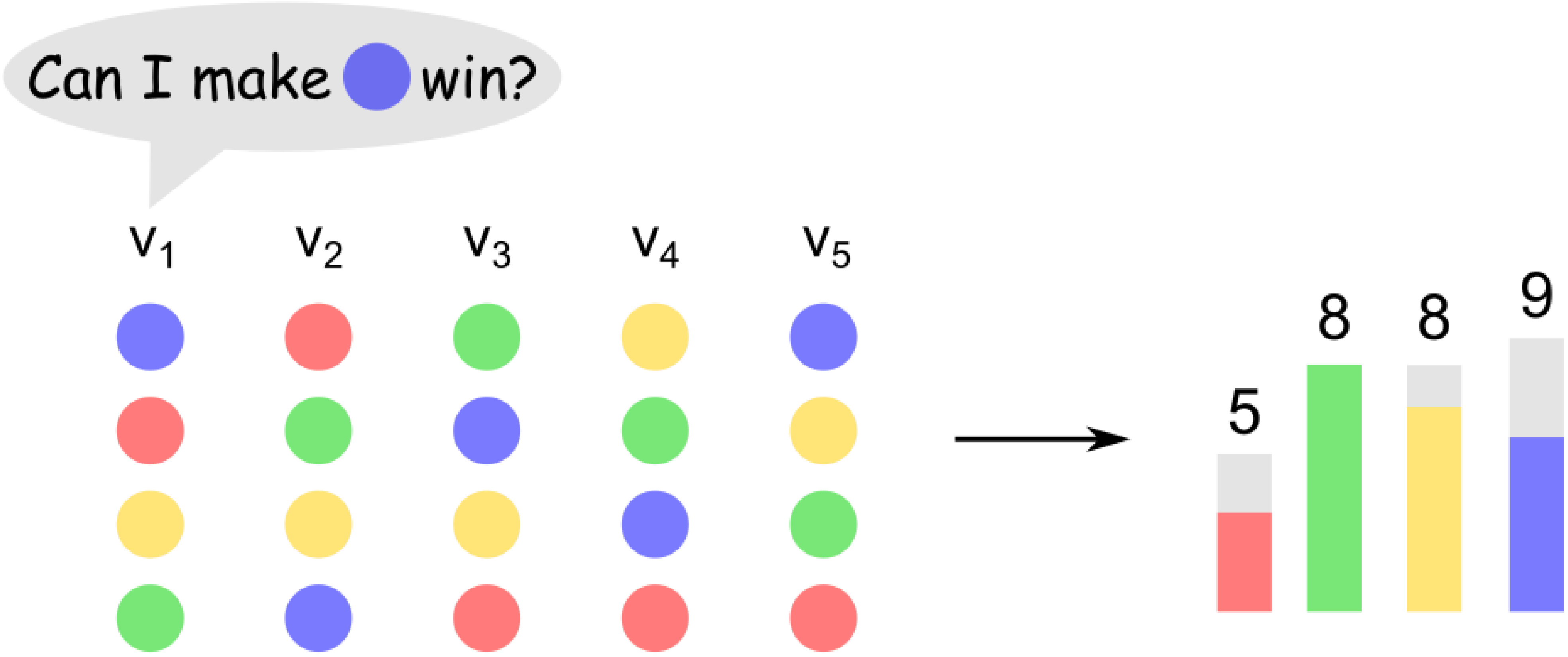

A Greedy Strategy

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A Greedy Strategy

- Rank c at the top position in v_1's vote

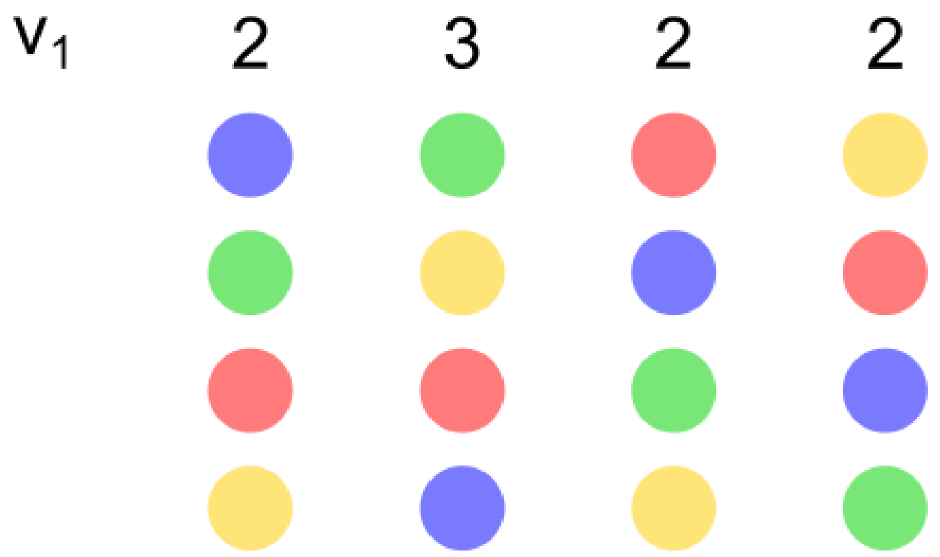
- While there is an unranked candidate:

  If a candidate, say x, can be "safely" placed in the next highest position in v_1's list without preventing c from winning, then place x in that position.

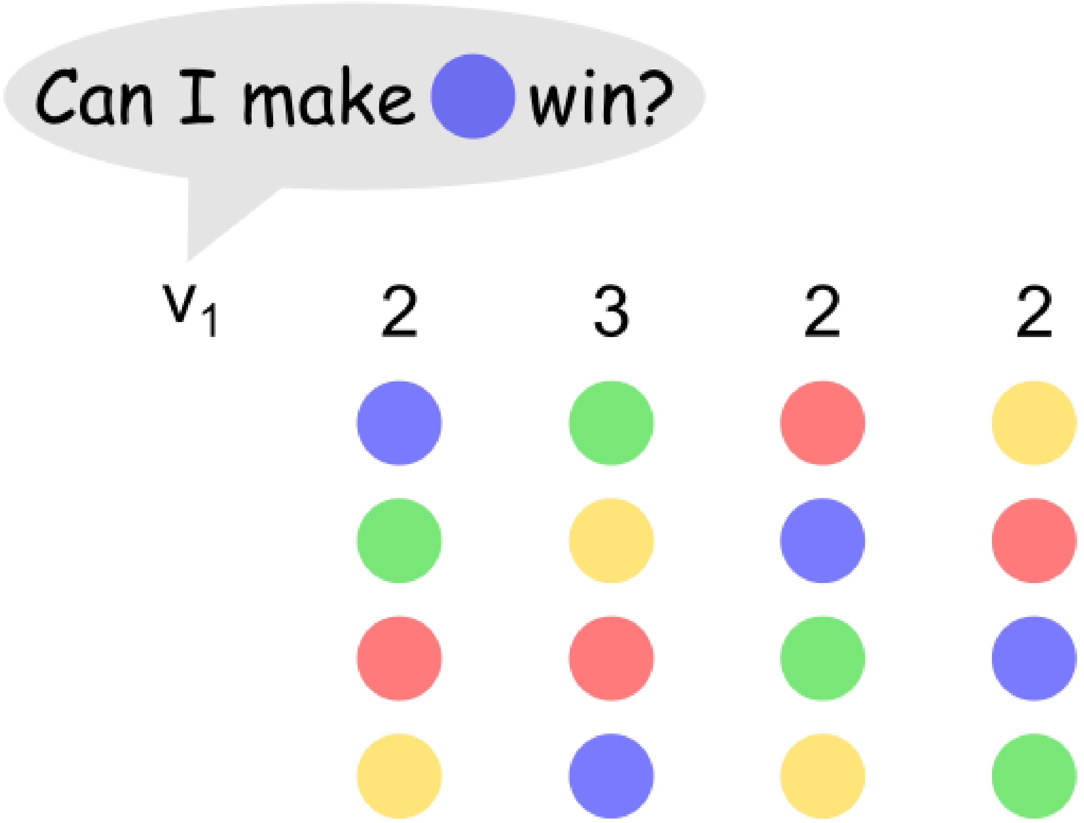
- Otherwise, return 'No'.

