STCS VIGYAN VIDUSHI 2025

NASH EQUILIBRIUM : COMPUTATION & CONSEQUENCES

ROHIT VAISH

Recall : Presentation game

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

Recall : Presentation game

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How to play this game?

Recall : Presentation game

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Two pure strategy Nash equilibria

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Two pure strategy Nash equilibria

$\Rightarrow$ Nash equilibrium may not be unique

Recall : Presentation game

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Any other Nash equilibrium ?

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Any other Nash equilibrium ? Yes !

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Any other Nash equilibrium? Yes!

puts effort with probability $\frac{9}{10}$, and slacks otherwise.

pay attention with probability $\frac{1}{10}$, and relax otherwise.

Recall : Presentation game

	Pay attention	Relax
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Any other Nash equilibrium? Yes!

MIXED STRATEGY EQUILIBRIUM

puts effort with probability $\frac{9}{10}$, and slacks otherwise.

pay attention with probability $\frac{1}{10}$, and relax otherwise.

Equilibria so far

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SPLIT or STEAL : (STEAL, STEAL)

Equilibria so far

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Presentation game : Two pure and one mixed equilibrium

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Presentation game : Two pure and one mixed equilibrium

$\frac{1}{2}$ - mean game : $(1, 1, \ldots, 1)$

Equilibria so far

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Presentation game : Two pure and one mixed equilibrium

$\frac{1}{2}$-mean game : $(1, 1, \dots, 1)$

Project game : $s_1^* = s_2^* = \frac{1}{1-b}$

Equilibria so far

SPLIT or STEAL : (STEAL, STEAL)

Presentation game : Two pure and one mixed equilibrium

$\frac{1}{2}$-mean game : $(1, 1, \dots, 1)$

Project game : $s_1^* = s_2^* = \frac{1}{1-b}$

Does a Nash equilibrium always exist?

ROCK PAPER SCISSORS

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Paper covers rock.

Scissor cuts paper.

Rock crushes scissors.

ROCK PAPER SCISSORS

	R	P	S
R	0,0	−1,1	1,−1
P	1,−1	0,0	−1,1
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Any pure strategy Nash eq. ? No !

Any mixed strategy Nash eq. ? Yes $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

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Does a mixed strategy Nash equilibrium always exist?

NASH'S THEOREM

(Nash 1951)

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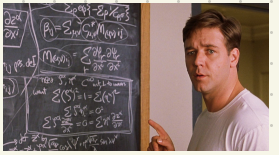

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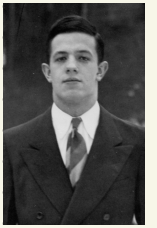

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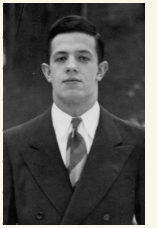

Thm: Every finite game has at least one mixed strategy equilibrium.

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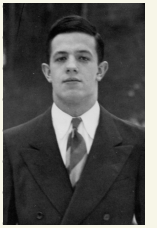

Thm: Every finite game has at least one mixed strategy equilibrium.

finite number of players

finite number of strategies per player

FINDING A NASH EQUILIBRIUM

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Zero-sum games :

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

FINDING A NASH EQUILIBRIUM

Zero-sum games: Easy (i.e., polynomial time)

via linear programming

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General-sum games :

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General-sum games : Unlikely to be easy

