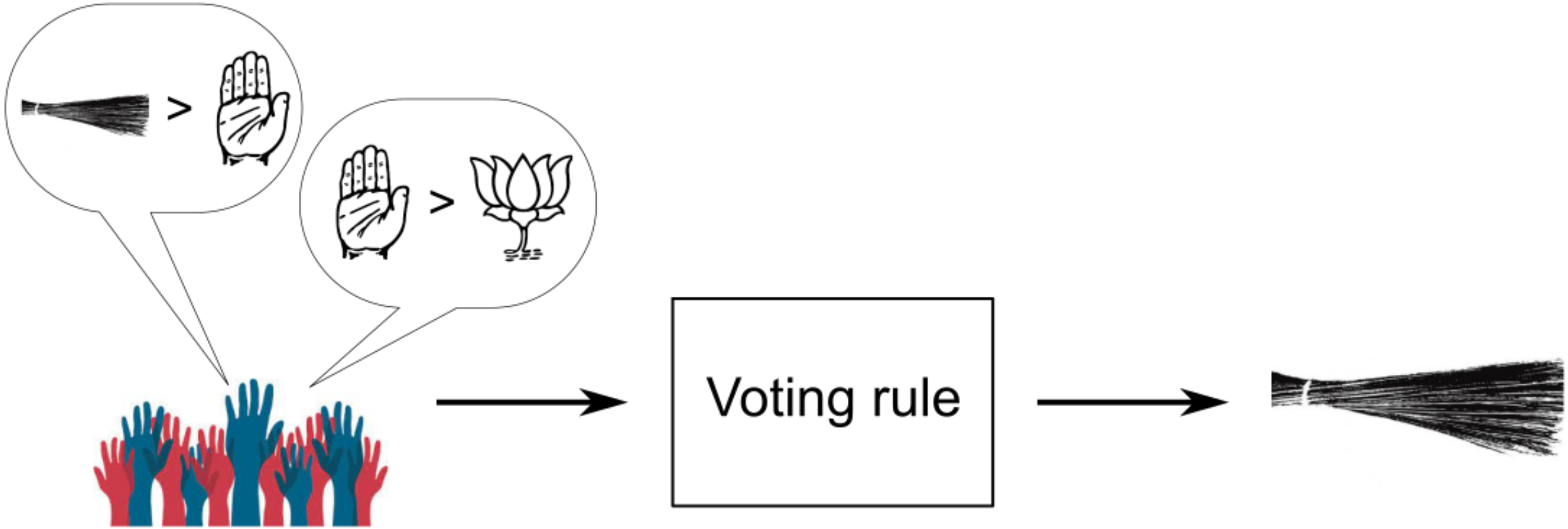
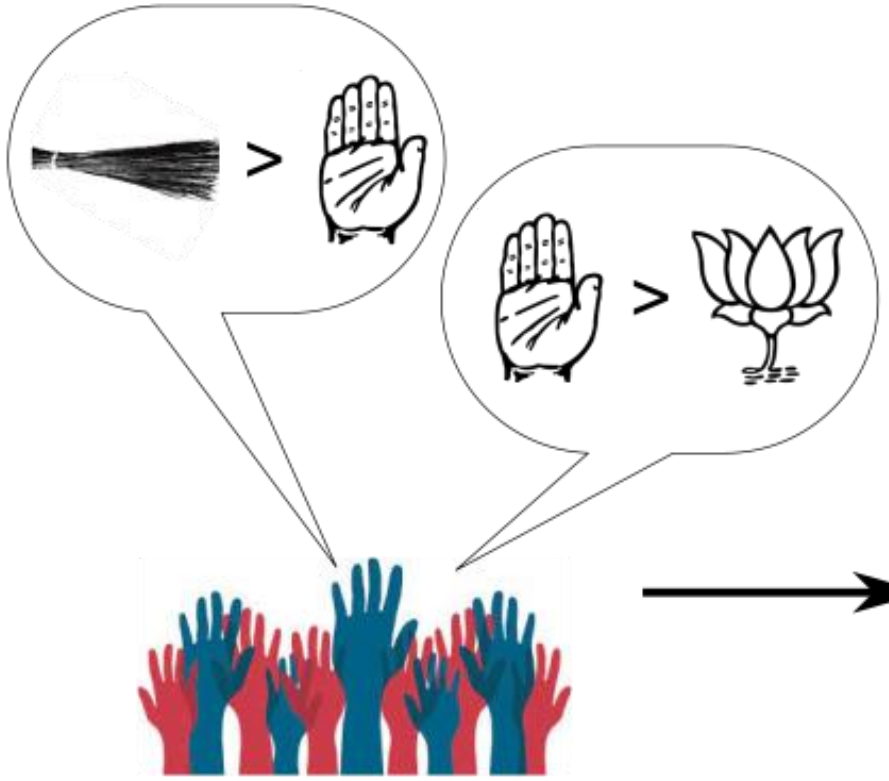
COL749: Computational Social Choice

# Lecture 16

# Voting Rules

March 06, 2025 | Rohit Vaish


Voting rule



Voting rule


amazon

# Voting Setup

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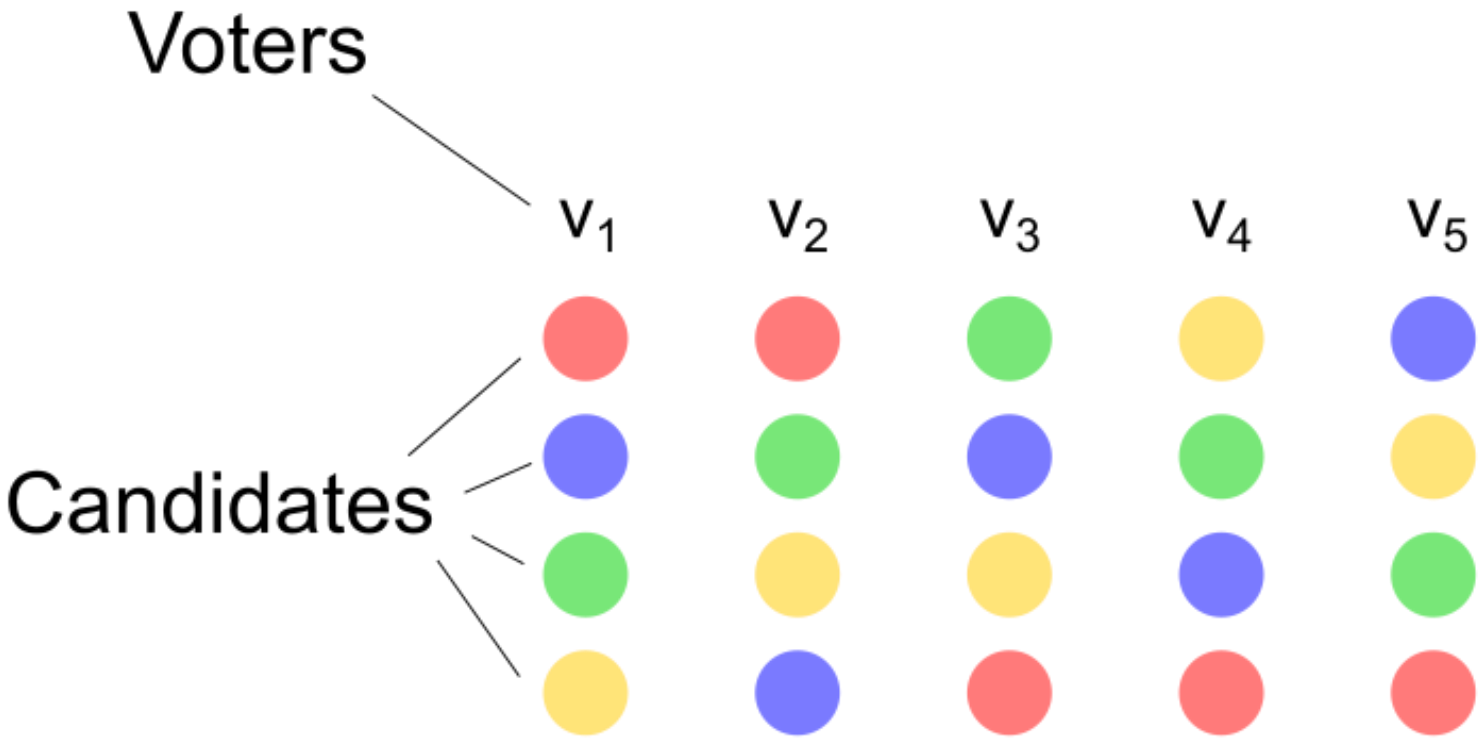

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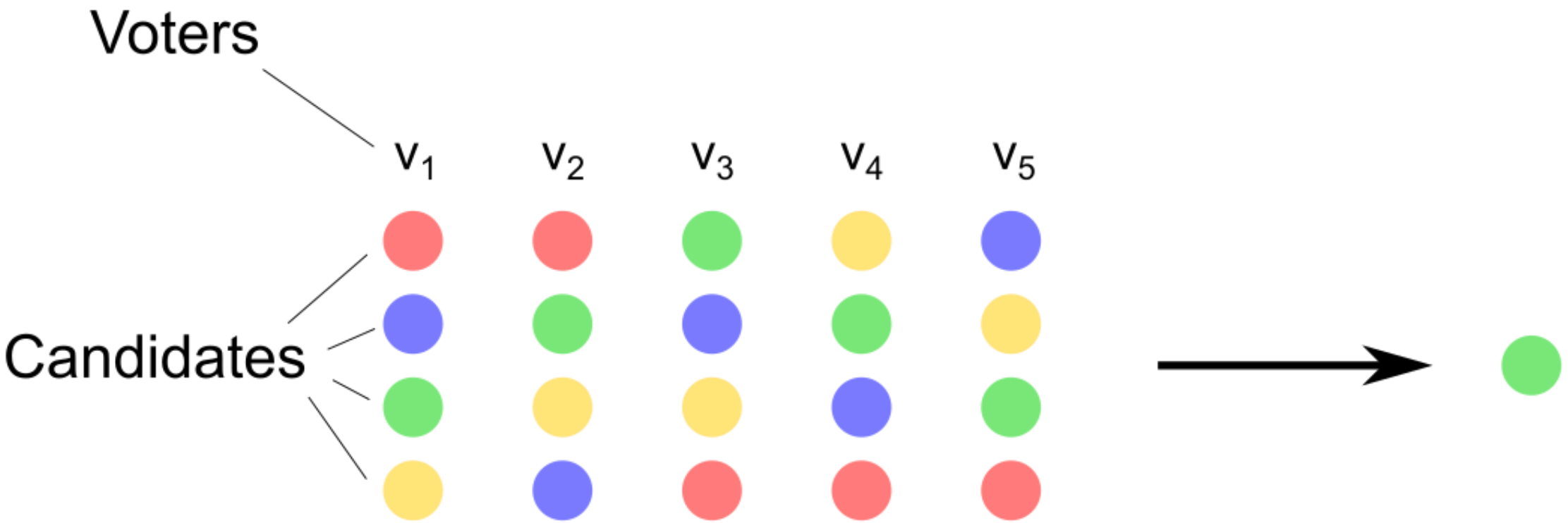


Goal: Pick exactly one winning candidate.

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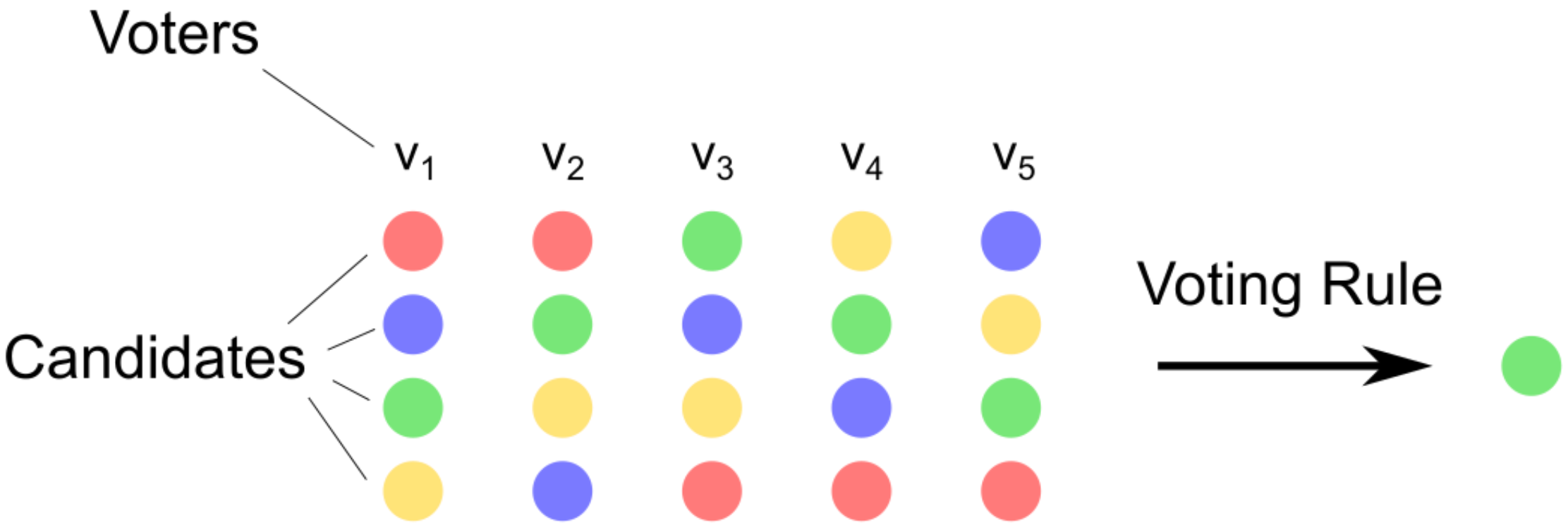


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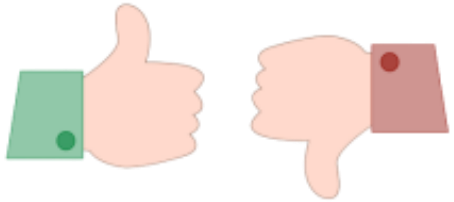

Voting Rule

# Voting with Two Candidates

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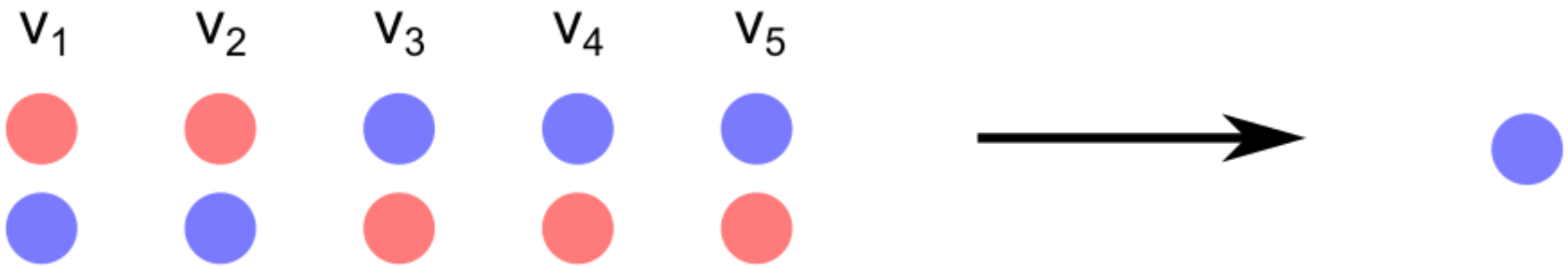
Majority!



# Voting with More Than Two Candidates

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No candidate may have a majority...

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# Plurality

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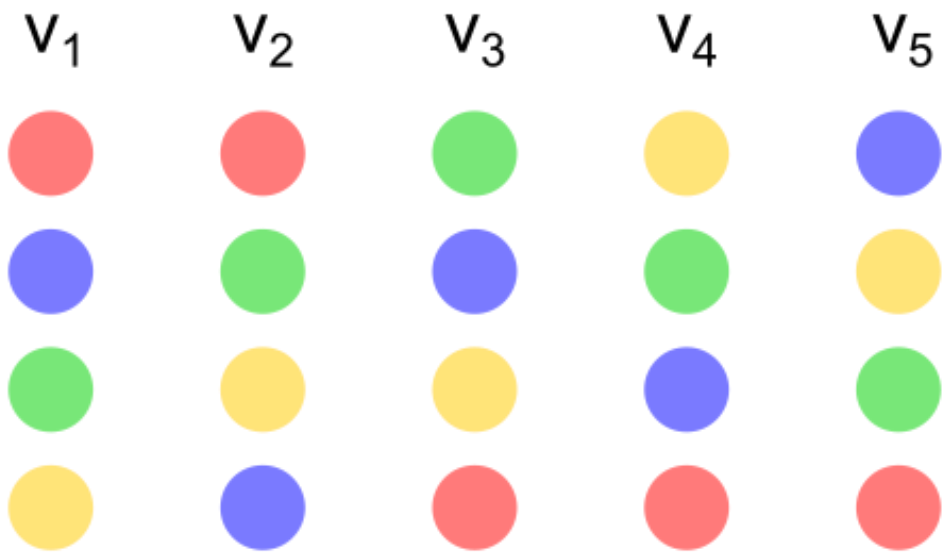
Plurality



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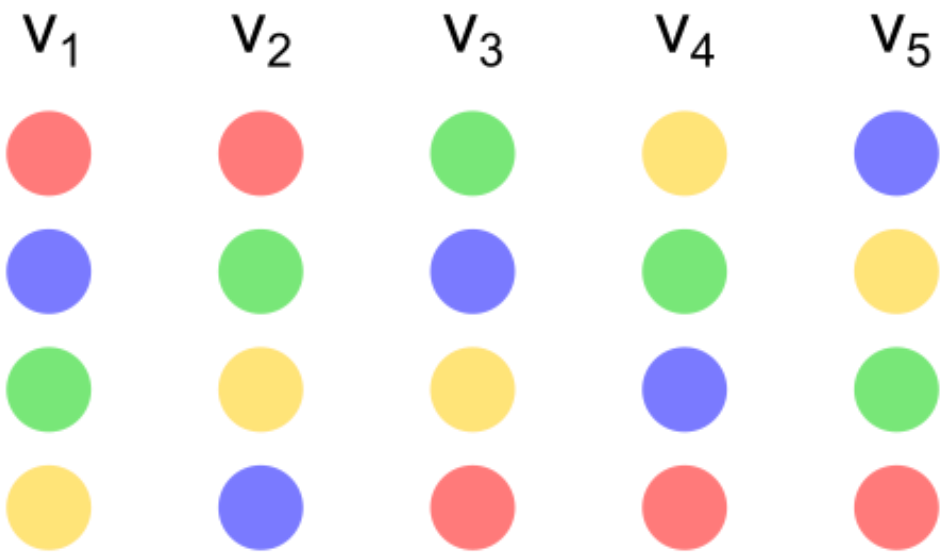
Candidate with the most first-place votes wins

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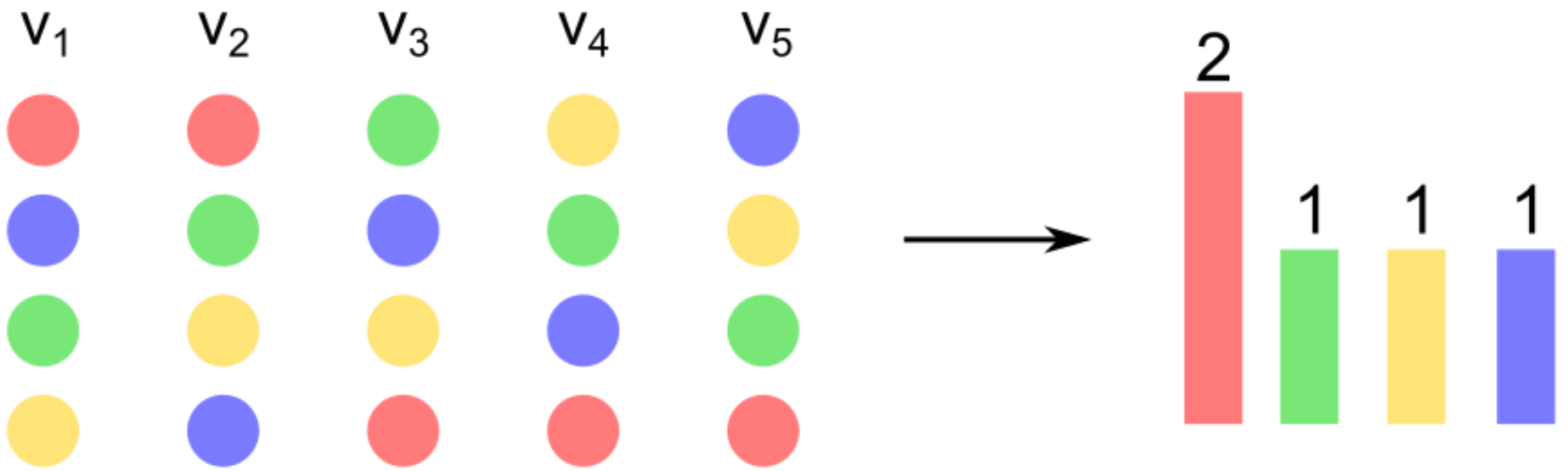



Plurality

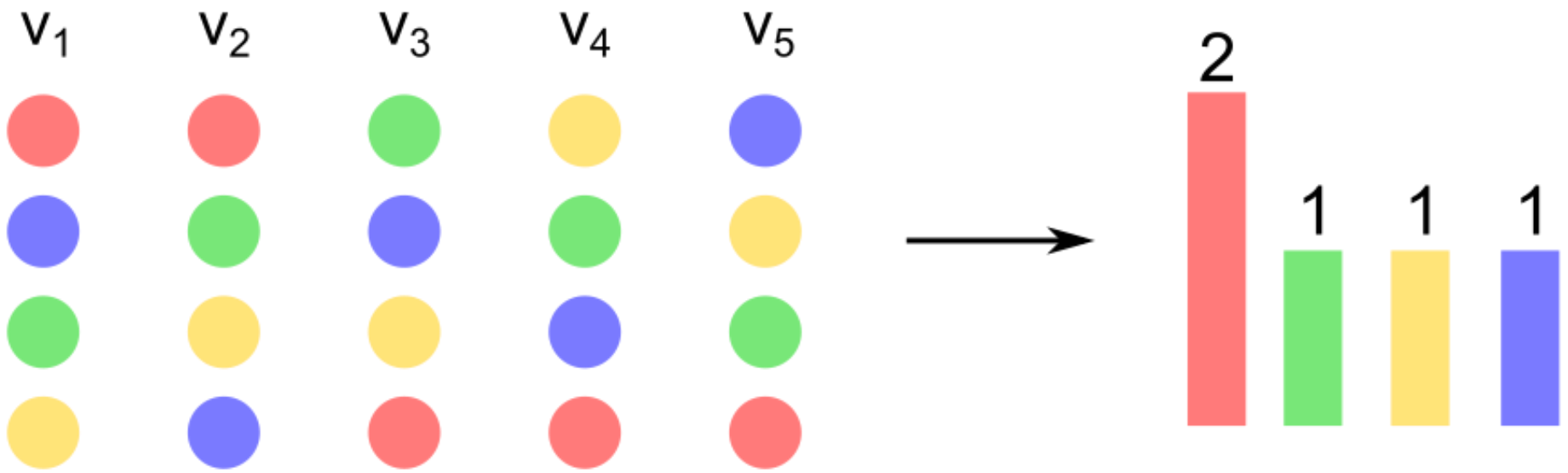
Plurality winner:

# 1

## Plurality

Candidate with the most first-place votes wins*

*subject to tie-breaking: lexicographic, random, ...




Plurality winner:

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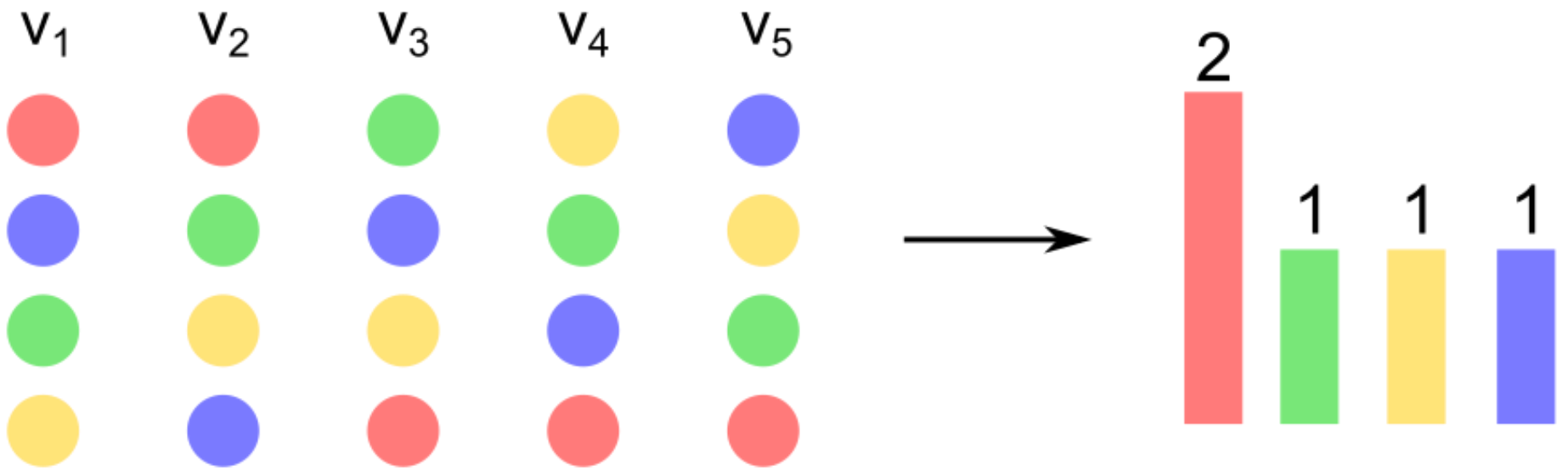
Problem: A majority prefers ● over the Plurality winner.

