STCS VIGYAN VIDUSHI 2025

RATIONALITY & COMMON KNOWLEDGE

ROHIT VAISH

Objective : maximize individual benefit

		Colin	
		SPLIT	STEAL
Rose	SPLIT	50, 50	0, 100
	STEAL	100, 0	1, 1

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Rational selfish choices can lead to inefficient outcomes

Prisoner's dilemma

Prisoner's dilemma

Two prisoners : A and B

Prisoner's dilemma

Two prisoners : A and B

Not enough evidence to convict them

Prisoner's dilemma

Two prisoners : A and B

Not enough evidence to convict them

Separate interrogation rooms — no communication.

Prisoner's dilemma

Two prisoners : A and B

Not enough evidence to convict them

Separate interrogation rooms — no communication.

Each offered two choices : Stay silent or defect.

Prisoner's dilemma

		B	
		Silent	Defect
A	Silent		
	Defect		

Prisoner's dilemma

		B	
		Silent	Defect
A	Silent	$-1, -1$	
	Defect		

Both stay silent — each gets a 1-year sentence

Prisoner's dilemma

		B	
		Silent	Defect
A	Silent	$-1, -1$	$-5, 0$
	Defect	$0, -5$	

One stays silent, other defects : Silent gets a 5-year sentence
Defect goes scot-free

Prisoner's dilemma

		B	
		Silent	Defect
A	Silent	−1 , −1	−5 , 0
	Defect	0 , −5	−3 , −3

Both defect —— each gets a 3-year sentence

Prisoner's dilemma

		B: Silent	B: Defect
A	Silent	$-1, -1$	$-5, 0$
	Defect	$0, -5$	$-3, -3$

"Silent" is strictly dominated $\Rightarrow$ both "defect"

Prisoner's dilemma

* Cleaning a shared flat

Prisoner's dilemma

* Cleaning a shared flat

* Limiting carbon emissions

Prisoner's dilemma

* Cleaning a shared flat

* Limiting carbon emissions

* Exploiting a common resource, eg., fishing or mining.

Prisoner's dilemma

* Cleaning a shared flat
* Limiting carbon emissions
* Exploiting a common resource, e.g., fishing or mining.
* Sports — performance enhancement drugs

and many more ...

Ingredients of a game

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* Players $N = \{1, 2, \ldots, n\}$

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* Strategies

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* Strategies S_i : set of player i's strategies

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  $s = (s_1, s_2, \ldots, s_n) \in S_1 \times S_2 \times \cdots \times S_n$

  Strategy profile

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  Strategy profile

  e.g., (Silent, Silent)

Ingredients of a game

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  $\forall i \in N$ utility function $u_i : S_1 \times \cdots \times S_n \to \mathbb{R}$

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equivalently, $u_i : S_i \times S_{-i} \to \mathbb{R}$

strategy sets of all players except i.

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  strategy sets of all players except i.

* Game tuple $T = \left(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N}\right)$

Dominant Strategy

Dominant Strategy

B

A	Silent	Defect
Silent	-1, -1	-5, 0
Defect	0, -5	-3, -3

Dominant Strategy

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For player i , strategy $s_i \in S_i$ is strictly dominant if

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For player i, strategy $s_i \in S_i$ is strictly dominant if

$\forall\, s_{-i} \in S_{-i}$, $\forall\, s_i' \in S_i$ s.t. $s_i' \neq s_i$

Dominant Strategy

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For player i, strategy $s_i \in S_i$ is strictly dominant if

$\forall\, s_{-i} \in S_{-i}$, $\forall\, s_i' \in S_i$ s.t. $s_i' \neq s_i$

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}).$$

Dominant Strategy

		B	
		Silent	Defect
A	Silent	$-1, -1$	$-5, 0$
	Defect	$0, -5$	$-3, -3$

For player i, strategy $s_i \in S_i$ is weakly dominant if

$\forall s_{-i} \in S_{-i}$, $\forall s_i' \in S_i$ s.t. $s_i' \neq s_i$

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}).$$

Dominant Strategy

		B	
		Silent	Defect
A	Silent	-1, -1	-5, 0
	Defect	0, -5	~~-3~~ -5, -3

Dominant Strategy

B

		Silent	Defect
A	Silent	-1 , -1	-5 , 0
	Defect	0 , -5	-5 , -3

weakly dominant

Recap

1. Do not play a strictly dominated strategy.

2. Put yourself in other people's shoes

The $1/2$-mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

Winner is one whose number is closest to $\frac{1}{2}$ of average.

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Players N = You all!

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Players $N =$ You all !

Strategies $S_i = \{1, 2, \ldots, 100\}$

The ½-mean game

(Secretly) pick a natural number in $\{1, 2, \dots, 100\}$.

Winner is one whose number is closest to $\frac{1}{2}$ of average.

Players $\quad N =$ You all !

Strategies $\quad S_i = \{1, 2, \dots, 100\}$

Payoffs $\quad u_i(s_i, s_{-i}) = -\left| s_i - \frac{1}{2}\text{avg}(s_i, s_{-i}) \right|$

The $\frac{1}{2}$-mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

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How to play this game?