PPAD-complete

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TRAFFIC ROUTING

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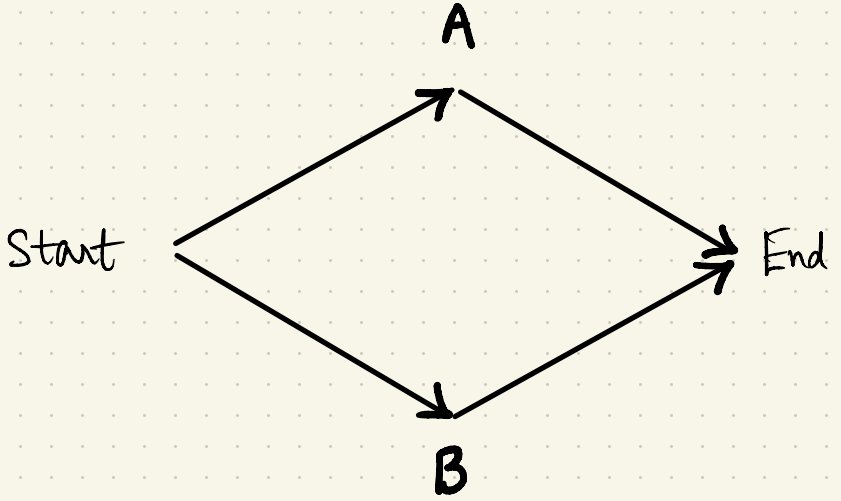
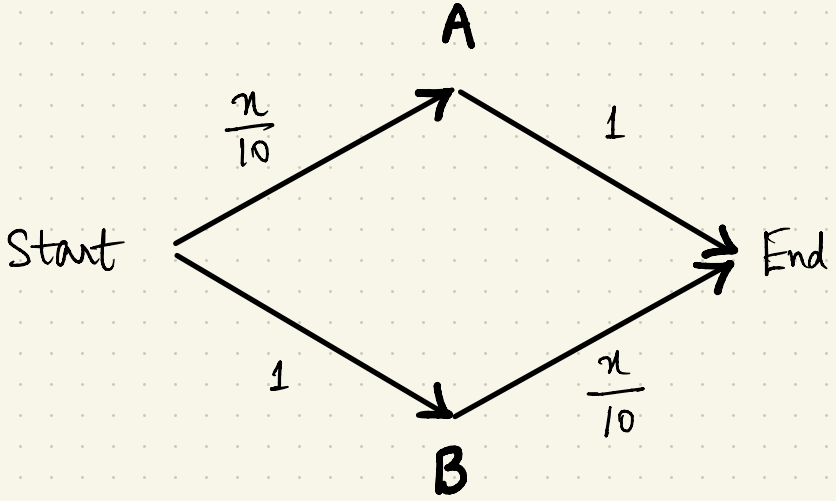


TRAFFIC ROUTING

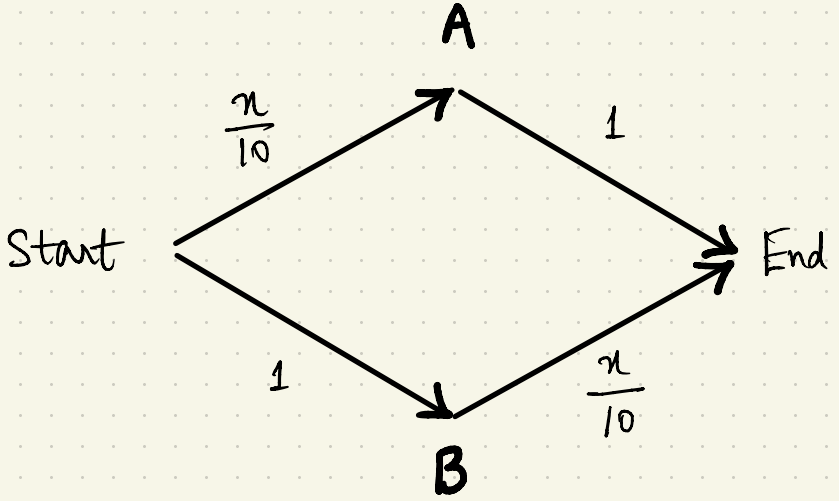
* 10 selfish vehicles want to go from Start to End