The greedy strategy does not always work.

Manipulation under STV



Manipulation under STV



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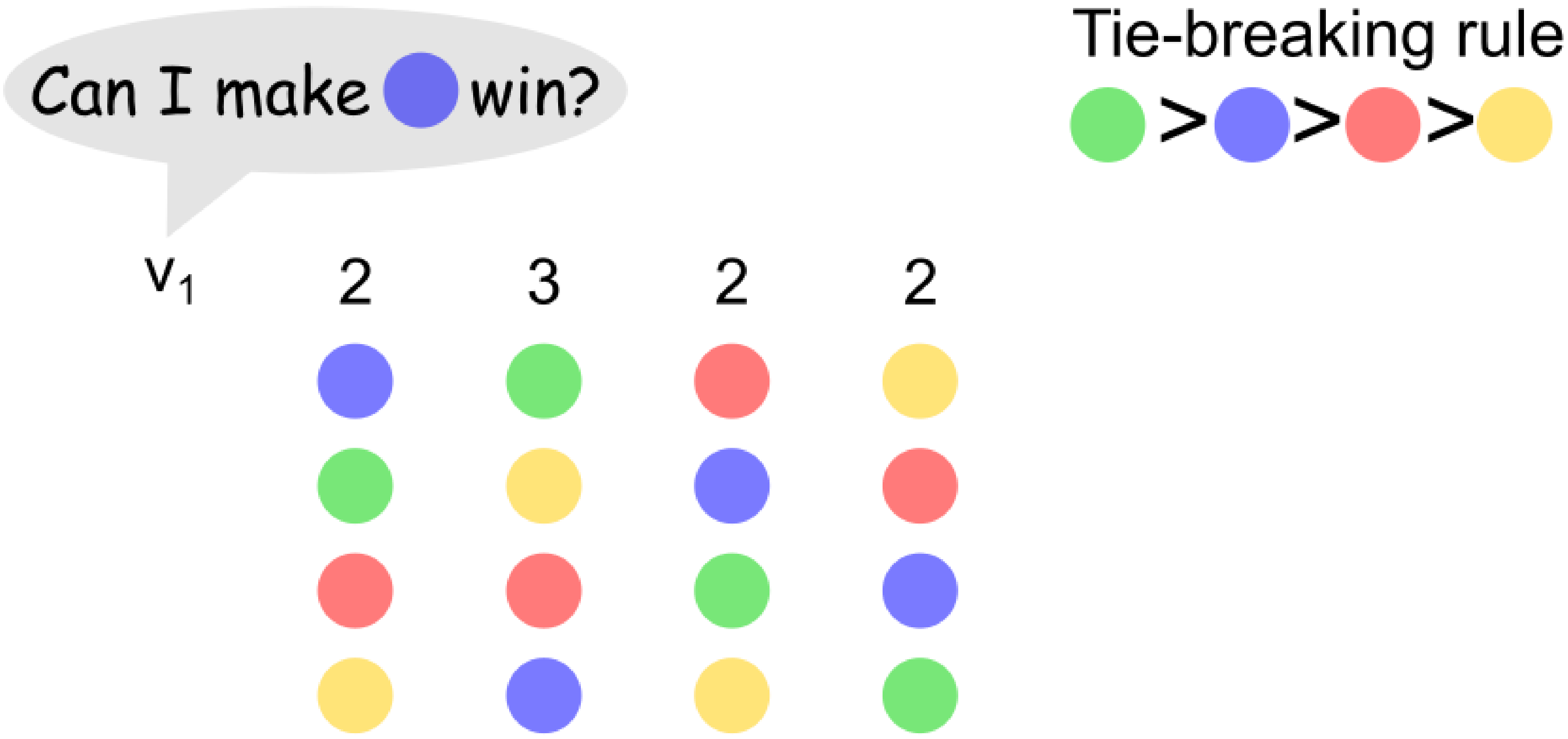
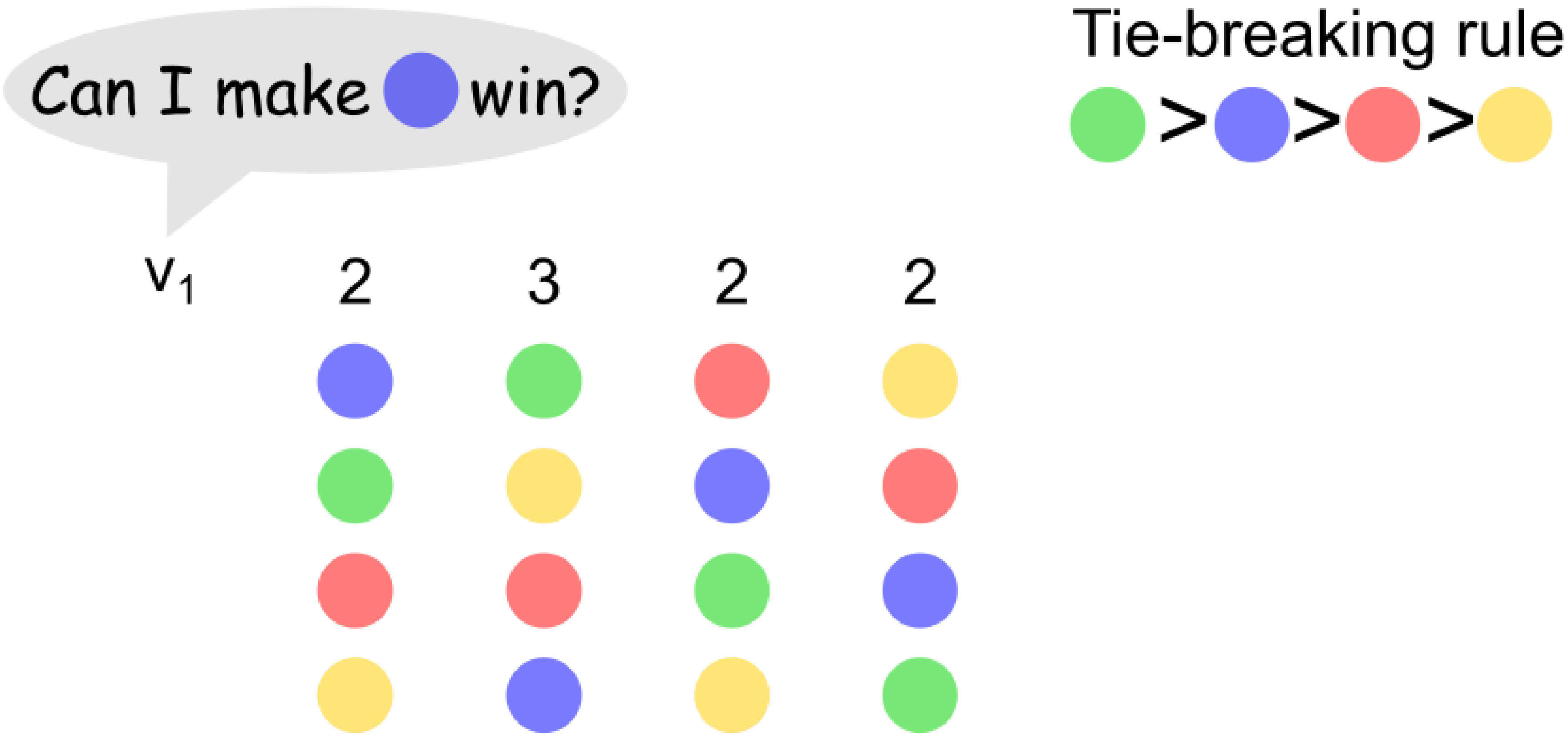
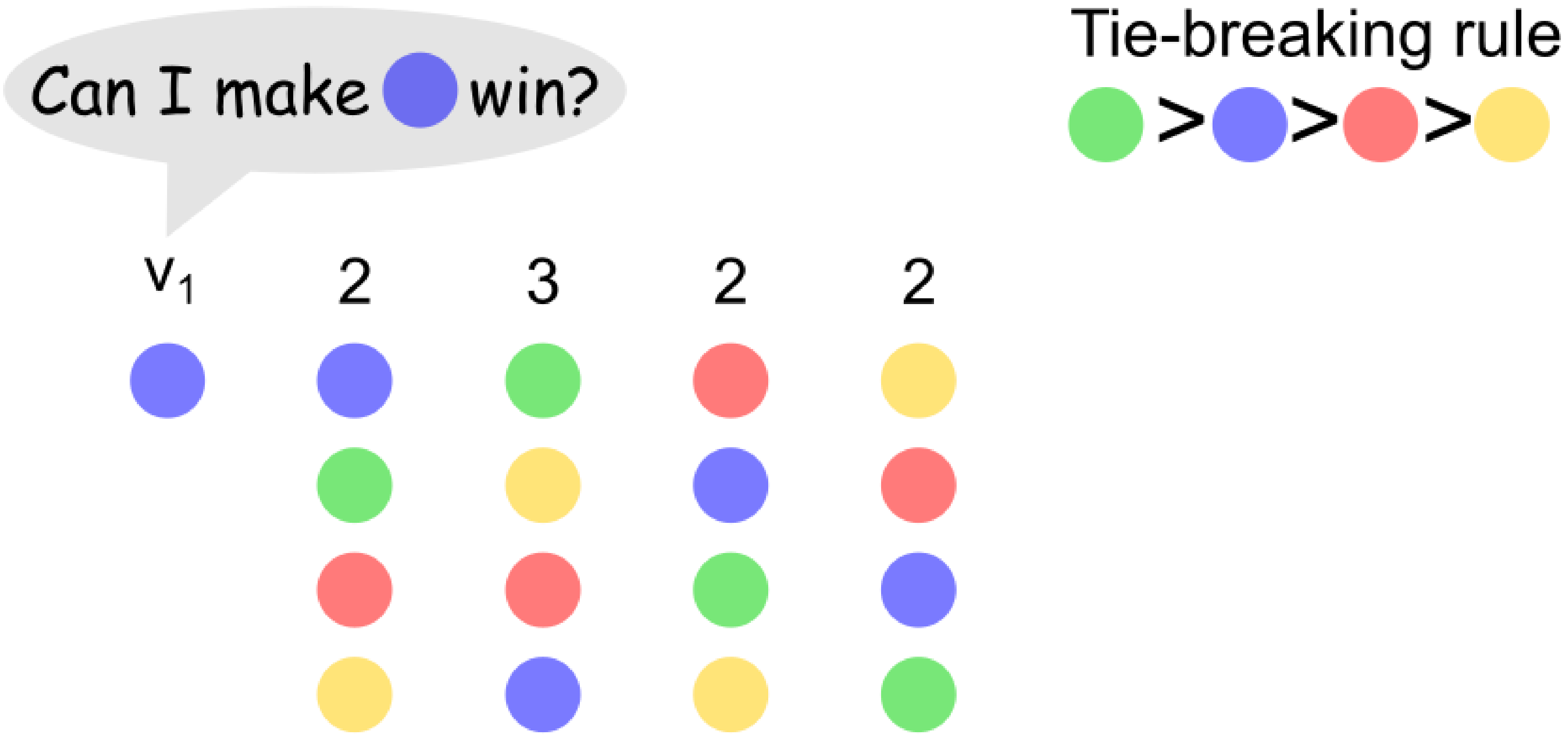


