

# Lecture 1

## Voting

Rohit Vaish (IIT Delhi)

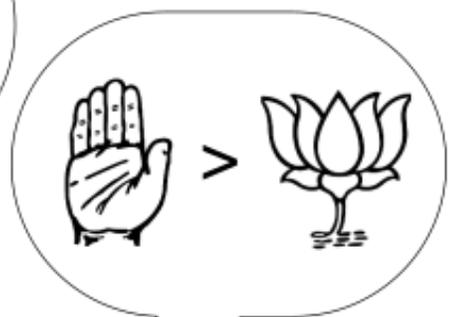
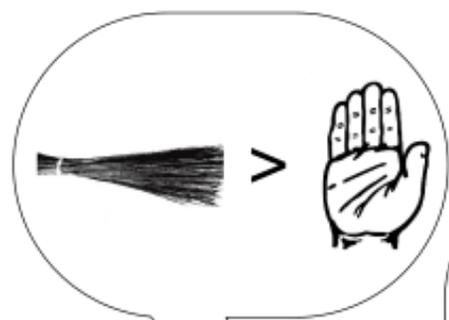
Slides available at



# What this course is about

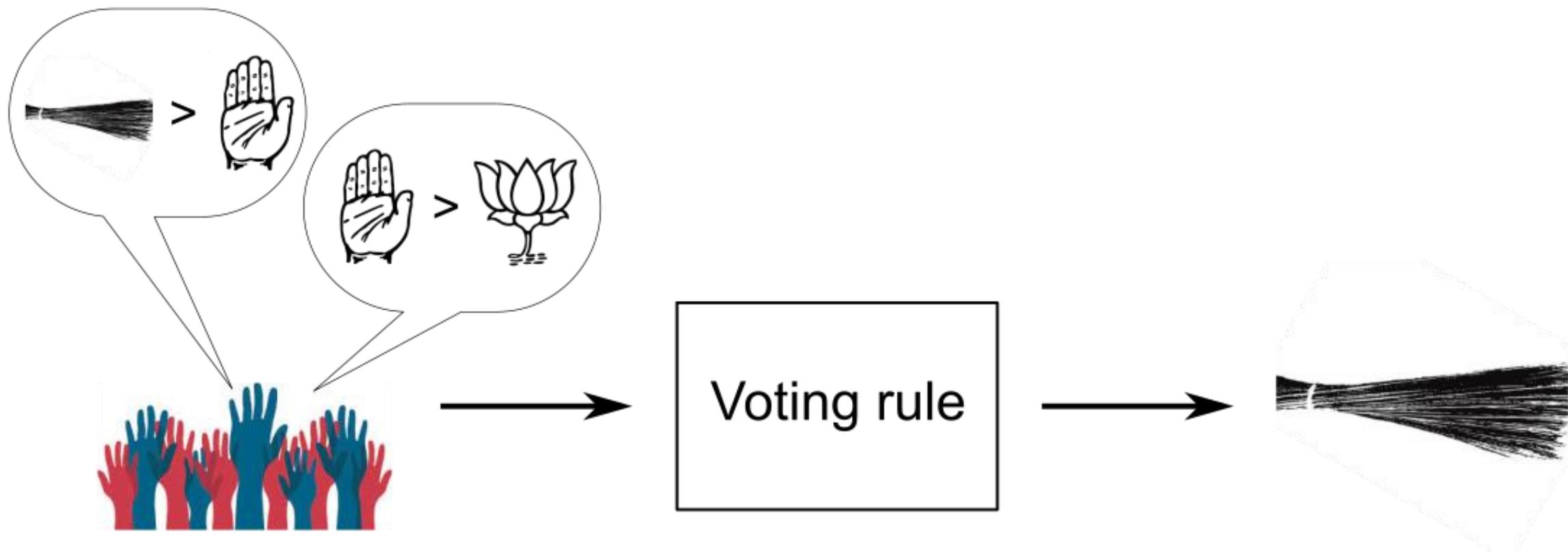


Understanding the role of **computation**  
in **collective decision-making** problems

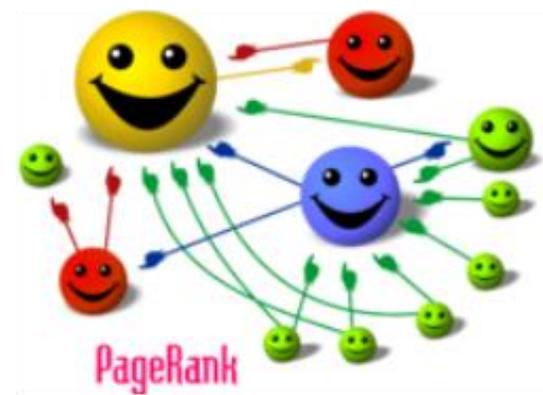


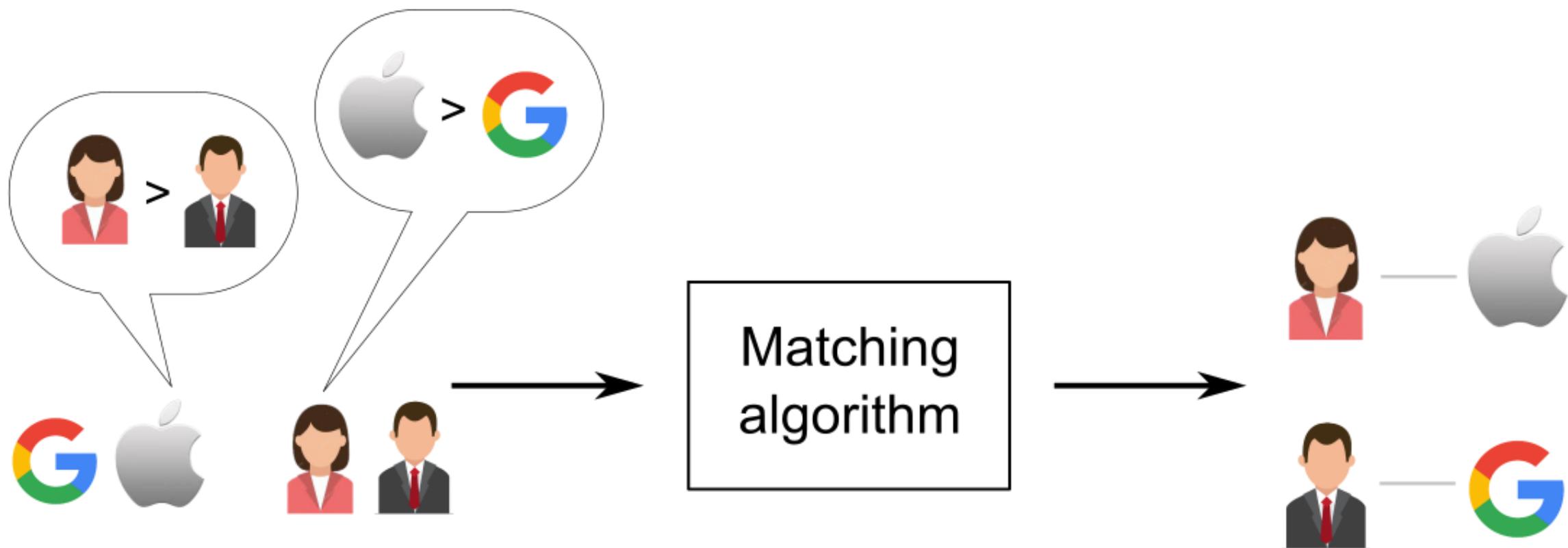
Voting rule

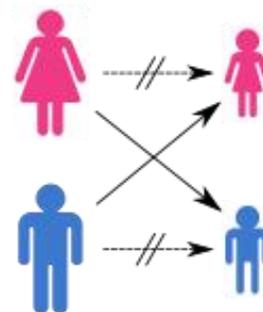
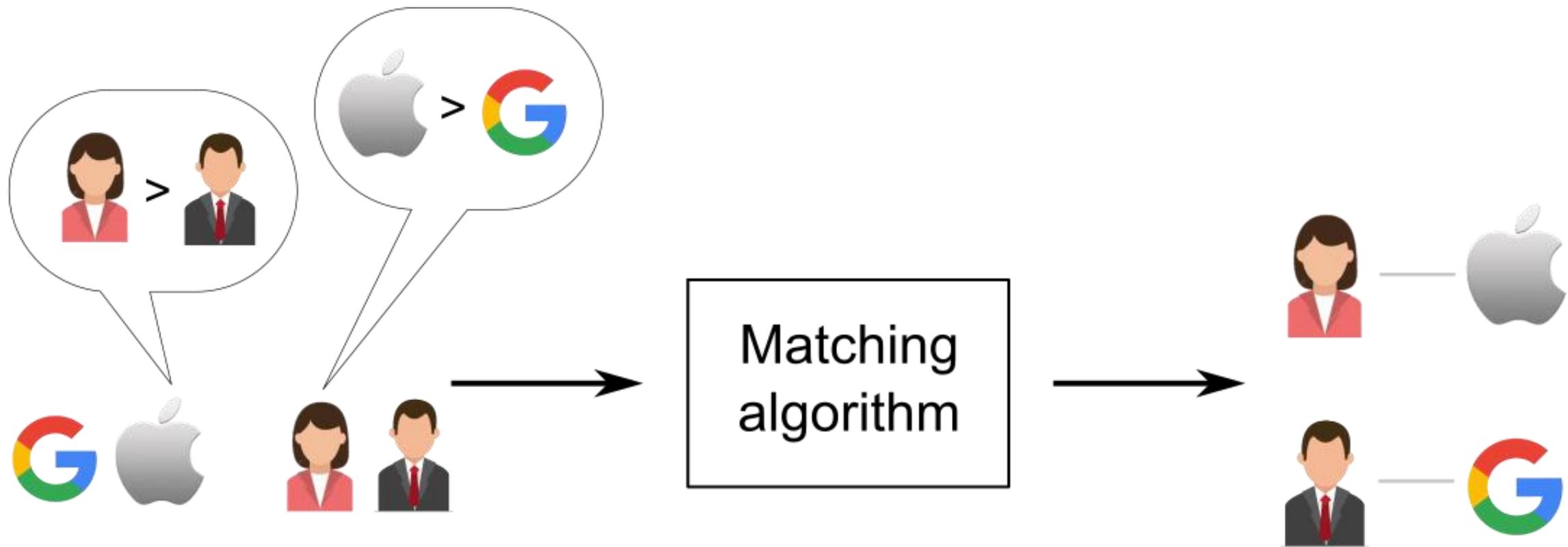




amazon









Allocation  
mechanism





Allocation  
mechanism

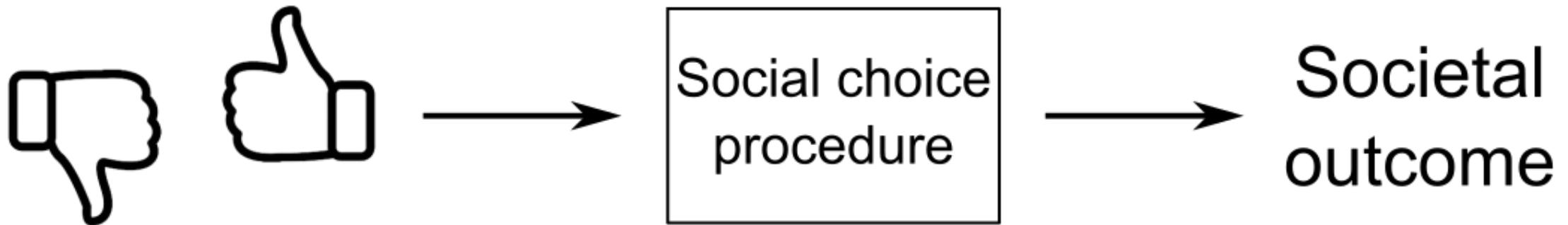


# Social Choice

Making a **collective** decision from **individual** preferences

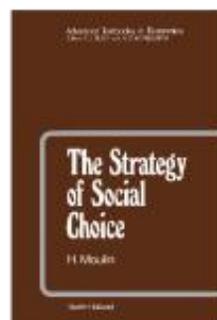
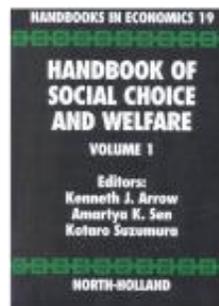
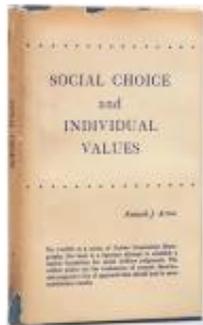
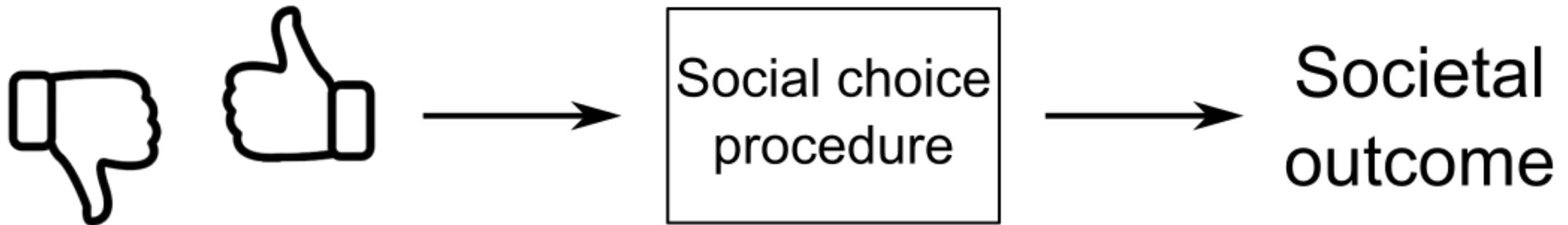
# Social Choice

Making a **collective** decision from **individual** preferences



# Social Choice

Making a **collective** decision from **individual** preferences



Arrow



Sen



Maskin



Roth



Shapley

## Classical Social Choice

Does there exist a social choice procedure  
with the desired economic properties?

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*Does there exist a "truthful" voting rule?*

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*Is there a matching procedure that is "stable"?*



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*Does there exist a "truthful" voting rule?*

*Is there a matching procedure that is "stable"?*



*Is there an allocation procedure that is "fair" and "economically efficient"?*

## Classical Social Choice

Does there **exist** a social choice procedure with the desired economic properties?



Does there **exist** a "truthful" voting rule?

*Is there* a matching procedure that is "stable"?



*Is there* an allocation procedure that is "fair" and "economically efficient"?

## Classical Social Choice

Does there **exist** a social choice procedure with the desired economic properties?

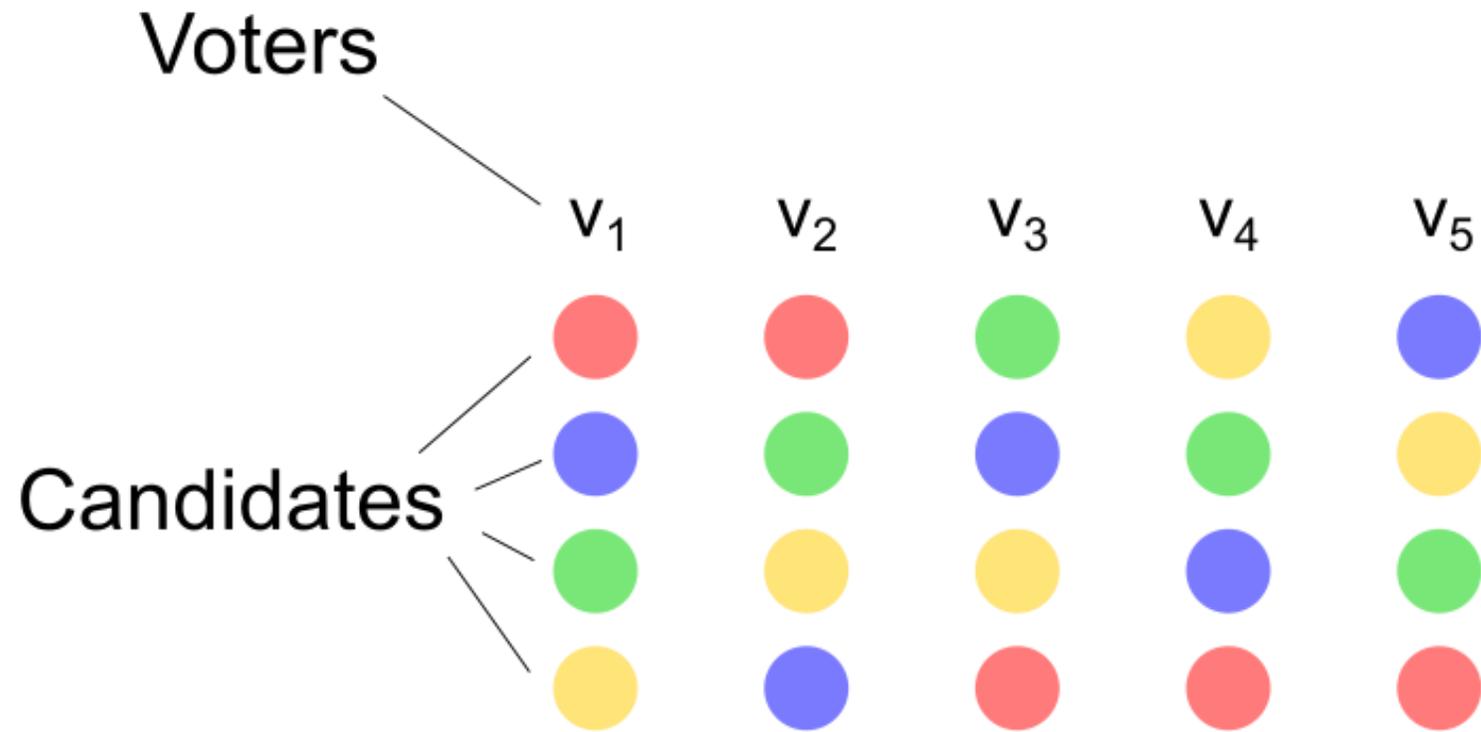
## Computational Social Choice

(This course)

How does **computation** influence the economic properties of social choice procedures?