TRAFFIC ROUTING

* 10 selfish vehicles want to go from Start to End

* Travel times

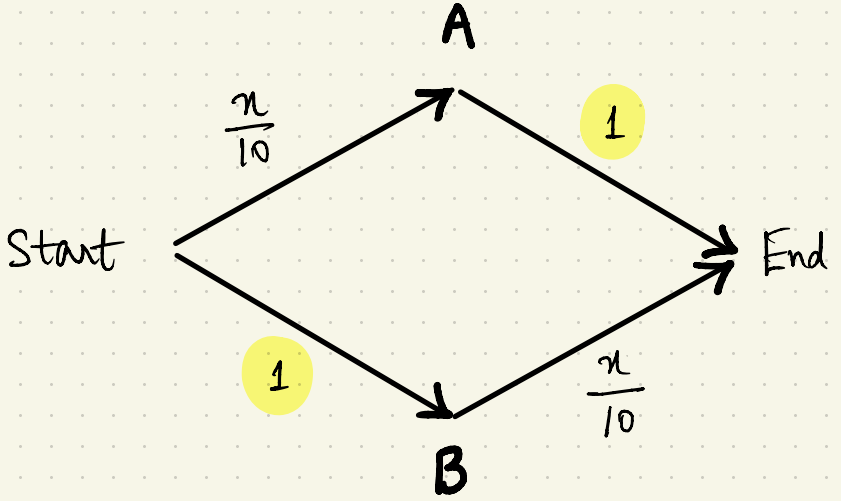


TRAFFIC ROUTING

* 10 selfish vehicles want to go from Start to End

* Travel times

  1 : One hr independent of traffic



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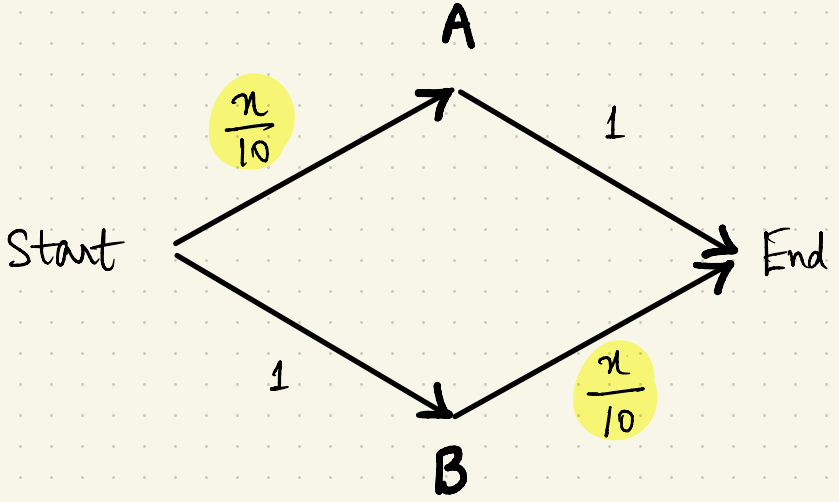

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  $\frac{x}{10}$: $\frac{\#\text{ vehicles}}{10}$ hr

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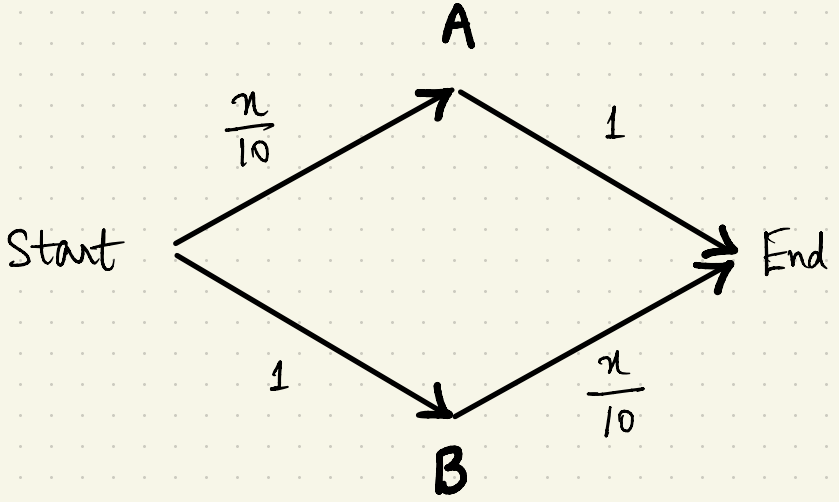


