COL749: Computational Social Choice

Lecture 19

Voting with Structured Preferences

March 24, 2025 | Rohit Vaish

Story So Far

Story So Far

No group-level transitivity

Condorcet paradox

Story So Far

No group-level transitivity

Condorcet paradox

No "reasonable" voting rules

Gibbard-Satterthwaite and Arrow's theorems

Story So Far

No group-level transitivity

Condorcet paradox

No "reasonable" voting rules

Gibbard-Satterthwaite and Arrow's theorems

Manipulation can be easy

Greedy strategy

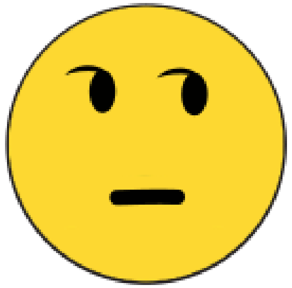
Story So Far

No group-level transitivity

Condorcet paradox

No "reasonable" voting rules

Gibbard-Satterthwaite and Arrow's theorems



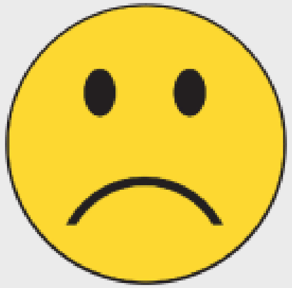
Manipulation can be easy

Greedy strategy

NP-hardness as a barrier to manipulation

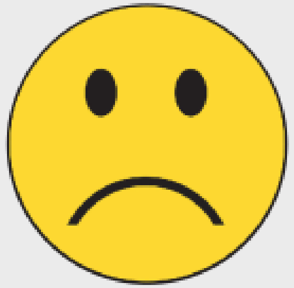
Bartholdi-Tovey-Trick

Story So Far



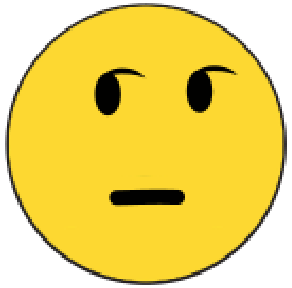
No group-level transitivity

Condorcet paradox



No "reasonable" voting rules

Gibbard-Satterthwaite and Arrow's theorems



Manipulation can be easy

Greedy strategy

NP-hardness as a barrier to manipulation

Bartholdi-Tovey-Trick

Single-Peaked Preferences

[Black'48]

Single-Peaked Preferences

[Black'48]

There is an ordering "<" over candidates.

$$a < b < c < d < e$$

Single-Peaked Preferences

[Black'48]

There is an ordering "<" over candidates.

a < b < c < d < e

a b c d e

Single-Peaked Preferences

[Black'48]

There is an ordering "<" over candidates.

a < b < c < d < e

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

a b c d e

Single-Peaked Preferences

[Black'48]

There is an ordering "<" over candidates.

a < b < c < d < e

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

- if top(v) ≤ x < y, then v prefers x over y

a b c d e

Single-Peaked Preferences

[Black'48]

There is an ordering "<" over candidates.

$a < b < c < d < e$

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

- if $top(v) \leq x < y$, then v prefers x over y
- if $y < x \leq top(v)$, then v prefers x over y

a b c d e

Single-Peaked Preferences

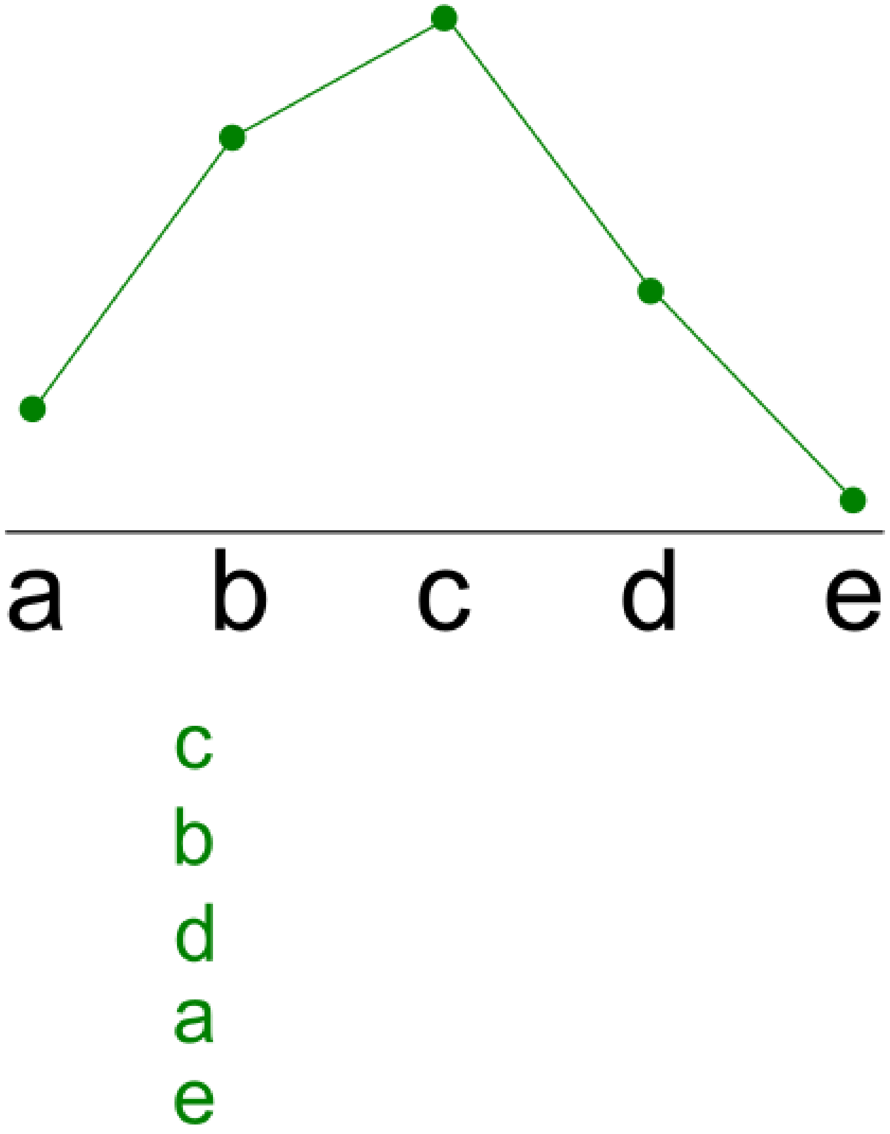
[Black'48]

There is an ordering "<" over candidates.

$a < b < c < d < e$

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

- if $\text{top}(v) \le x < y$, then v prefers x over y
- if $y < x \le \text{top}(v)$, then v prefers x over y



Single-Peaked Preferences

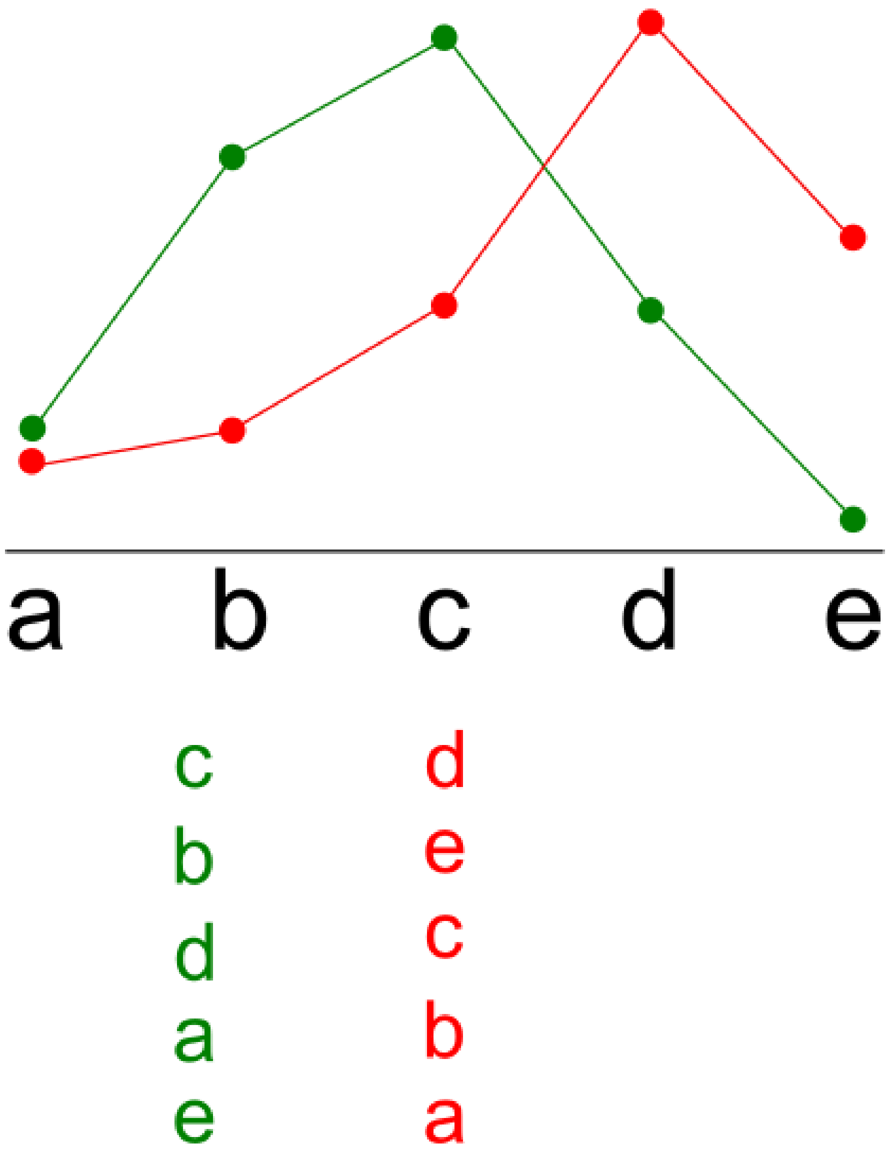
[Black'48]

There is an ordering "<" over candidates.

a < b < c < d < e

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

- if top(v) ≤ x < y, then v prefers x over y
- if y < x ≤ top(v), then v prefers x over y



Single-Peaked Preferences

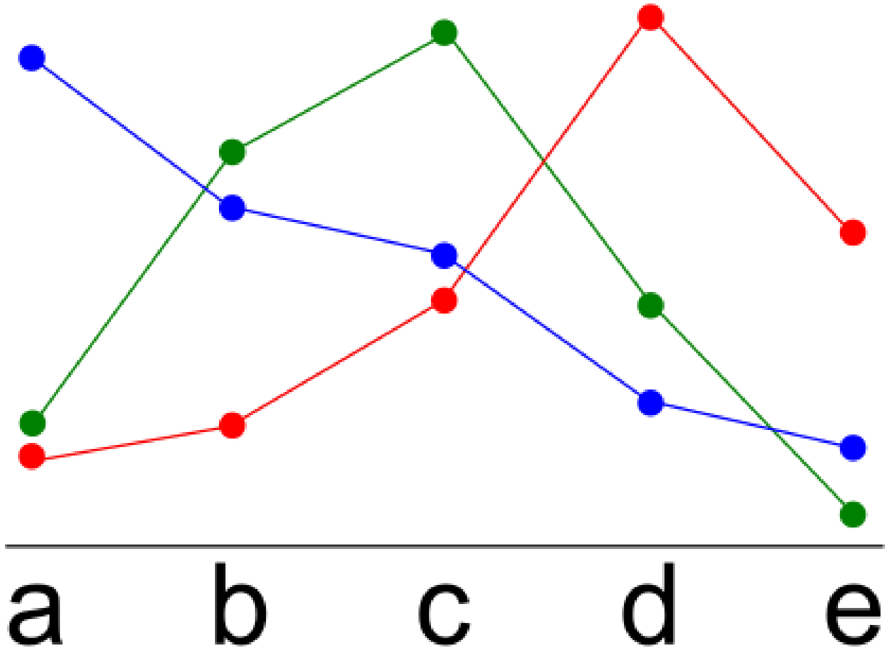
[Black'48]

There is an ordering "<" over candidates.

$a < b < c < d < e$

A preference profile is "single-peaked" w.r.t. the ordering "<" if for each voter v:

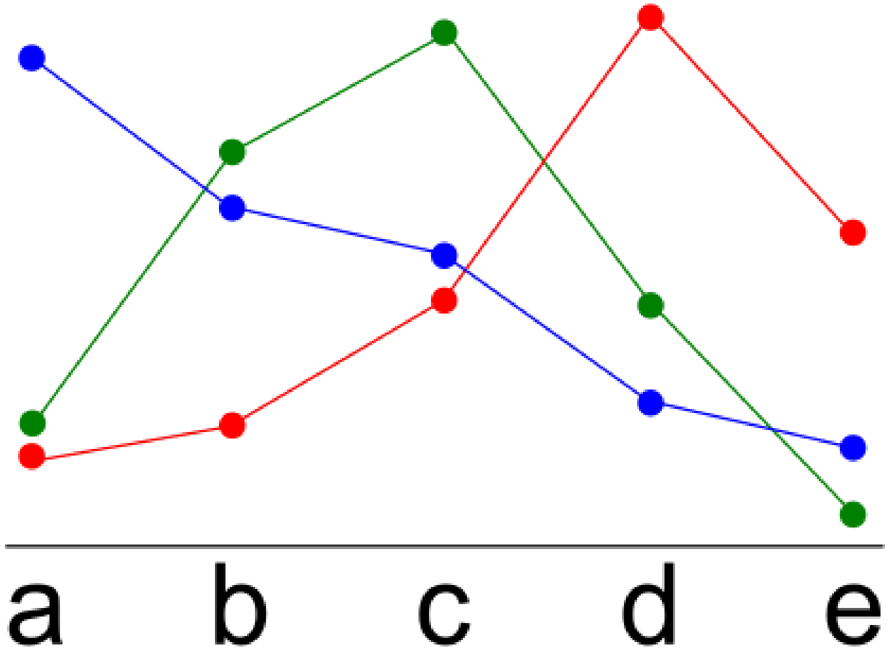
- if $\text{top}(v) \leq x < y$, then v prefers x over y
- if $y < x \leq \text{top}(v)$, then v prefers x over y



c	d	a
b	e	b
d	c	c
a	b	d
e	a	e

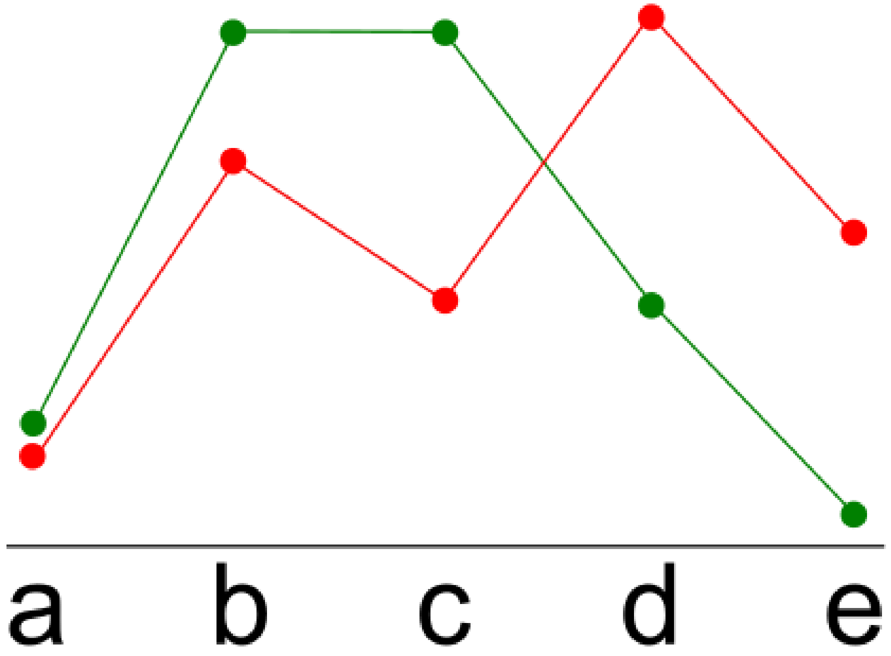
Single-Peaked Preferences

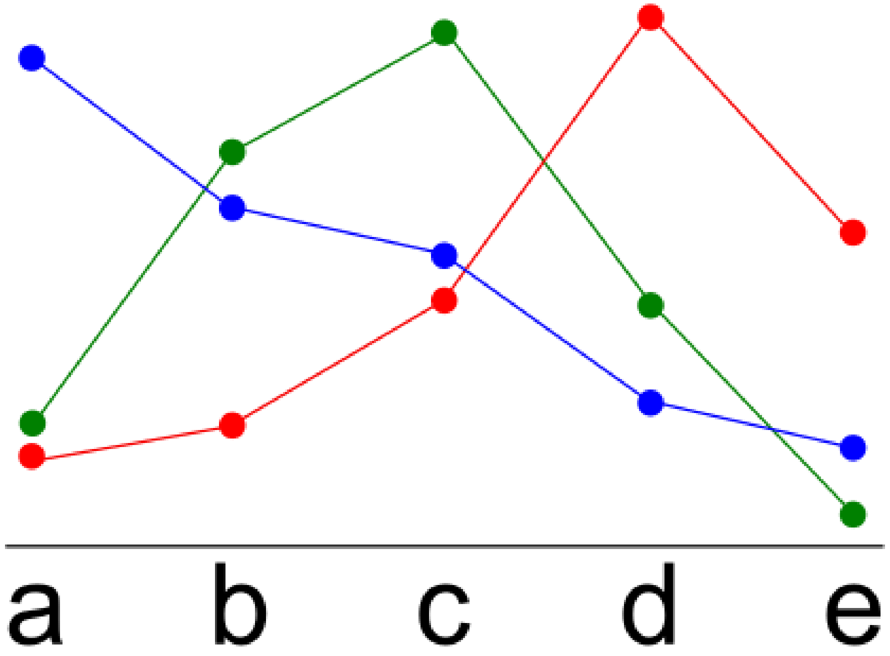
[Black'48]