Manipulation under STV



Let's follow the greedy strategy and put at the top.

Manipulation under STV



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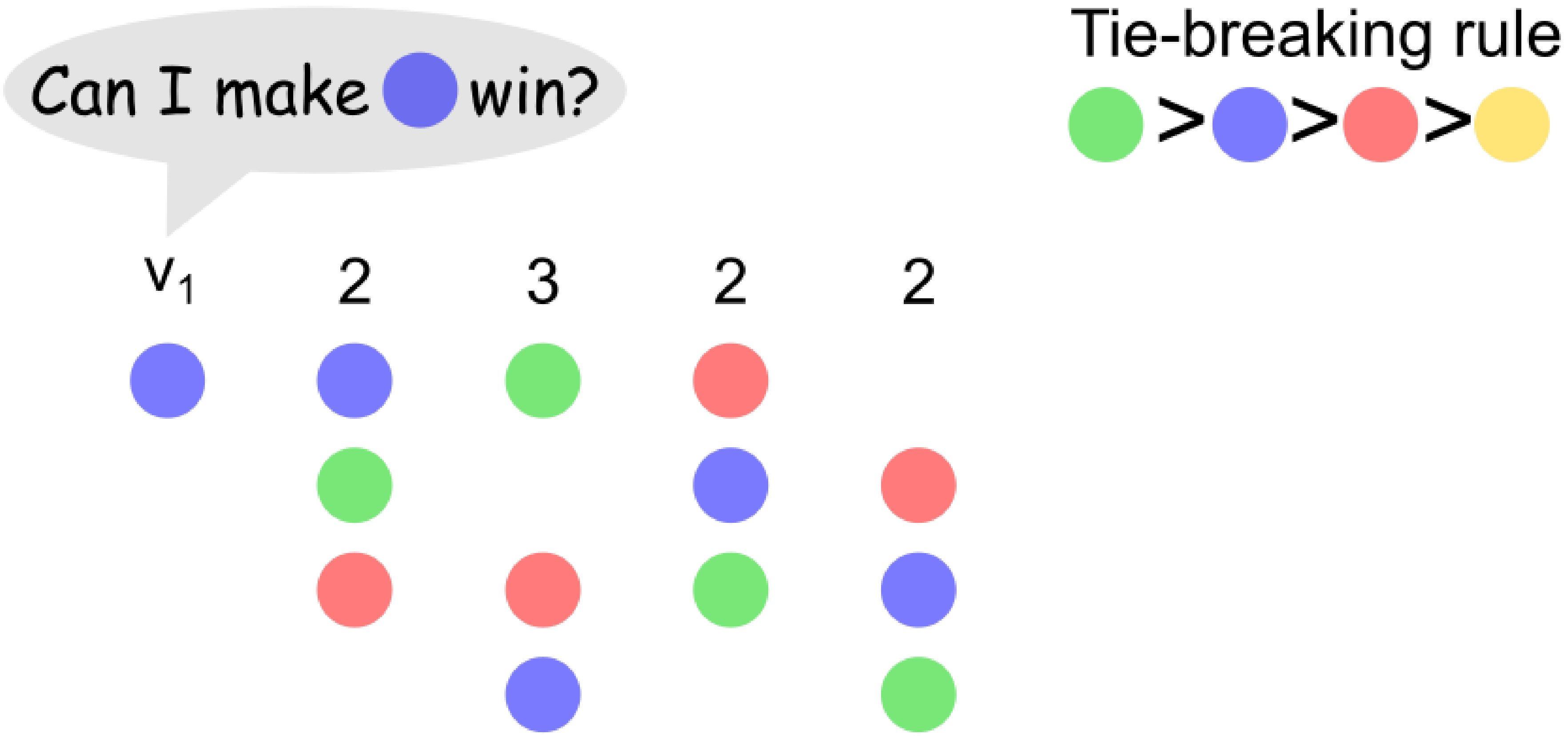
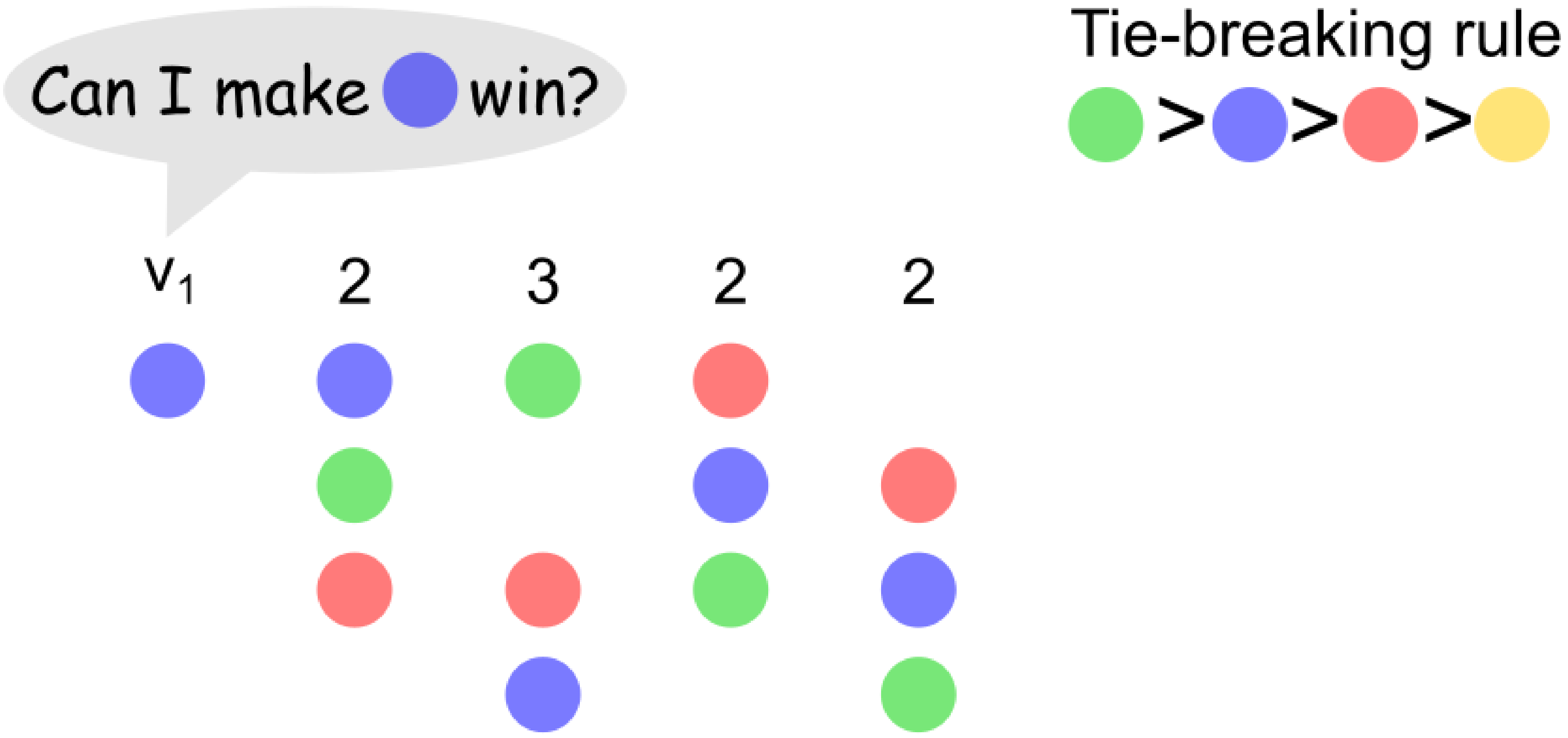