* Travel times

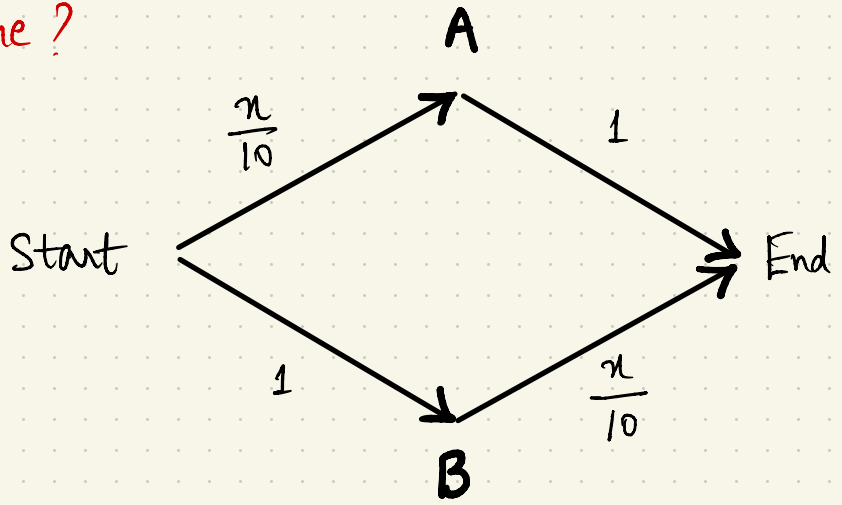
  1 : One hr independent of traffic

  $\frac{x}{10}$: $\frac{\#\text{ vehicles}}{10}$ hr

* Objective : minimize individual travel time

TRAFFIC ROUTING

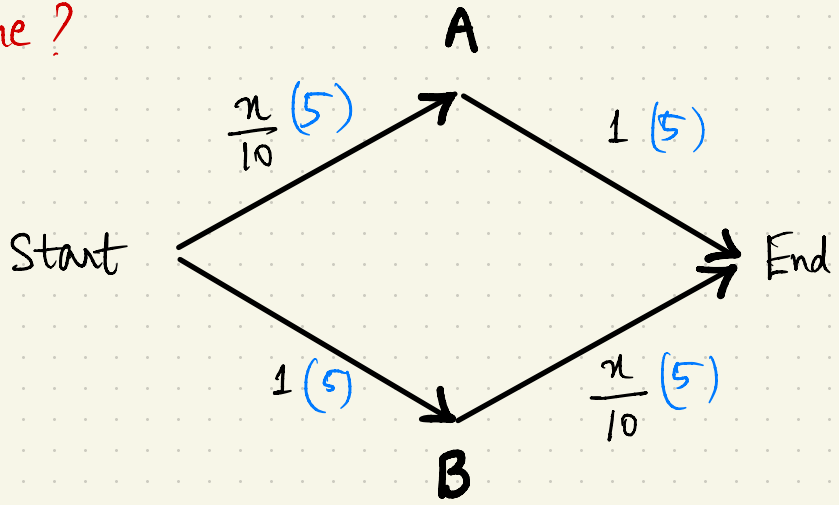
How should each vehicle play this game?



TRAFFIC ROUTING

How should each vehicle play this game?

Equal split is a Nash eq.

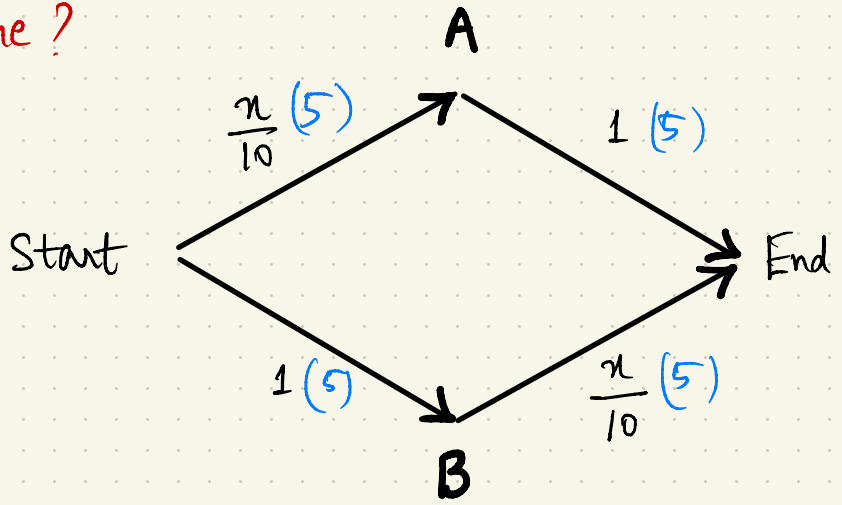


TRAFFIC ROUTING

How should each vehicle play this game?

Equal split is a Nash eq.

Individual travel time = 1.5 hr



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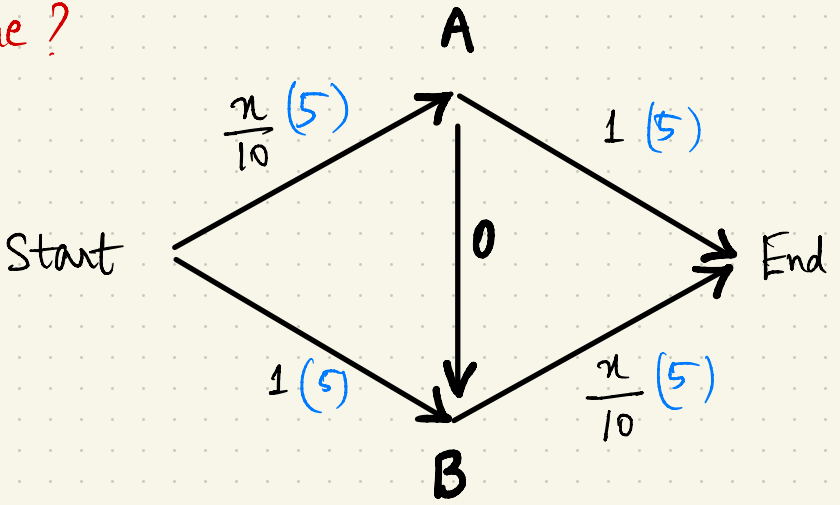