Single-Peaked Preferences

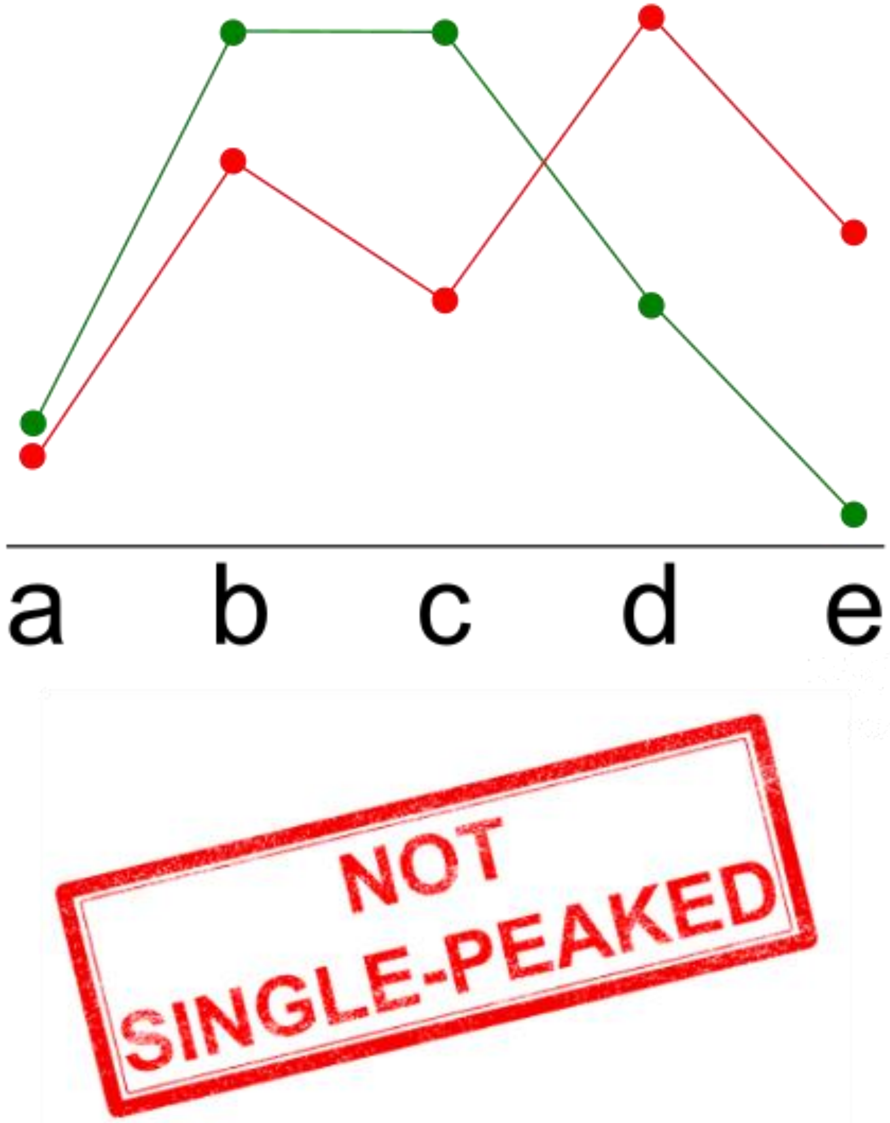
[Black'48]

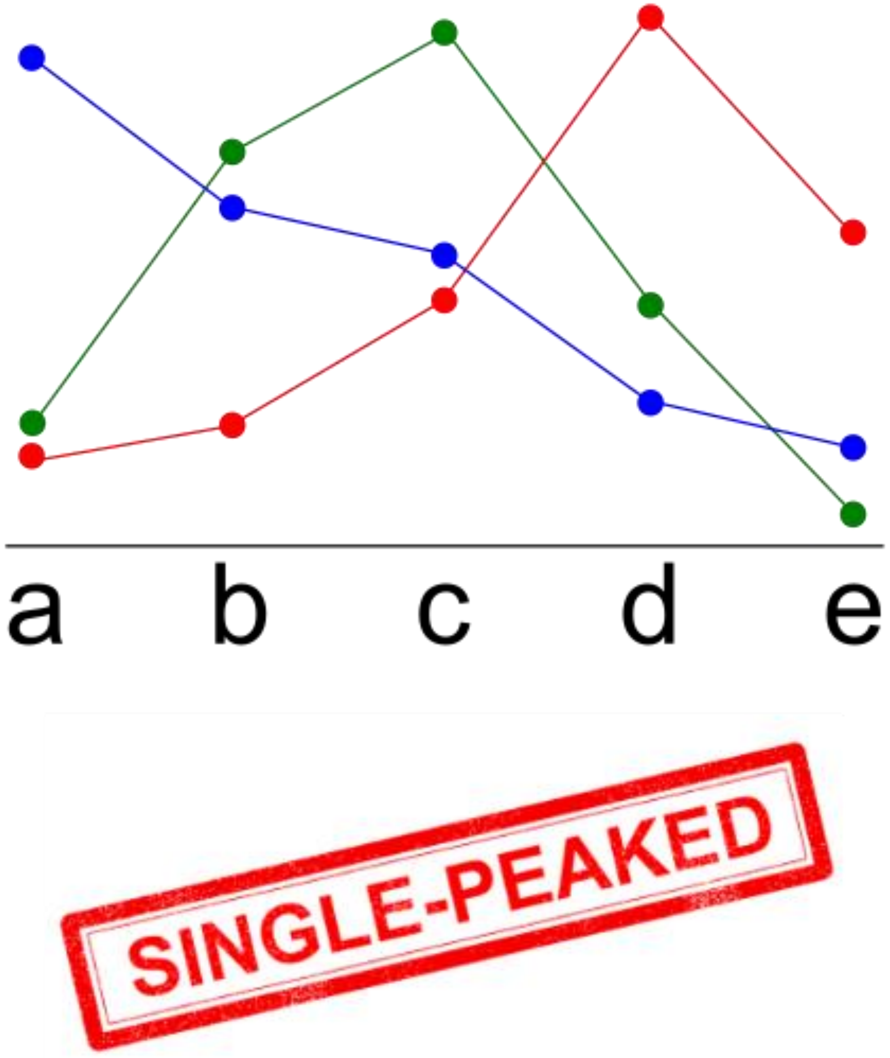


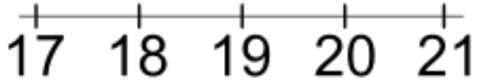


Single-Peaked Preferences

[Black'48]






17 18 19 20 21

17 18 19 20 21

5% 10% 15% 20%

17 18 19 20 21

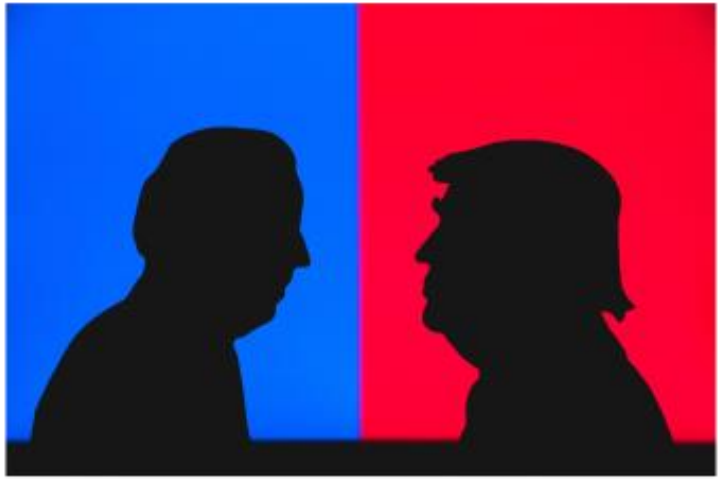
5% 10% 15% 20%

20° 21° 22° 23°

17 18 19 20 21

5% 10% 15% 20%

20° 21° 22° 23°

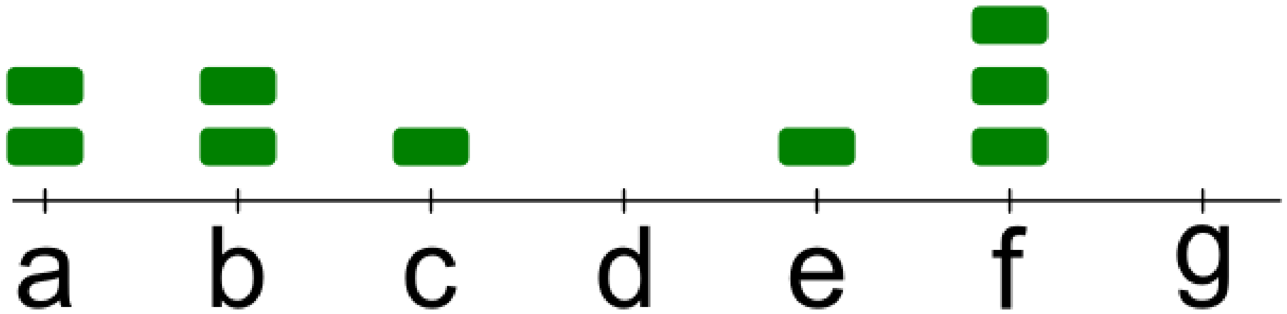


Liberal Conservative

Single-peaked profiles always admit a (weak) Condorcet winner.

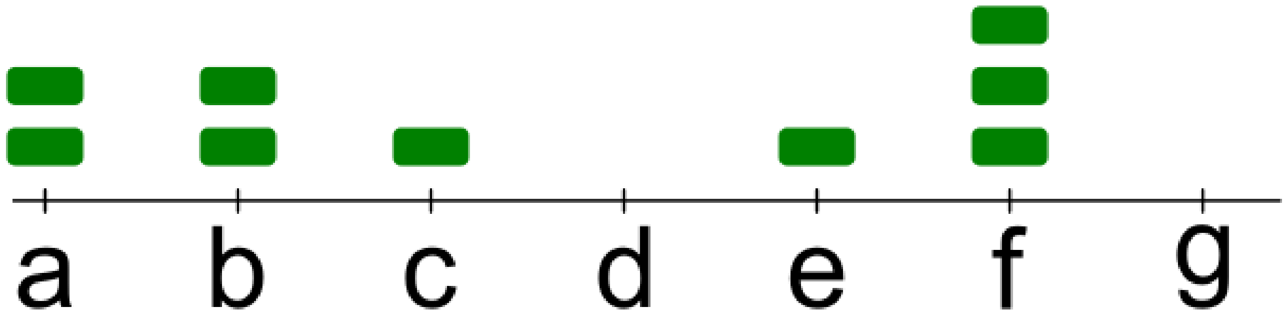
Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices



Single-peaked profiles always admit a (weak) Condorcet winner.

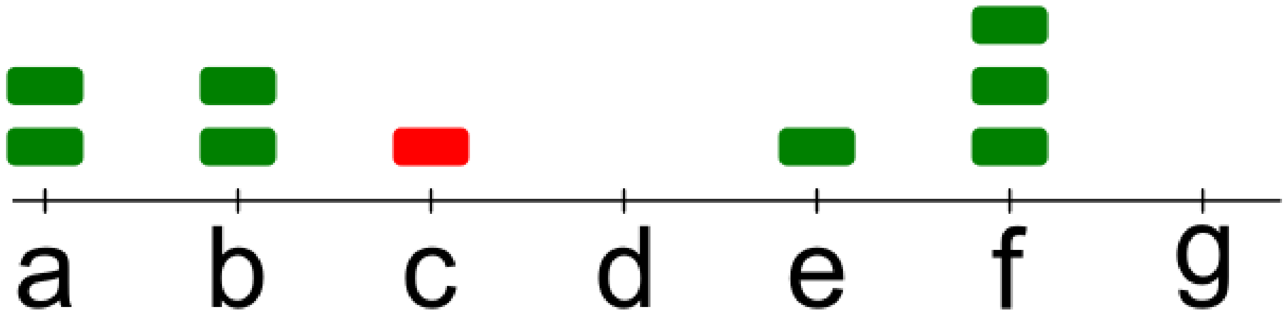
Order the voters according to their top choices



If no. of voters is odd (say, 2k+1)

Single-peaked profiles always admit a (weak) Condorcet winner.

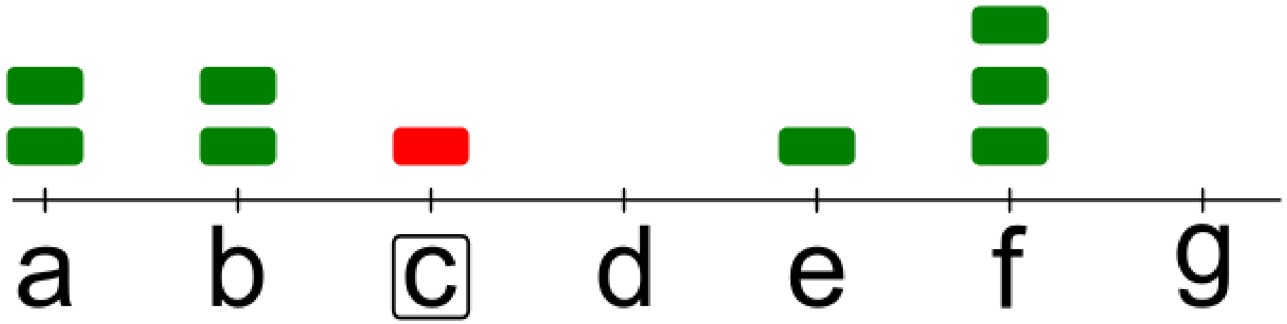
Order the voters according to their top choices



If no. of voters is odd (say, 2k+1)

Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices

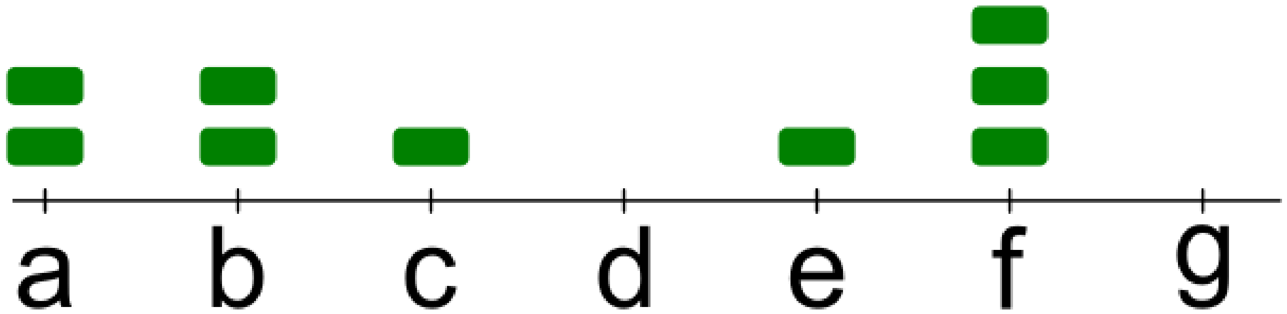


If no. of voters is odd (say, 2k+1)

top(v_{k+1}) is a Condorcet winner

Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices

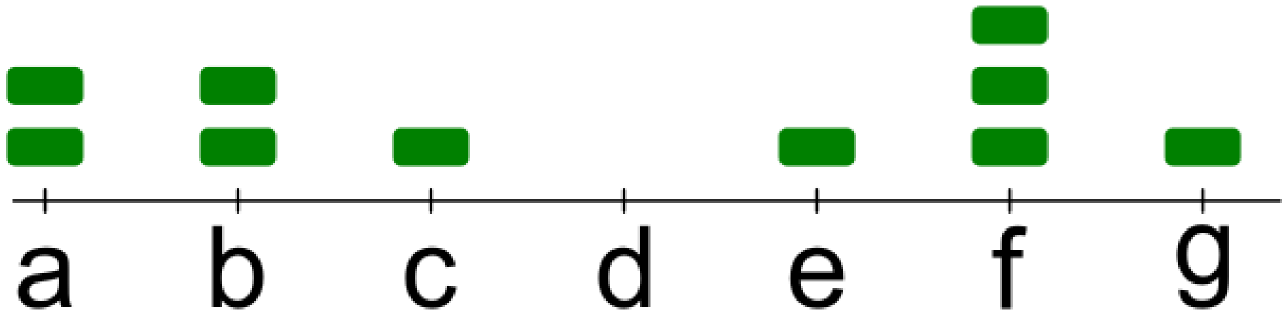


If no. of voters is odd (say, 2k+1)

top(v_{k+1}) is a Condorcet winner

Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices



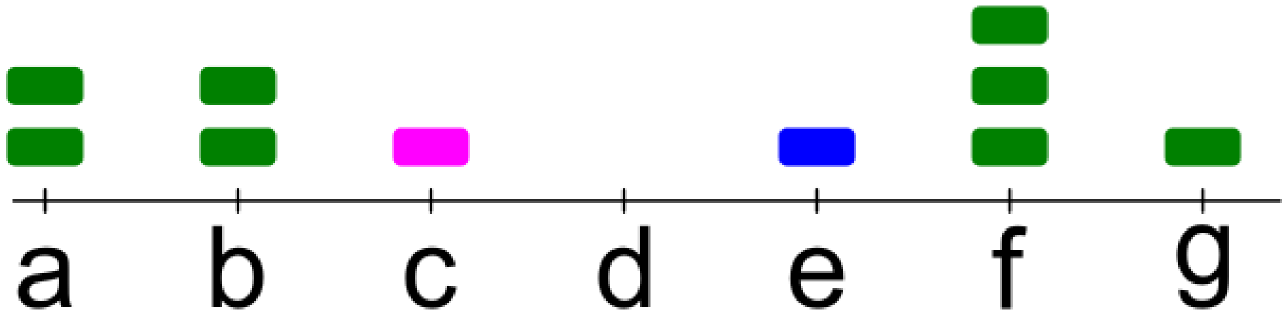
If no. of voters is odd (say, 2k+1)

If no. of voters is even (say, 2k)

top(v_{k+1}) is a Condorcet winner

Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices



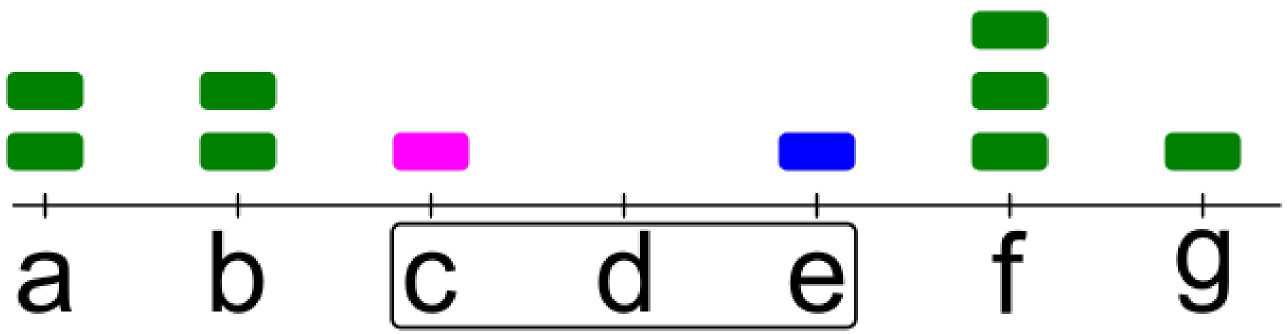
If no. of voters is odd (say, 2k+1)

If no. of voters is even (say, 2k)

top(v_{k+1}) is a Condorcet winner

Single-peaked profiles always admit a (weak) Condorcet winner.

