COL749: Computational Social Choice

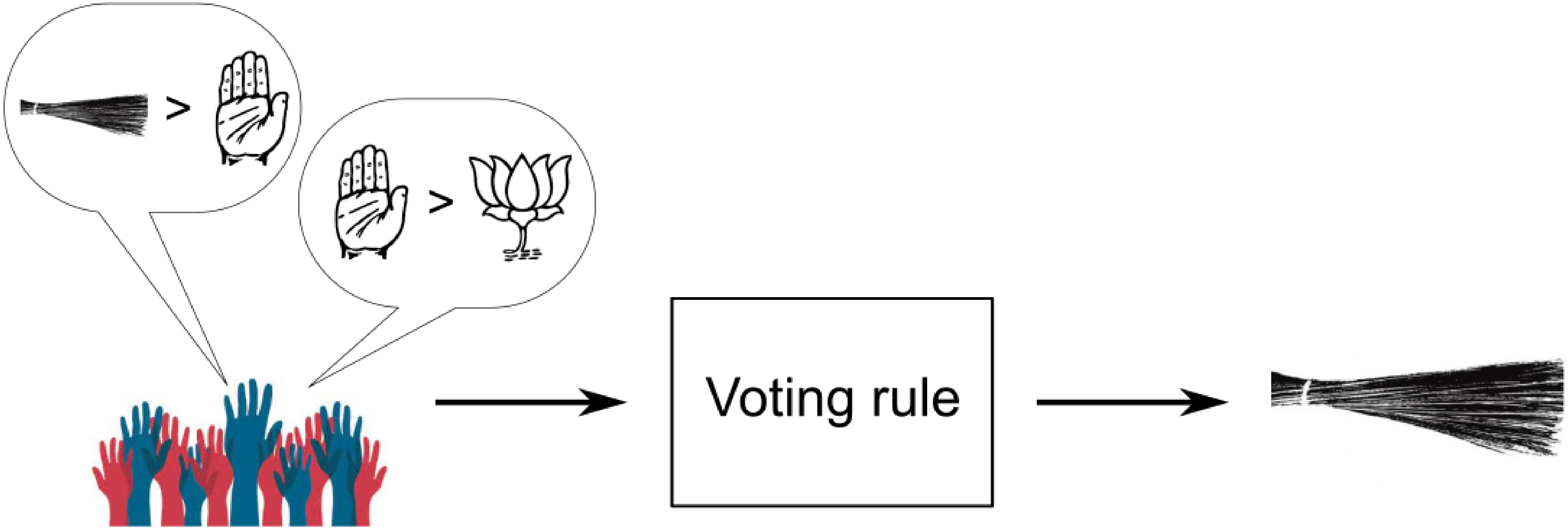
Lecture 1

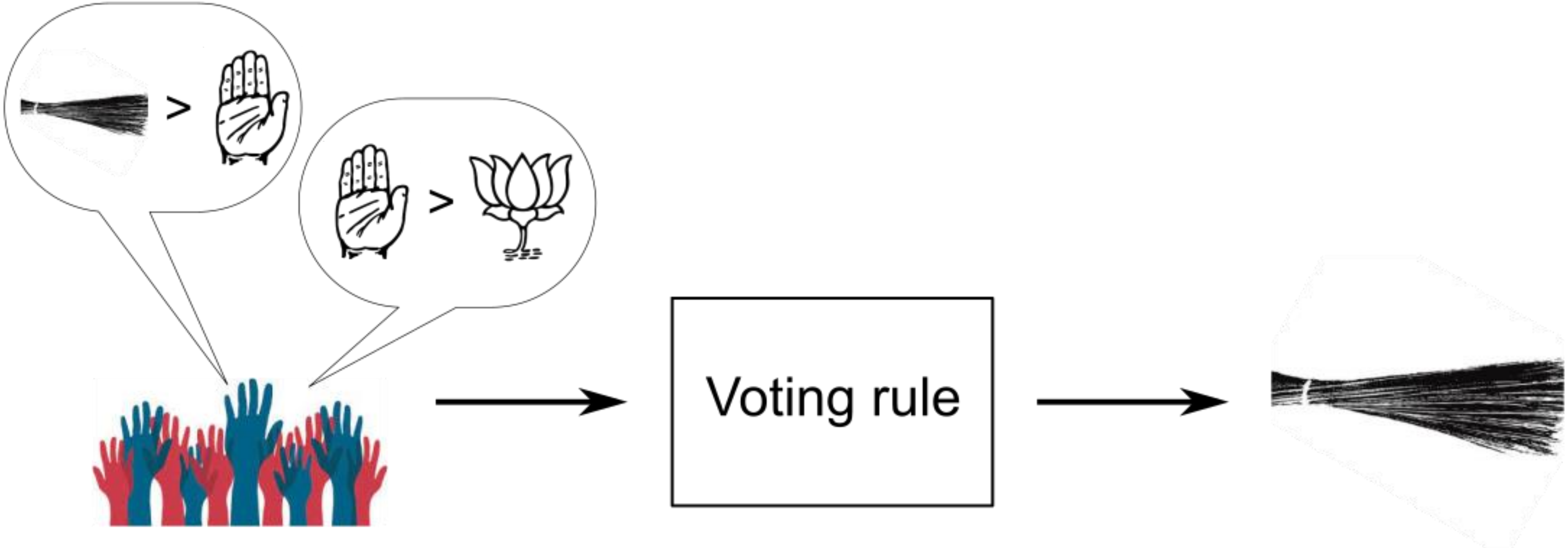
Introduction to The Course and Stable Matching Problem