... more roads = faster travel

TRAFFIC ROUTING

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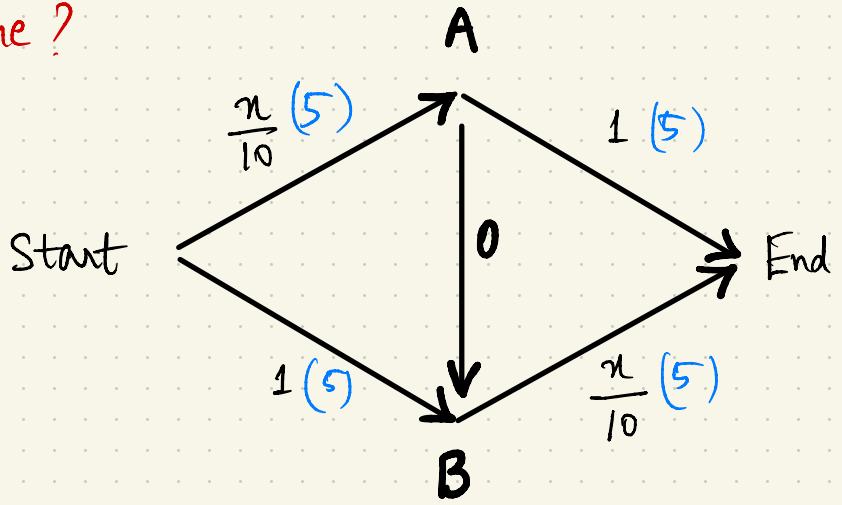

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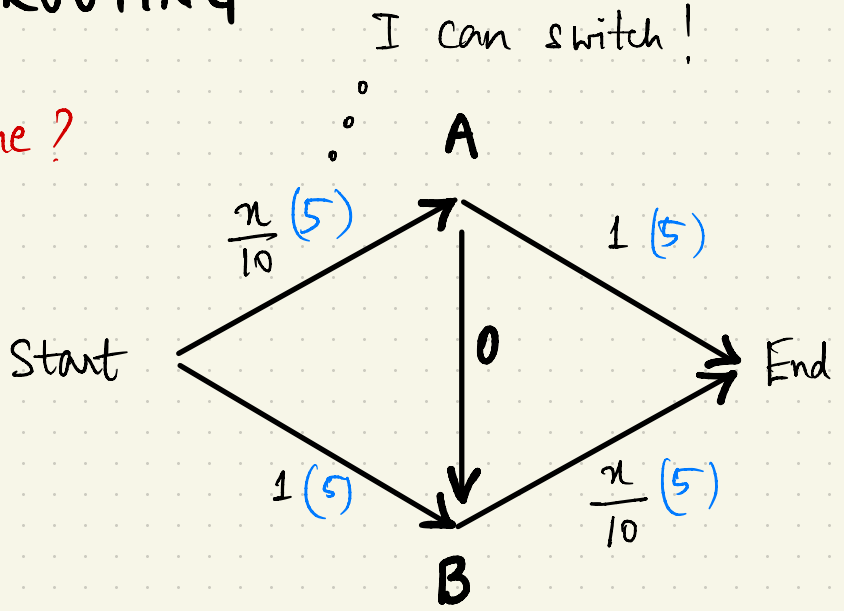