# Voting Setup

# Voting Setup

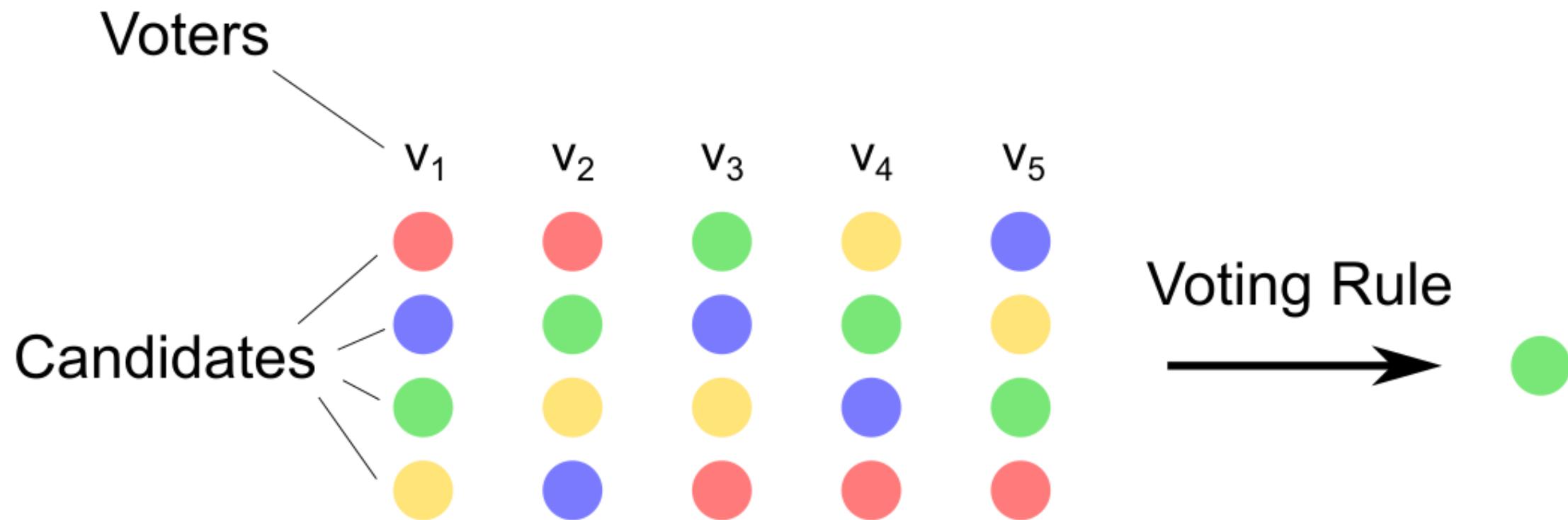


# Voting Setup



**Goal:** Pick exactly one winning candidate.

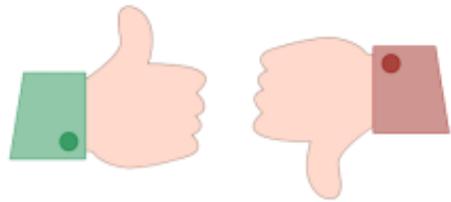
# Voting Setup



**Goal:** Pick exactly one winning candidate.

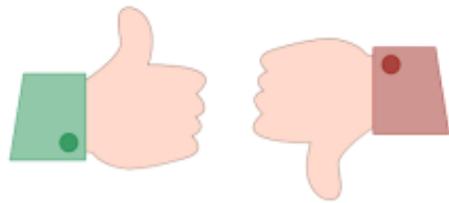


Voting Rule





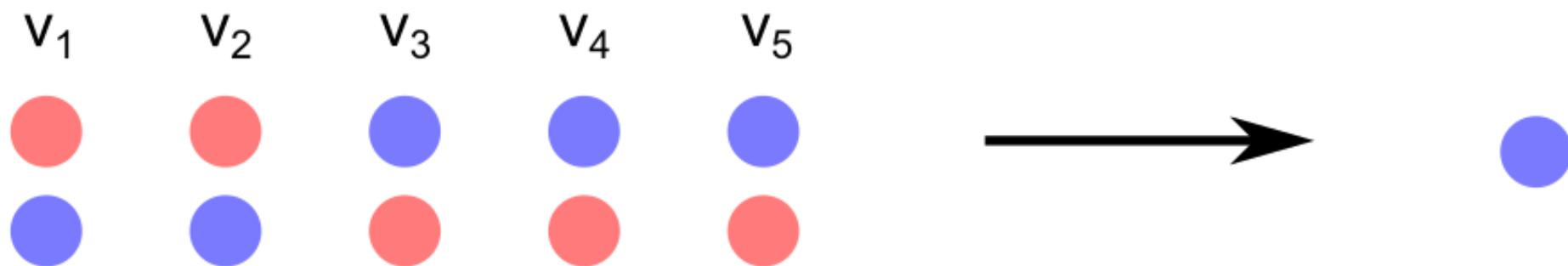
Voting Rule



# Voting with Two Candidates

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Majority!



# Voting with More Than Two Candidates

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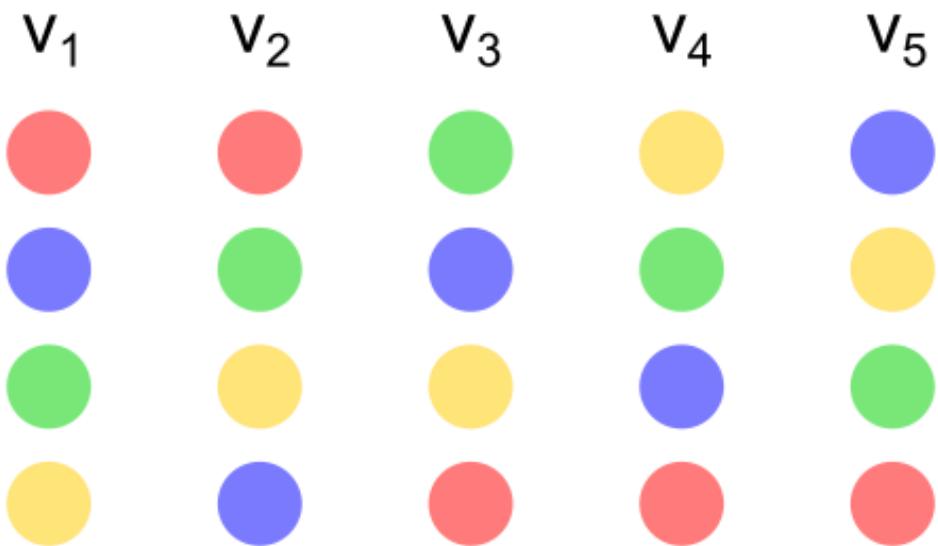
No candidate may have a majority...

1

Plurality

# 1

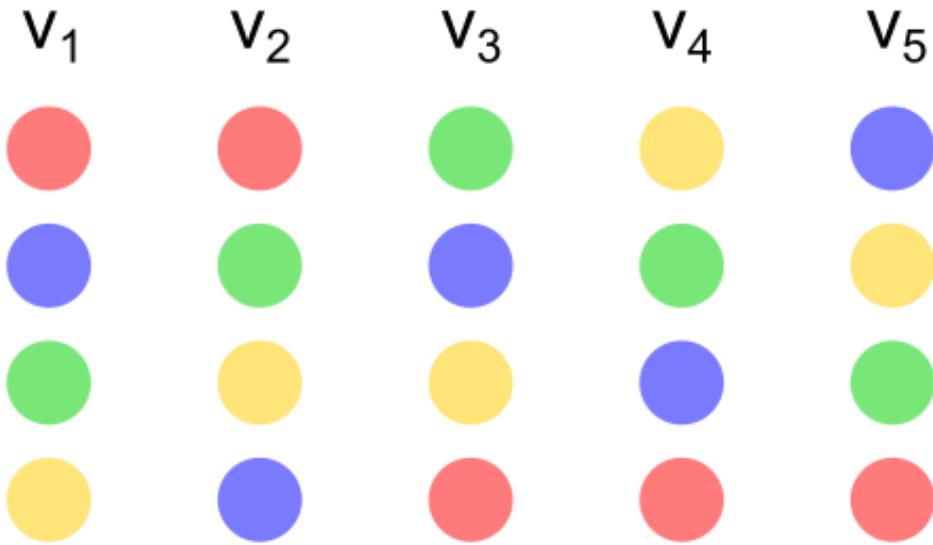
Plurality



# 1

Candidate with the most first-place votes wins

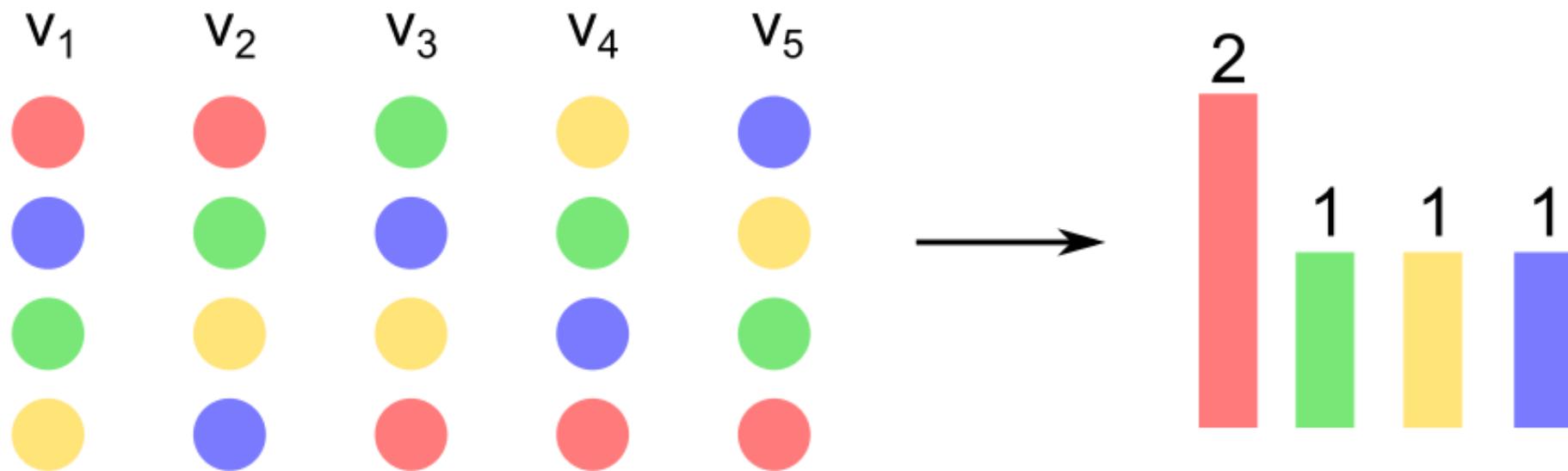
Plurality



# 1

Candidate with the most first-place votes wins

Plurality



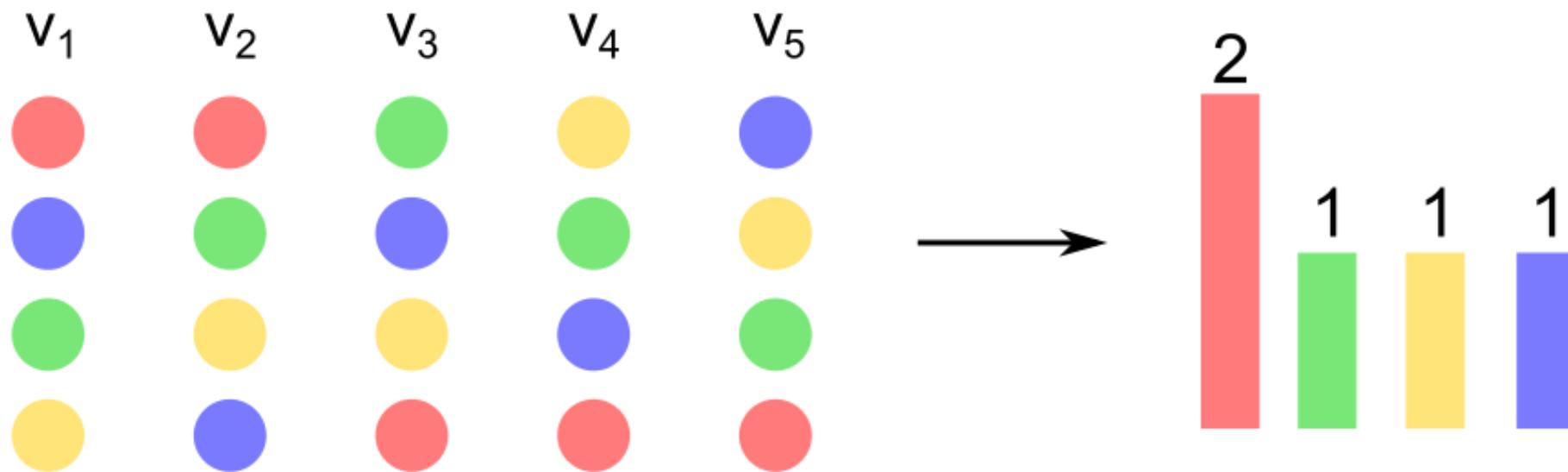
Plurality winner: ●

# 1

Candidate with the most first-place votes wins\*

\*subject to tie-breaking: lexicographic, random, ...

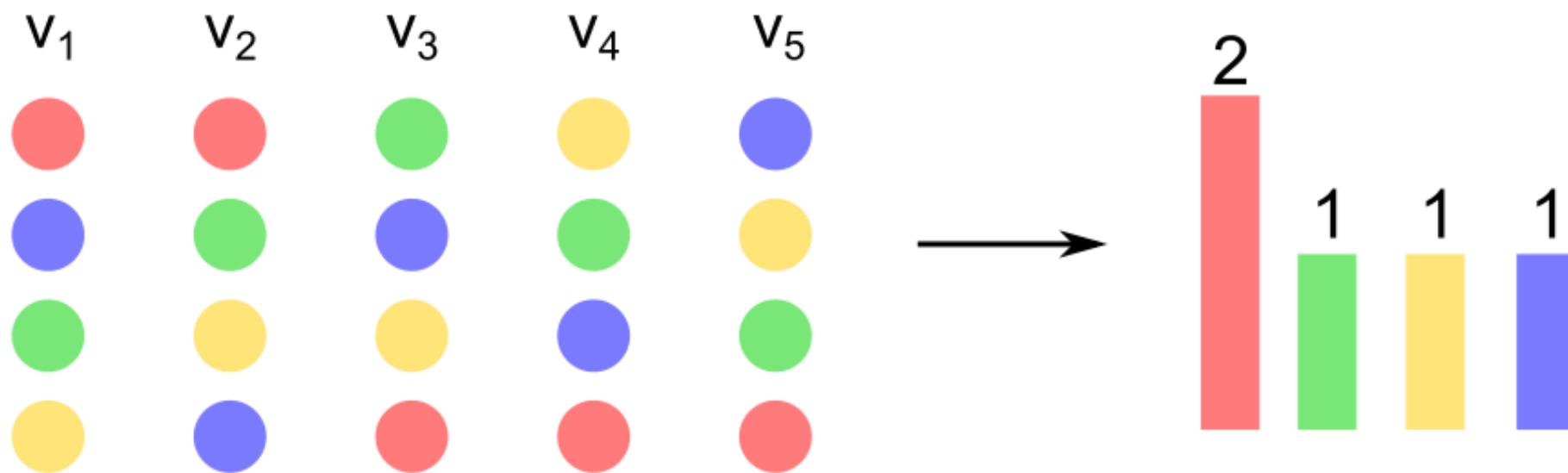
Plurality



Plurality winner: ●

# 1

**Problem:** A majority prefers ● over the Plurality winner.



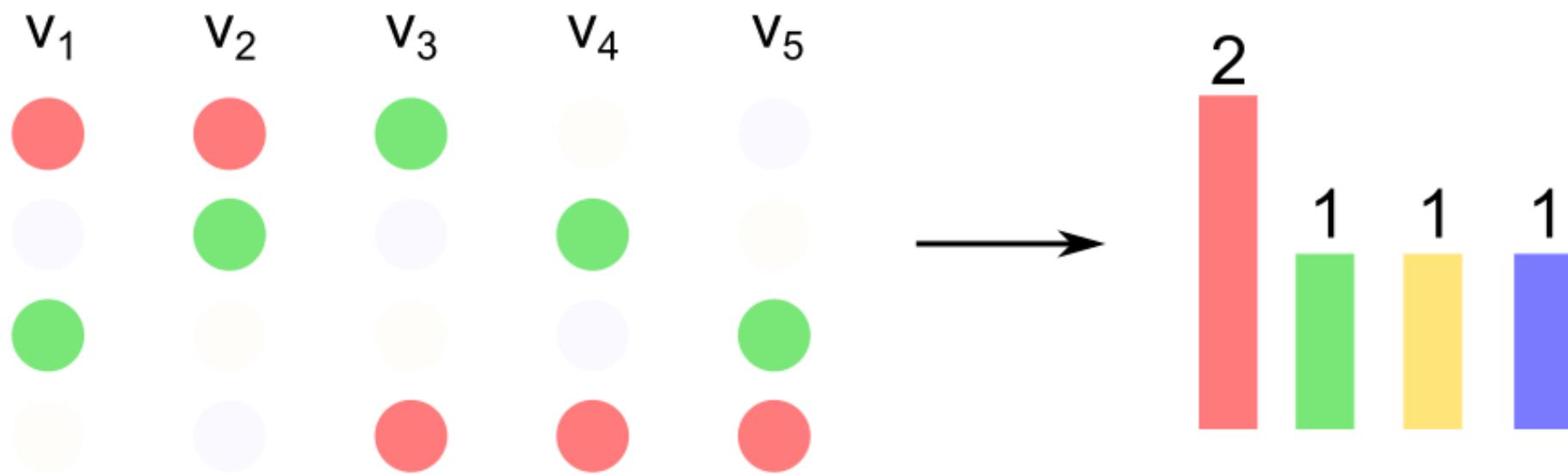
Plurality winner: ●

Plurality

# 1

**Problem:** A majority prefers ● over the Plurality winner.

Plurality



Plurality winner: ●

# 1

Plurality

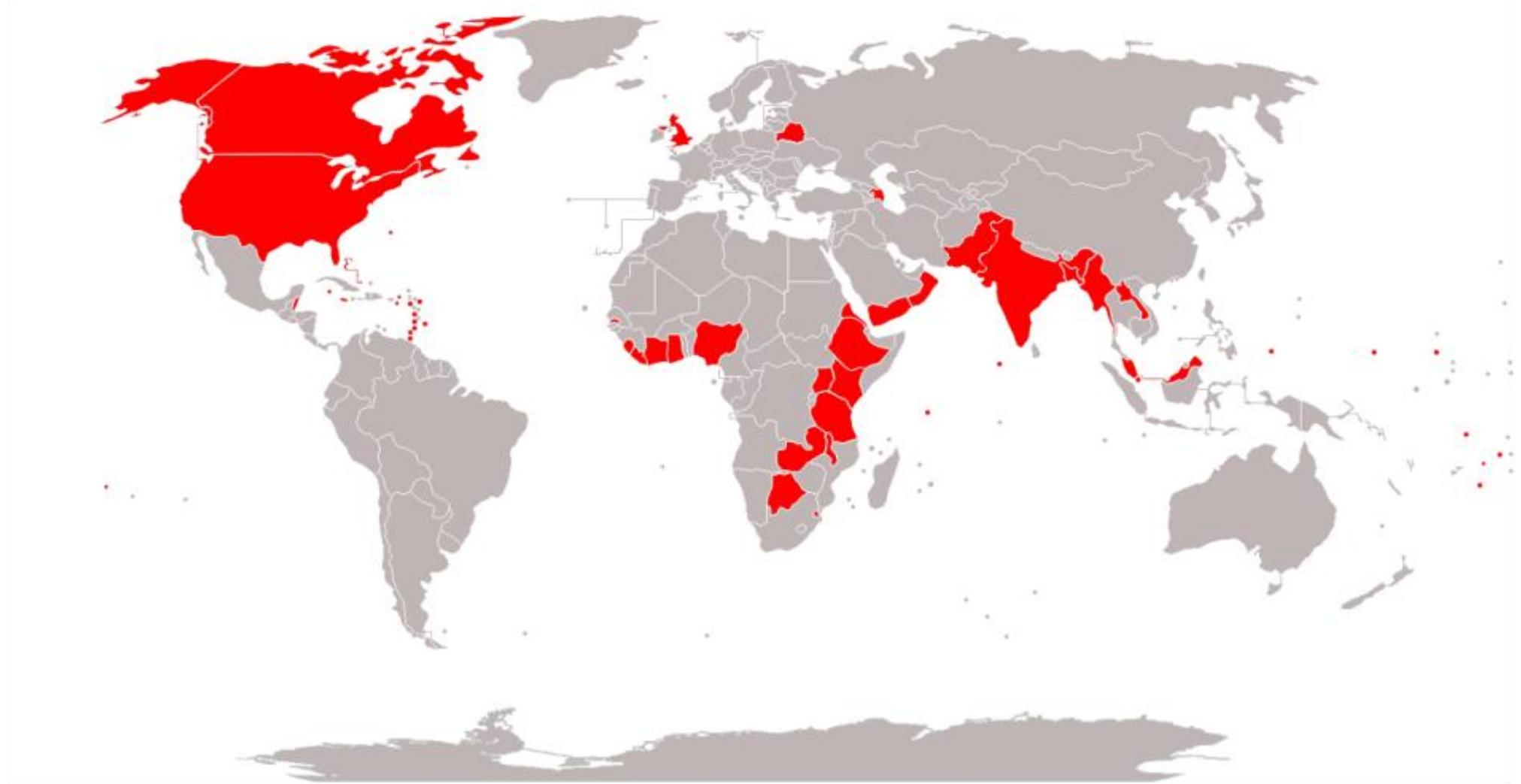


Image Source: Wikipedia article on "Electoral system" (Jan 2022)