## Plurality

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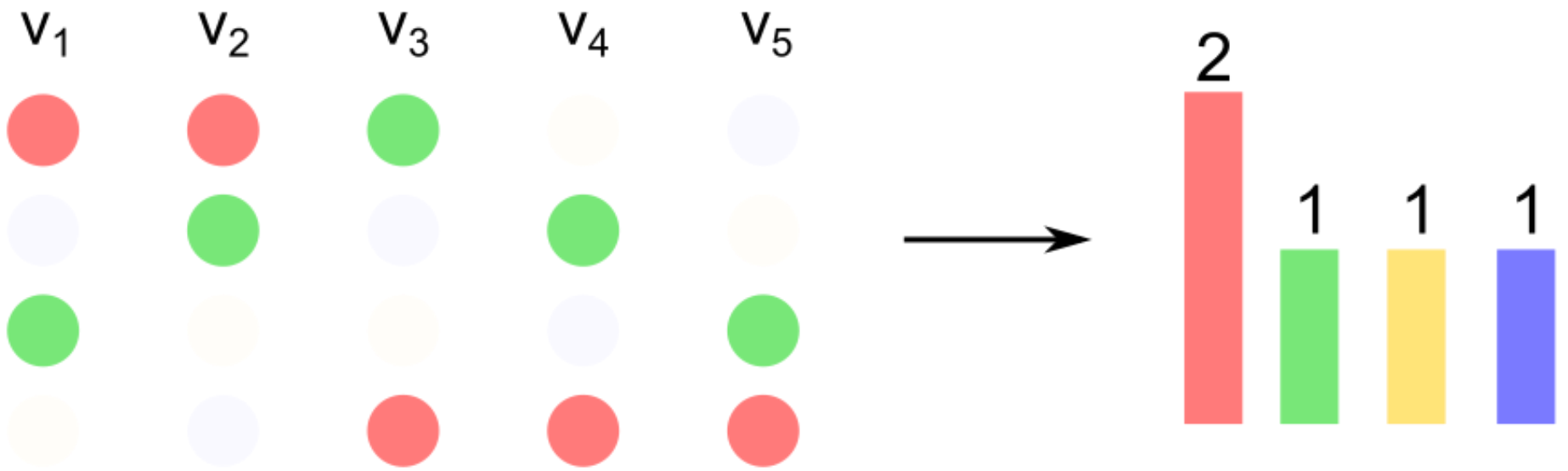


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## Plurality

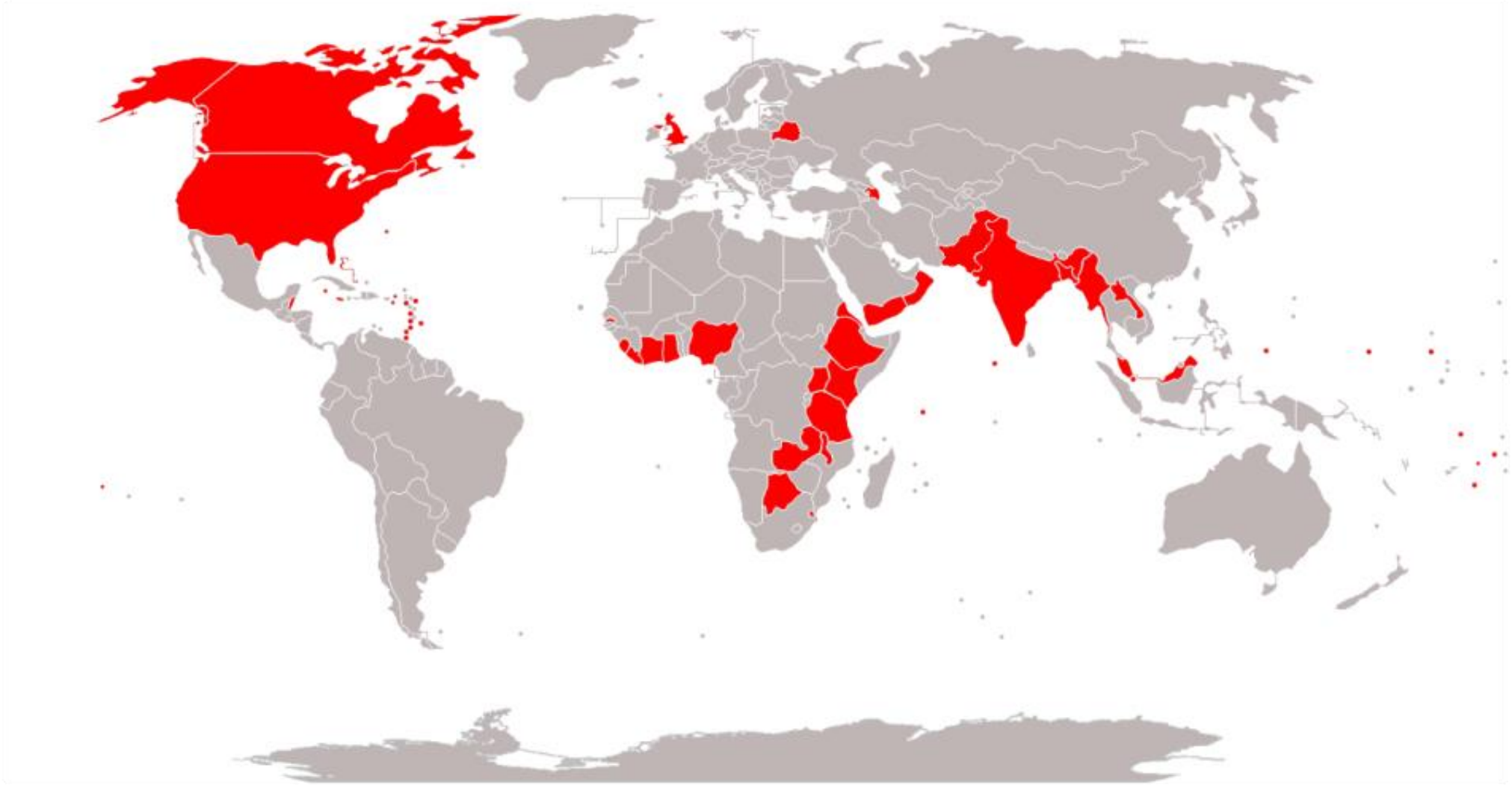

Image Source: Wikipedia article on "Electoral system" (Jan 2022)

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Borda Count

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Jean-Charles de Borda
(1733-1799)

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Each voter gives its $k^{th}$ ranked candidate m-k points, where m is the number of candidates.

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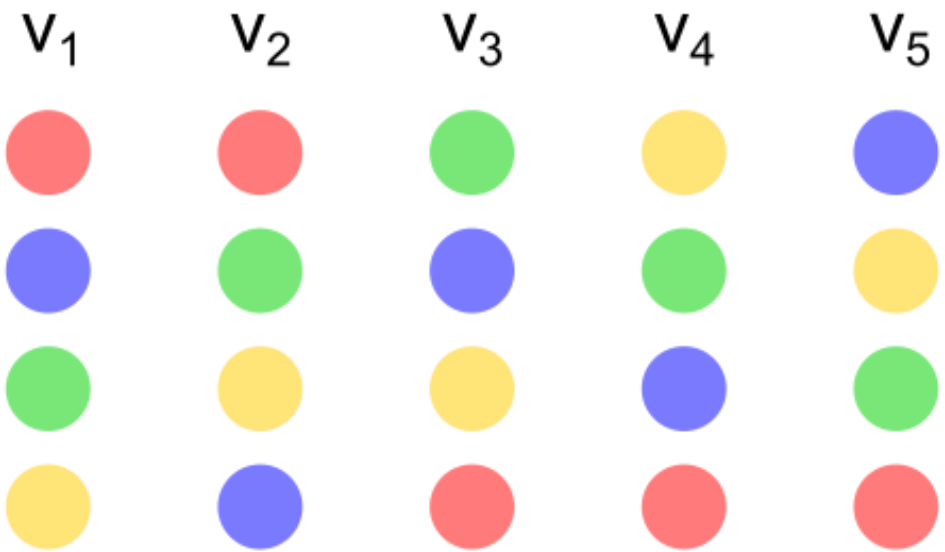

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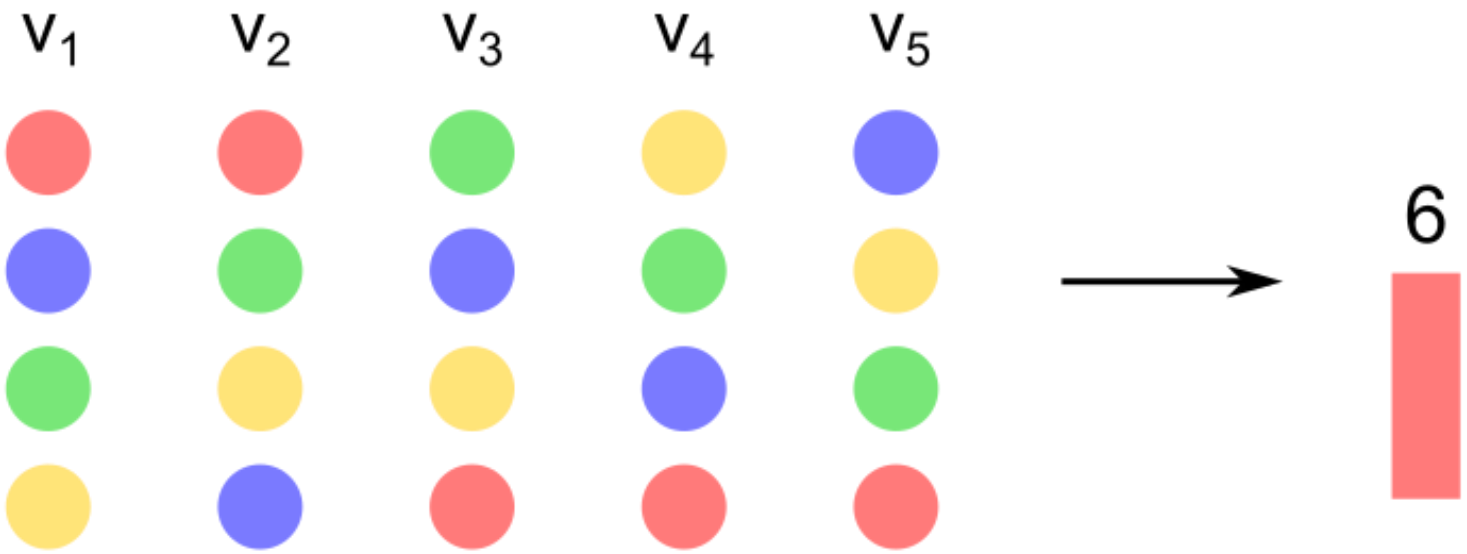

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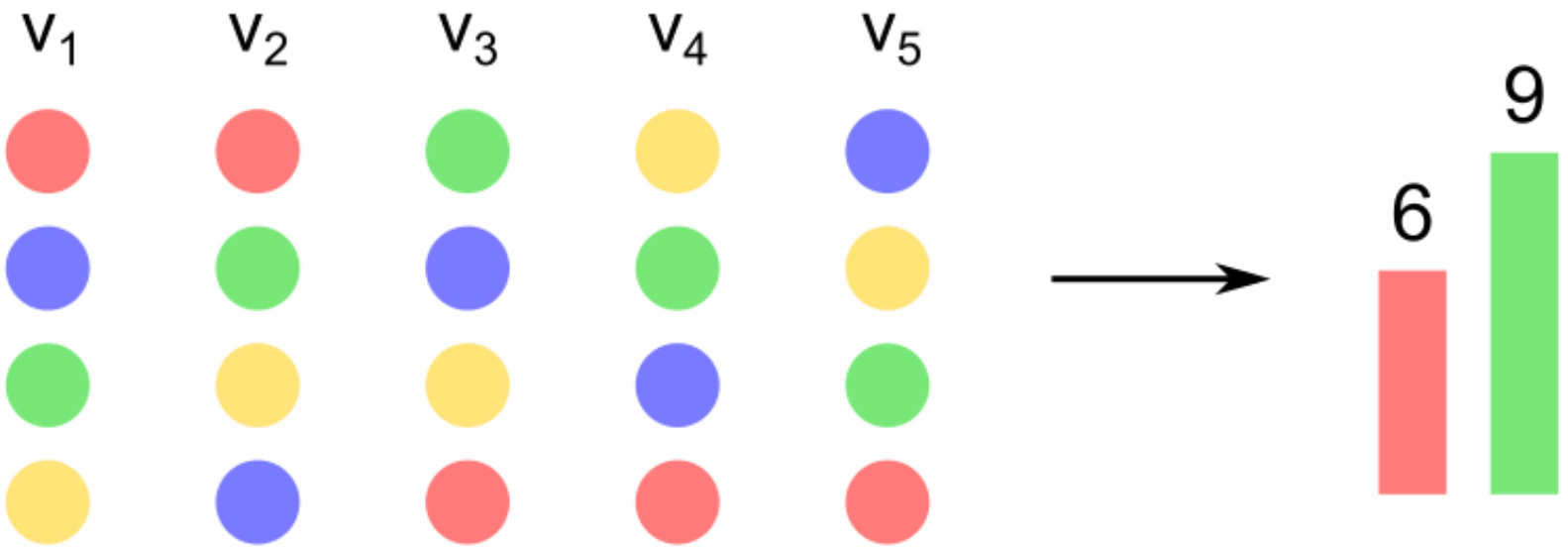

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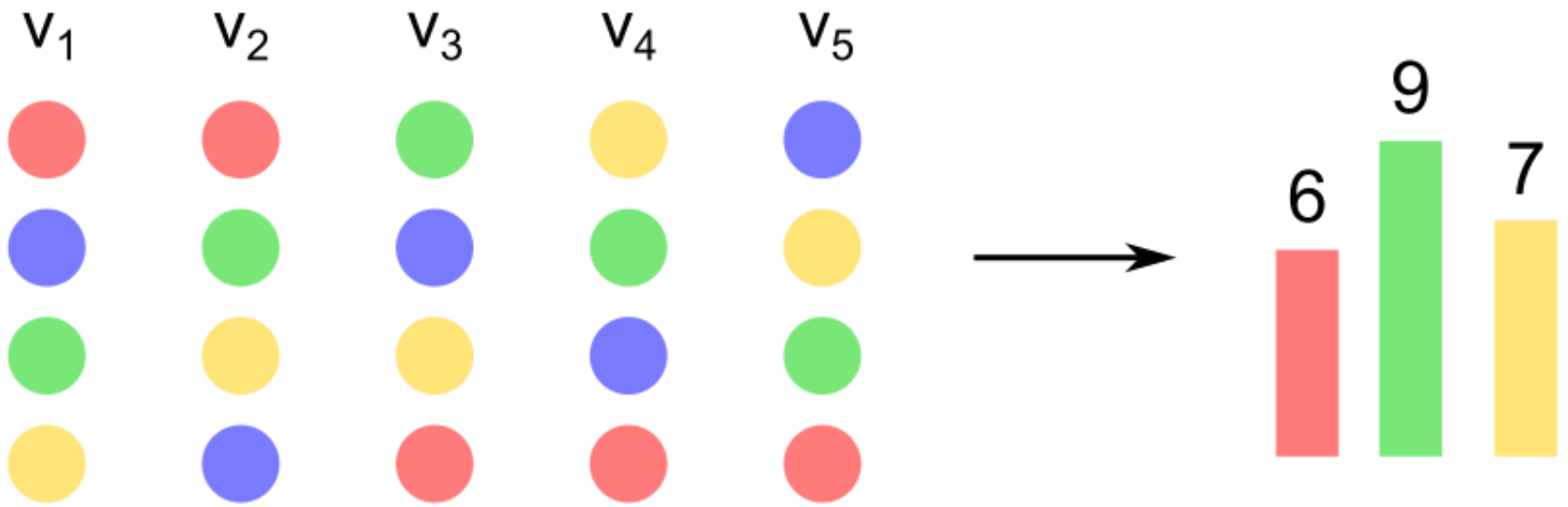

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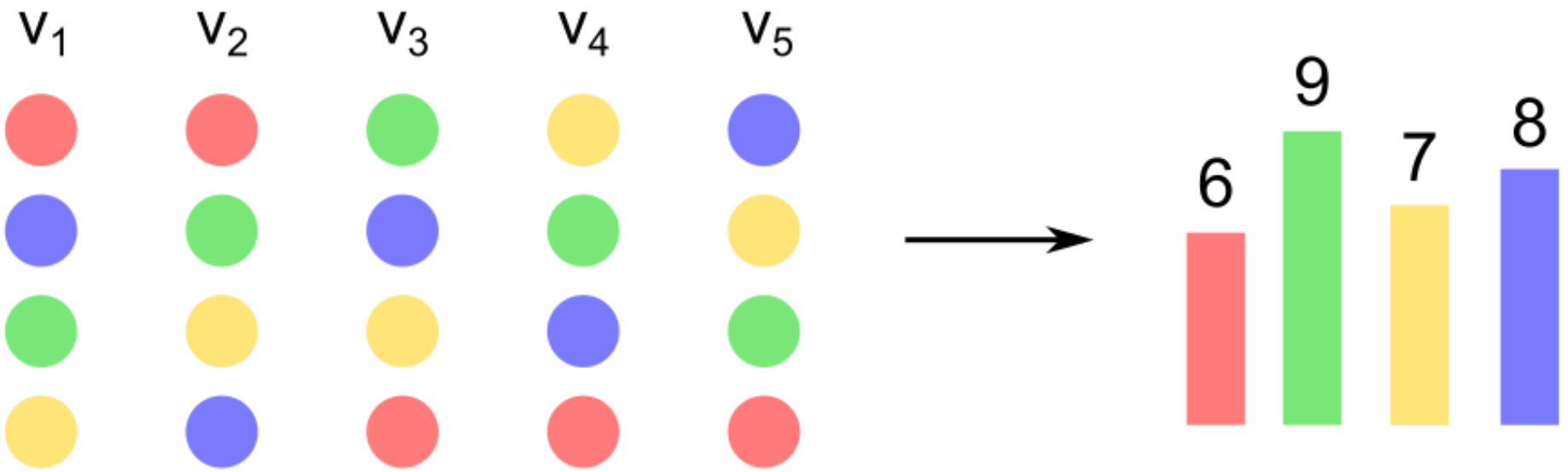

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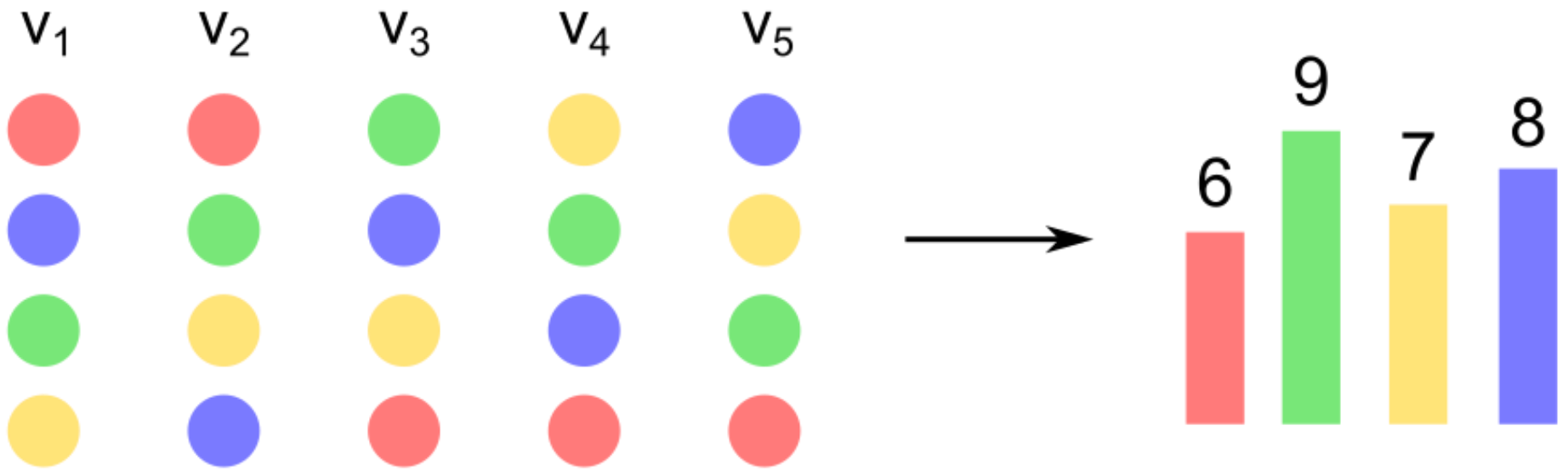


Borda winner:

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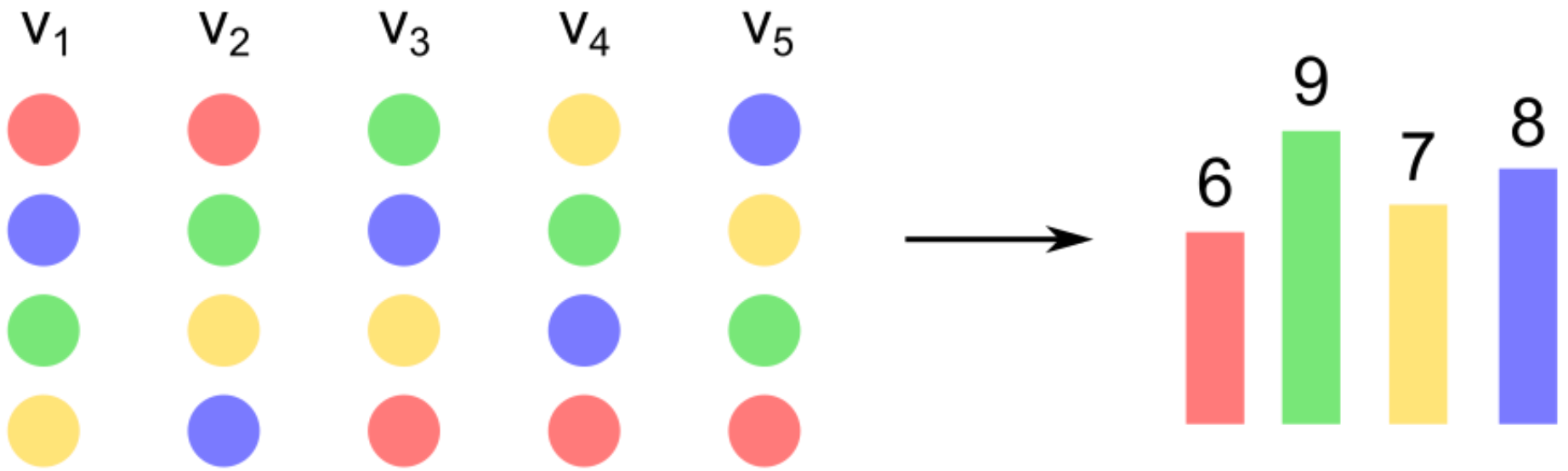


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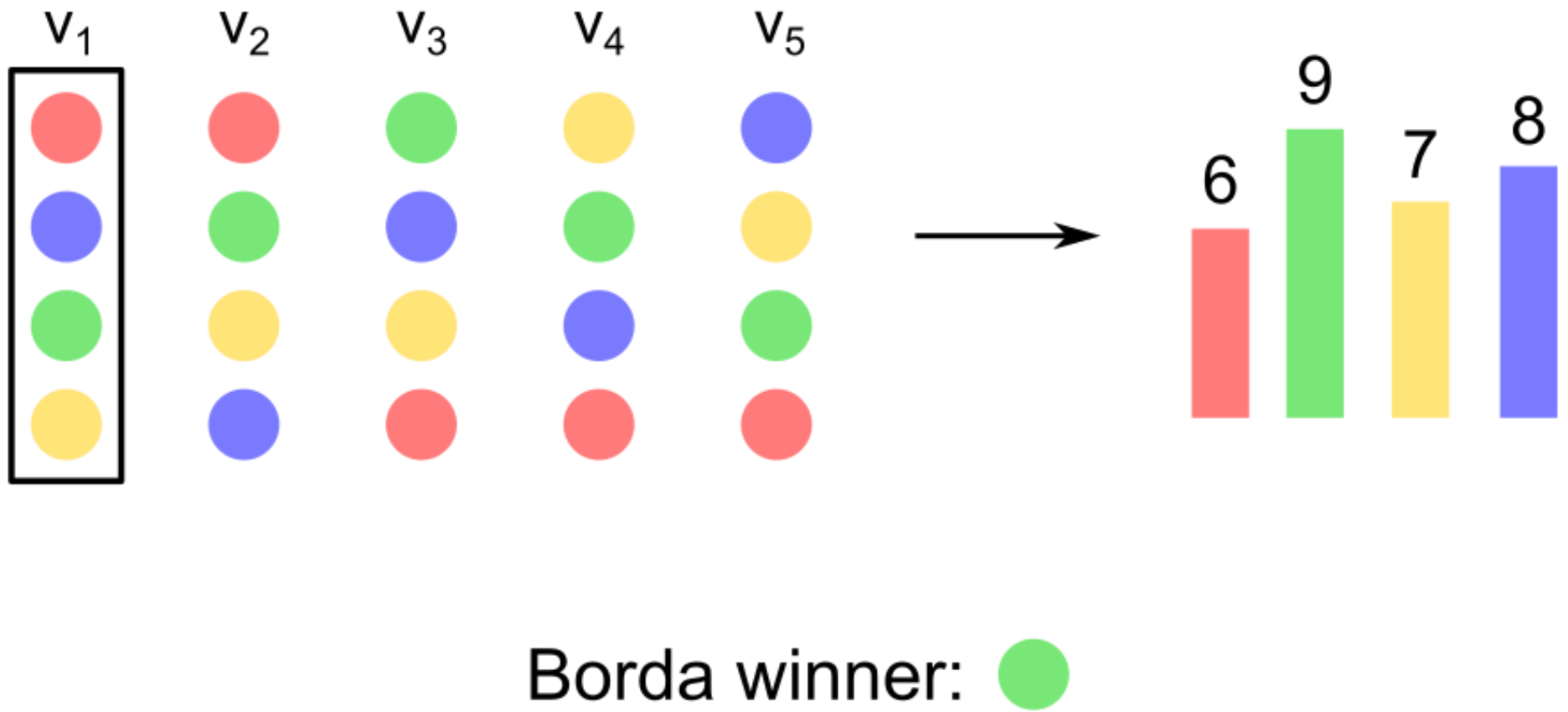