Order the voters according to their top choices



If no. of voters is odd (say, 2k+1)

top(v_{k+1}) is a Condorcet winner

If no. of voters is even (say, 2k)

All candidates between top(v_k) and top(v_{k+1}) are weak Condorcet winners

Single-peaked preferences admit a strategyproof voting rule.

Single-peaked preferences admit a strategyproof voting rule.

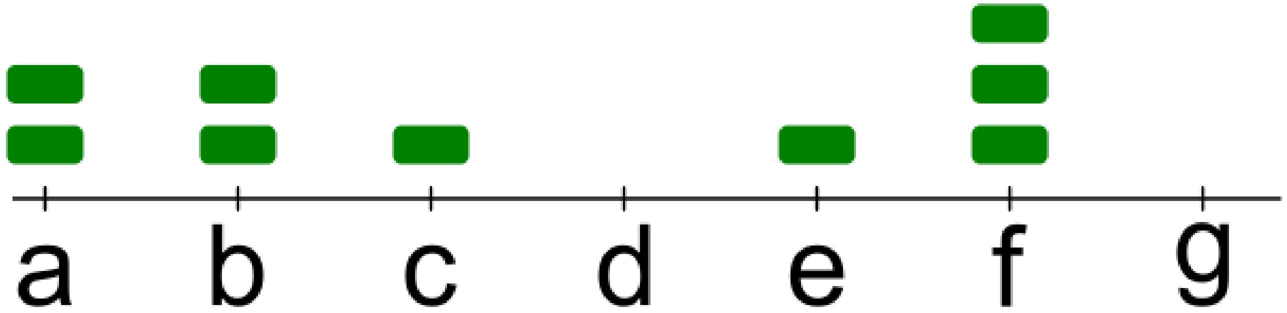
Median voter rule

1. Each voter reports its favorite candidate (or "peak").
2. The* median of the reported peaks is the winner.

Single-peaked preferences admit a strategyproof voting rule.

Median voter rule

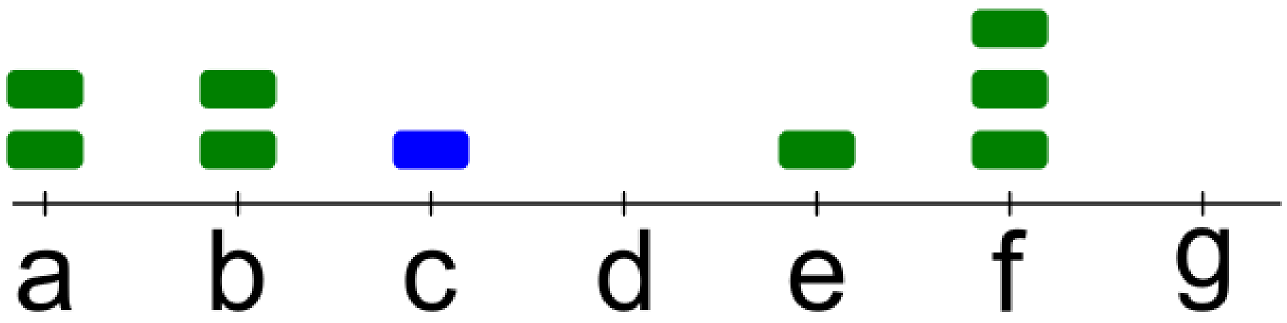
1. Each voter reports its favorite candidate (or "peak").
2. The* median of the reported peaks is the winner.



Single-peaked preferences admit a strategyproof voting rule.

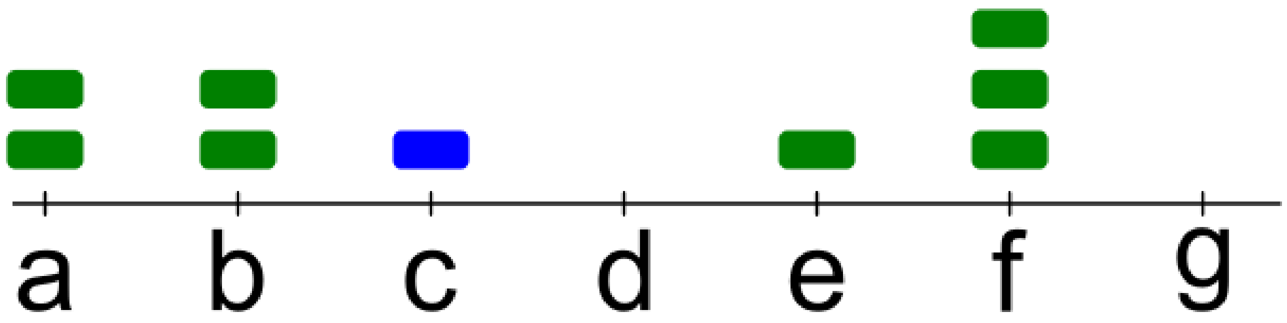
Median voter rule

1. Each voter reports its favorite candidate (or "peak").
2. The* median of the reported peaks is the winner.



Single-peaked preferences admit a strategyproof voting rule.

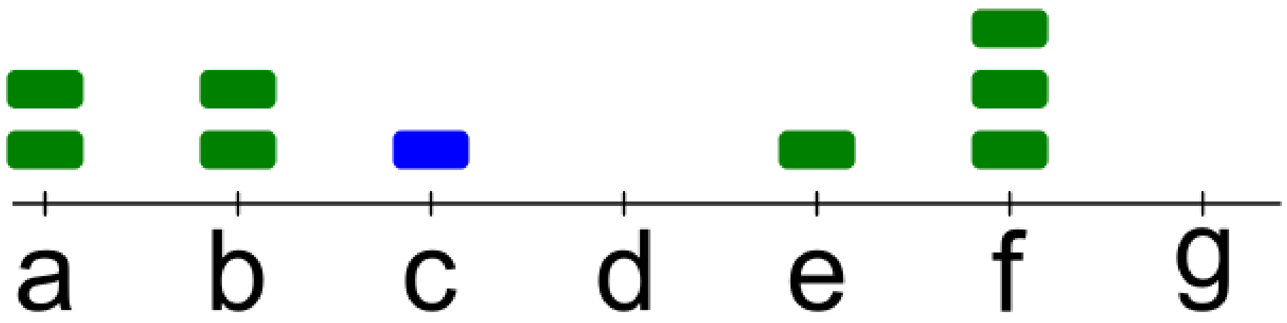
Why is the median voter rule strategyproof?



Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

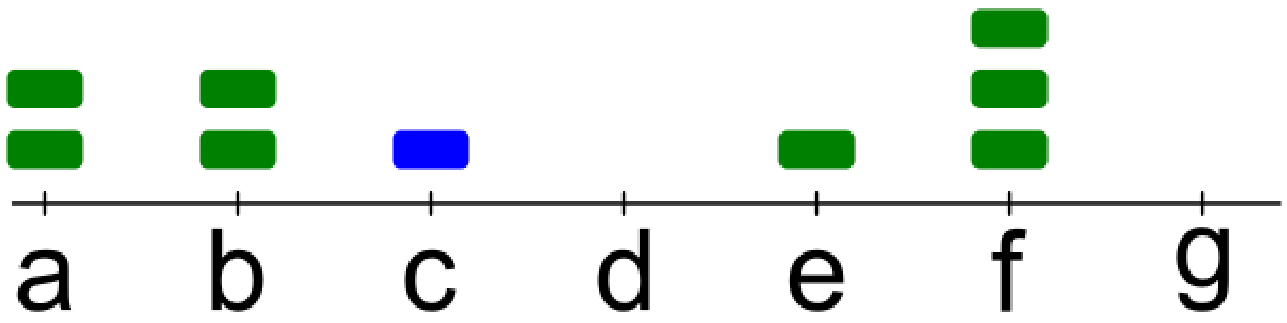


Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.

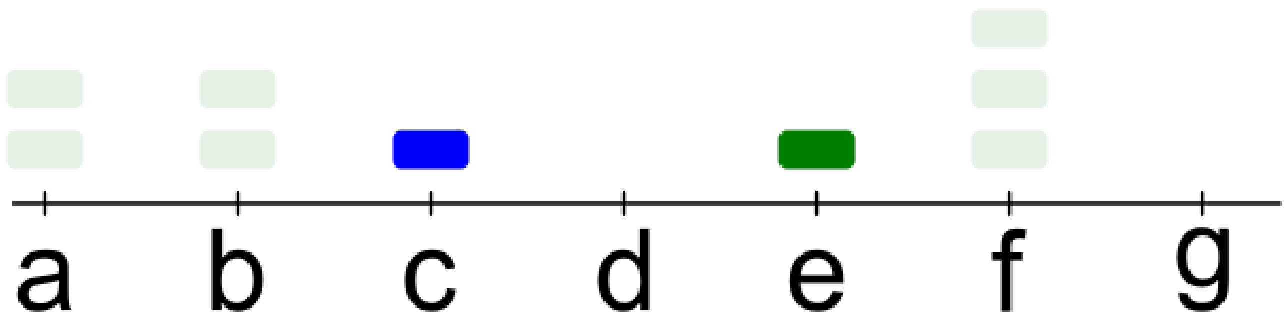


Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.

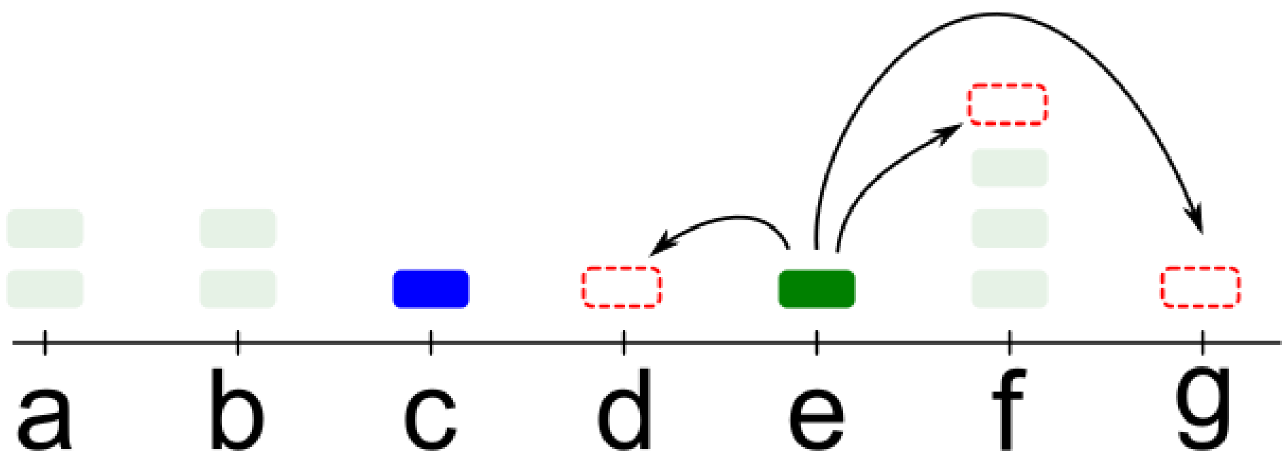


Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.

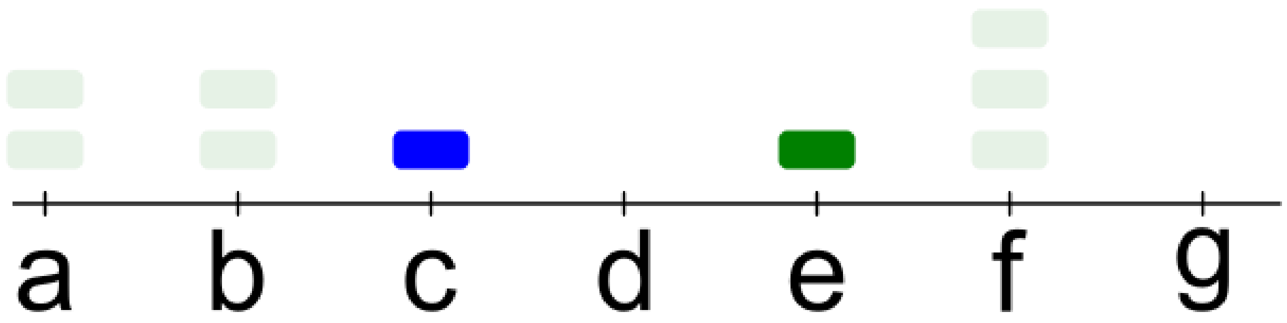


Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.



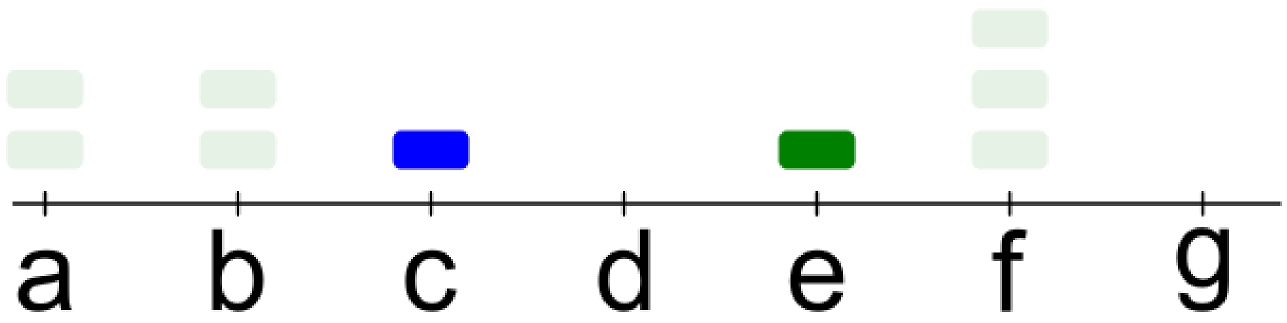
Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.

Any misreport on the *other* side moves the outcome away from the peak.



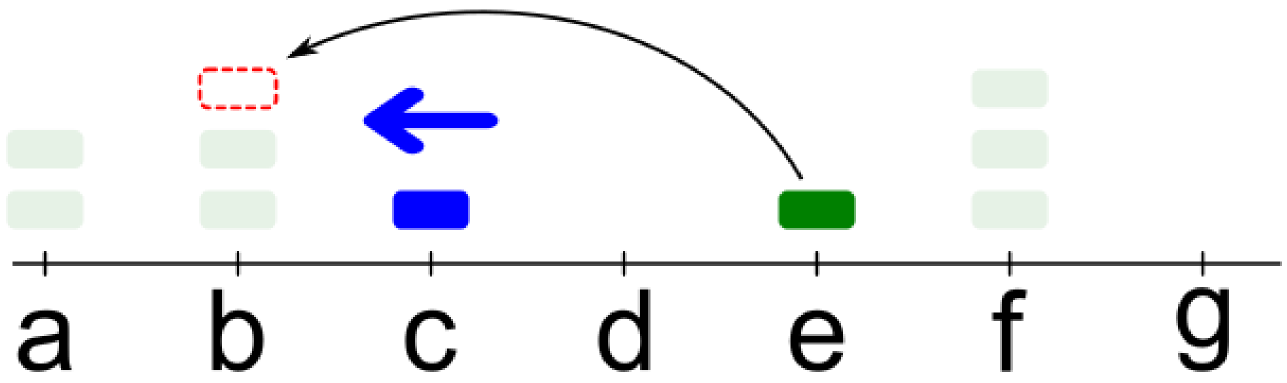
Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is odd.

For any misreport on the *same* side, the outcome doesn't change.

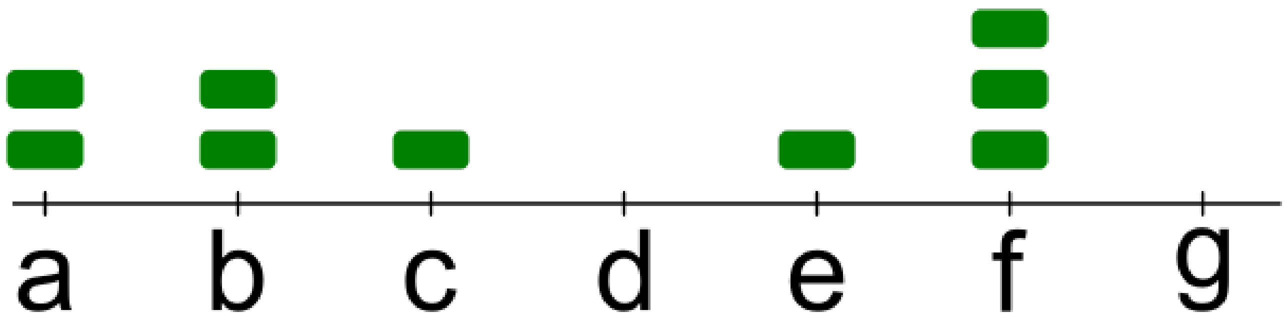
Any misreport on the *other* side moves the outcome away from the peak.



Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

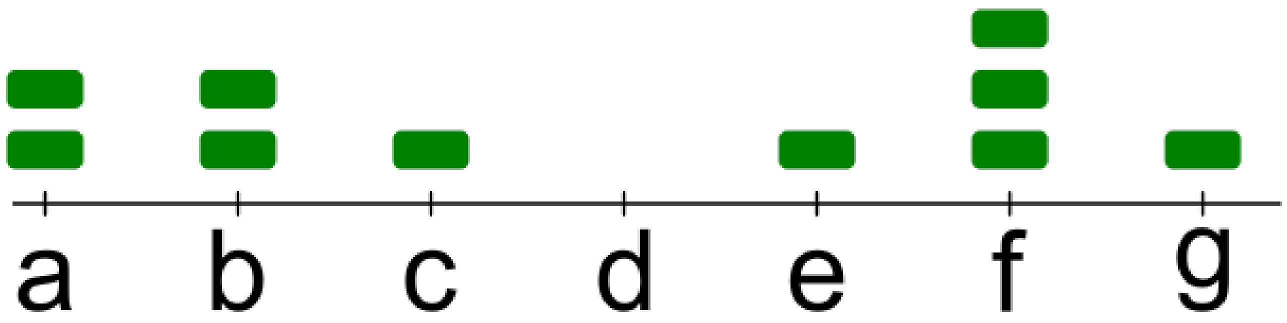
Suppose the no. of voters is odd.



Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is even.

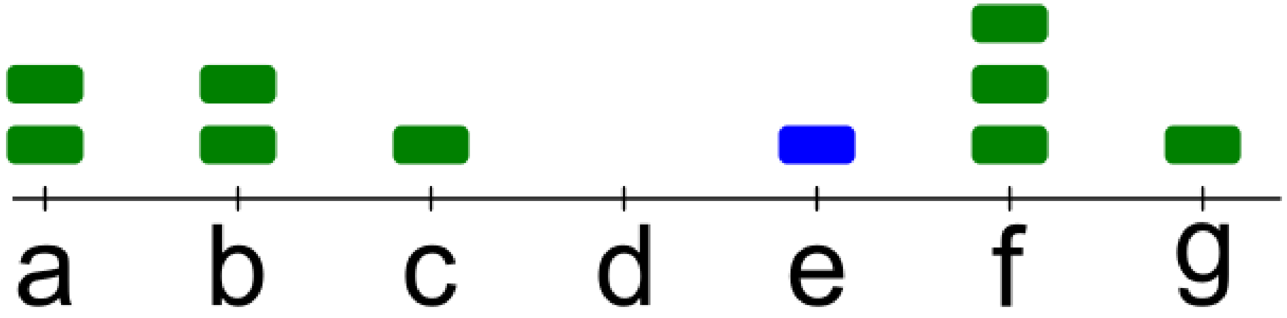


Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is even.

Deterministically pick a fixed median (either left or right).



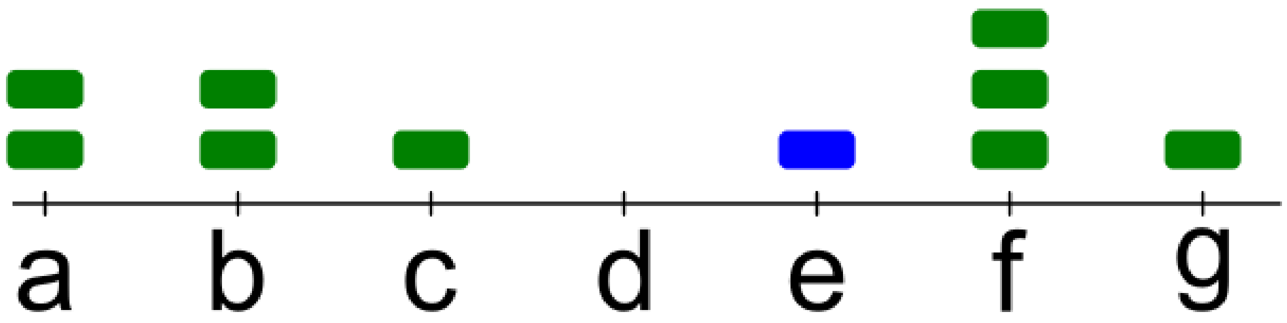
Single-peaked preferences admit a strategyproof voting rule.

Why is the median voter rule strategyproof?

Suppose the no. of voters is even.

Deterministically pick a fixed median (either left or right).

In fact, any k^{th} order statistic works [Moulin, 1980].



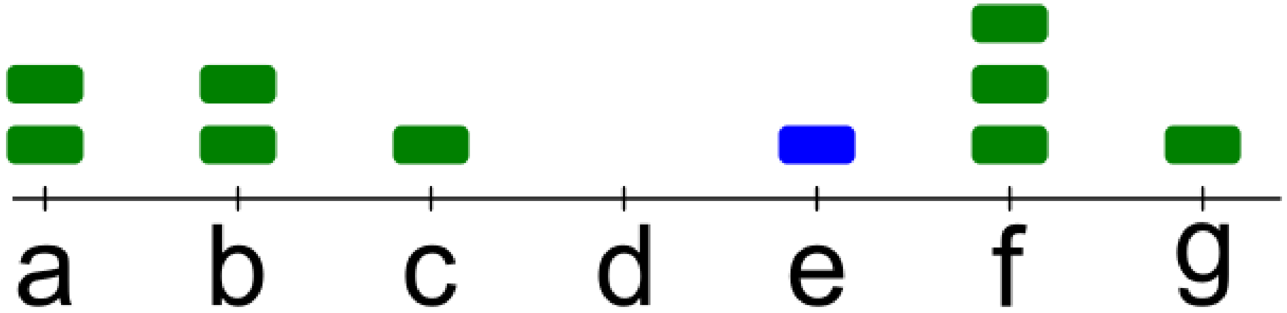
Single-peaked preferences admit ~~a~~ many strategyproof voting rules

Why is the median voter rule strategyproof?

Suppose the no. of voters is even.

Deterministically pick a fixed median (either left or right).

In fact, any k^{th} order statistic works [Moulin, 1980].



Ok, single-peaked preferences are great.

But how to find out if a given set of preferences are single-peaked?

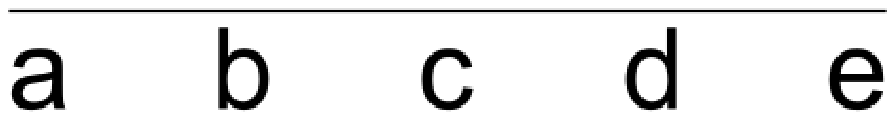
Ok, single-peaked preferences are great.
But how to find out if a given set of preferences are single-peaked?

w.r.t. a given axis

w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. a given axis

Recognizing Single-Peaked Prefs w.r.t. a given axis



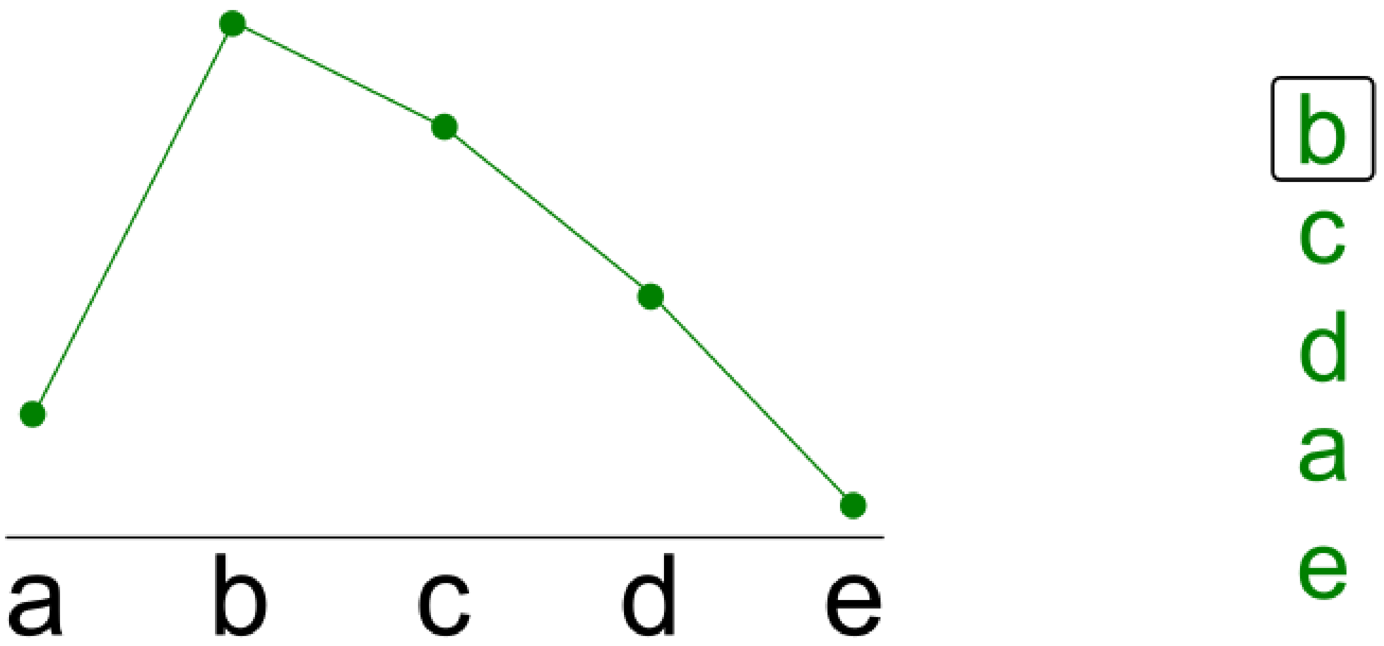
Recognizing Single-Peaked Prefs w.r.t. a given axis

a b c d e

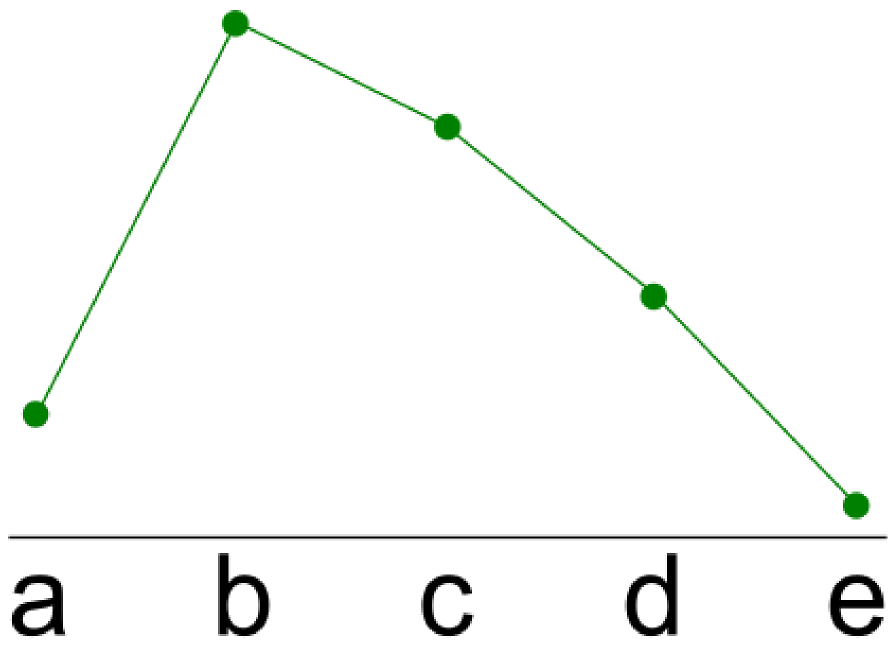
b
c
d
a
e

Recognizing Single-Peaked Prefs w.r.t. a given axis

Recognizing Single-Peaked Prefs w.r.t. a given axis



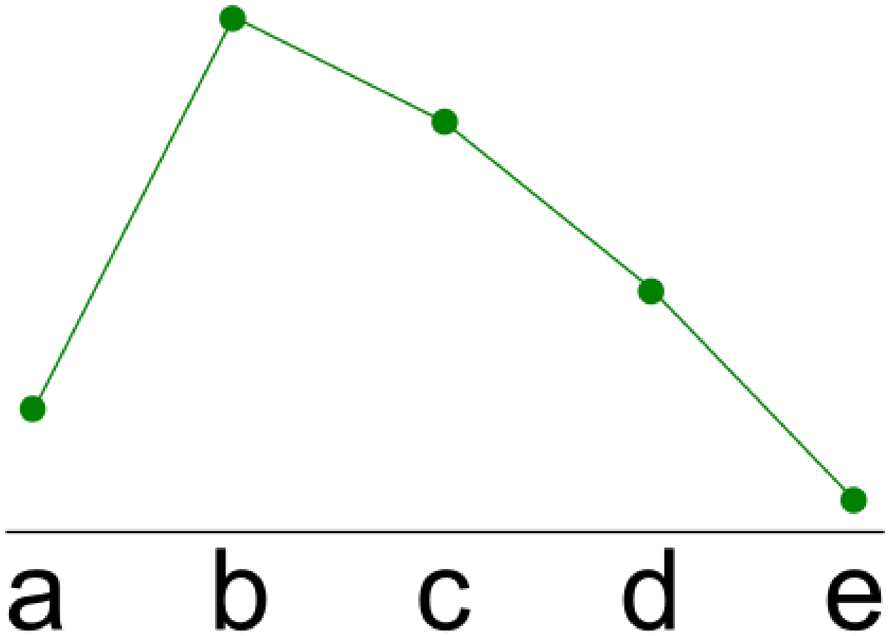
Recognizing Single-Peaked Prefs w.r.t. a given axis



✓

b
c
d
a
e

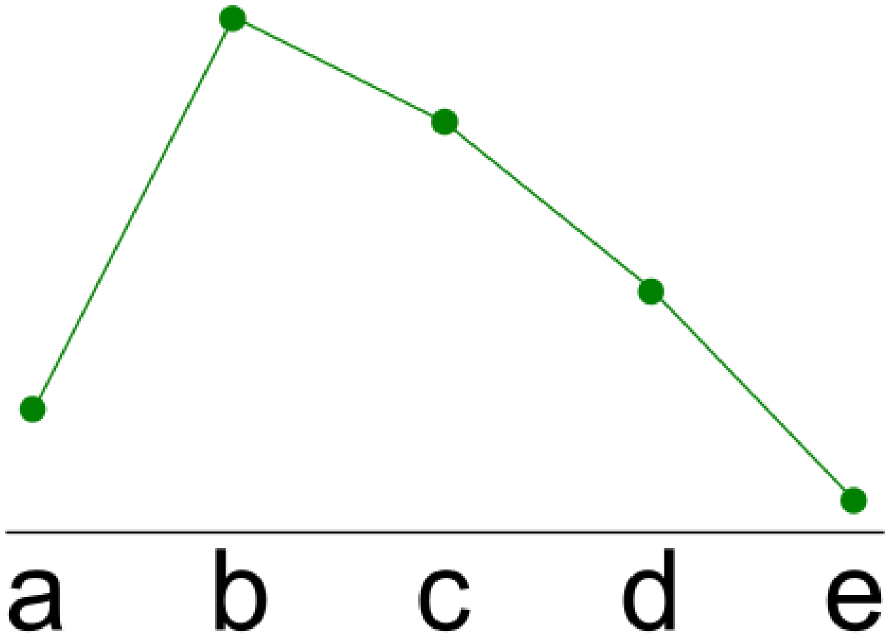
Recognizing Single-Peaked Prefs w.r.t. a given axis



✓

b	d
c	e
d	c
a	b
e	a

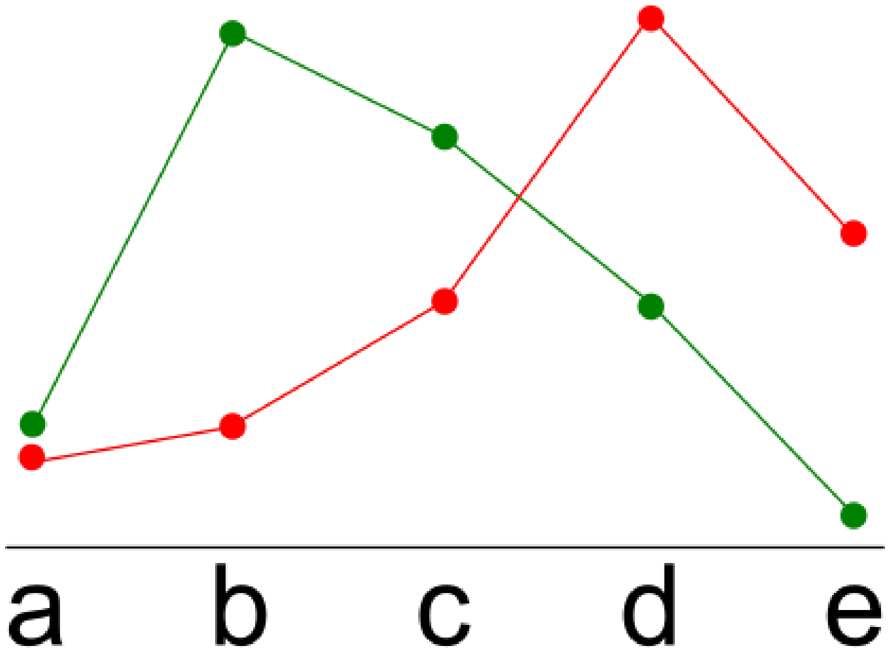
Recognizing Single-Peaked Prefs w.r.t. a given axis



