STCS VIGYAN VIDUSHI 2025

BEST RESPONSE & NASH EQUILIBRIUM

ROHIT VAISH

* Play strictly dominant strategy (if it exists)

- Play strictly dominant strategy (if it exists)
- Remove strictly dominated strategy (if it exists)

- Play strictly dominant strategy (if it exists)
- Remove strictly dominated strategy (if it exists)
- Put yourself in others' shoes

* Play strictly dominant strategy (if it exists)

* Remove strictly dominated strategy (if it exists)

* Put yourself in others' shoes

* Common knowledge

BEST RESPONSE

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Does player 1 have a dominated strategy?

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Does player 1 have a dominated strategy?

No!

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Does player 1 have a dominated strategy?

No!

Does player 2 have a dominated strategy?

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Does player 1 have a dominated strategy?

No!

Does player 2 have a dominated strategy?

No!

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 2's strategy is known

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 2's strategy is known

If 2 → Right , then 1 → Midde

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
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Suppose player 2's strategy is known

If 2 → Right , then 1 → Midde

If 2 → Left , then 1 → Up

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 2's strategy is known

If 2 → Right , then 1 → Midde

If 2 → Left , then 1 → Up

So, should player 1 eliminate "Down" ?

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

But, player 1 may not know player 2's strategy.

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

But, player 1 may not know player 2's strategy.

How to play the game?

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
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But, player 1 may not know player 2's strategy.

How to play the game?

Guess player 2's strategy!

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 1 believes that player 2 picks a strategy uniformly at random.

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 1 believes that player 2 picks a strategy uniformly at random.

Expected payoff of Up =

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose player 1 believes that player 2 picks a strategy uniformly at random.

Expected payoff of Up $= \frac{5}{2} + \frac{0}{2} = 2.5$

BEST RESPONSE

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		Left	Right
	Up	5, 1	0, 2
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" " Middle =

BEST RESPONSE

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		Left	Right
	Up	5, 1	0, 2
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Expected payoff of Up $= \frac{5}{2} + \frac{0}{2} = 2.5$

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BEST RESPONSE

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" " Middle $= \frac{1}{2} + \frac{4}{2} = 2.5$

" " Down $= \frac{4}{2} + \frac{2}{2} = 3$

If player 2 is unif. random, then Down is player 1's best response.

BEST RESPONSE

		2	
		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose $\Pr(\text{left}) = \frac{3}{4}$ $\Pr(\text{right}) = \frac{1}{4}$

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose $Pr(\text{left}) = \frac{3}{4}$ $\quad Pr(\text{right}) = \frac{1}{4}$

Expected payoff of Up $= \frac{15}{4} + \frac{0}{4} = 15/4$

" " Middle $= \frac{3}{4} + \frac{4}{4} = 7/4$

" " Down $= \frac{12}{4} + \frac{2}{4} = 14/4$

BEST RESPONSE

2

		Left	Right
	Up	5, 1	0, 2
1	Middle	1, 3	4, 1
	Down	4, 2	2, 3

Suppose Pr(left) $= \frac{3}{4}$ Pr(right) $= \frac{1}{4}$

Expected payoff of Up $= \frac{15}{4} + \frac{0}{4} = 15/4$

" " Middle $= \frac{3}{4} + \frac{4}{4} = 7/4$

" " Down $= \frac{12}{4} + \frac{2}{4} = 14/4$

Now "Up" is the best response.

BEST RESPONSE

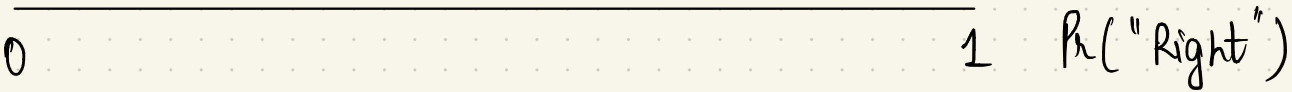
Let's examine player 1's best response for every possible belief about player 2's strategy.

BEST RESPONSE

	Left	Right
Up	5,1	0,2
Middle	1,3	4,1
Down	4,2	2,3

BEST RESPONSE

	Left	Right
Up	5,1	0,2
Middle	1,3	4,1
Down	4,2	2,3



BEST RESPONSE

	Left	Right
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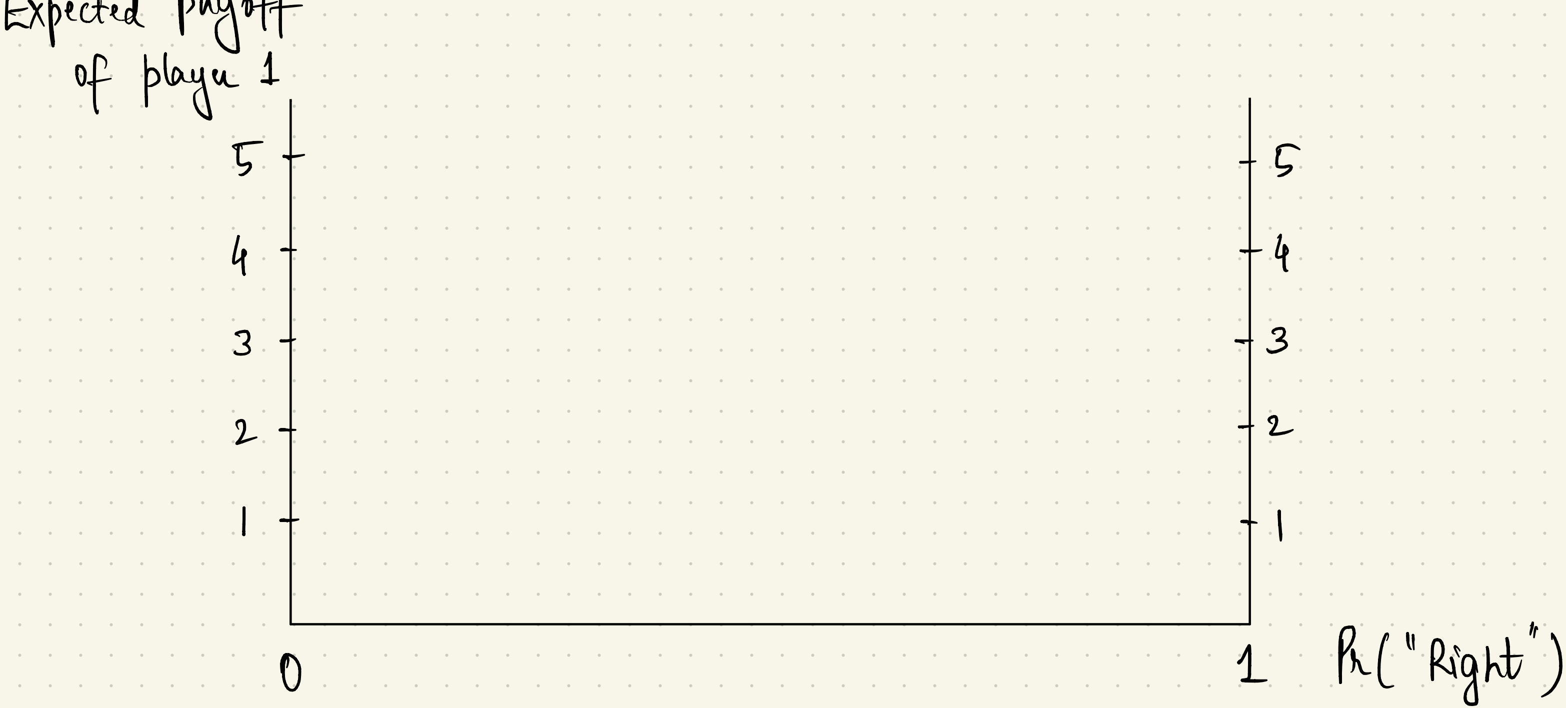

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	Left	Right
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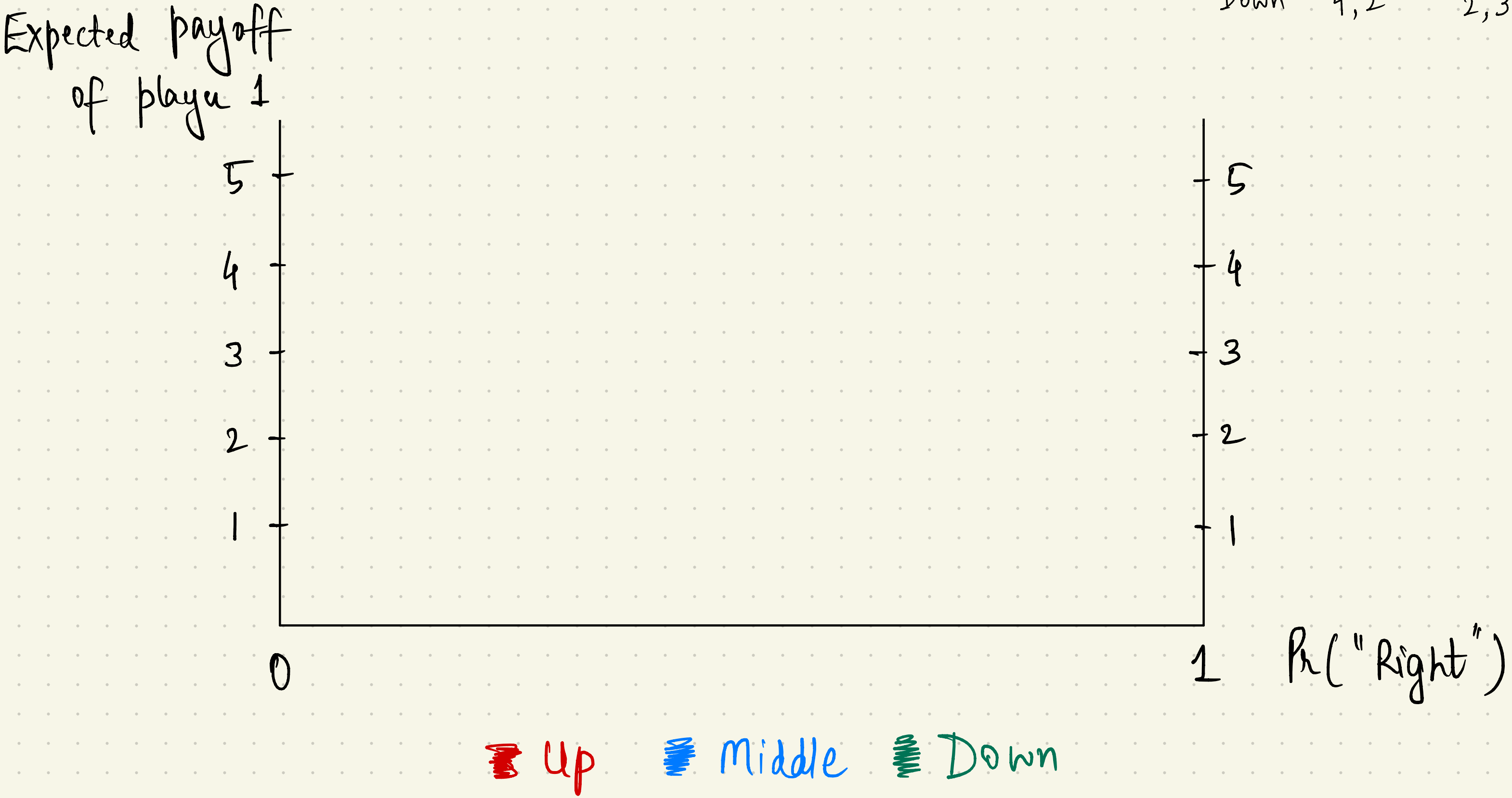