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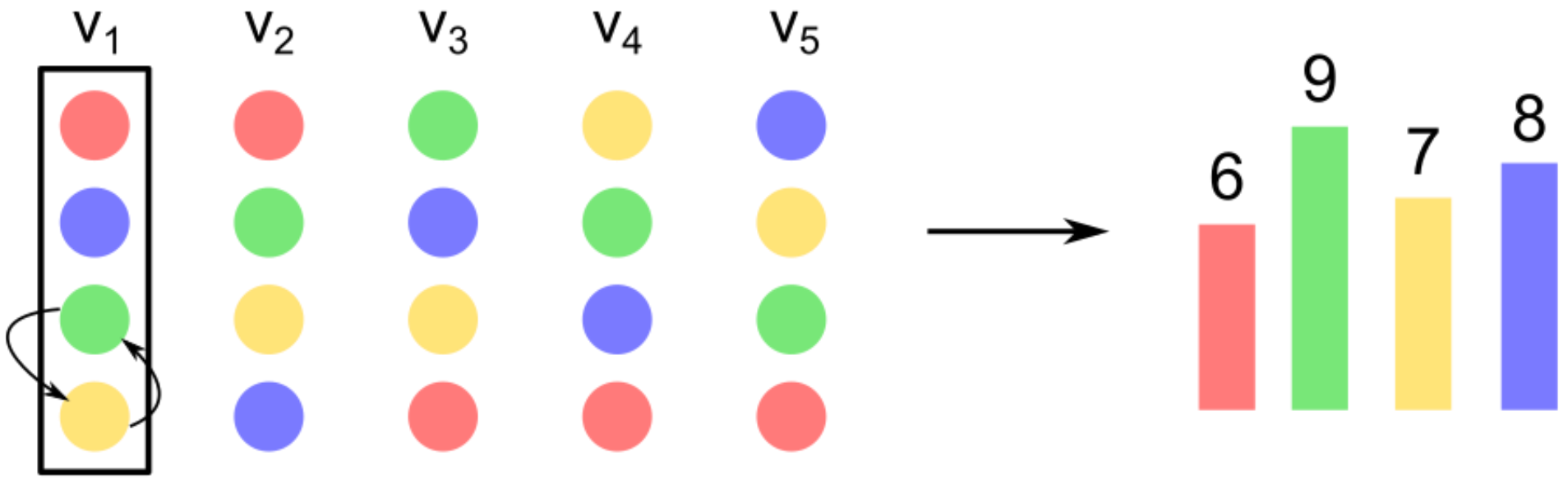


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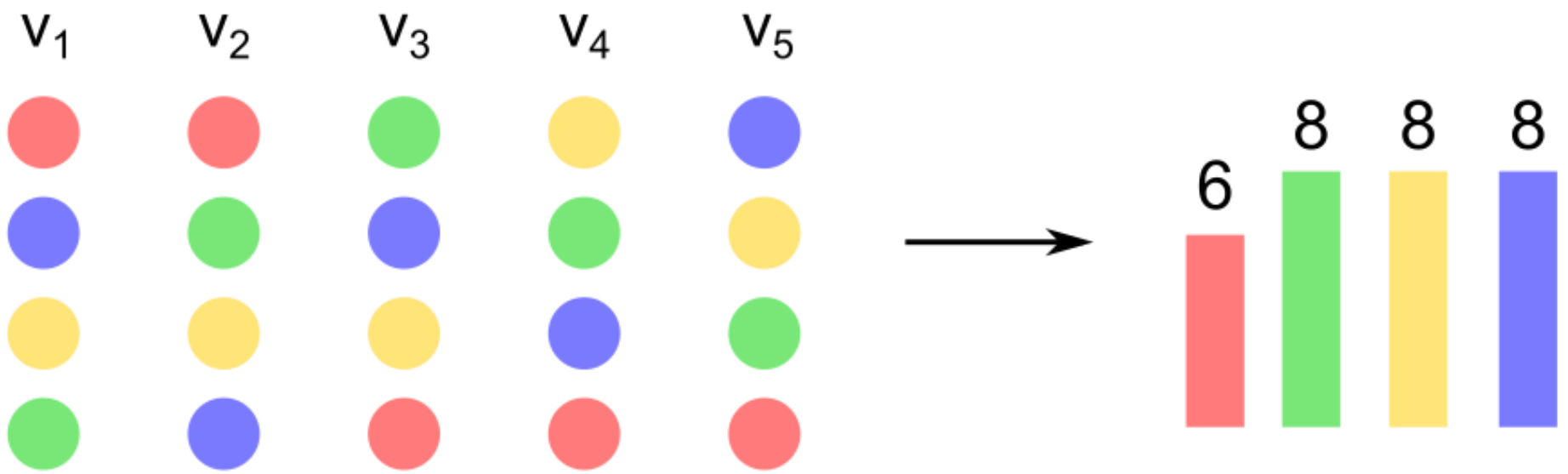


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"My scheme is intended for only honest men."

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Borda Count


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Plurality With Runoff

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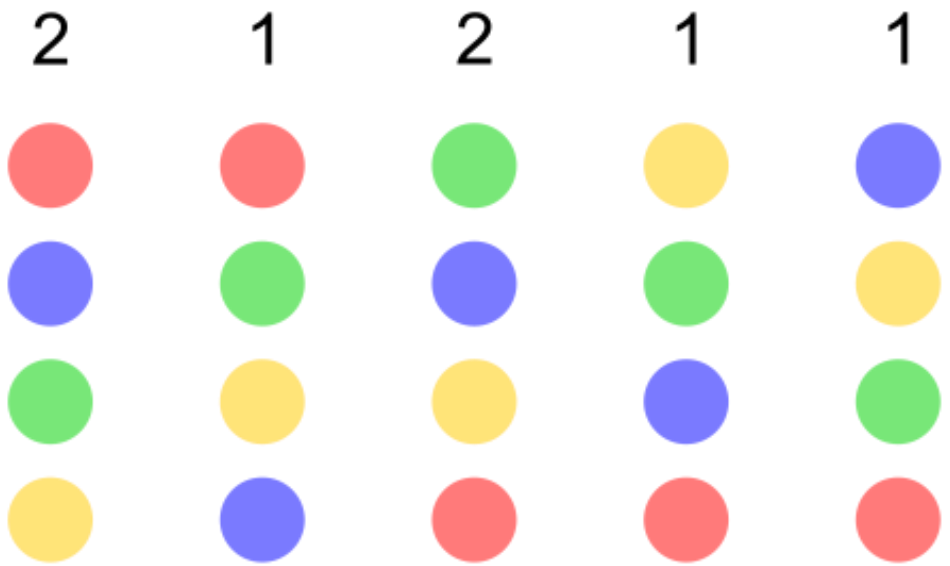

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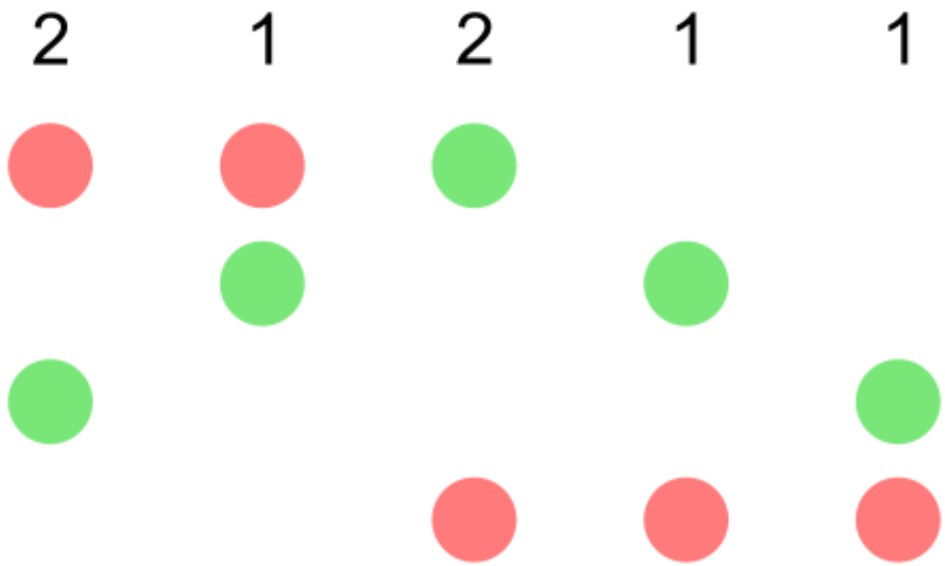

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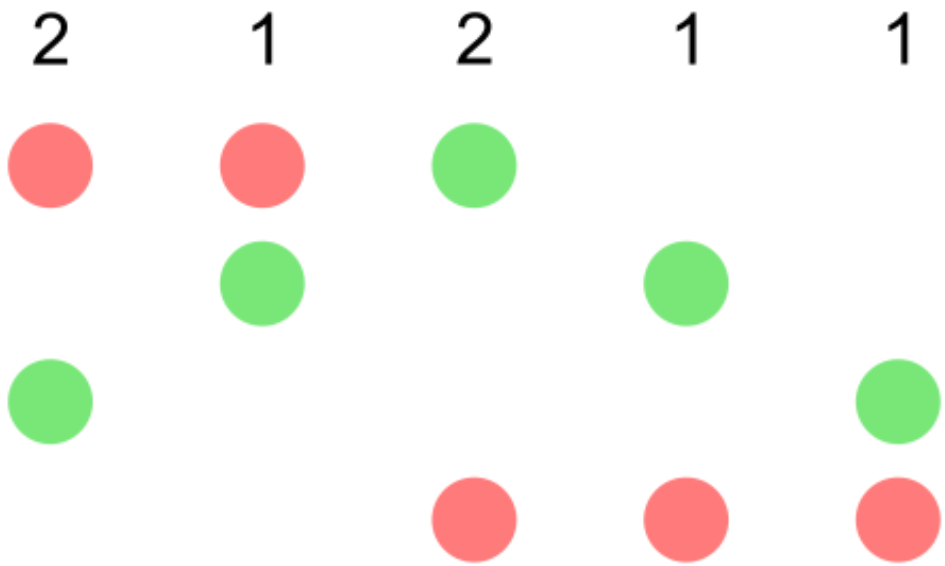


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Single Transferable Vote

(Instant-Runoff)

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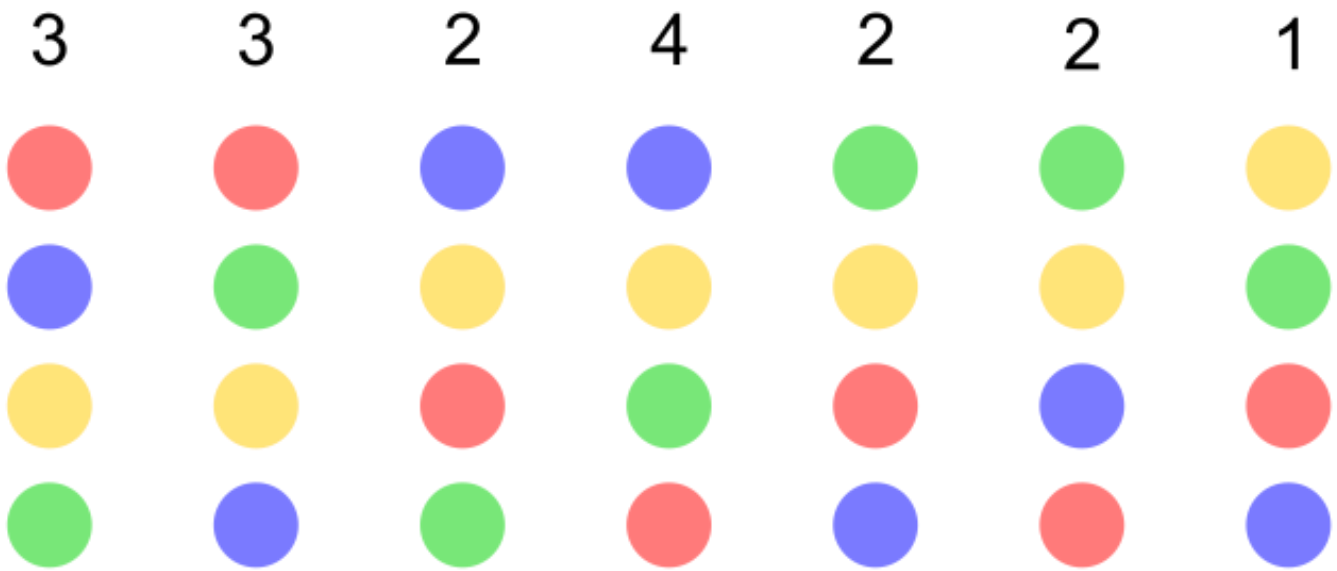

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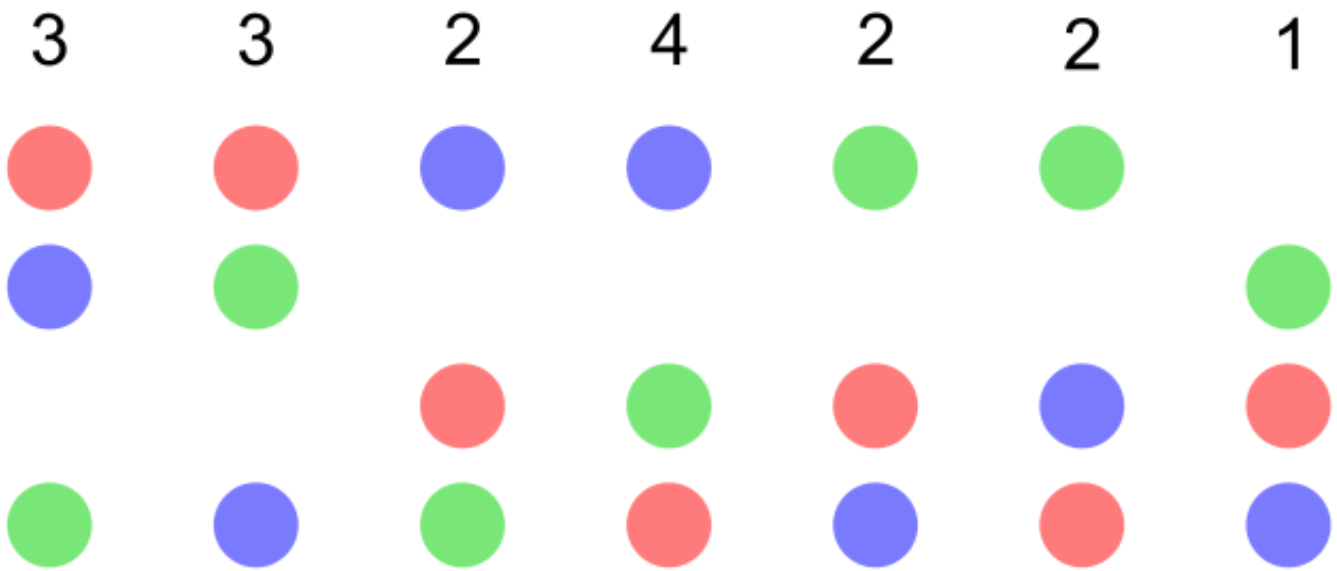

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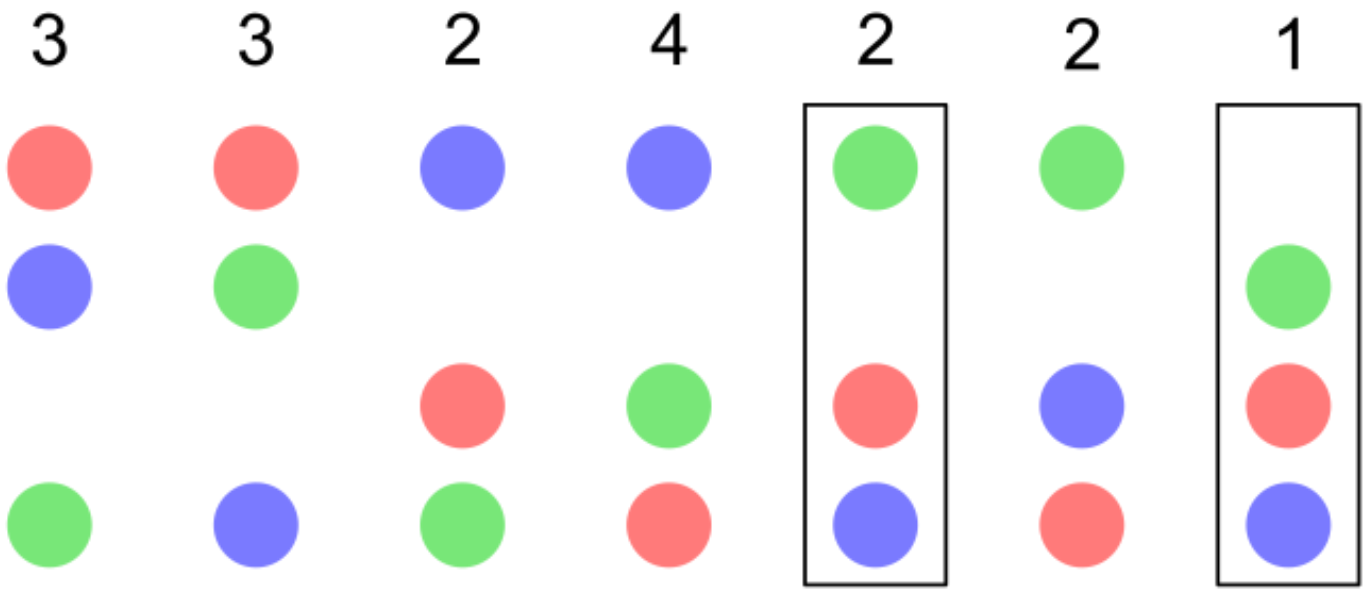

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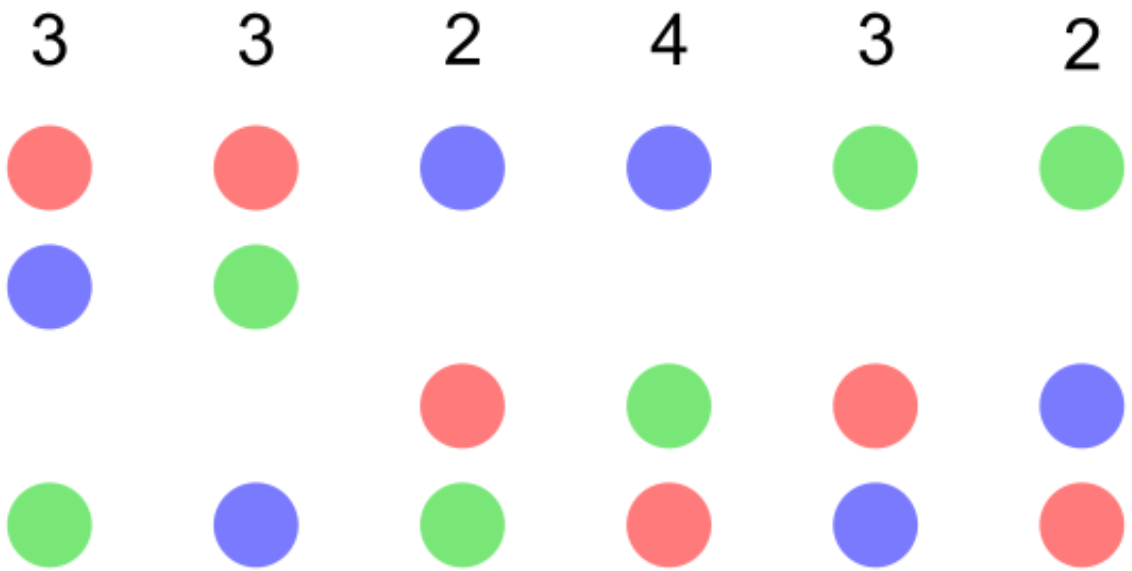

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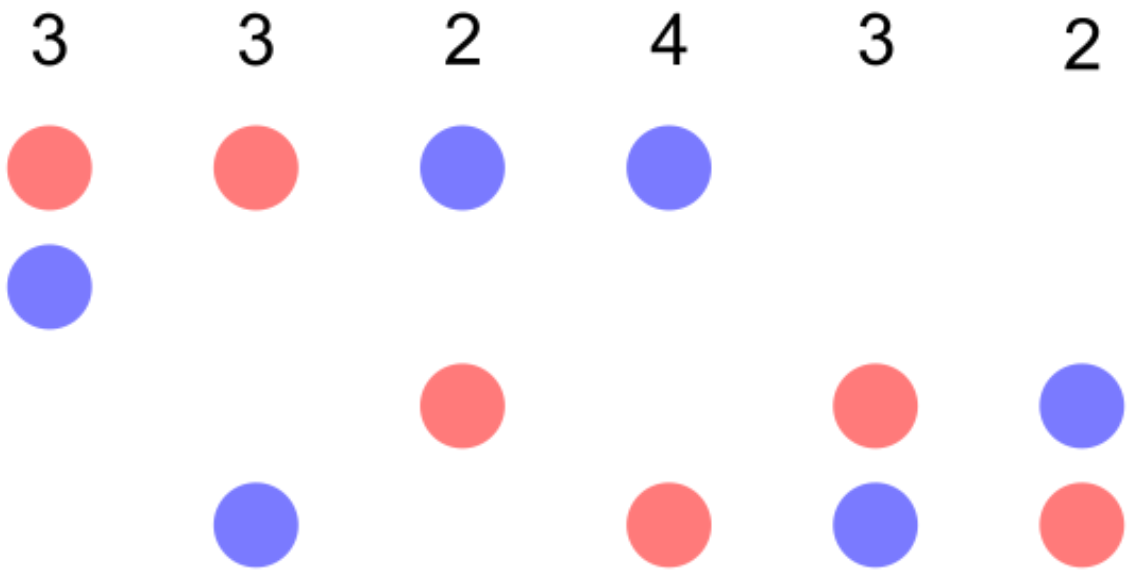

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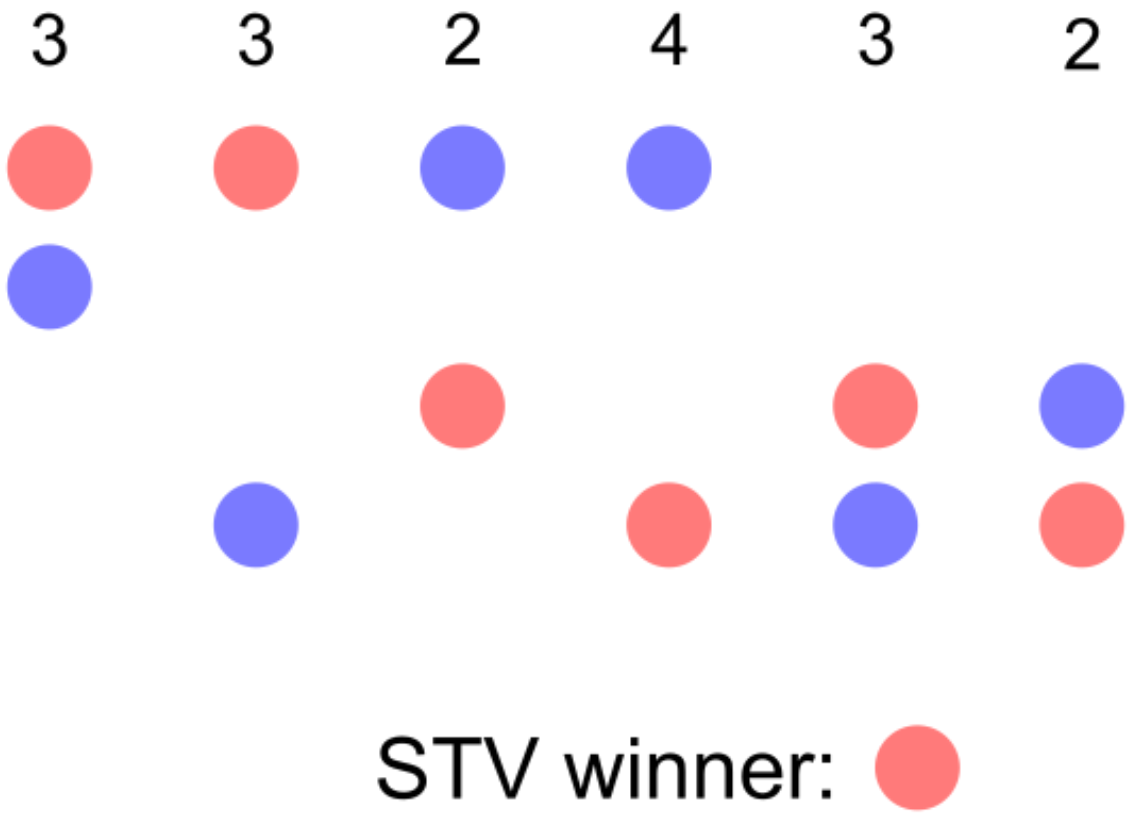