✓

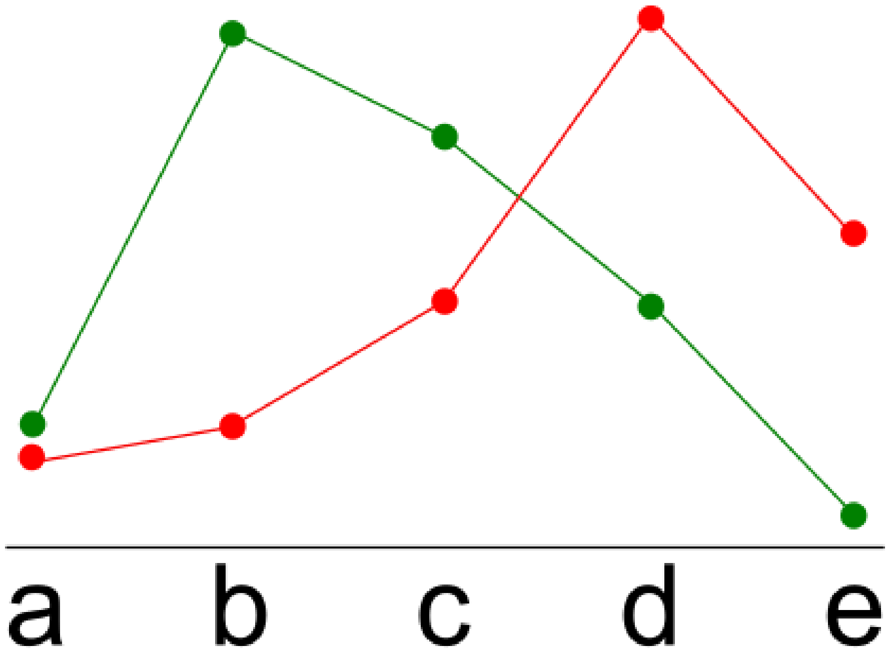
b	d
c	e
d	c
a	b
e	a

Recognizing Single-Peaked Prefs w.r.t. a given axis



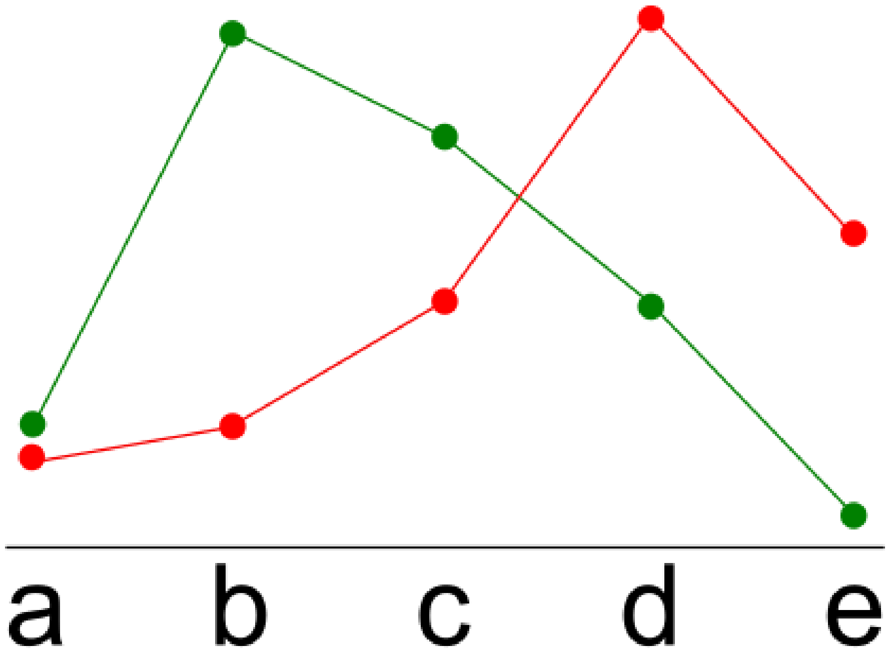
✓	✓
b	d
c	e
d	c
a	b
e	a

Recognizing Single-Peaked Prefs w.r.t. a given axis



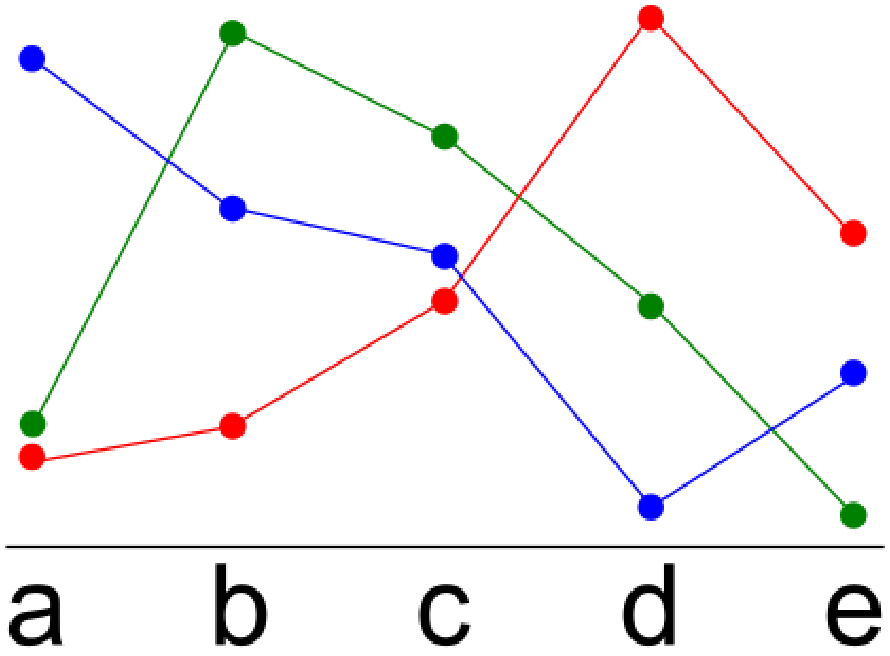
✓	✓	
b	d	a
c	e	b
d	c	c
a	b	e
e	a	d

Recognizing Single-Peaked Prefs w.r.t. a given axis



✓	✓	
b	d	a
c	e	b
d	c	c
a	b	e
e	a	d

Recognizing Single-Peaked Prefs w.r.t. a given axis



✓	✓	✗
b	d	a
c	e	b
d	c	c
a	b	e
e	a	d

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

For this, let us discuss an equivalent definition of single-peaked preferences.

Contiguous Segments Property

Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. $<$ if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. $<$.

Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.

a b c d e

Contiguous Segments Property

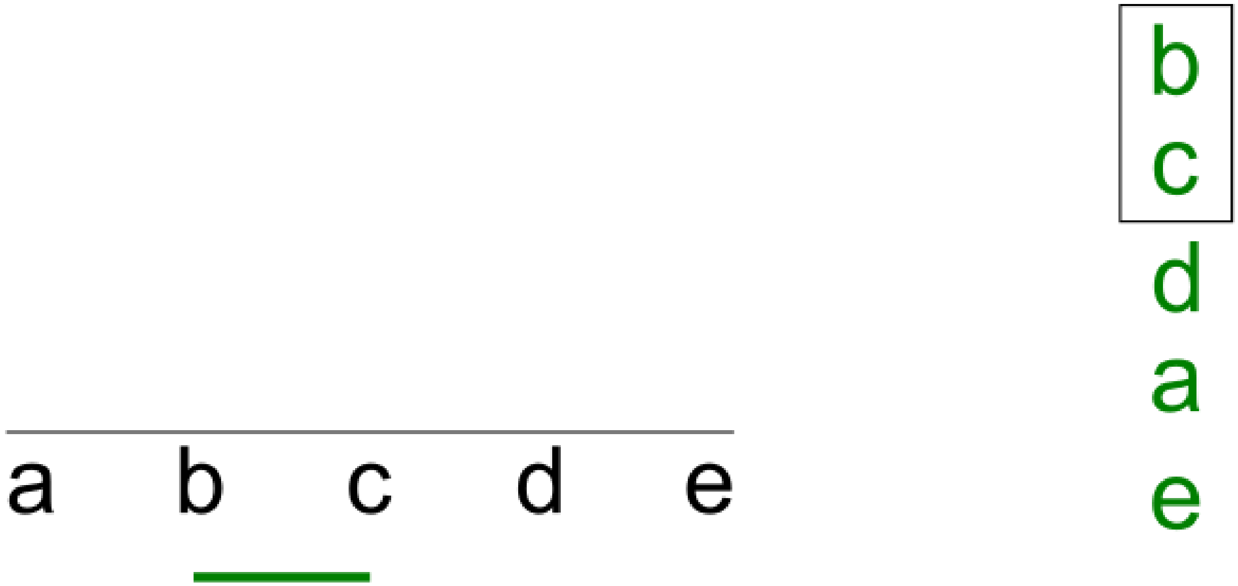
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.

a b c d e

b
c
d
a
e

Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. < if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. <.

Contiguous Segments Property

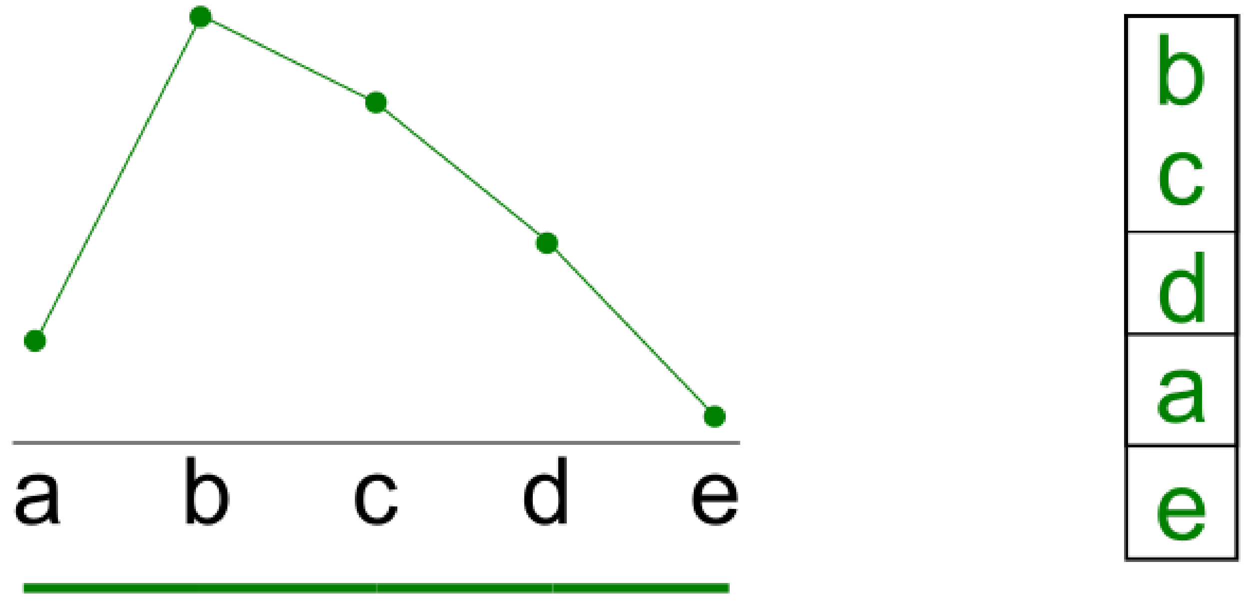
A preference profile satisfies contiguous segments property w.r.t. < if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. <.

Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.

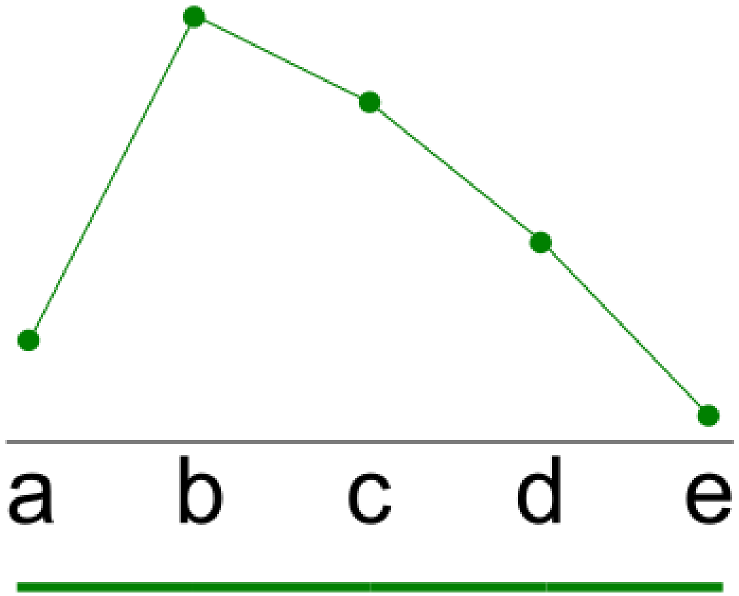
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

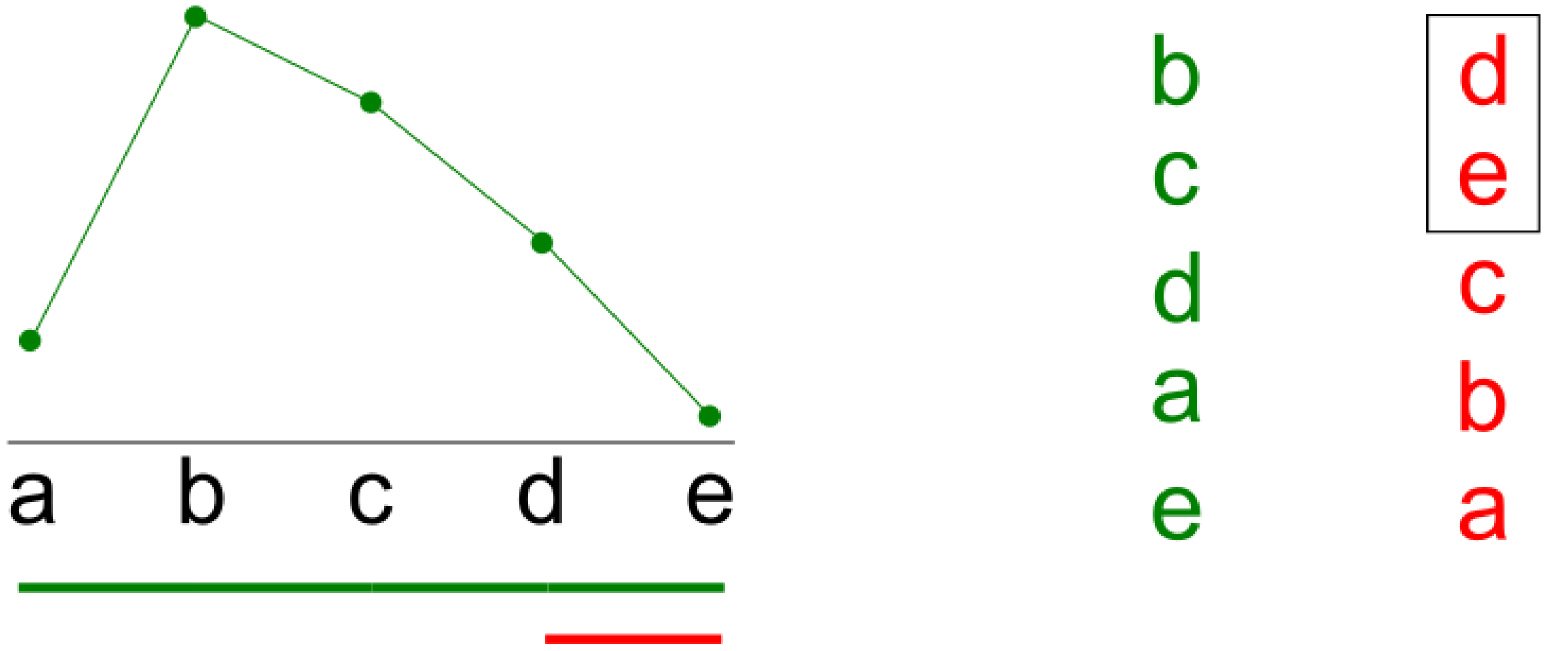
A preference profile satisfies contiguous segments property w.r.t. < if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. <.



b	d
c	e
d	c
a	b
e	a

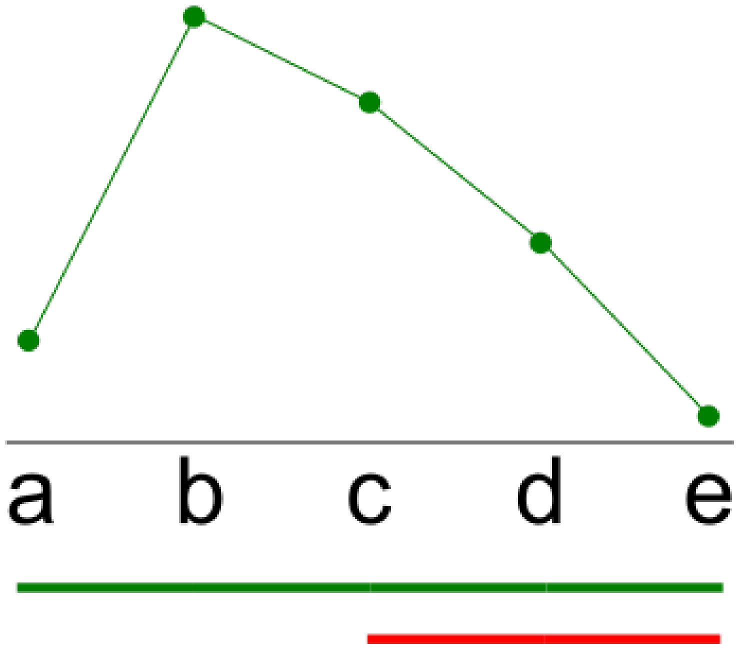
Contiguous Segments Property

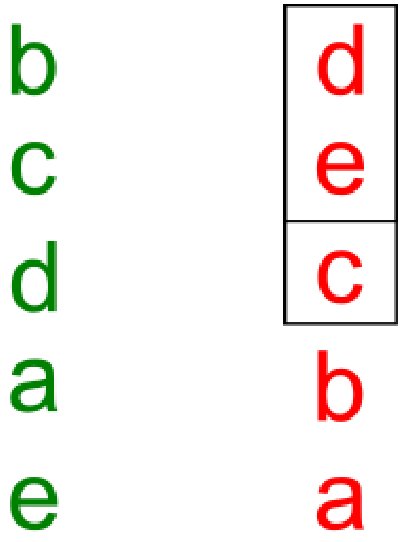
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

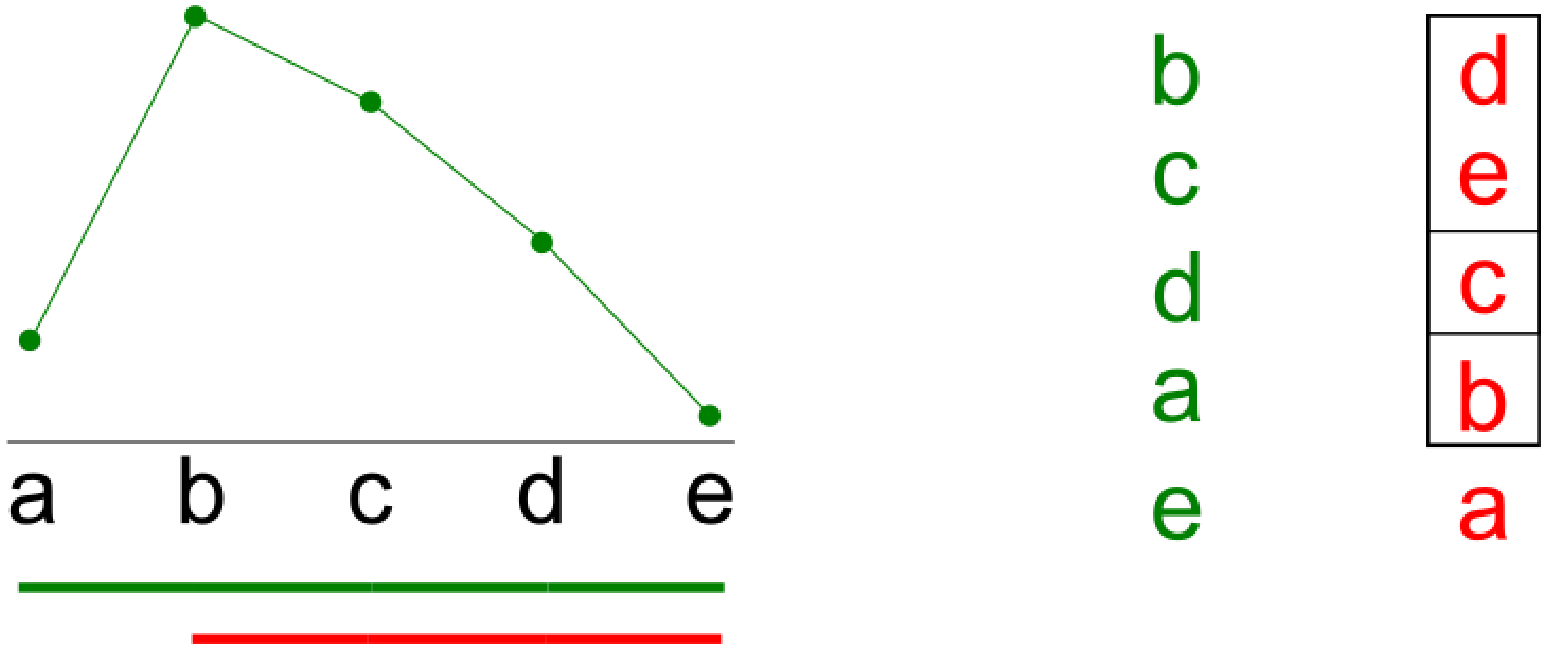
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.