Jan 02, 2025 | Rohit Vaish

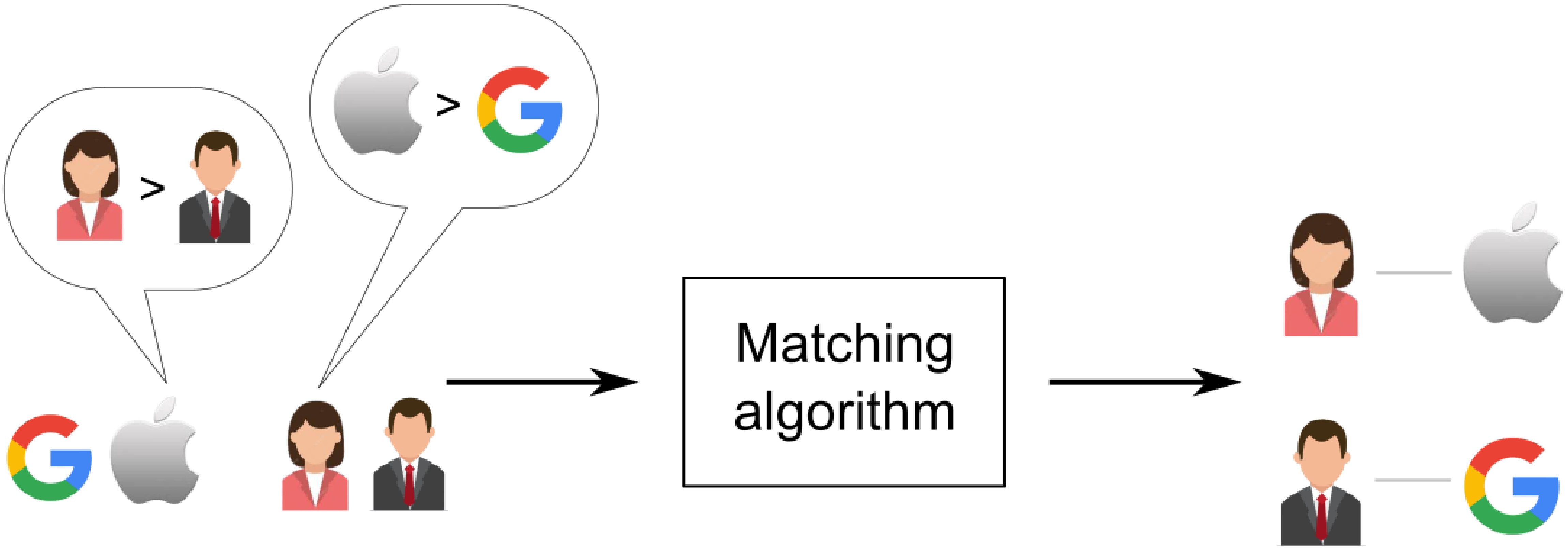
What this course is about

Understanding the role of **computation** in **collective decision-making** problems