The $\frac{1}{2}$-mean game

Q. Is there a strictly dominant strategy?

The ½ - mean game

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No.

The $\frac{1}{2}$-mean game

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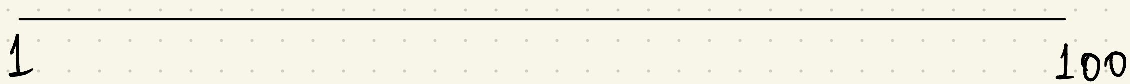

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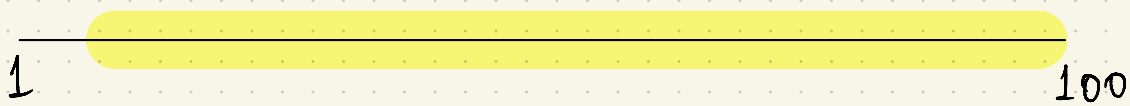
if you play $s_1 \geq 2$



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everyone here

$\Rightarrow \frac{1}{2}$ avg here

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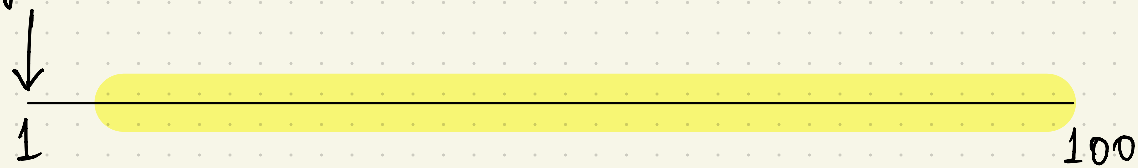

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if you play $s_1 \geq 2$

then dominated by $s_1 = 1$

The 1/2 - mean game

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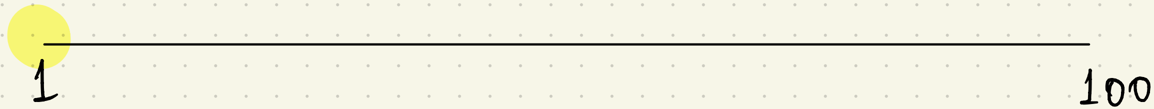
if you play $s_1 = 1$



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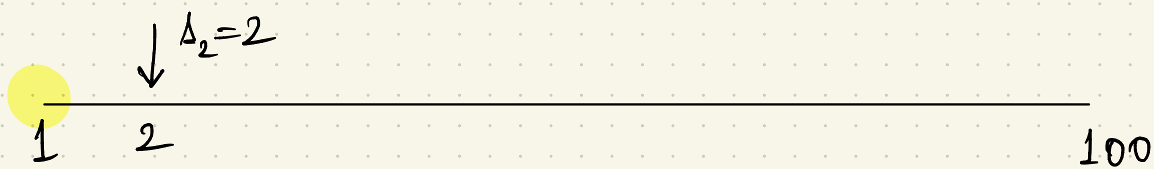

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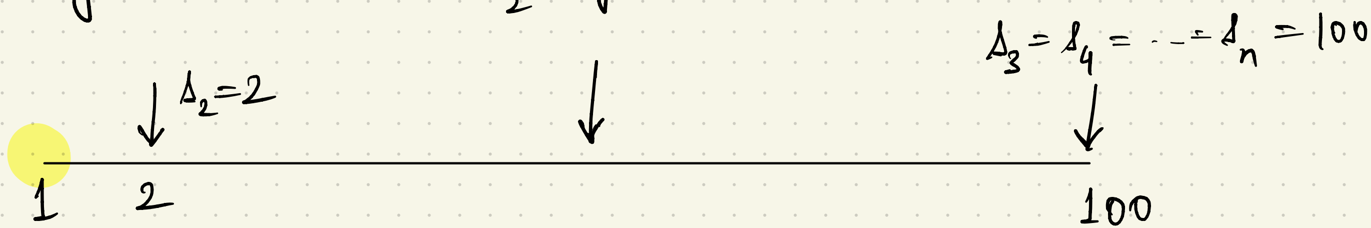
if you play $s_1 = 1$ $\qquad$ $\frac{1}{2}$ avg is here



The $\frac{1}{2}$- mean game

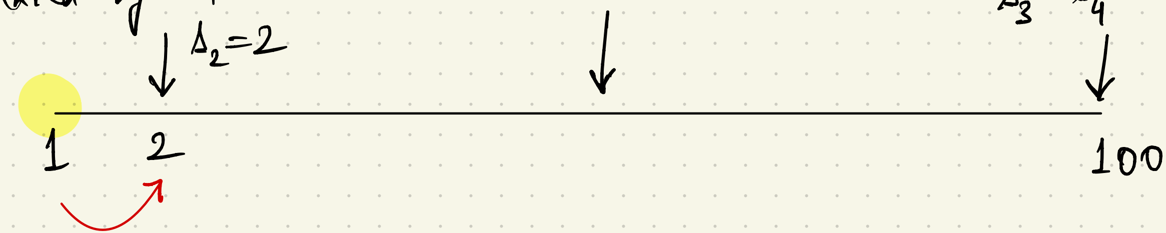
Q. Is there a strictly dominant strategy?