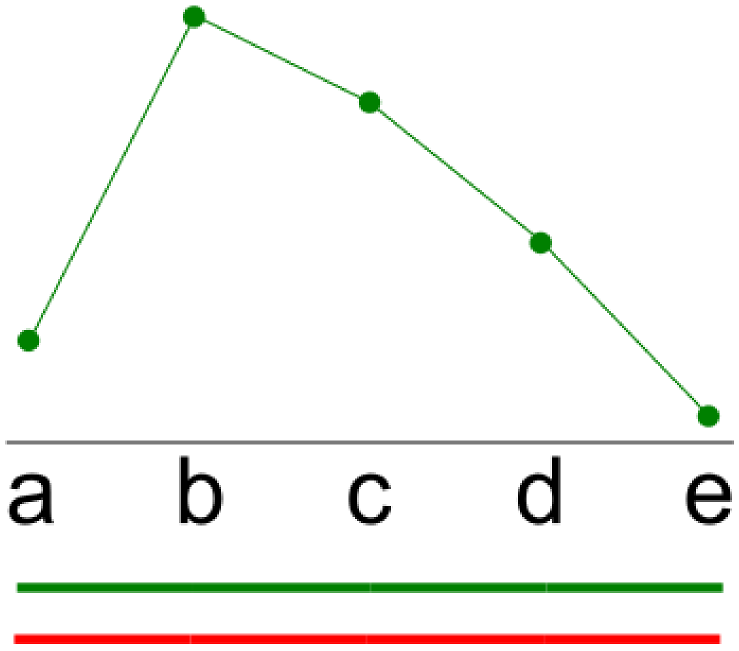
Contiguous Segments Property

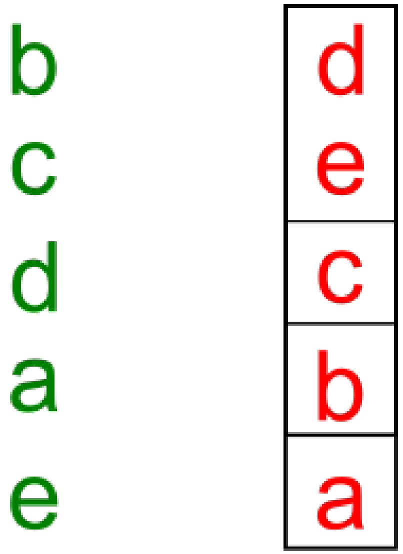
A preference profile satisfies contiguous segments property w.r.t. < if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

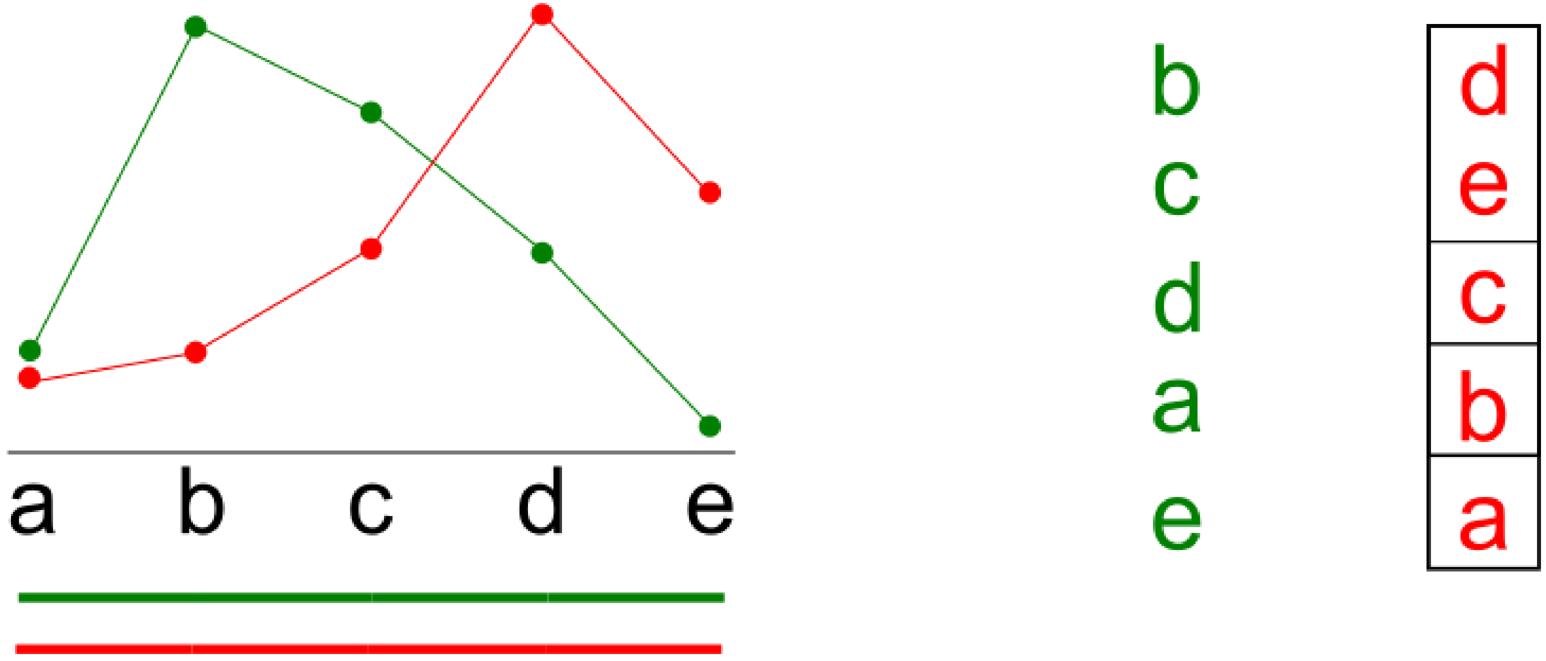
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.





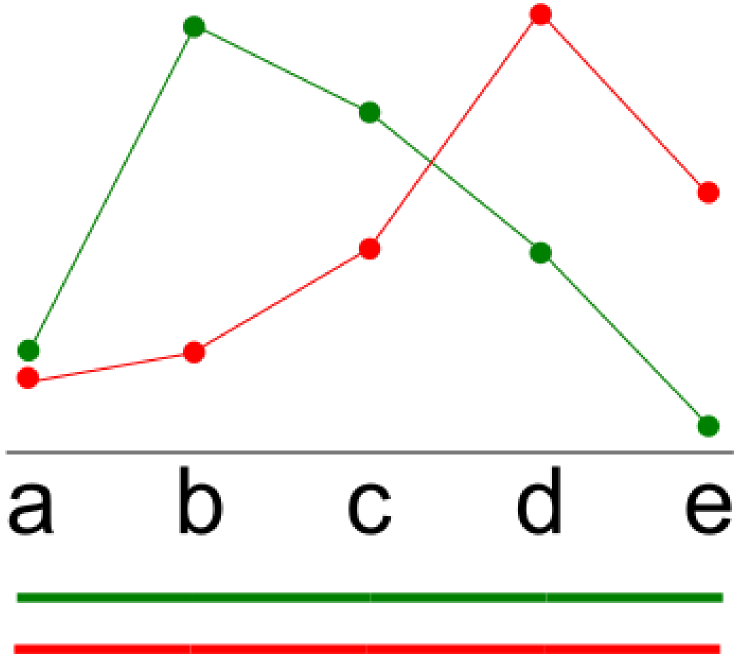
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. < if, for each vote and for every k, the set of top-k candidates in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

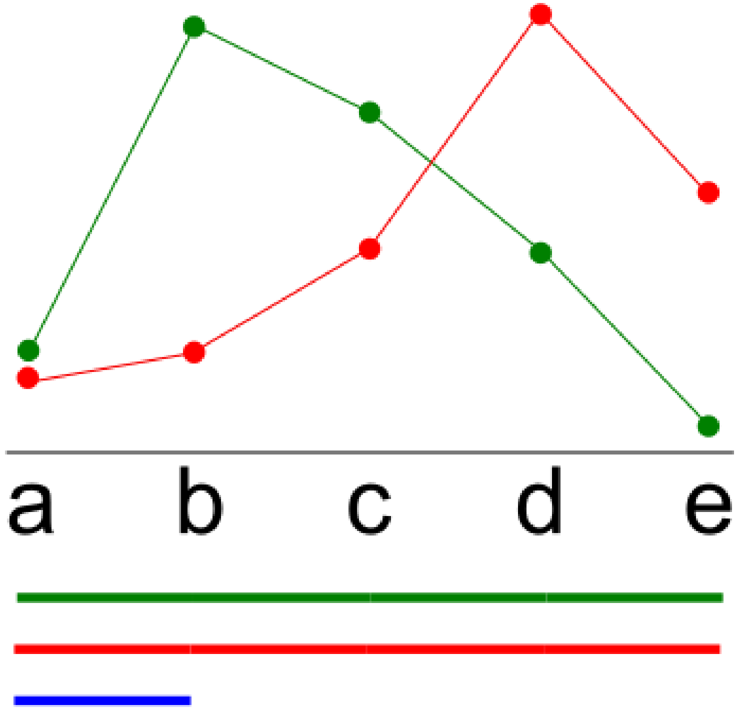
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



b	d	a
c	e	b
d	c	c
a	b	d
e	a	e

Contiguous Segments Property

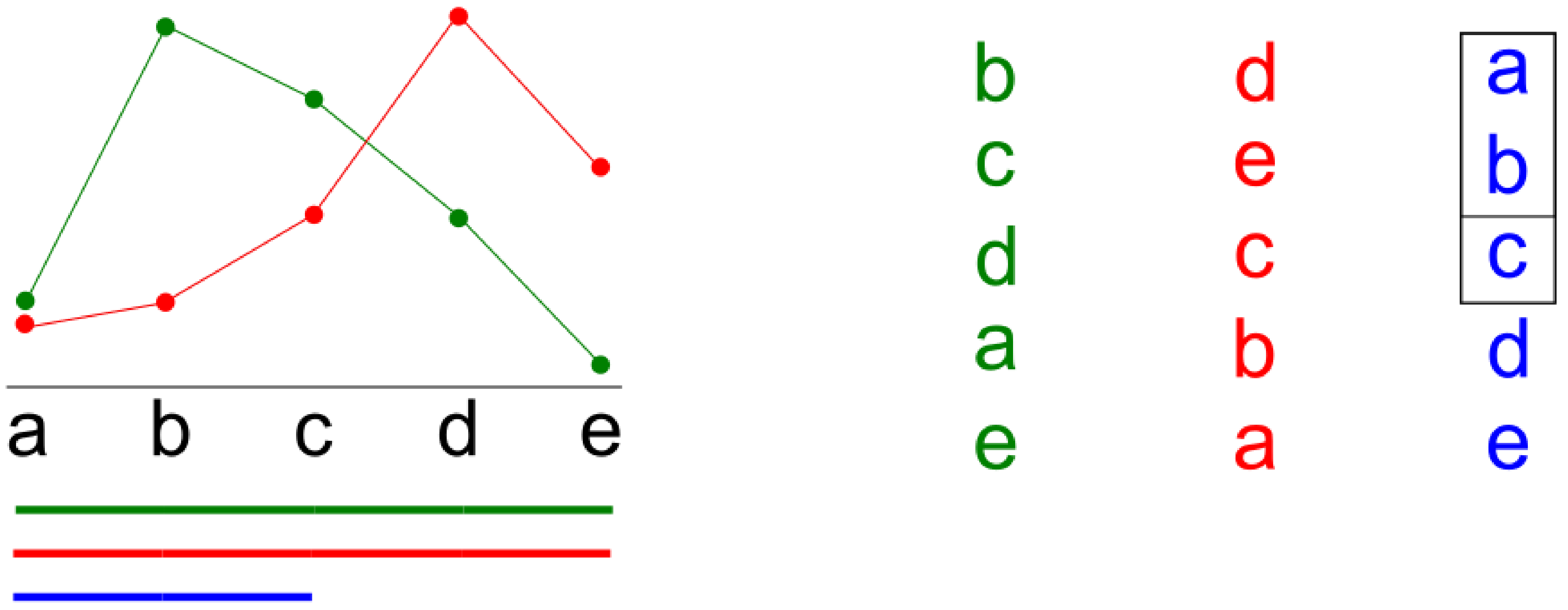
A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



b	d	a
c	e	b
d	c	c
a	b	d
e	a	e

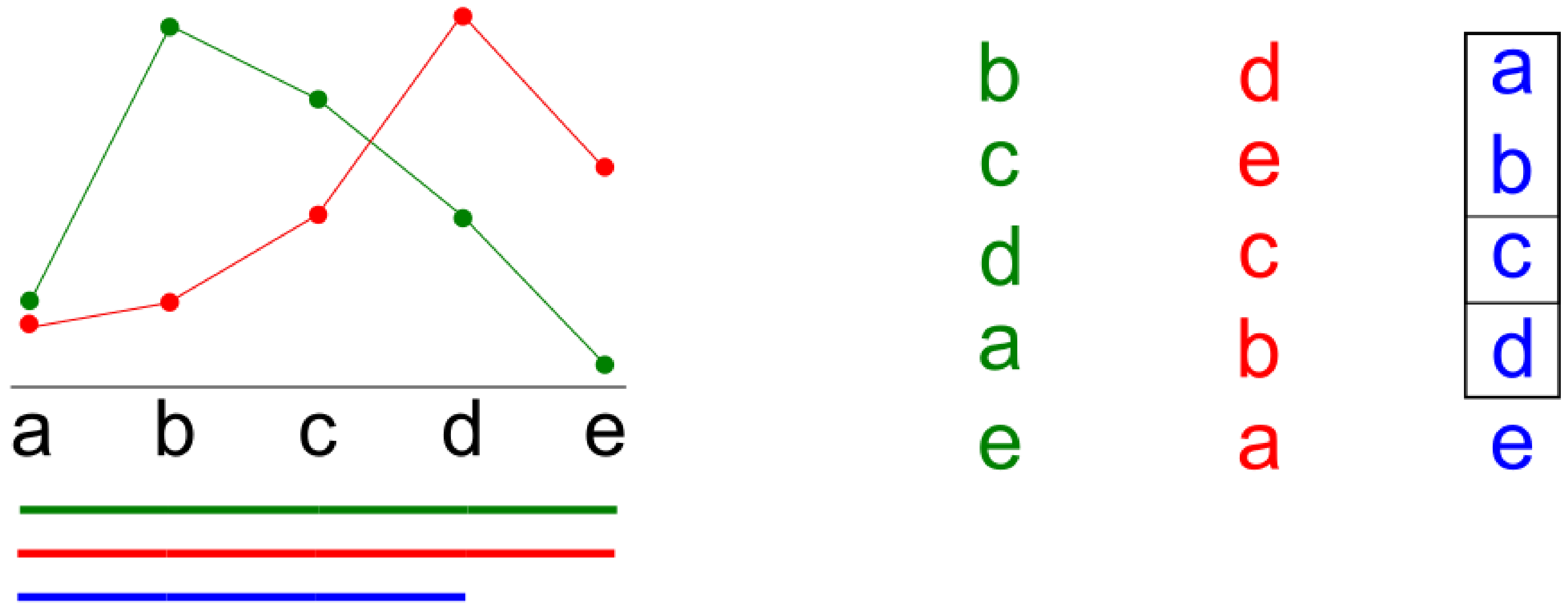
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



