COL 351 : Analysis & Design of Algorithms

LECTURE 37

NP-COMPLETENESS I : INTRODUCTION

NOV 06, 2026 | ROHIT VAISH

STORY SO FAR

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Problem	Algorithm	Running time
Sorting	Merge Sort	$O(n \log n)$
Strongly Connected Components	Kosaraju	$O(n+m)$
Shortest Paths	Dijkstra	$O((m+n) \log n)$
Minimum Spanning Tree	Kruskal	$O(m \log n)$
Sequence Alignment	Needleman-Wunsch	$O(mn)$
All Pairs Shortest Paths	Floyd-Warshall	$O(n^3)$

An Algorithmic Mystery

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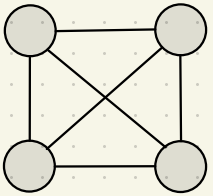
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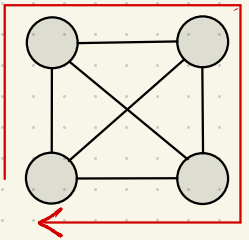
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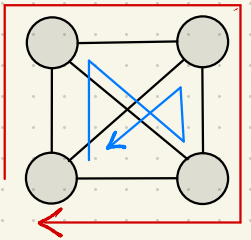
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* many more!

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Fact

No known fast (i.e., poly time) algorithm for TSP (despite decades of effort)