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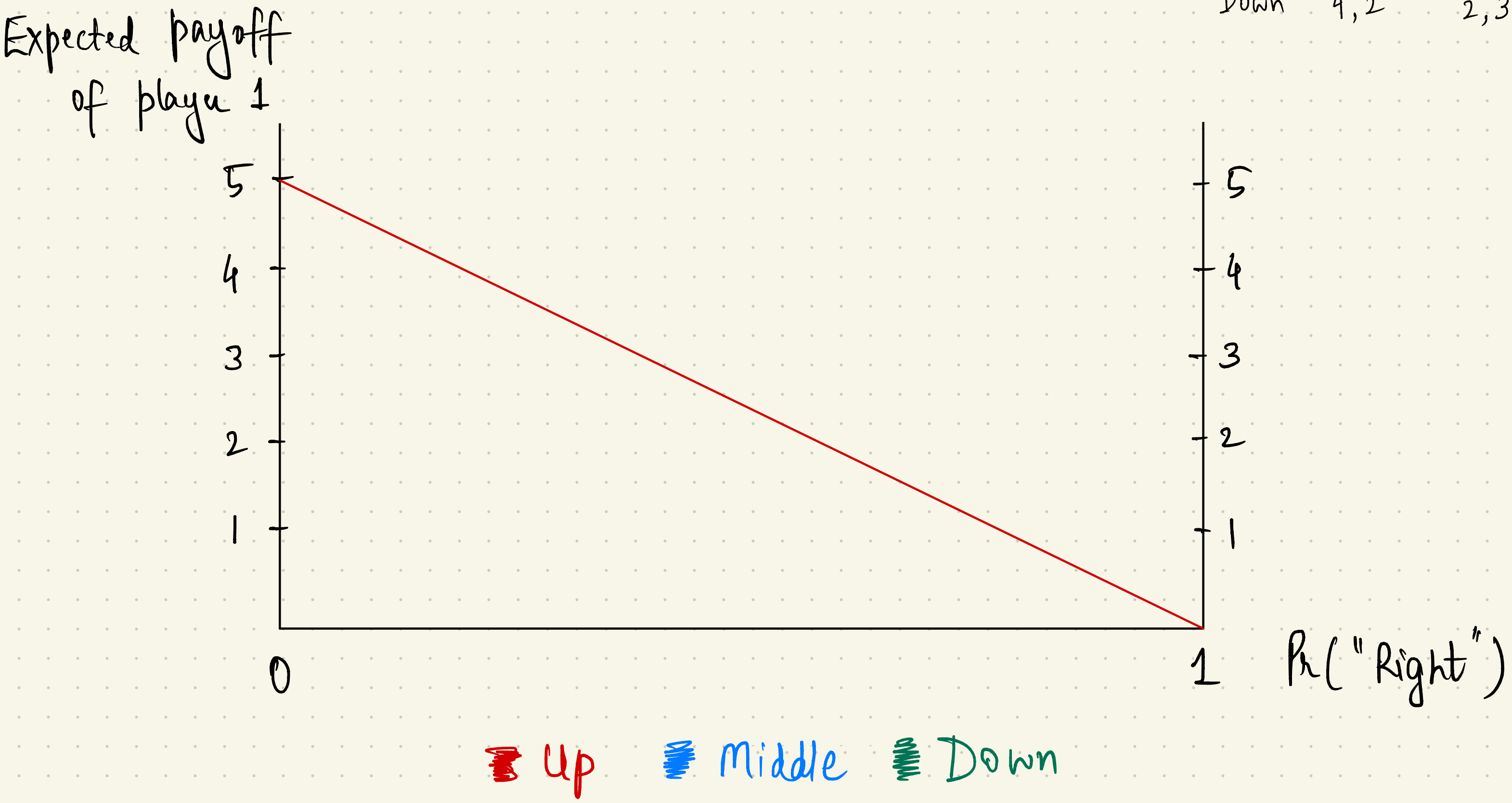

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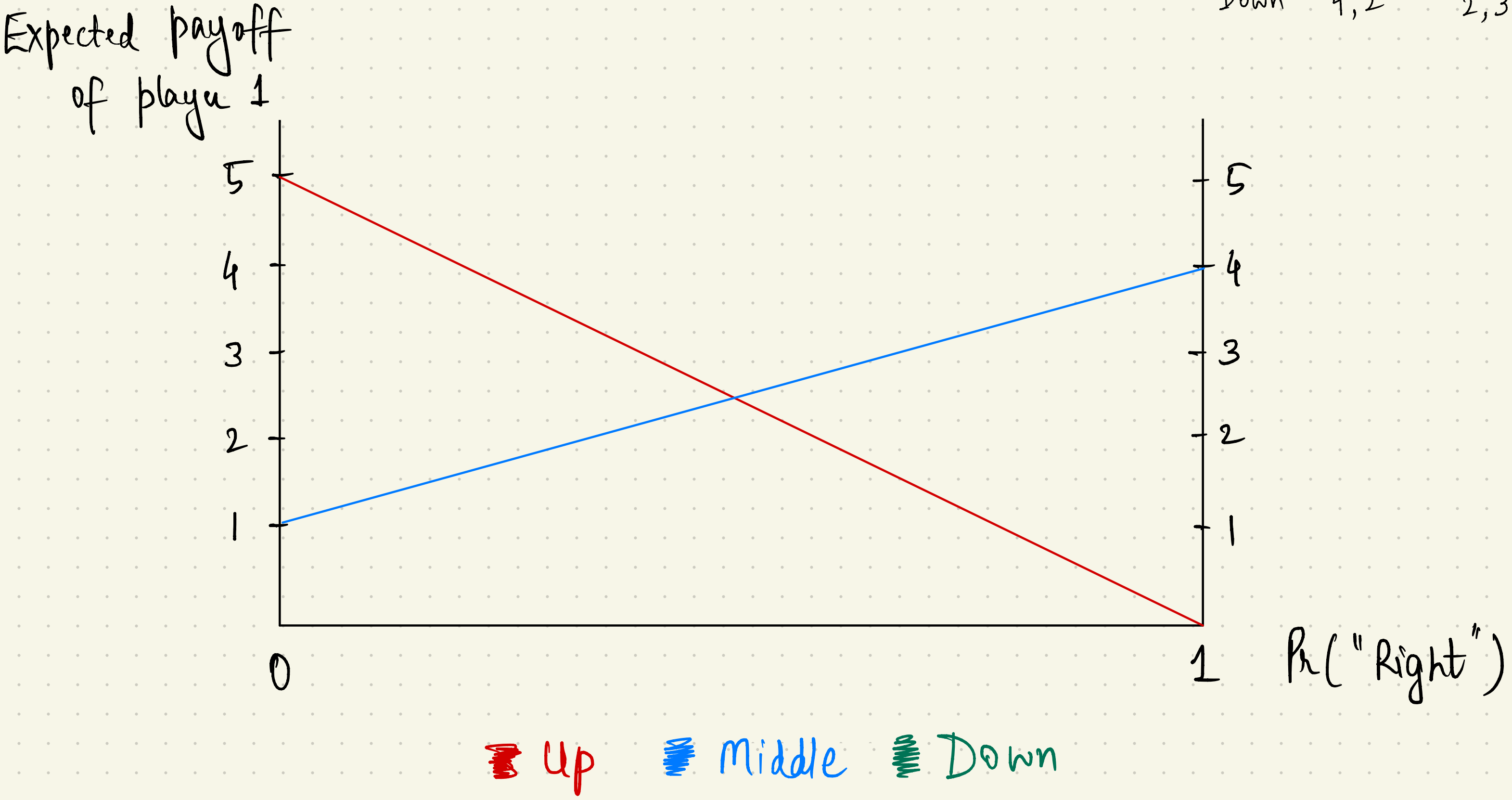

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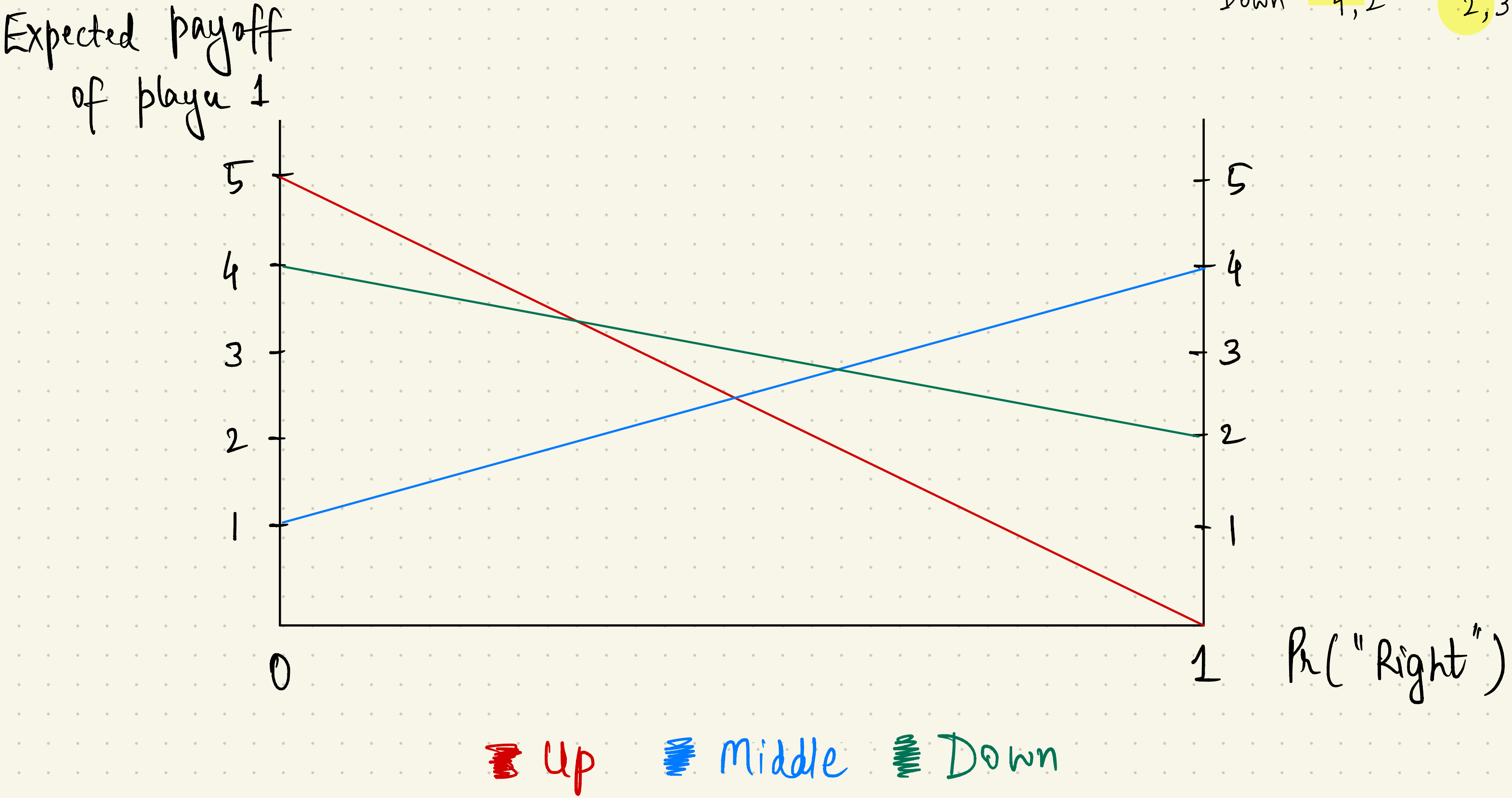