2

Borda  
Count

# 2

Borda  
Count



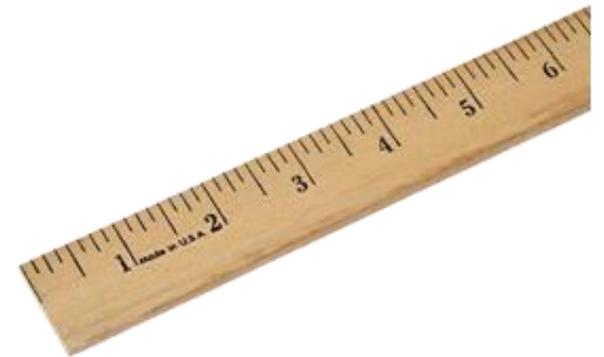
Jean-Charles de Borda  
(1733-1799)

# 2

## Borda Count



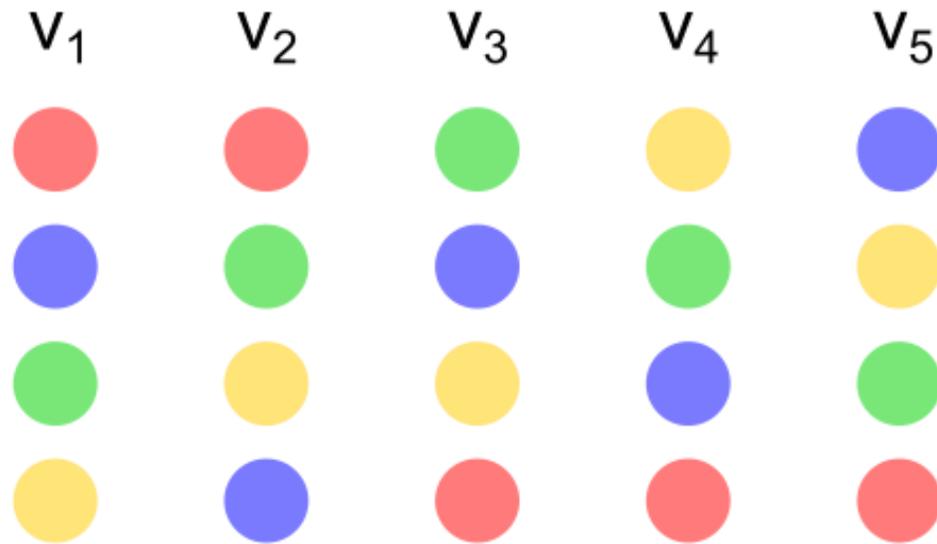
Jean-Charles de Borda  
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# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

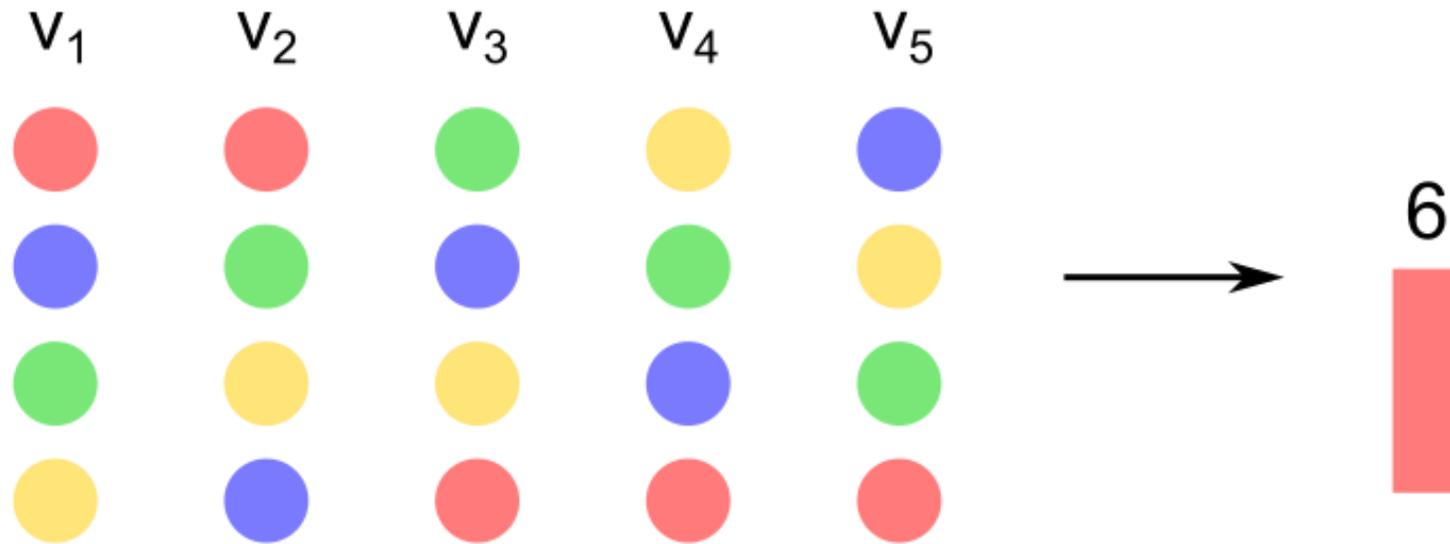
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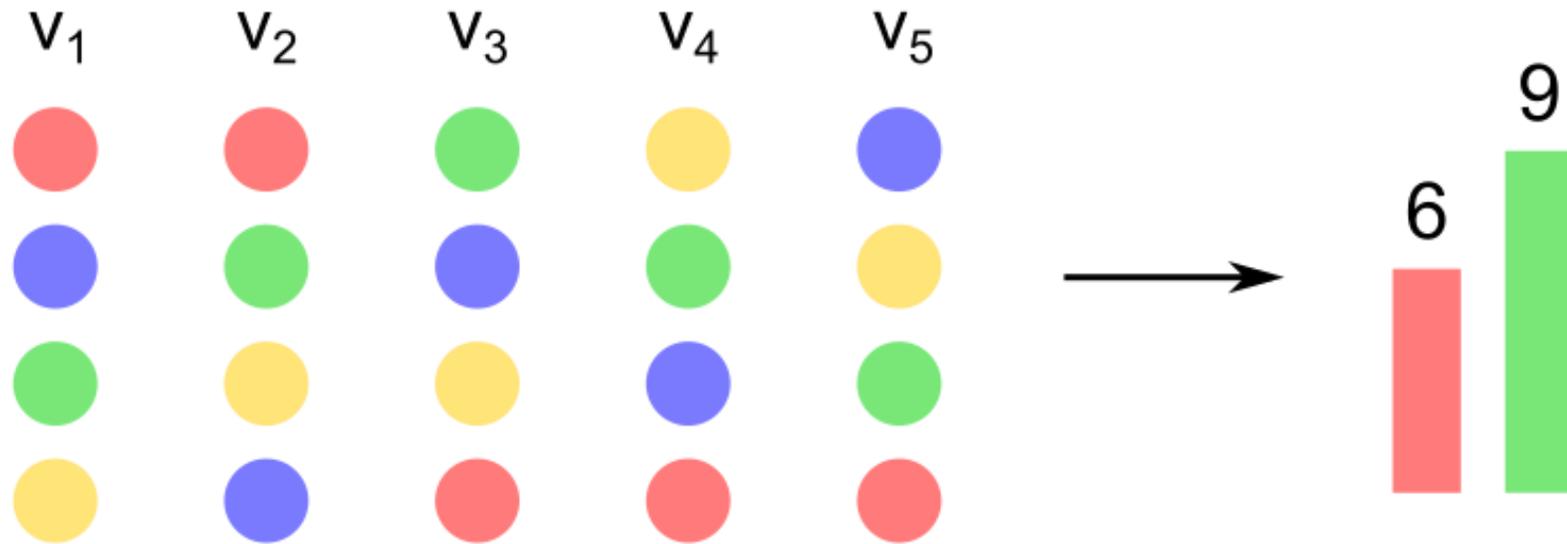
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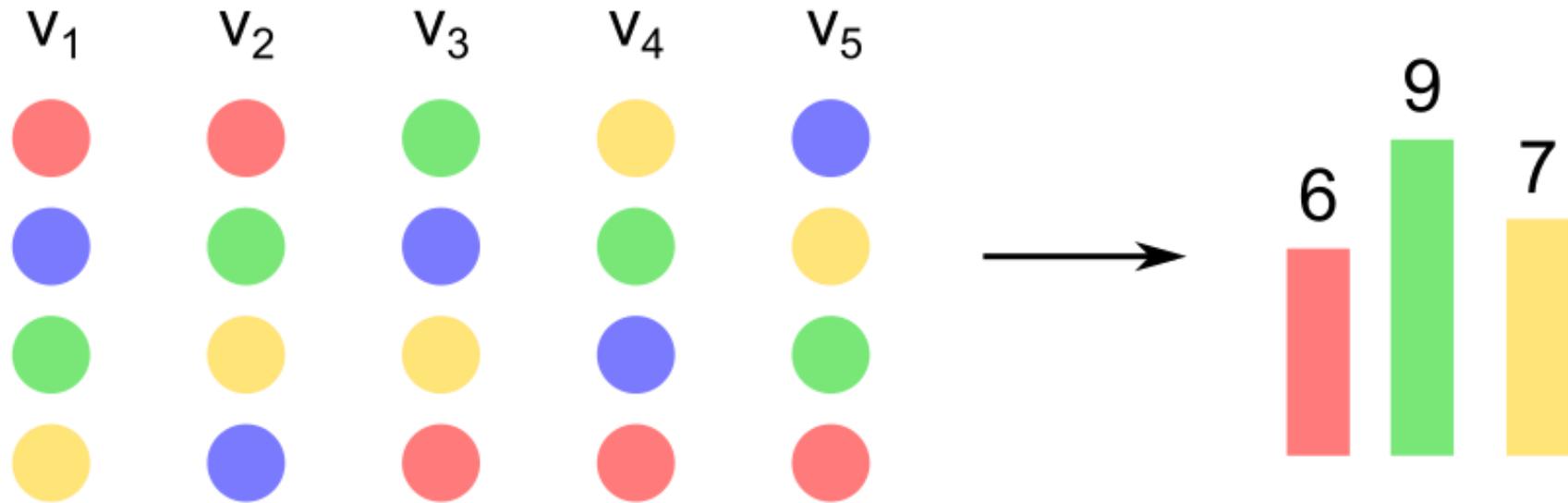
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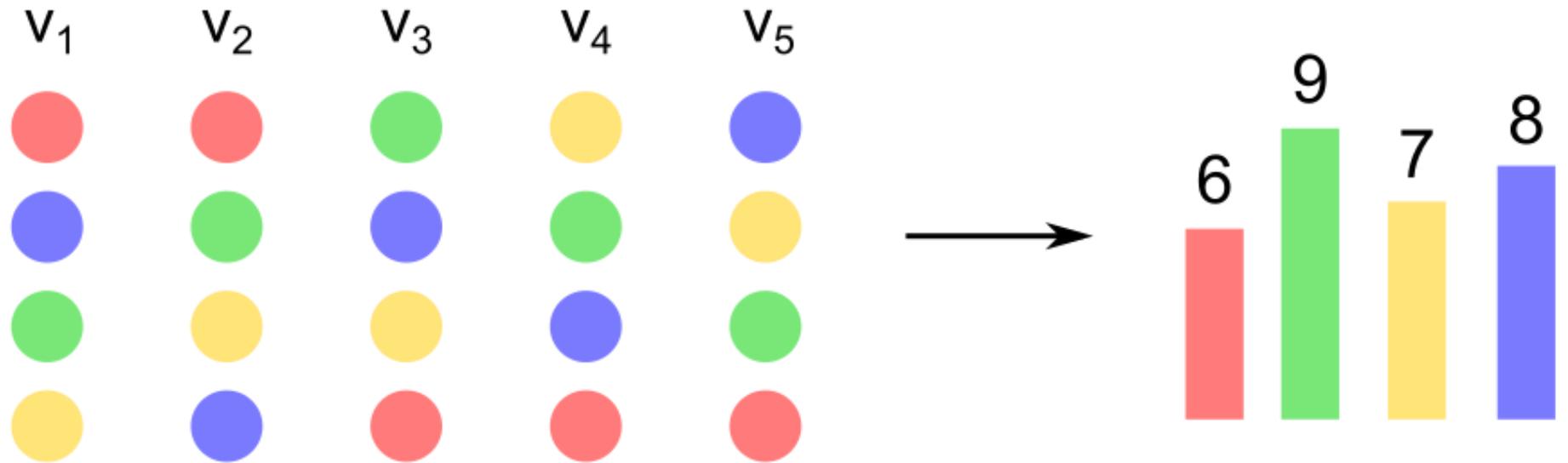
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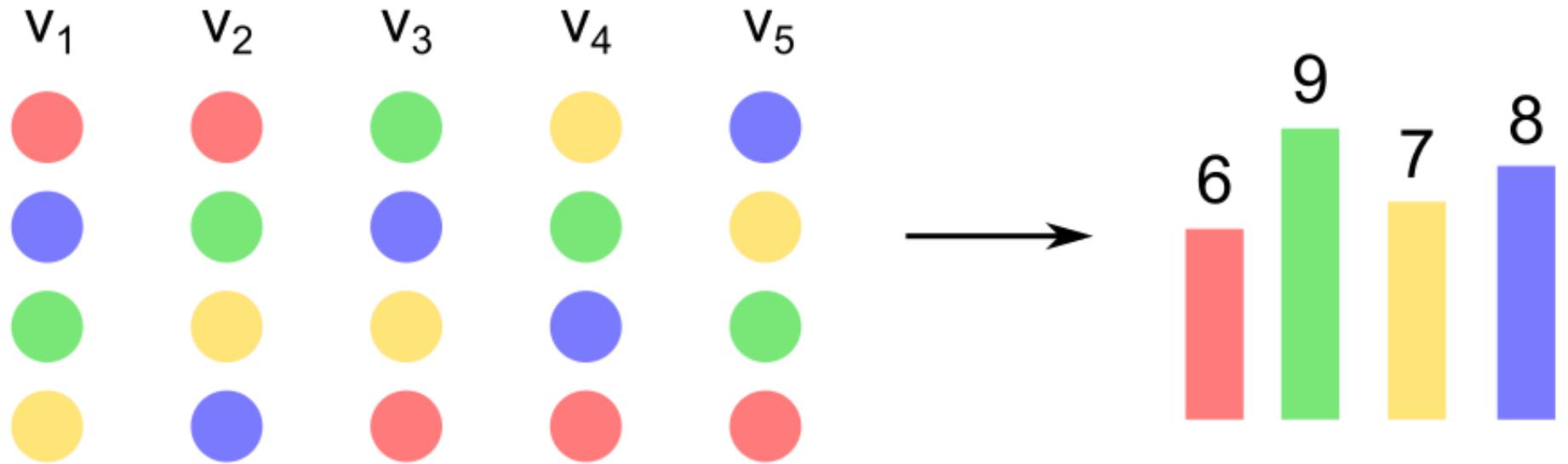
Borda  
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Borda  
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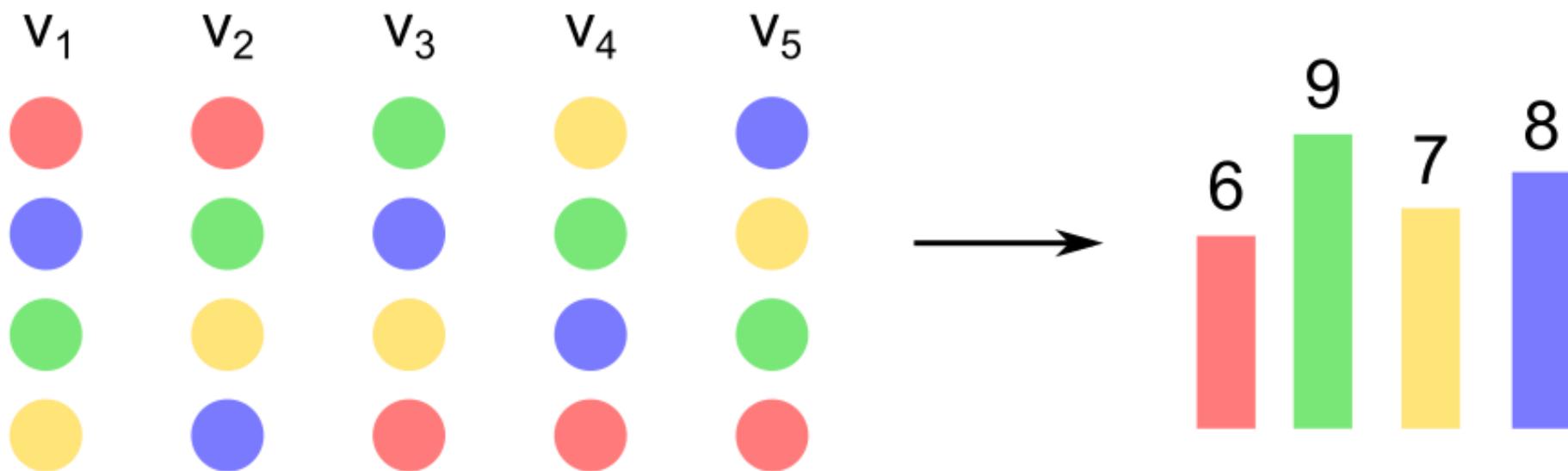