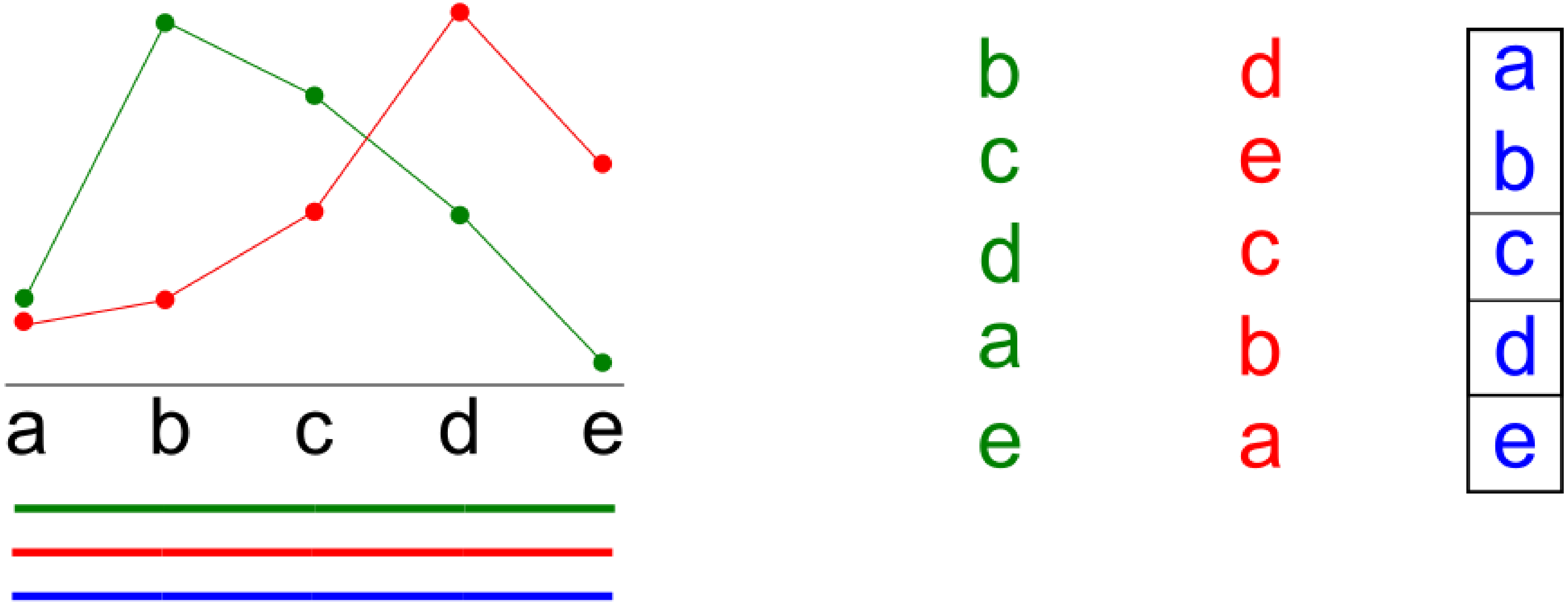
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



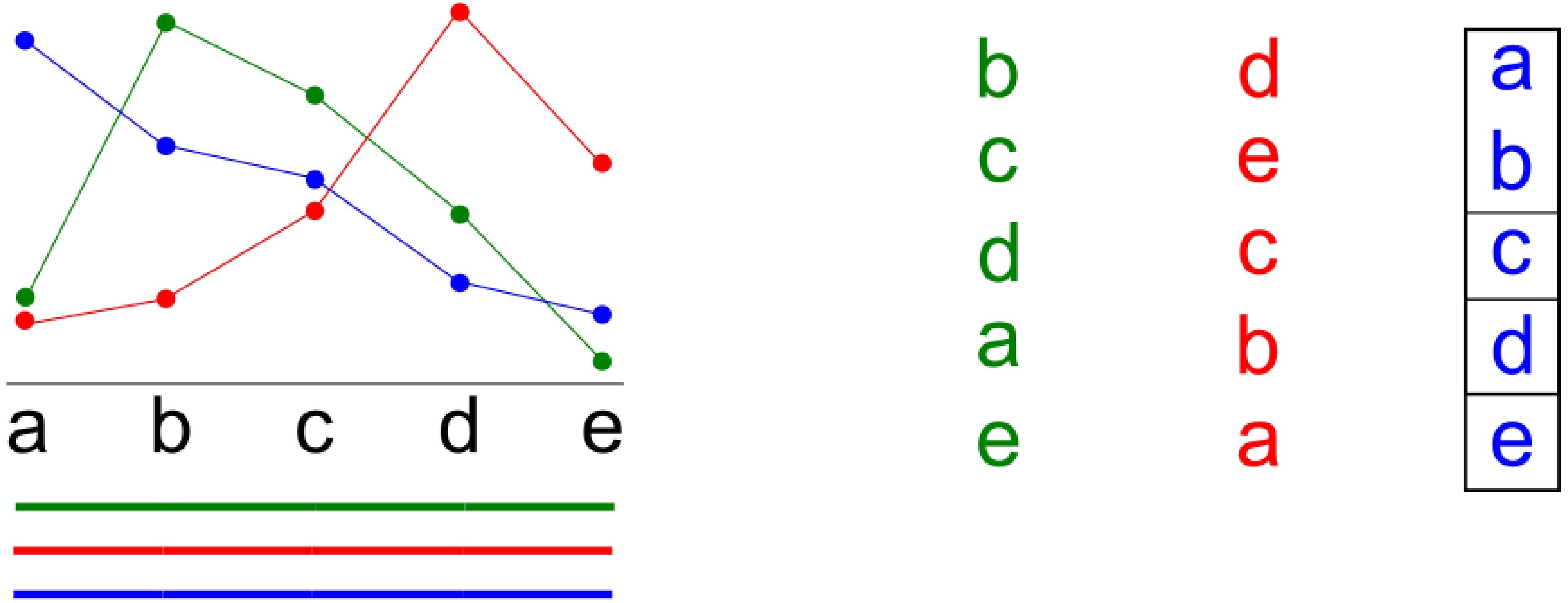
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



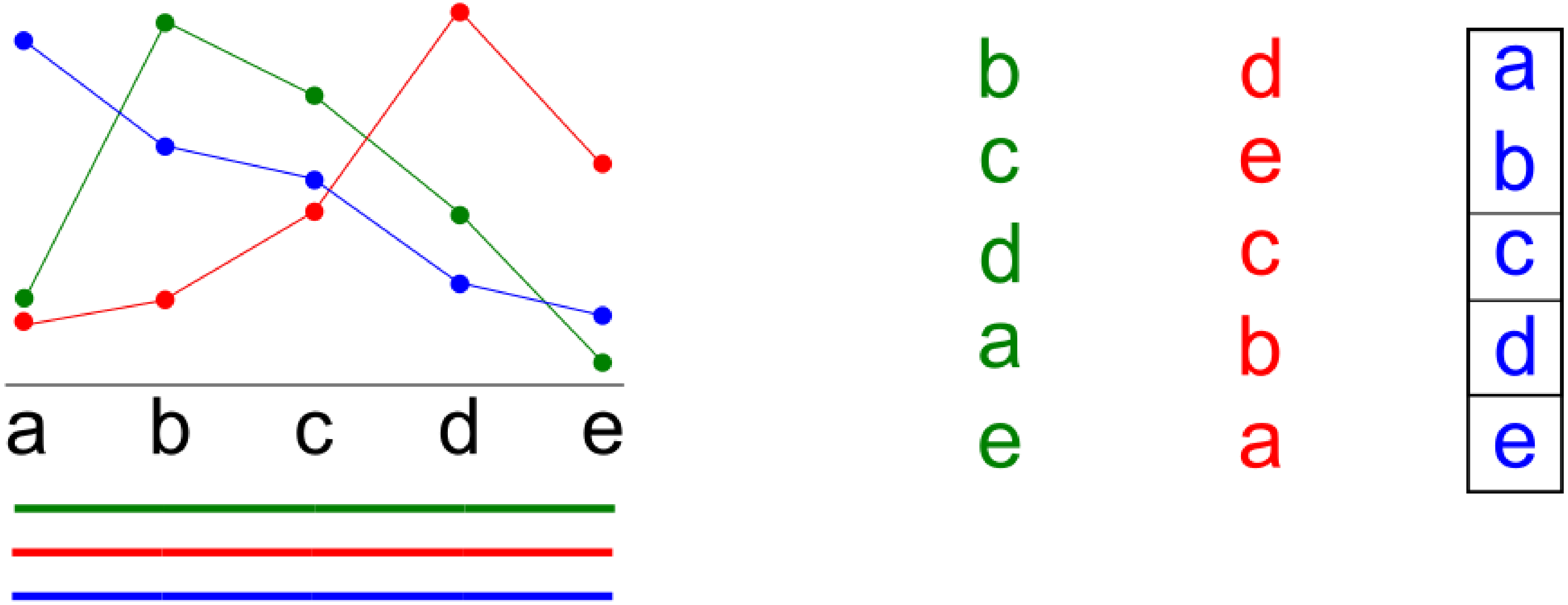
Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



Contiguous Segments Property

A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
in that vote forms a contiguous segment w.r.t. <.



Contiguous segments (w.r.t. <) ⇔ Single-peaked (w.r.t. <)

Contiguous Segments ⇒ Single-Peaked

Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

Contiguous Segments ⇒ Single-Peaked

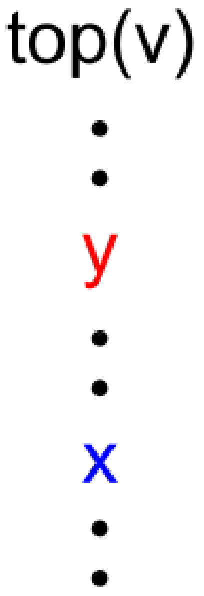
Suppose the contiguous segments property holds w.r.t. the axis <.

Suppose, for contradiction, that for some pair of candidates x,y and some voter v, $top(v) < x < y$ but v prefers y over x.

Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

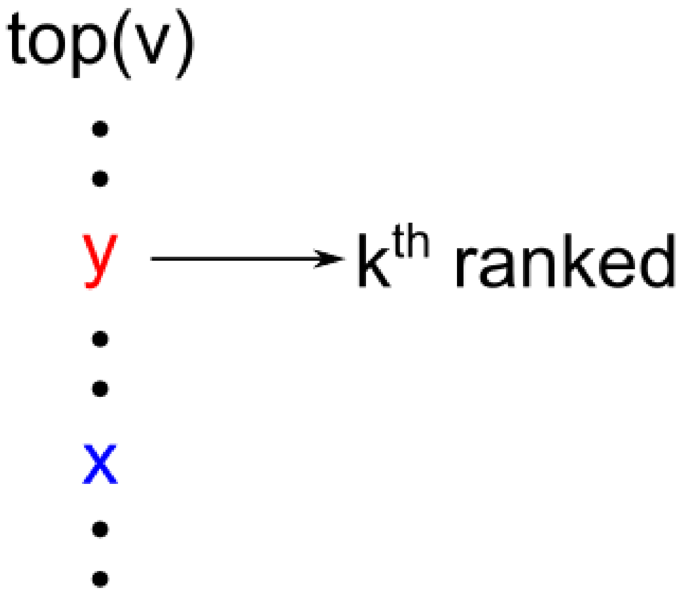
Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.



Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

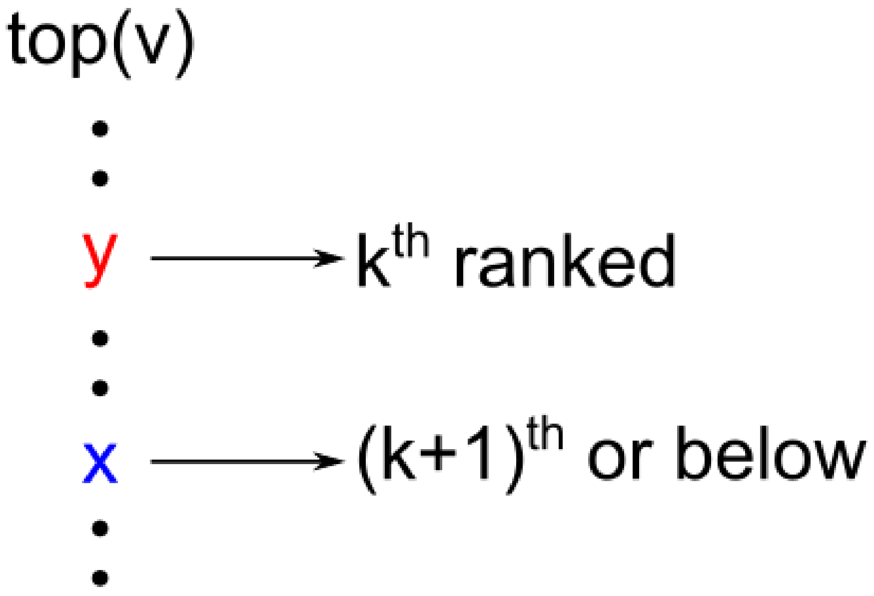
Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.



Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

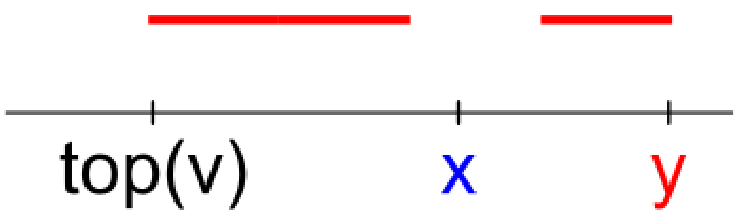
Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.

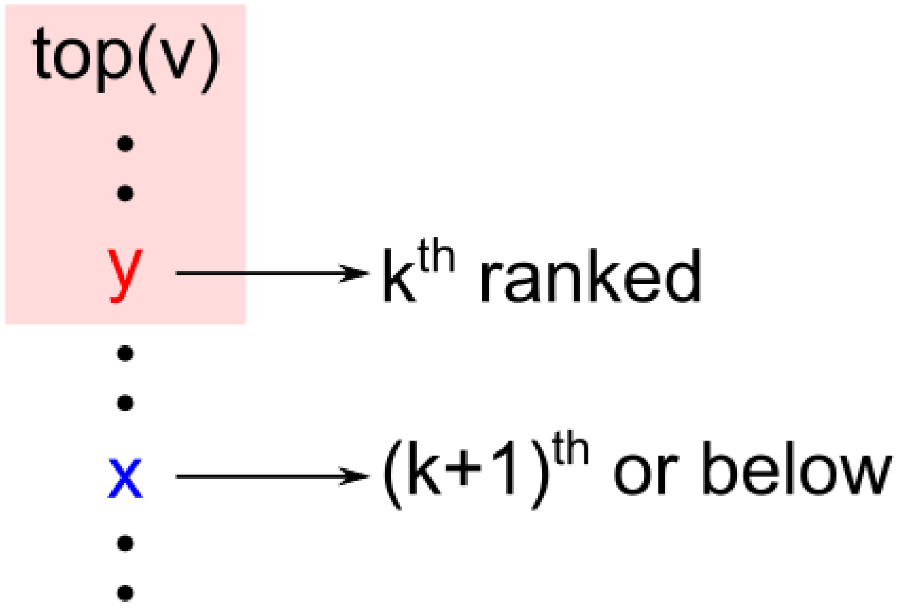


Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis $<$.

Suppose, for contradiction, that for some pair of candidates x,y and some voter v, $top(v) < x < y$ but v prefers y over x.

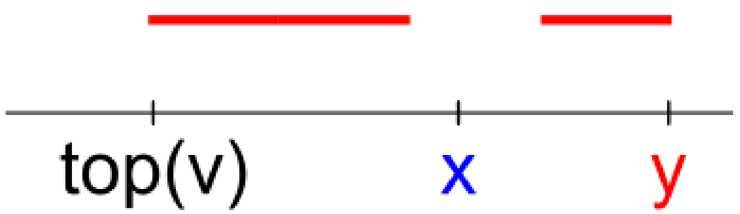




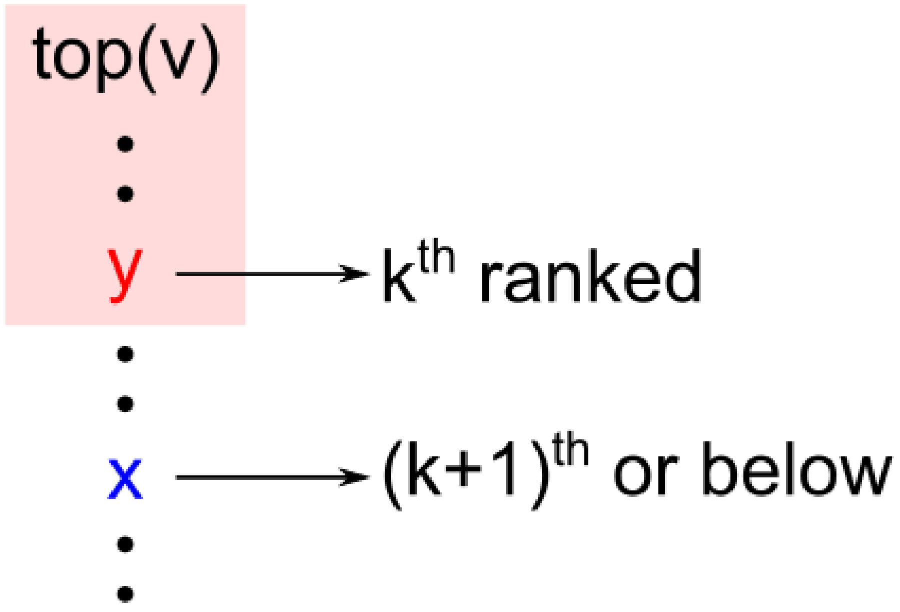
Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.