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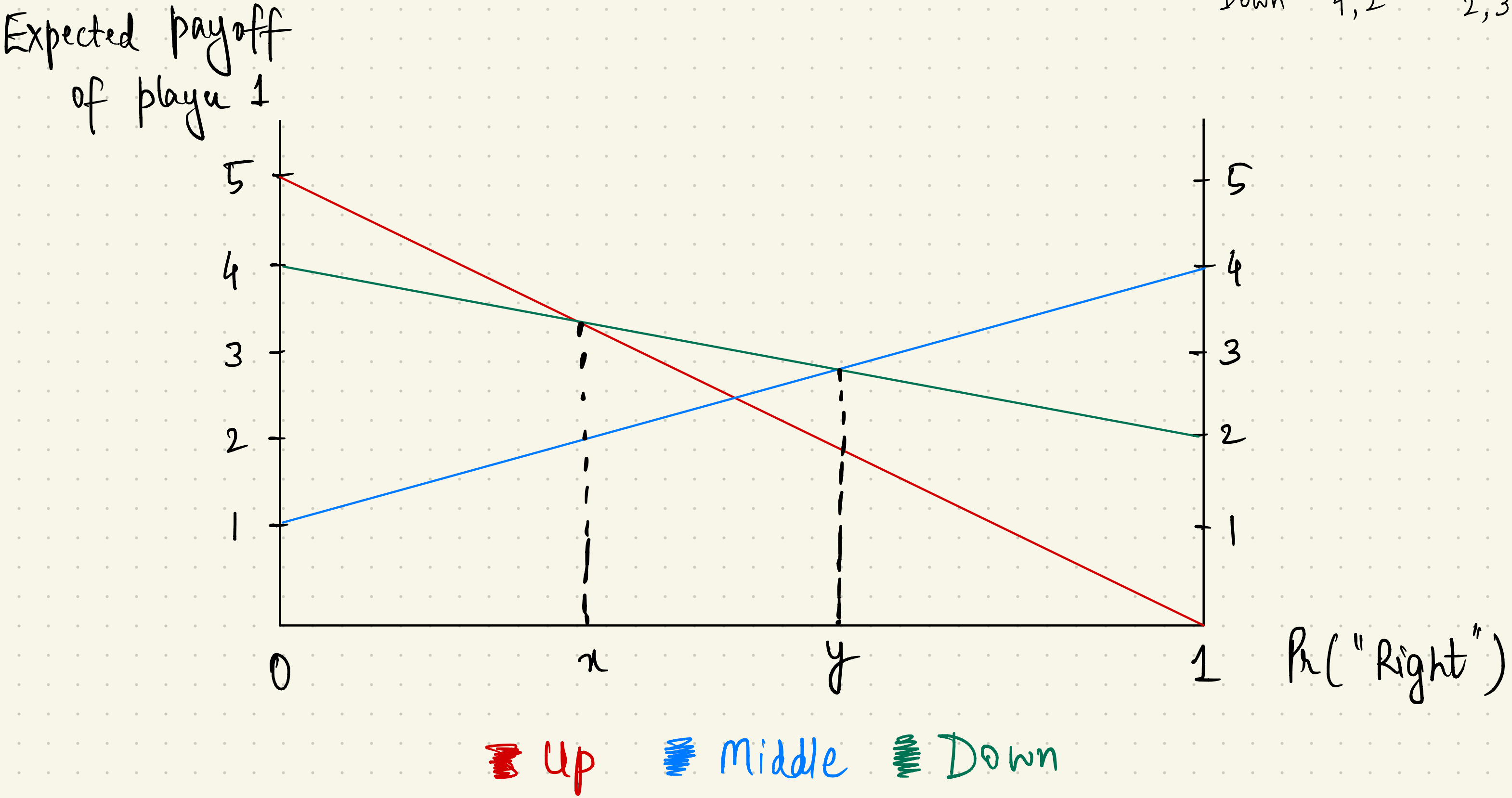
	Left	Right
Up	5,1	0,2
Middle	1,3	4,1
Down	4,2	2,3



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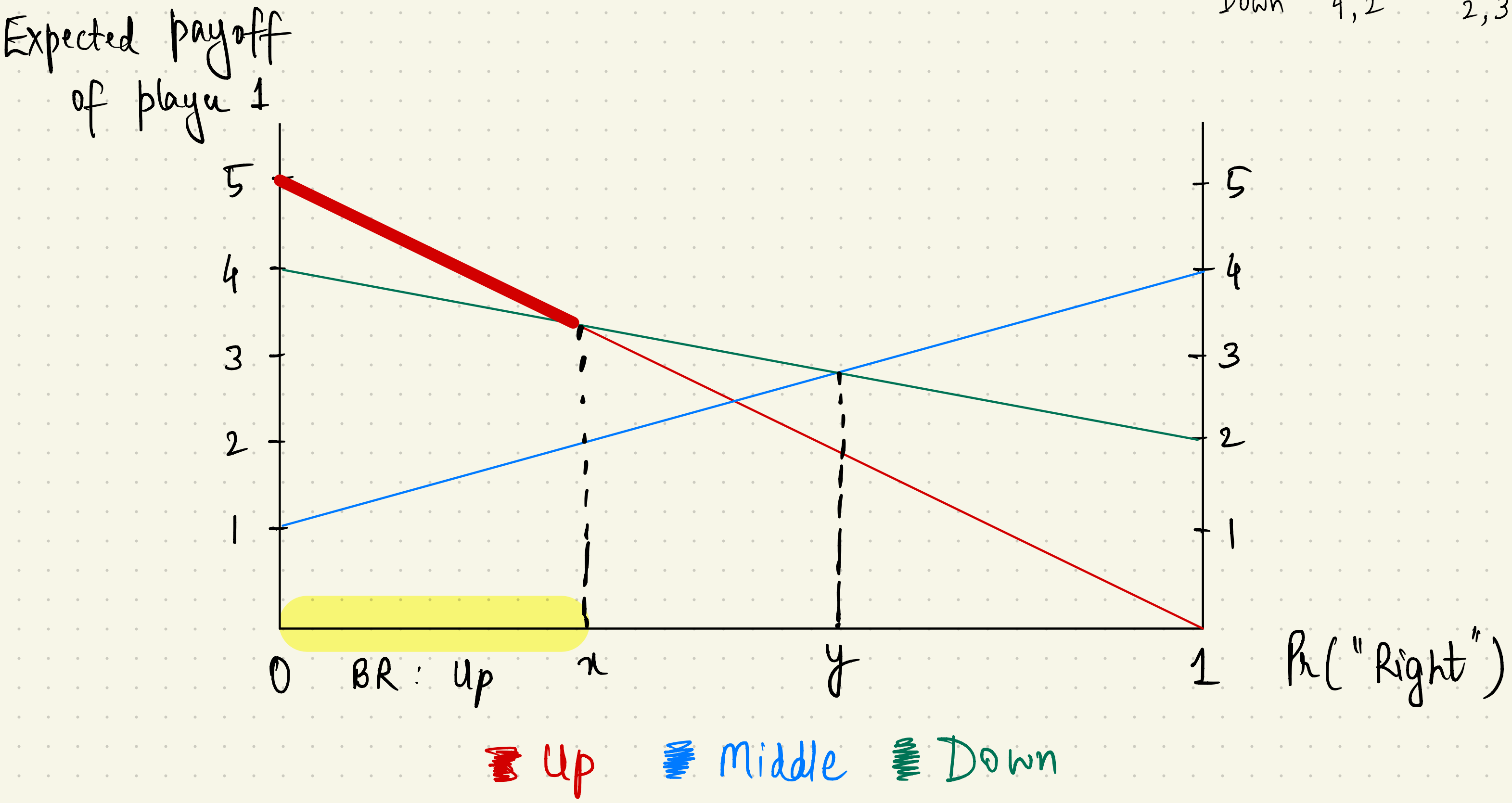

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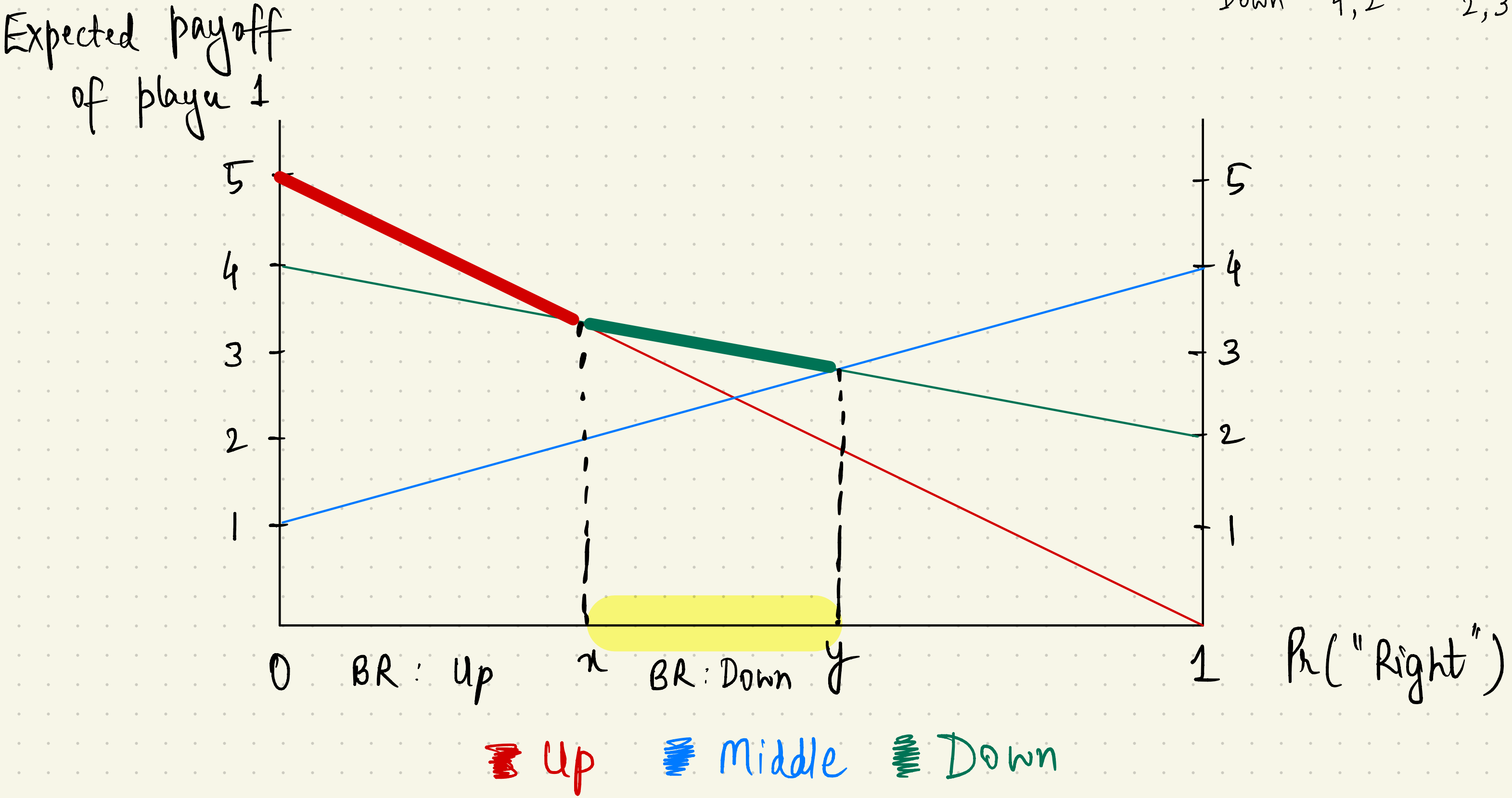