Borda winner: ●

# 2

**Problem:** Susceptible to strategic voting (manipulation).

Borda  
Count

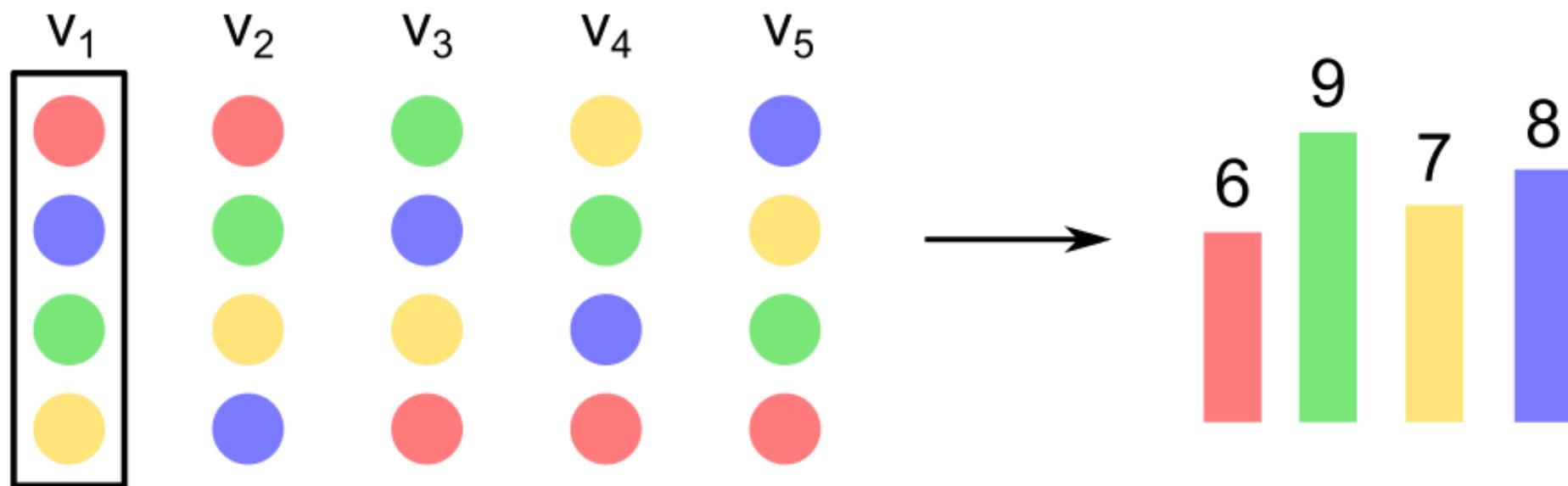


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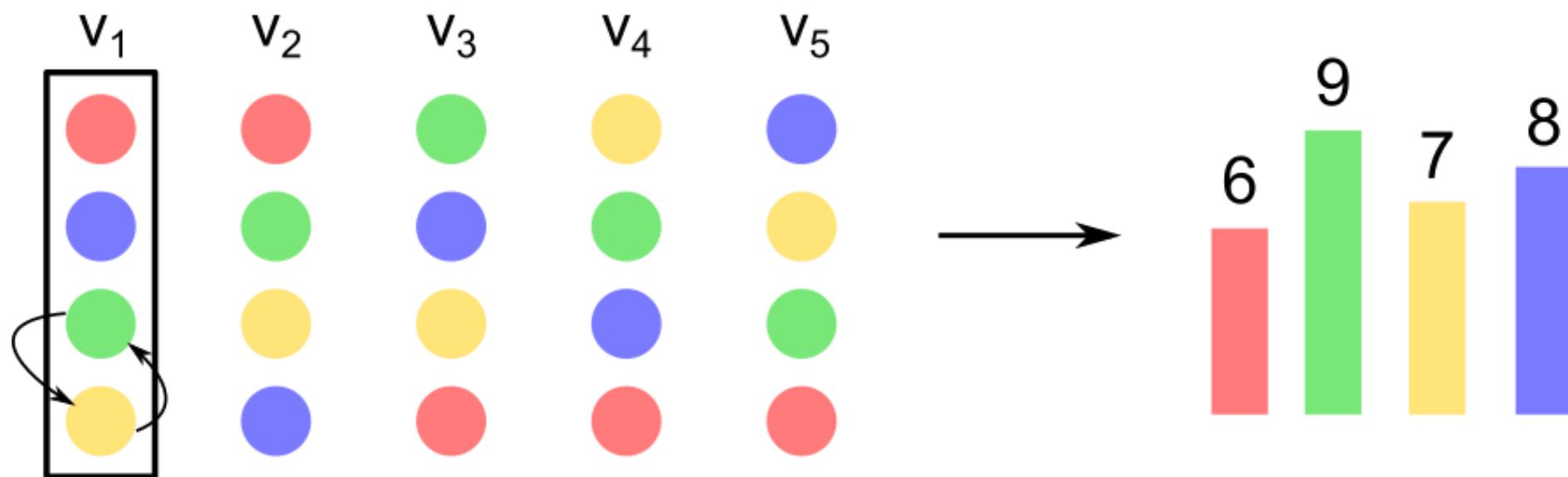


Borda winner: ●

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Borda  
Count

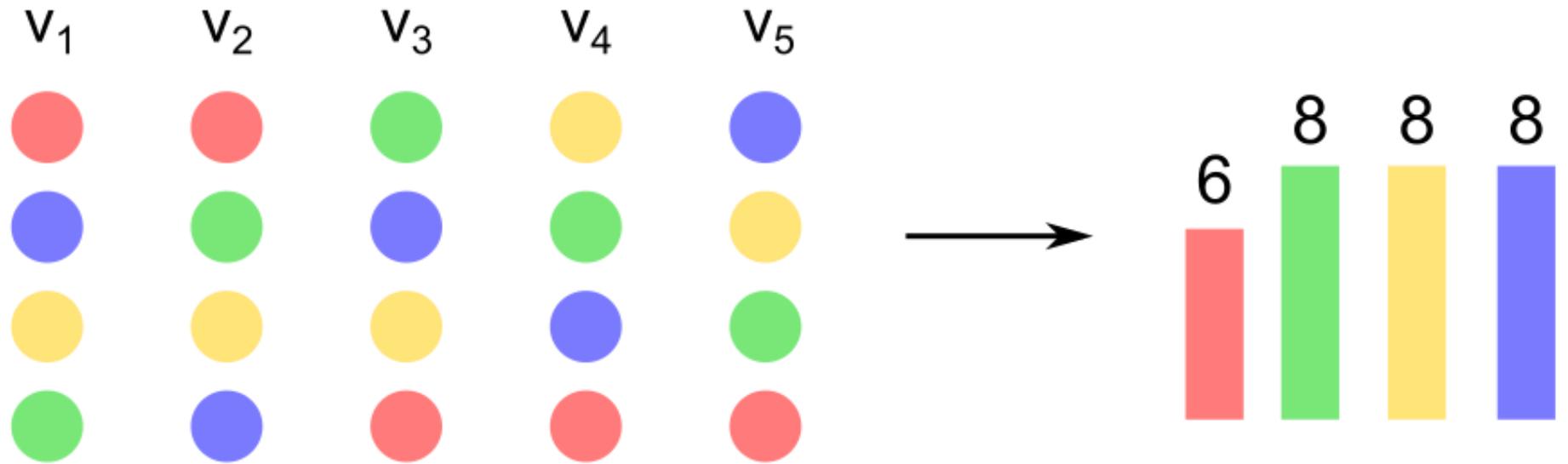


Borda winner: ●

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Borda  
Count



Borda winner: ●

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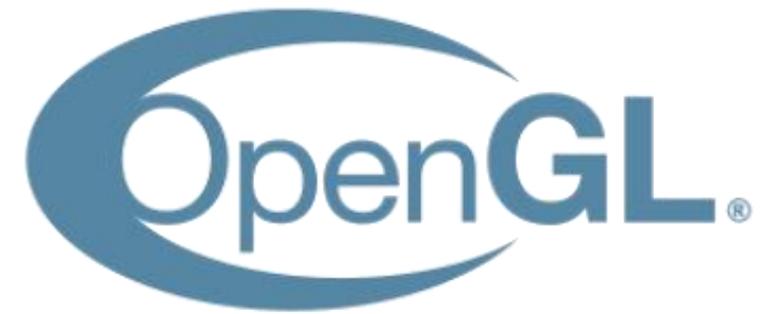
"My scheme is intended for only honest men."



Borda  
Count

# 2

Borda  
Count



# 3

Plurality  
With  
Runoff

# 3

## Plurality With Runoff



# 3

Plurality  
With  
Runoff

# 3

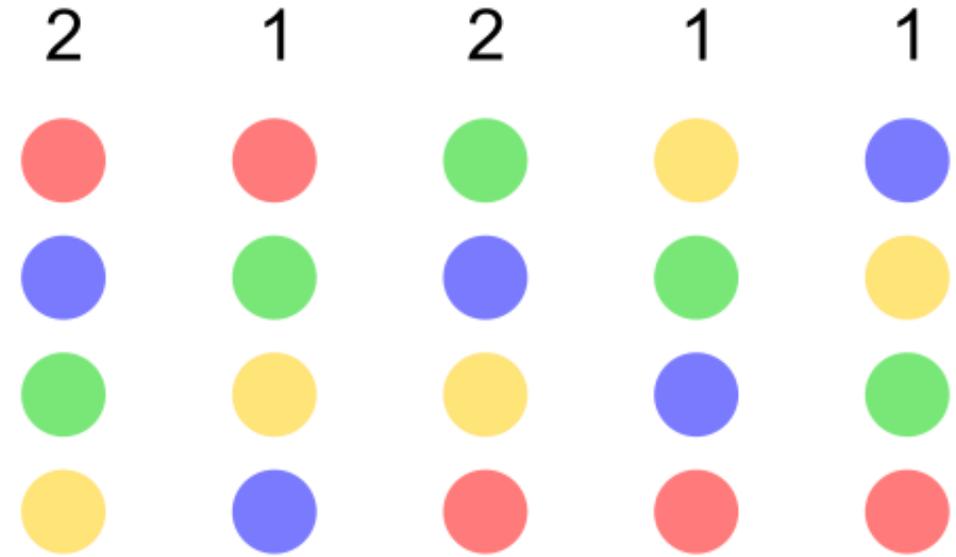
The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

Plurality  
With  
Runoff

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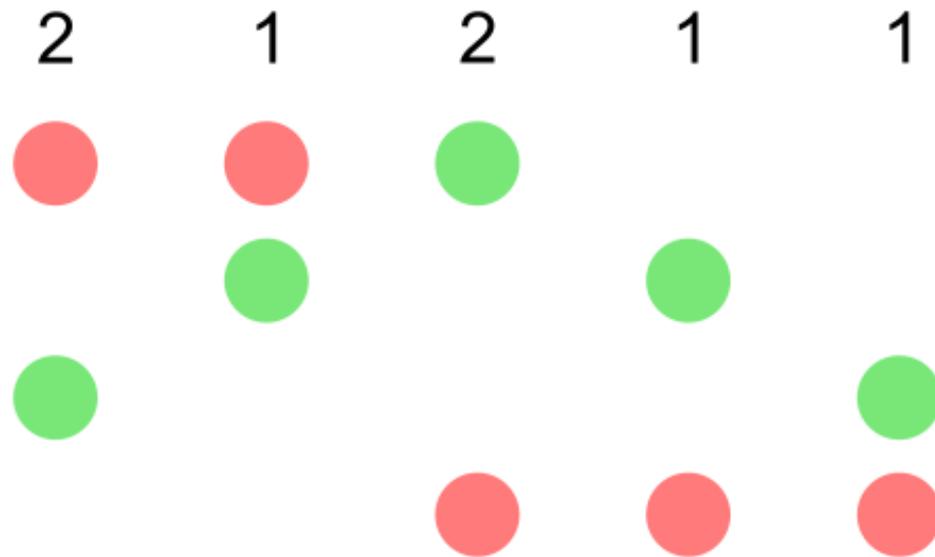
Plurality  
With  
Runoff



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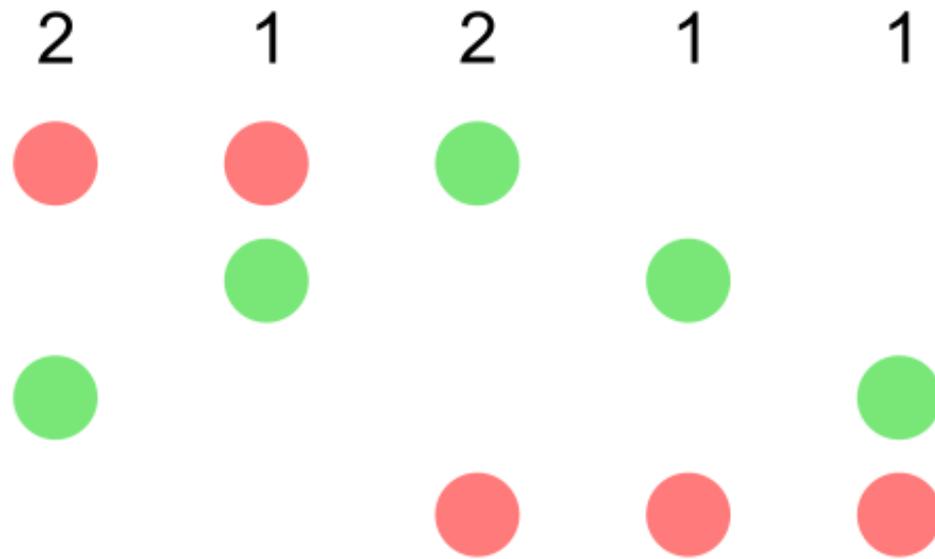
Plurality  
With  
Runoff



# 3

The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

Plurality  
With  
Runoff



Plurality with runoff winner: ●

4

Single  
Transferable  
Vote

(Instant-Runoff)

# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

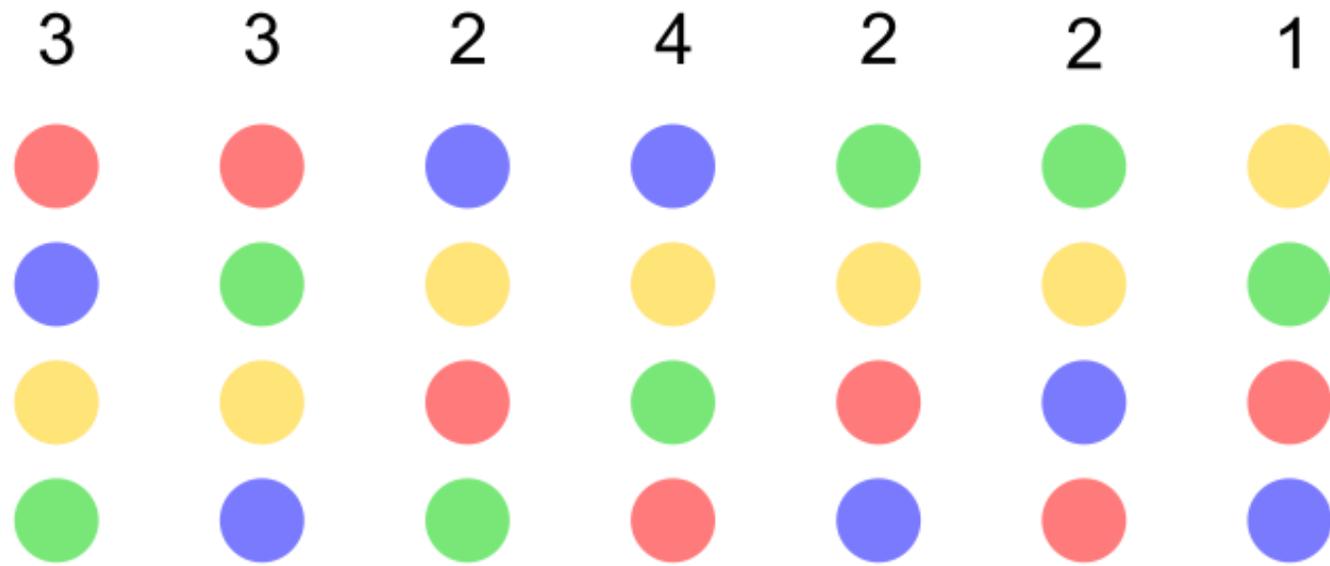
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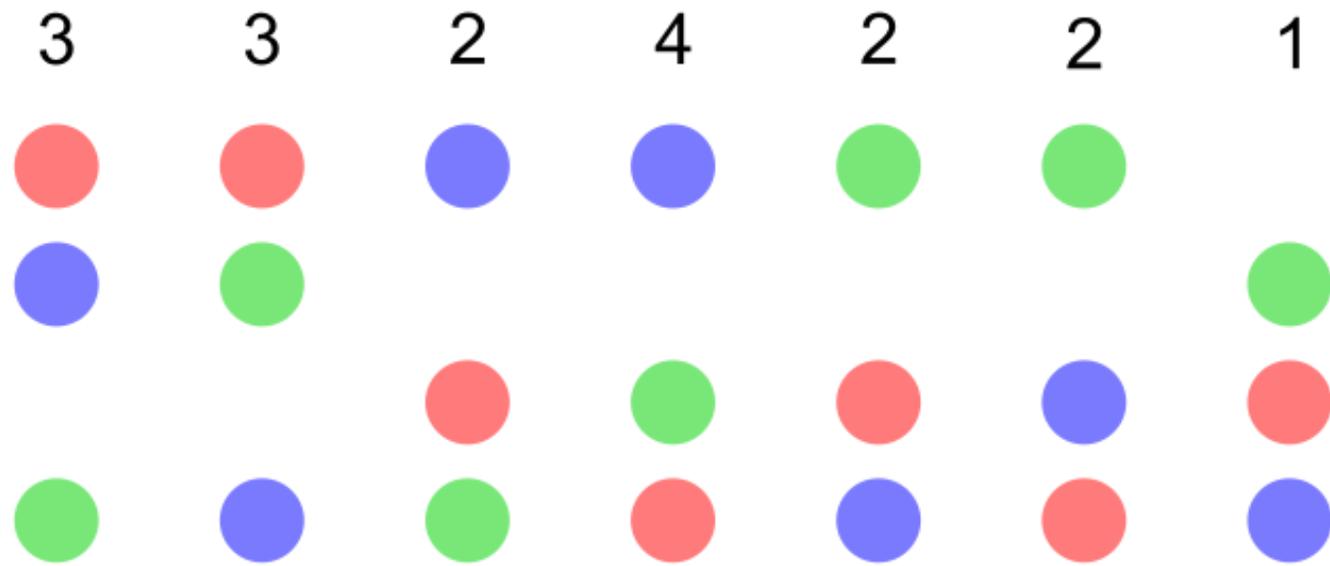
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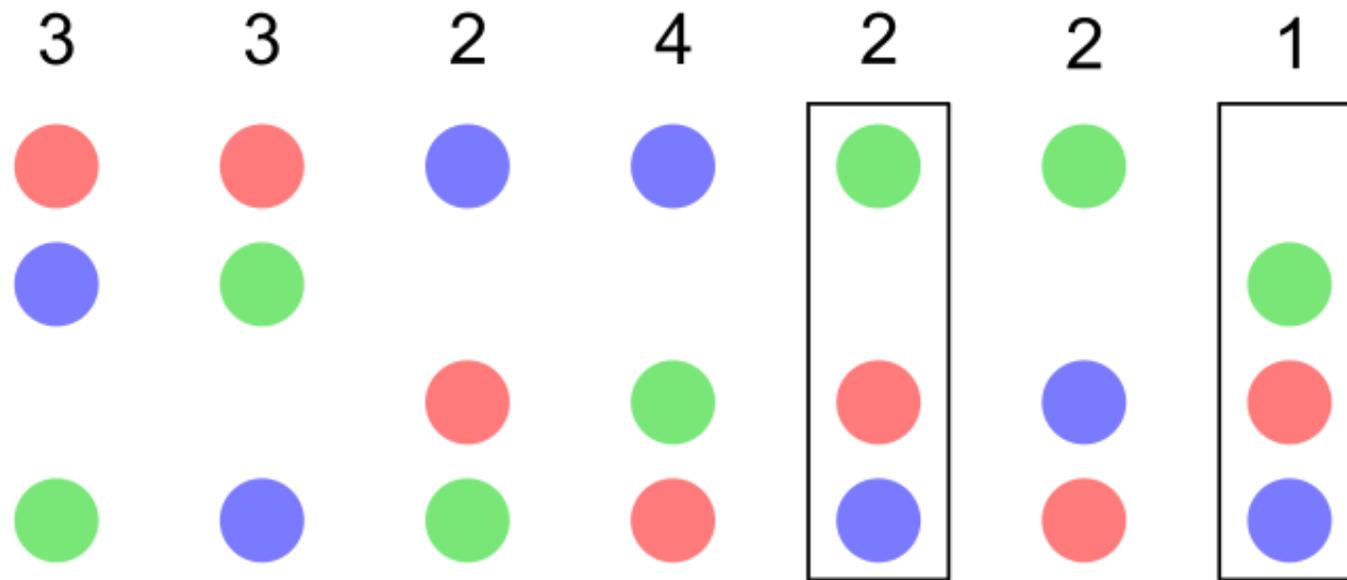
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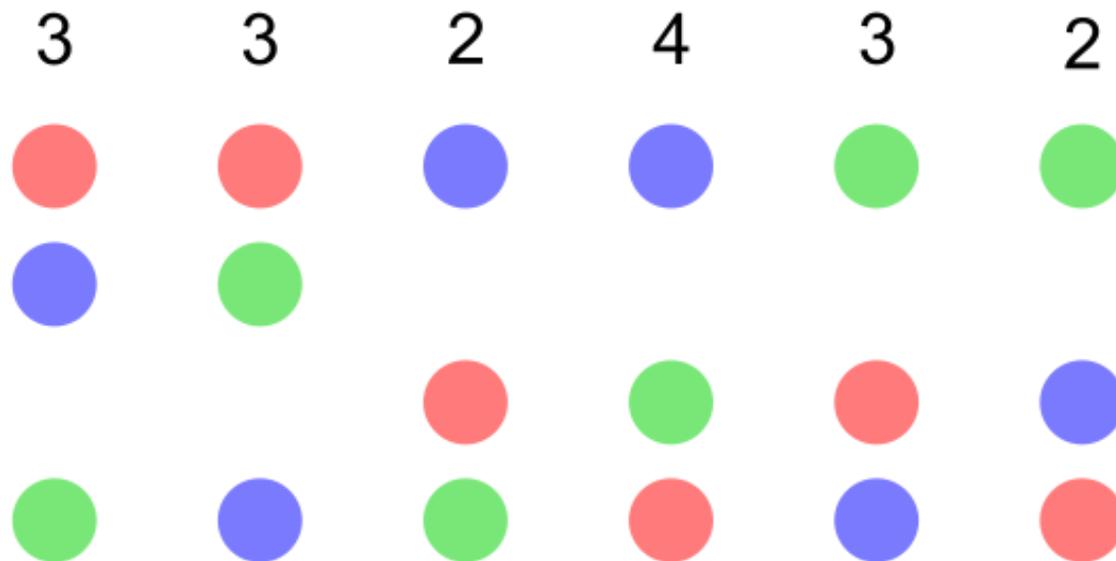
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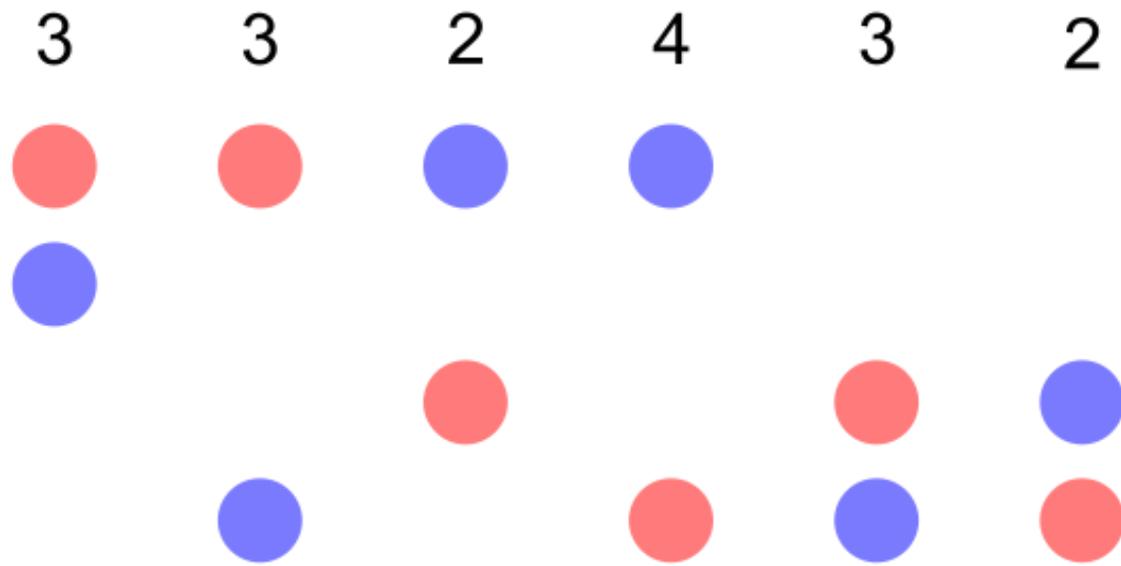
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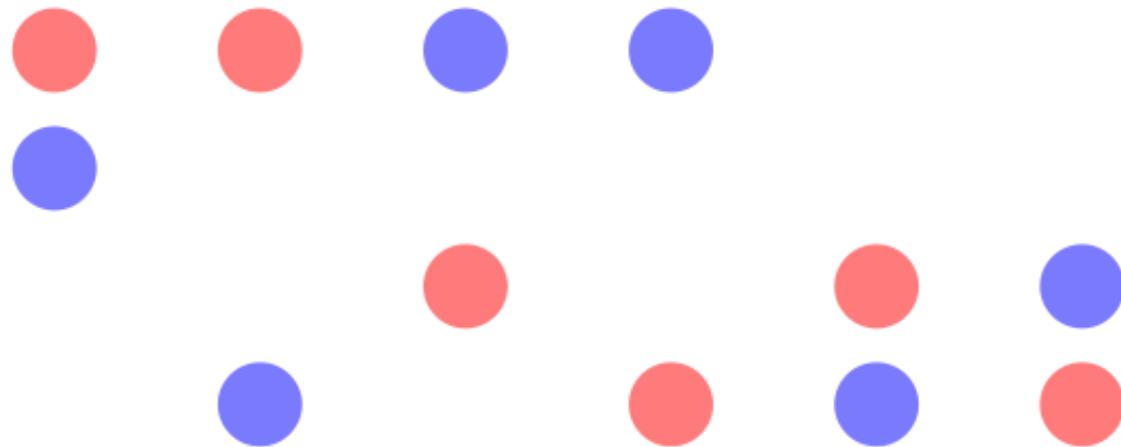
Single  
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Vote  
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In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

3      3      2      4      3      2



STV winner: ●

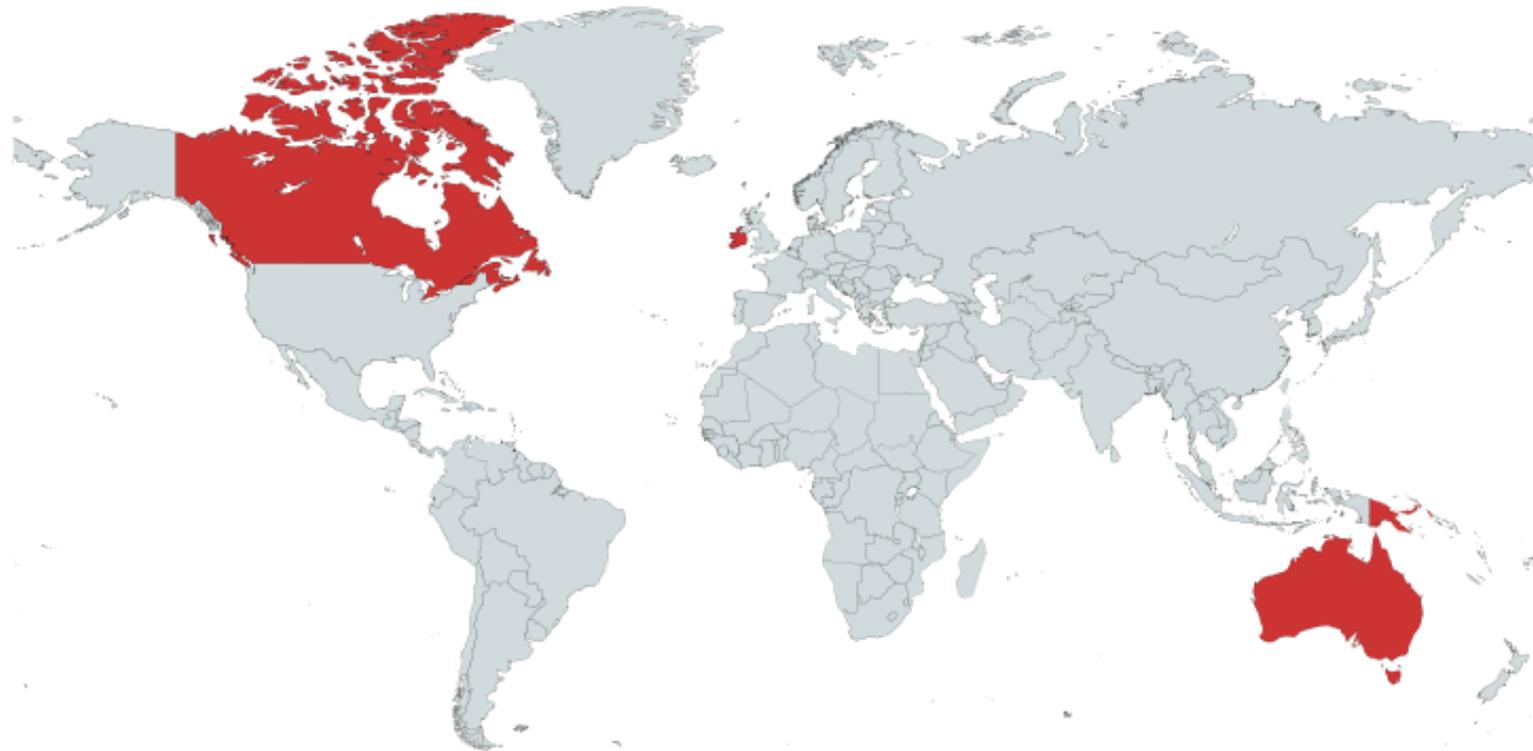
Single  
Transferable  
Vote

(Instant-Runoff)

# 4

Single  
Transferable  
Vote

(Instant-Runoff)



4

Single  
Transferable  
Vote

(Instant-Runoff)



4

Single  
Transferable  
Vote

(Instant-Runoff)



# 4

Single  
Transferable  
Vote

(Instant-Runoff)

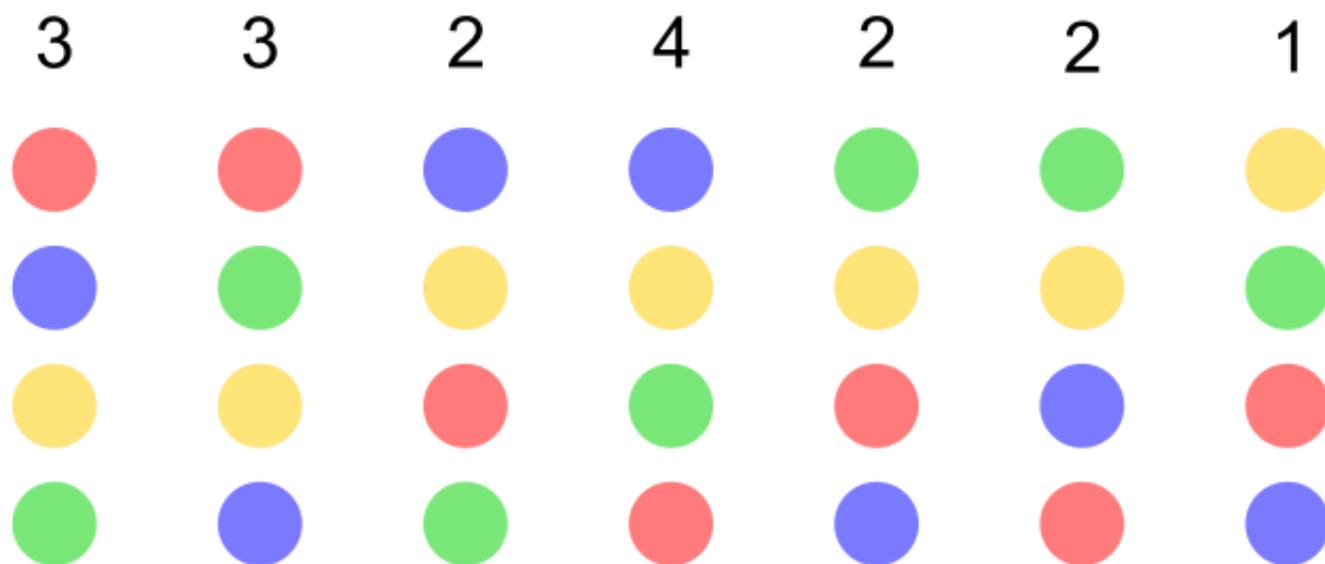
**Problem:** Failure of *monotonicity*  
(improving a candidate's support could make it worse off)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)



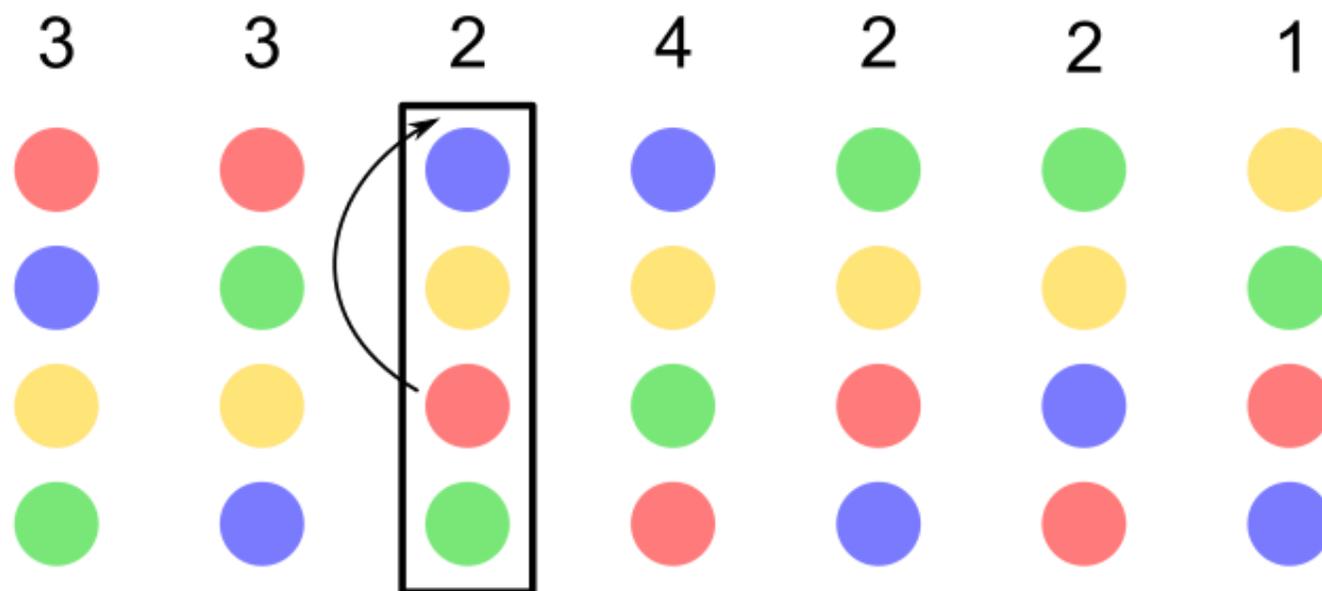
Recall that STV winner was 

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
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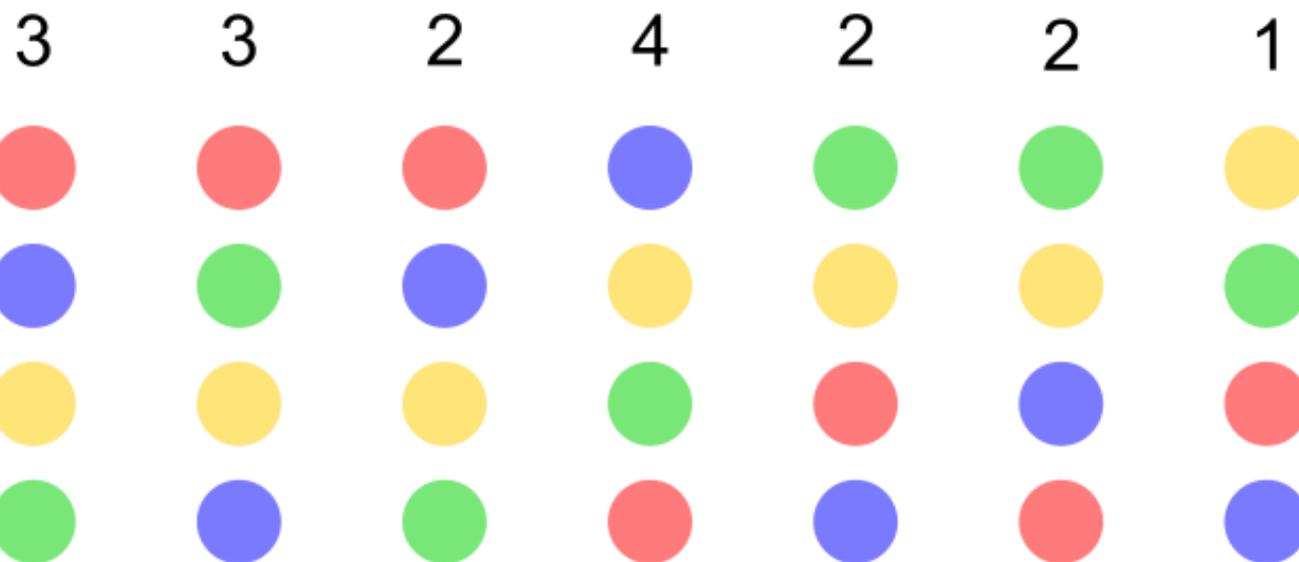


