STCS VIGYAN VIDUSHI 2025

INTRODUCTION TO GAME THEORY

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What this course is NOT about

Let's play!

Split or Steal ?

Split or Steal ?

Money at stake : Rs 100

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Two participants : Rose and Colin

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Each picks one of two actions : SPLIT or STEAL

Split or Steal ?

Money at stake : Rs 100

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Each picks one of two actions : SPLIT or STEAL

Rules :

Split or Steal ?

Money at stake : Rs 100

Two participants : Rose and Colin

Each picks one of two actions : SPLIT or STEAL

Rules : Both SPLIT : Each gets Rs 50

Split or Steal ?

Money at stake : Rs 100

Two participants : Rose and Colin

Each picks one of two actions : SPLIT or STEAL

Rules : Both SPLIT : Each gets Rs 50

Both STEAL : Each gets Rs 1

Split or Steal ?

Money at stake : Rs 100

Two participants : Rose and Colin

Each picks one of two actions : SPLIT or STEAL

Rules :

Both SPLIT : Each gets Rs 50

Both STEAL : Each gets Rs 1

One SPLIT, other STEAL : STEAL ← Rs 100
SPLIT ← Rs 0

Compact Representation

Compact Representation

Rose's payoffs

Colin's payoffs

Compact Representation

		Colin	
		SPLIT	STEAL
Rose	SPLIT		
	STEAL		

Rose's payoffs

Colin's payoffs

Compact Representation

		Colin	
		SPLIT	STEAL
Rose	SPLIT	50	
	STEAL		

Rose's payoffs

Colin's payoffs

Compact Representation

		Colin	
		SPLIT	STEAL
Rose	SPLIT	50	
	STEAL	100	

Rose's payoffs

Colin's payoffs

Compact Representation

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50	0
	STEAL	100	

Rose's payoffs

Colin's payoffs

Compact Representation

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50	0
	STEAL	100	1

Rose's payoffs

Colin's payoffs

Compact Representation

Rose's payoffs

Rose \ Colin	SPLIT	STEAL
SPLIT	50	0
STEAL	100	1

Colin's payoffs

Rose \ Colin	SPLIT	STEAL
SPLIT	50	100
STEAL	0	1

Compact Representation

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50, 50	0, 100
	STEAL	100, 0	1, 1

Compact Representation

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50, 50	0, 100
	STEAL	100, 0	1, 1

Strategic Form / Normal Form

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. individual benefit ("selfish")

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STEAL	100, 0	1, 1

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Solution :

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. individual benefit ("selfish")

Solution : STEAL regardless of opponent's strategy

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↳ Dominant strategy

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. individual benefit ("selfish")

Solution : STEAL regardless of opponent's strategy

↳ Dominant strategy

> Takeaway # 1
>
> If a dominant strategy is available, play it.

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. individual benefit ("selfish")

Solution : STEAL regardless of opponent's strategy

→ Strictly Dominant strategy

> Takeaway # 1
>
> If a dominant strategy is available, play it.

How to play?

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. individual benefit ("selfish")

Solution : STEAL regardless of opponent's strategy

↳ Strictly Dominant strategy

> Takeaway # 1
>
> If a dominant strategy is available, play it.
>
> Don't play a dominated strategy.

A different objective

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution :

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution :

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution :

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution : SPLIT regardless of opponent's strategy

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution : SPLIT regardless of opponent's strategy

↳ Weakly Dominant strategy

A different objective

	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

Objective : max. social benefit ("utilitarian")

Solution : SPLIT regardless of opponent's strategy

↳ Weakly Dominant strategy

Takeaway #2

A strictly dominant strategy may not exist.

A different game

A different game

Pay attention Relax

Put effort

Slack

A different game

Costs you attention,
but you get to
learn something new.

Pay attention Relax

Put effort 2, 2

Slack

Costs me time,
but ultimately I am happy
that you learned something new

A different game

Costs you nothing

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack		

Costs me time,
and ultimately I am unhappy
that no learning happened

A different game

Wasted effort for you

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	

I don't waste effort,

but bad publicity!

A different game

You don't waste effort

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

I don't waste effort

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

How to play this game?

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Best response now depends on opponent's strategy

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Takeaway # 3

Even a weakly dominant strategy may not exist.

A different game

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

But then, how to play this game?

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Suppose opponent's strategy is known.

Solution : ?

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Suppose opponent's strategy is known.

Solution : Mimic the opponent.

A different game

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

No player wants to unilaterally deviate from solution states.

A different game

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

No player wants to unilaterally deviate from solution states.

EQUILIBRIUM !

A different game

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

But, what if opponent's strategy is not known.

A different game

	Pay attention	Relax
Put effort	2, 2	-1, 0
Slack	-7, -8	0, 0

But, what if opponent's strategy is not known.

We will return to this question.

Story so far

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50, 50	0, 100
	STEAL	100, 0	1, 1

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Story so far

		Colin: SPLIT	Colin: STEAL
Rose	SPLIT	50, 50	0, 100
	STEAL	100, 0	1, 1

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

What you get depends not only on your own actions but also on the actions of others.

Story so far

Rose \ Colin	SPLIT	STEAL
SPLIT	50, 50	0, 100
STEAL	100, 0	1, 1

	Pay attention	Relax
Put effort	2, 2	−1, 0
Slack	−7, −8	0, 0

Essence of game theory

What you get depends not only on your own actions but also on the actions of others.

Presentation game with morals

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

The act of paying attention in class brings you joy!

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	-1, 0
Slack	-7, 1	0, 0

How to play this game?

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

Still no weakly dominant strategy for me.

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

Still no weakly dominant strategy for me.

But I know how you are going to play!

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

Still no weakly dominant strategy for me.

But I know how you are going to play!

Solution : I should put effort.

Presentation game with morals

	Pay attention	Relax
Put effort	2, 11	−1, 0
Slack	−7, 1	0, 0

Takeaway # 4

Put yourself in others' shoes to figure out what they want to do.

The $\frac{1}{2}$-mean game

The $1/2$-mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

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Winner is one whose number is closest to $\frac{1}{2}$ of average.

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E.g., 3 participants : 15 4 90

The $\frac{1}{2}$-mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

Winner is one whose number is closest to $\frac{1}{2}$ of average.

E.g., 3 participants : 15 4 90

$$\text{Average} = \frac{15 + 4 + 90}{3} = 36.33$$

The $\frac{1}{2}$-mean game

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Winner is one whose number is closest to $\frac{1}{2}$ of average.

E.g., 3 participants : 15 4 90

$$\text{Average} = \frac{15 + 4 + 90}{3} = 36.33$$

$$\frac{1}{2} \times \text{Average} = 18.17$$

The $\frac{1}{2}$-mean game

(Secretly) pick a natural number in $\{1, 2, \ldots, 100\}$.

Winner is one whose number is closest to $\frac{1}{2}$ of average.

E.g., 3 participants : 15 4 90

$$\text{Average} = \frac{15 + 4 + 90}{3} = 36.33$$

$$\frac{1}{2} \times \text{Average} = 18.17$$

"15" wins!