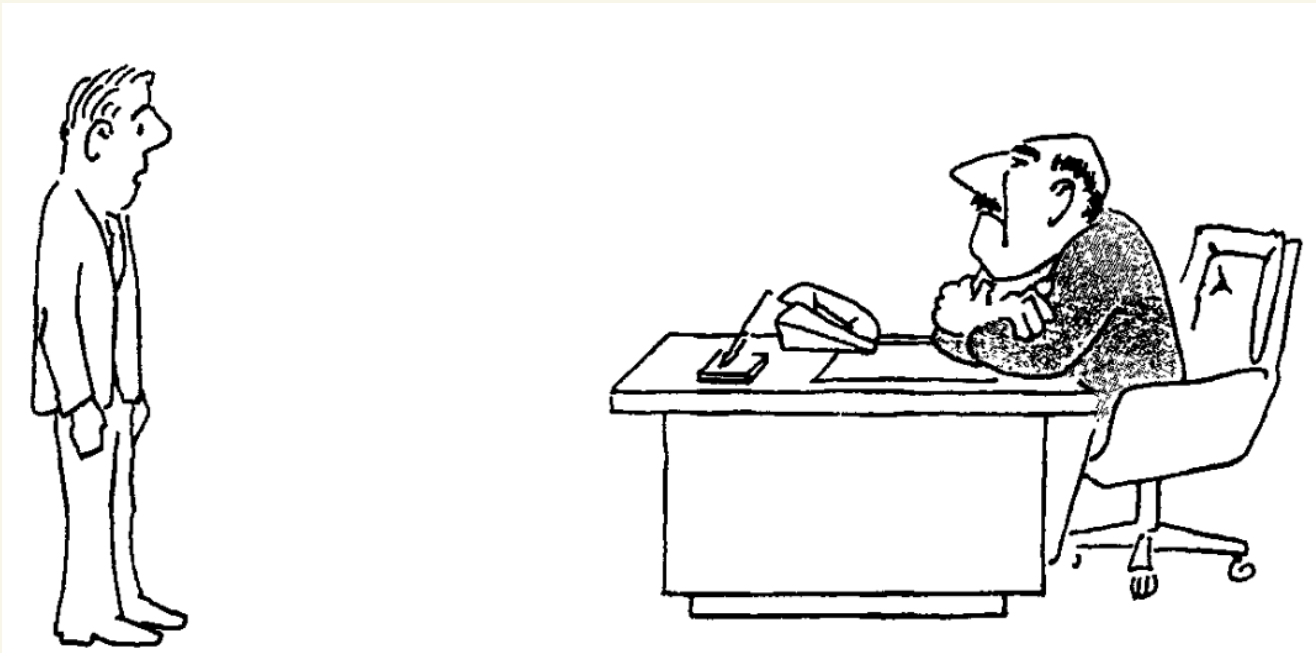
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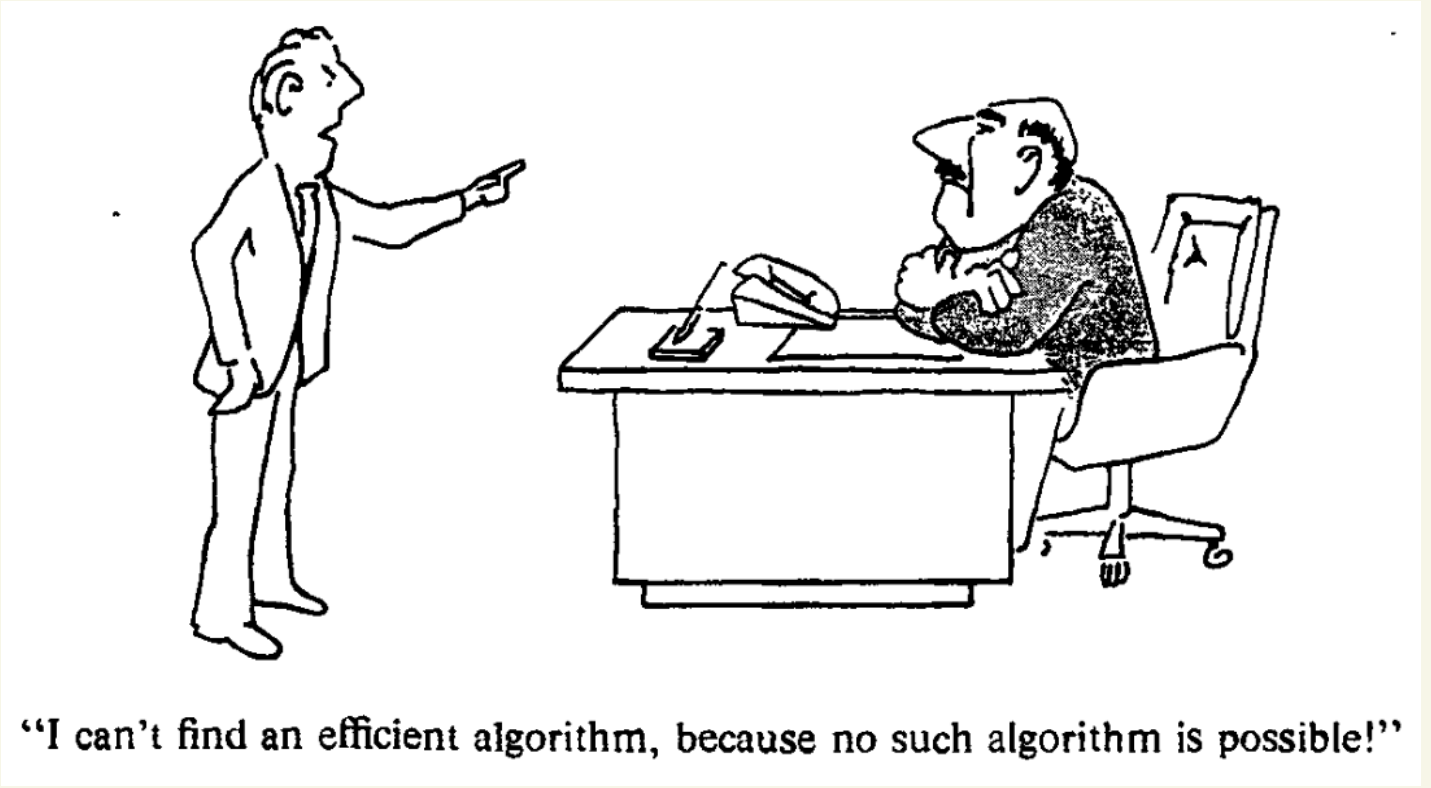
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algorithms: 



"I can't find an efficient algorithm, I guess I'm just too dumb."

Source : Garey and Johnson (1979)



"I can't find an efficient algorithm, because no such algorithm is possible!"

Source : Garey and Johnson (1979)

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematics and Mathematical Physics
Vol. 71B, No. 4, October–December 1967

Optimum Branchings*

Jack Edmonds

traveling saleman problem [cf. 4]. I conjecture that there is no good algorithm for the traveling saleman problem. My reasons are the same as for any mathematical conjecture: (1) It is a legitimate mathematical possibility, and (2) I do not know.

good = polynomial-time

So far, we haven't managed to prove Edmonds' conjecture.

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We have identified thousands of "TSP-like" problems:

* Graph coloring
* Integer Programming
* Ising Models
* Longest Paths
* Protein Folding
* Rubik's cube
* Number Partitioning
* Boolean Satisfiability
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RELATIVE INTRACTABILITY