Recall that STV winner was ●

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was 

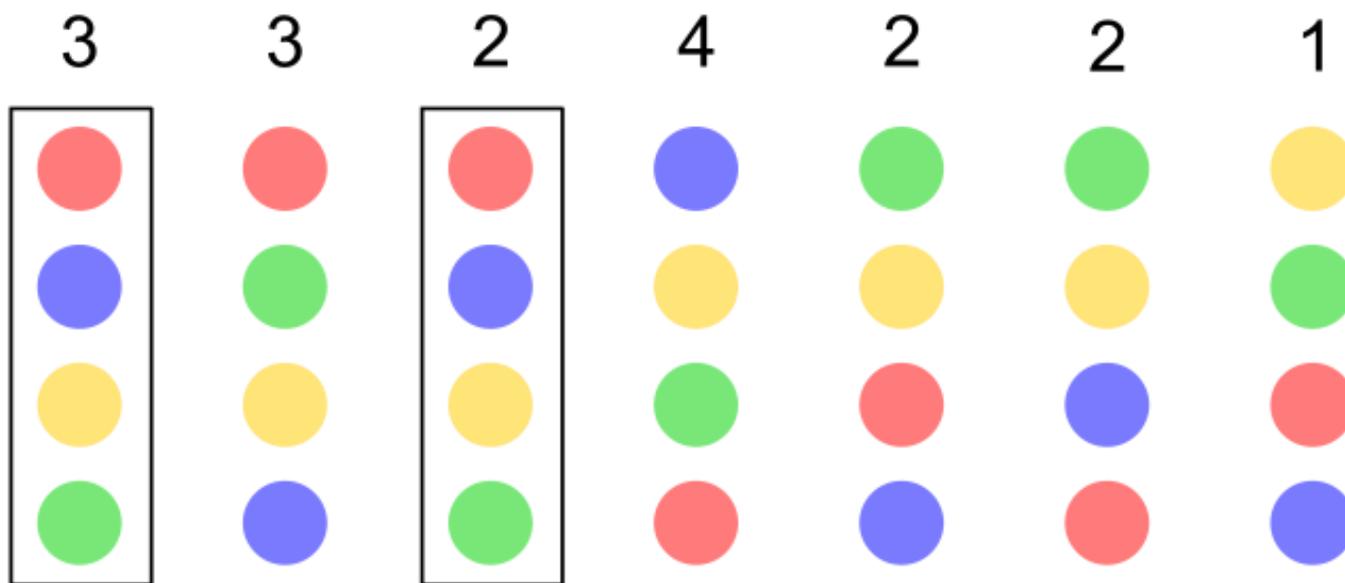
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Single  
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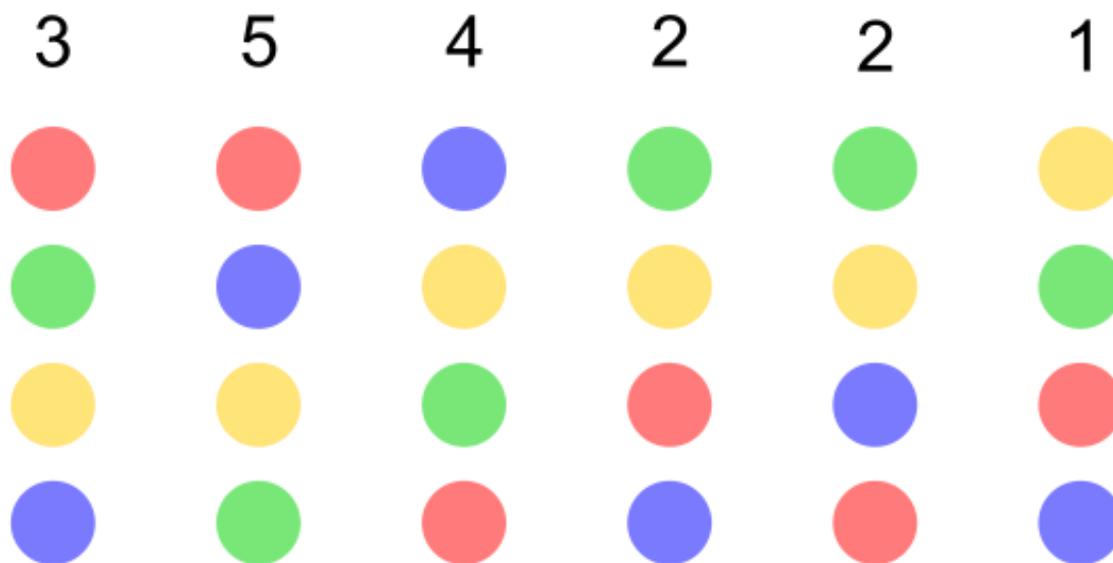
Recall that STV winner was ●

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(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)

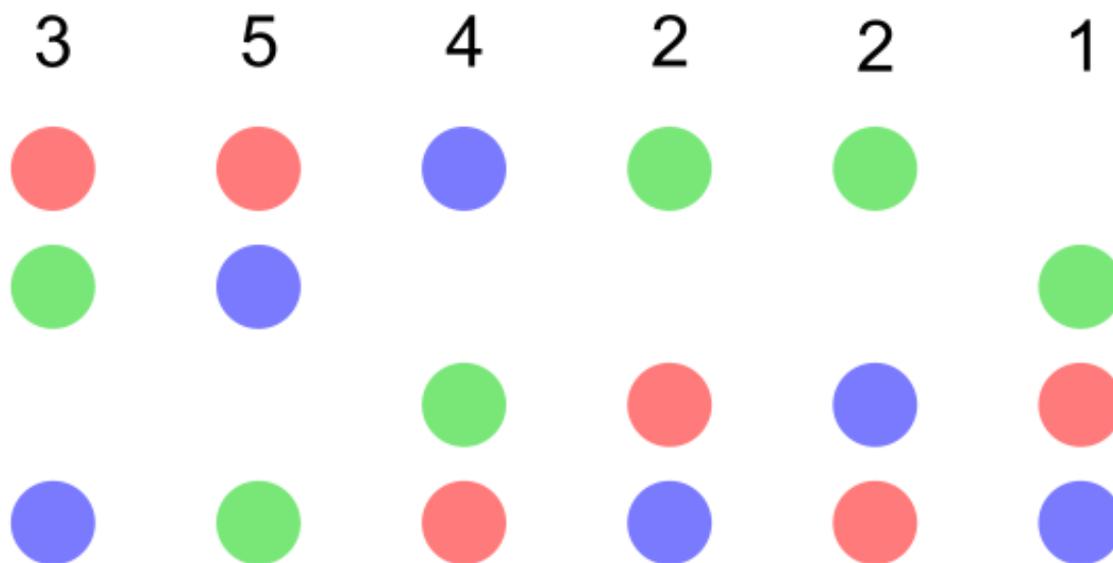


Recall that STV winner was 

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## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



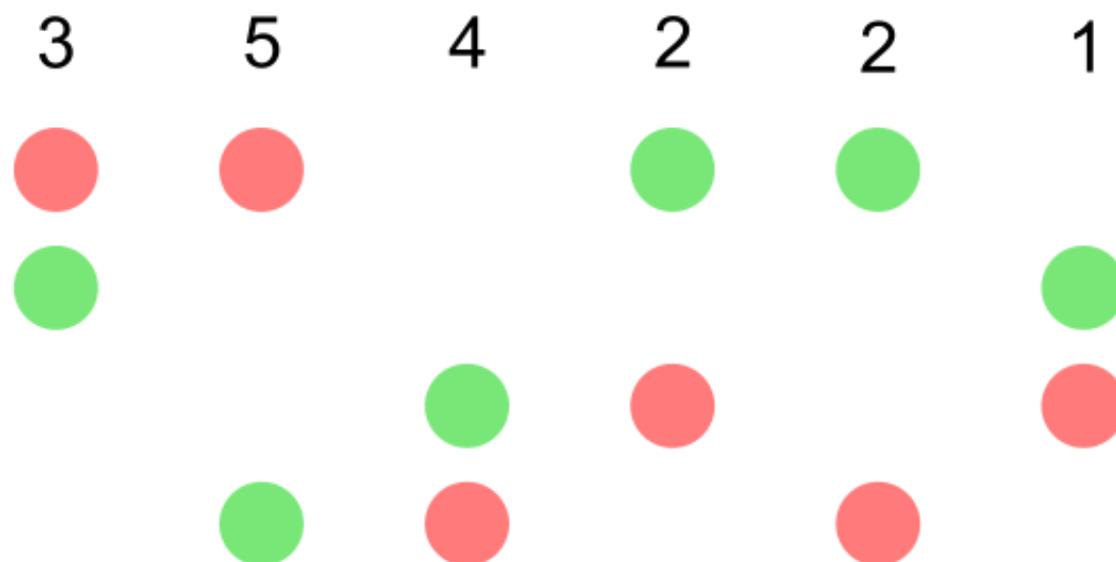
Recall that STV winner was 

Single  
Transferable  
Vote  
(Instant-Runoff)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was ●

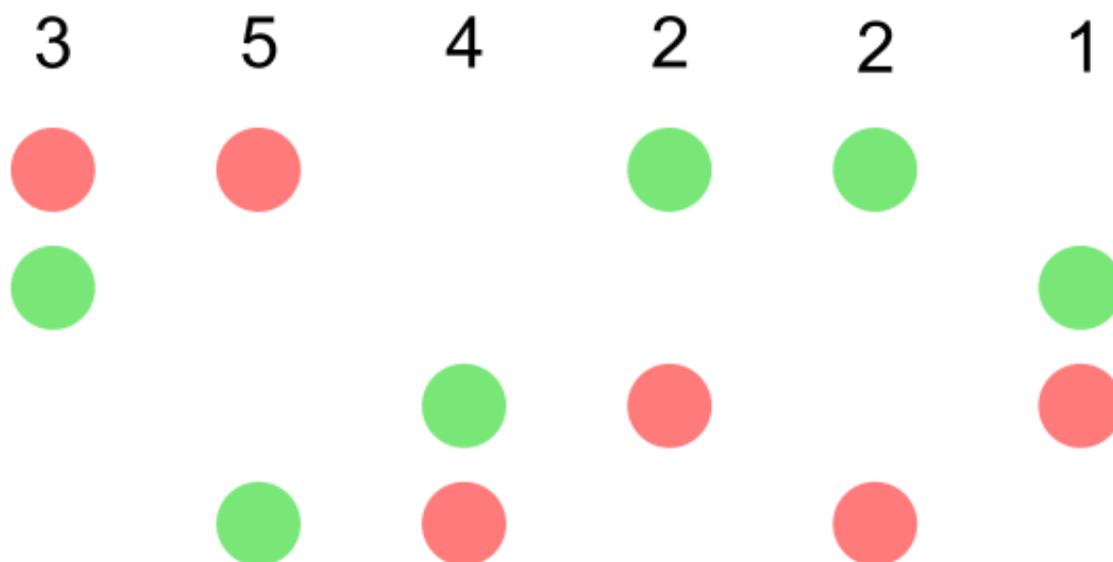
Single  
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Vote

(Instant-Runoff)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was   
but now it's 

Single  
Transferable  
Vote  
(Instant-Runoff)

# 4

Single  
Transferable  
Vote

(Instant-Runoff)

## 2016 host city election ballots results [\[ edit \]](#)

City	NOC	Round 1	Round 2	Round 3
<a href="#">Rio de Janeiro</a>	 <a href="#">Brazil</a> (COB)	26	<b>46</b>	<b>66</b>
<a href="#">Madrid</a>	 <a href="#">Spain</a> (COE)	<b>28</b>	29	32
<a href="#">Tokyo</a>	 <a href="#">Japan</a> (JOC)	22	20	—
<a href="#">Chicago</a>	 <a href="#">United States</a> (USOC)	18	—	—

Venue
<a href="#">Bella Center</a> <b>121st IOC Session</b> October 2, 2009  <a href="#">Copenhagen</a>

Vote details	Round 1	Round 2	Round 3
Eligible members	95	97	99
Participants	94	96	98
Abstentions	0	1	0
Valid ballots	94	95	98

# 4

## 2016 host city election ballots results [\[ edit \]](#)

City	NOC	Round 1	Round 2	Round 3
Rio de Janeiro	 Brazil (COB)	26	46	66
Madrid	 Spain (COE)	28	29	32
Tokyo	 Japan (JOC)	22	20	—
Chicago	 United States (USOC)	18	—	—

Venue
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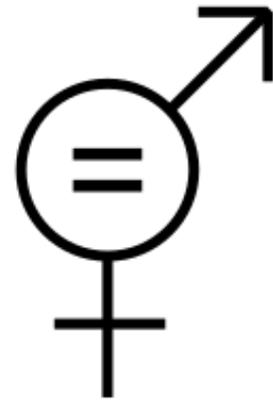
Single  
Transferable  
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(Instant-Runoff)



Nicolas de Condorcet  
(1743-1794)



Nicolas de Condorcet  
(1743-1794)



If a candidate beats every other candidate  
in a head-to-head election, select it!

If a candidate beats every other candidate  
in a head-to-head election, select it!

3



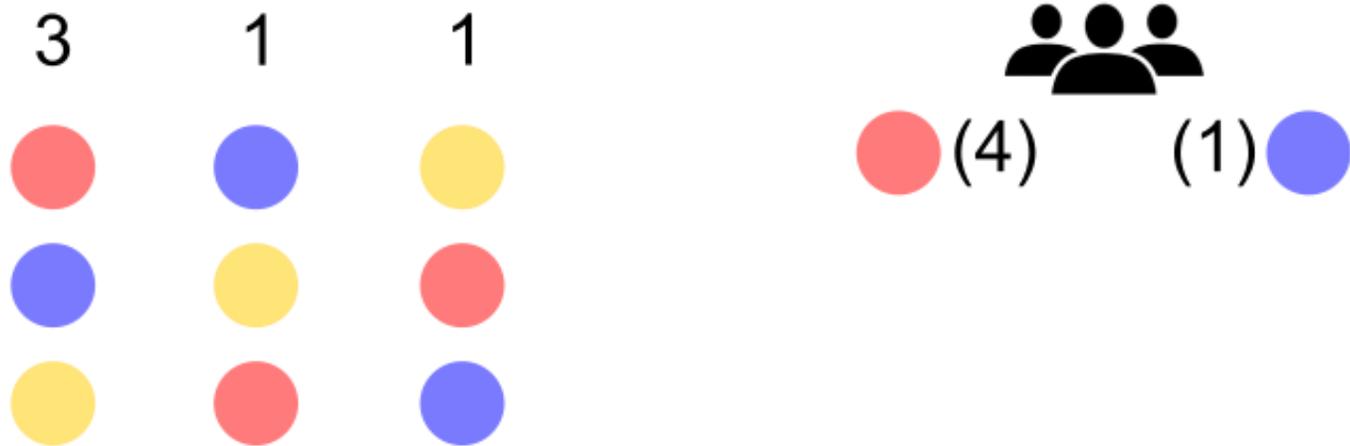
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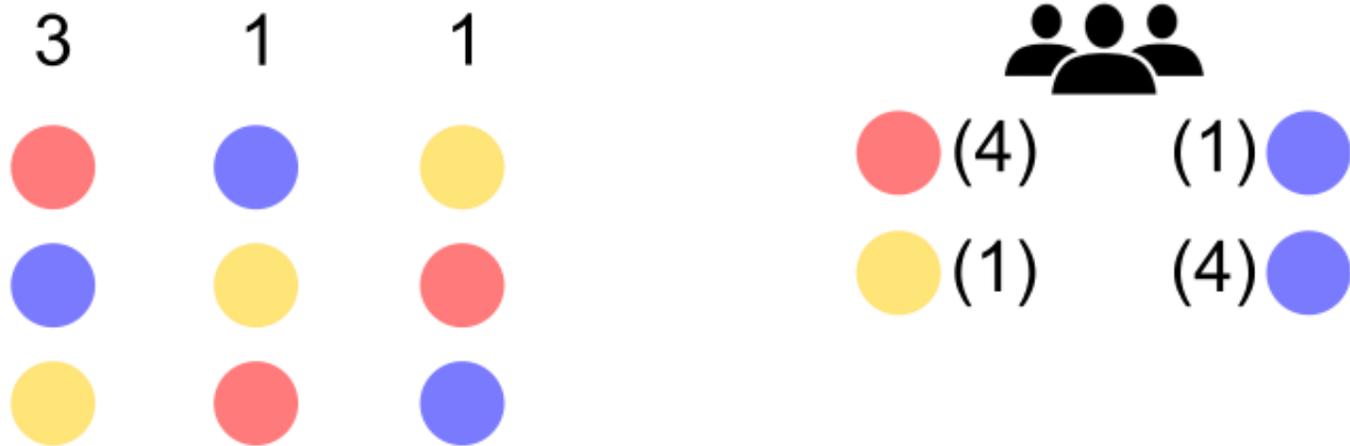
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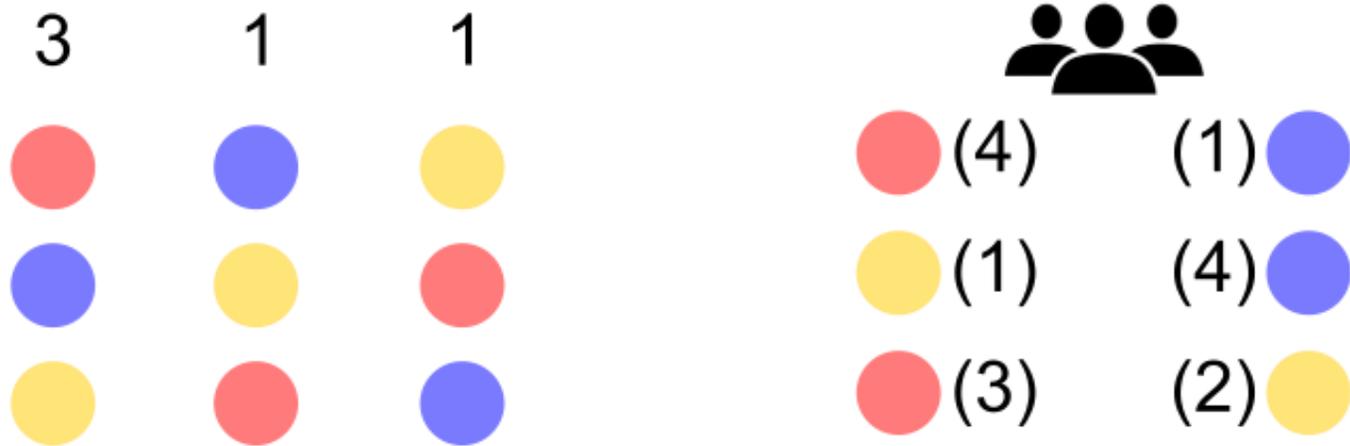
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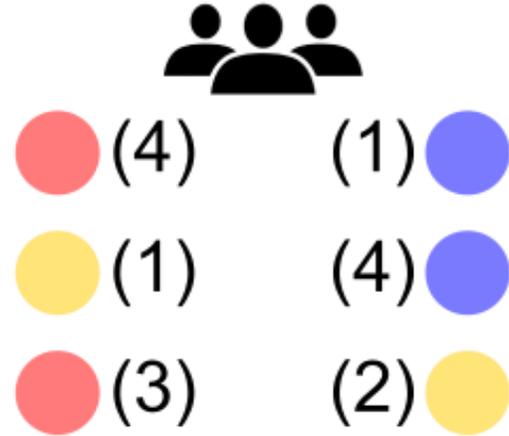
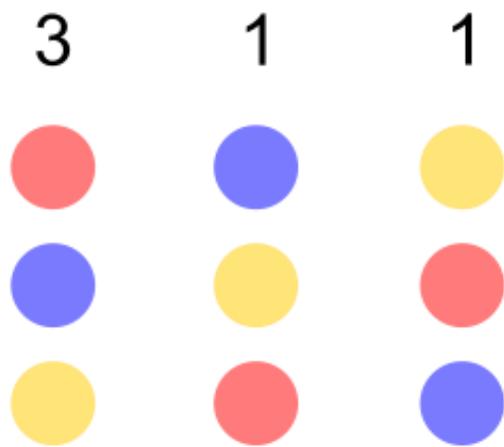
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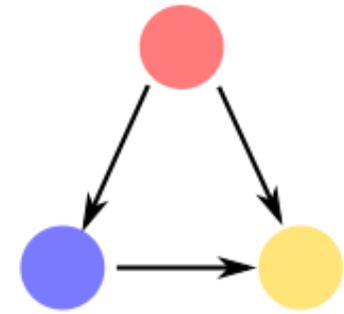
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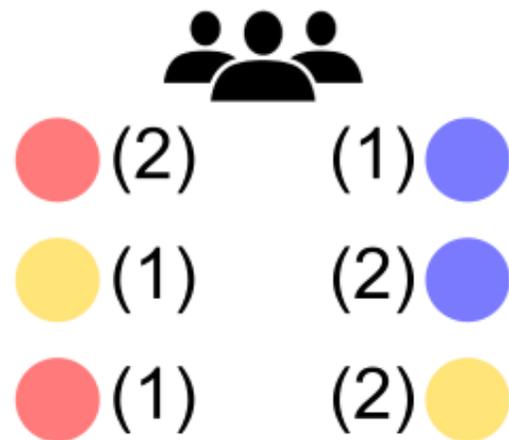
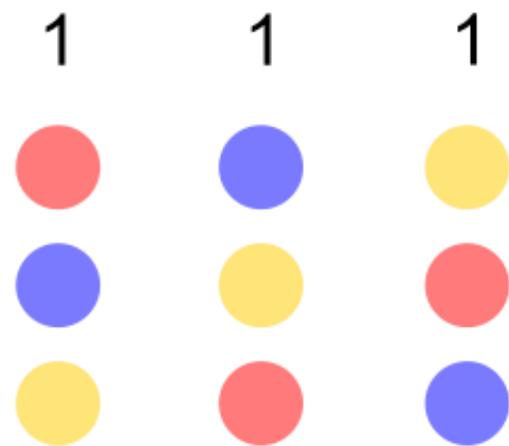
Condorcet winner