Voting rule



amazon



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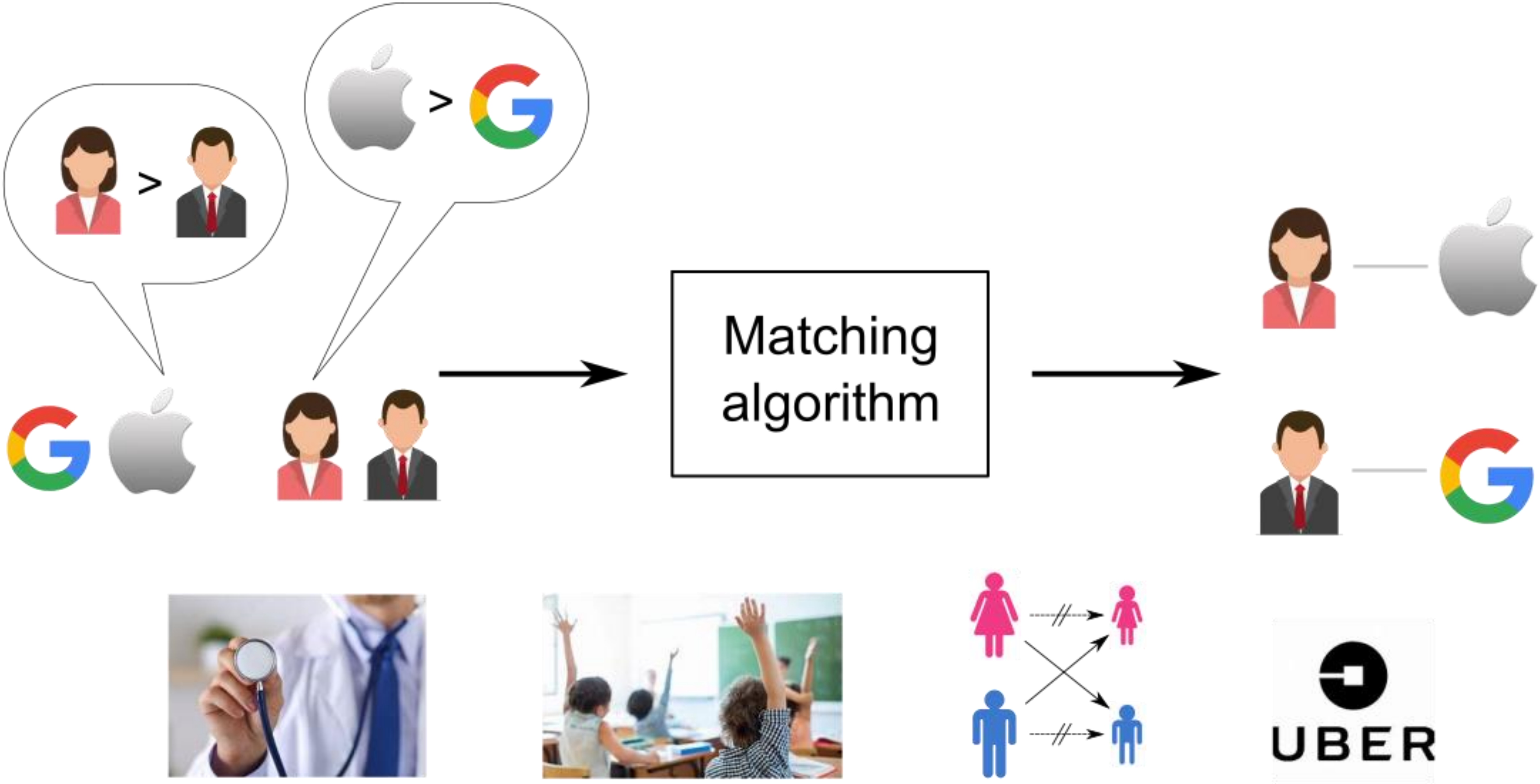
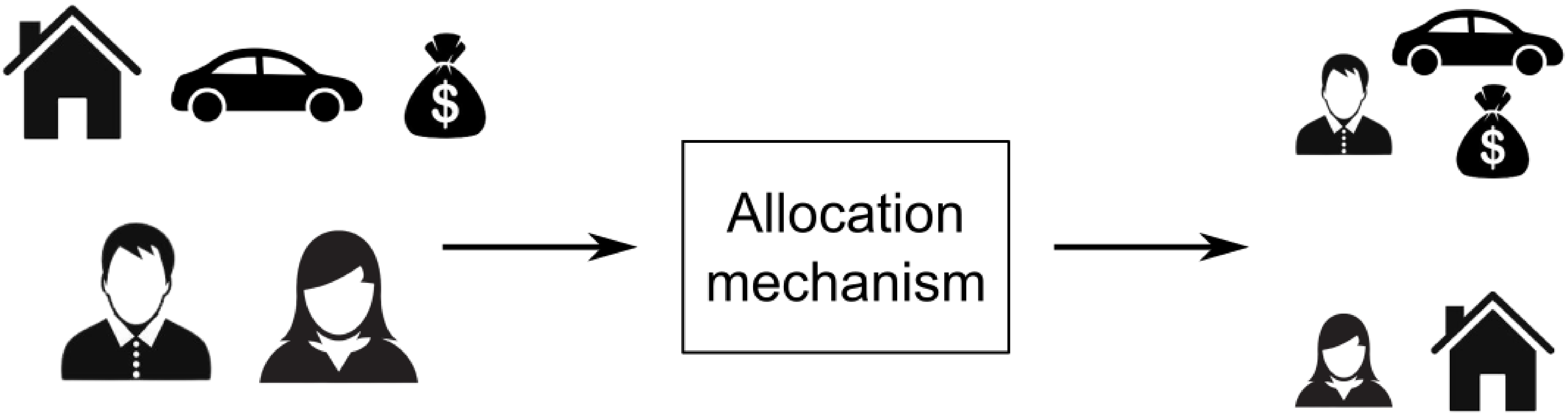
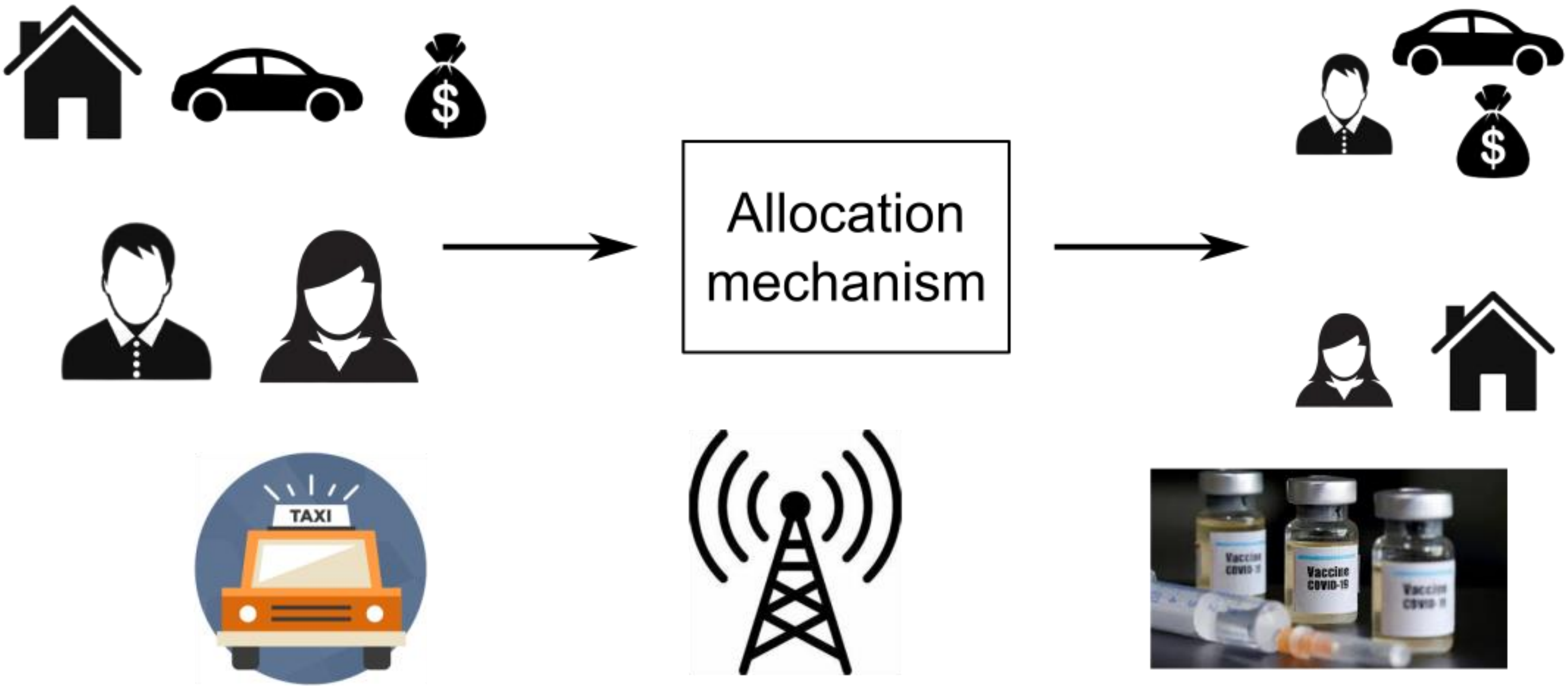
Matching
algorithm