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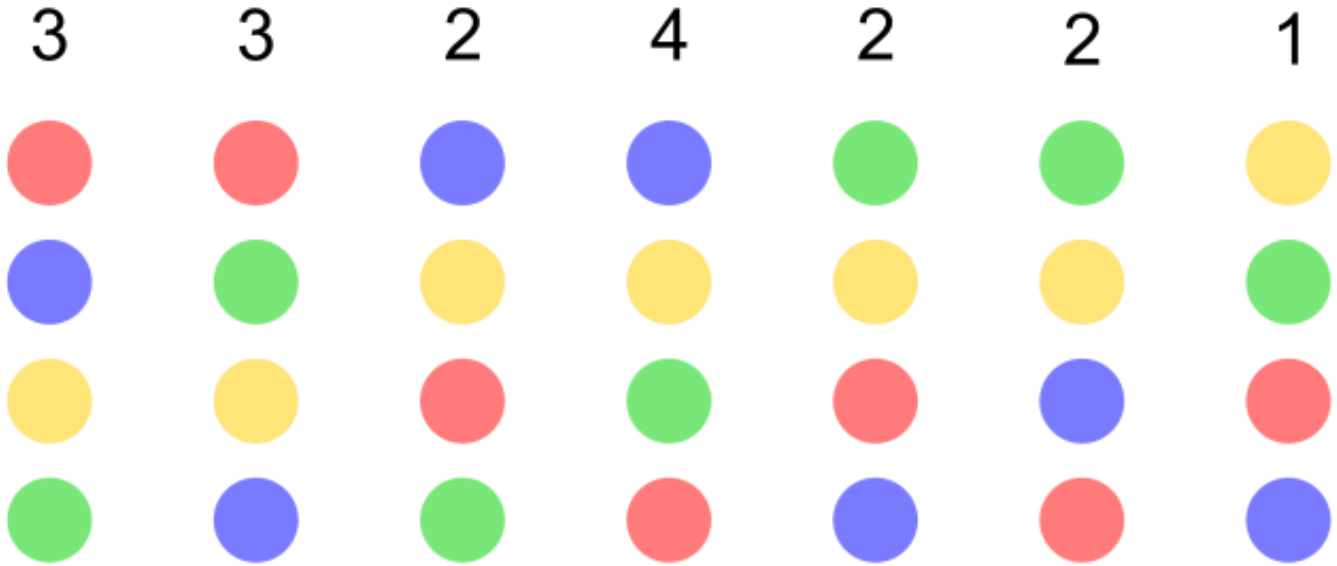


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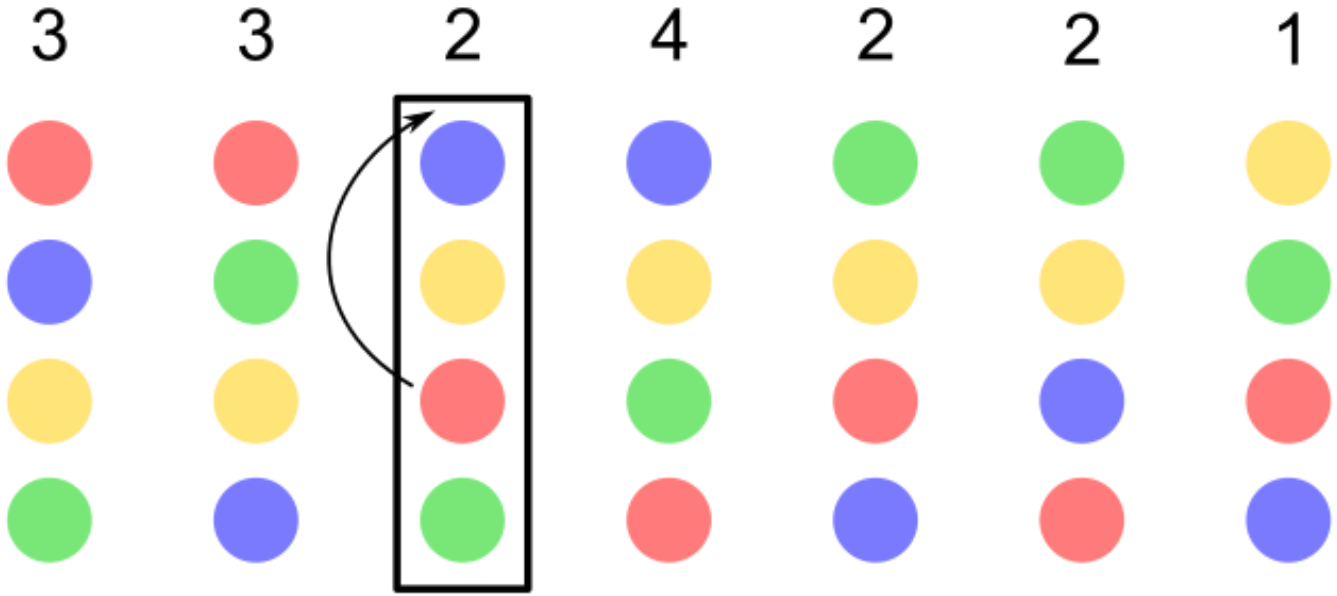


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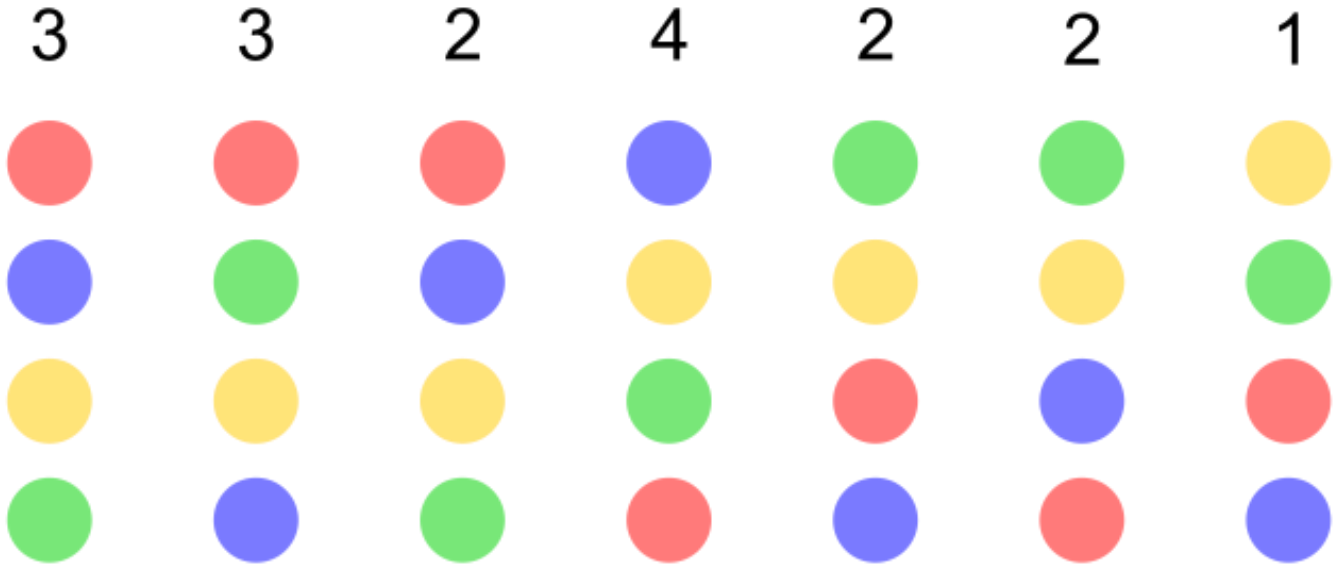


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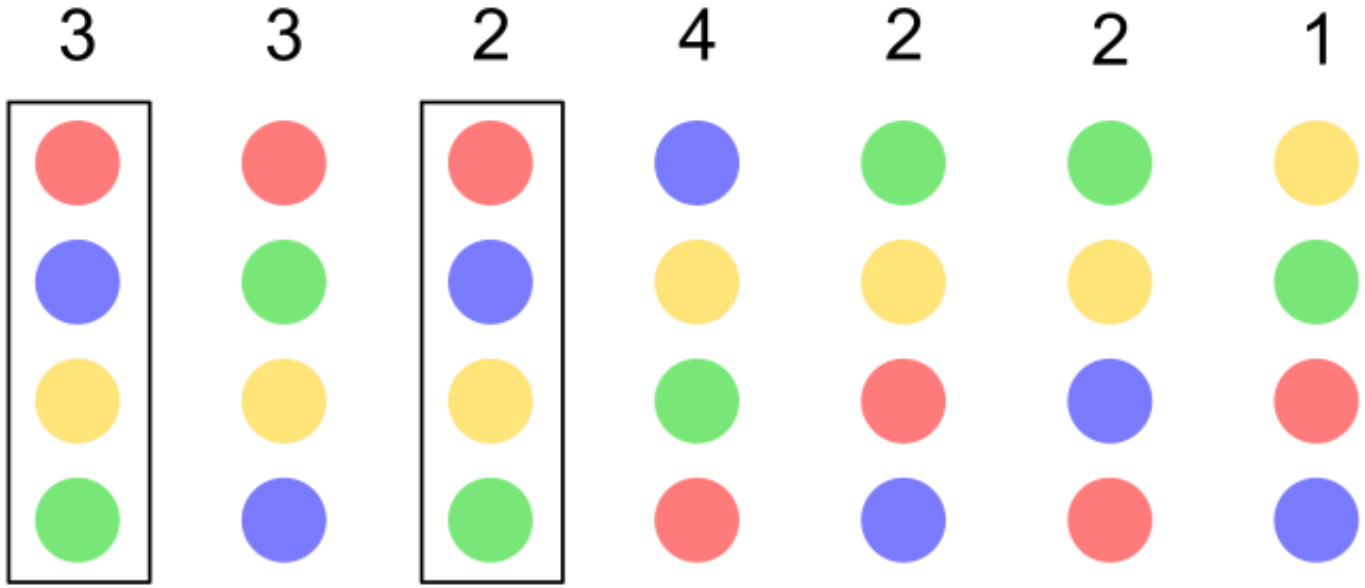


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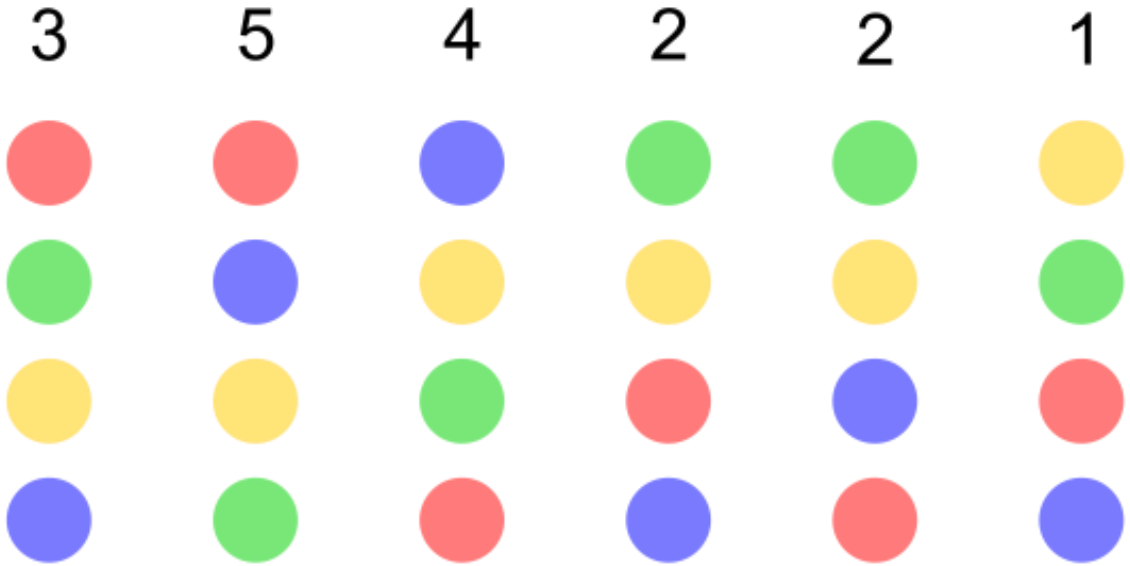


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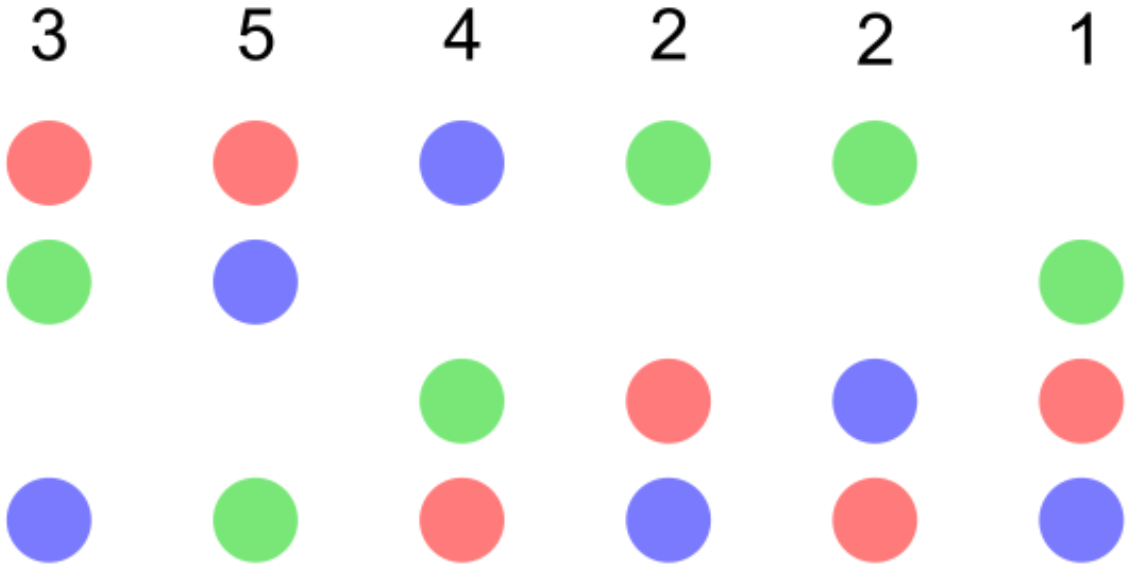


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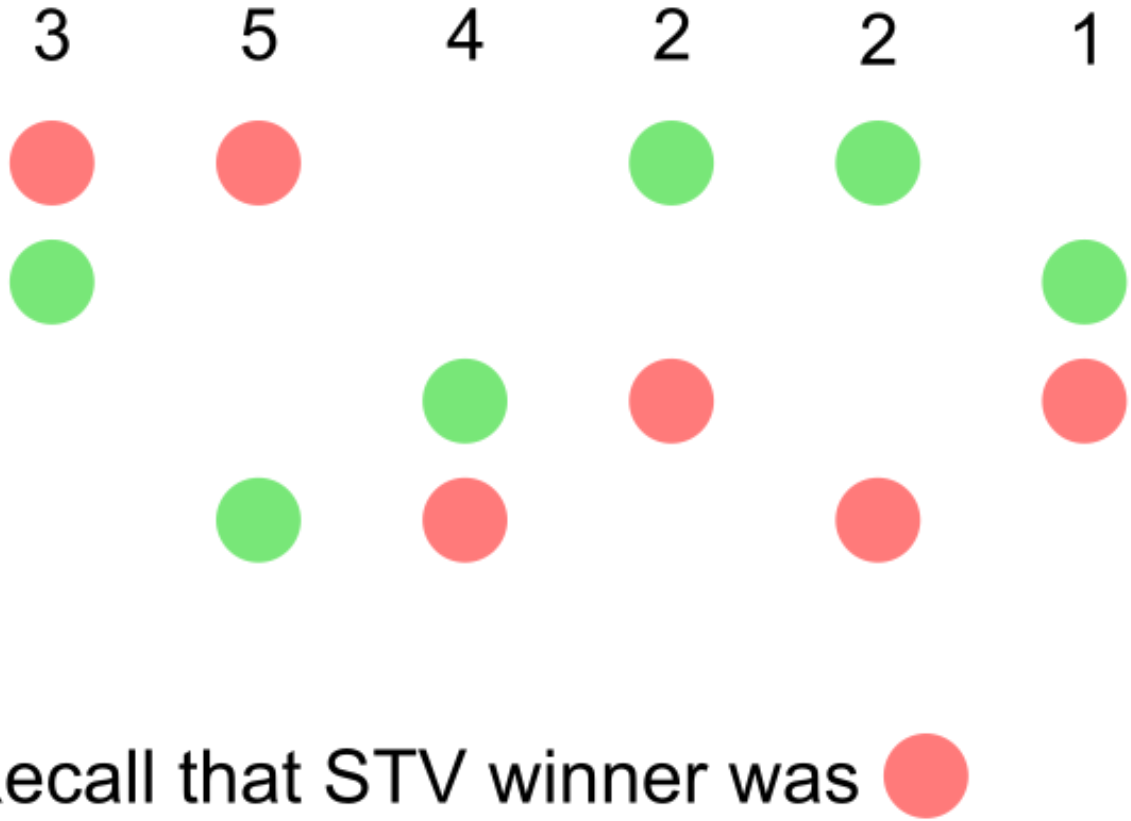


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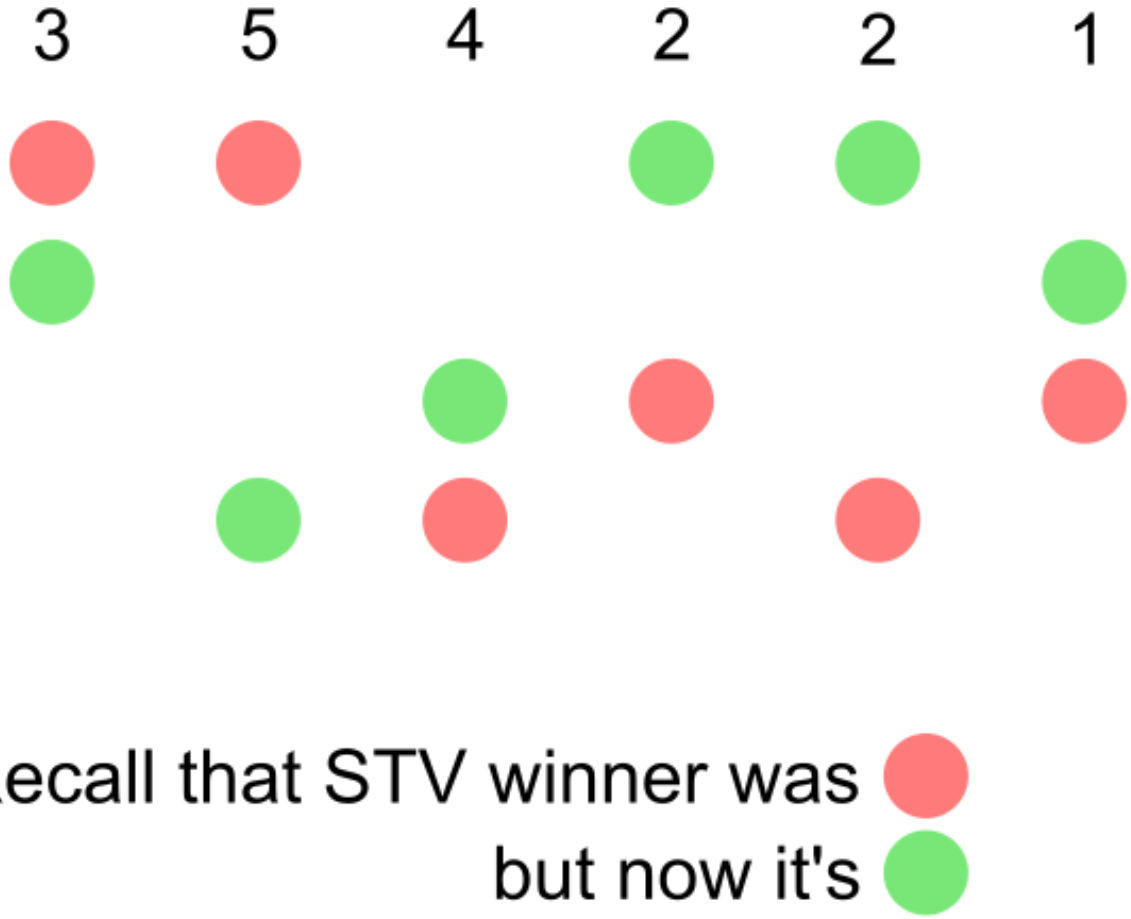

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## 2016 host city election ballots results [ edit ]

| City | NOC | Round 1 | Round 2 | Round 3 |
|---|---|---|---|---|
| Rio de Janeiro | Brazil (COB) | 26 | **46** | **66** |
| Madrid | Spain (COE) | **28** | 29 | 32 |
| Tokyo | Japan (JOC) | 22 | 20 | — |
| Chicago | United States (USOC) | 18 | — | — |

| Venue |
|---|
| Bella Center<br>**121st IOC Session**<br>October 2, 2009<br>Copenhagen |

| Vote details | Round 1 | Round 2 | Round 3 |
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| Eligible members | 95 | 97 | 99 |
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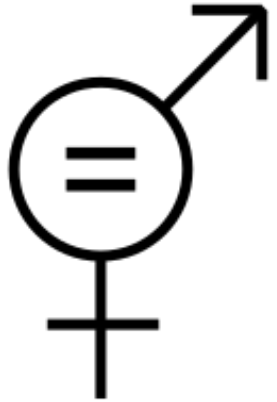

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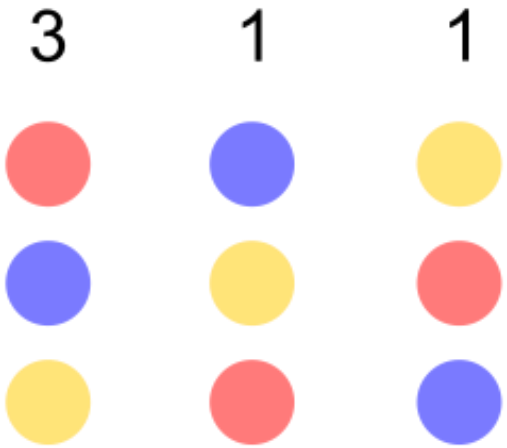

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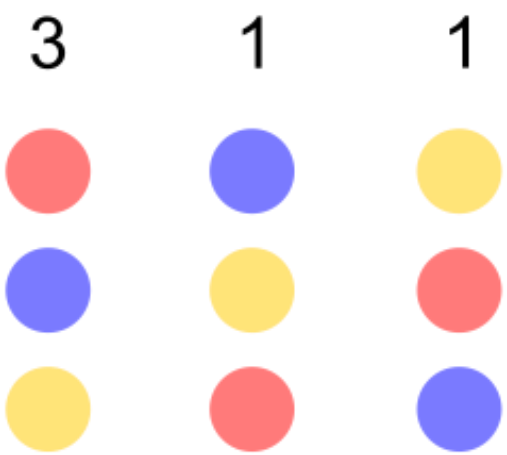


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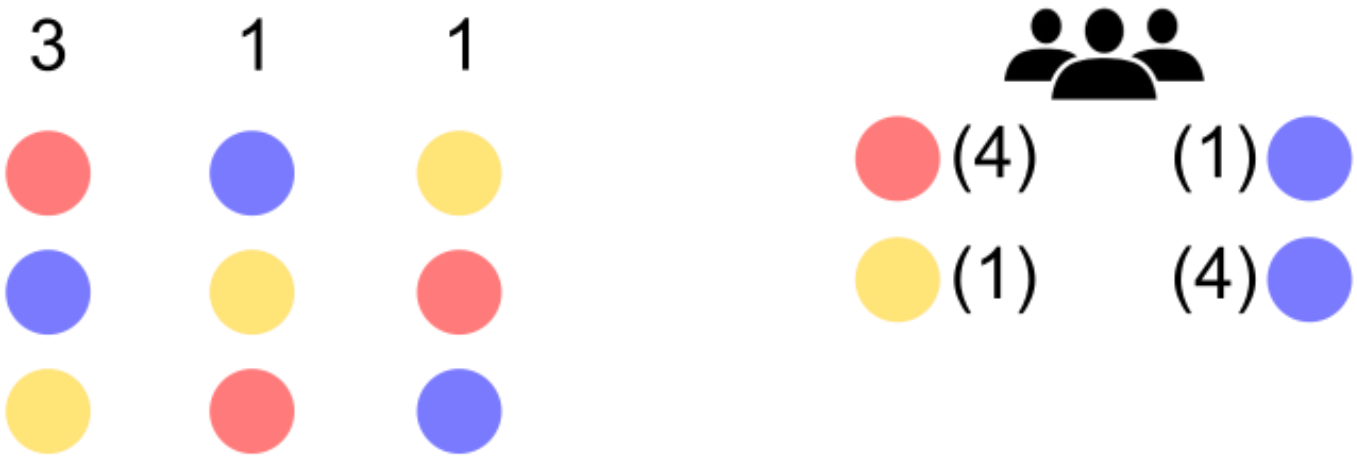