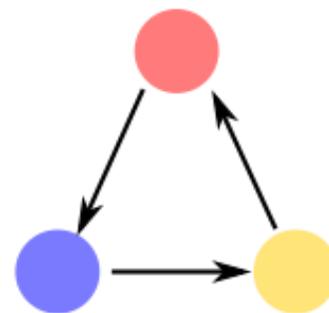
# Condorcet Criterion

**Problem:** A Condorcet winner may not exist

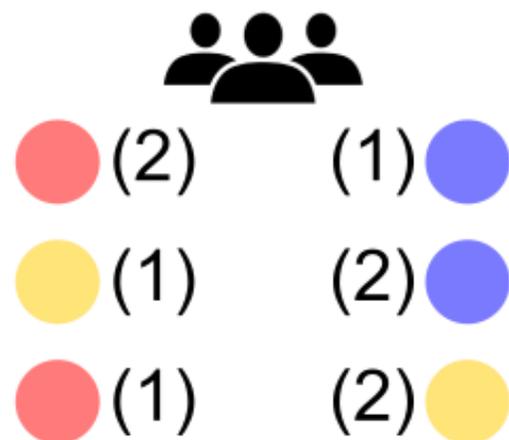
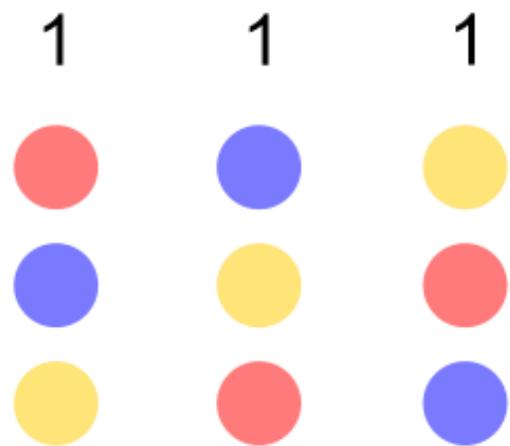
**Problem:** A Condorcet winner may not exist



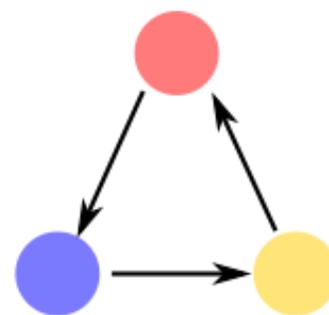
Condorcet cycle



**Problem:** A Condorcet winner may not exist



Condorcet cycle



**Condorcet paradox**

Transitivity of individual preferences  $\not\Rightarrow$  Transitivity of societal preferences

5

Copeland

# 5

For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie

Copeland

# 5

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3



1



1



4



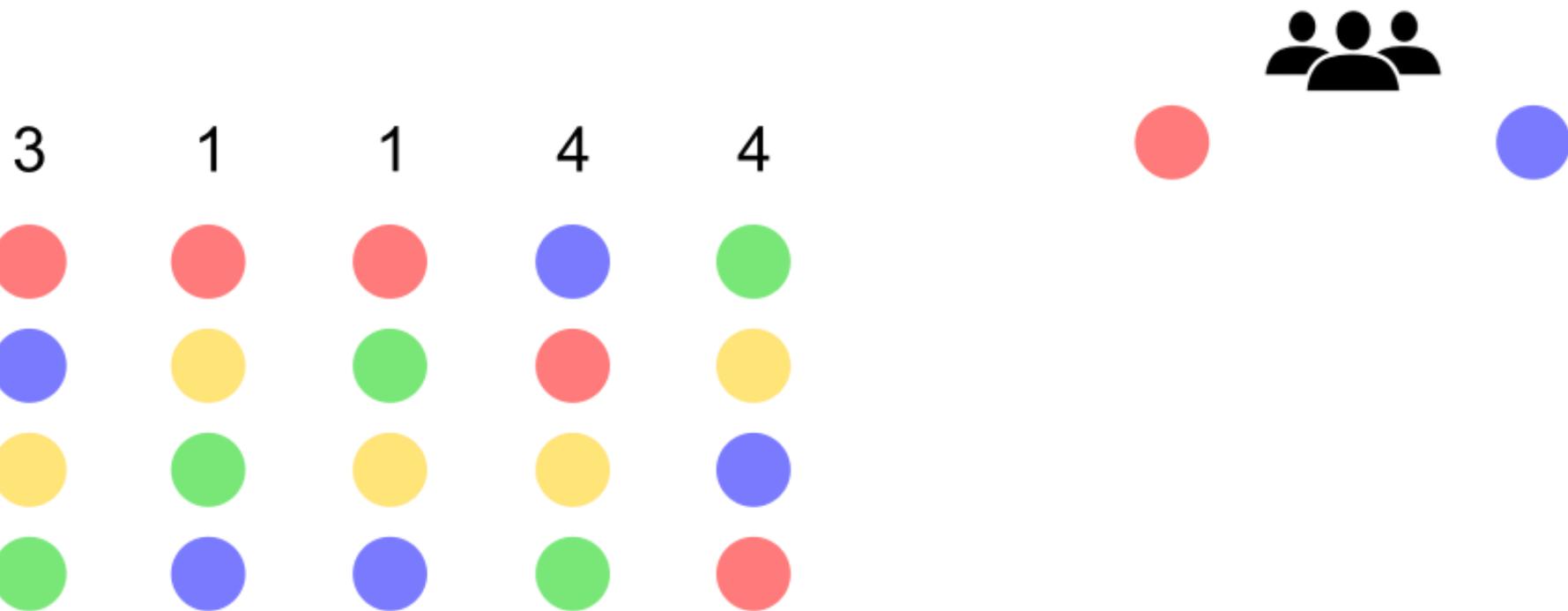
4



Copeland

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Copeland

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1



1



4



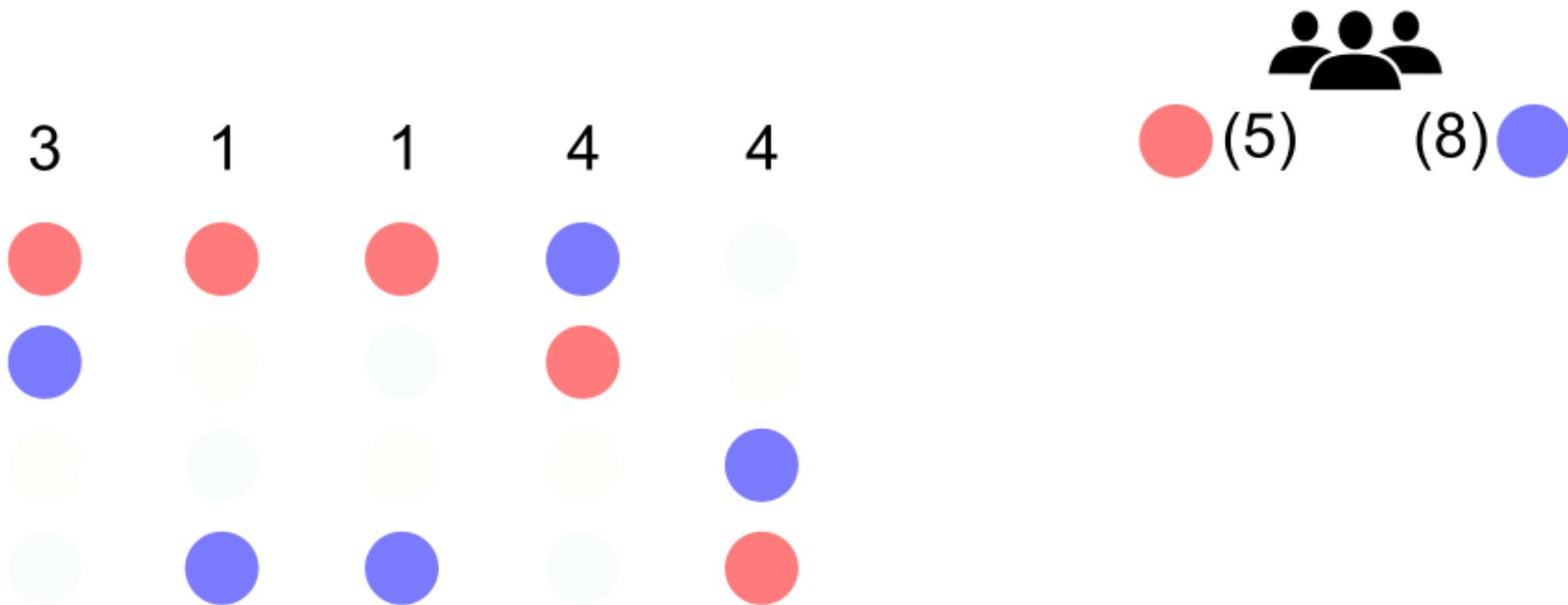
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Copeland

# 5

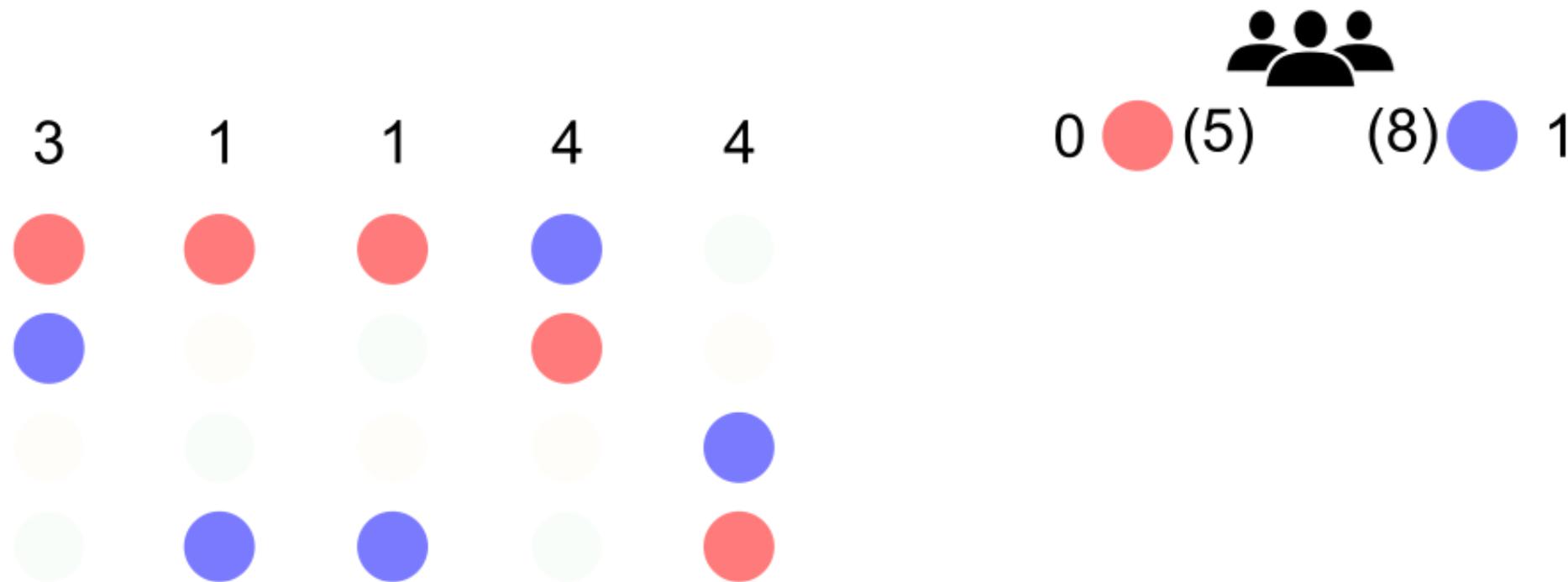
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Copeland