Matching algorithm
UBER


Allocation
mechanism

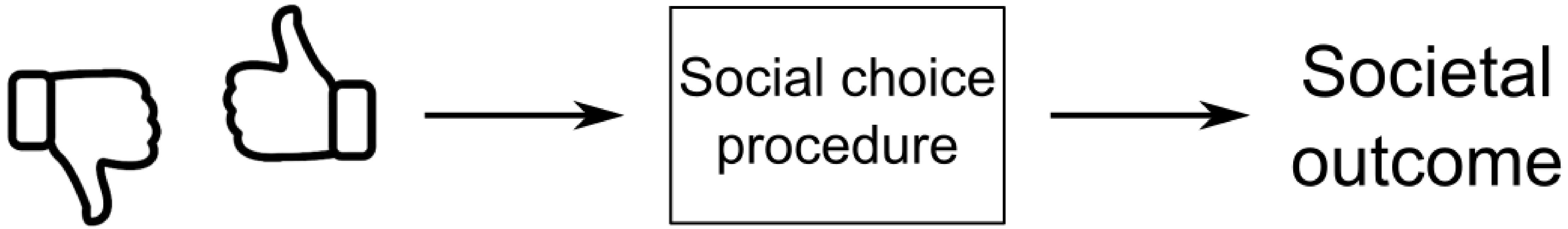

Allocation mechanism
TAXI
Vaccine COVID-19

Social Choice

Making a collective decision from individual preferences

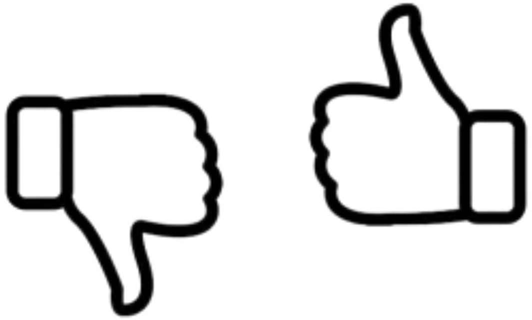
Social Choice

Making a collective decision from individual preferences



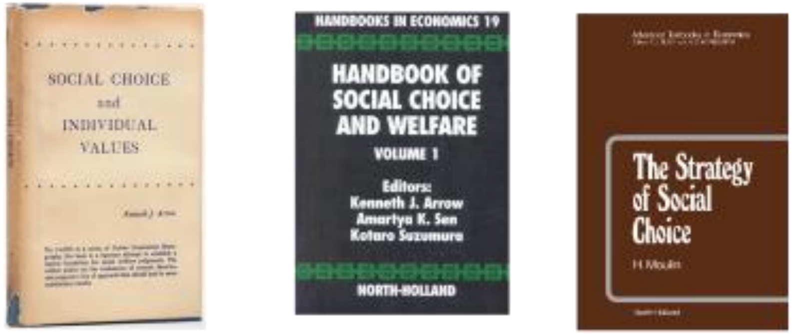
Social Choice

Making a collective decision from individual preferences



Social choice procedure

Societal outcome



Arrow Sen Maskin Roth Shapley

Classical Social Choice

Does there exist a social choice procedure with the desired economic properties?

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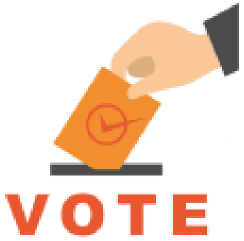

Does there exist a "truthful" voting rule?

Classical Social Choice

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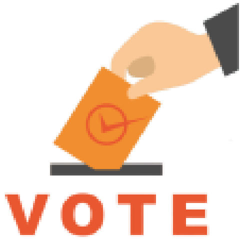

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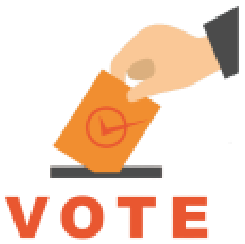

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Is there an allocation procedure that is "fair" and "economically efficient"?

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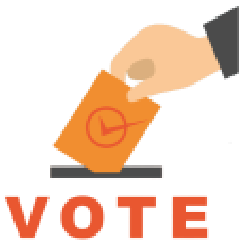

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Is there a matching procedure that is "stable"?