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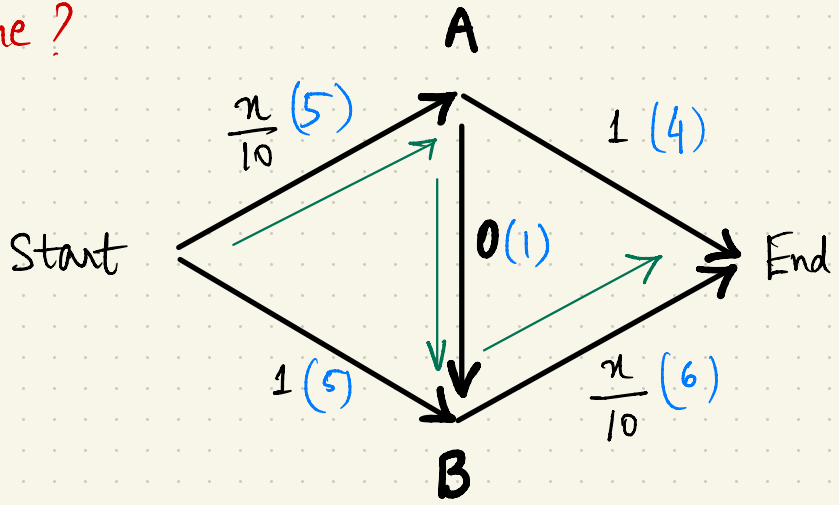
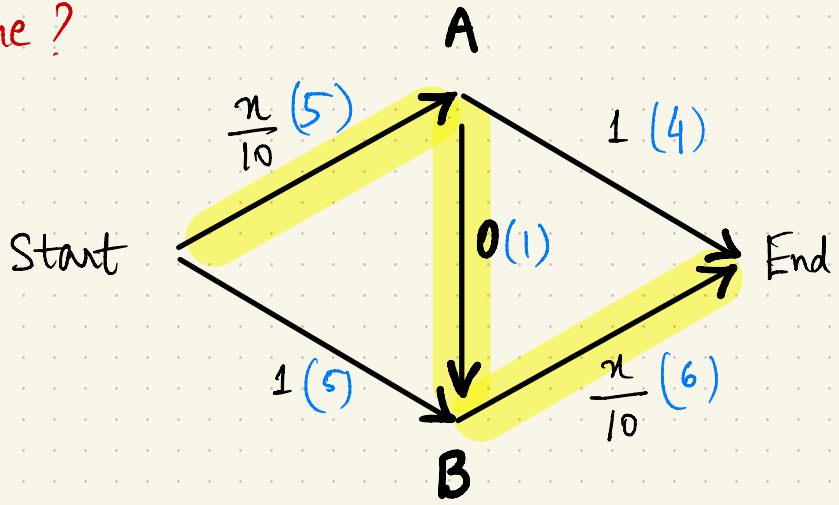


TRAFFIC ROUTING

How should each vehicle play this game?

Travel time of deviator

1.5 ⟶ 1.1 👍



TRAFFIC ROUTING

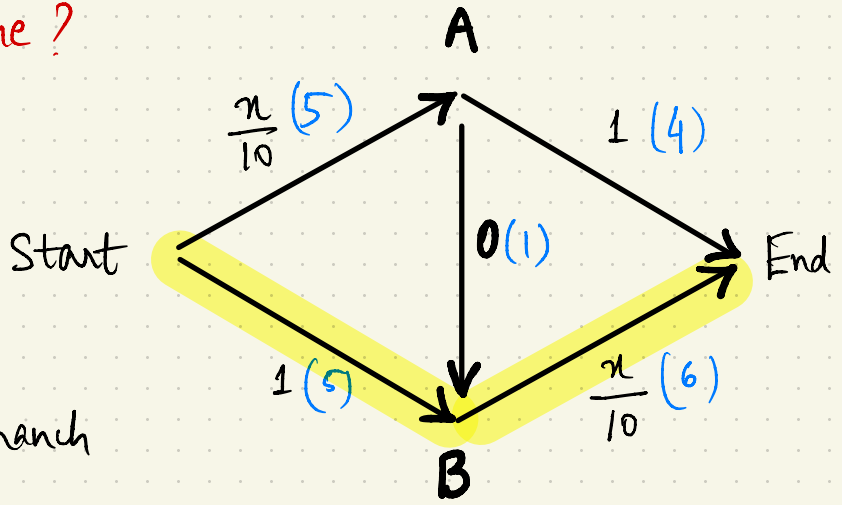
How should each vehicle play this game?