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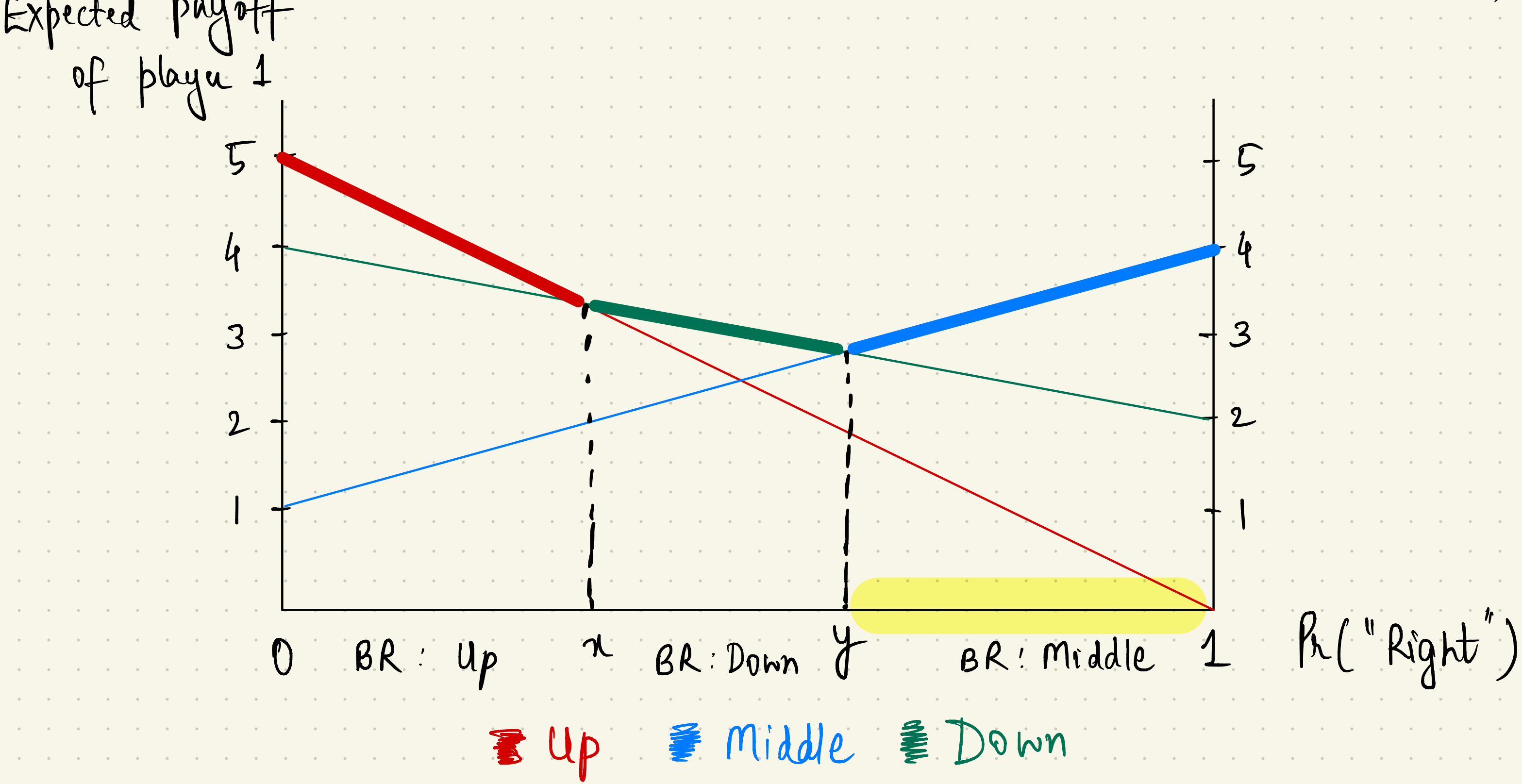
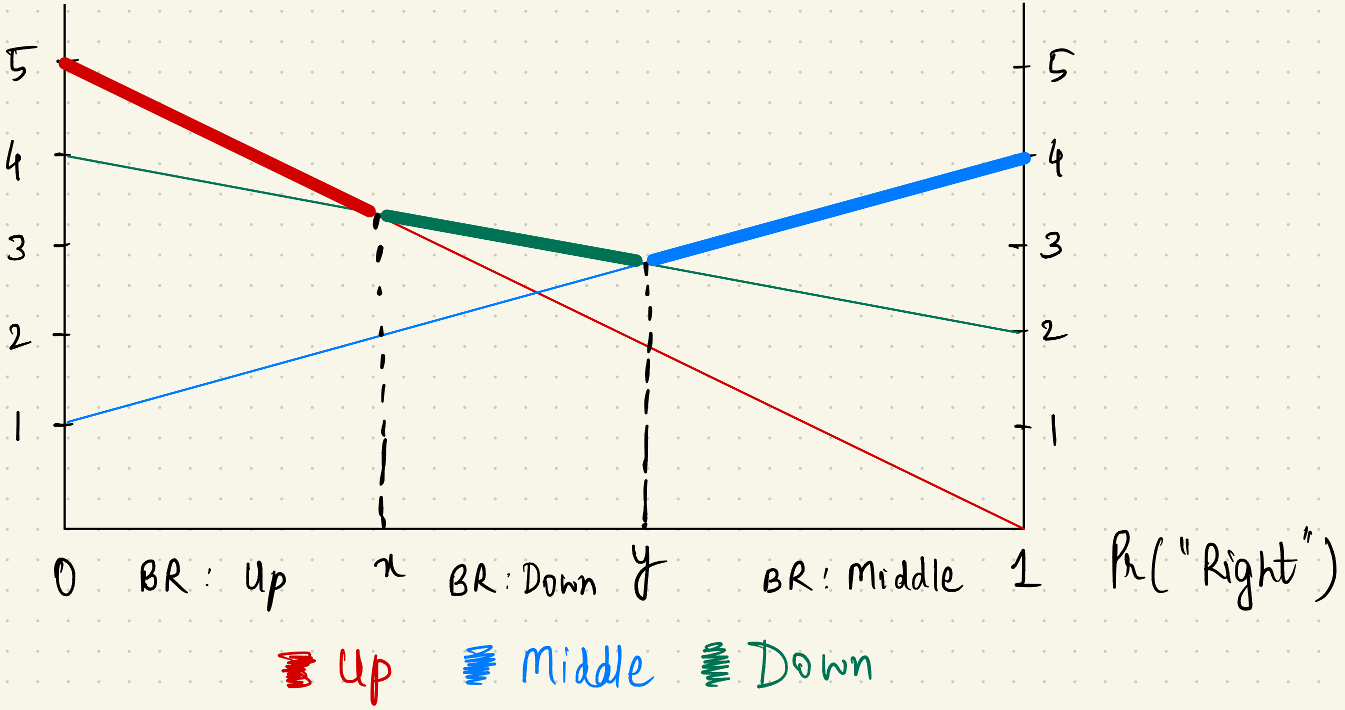


BEST RESPONSE

Lesson : Best response depends on the belief.



BEST RESPONSE

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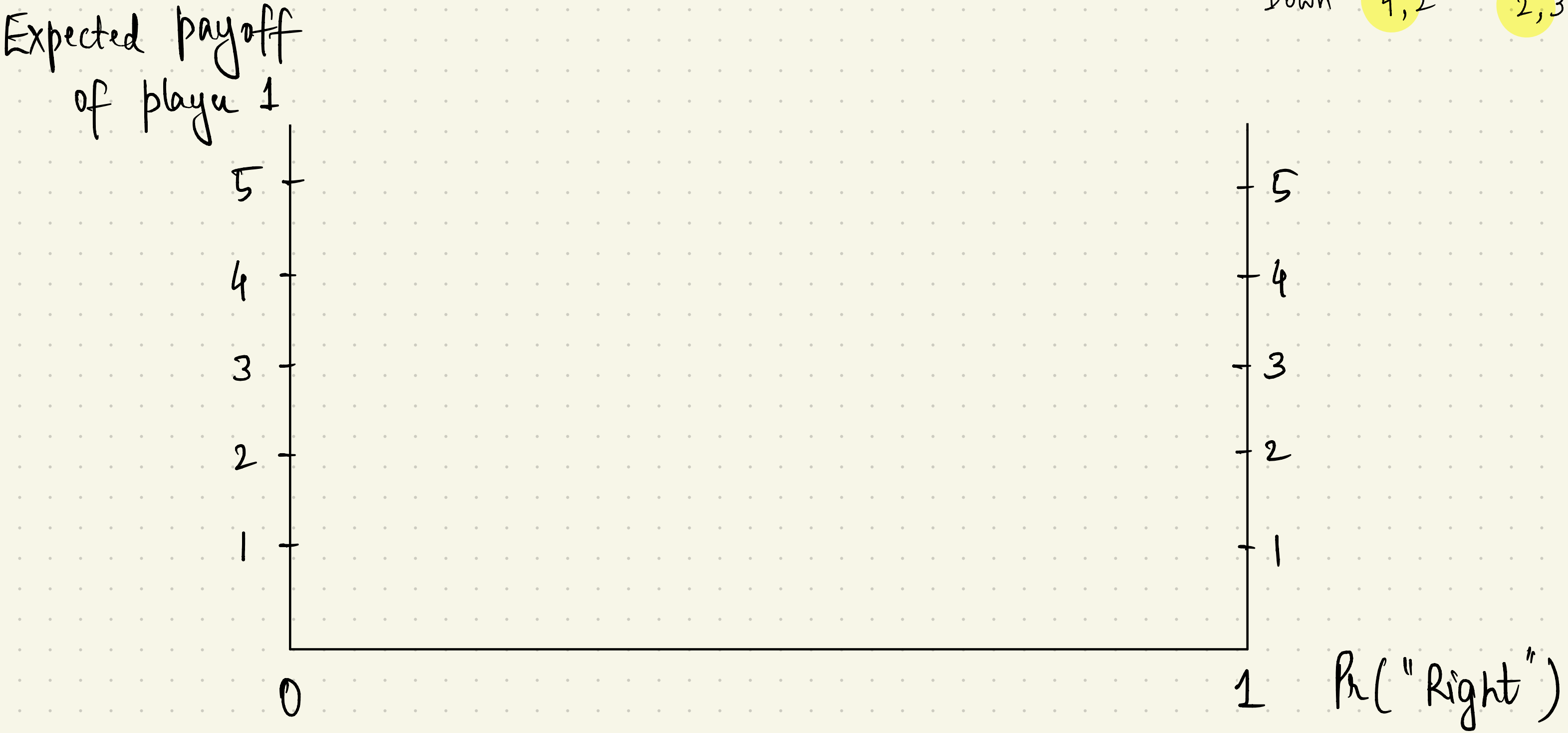
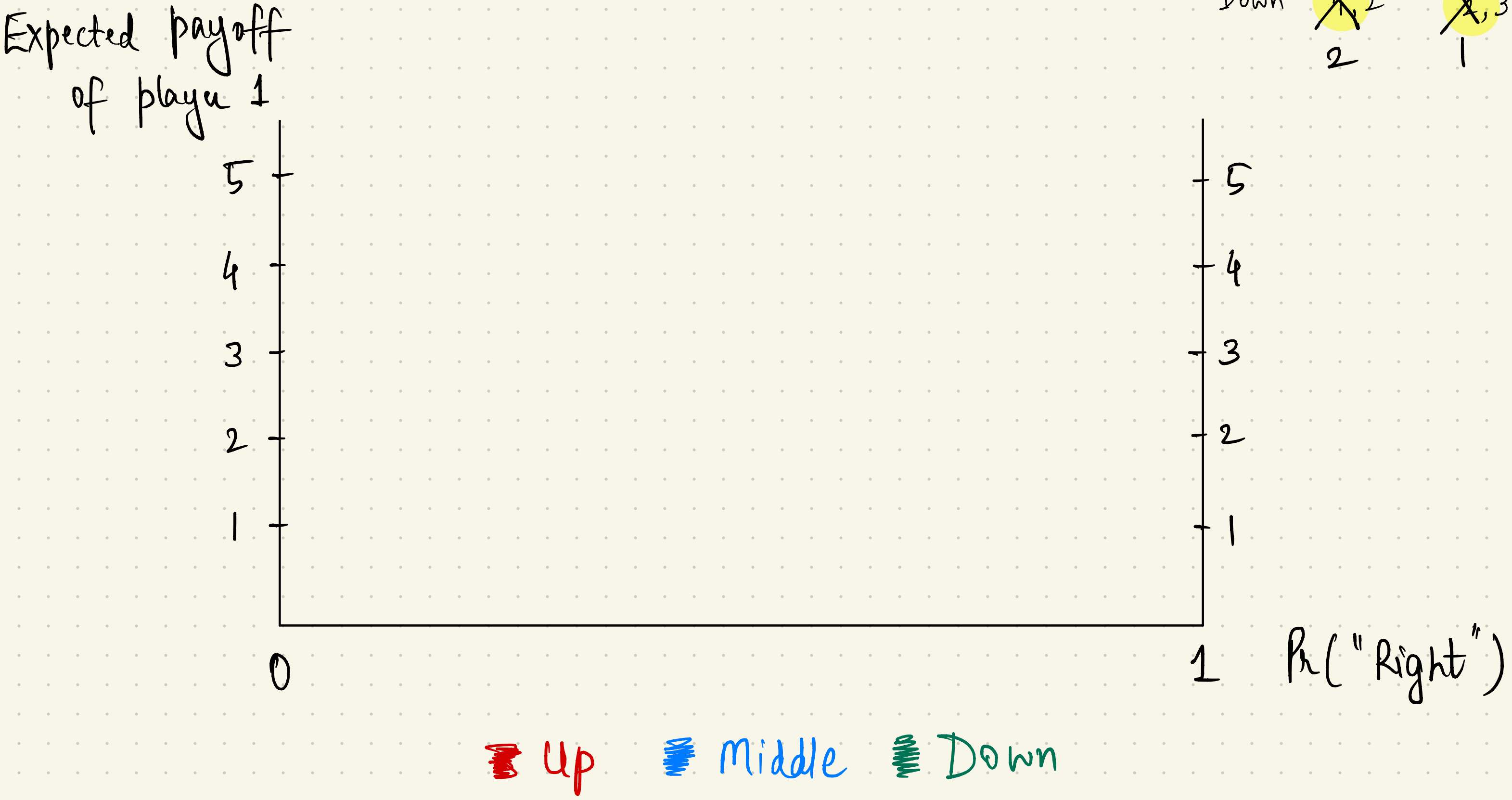