Let's follow the greedy strategy and put at the top.

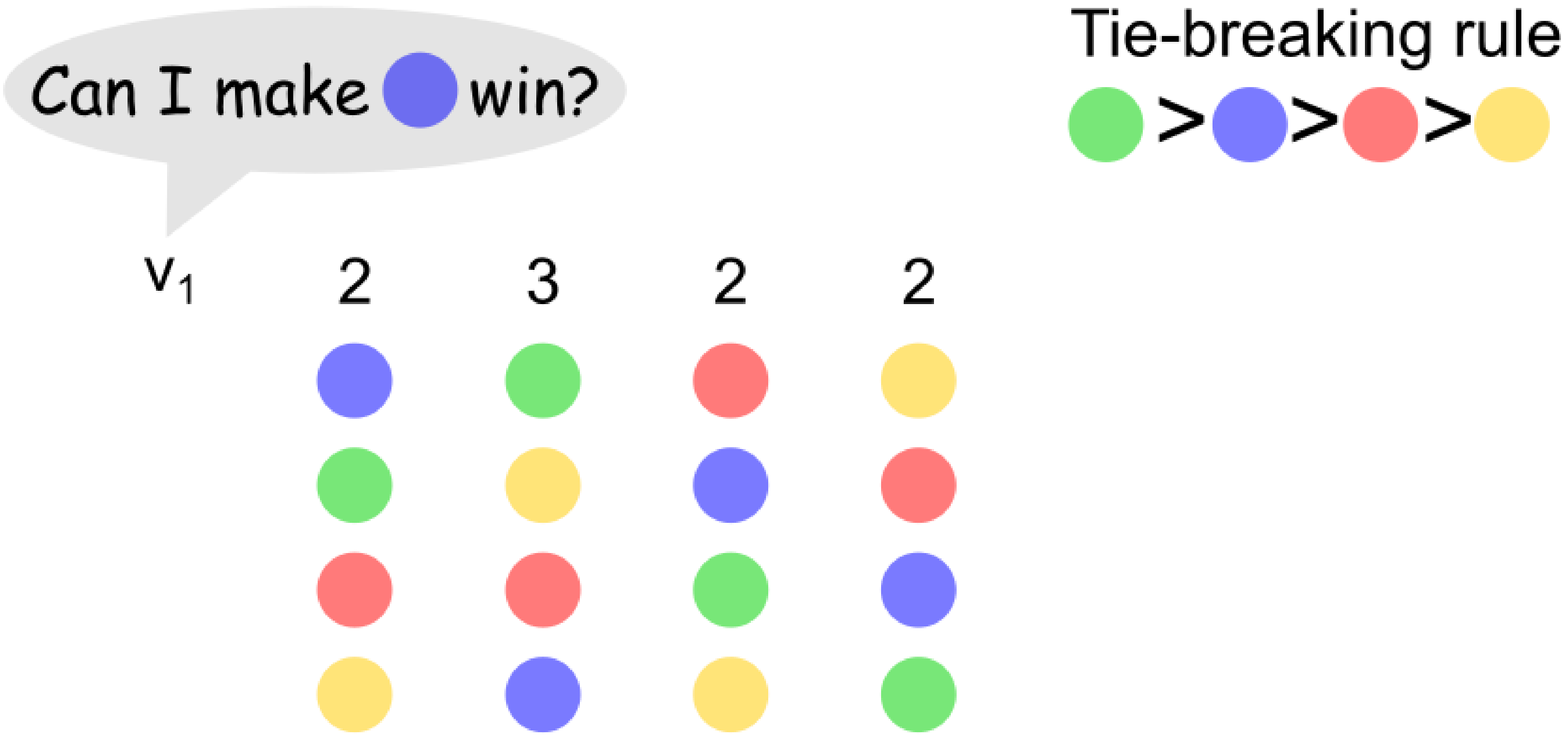
Manipulation under STV



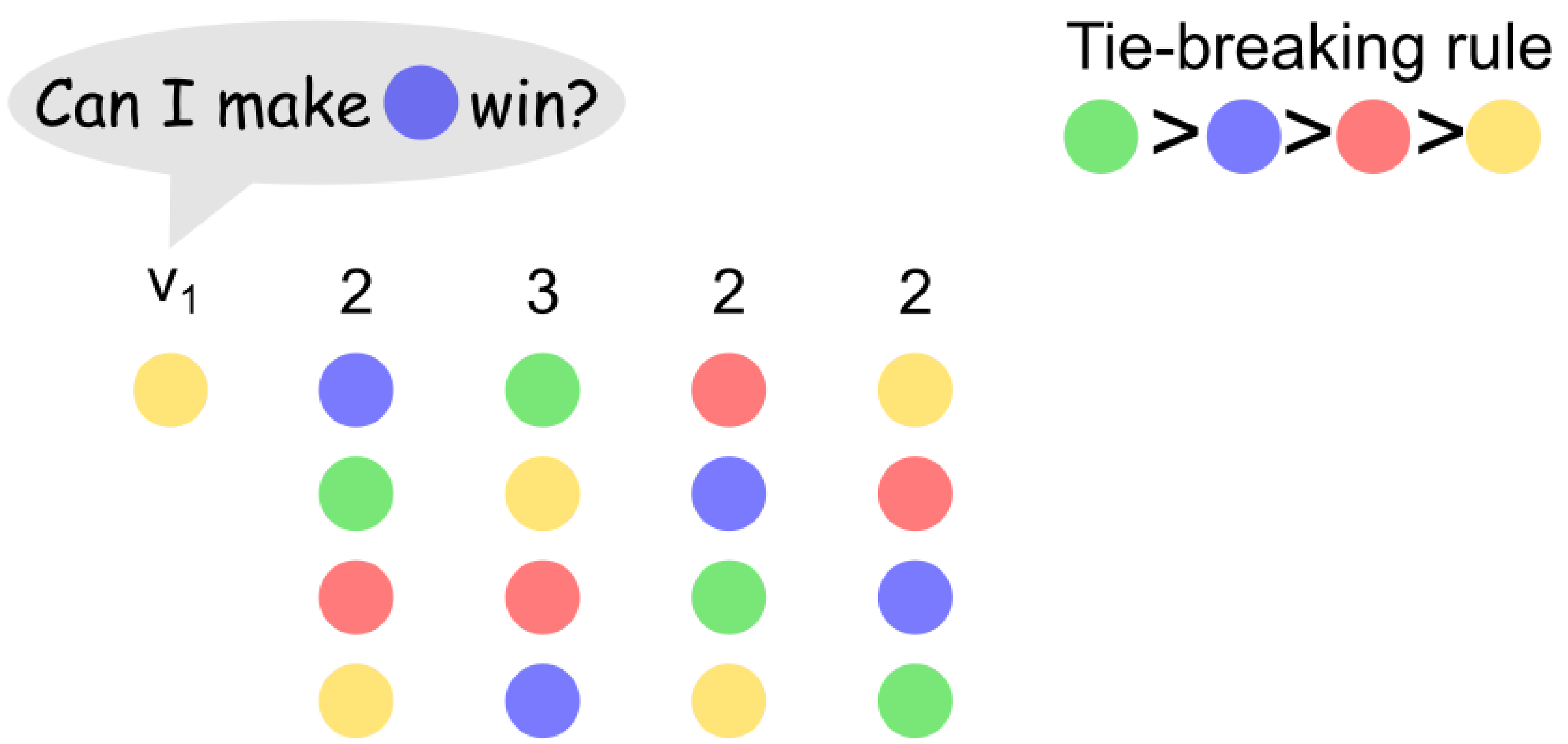
Let's follow the greedy strategy and put at the top.

is eliminated in the next round (due to tie-breaking rule).