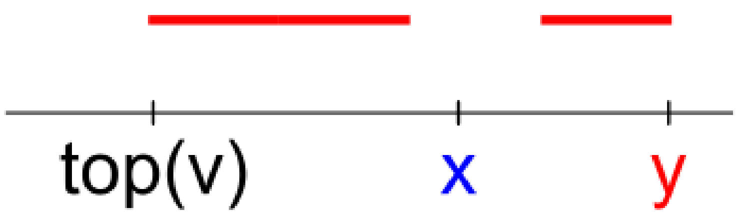
x disconnects the top-k segment

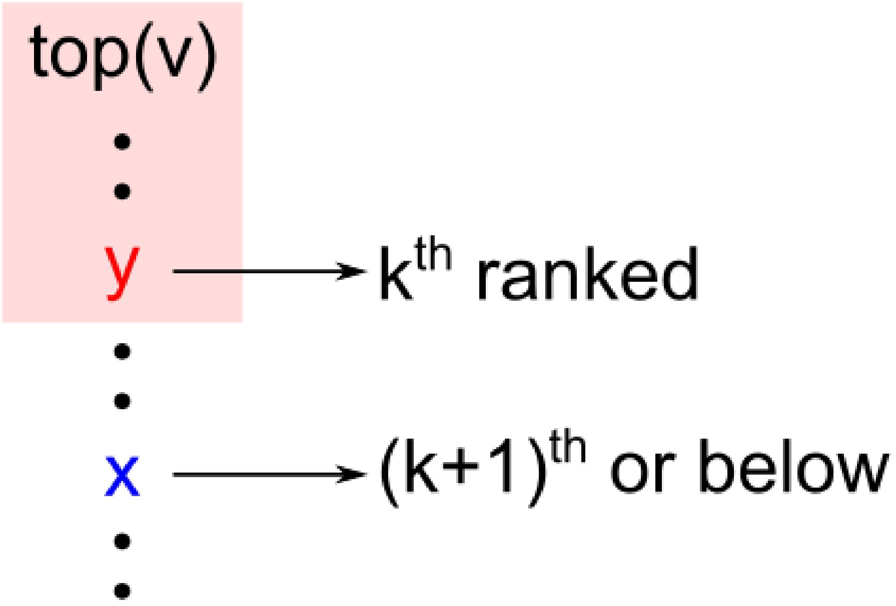


Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.





x disconnects the top-k segment

Single-Peaked ⇒ Contiguous Segments

Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Single-Peaked ⇒ Contiguous Segments

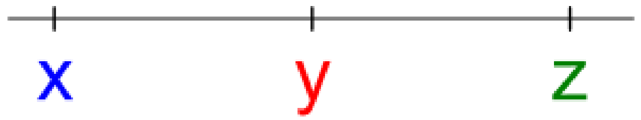
Suppose single-peaked property holds w.r.t. the axis $<$.

Then, for any $x < y < z$, a voter v will never rank y below x and z.
("**no valleys**" property)

Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Then, for any x < y < z, a voter v will never rank y below x and z.
("**no valleys**" property)

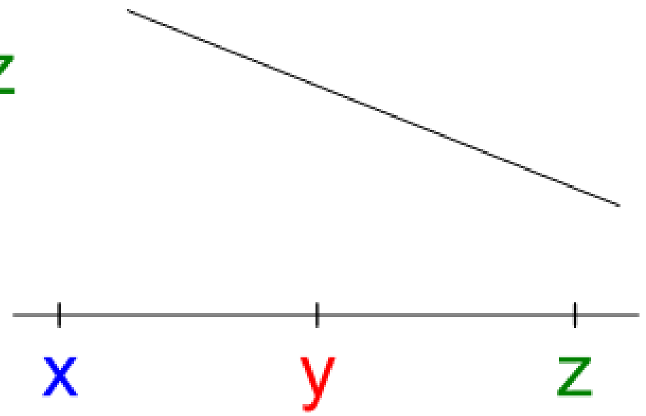


Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Then, for any $x < y < z$, a voter v will never rank y below x and z.
("**no valleys**" property)

if top(v) is to the left of y
then y is preferred over z



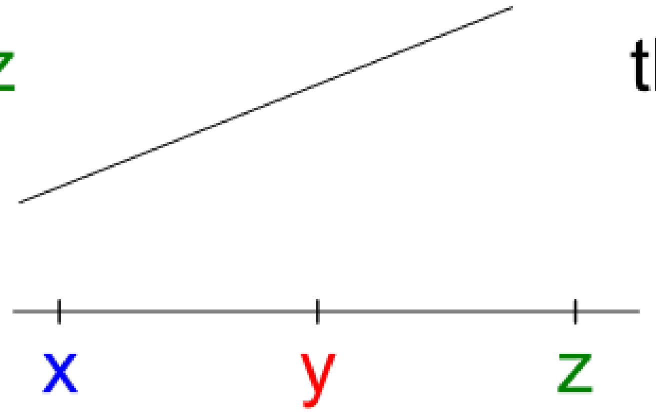
Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Then, for any $x < y < z$, a voter v will never rank y below x and z.
("**no valleys**" property)

if top(v) is to the left of y
then y is preferred over z

if top(v) is to the right of y
then y is preferred over x



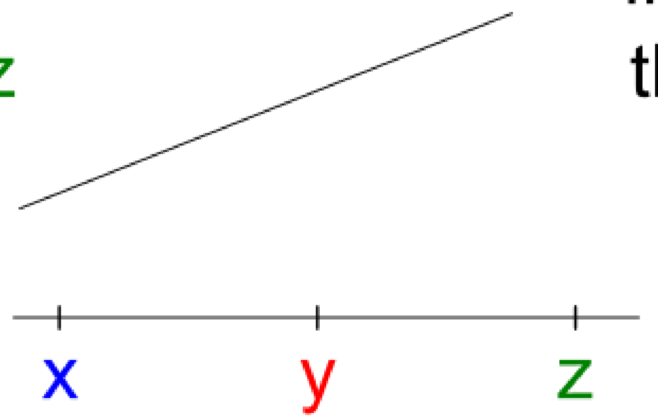
Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Then, for any x < y < z, a voter v will never rank y below x and z.
("**no valleys**" property)

if top(v) is to the left of y
then y is preferred over z

if top(v) is to the right of y
then y is preferred over x



Thus, single peaked (w.r.t. <) ⇒ no valleys (w.r.t. <).

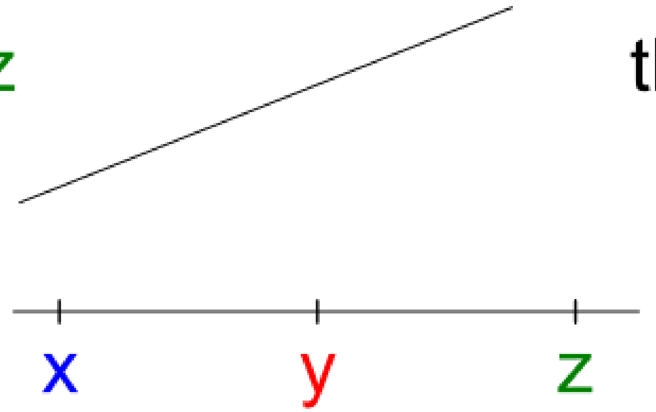
Single-Peaked ⇒ Contiguous Segments

Suppose single-peaked property holds w.r.t. the axis <.

Then, for any x < y < z, a voter v will never rank y below x and z.
("**no valleys**" property)

if top(v) is to the left of y
then y is preferred over z

if top(v) is to the right of y
then y is preferred over x



Thus, single peaked (w.r.t. <) ⇒ no valleys (w.r.t. <).

Let us now show that no valleys (w.r.t. <) ⇒ contiguous segments (w.r.t. <).

Single-Peaked ⇒ Contiguous Segments

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Pick the smallest such k.

Single-Peaked ⇒ Contiguous Segments

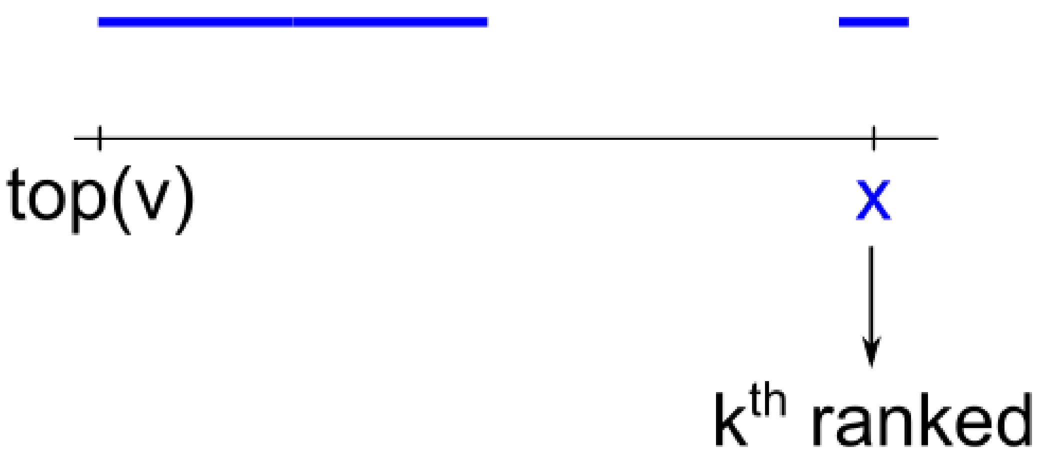
Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Pick the smallest such k.

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

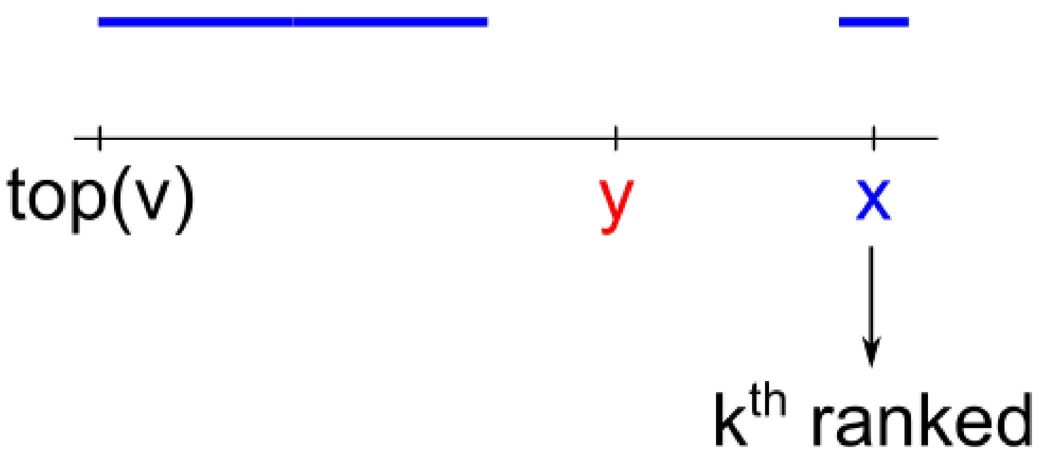
Pick the smallest such k.



Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Pick the smallest such k.

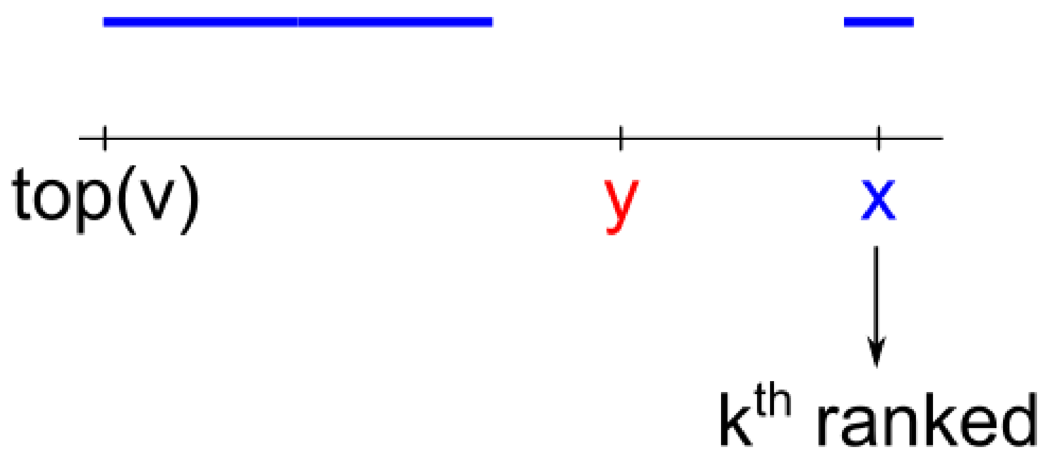


Let y be a candidate that separates x from the top (k-1) candidates

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Pick the smallest such k.



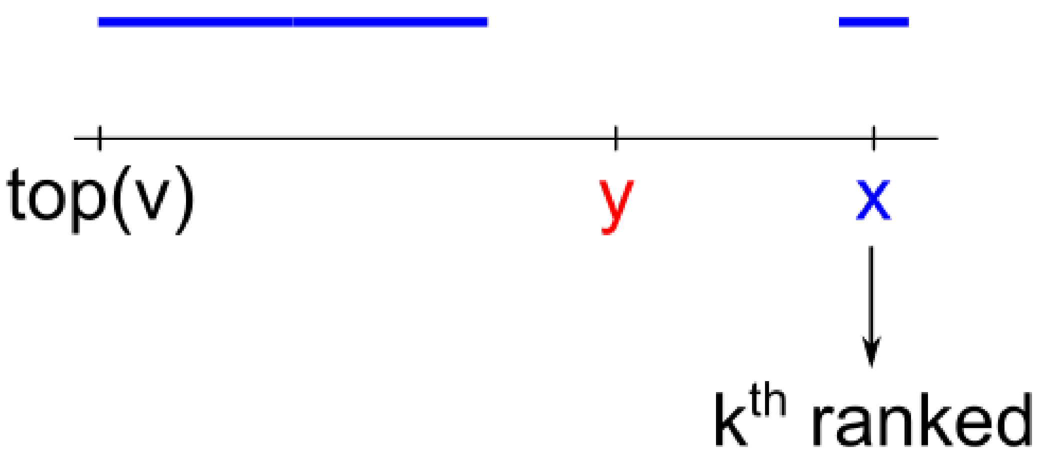
Let y be a candidate that separates x from the top (k-1) candidates

Then, top(v), y and x constitute a valley.

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

Pick the smallest such k.



Let y be a candidate that separates x from the top (k-1) candidates

Then, top(v), y and x constitute a valley.

■

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

We will use the contiguous segments property
to design an algorithm
for recognizing single-peaked preferences.

Recognizing Single-Peaked Prefs w.r.t. some axis

We will use the contiguous segments property
to design an algorithm
for recognizing single-peaked preferences.

But, before that, another digression.

Consecutive 1's Problem

Consecutive 1's Problem

Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

Consecutive 1's Problem

Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0

Consecutive 1's Problem

Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0

→

Consecutive 1's Problem

Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

$$\begin{matrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{matrix} \quad \longrightarrow \quad \begin{matrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{matrix}$$

Consecutive 1's Problem

Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0

→

0	1	1	0
1	1	0	0
0	0	1	1
0	1	0	0

[Booth and Leuker, JCSS 1976]

The consecutive 1's problem can be solved in polynomial time.

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a b c d e

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
its prefix matrix satisfies consecutive 1's property.

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
its prefix matrix satisfies consecutive 1's property.

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

b	a	c	d	e
0	1	1	0	0
0	1	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
0	1	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
its prefix matrix satisfies consecutive 1's property.

Do single-peaked preferences occur in real world?

https://www.preflib.org/

PrefLib: A Library for Preferences

Data ▾ Elections Matchings Ratings Search About

PrefLib

PrefLib is a reference library of preference data and links assembled by *Nicholas Mattei*, *Toby Walsh* and lately *Simon Rey*. This site and library is proudly supported by the *Algorithmic Decision Theory group* at *Data61* and the *The COMSOC Group at the University of Amsterdam*.

We want to provide a comprehensive resource for the multiple research communities that deal with preferences, including computational social choice, recommender systems, data mining, machine learning, and combinatorial optimization, to name just a few.

For more information on PrefLib and some helpful tips on using it, please see Nick's Tutorial *Slides and Code* from *EXPLORE 2014*. Check out the *data type* page to learn more about the kind of data we provide.

Please see the *about* page for information about the site, contacting us, and our citation policy. We rely on the support of the community in order to grow the usefulness of this site. To contribute, please contact *Nicholas Mattei* at: nsmattei{at}gmail or *Simon Rey* at: s.j.rey{at}uva{dot}nl.

In Brief

We currently host:

- 11 types of data
- 38 datasets
- 3668 data files
- More than 3.37 Gb. of data

Other Links

Here are some links that you might find relevant as well.

- *DEMOCRATIX:* A Declarative Approach to Winner Determination

CHAPTER 15

A PrefLib.org Retrospective: Lessons Learned and New Directions

Nicholas Mattei and Toby Walsh

Trends in Computational Social Choice

Realism. Perhaps the key motivating factor behind assembling PrefLib was a desire to have realistic data. Many of the models studied in classical social choice seem to be chosen because they *seem* reasonable or were explicitly chosen for mathematical expediency. Perhaps nothing is more of an exemplar here than the fact that out of over 300 profiles containing strict, complete preference relations, absolutely none are single-peaked, a common profile restriction that has been called "natural" or "well-motivated" numerous times since its introduction by Black (1948). Collecting data has helped us to quantify what is reasonable. Now we have to start using the data.

Next Time

Rank Aggregation

Quiz

Quiz

Given an axis with n candidates,
what is the maximum no. of distinct single-peaked votes with respect to that axis?

References

- Single-peaked preferences in theory:

  Duncan Black
  "*On the Rationale of Group Decision-Making*"
  Journal of Political Economy, Feb 1948, 56(1), pg 23-34
  https://www.journals.uchicago.edu/doi/10.1086/256633

- Single-peaked preferences in the real-world:

  Nicholas Mattei and Toby Walsh
  "*A PREFLIB.ORG Retrospective: Lessons Learned and New Directions*"
  Chapter 15 in Trends in Computational Social Choice
  https://research.illc.uva.nl/COST-IC1205/BookDocs/TrendsCOMSOC.pdf

References

- Lecture by Edith Elkind on restricted preference domains:
  https://www.youtube.com/watch?v=q8vc8Znoev0
  https://www.youtube.com/watch?v=vL_U-5tlQu4

- Strategyproof voting rules using “phantom” voters:

  Hervé Moulin
  “*On Strategyproofness and Single-Peakedness*”
  Public Choice, 35(4), 1980, pp. 437-455
  https://www.jstor.org/stable/30023824

References

- Recognizing single-peaked preferences:

  John Bartholdi III and Michael A. Trick
  "*Stable Matching with Preferences Derived from a Psychological Model*"
  Operations Research Letters, 5(4), 1986, pp. 165-169
  https://www.sciencedirect.com/science/article/pii/0167637786900726

- Consecutive 1's Problem:

  Kellogg S. Booth and George S. Lueker
  "*Testing for the Consecutive One Property, Interval Graphs, and Graph Planarity using PQ Tree Algorithms*"
  JCSS, 13(3), 1976, pp. 335-379
  https://www.sciencedirect.com/science/article/pii/S0022000076800451