No.

if you play $s_1 = 1$
then dominated by $s_1 = 2$

$\frac{1}{2}$ avg is here

$s_3 = s_4 = \cdots = s_n = 100$



The $1/2$-mean game

No strategy is strictly dominant.

The $1/2$- mean game

No strategy is strictly dominant.

But there is a strictly dominated strategy

The ½-mean game

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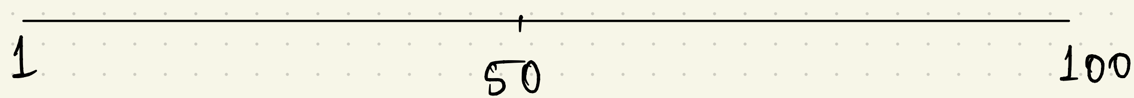

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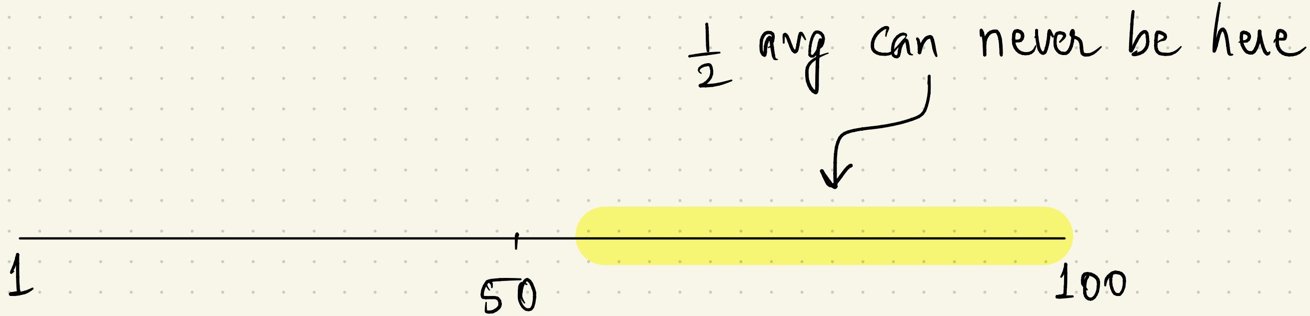
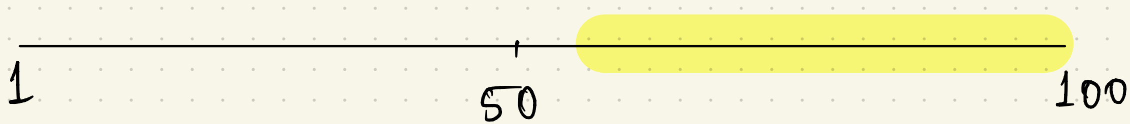


The ½-mean game

No strategy is strictly dominant.

But there is a strictly dominated strategy

$\frac{1}{2}$ avg can never be here



$\{51, \ldots, 100\}$: strictly dominated strategies

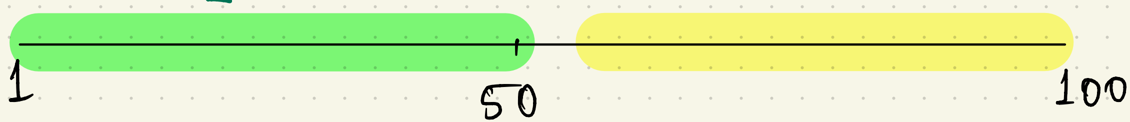
The ½-mean game

No strategy is strictly dominant.

But there is a strictly dominated strategy

NOT strictly dominated
(each of them could win sometimes!)

$\frac{1}{2}$ avg can never be here

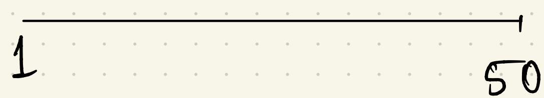


$\{51, \ldots, 100\}$: strictly dominated strategies

The $\frac{1}{2}$-mean game

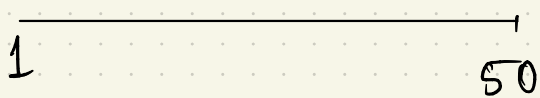
So, if everyone's rational, we can "delete" strategies $\{51, \ldots, 100\}$

and consider a reduced game.



The ½-mean game

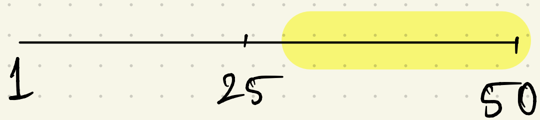
How to play this reduced game?



The ½-mean game

How to play this reduced game?