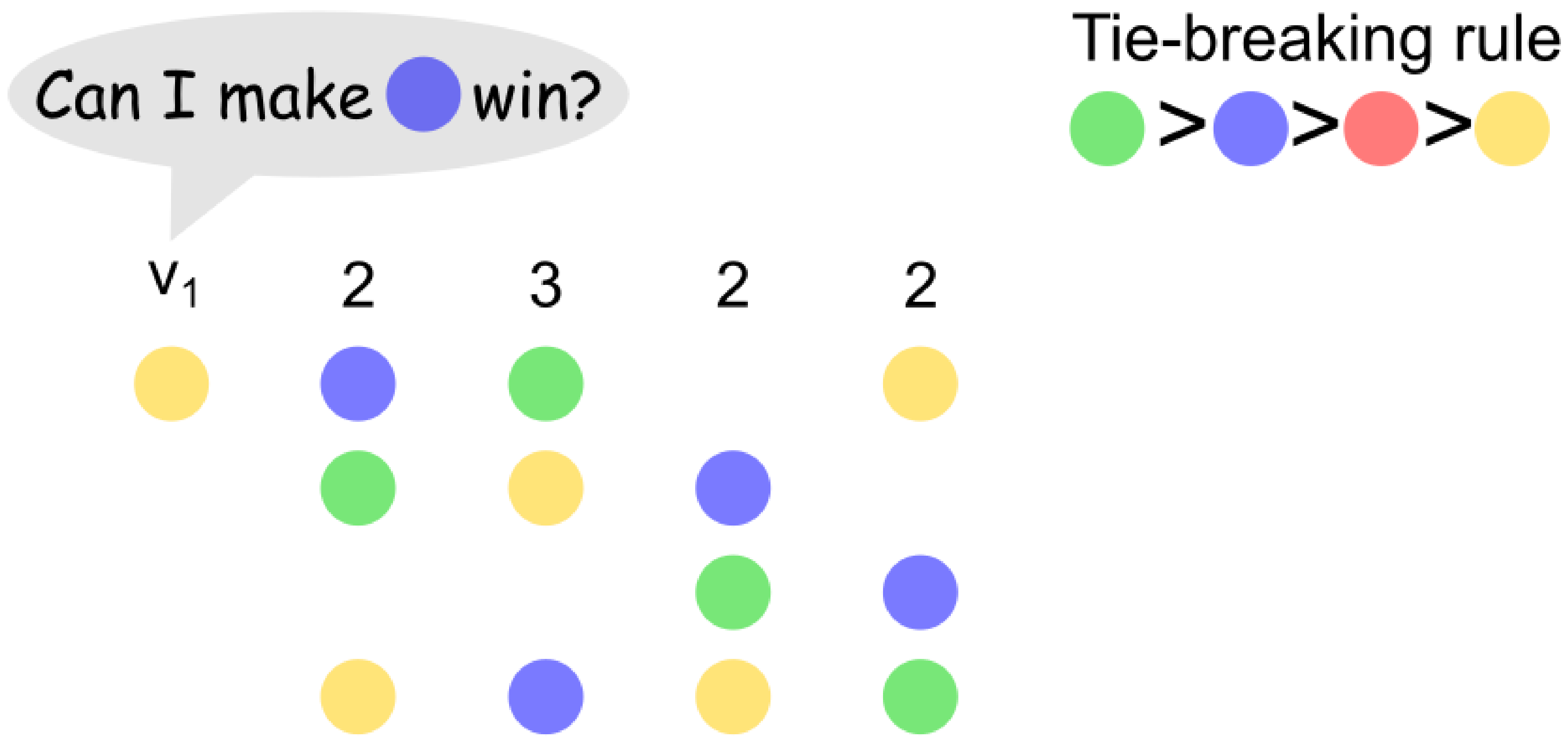
Manipulation under STV



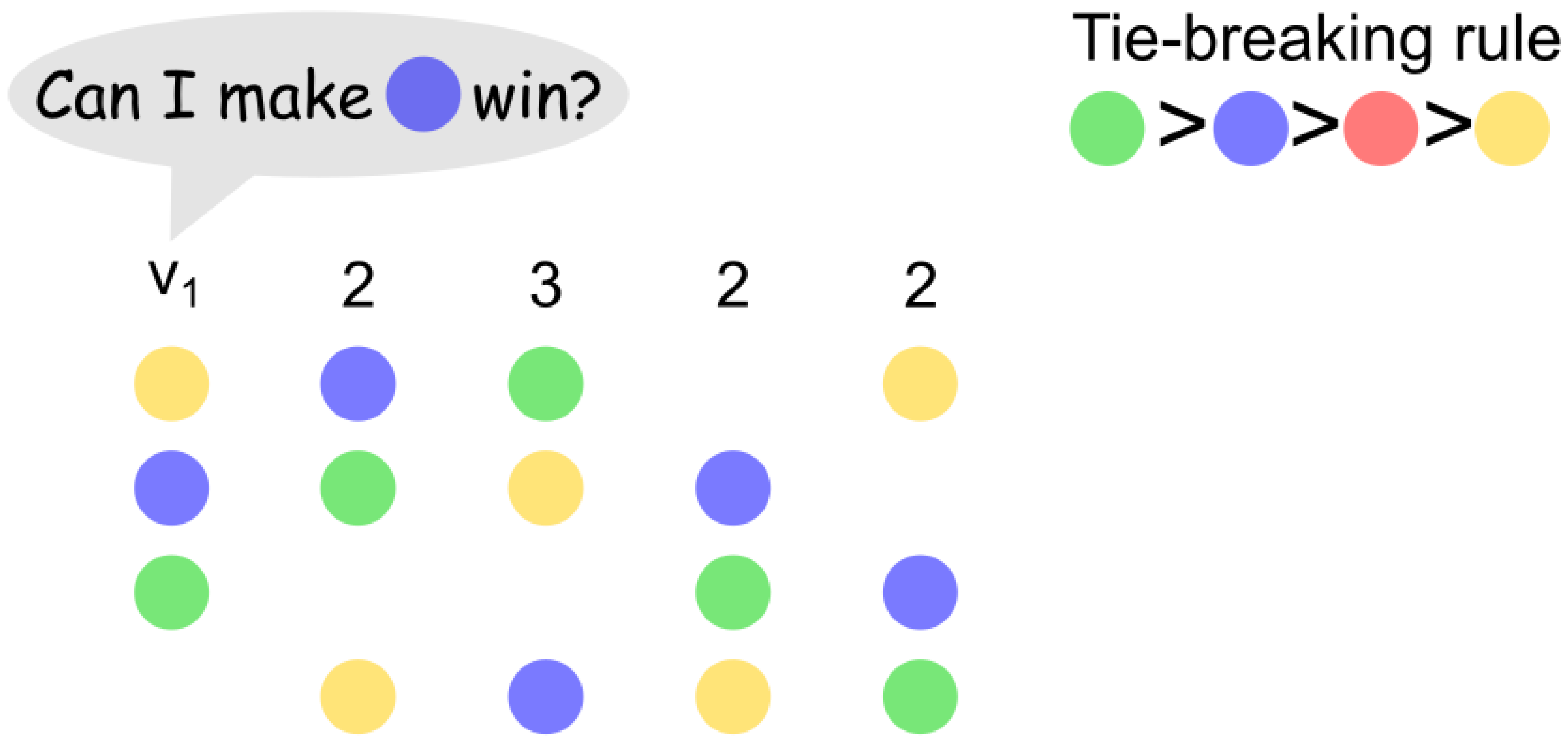
Manipulation under STV



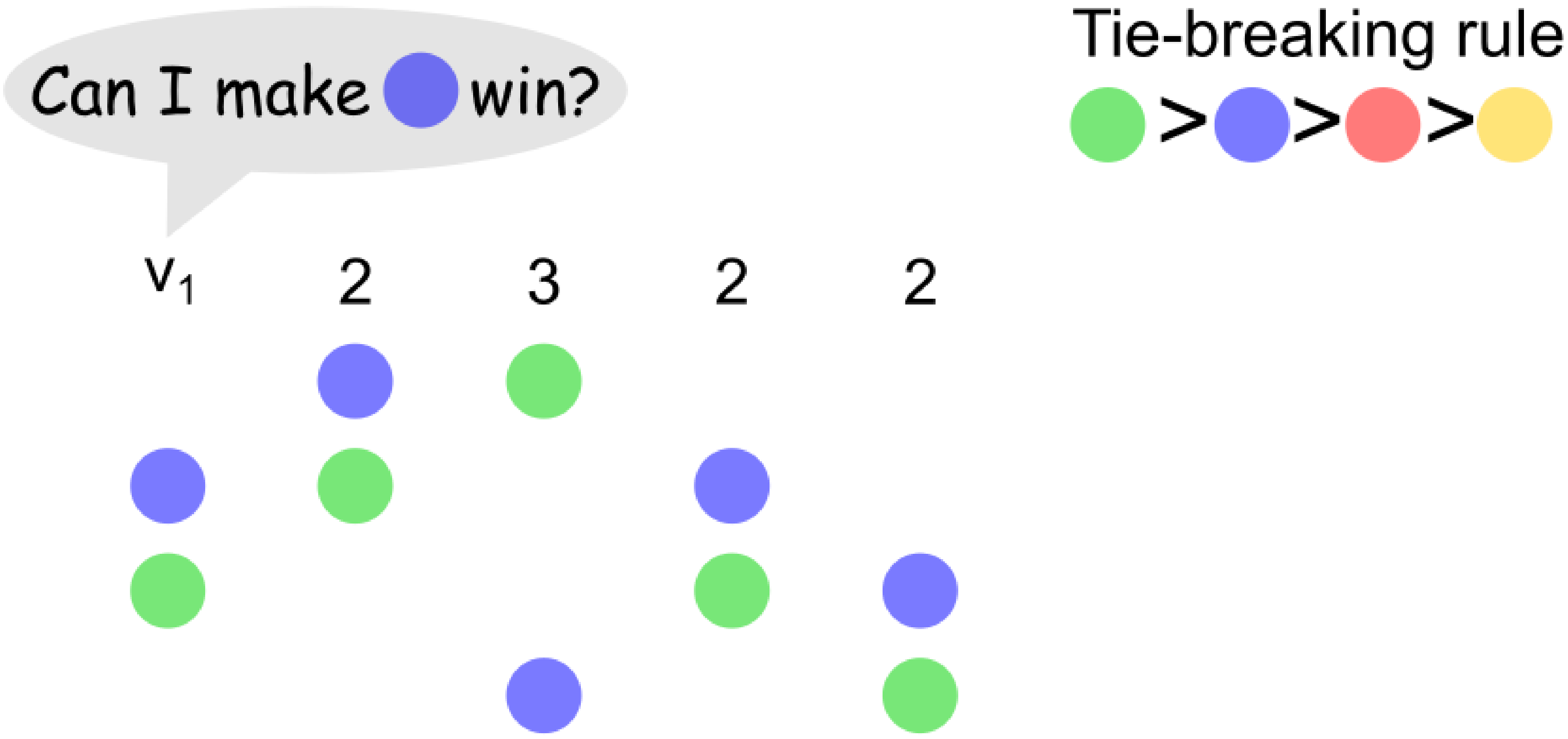
Manipulation under STV



Manipulation under STV



Manipulation under STV



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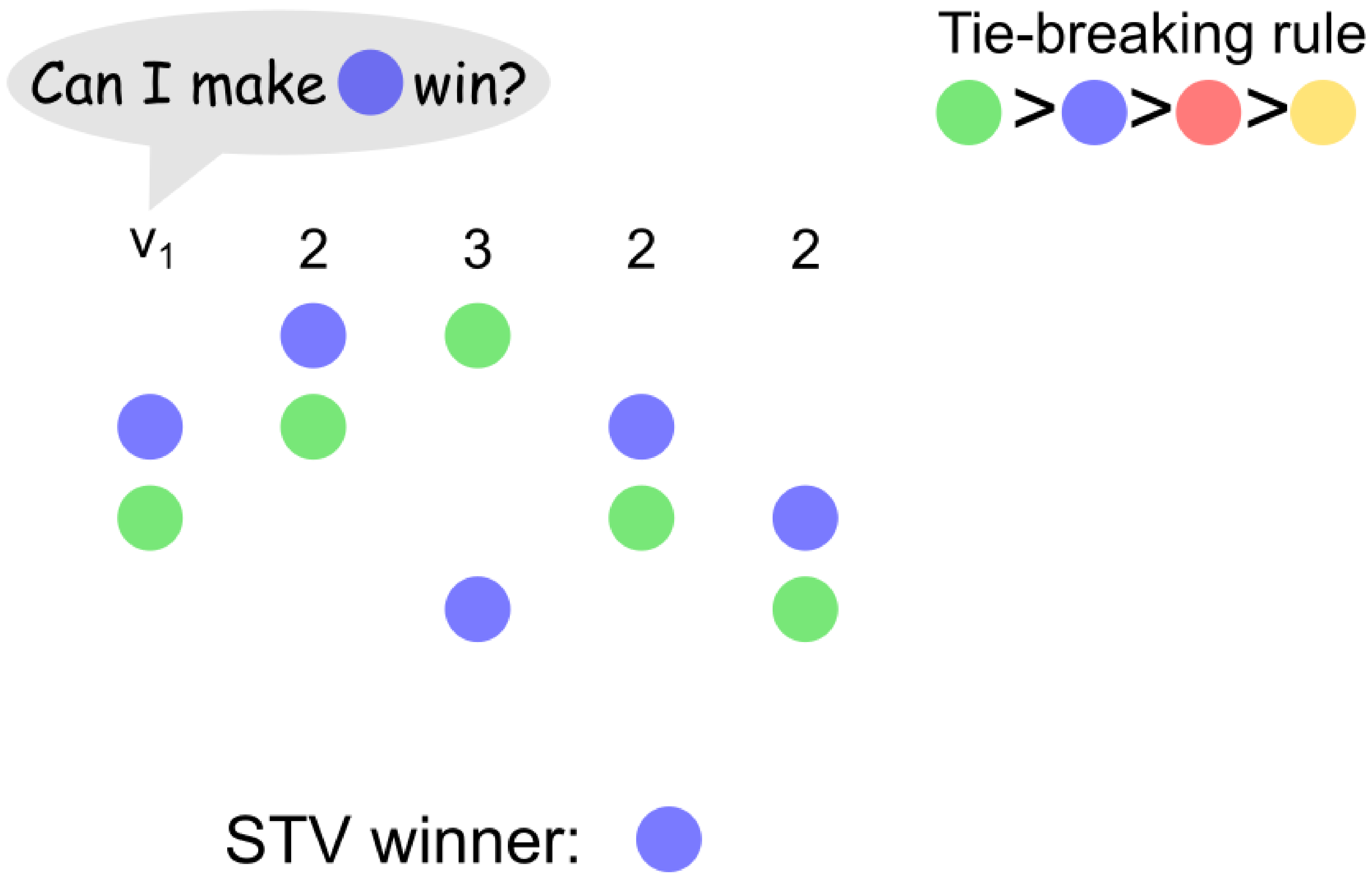

Is manipulation *always* easy?

Soc Choice Welfare (1989) 6:227–241

Social Choice and Welfare

The Computational Difficulty of Manipulating an Election*

J. J. Bartholdi III, C. A. Tovey, and M. A. Trick**

School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

Received June 9, 1987 / Accepted July 29, 1988

Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.

For many voting rules, f-Manipulation is NP-hard.

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[Bartholdi, Tovey, and Trick, SCW 1989]

Copeland with second-order tie-breaking

In case of a tie, winner is the candidate whose defeated competitors have the highest sum of Copeland scores.

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Single Transferable Vote (STV)

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[Bartholdi and Orlin, SCW 1991]

Single Transferable Vote (STV)

[Xia, Zuckerman, Procaccia, Conitzer, Rosenschein, IJCAI 2009]

Ranked Pairs

Consider candidate pairs according to the margin of head-to-head victories, and create a ranking based on it while avoiding cycles.

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NP-hardness is good news!