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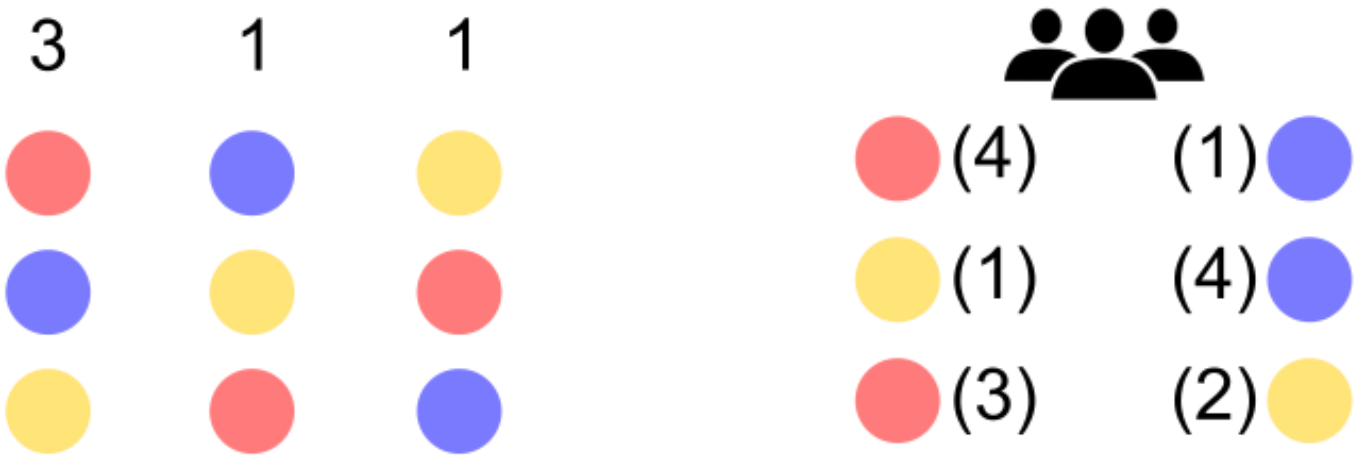

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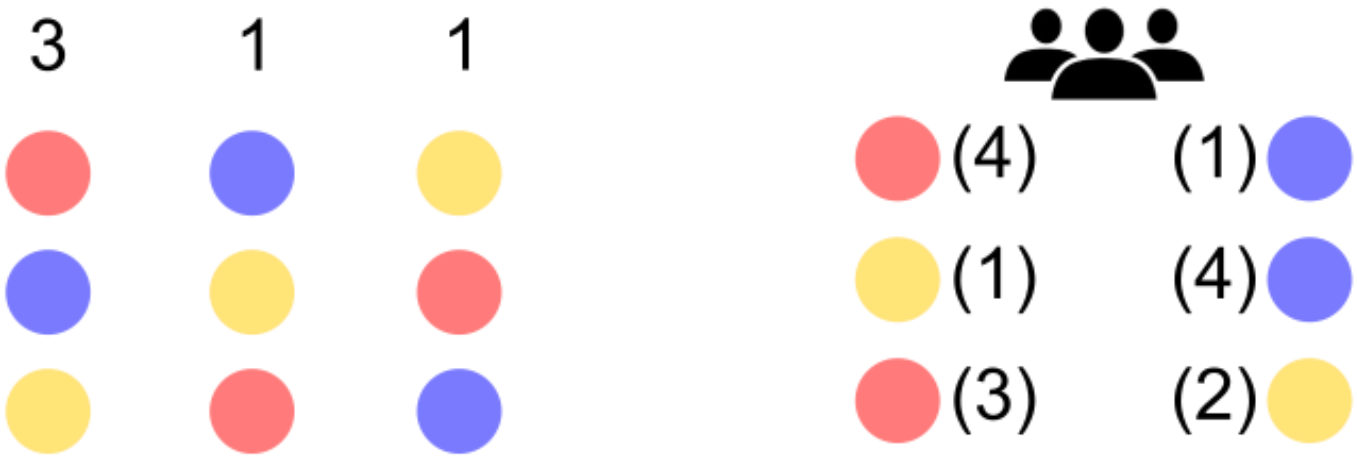
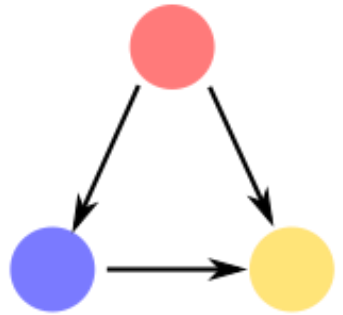



Condorcet winner



# Condorcet Criterion

Problem: A Condorcet winner may not exist

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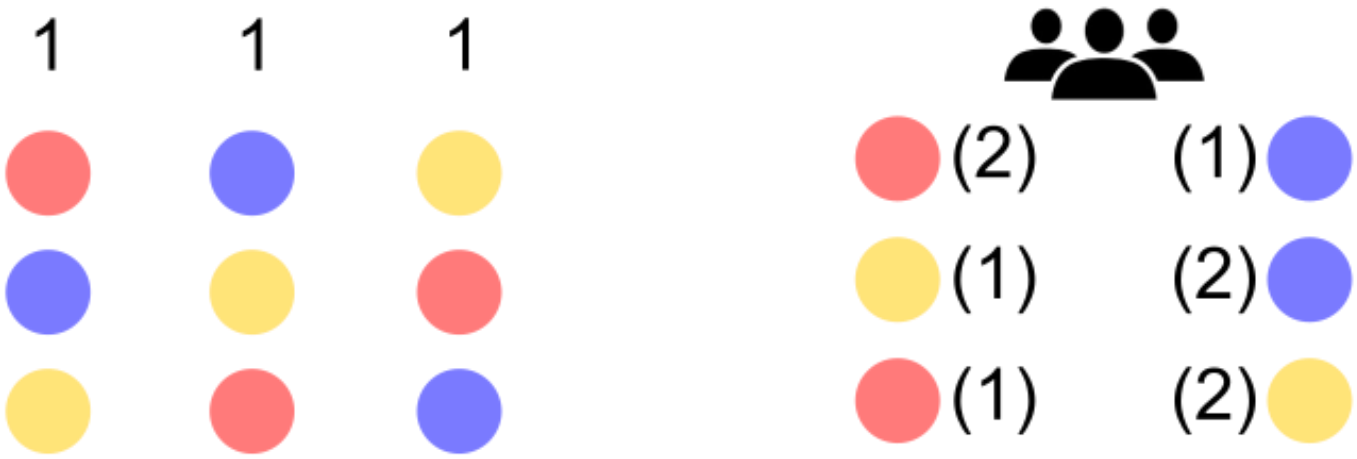
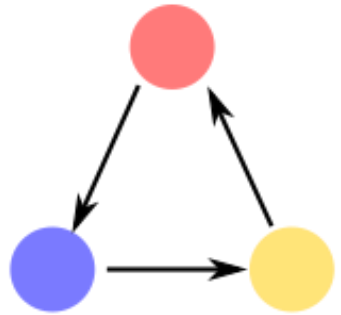
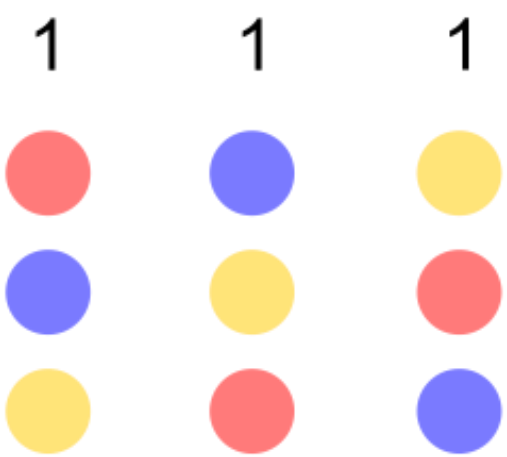
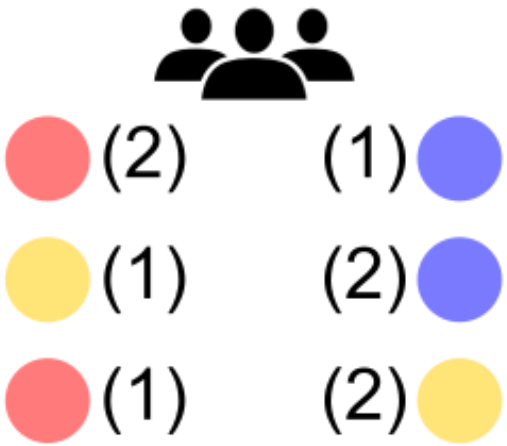



Condorcet cycle

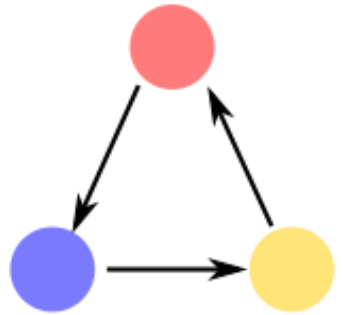


# Problem: A Condorcet winner may not exist




## Condorcet cycle



## Condorcet paradox

Transitivity of individual preferences $\not\Rightarrow$ Transitivity of societal preferences

5

Copeland

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For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie

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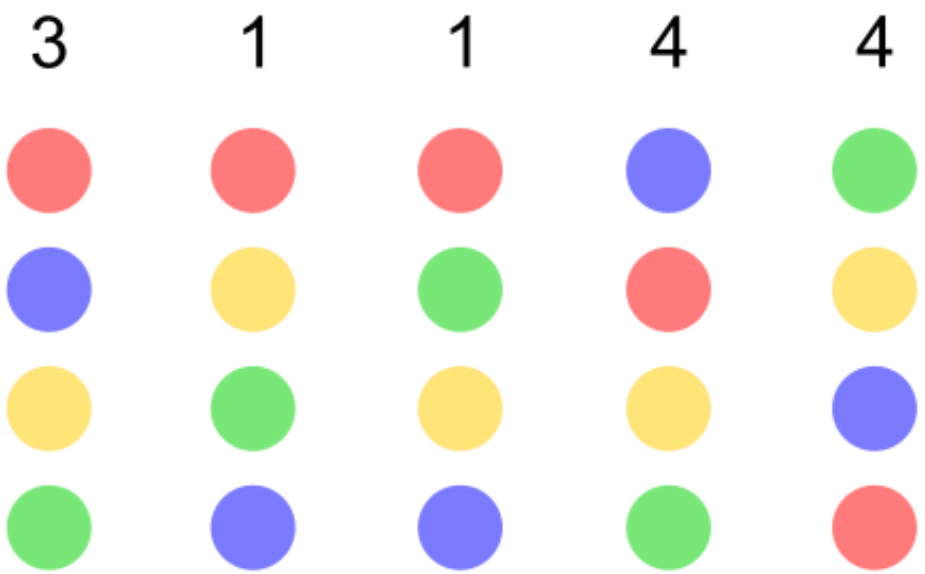

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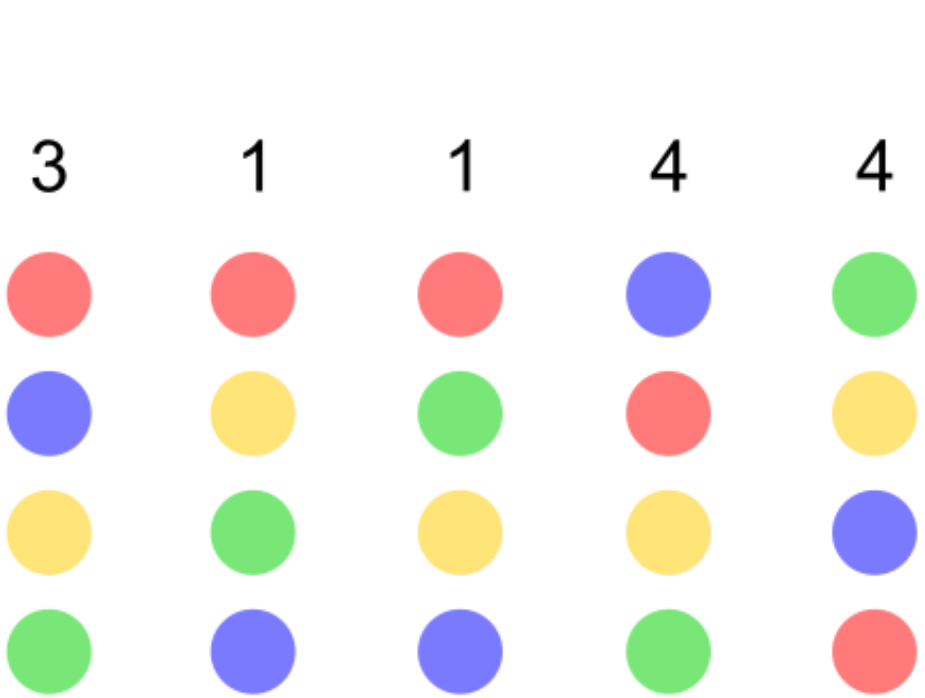


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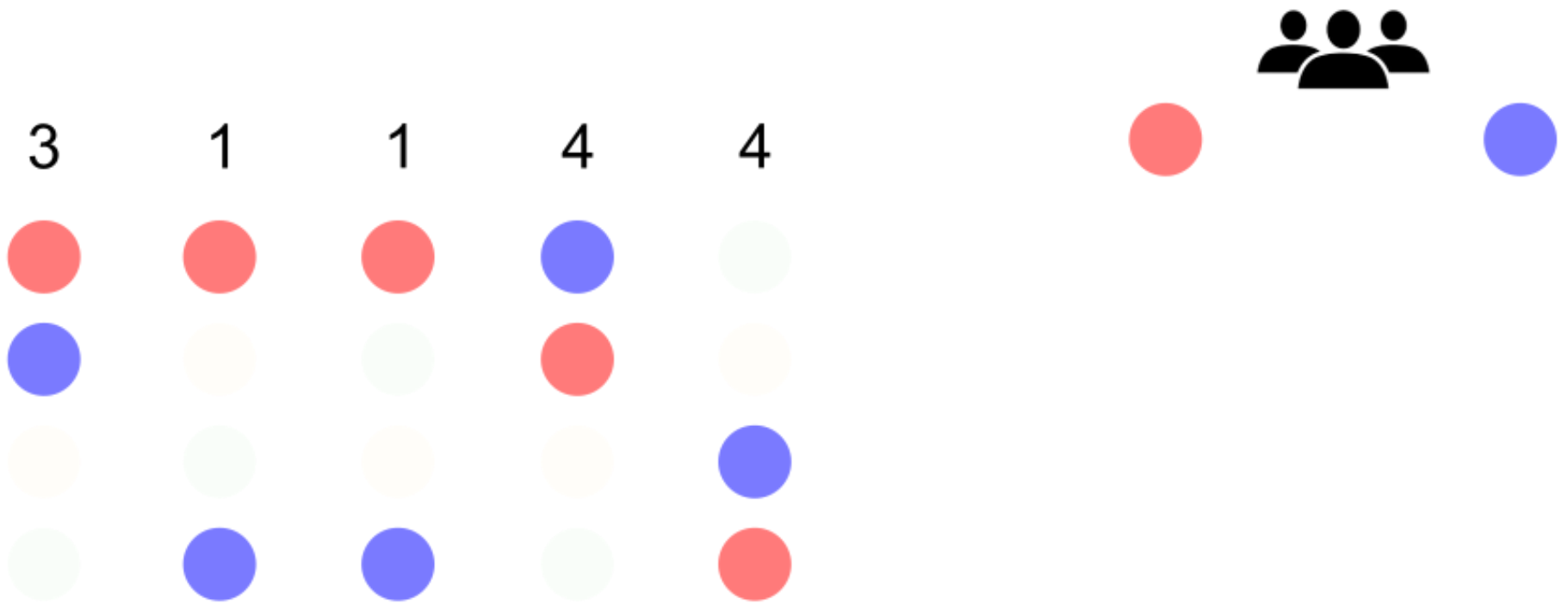

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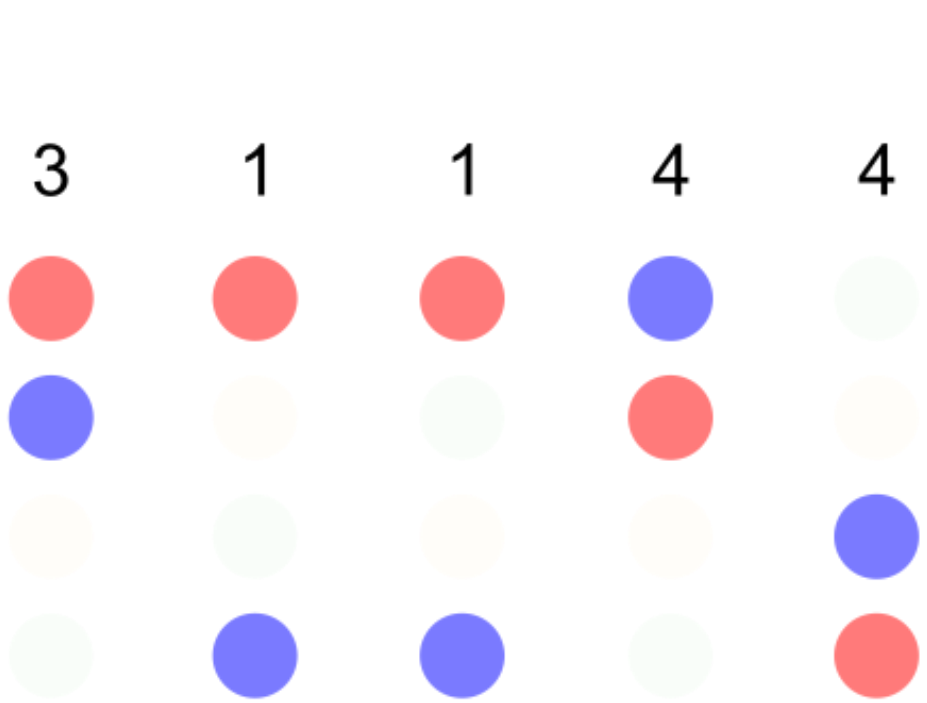


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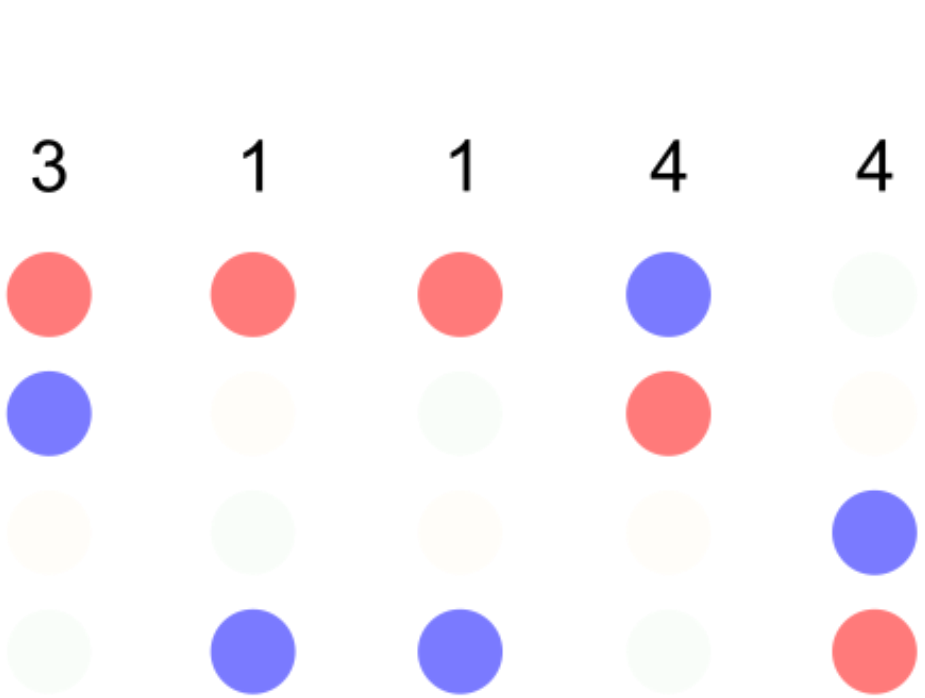


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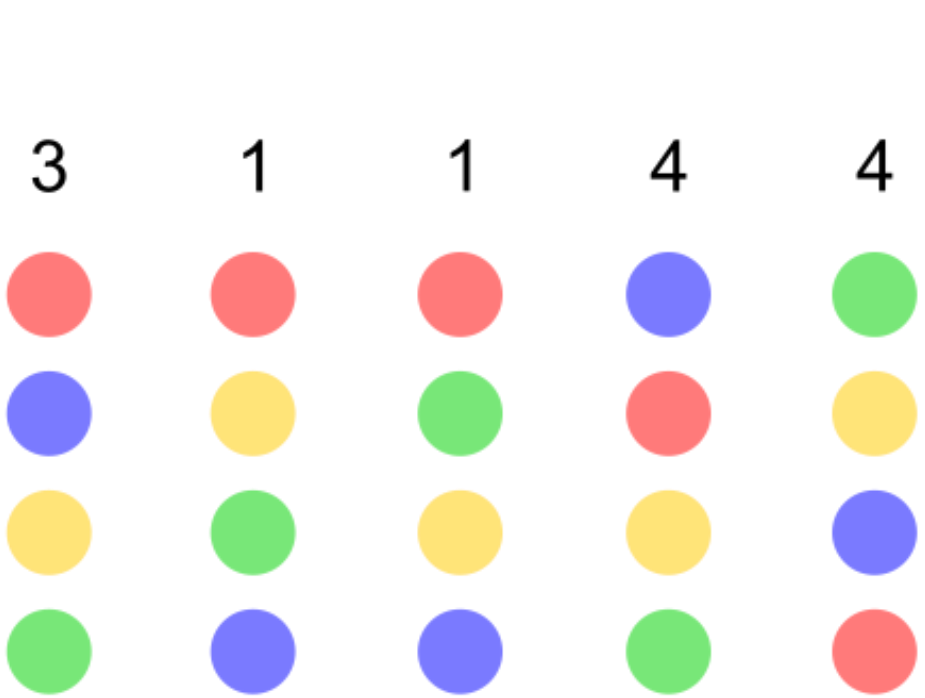


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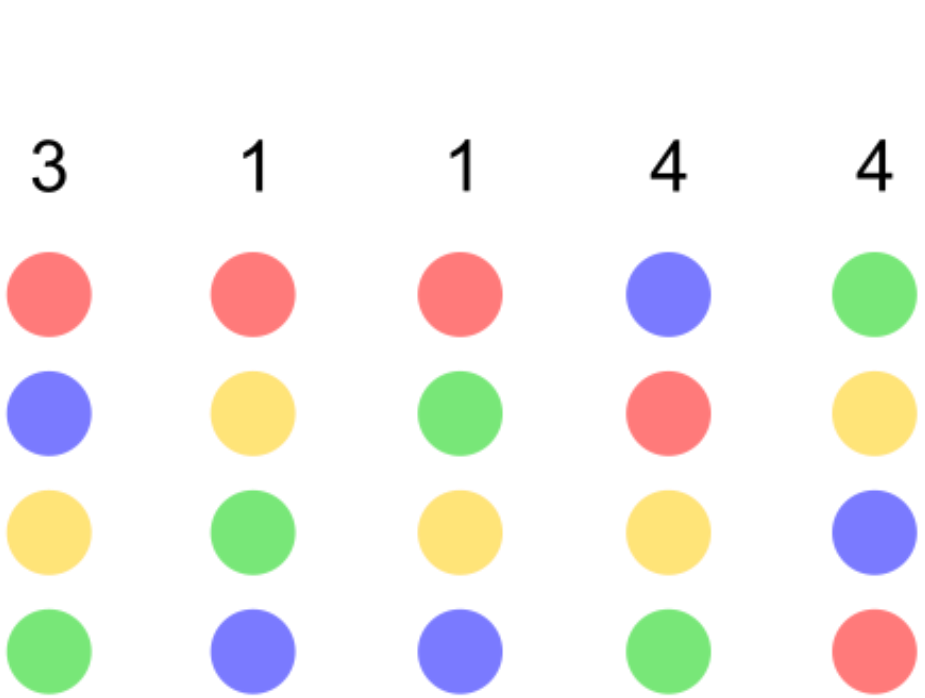