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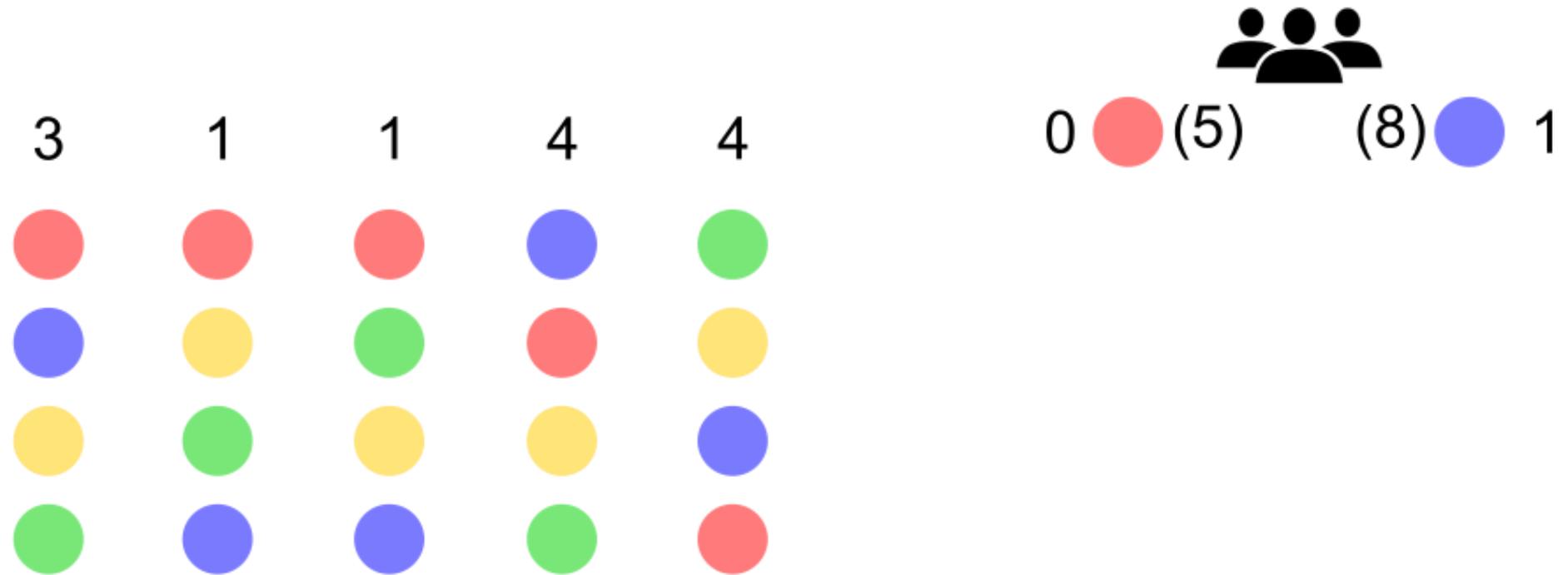
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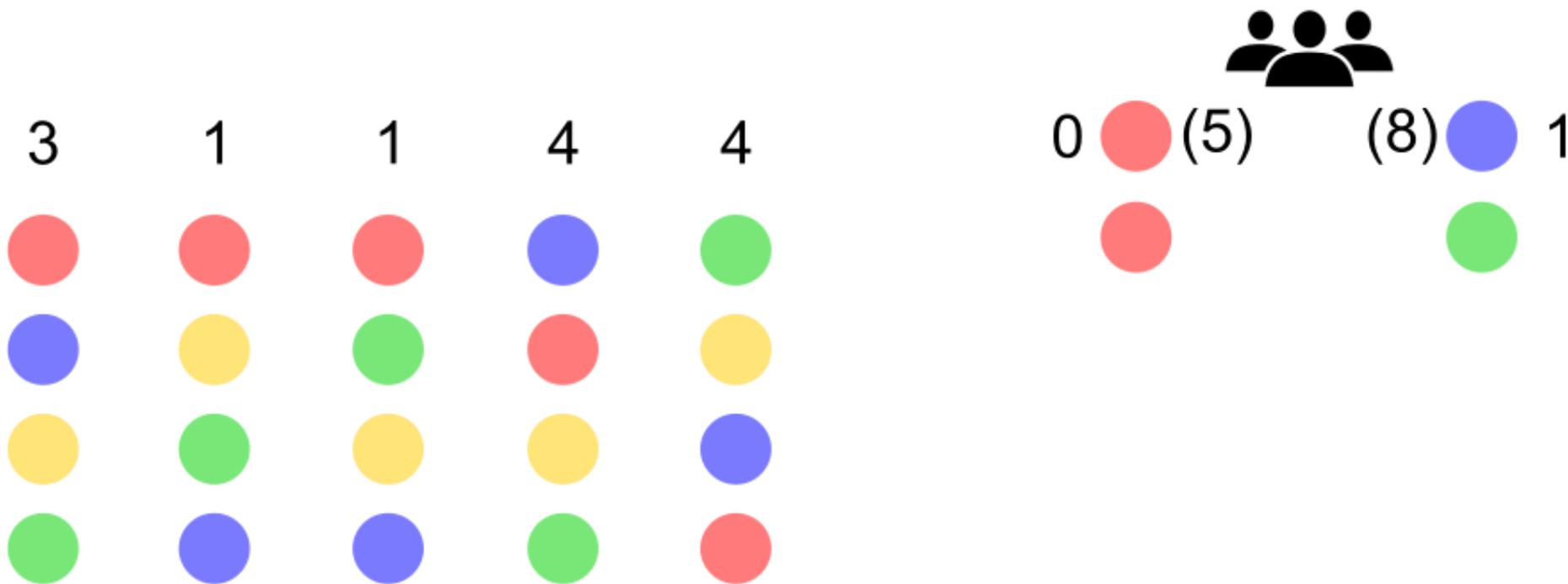
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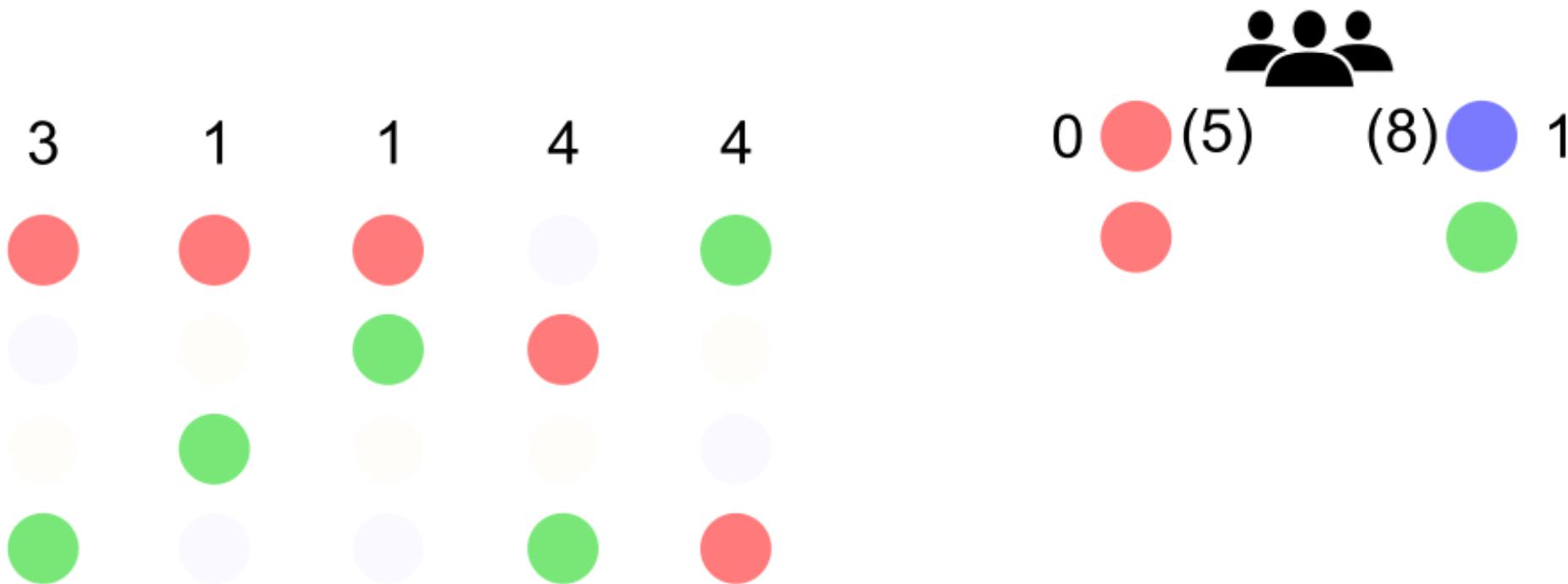
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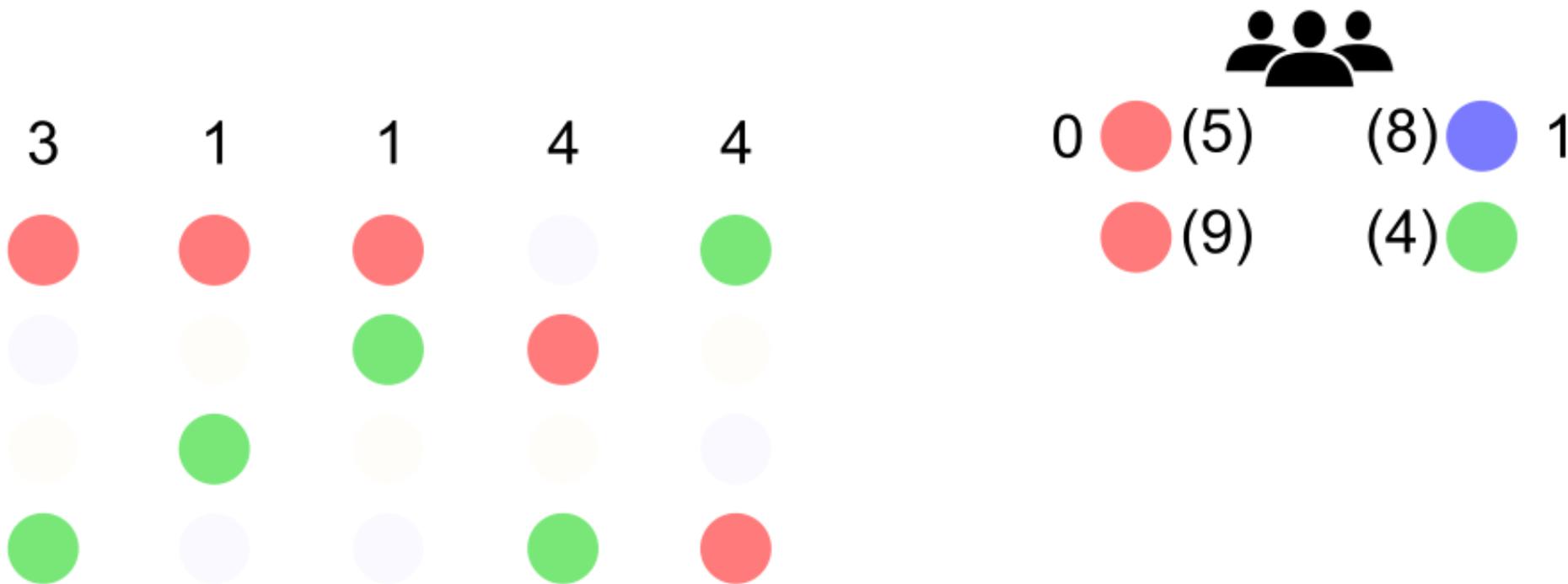
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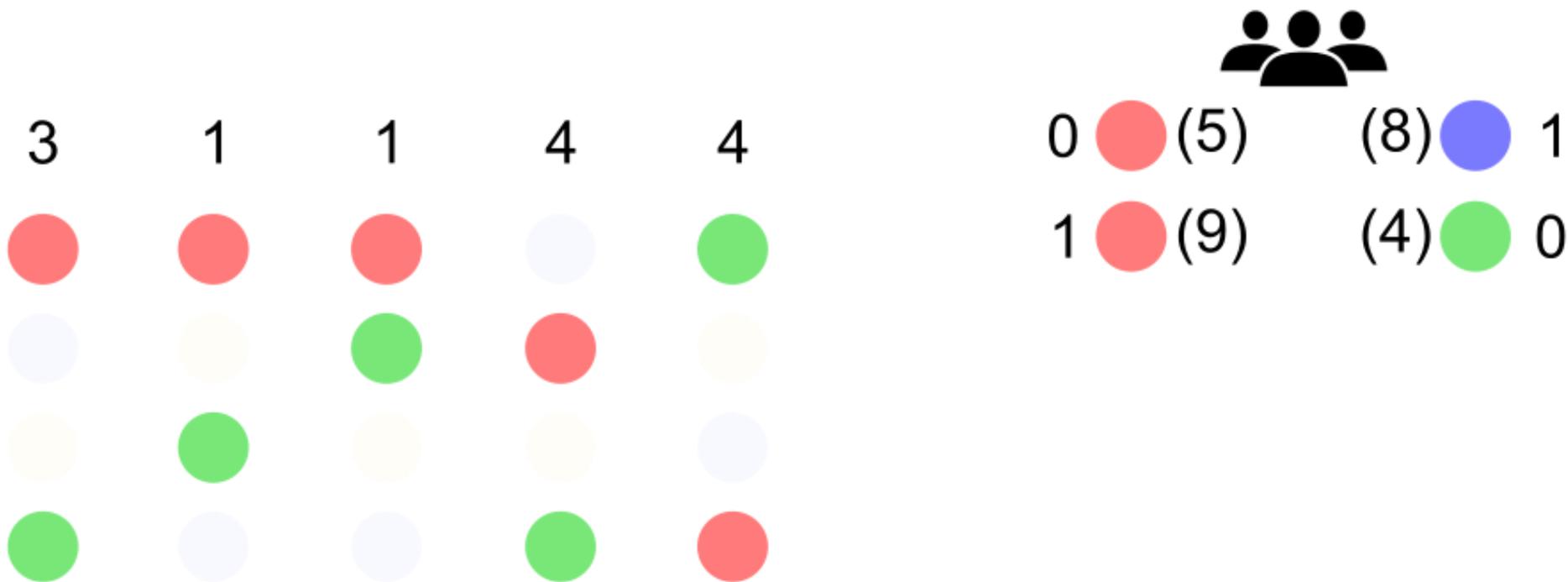
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Copeland

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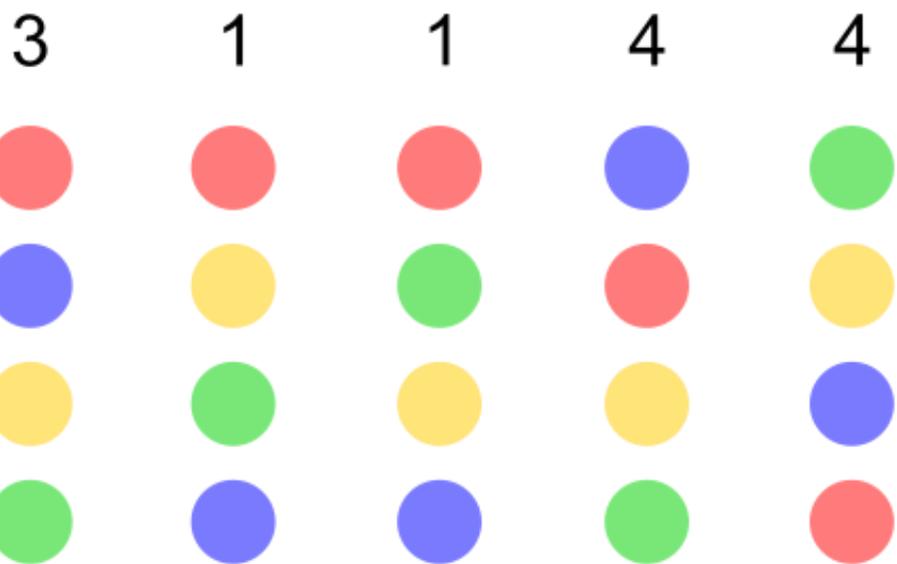
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Copeland

# 5



# LaLiga



**BUNDESLIGA**



# Premier League



**ICC WOMEN'S  
CRICKET WORLD CUP  
NEW ZEALAND 2022**



# INDIAN PREMIER LEAGUE

Copeland

# 5

**Problem:** Voters are sometimes better off not voting

Copeland

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: 

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: ●

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: ●

Which Voting Rule is the Best?

# Which Voting Rule is the Best?

**Voting Power in Practice Summer Workshop**

**Assessing Alternative Voting Procedures**

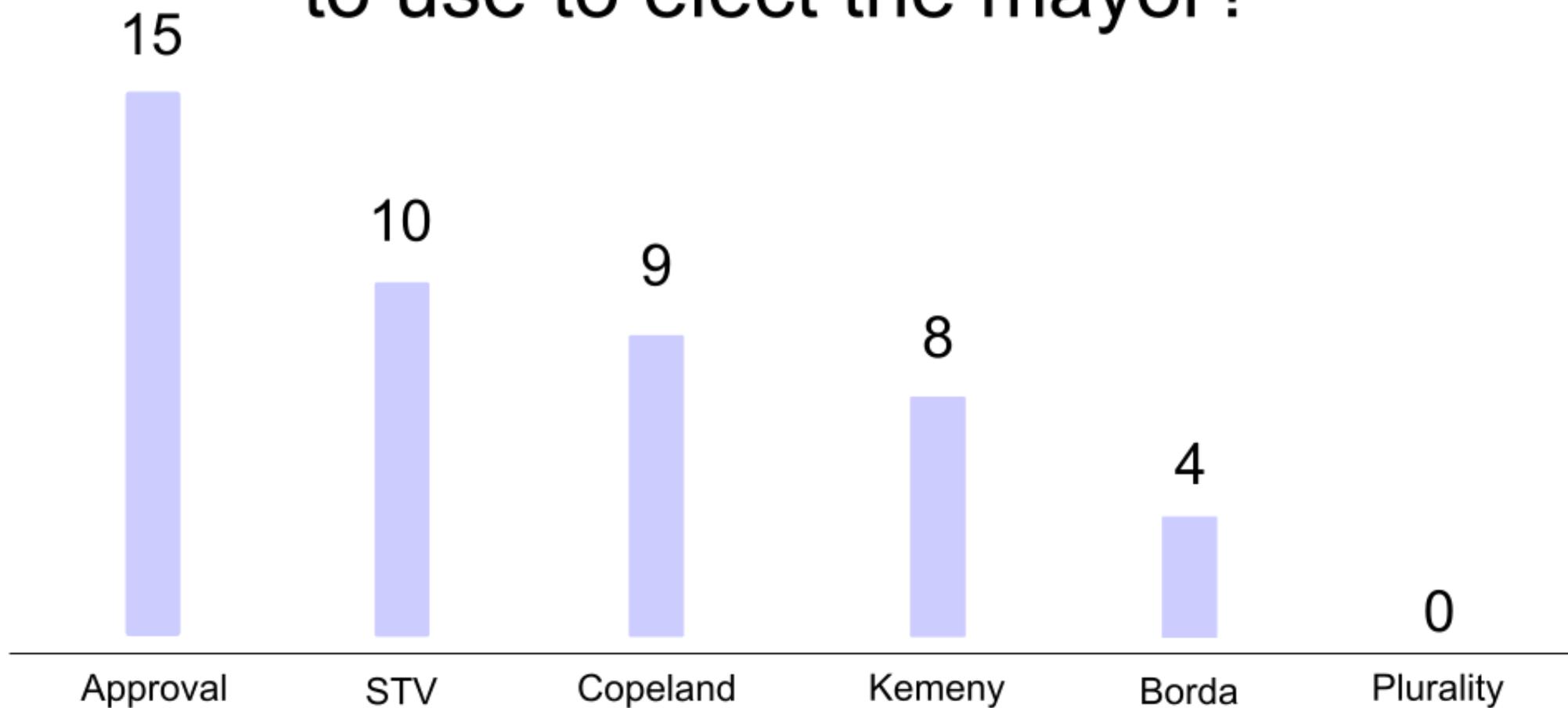
**Sponsored by The Leverhulme Trust**

Chateau du Baffy, Normandy, France  
30 July – 2 August 2010

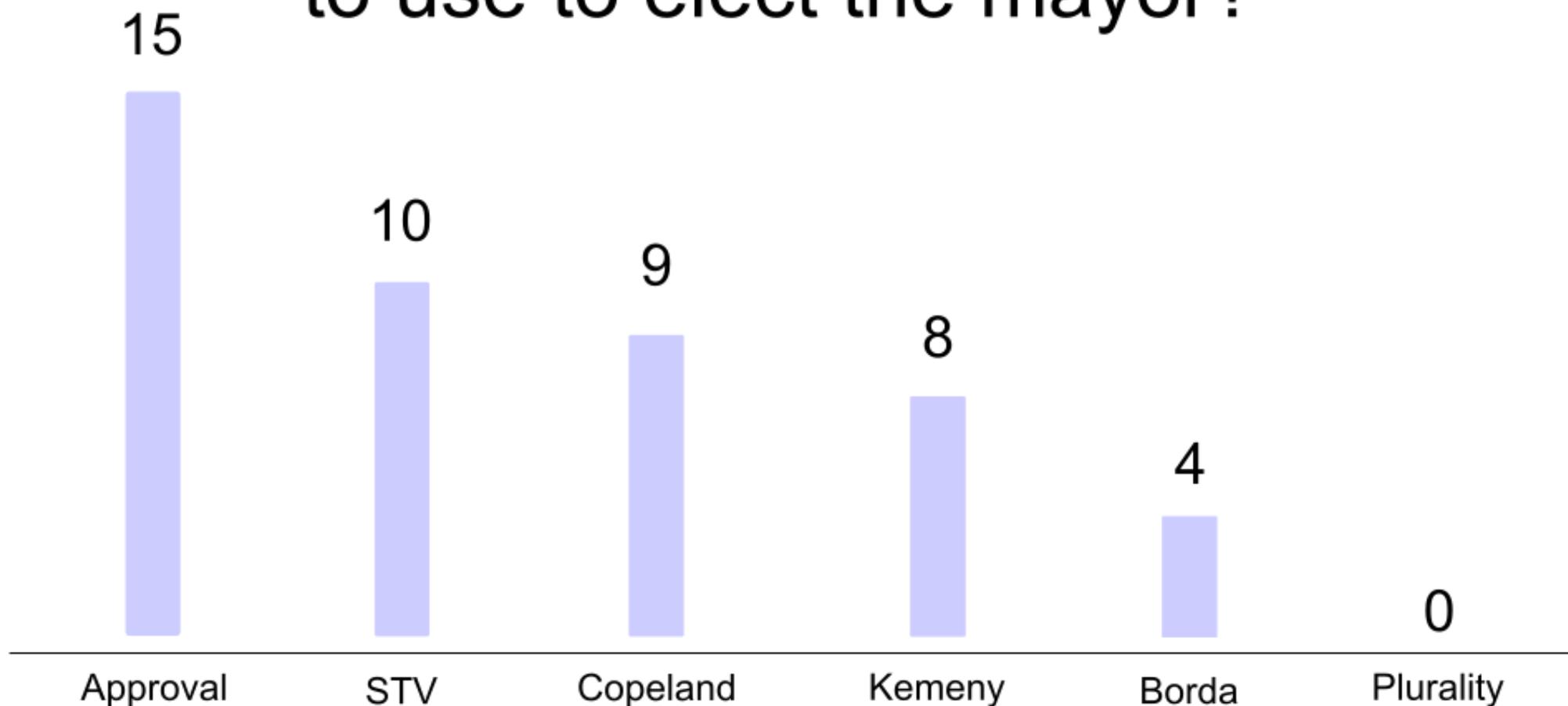
22 voting theorists

"What is the best voting rule for your town to use to elect the mayor?"

"What is the best voting rule for your town to use to elect the mayor?"



"What is the best voting rule for your town to use to elect the mayor?"



Votes were counted via approval voting.

18

12

10

9

4

2

Plurality



Plurality  
with runoff



Borda



Condorcet



(Copeland, Schulze)



STV