No general-purpose efficient algorithm that correctly works on all preference profiles (unless P=NP).

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Using worst-case computational hardness
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For many voting rules, f-Manipulation is NP-hard.

NP-hardness is good news!
No general-purpose efficient algorithm that correctly works on all preference profiles (unless P=NP).

Using worst-case computational hardness as a barrier to manipulation.

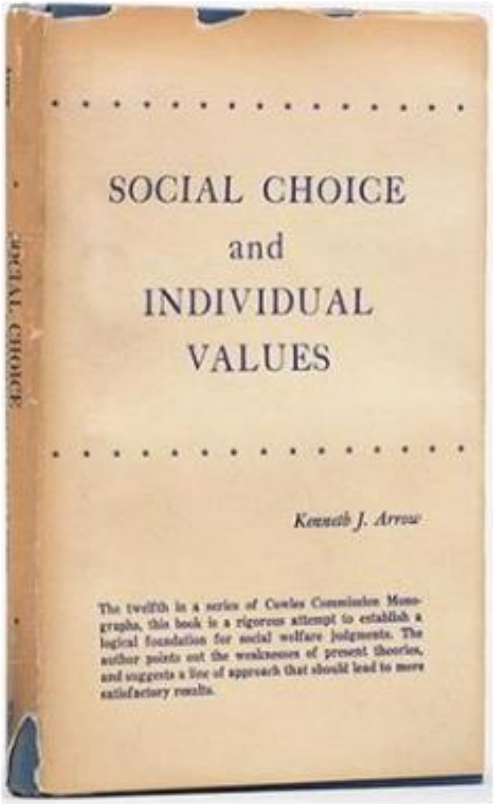
Note: NP-hard *even with* full information.

Remember this?

Method \ Criterion	Majority	Maj. loser	Mutual maj.	Condorcet	Cond. loser	Smith/ ISDA	LIIA	IIA	Cloneproof	Monotone	Consistency	Participation	Reversal symmetry	Polytime/ resolvable		Summable	Later-no-		No favorite betrayal	Ballot type	Ranks	
																	Harm	Help			=	>2
Approval	Rated[a]	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes[e]	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes[f]	Yes	Approvals	Yes	No
Borda count	No	Yes	No	No[b]	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	Yes	Yes
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Teams, crowds	Yes	No[b]	No[b]	Yes	$O(N^2)$	No	$O(N^2)$	No[b]	No	No[b]	Ranking	Yes	Yes
IRV (AV)	Yes	Yes	Yes	No[b]	Yes	No[b]	No	No	Yes	No	No	No	No	$O(N^2)$	Yes[g]	O(N!)[h]	Yes	Yes	No	Ranking	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Spoilers	Yes	No[b][i]	No[b]	Yes	O(N!)	Yes	$O(N^2)$[j]	No[b]	No	No[b]	Ranking	Yes	Yes