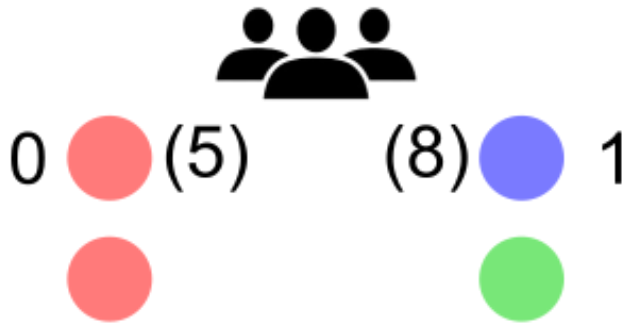

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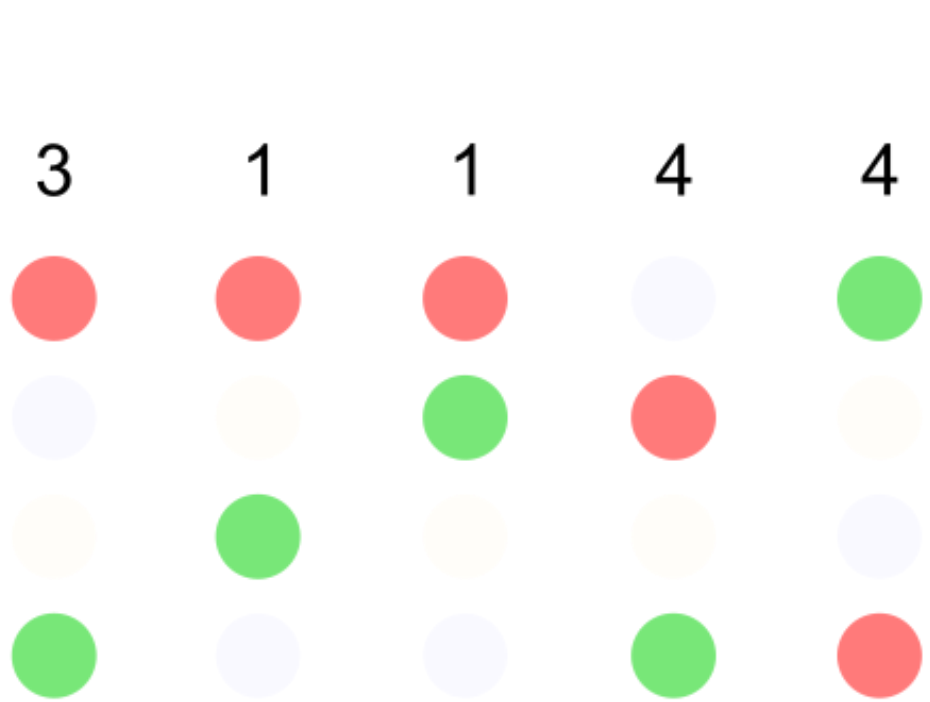


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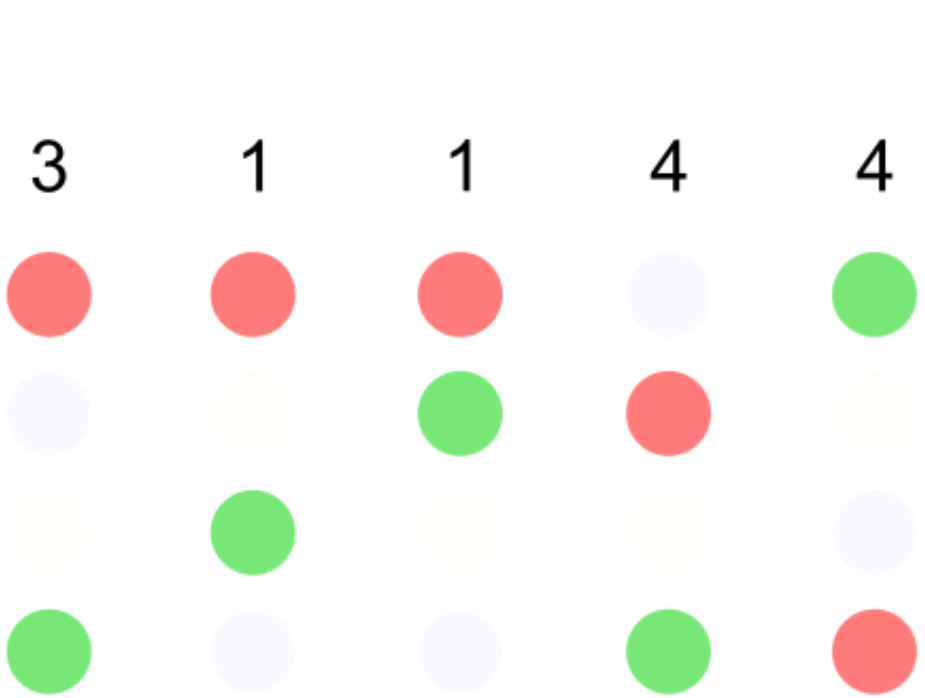


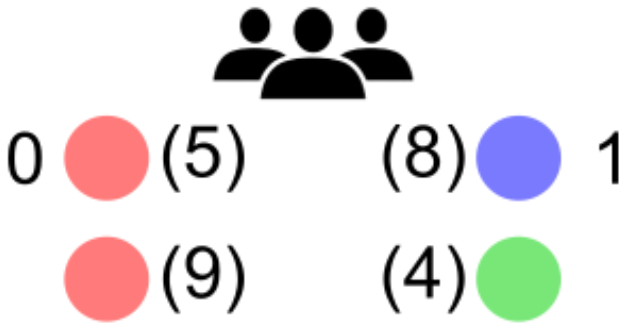

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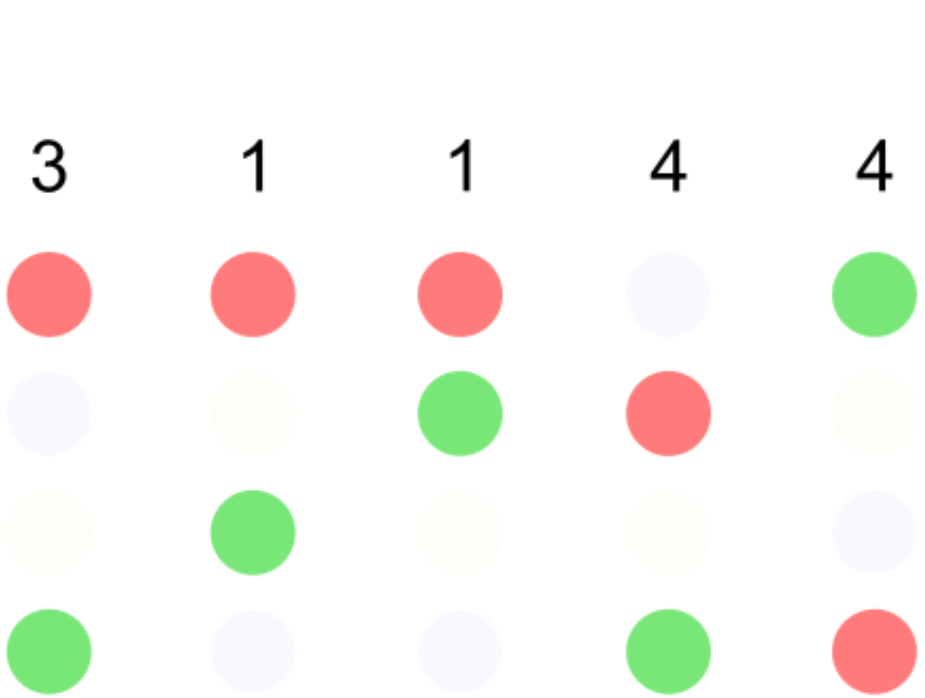


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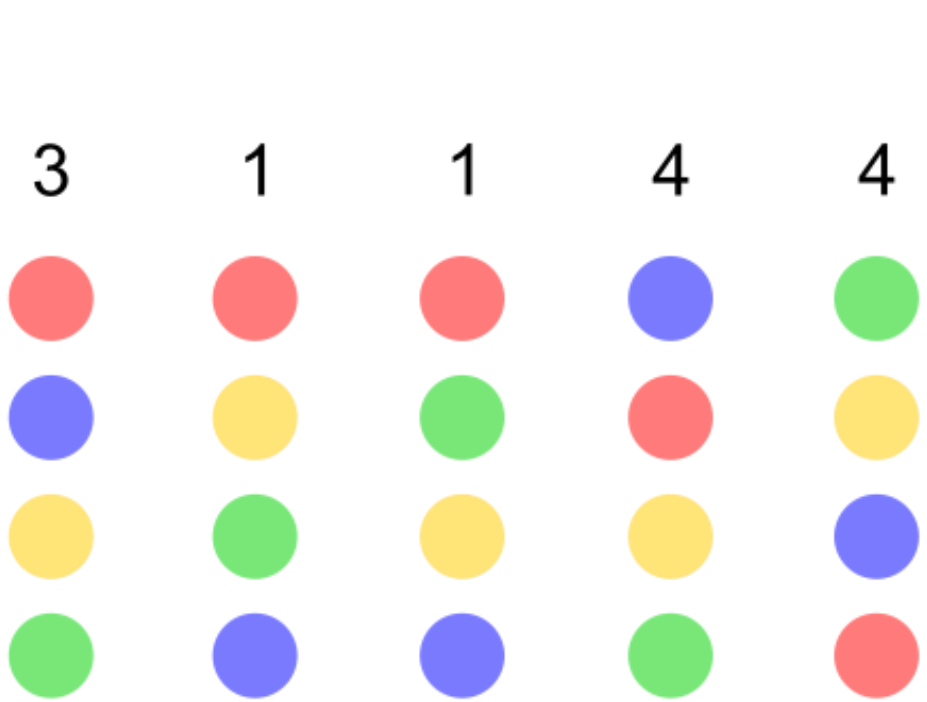


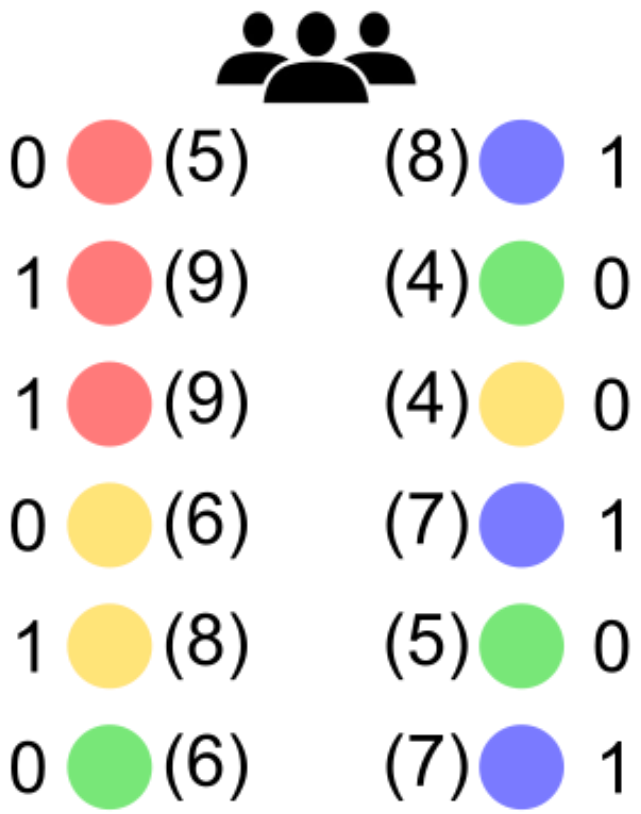

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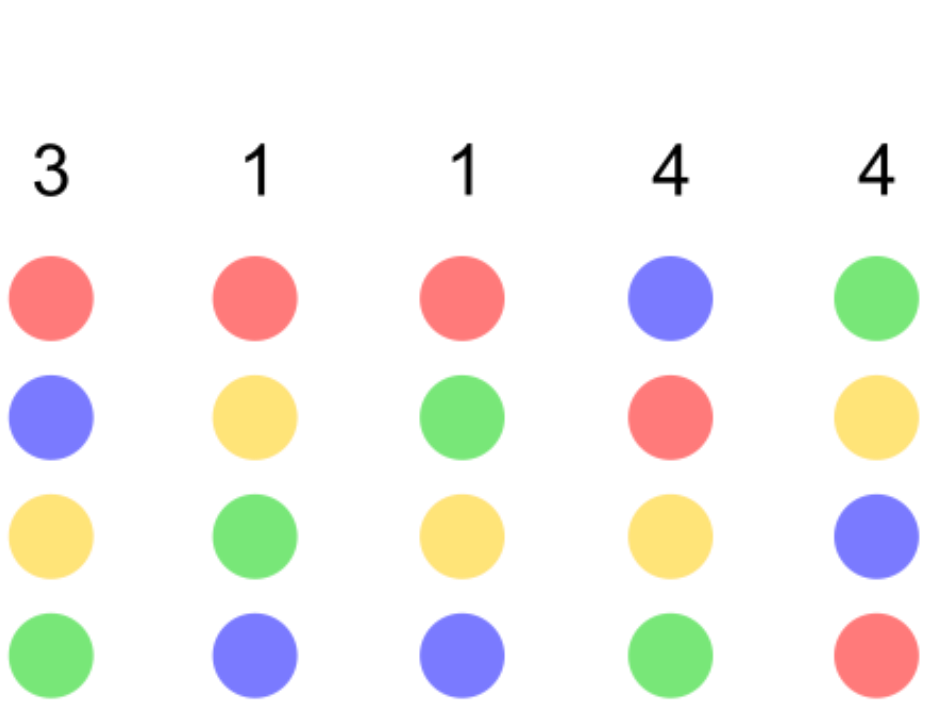


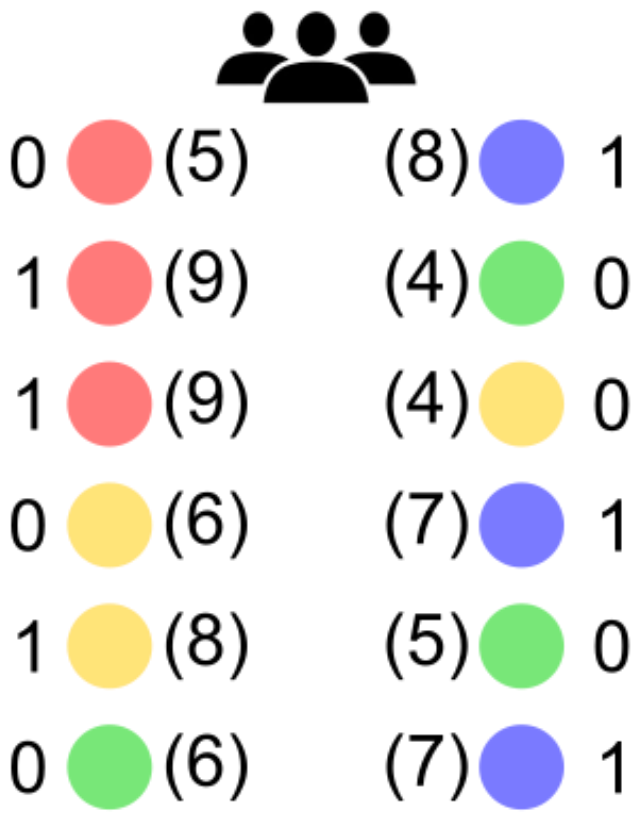


Copeland winner:

5

Copeland

LaLiga

BUNDESLIGA

ICC WOMEN'S CRICKET WORLD CUP NEW ZEALAND 2022

Premier League

INDIAN PREMIER LEAGUE ®

# 5

Problem: Voters are sometimes better off not voting

Copeland

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Copeland


Copeland winner:

# 5

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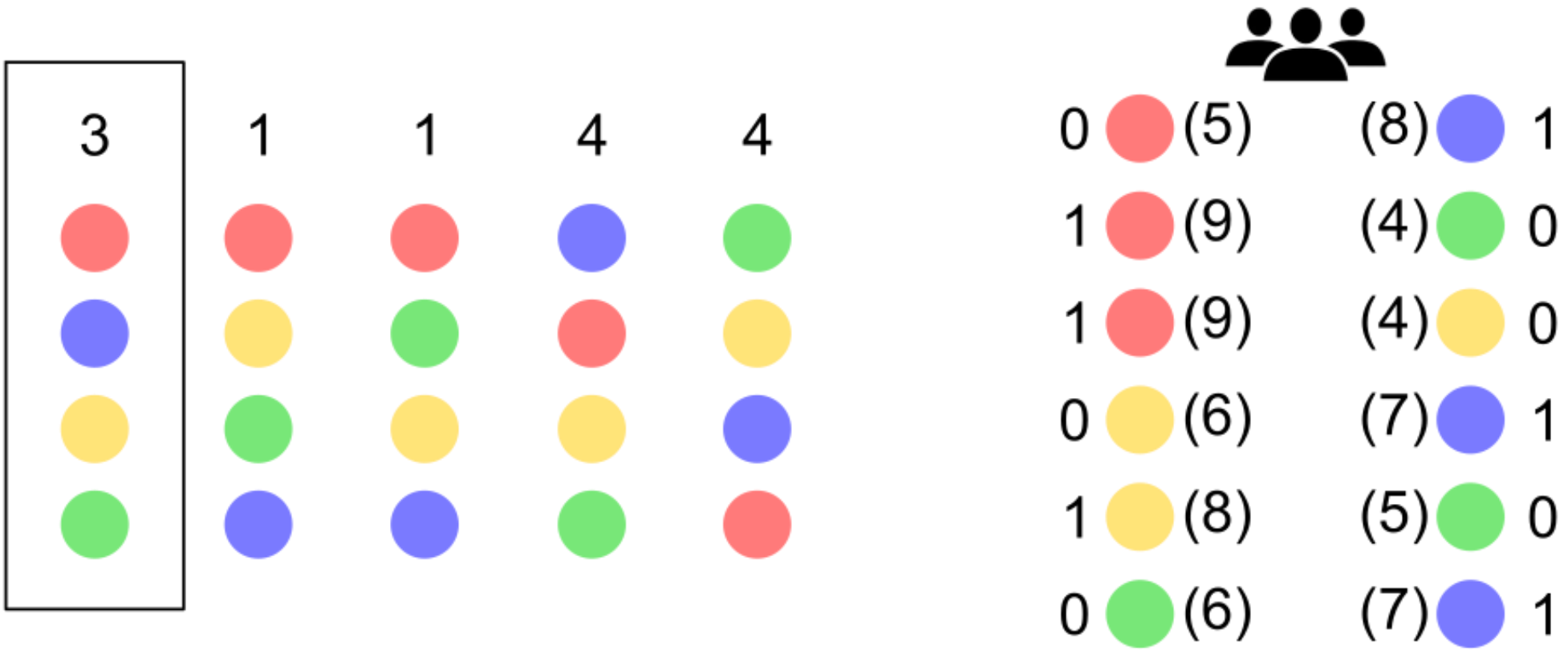


Copeland winner:

# 5

Problem: Voters are sometimes better off not voting

Copeland


Copeland winner:

6

Schulze

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Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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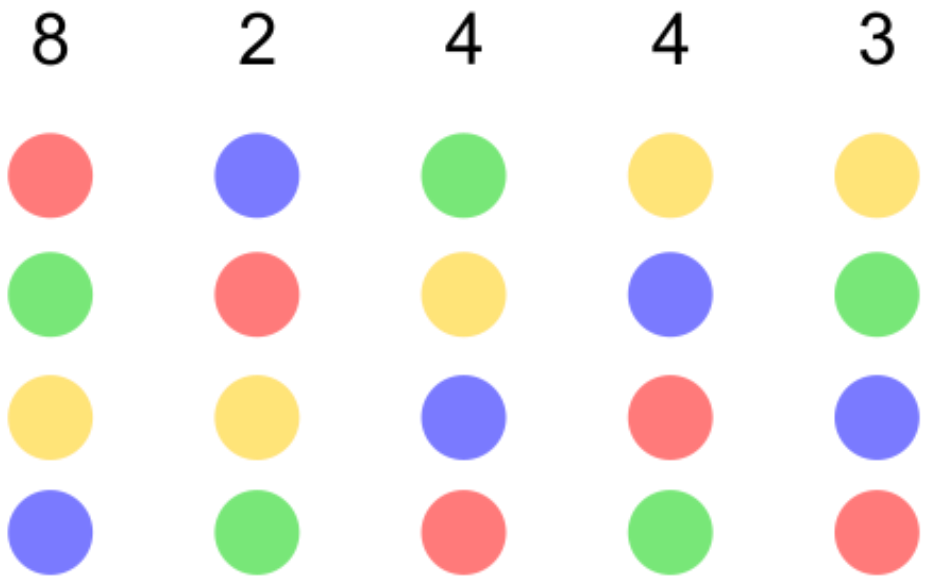

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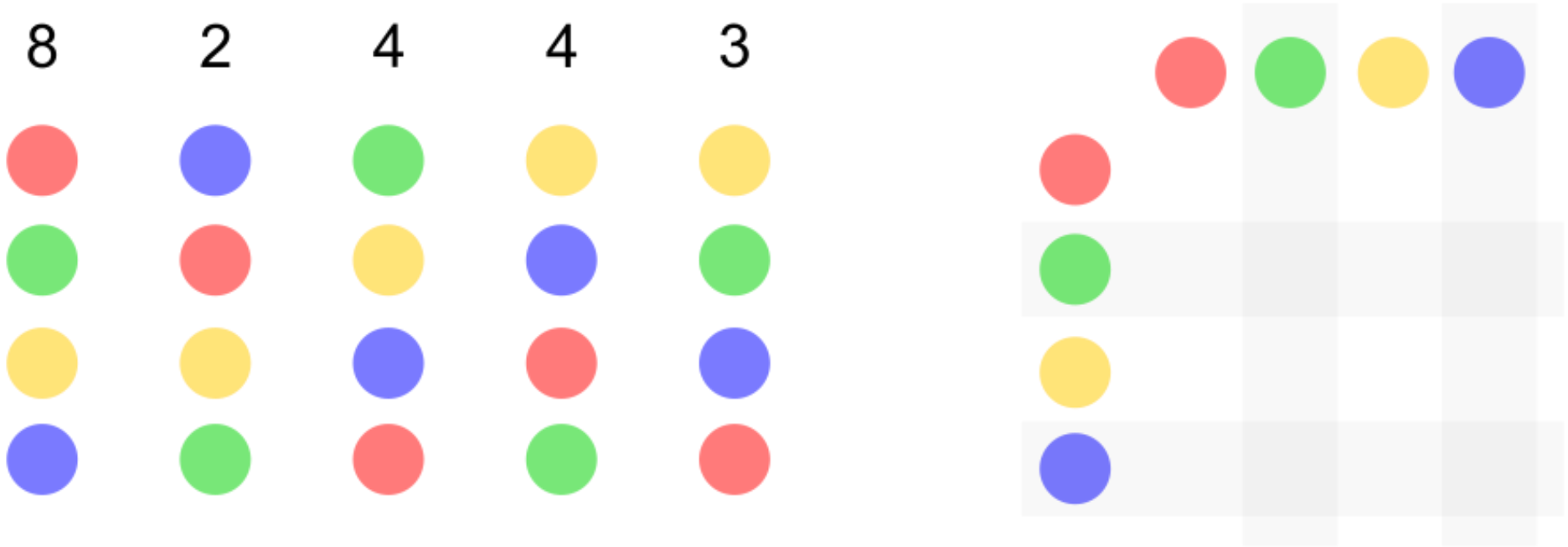
Schulze



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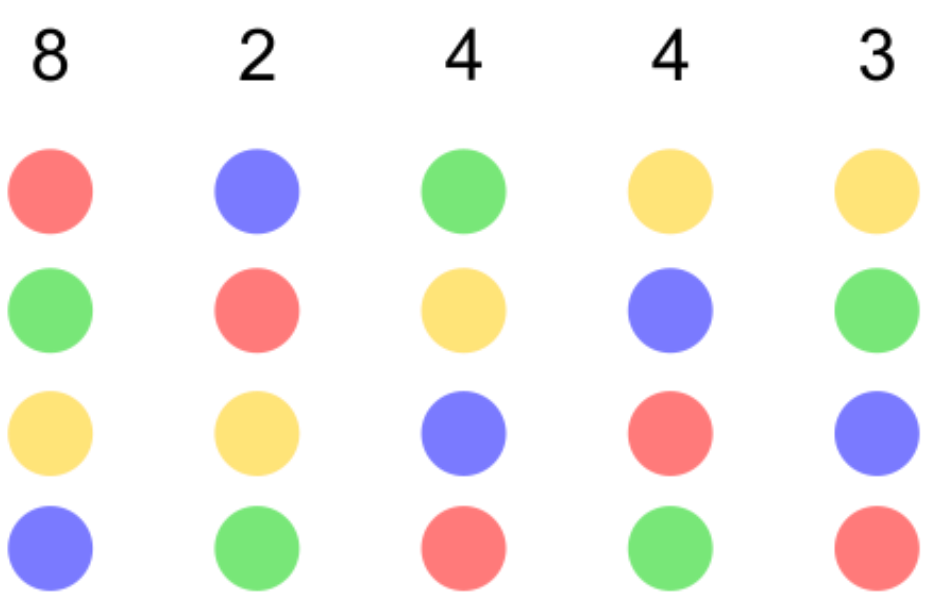