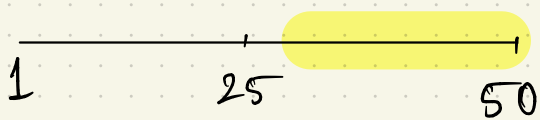
$\frac{1}{2}$-avg cannot be here



The $\frac{1}{2}$-mean game

How to play this reduced game?

$\frac{1}{2}$-avg cannot be here



$\{26, \ldots, 50\}$ strictly dominated in reduced game

NOT " " original "

The $\frac{1}{2}$-mean game

A further reduced game.

The ½-mean game

$s > 50$

dominated

The $\frac{1}{2}$-mean game

$s > 50$

dominated

Lesson 1: Do not play dominated strategies

The $\frac{1}{2}$-mean game

$s > 50$ dominated

Lesson 1: Do not play dominated strategies

$50 \geq s > 25$ not dominated in the original game

but become dominated after deleting $\{51, \ldots, 100\}$

The $\frac{1}{2}$-mean game

$S > 50$ dominated

Lesson 1: Do not play dominated strategies

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Lesson 2: Put yourself in the shoes of others

The $\frac{1}{2}$-mean game

$s > 50$ dominated

Lesson 1: Do not play dominated strategies

$50 \geq s > 25$ not dominated in the original game but become dominated after deleting $\{51, \dots, 100\}$

Lesson 2: Put yourself in the shoes of others

$25 \geq s > 12$ not dominated in original or reduced game but become dominated after deleting $\{26, \dots, 50\}$

The $1/2$-mean game

$s > 50$ dominated

Lesson 1: Do not play dominated strategies

$50 \geq s > 25$ not dominated in the original game

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$25 \geq s > 12$ not dominated in original or reduced game

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Lesson 3: Put yourself in the shoes of others

while they put themselves in others' shoes

The ½-mean game

$S > 50$ dominated

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Lesson 3 : Put yourself in the shoes of others while they put themselves in others' shoes

⋮

The ½-mean game

$S > 50$ Rationality

$50 \geq S > 25$ Rational and know that others are rational

$25 \geq S > 12$ Rational, know others are rational,

know that others know that others are rational

$\vdots$

The $\frac{1}{2}$-mean game : RESULTS

The $1/2$-mean game : RESULTS

100 "We do not like samosas (especially at Jagdish). Our utility is maximized by NOT eating Jagdish samosas."

The ½-mean game : RESULTS

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"We do not like samosas (especially at Jagdish).

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25, 24

The ½-mean game : RESULTS

100 "We do not like samosas (especially at Jagdish). Our utility is maximized by NOT eating Jagdish samosas."

25, 24

15 "If all choose ~100, answer ~ 47. But if all ~ 50, answer ~ 25. But if anyone goes < 25, answer ~ 15 or 20."

The ½-mean game : RESULTS

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answer ~25. But if anyone goes < 25, answer ~ 15 or 20."

9 " Unlikely that average exceeds 25. So, to win,
best to pick ~ $\frac{1}{2} \times 25$. "

The $\frac{1}{2}$-mean game : RESULTS

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5 "We know everyone would pick 1. Allowing for deviations and other factors, 5 is safer."

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The $\frac{1}{2}$-mean game : RESULTS

100 "We do not like samosas (especially at Jagdish). Our utility is maximized by NOT eating Jagdish samosas."

25, 24 –

15 "If all choose ~ 100, answer ~ 47. But if all ~ 50, answer ~ 25. But if anyone goes < 25, answer ~ 15 or 20."

9 "Unlikely that average exceeds 25. So, to win, best to pick $\sim \frac{1}{2} \times 25$."

5 "We know everyone would pick 1. Allowing for deviations and other factors, 5 is safer."

2 –

3 x 1 "We are ok with $1/13^{th}$ of the samosa."

The $1/2$-mean game : RESULTS

And the winner is . . .

The $\frac{1}{2}$ - mean game : RESULTS

100 — "We do not like samosas (especially at Jagdish). Our utility is maximized by NOT eating Jagdish samosas."

25, 24

15 — "If all choose ~ 100, answer ~ 47. But if all ~ 50, answer ~ 25. But if anyone goes < 25, answer ~ 15 or 20."

$\frac{1}{2}$ avg $= 9.15$

9  — "Unlikely that average exceeds 25. So, to win, best to pick $\sim \frac{1}{2} \times 25$."

5 — "We know everyone would pick 1. Allowing for deviations and other factors, 5 is safer."

2

3 x 1 — "We are ok with $1/13^{th}$ of the samosa."

Common Knowledge

Common Knowledge

X is common knowledge if

everyone knows X and

everyone knows that everyone knows X and

everyone knows that everyone knows that
everyone knows X

and so on.

Common Knowledge

If rationality is common knowledge,

then everyone should report 1 in the $\frac{1}{2}$-mean game.

Common Knowledge

Puzzle: Are You Smarter Than 61,139 Other New York Times Readers?