🟥 Up 🟦 Middle 🟩 Down

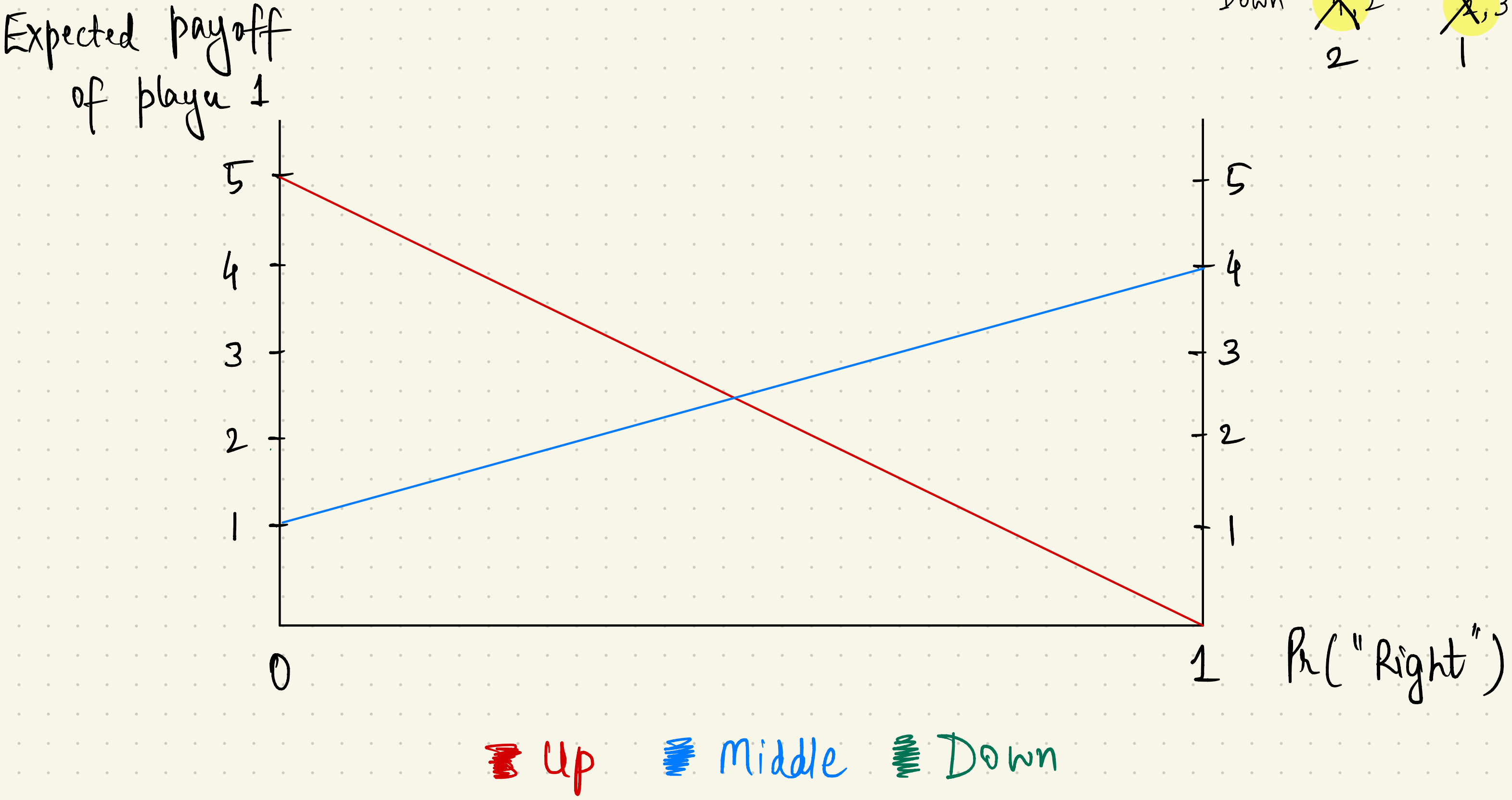
BEST RESPONSE

	Left	Right
Up	5, 1	0, 2
Middle	1, 3	4, 1
Down	~~1~~, 2 2	~~2~~, 3 1



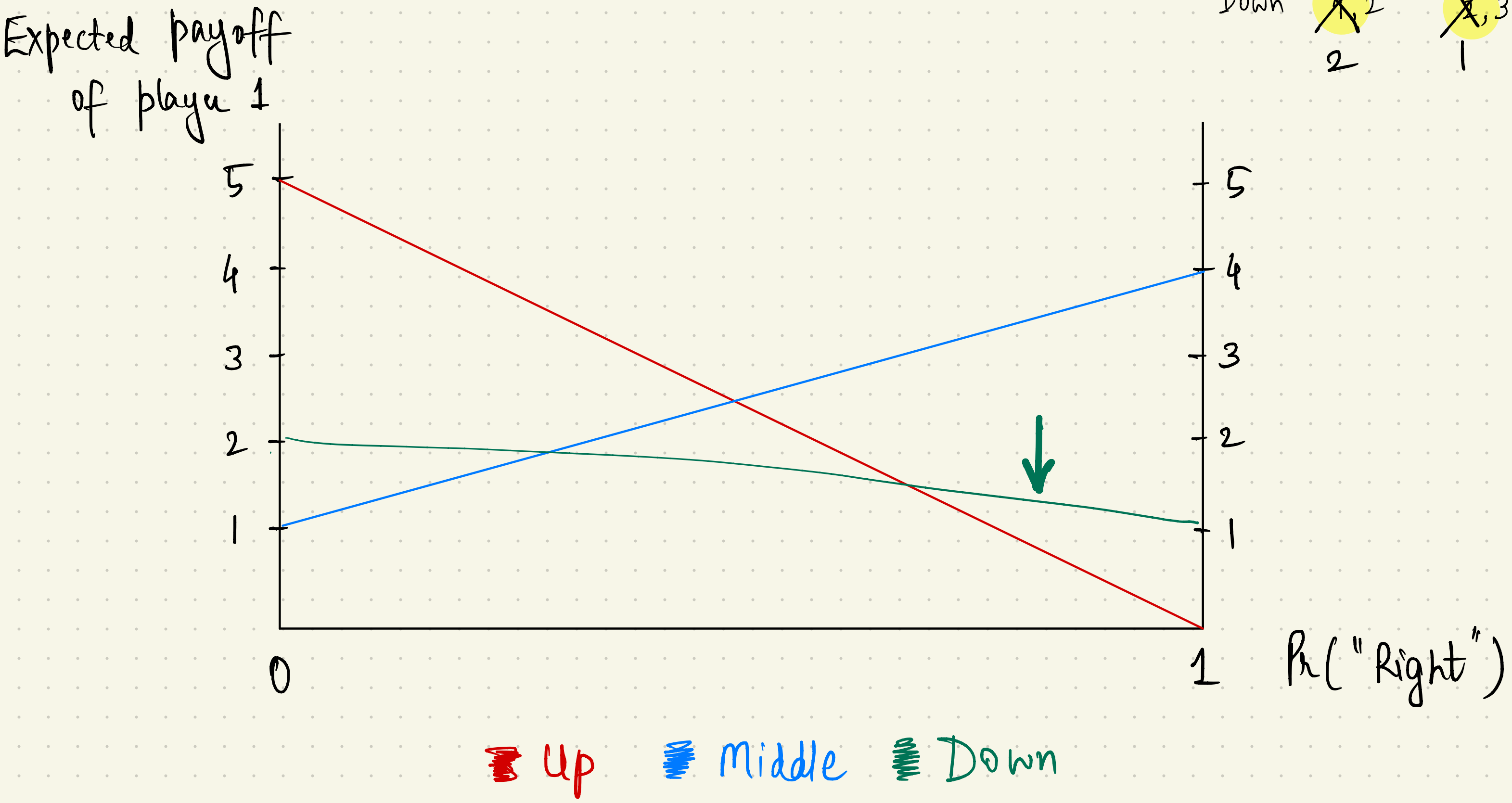
BEST RESPONSE

	Left	Right
Up	5,1	0,2
Middle	1,3	4,1
Down	~~1~~,2	~~2~~,3
	2	1



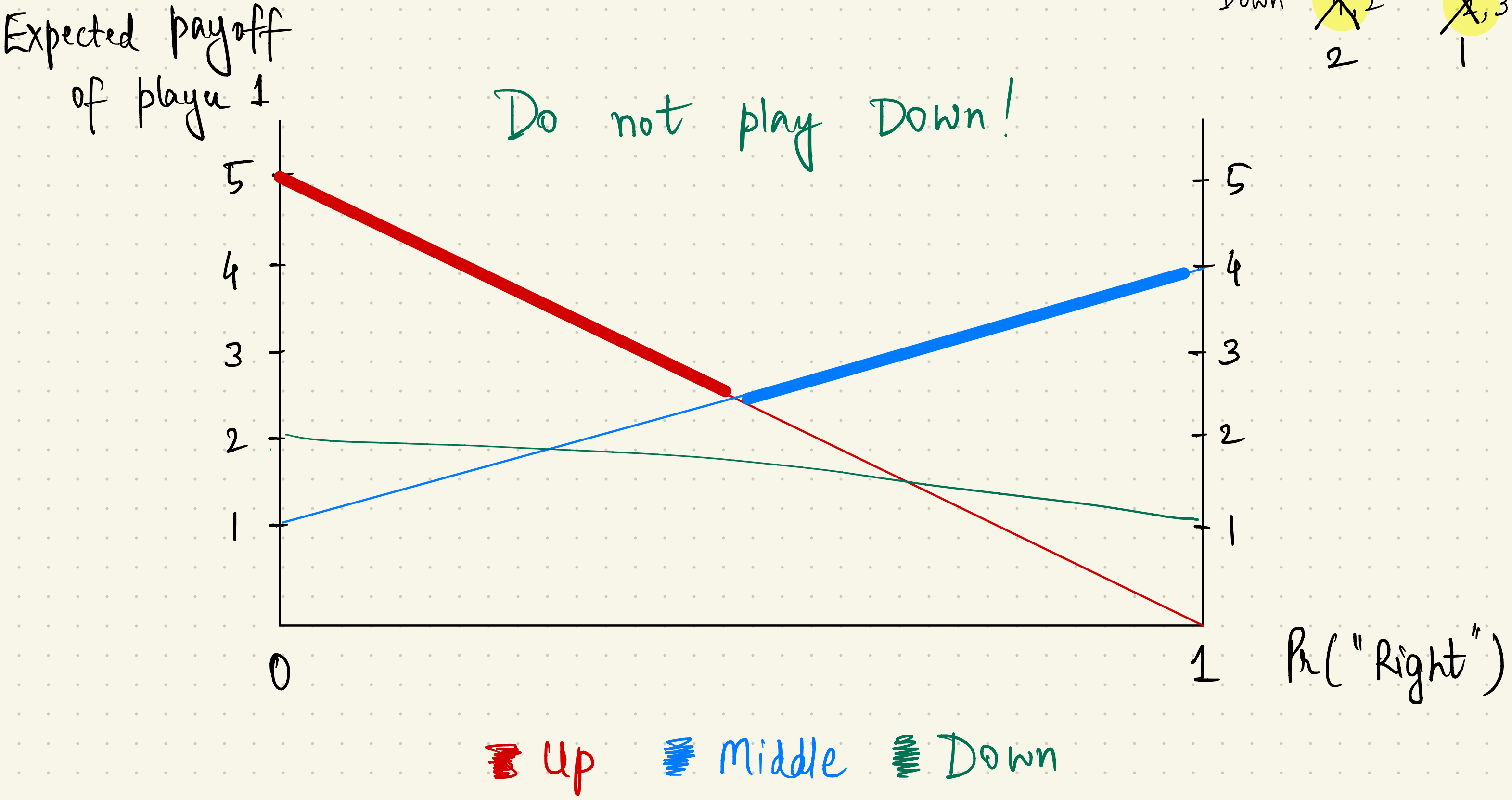
BEST RESPONSE

	Left	Right
Up	5, 1	0, 2
Middle	1, 3	4, 1
Down	~~4~~, 2	~~2~~, 3
	2	1



BEST RESPONSE

	Left	Right
Up	5,1	0,2
Middle	1,3	4,1
Down	~~4~~,2	~~2~~,3
	2	1



Do not play Down!