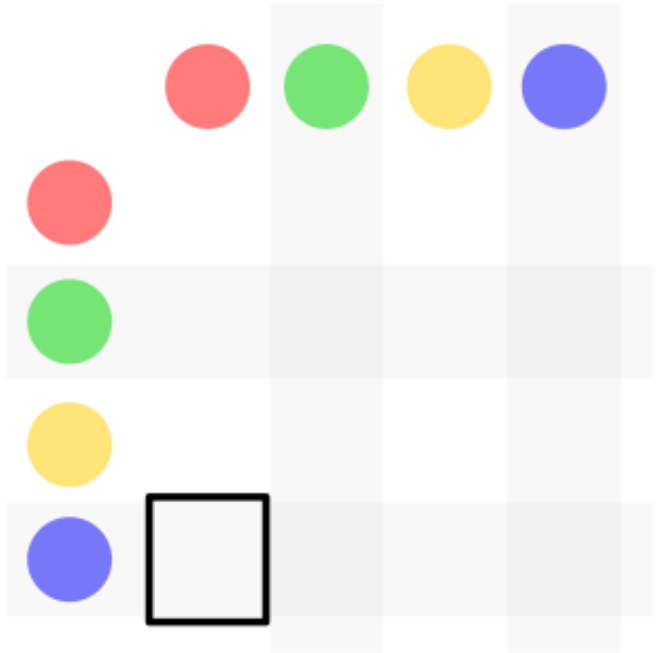

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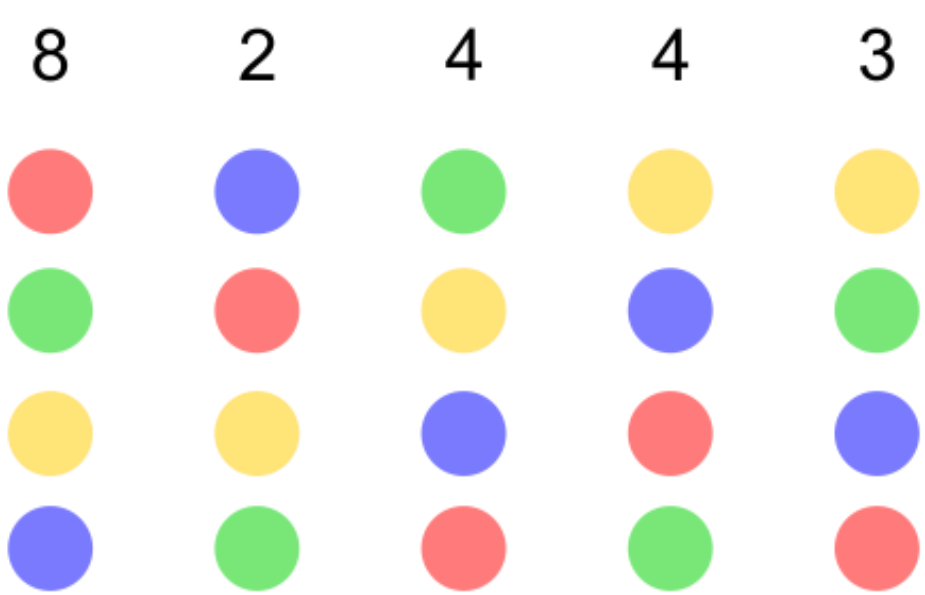


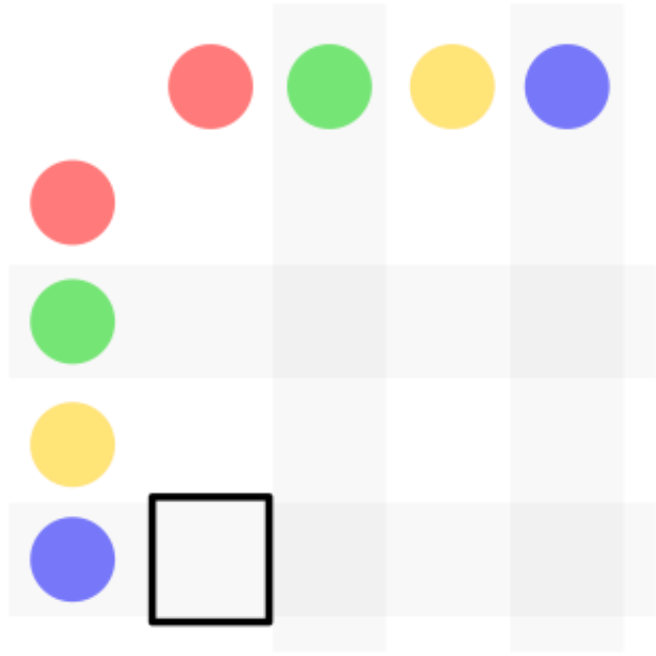

(2+4+4+3) - 8

#voters who think > 

#voters who think > 

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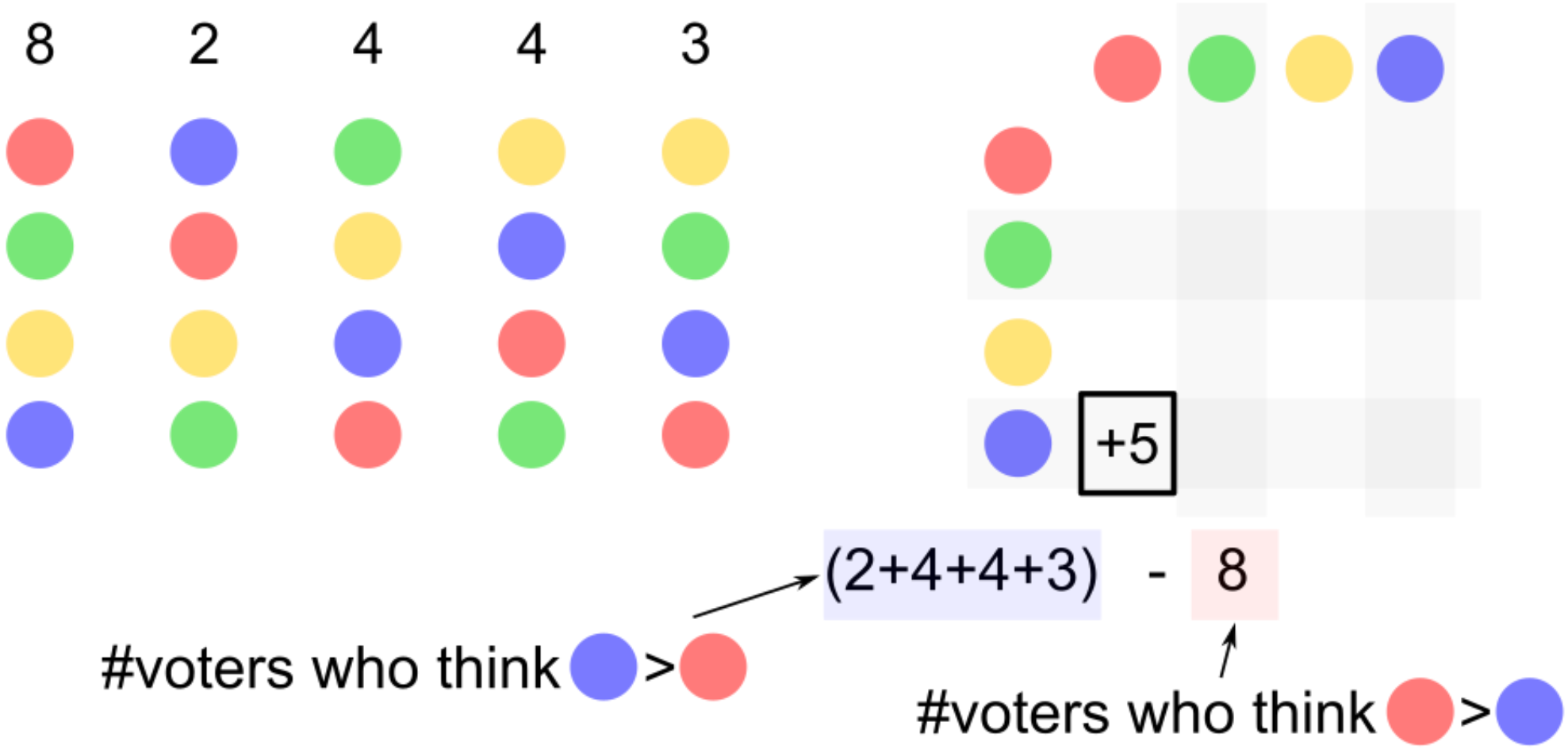
Schulze



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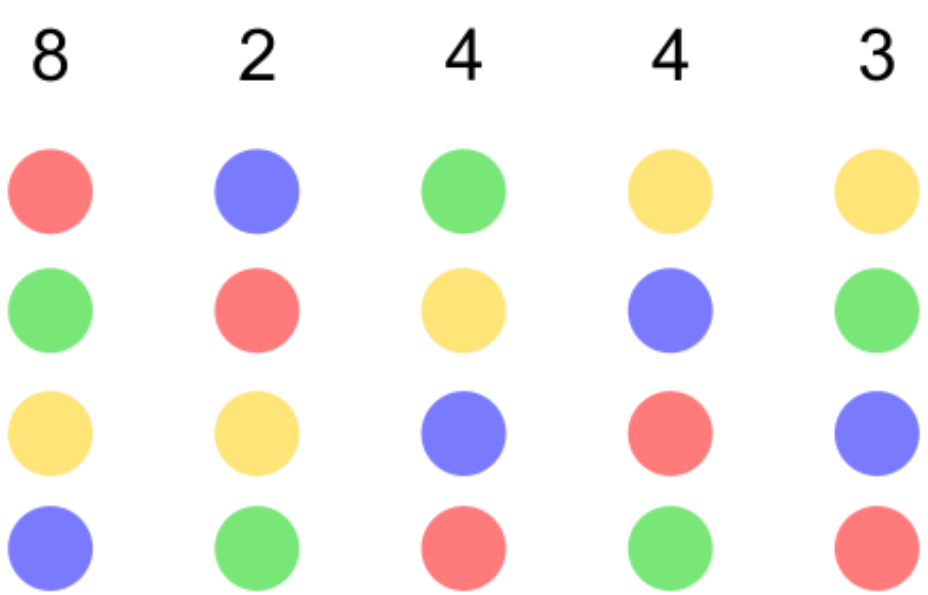


| | | | | |
|---|---|---|---|---|
| | | +7 | -1 | -5 |
| | -7 | | +3 | +9 |
| | +1 | -3 | | +17 |
| | +5 | -9 | -17 | |

#voters who think > 

(2+4+4+3) - 8

#voters who think > 

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Schulze

| 8 | 2 | 4 | 4 | 3 |
|---|---|---|---|---|
| 🔴 | 🔵 | 🟢 | 🟡 | 🟡 |
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| 🔵 | 🟢 | 🔴 | 🟢 | 🔴 |

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| 🔴 | 0 | +7 | -1 | -5 |
| 🟢 | -7 | 0 | +3 | +9 |
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| 🔵 | +5 | -9 | -17 | 0 |

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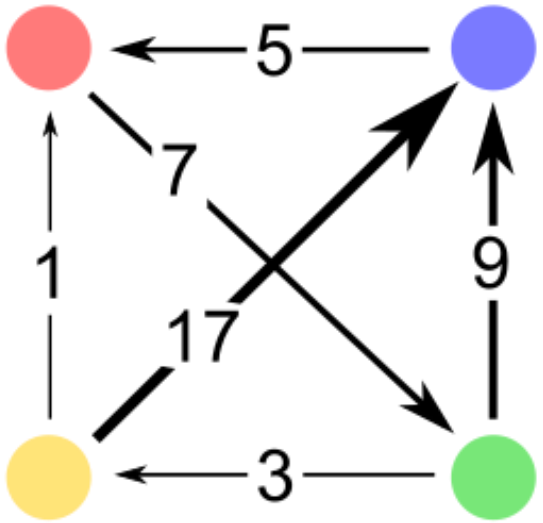


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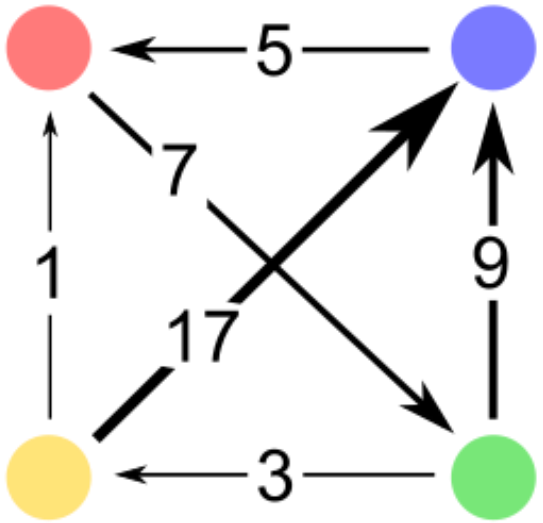

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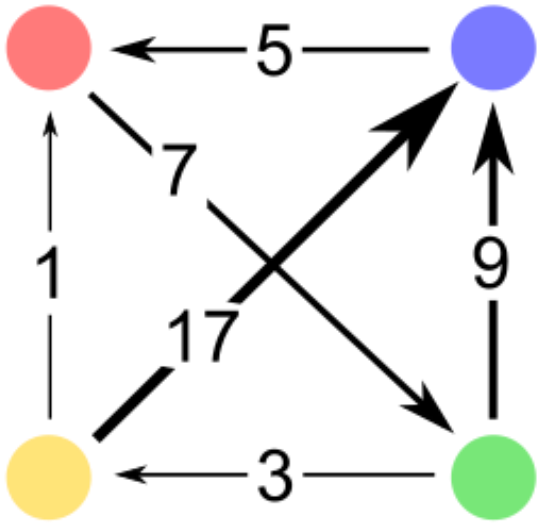


The strength of a path/chain is the weight of the weakest link in it

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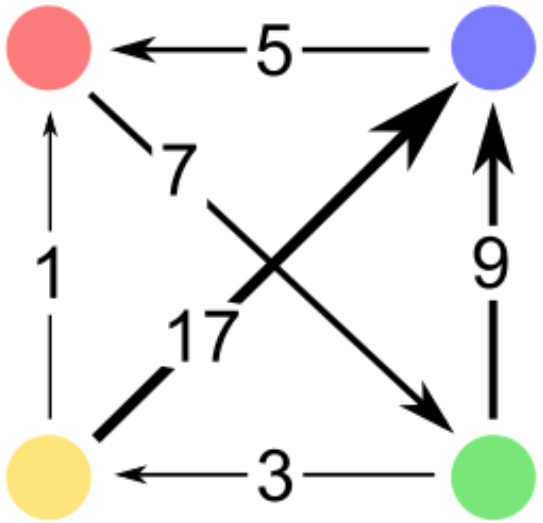


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What's the strongest path from ● to ●?

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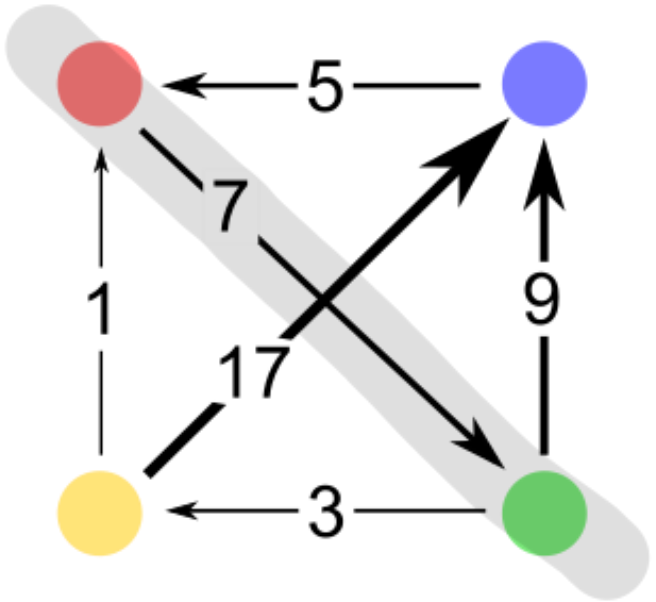


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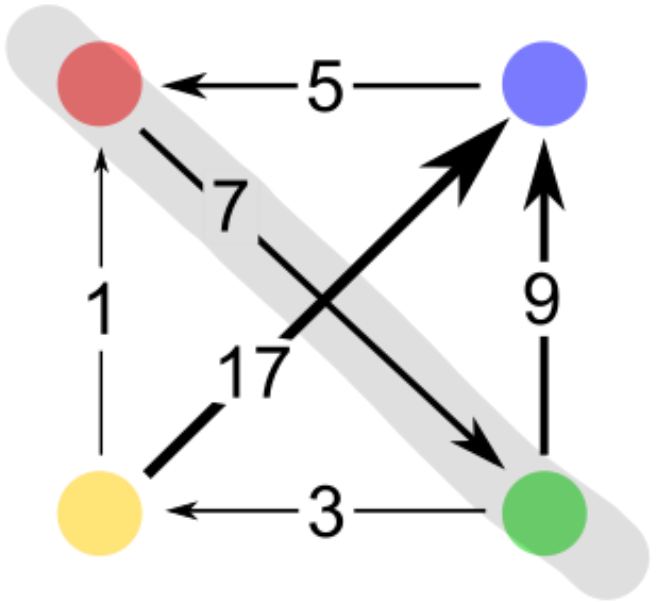


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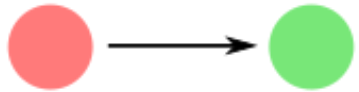

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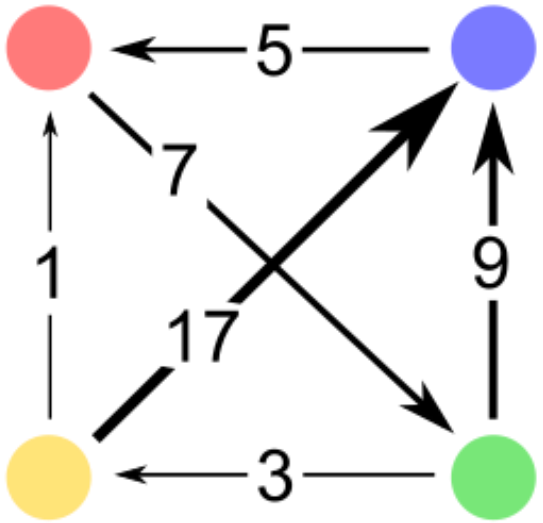


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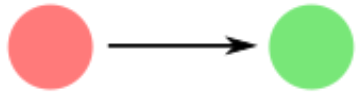

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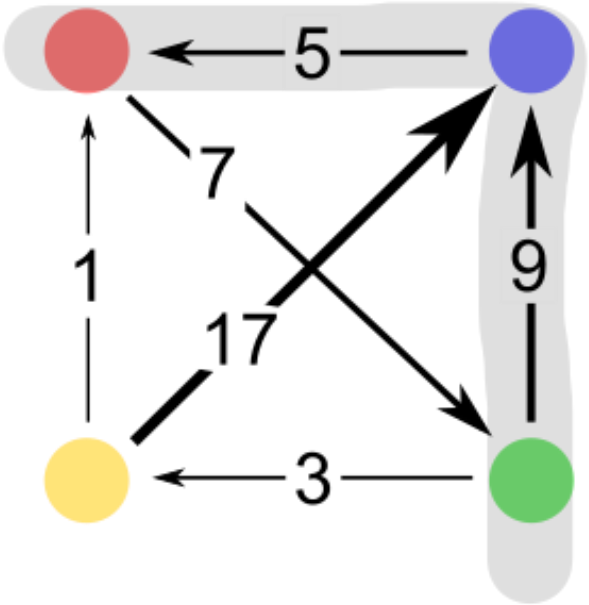


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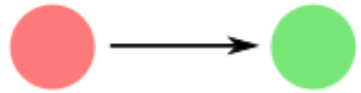

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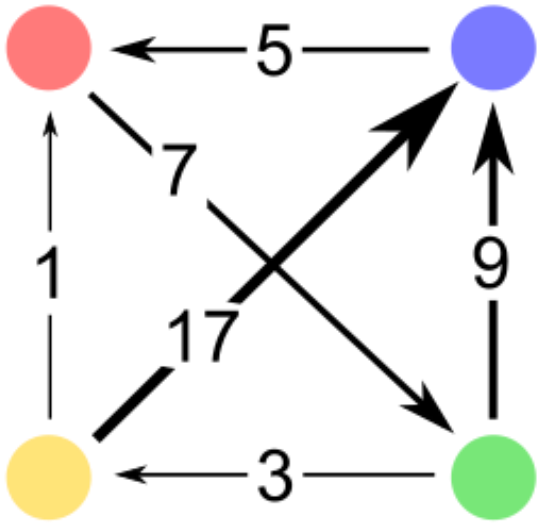

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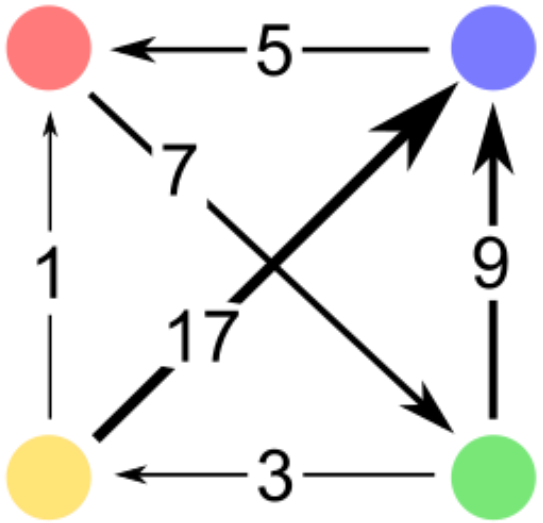


ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
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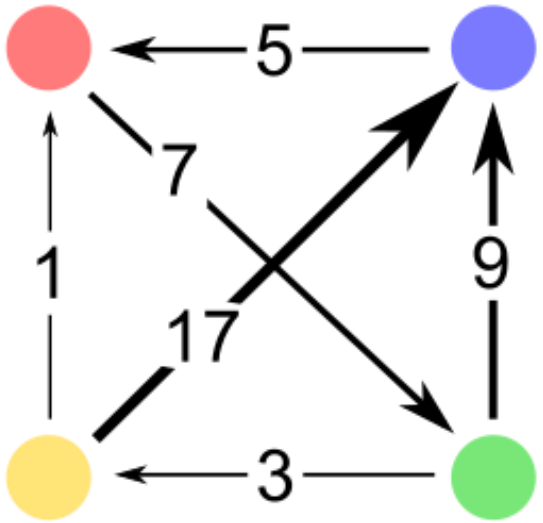


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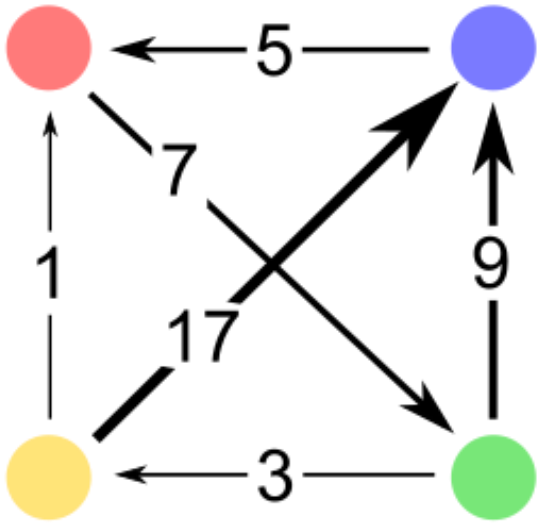



Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
the *strongest* path from Ⓐ to Ⓑ
is *stronger* than
the *strongest* path from Ⓑ to Ⓐ

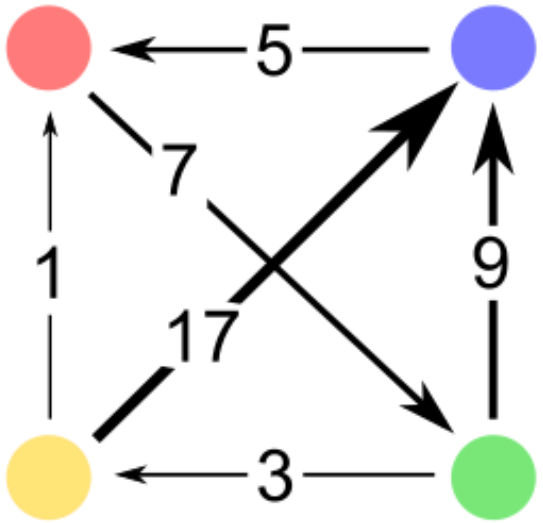
● >> ●

the strongest path from ● to ● has strength 7

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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● >> ●

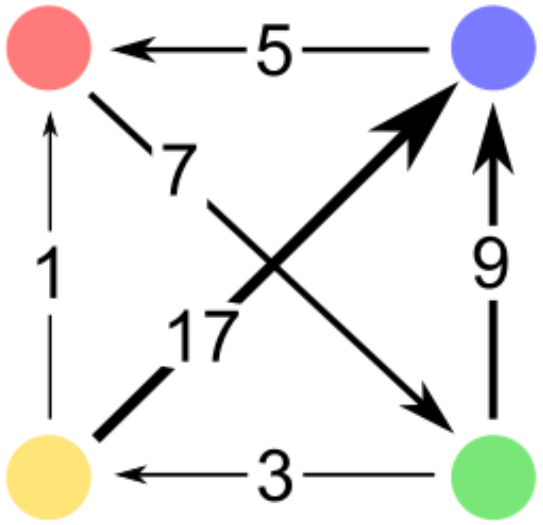
the strongest path from ● to ● has strength 7

the strongest path from ● to ● has strength 5

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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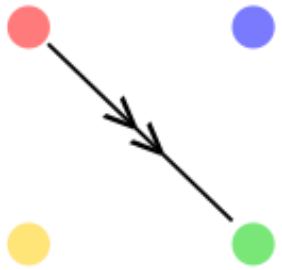


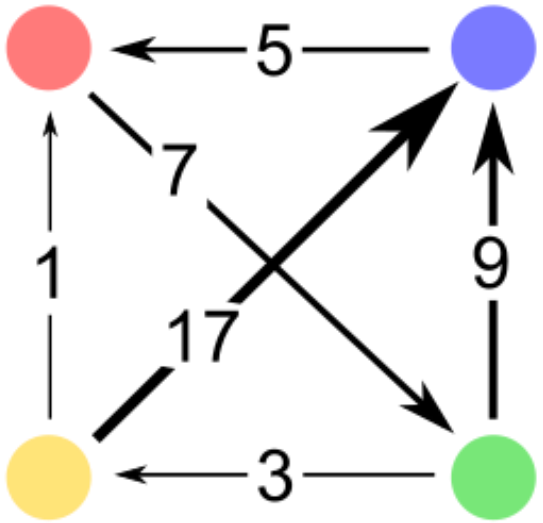

the strongest path from  to  has strength 7

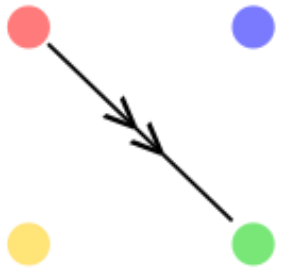
the strongest path from  to  has strength 5