Is there an allocation procedure that is "fair" and "economically efficient"?

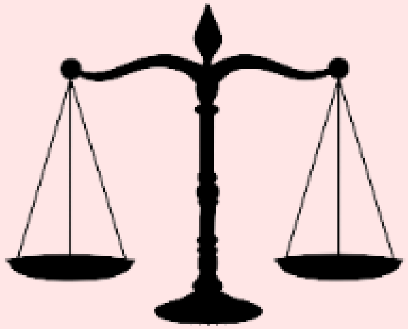
Classical Social Choice

Does there exist a social choice procedure with the desired economic properties?

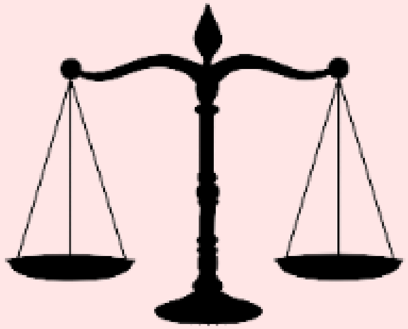
Computational Social Choice
(This course)

How does computation influence the economic properties of social choice procedures?


VOTE



LEC 1-6



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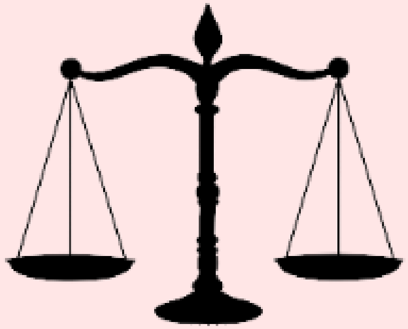
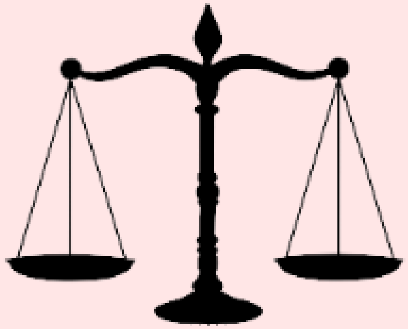


LEC 7-15

LEC 16-21

LEC 1-6



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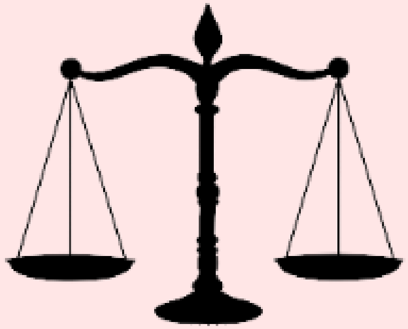

LEC 16-21

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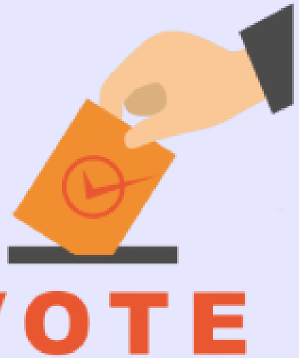
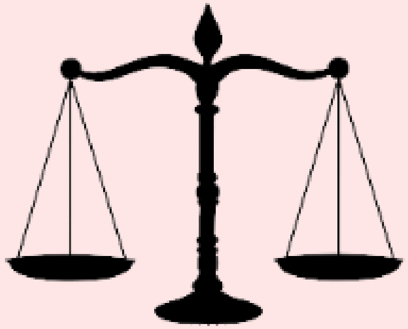
LEC 7-15