BEST RESPONSE

Player i's strategy $s_i \in S_i$ is a best responce to the strategy $s_{-i} \in S_{-i}$ of other players if $\forall s_i' \in S_i \quad s_i' \neq s_i$

$$u_i(s_i, s_{-i}) \geqslant u_i(s_i', s_{-i}).$$

BEST RESPONSE

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$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) .$$

Player i's strategy $s_i \in S_i$ is a best responce to the belief p about the other players if $\forall s_i' \in S_i \quad s_i' \neq s_i$

$$\mathbb{E}[u_i(s_i, p)] \geq \mathbb{E}[u_i(s_i', p)]$$

distribution over S_{-i}

PROJECT GAME

PROJECT GAME

* Two students working on a joint project

PROJECT GAME

- Two students working on a joint project

- Each chooses an effort level to put into the project
  $S_i = [0, 4]$ e.g., up to 4 hours a day

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PROJECT GAME

* Two students working on a joint project

* Each chooses an effort level to put into the project

$S_i = [0, 4]$ e.g., up to 4 hours a day

* Final score $= 4\left[s_1 + s_2 + b\, s_1 s_2\right]$ $b \in [0, 1/4]$

Synergy / Complementarity

PROJECT GAME

- Two students working on a joint project

- Each chooses an effort level to put into the project
  $S_i = [0, 4]$ e.g., up to 4 hours a day

- Final score $= 4 [s_1 + s_2 + b s_1 s_2]$ $b \in [0, 1/4]$

- Payoffs

PROJECT GAME

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- Each chooses an effort level to put into the project
  $S_i = [0, 4]$ e.g., up to 4 hours a day

- Final score $= 4[s_1 + s_2 + b s_1 s_2]$ $b \in [0, 1/4]$

- Payoffs $u_1(s_1, s_2) = \frac{1}{2}\left(4[s_1 + s_2 + b s_1 s_2]\right) - \underbrace{s_1^2}_{\text{effort}}$

PROJECT GAME

- Two students working on a joint project

- Each chooses an effort level to put into the project
  $S_i = [0, 4]$ eg., up to 4 hours a day

- Final score $= 4\left[s_1 + s_2 + b s_1 s_2\right]$ $b \in [0, 1/4]$

- Payoffs $u_1(s_1, s_2) = \frac{1}{2}\left(4\left[s_1 + s_2 + b s_1 s_2\right]\right) - s_1^2$
  $u_2(s_1, s_2) = \frac{1}{2}\left(4\left[s_1 + s_2 + b s_1 s_2\right]\right) - s_2^2$

PROJECT GAME

Analyze player 1's best response for every possible belief about player 2's strategy.

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But $S_2 = [0,4]$

PROJECT GAME

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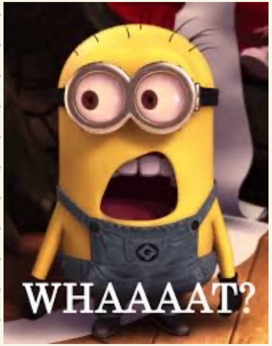
But $S_2 = [0,4]$



Too many probability distributions!

PROJECT GAME

Given s_2 ,
$$\max_{s_1} \quad 2(s_1 + s_2 + b s_1 s_2) - s_1^2$$
$$\text{s.t.} \quad s_1 \in [0, 4]$$

PROJECT GAME

Given s_2,

$$\begin{aligned} \max_{s_1} \quad & 2(s_1 + s_2 + b s_1 s_2) - s_1^2 \\ \text{s.t.} \quad & s_1 \in [0,4] \end{aligned}$$

$$\Bigg\downarrow \quad \text{some calculus later}$$

PROJECT GAME

Given s_2 , $$\max_{s_1} \quad 2(s_1 + s_2 + b s_1 s_2) - s_1^2$$
$$\text{s.t.} \quad s_1 \in [0, 4]$$

$$\Big\downarrow \quad \text{some calculus later}$$

player 1's best responce $\hat{s}_1 = 1 + b s_2$

PROJECT GAME

Given s_2,

$$\max_{s_1} \quad 2(s_1 + s_2 + b s_1 s_2) - s_1^2$$

$$\text{s.t.} \quad s_1 \in [0, 4]$$

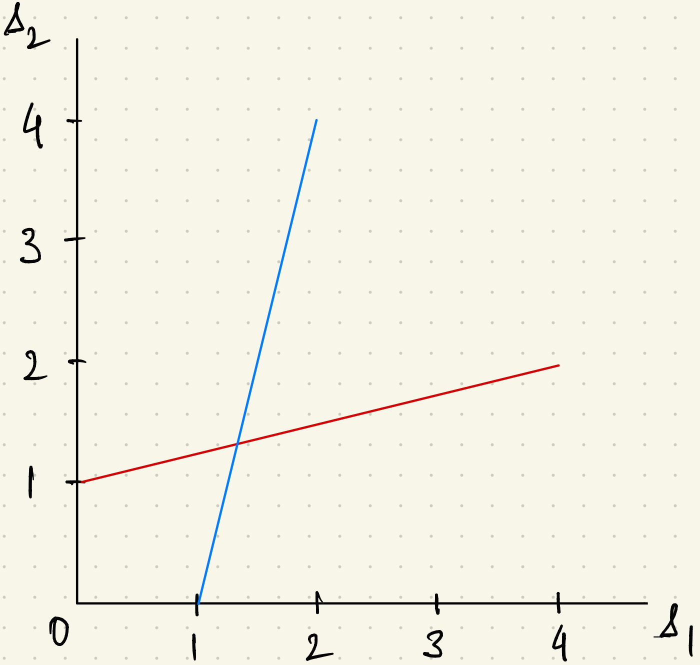
$$\Big\downarrow \quad \text{some calculus later}$$

player 1's best responce $\hat{s}_1 = 1 + b s_2$

Similarly, given s_1,

player 2's best responce $\hat{s}_2 = 1 + b s_1$

PROJECT GAME



say $b = 1/4$

$\hat{s}_1 = 1 + s_2/4$

$\hat{s}_2 = 1 + s_1/4$

PROJECT GAME

say $b = 1/4$

$$\hat{s}_1 = 1 + s_2/4$$

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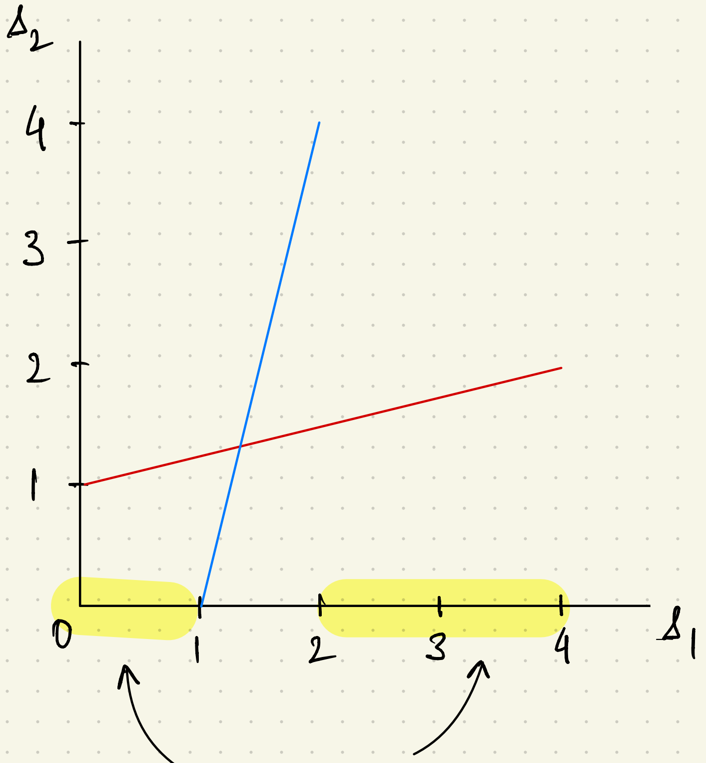
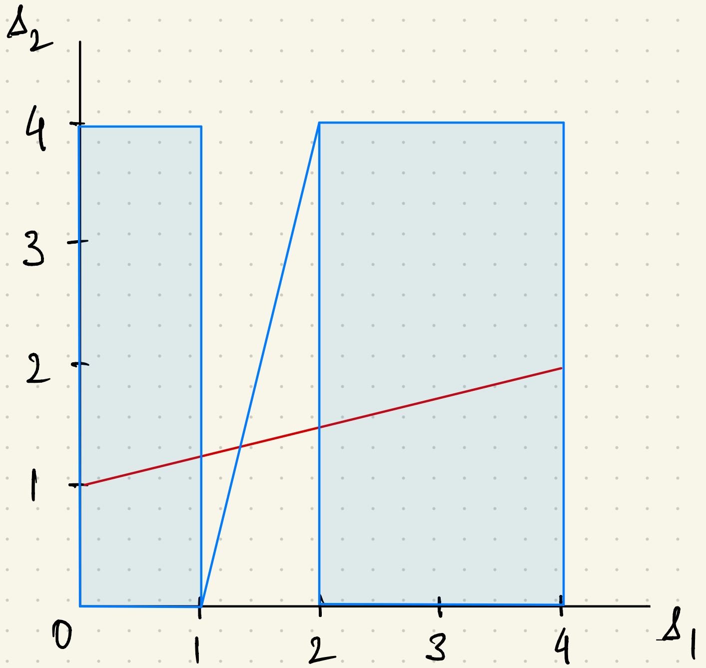


these strategies are never a best response for player 1

PROJECT GAME



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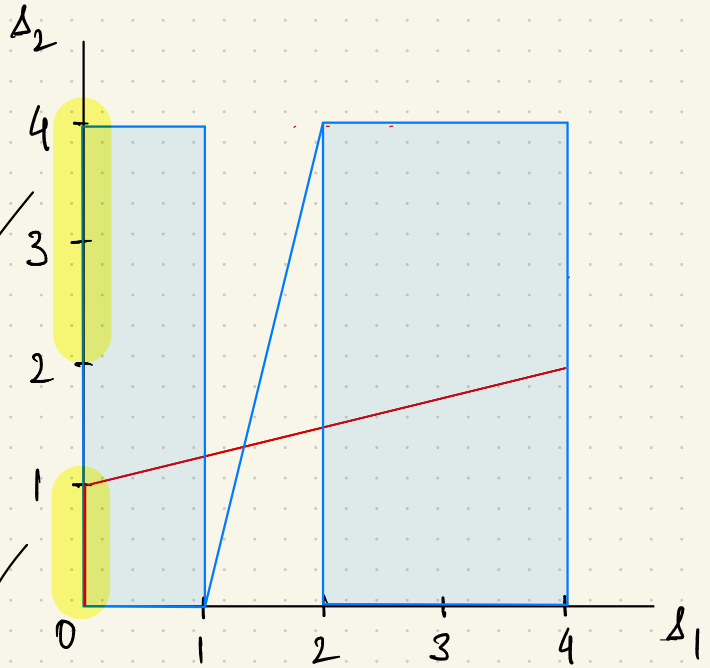

these strategies are never best response for player 2

PROJECT GAME

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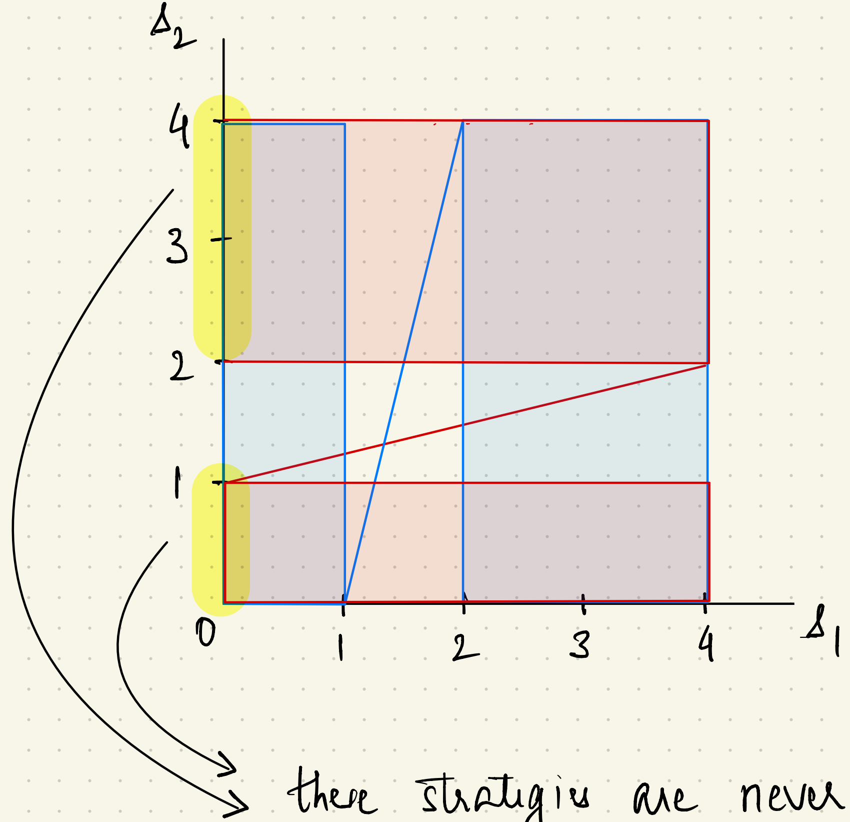
$\hat{s}_1 = 1 + s_2/4$