# 6

## Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



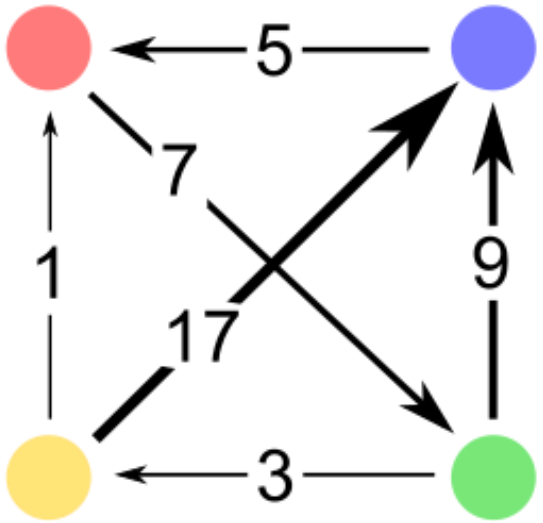



ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
the *strongest* path from ⓐ to ⓑ
is *stronger* than
the *strongest* path from ⓑ to ⓐ

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph




Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
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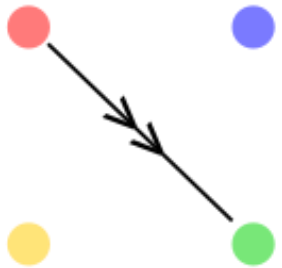
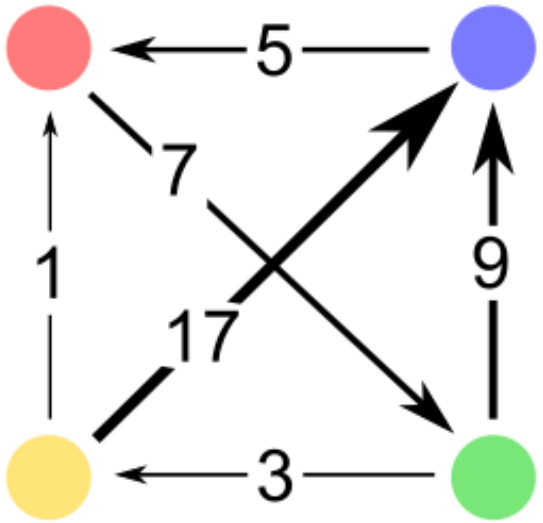


# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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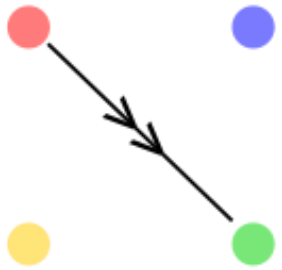
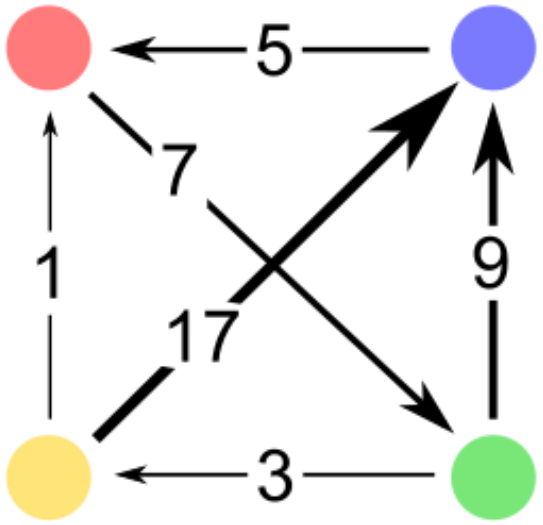



the strongest path from  to  has strength 7

# 6

## Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph




Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
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is *stronger* than
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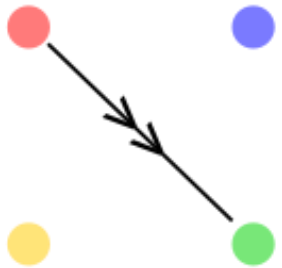


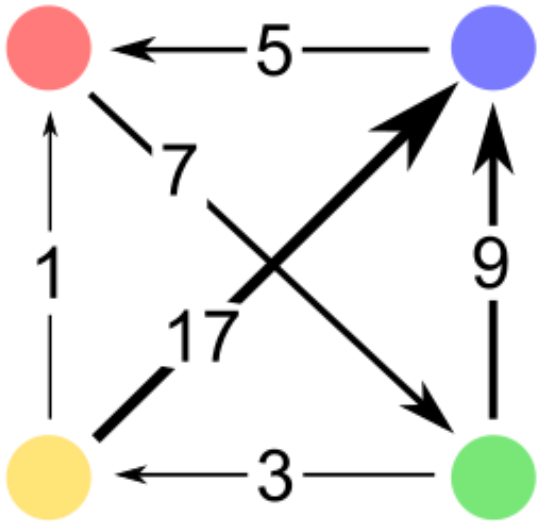
the strongest path from  to  has strength 7

the strongest path from  to  has strength 5

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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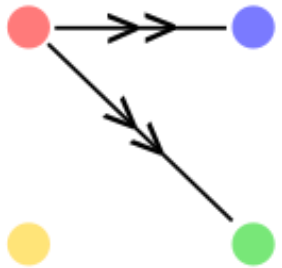


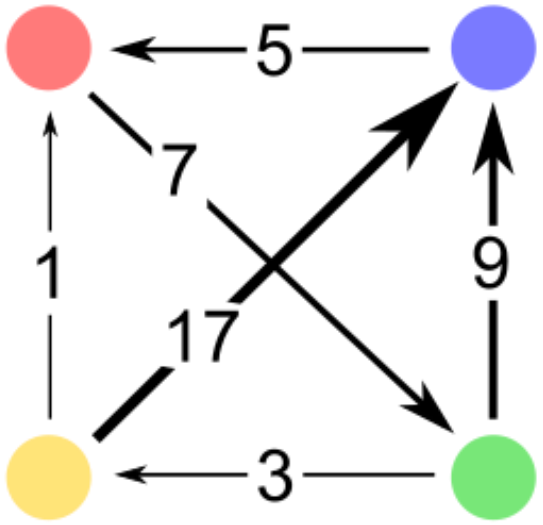

the strongest path from  to  has strength 7

the strongest path from  to  has strength 5

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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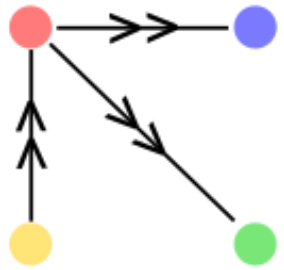


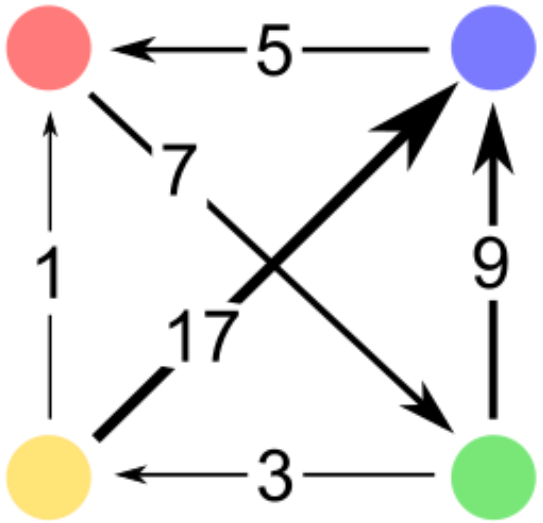

the strongest path from  to  has strength 5

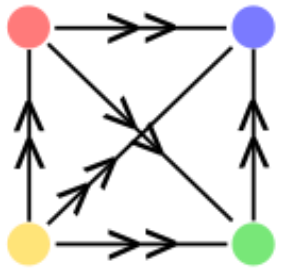
the strongest path from  to  has strength 3

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



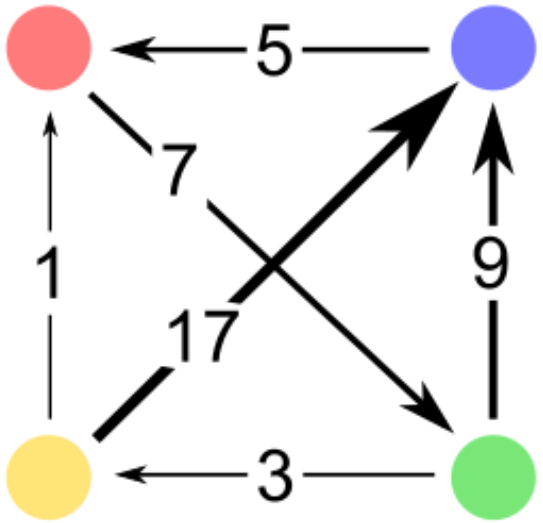
ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
the *strongest* path from ⓐ to ⓑ
is *stronger* than
the *strongest* path from ⓑ to ⓐ

● >> all others

# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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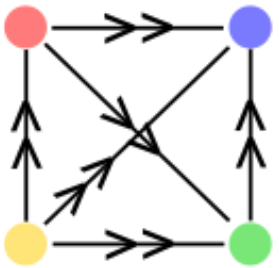
is *stronger* than
the *strongest* path from Ⓑ to Ⓐ



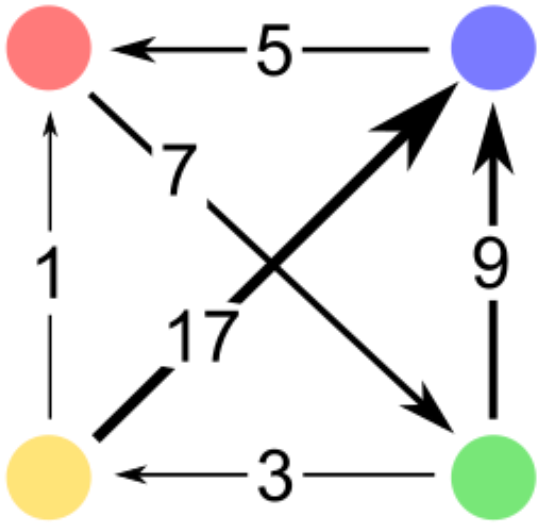
● >> all others

Schulze winner: ●

# 6

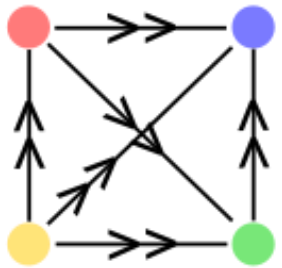
Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph




Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
the *strongest* path from Ⓐ to Ⓑ
is *stronger* than
the *strongest* path from Ⓑ to Ⓐ

A Schulze winner always exists!



The "chain beats" relation is transitive
Ⓐ>>Ⓑ and Ⓑ>>Ⓒ, then Ⓐ>>Ⓒ

6

Schulze


ubuntu


debian
GNU/Linux


gentoo linux™


IEEE


MTV


acm
Association for
Computing Machinery


topcoder™

# Which Voting Rule is the Best?

# Which Voting Rule is the Best?

**Voting Power in Practice Summer Workshop**

**Assessing Alternative Voting Procedures**

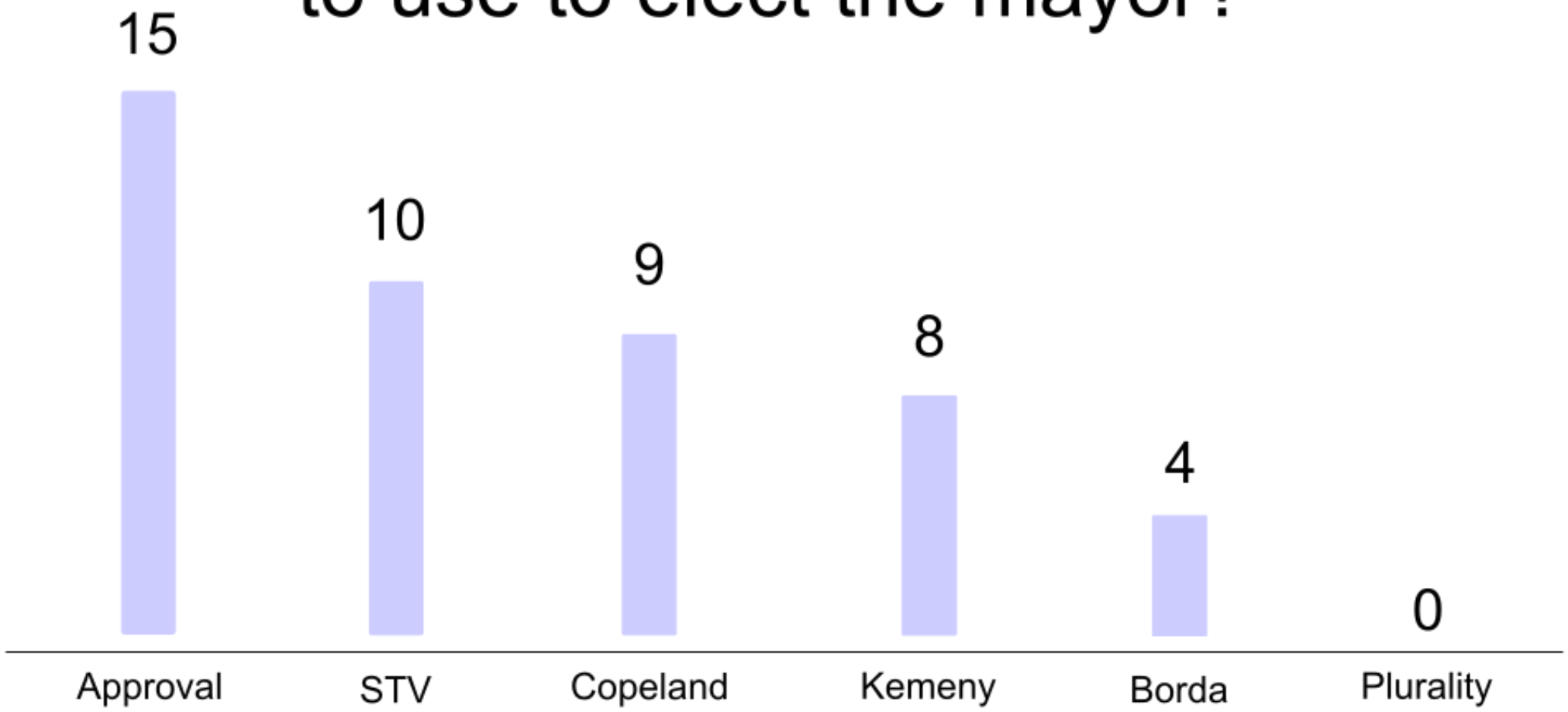
**Sponsored by The Leverhulme Trust**

Chateau du Baffy, Normandy, France
30 July – 2 August 2010

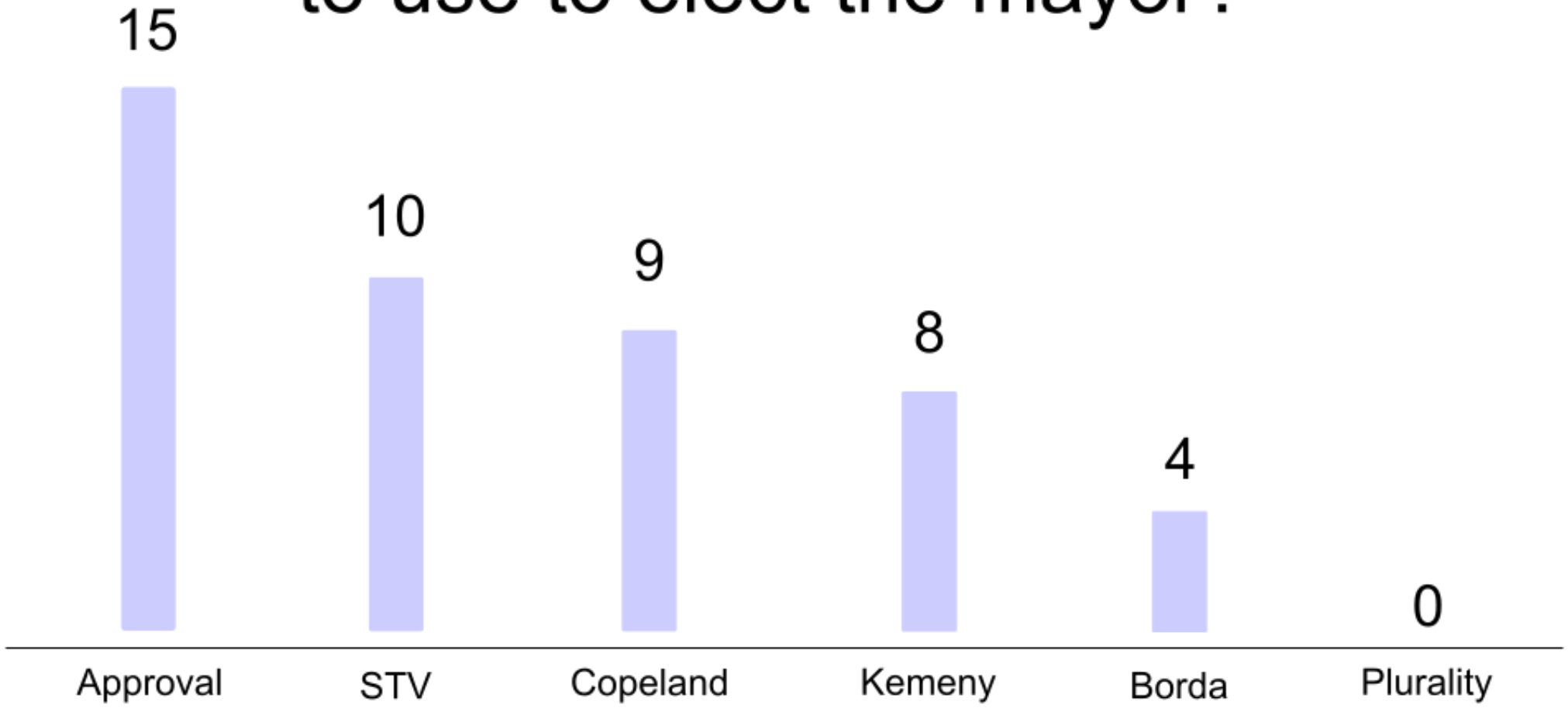
22 voting theorists

"What is the best voting rule for your town to use to elect the mayor?"

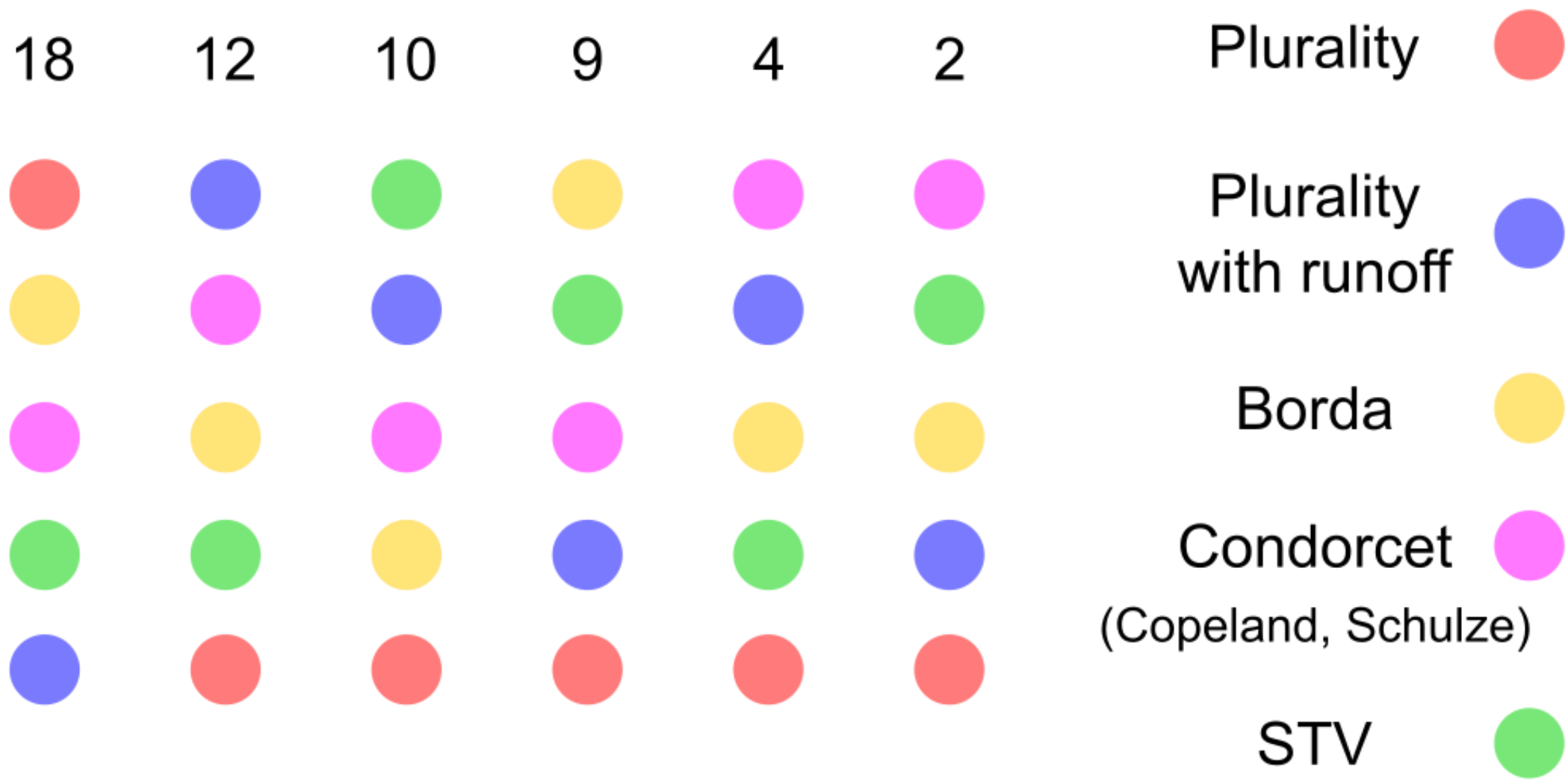
# "What is the best voting rule for your town to use to elect the mayor?"



# "What is the best voting rule for your town to use to elect the mayor?"

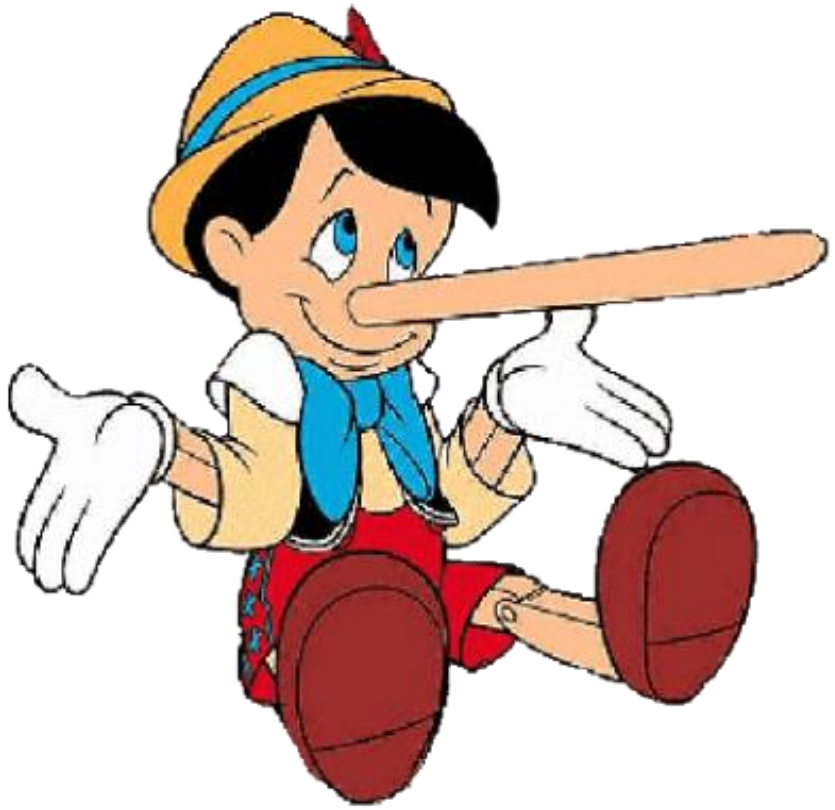



Votes were counted via approval voting.


18
12
10
9
4
2
Plurality
Plurality
with runoff
Borda
Condorcet
(Copeland, Schulze)
STV

# Next Time

Manipulation in voting



# Quiz

# Quiz

Suggest a voting rule you think is "reasonable"
(other than the ones we discussed today)
and justify why it is reasonable.

# References

- “Disagreement between voting rules” example: http://www.ams.org/publicoutreach/feature-column/fcarc-voting-decision
- The “chain beats” terminology in the description of Schulze rule was borrowed from Hubert Bray’s explanatory video: https://www.youtube.com/watch?v=_HVeN0GnnuA
- The Schulze rule example is from the paper “The Schulze Method of Voting” by Markus Schulze: https://arxiv.org/abs/1804.02973
- (Possibly) strategic voting in the selection of Olympics host: https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic_2_Plurality_Runoff.pdf

# References

- Non-monotonicity of STV:

  Gideon Doron and Richard Kronick
  *Single Transferable Vote: An Example of a Perverse Social Choice Function*
  American Journal of Political Science, Vol. 21, No. 2 (May, 1977), pg 303-311
  https://www.jstor.org/stable/2110496

- “Which voting rule is the best” poll
  https://hal.archives-ouvertes.fr/hal-00609810/document