We are asking them – and you – to pick a number from 0 to 100, with that number representing your best guess of **two-thirds of the average of all numbers** chosen in the contest.

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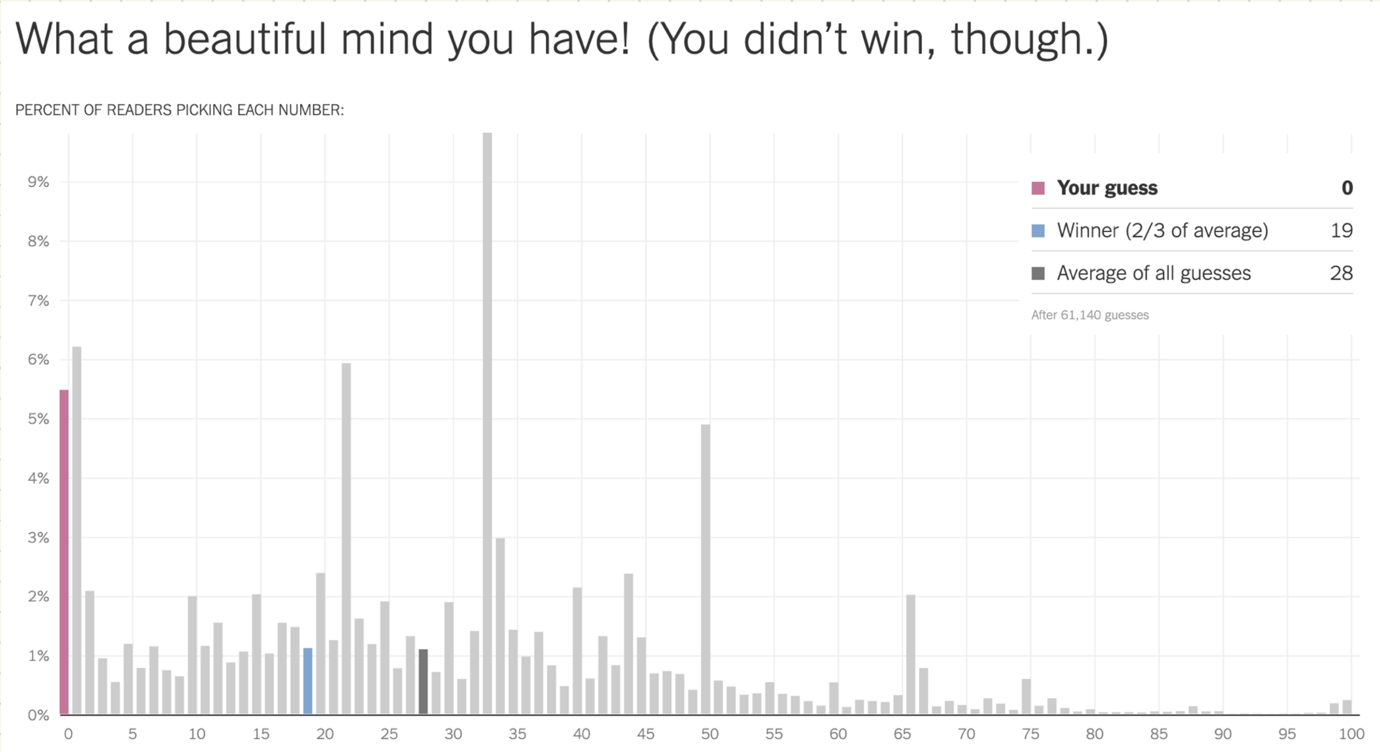
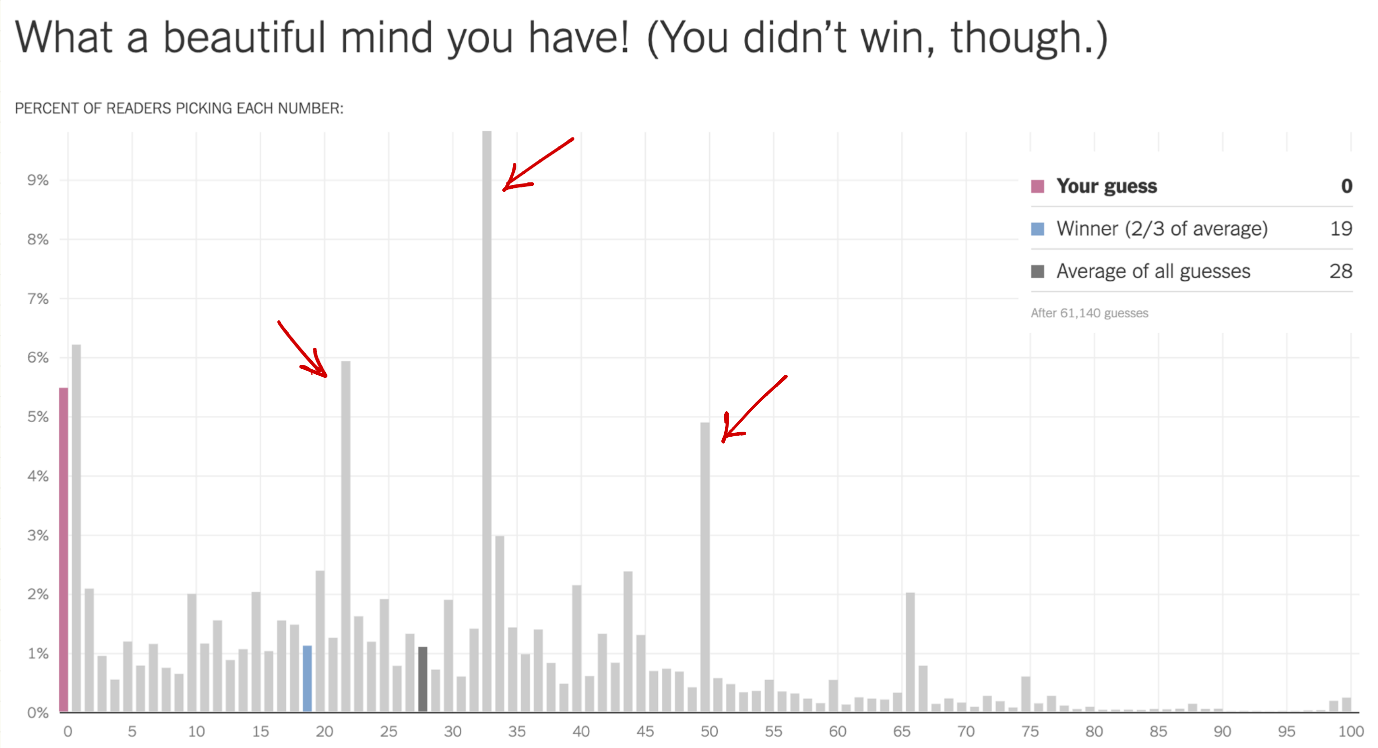
0, of course!