Highest median/Majority judgment[k]	Rated[l]	Yes[m]	No[n]	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	No[o]	No[p]	Depends[q]	O(N)	Yes	O(N)[r]	No[s]	Yes	Yes	Scores[t]	Yes	Yes
Minimax	Yes	No	No	Yes[u]	No	No	No	No[b]	Spoilers	Yes	No[b]	No[b]	No	$O(N^2)$	Yes	$O(N^2)$	No[b][u]	No	No[b]	Ranking	Yes	Yes
Plurality/FPTP	Yes	No	No	No[b]	No	No[b]	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A[v]	N/A[v]	No	Single mark	N/A	No
Score voting	No	No	No	No[b][c]	No	No[b]	Yes	Yes[d]	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
Runoff voting	Yes	Yes	No	No[b]	Yes	No[b]	No	No	Spoilers	No	No	No	No	O(N)[w]	Yes	O(N)[w]	Yes	Yes[x]	No	Single mark	N/A	No[y]
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No[b]	Yes	Yes	No[b]	No[p][b]	Yes	$O(N^3)$	Yes	$O(N^2)$	No[b]	No	No[p][b]	Ranking	Yes	Yes
STAR voting	No[z]	Yes	No[aa]	No[b][c]	Yes	No[b]	No	No	No	Yes	No	No	Depends[ab]	O(N)	Yes	$O(N^2)$	No	No	No[ac]	Scores	Yes	Yes
Sortition, arbitrary winner[ad]	No	No	No	No[b]	No	No[b]	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A
Random ballot[ae]	No	No	No	No[b]	No	No[b]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No

	Single manipulator
Plurality	P [Bartholdi, Tovey and Trick, SCW 1989]
Borda	P [Bartholdi, Tovey and Trick, SCW 1989]
Copeland$^{\alpha}$ (friendly tie-breaking)	P [Bartholdi, Tovey and Trick, SCW 1989]
Ranked pairs	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]
Schulze	P [Parkes and Xia, AAAI 2012]