LEC 22-26





LEC 16-21

LEC 1-6

LEC 7-15

LEC 22-26

LEC 11 and 27-28: Project Presentations

LEC 16-21

LEC 1-6

LEC 7-15

LEC 22-26

LEC 11 and 27-28: Project Presentations

Stable Matching Problem

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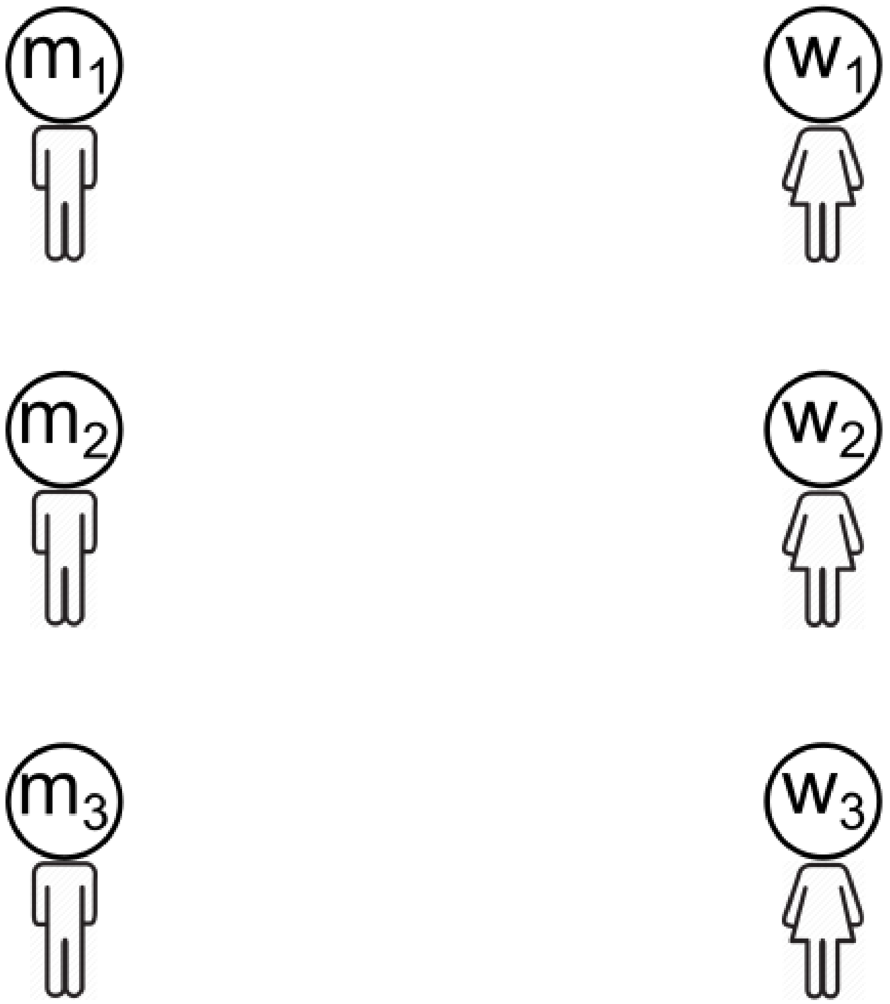
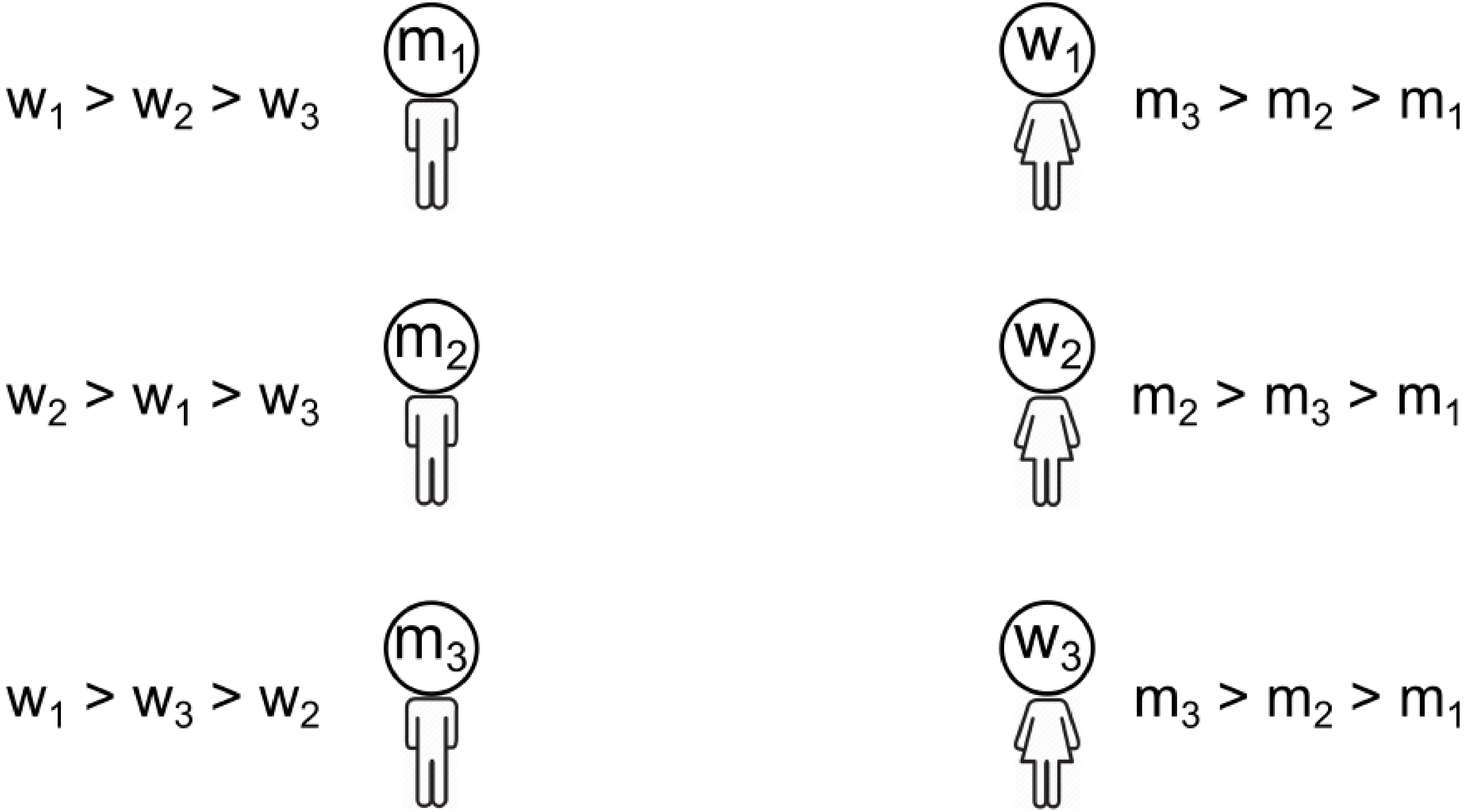
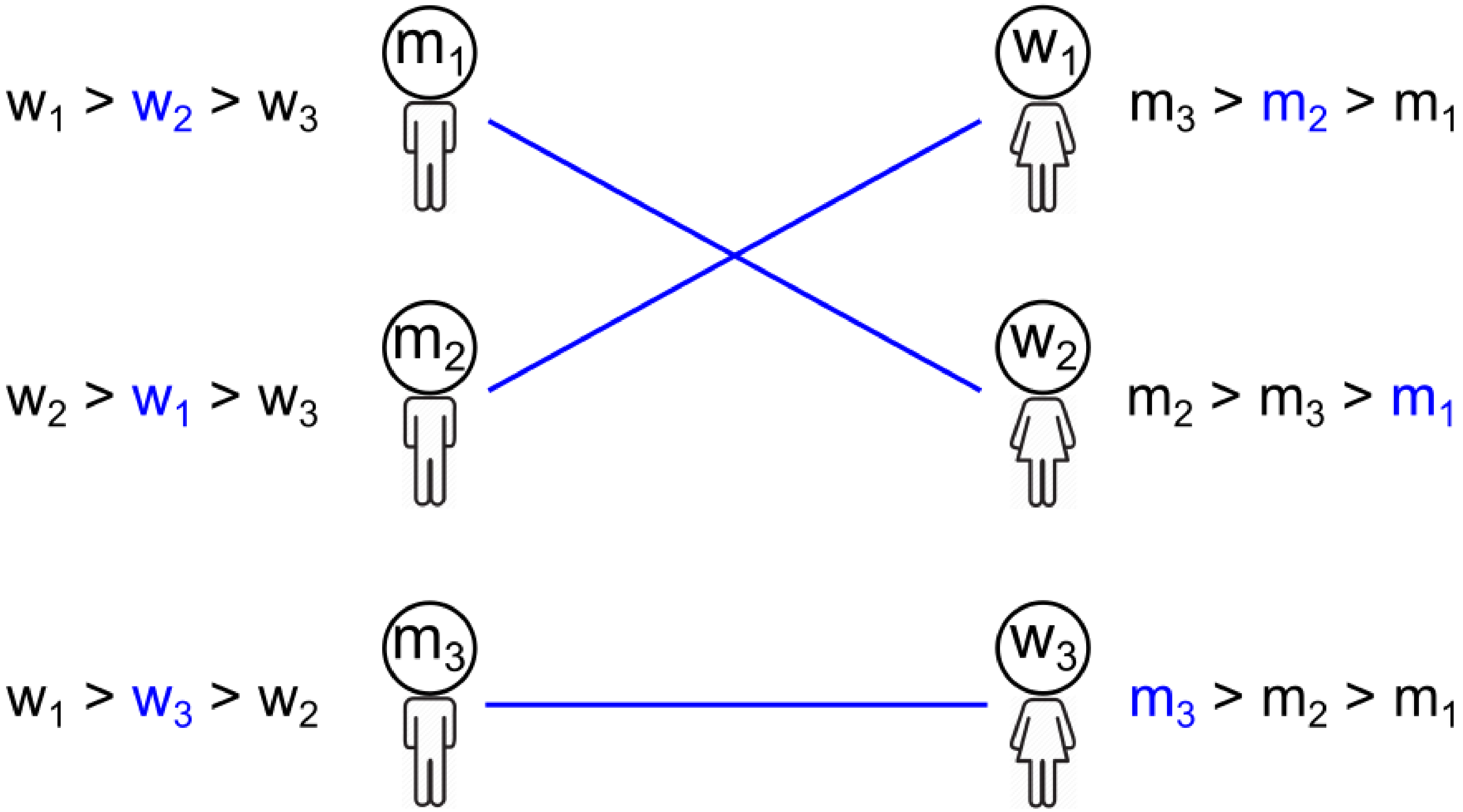