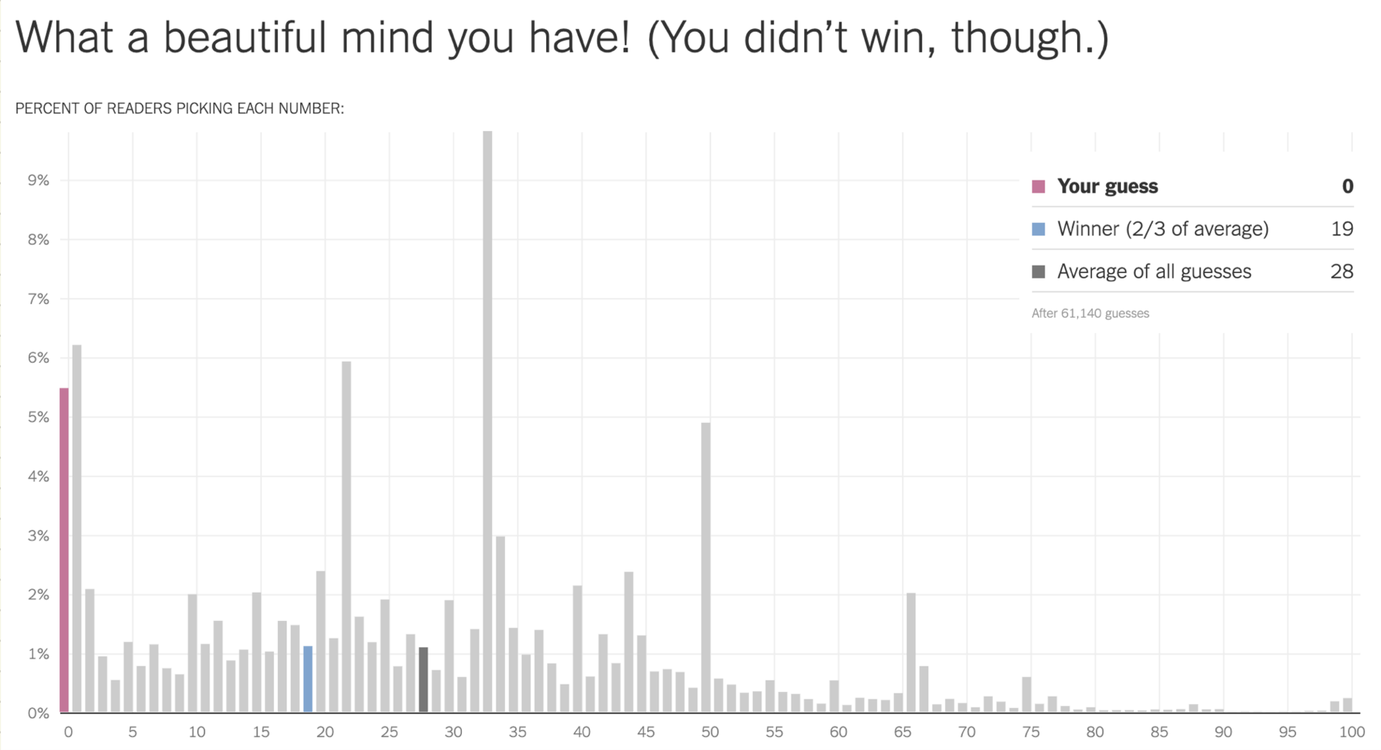
Common Knowledge



Common Knowledge

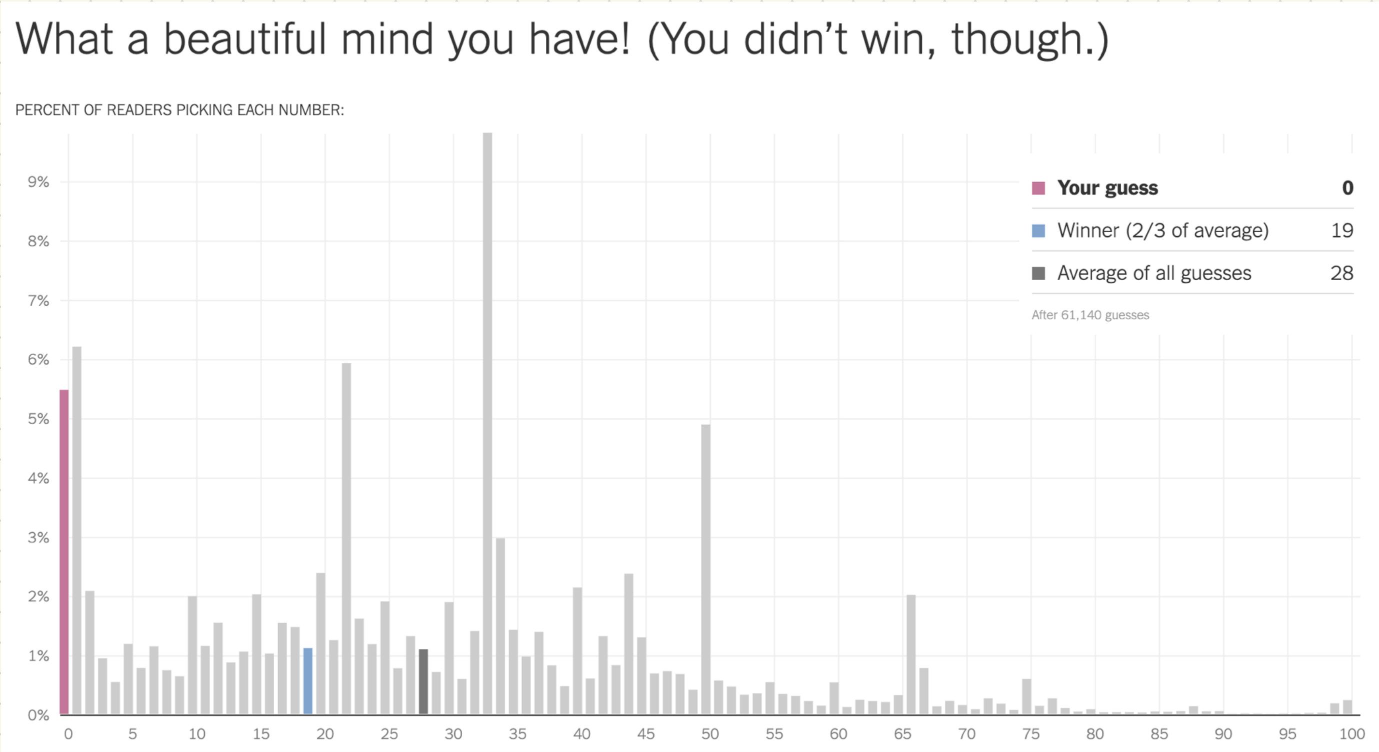


Common Knowledge



↳ Naive

Common Knowledge



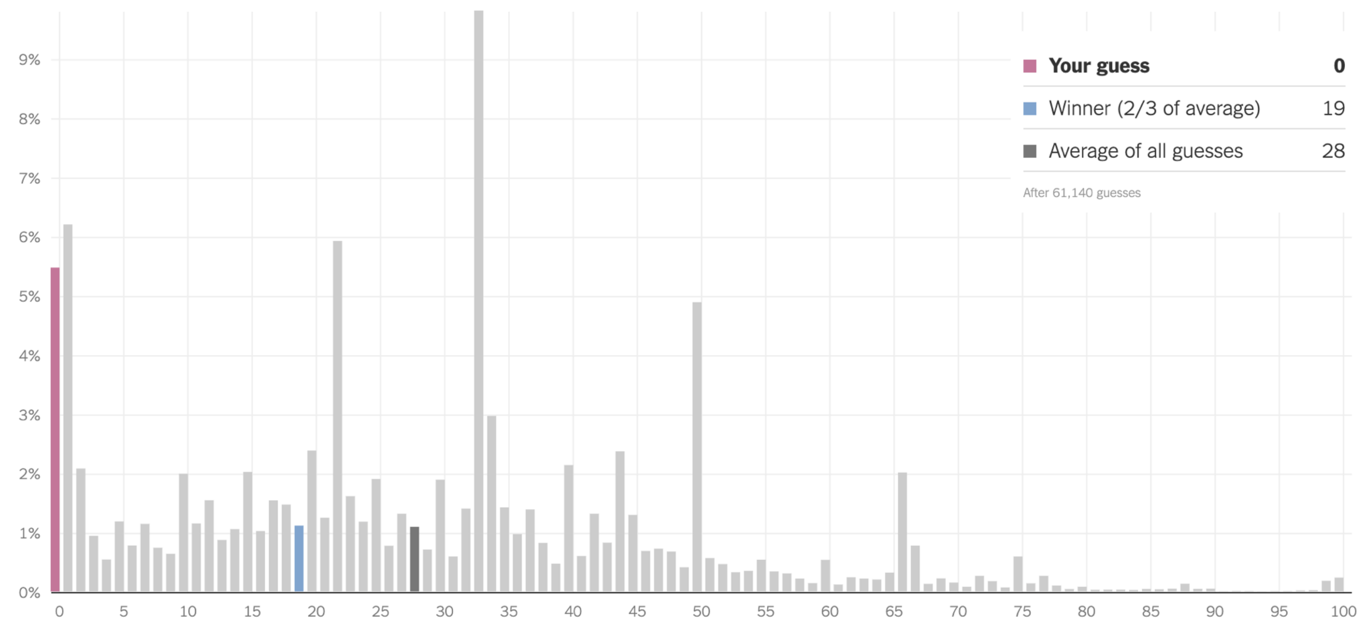
$\frac{2}{3} \times 50 = 33$

↳ Think that everyone else is naive (~50)
so best to play $\frac{2}{3}$rd of that

Common Knowledge

What a beautiful mind you have! (You didn't win, though.)