	Single manipulator	Two manipulators
Plurality	P [Bartholdi, Tovey and Trick, SCW 1989]	P
Borda	P [Bartholdi, Tovey and Trick, SCW 1989]	NP-hard [Betzler, Niedermeier and Woeginger, IJCAI 2011; Davies, Katsirelos, Narodytska and Walsh, AAAI 2011]
Copeland$^{\alpha}$ (friendly tie-breaking)	P [Bartholdi, Tovey and Trick, SCW 1989]	NP-hard [Faliszewski, Hemaspaandra and Schnoor, AAMAS 2008]
Ranked pairs	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]	NP-hard [Xia, Zuckerman, Procaccia, Conitzer, and Rosenschein, IJCAI 2009]
Schulze	P [Parkes and Xia, AAAI 2012]	P [Gaspers, Kalinowski, Narodytska and Walsh, AAMAS 2013]



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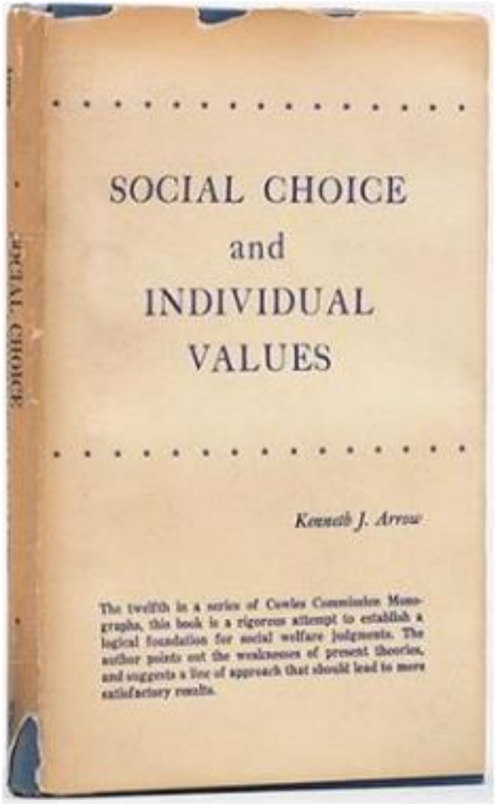

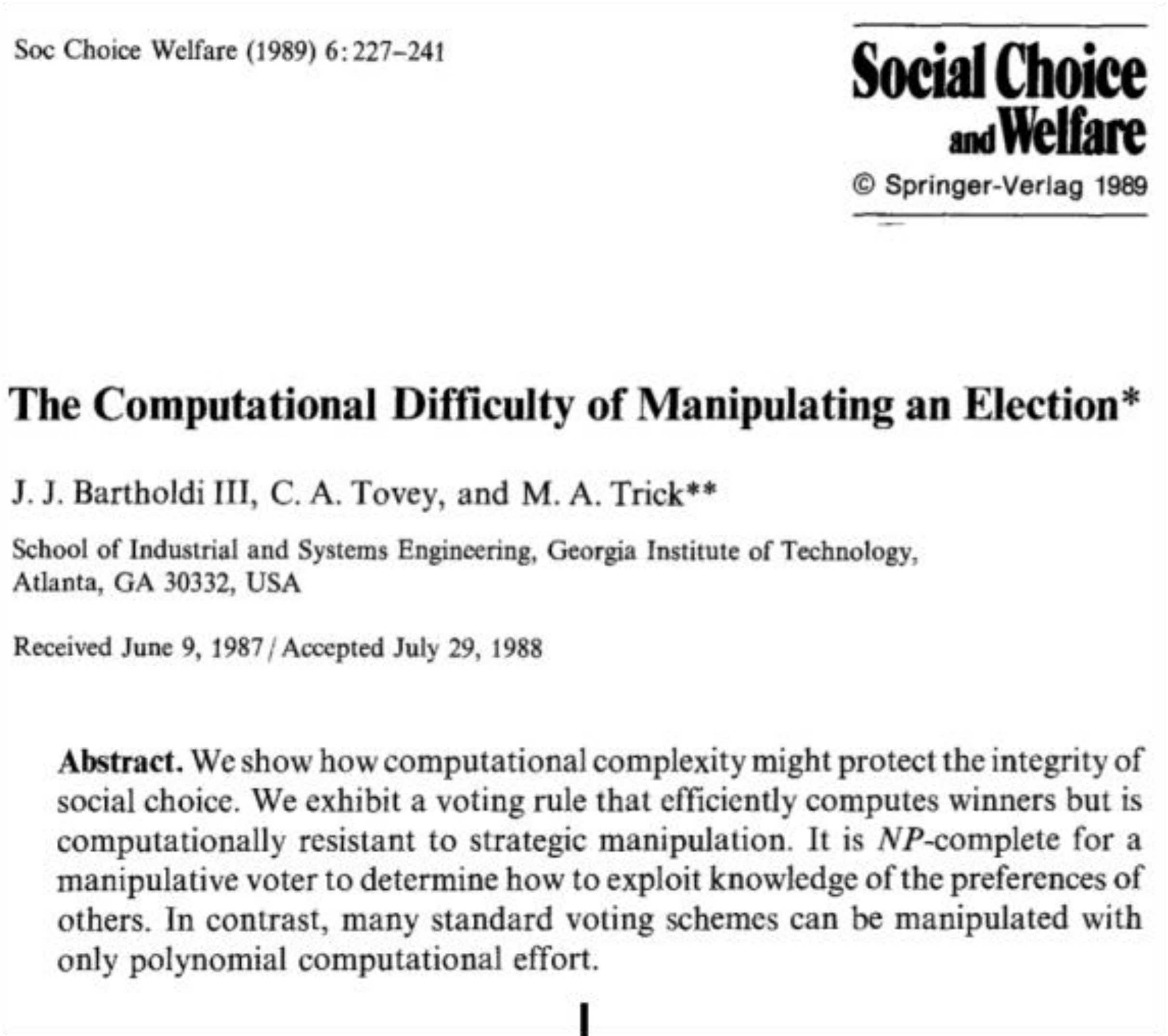

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The Computational Difficulty of Manipulating an Election*

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Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.

Social Choice Theory

Computational Social Choice

References

- “Disagreement between voting rules” example: http://www.ams.org/publicoutreach/feature-column/fcarc-voting-decision

- The “chain beats” terminology in the description of Schulze rule was borrowed from Hubert Bray’s explanatory video: https://www.youtube.com/watch?v=_HVeN0GnnuA

- The Schulze rule example is from the paper “The Schulze Method of Voting” by Markus Schulze: https://arxiv.org/abs/1804.02973

- (Possibly) strategic voting in the selection of Olympics host: https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic_2_Plurality_Runoff.pdf

References

- Non-monotonicity of STV:

  Gideon Doron and Richard Kronick
  Single Transferable Vote: An Example of a Perverse Social Choice Function
  American Journal of Political Science, Vol. 21, No. 2 (May, 1977), pg 303-311
  https://www.jstor.org/stable/2110496

- “Which voting rule is the best” poll
  https://hal.archives-ouvertes.fr/hal-00609810/document

References

- "Sports elimination via max flow" with IPL teams: https://www.youtube.com/watch?v=XK6qZjHWo9A

- When it's easy to recognize the *existence* of a beneficial manipulation but hard to *find* a manipulative vote.

  "Search versus Decision for Election Manipulation Problems"
  Hemaspaandra, Hemaspaandra, and Menton
  https://dl.acm.org/doi/10.1145/3369937

References

- A shared proof of Arrow's and Gibbard-Satterthwaite theorems: "Arrow's Theorem and the Gibbard-Satterthwaite Theorem: A Unified Approach" by Philip J. Reny. https://www.sciencedirect.com/science/article/pii/S0165176500003323

- Another nice proof of Arrow's theorem: https://www.youtube.com/watch?v=QLi_5LCwJ20

- The "big table" of voting rules is from the Wikipedia article "Comparison of Electoral Systems".

- A tribute to Arrow (see the end of the article for a nice anecdote): https://www.nytimes.com/2017/02/21/business/economy/kenneth-arrow-dead-nobel-laureate-in-economics.html