Travel time of deviator

$1.5 \longrightarrow 1.1$ 👍

Travel time of Start → B → End branch

$1.5 \longrightarrow 1.6$ 👎



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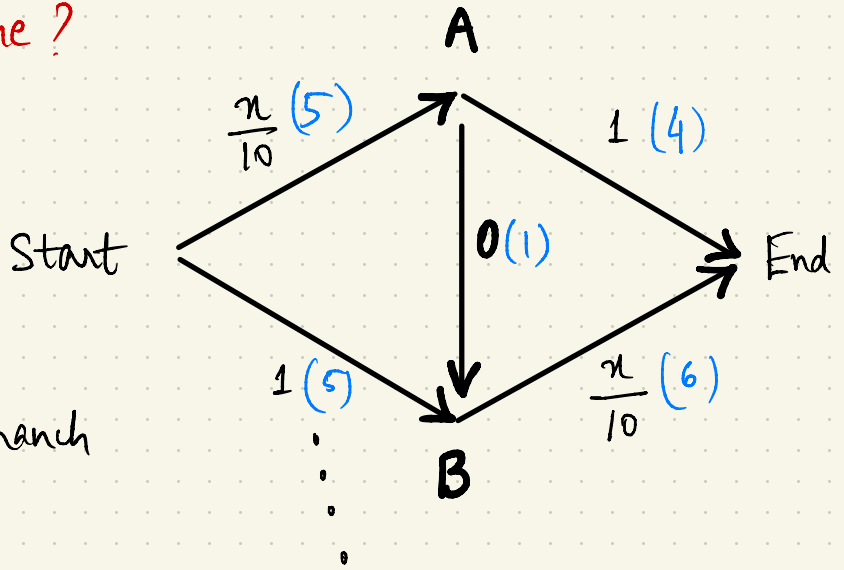
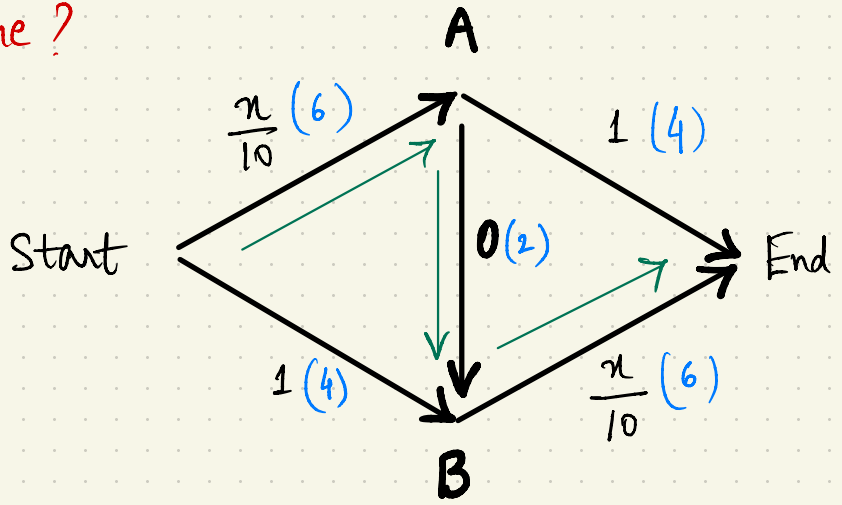


Maybe I should switch?

TRAFFIC ROUTING

How should each vehicle play this game?



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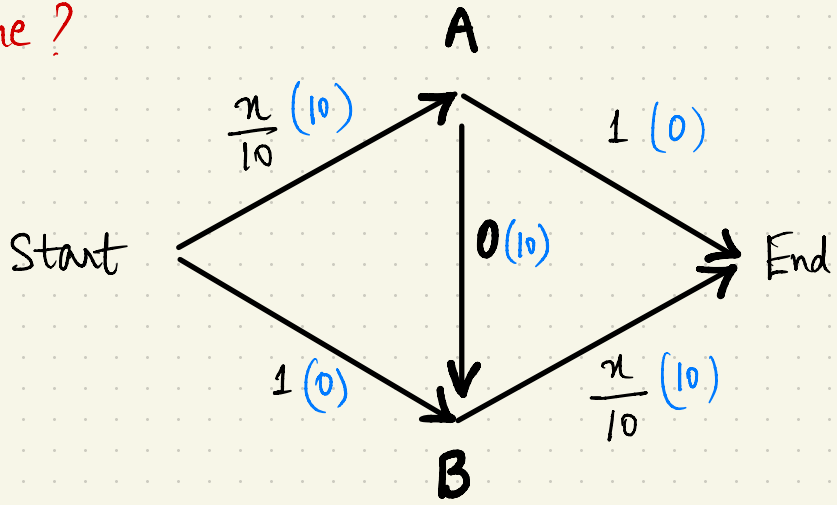