$\hat{s}_2 = 1 + s_1/4$



these strategies are never best response for player 2

PROJECT GAME

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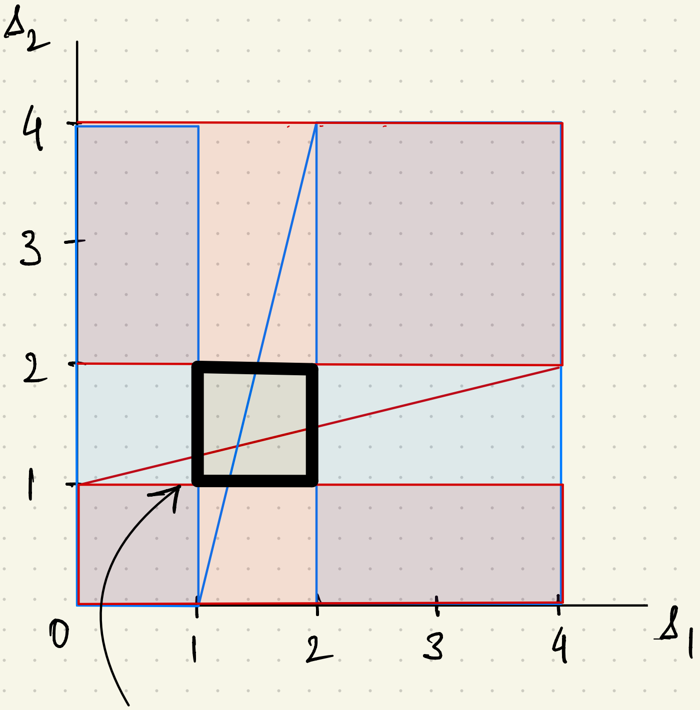
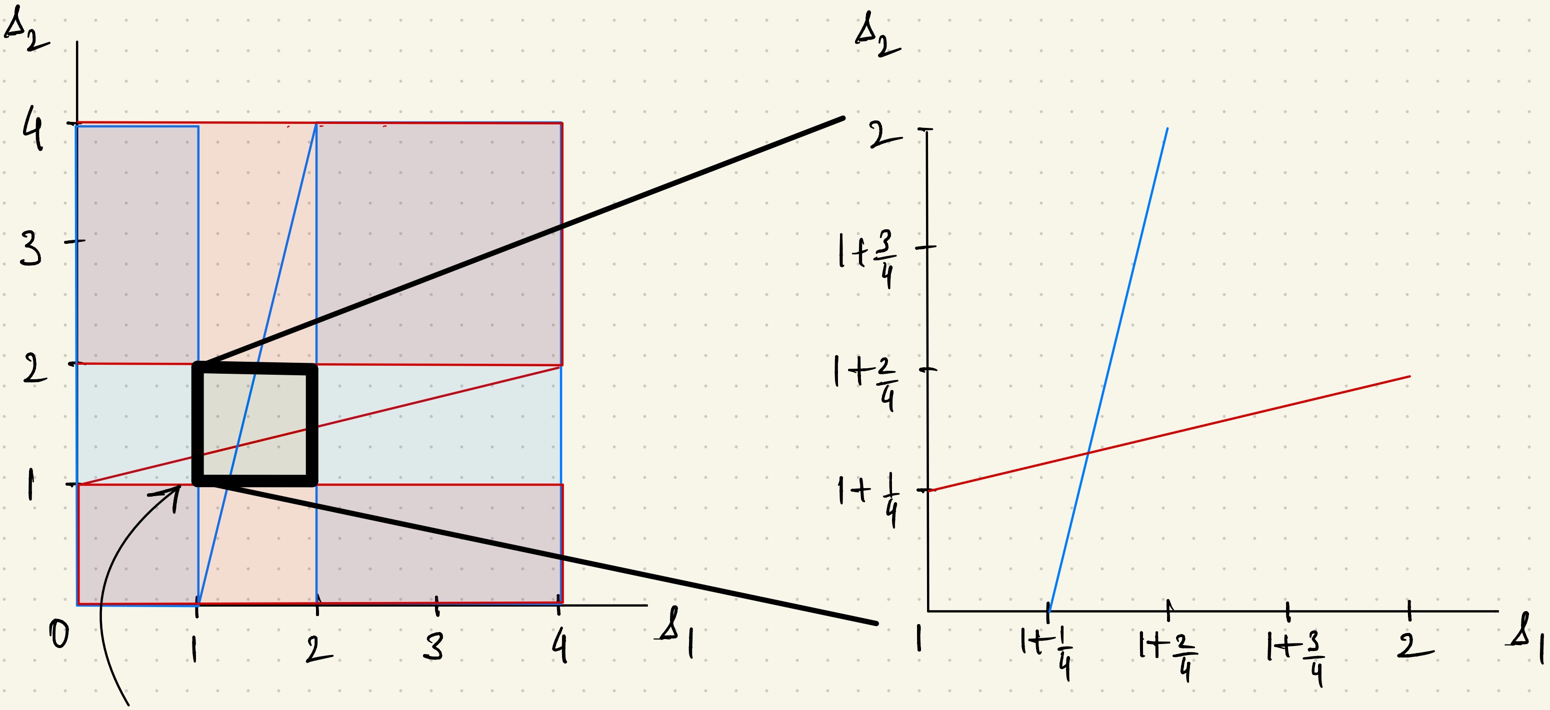


So, we are left with this region. Let's zoom in.

PROJECT GAME



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say $b = 1/4$

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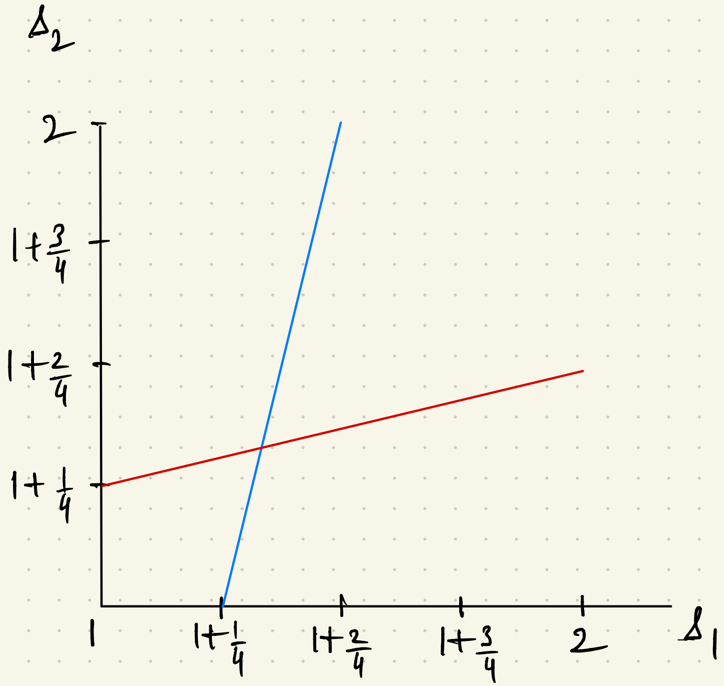

PROJECT GAME

say $b = 1/4$

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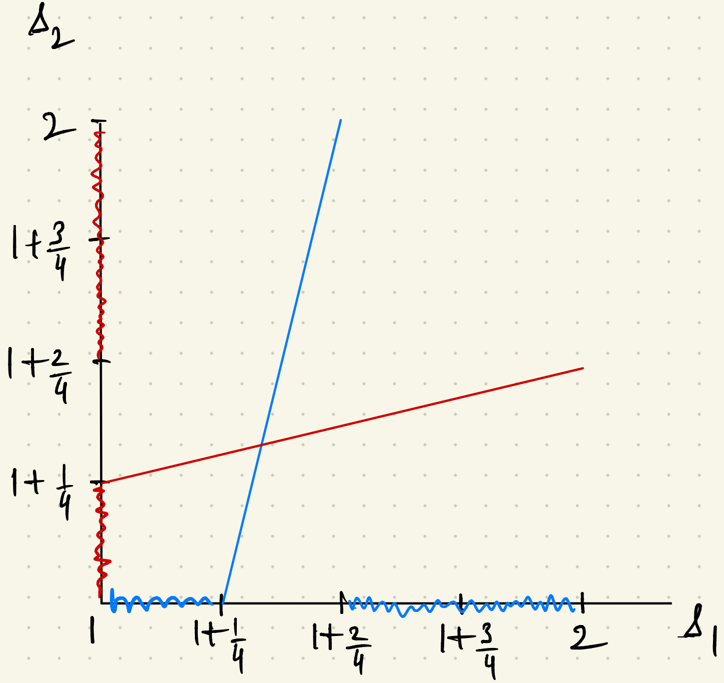

Apply the same argument!

PROJECT GAME

say $b = 1/4$

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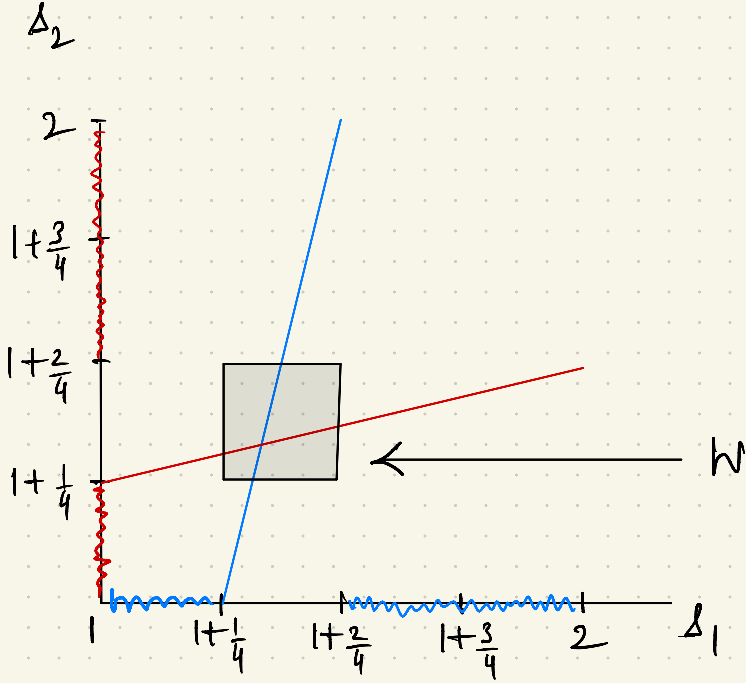

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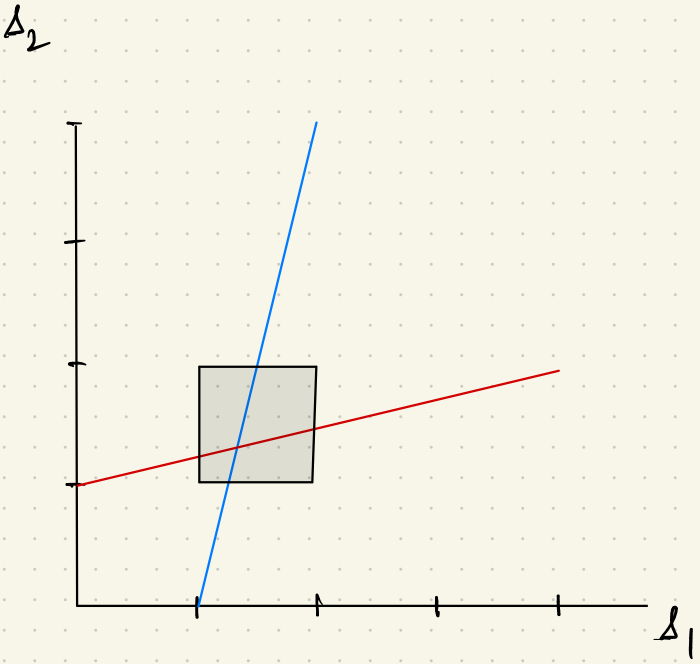
PROJECT GAME

say $b = 1/4$



$\hat{s}_1 = 1 + s_2/4$

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Eventually, we are left with : $s_1^* = 1 + b s_2^*$