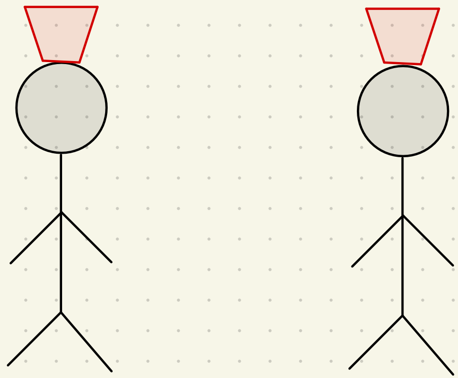
PERCENT OF READERS PICKING EACH NUMBER:



$$\frac{2}{3} \times 33 = 22$$

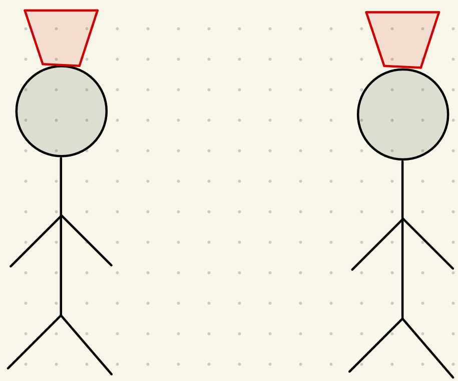
Common Knowledge $\neq$ Mutual Knowledge

Common Knowledge ≠ Mutual Knowledge



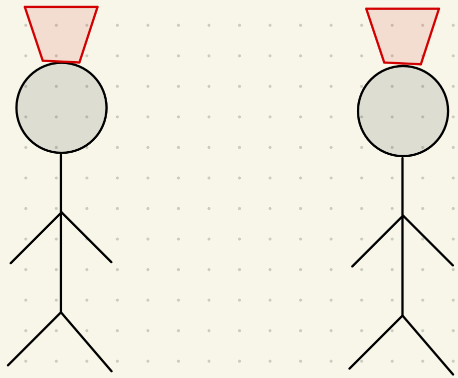
* Two people with colored hats

Common Knowledge ≠ Mutual Knowledge



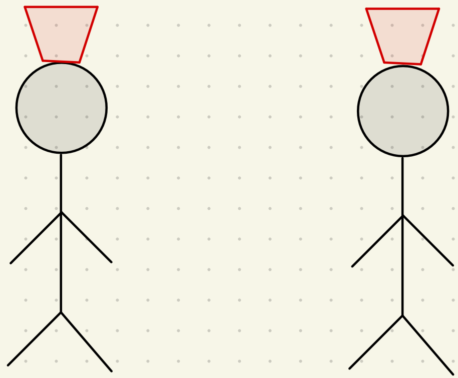
* Two people with colored hats
* Can see the hats of others but not their own

Common Knowledge ≠ Mutual Knowledge



* Two people with colored hats
* Can see the hats of others but not their own
* Cannot communicate

Common Knowledge ≠ Mutual Knowledge



* Two people with colored hats
* Can see the hats of others but not their own
* Cannot communicate

"There is at least one red hat in the room."
is mutual knowledge but not common knowledge.