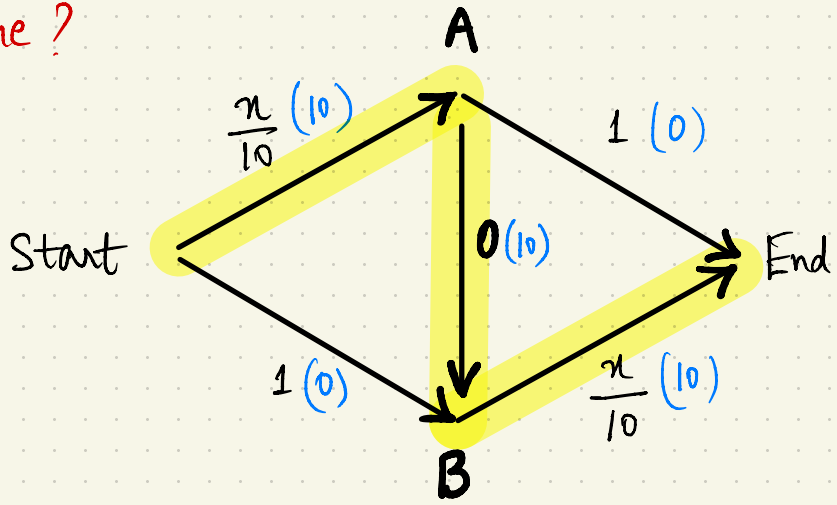
eventually, everyone switches

TRAFFIC ROUTING

How should each vehicle play this game?

Individual travel time

1.5 ⟶ 2 ☹



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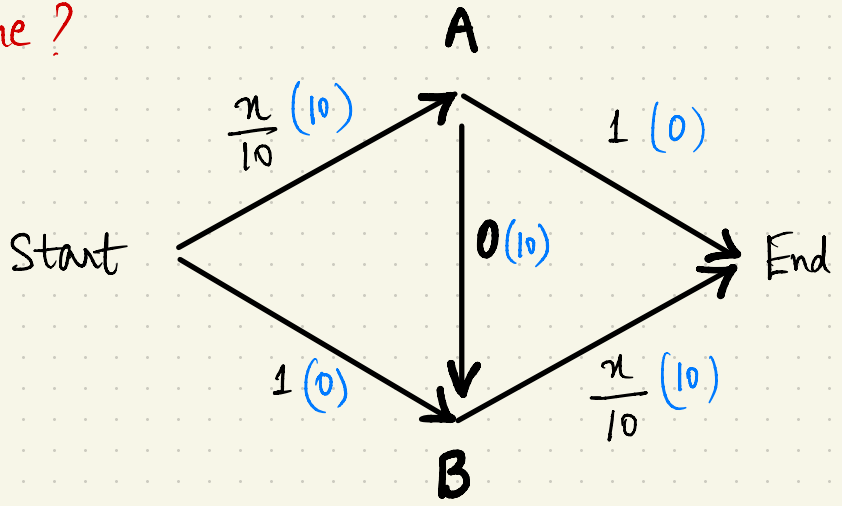

Individual travel time

1.5 ⟶ 2 ☹

Braess's paradox

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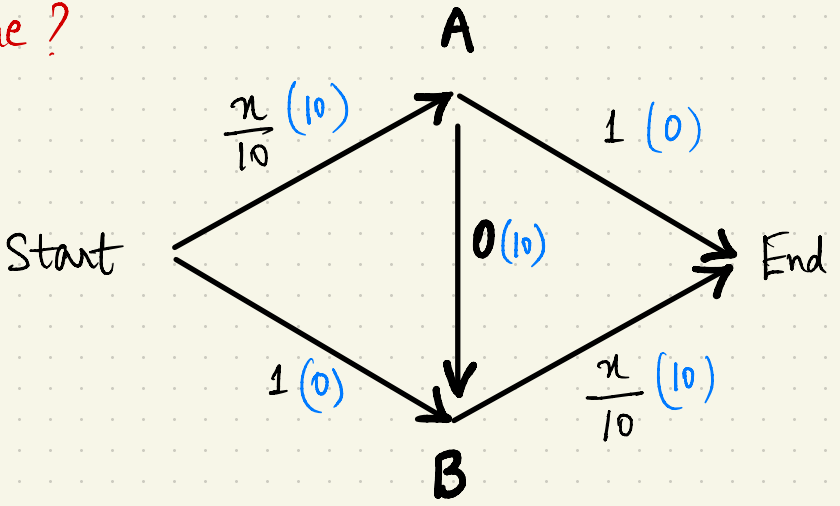

Individual travel time

1.5 ⟶ 2 ☹

Braess's paradox

Well-intentioned changes could have the opposite effect if agents' incentives are not accounted for.