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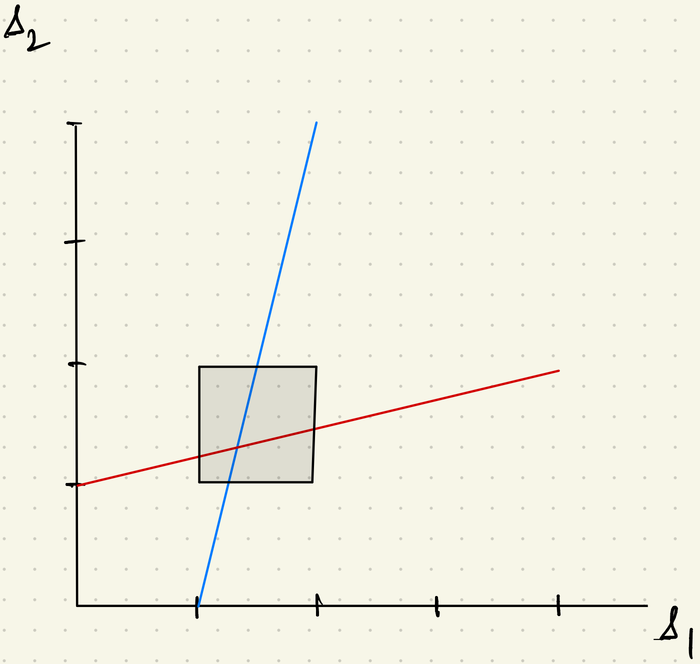

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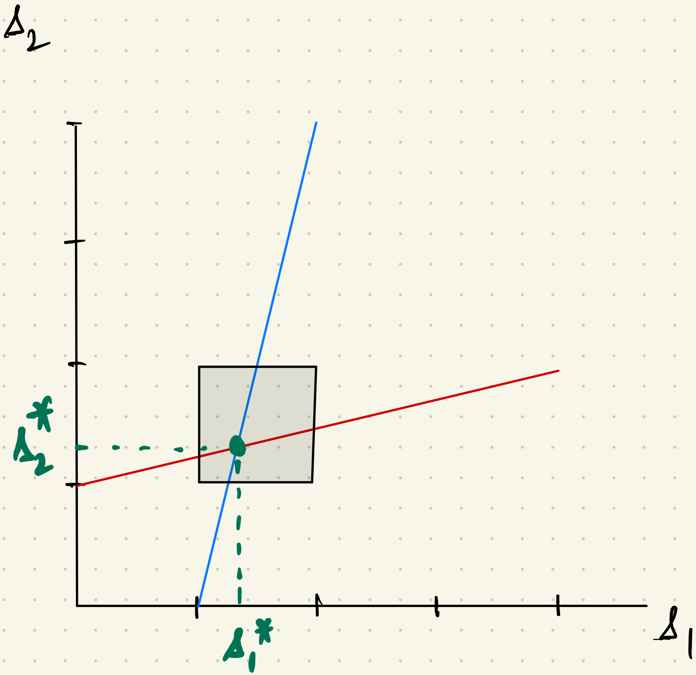

NASH EQUILIBRIUM

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Self-fulfilling : If everyone's playing their Nash eq. strategy, then my best response is to play mine.

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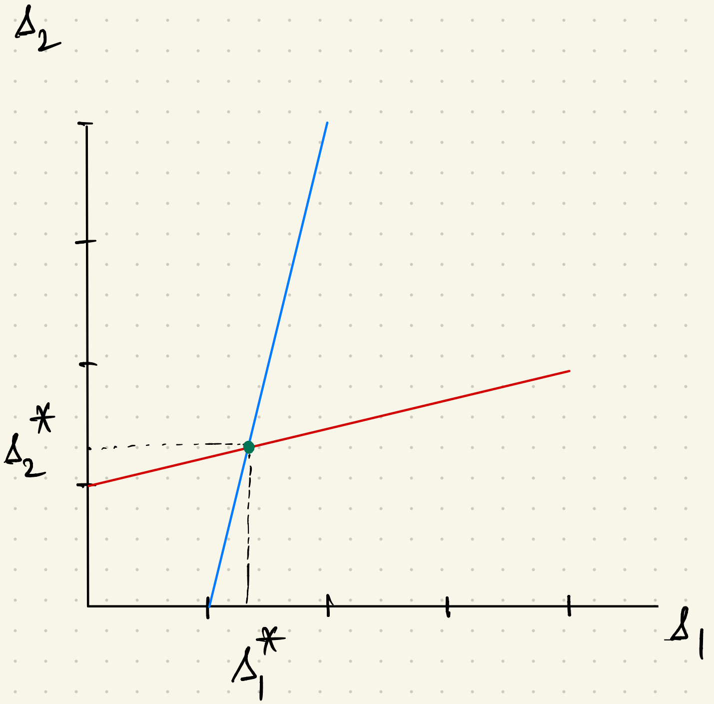

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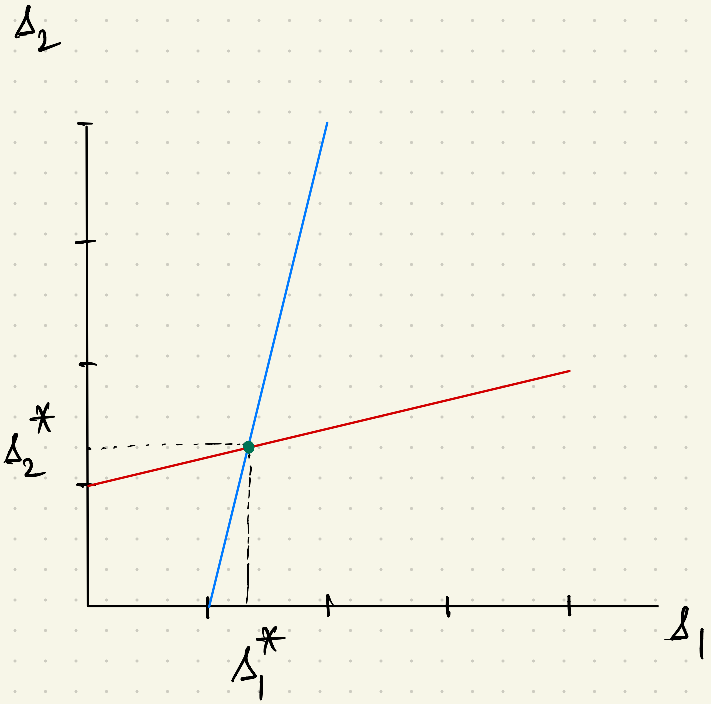

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What if we lower "b" ?

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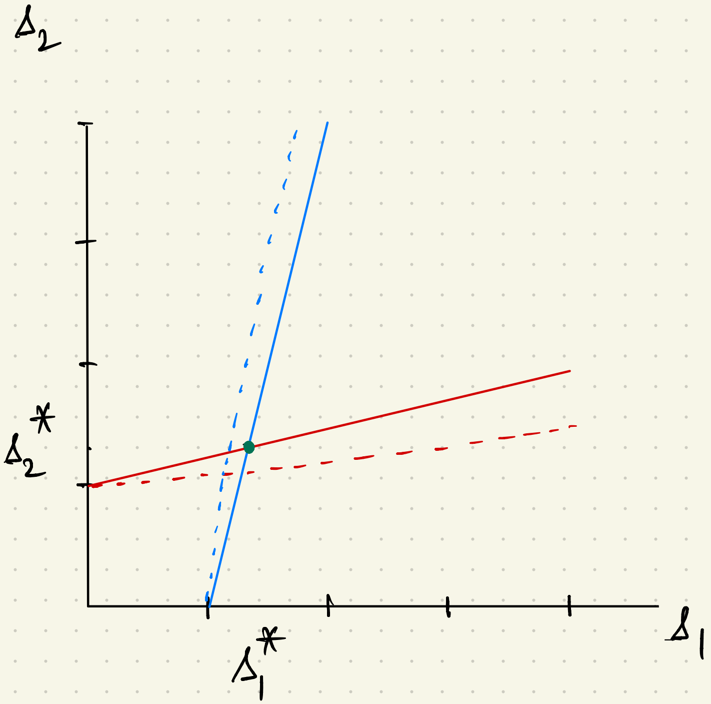

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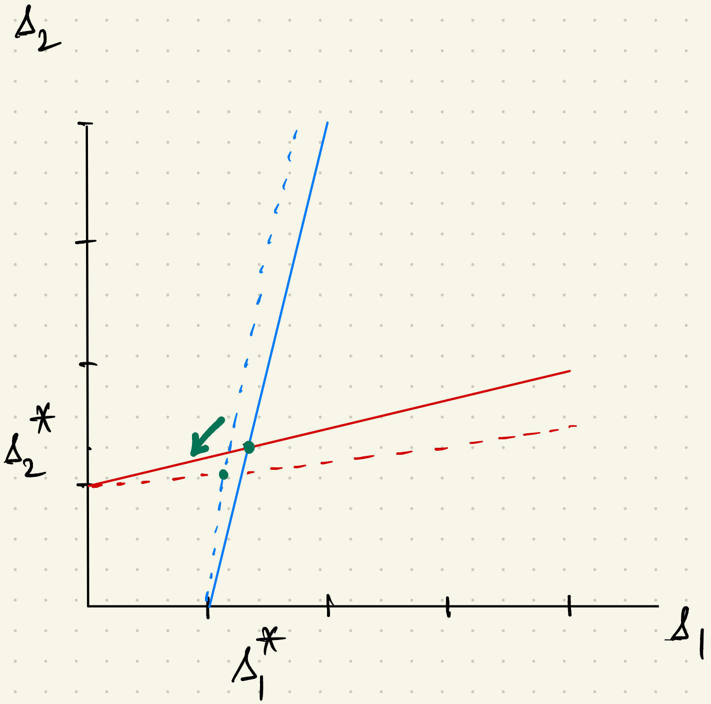

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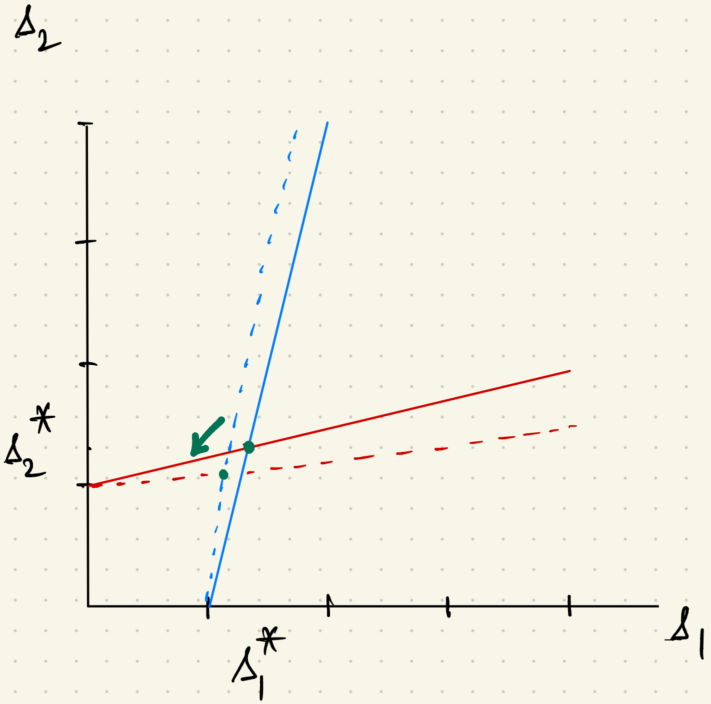

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What if we lower "b"?

Equilibrium effort decreases.

Recall: The $\frac{1}{2}$- mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

Winner is one whose number is closest to $\frac{1}{2}$ of average.

Is there a Nash equilibrium?

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Recall: The ½ - mean game

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Winner is one whose number is closest to $\frac{1}{2}$ of average.

Is there a Nash equilibrium? Yes, 1.

Doesn't mean people will necessarily play it.