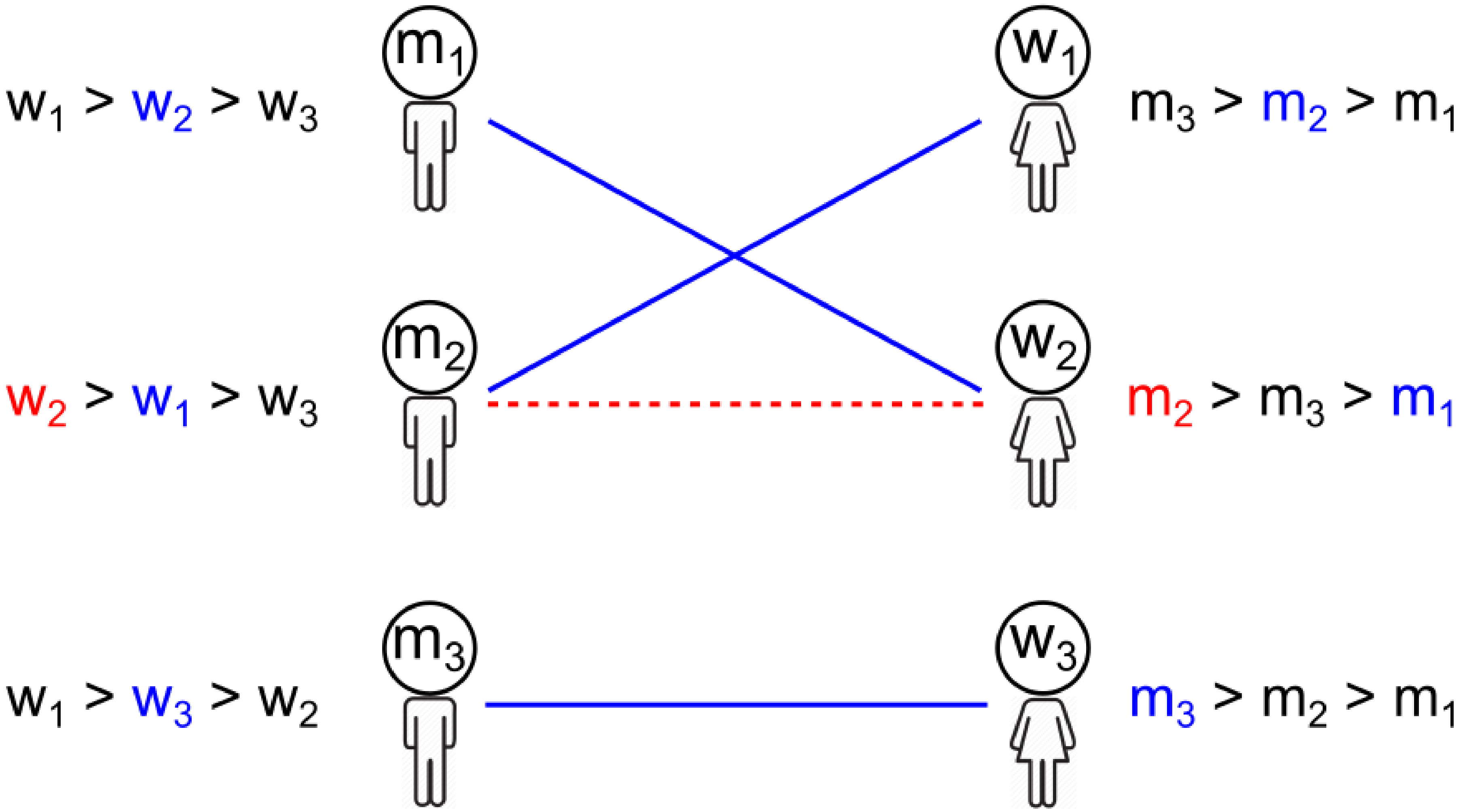
Stable Matching Problem



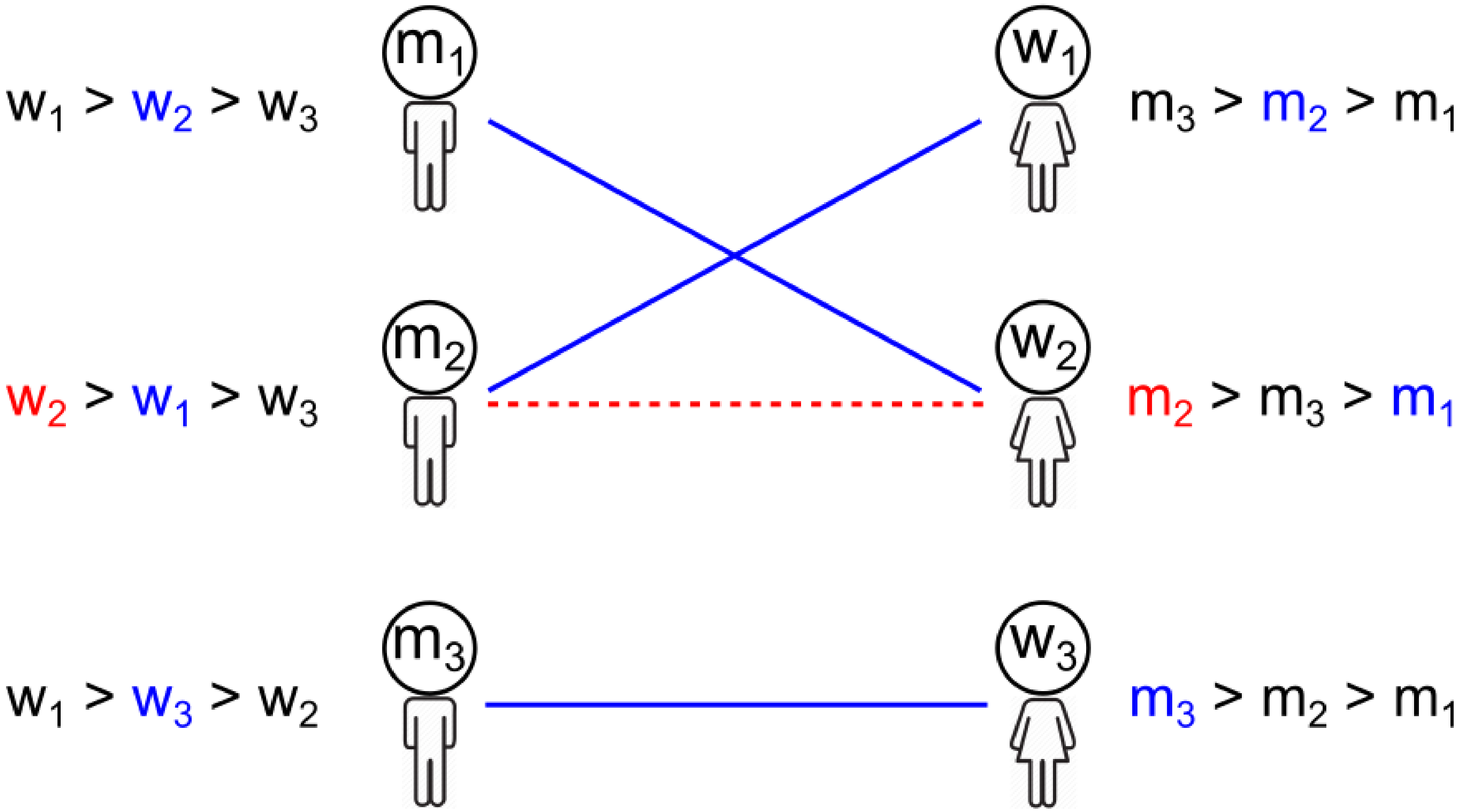
Stable Matching Problem



Stable Matching Problem

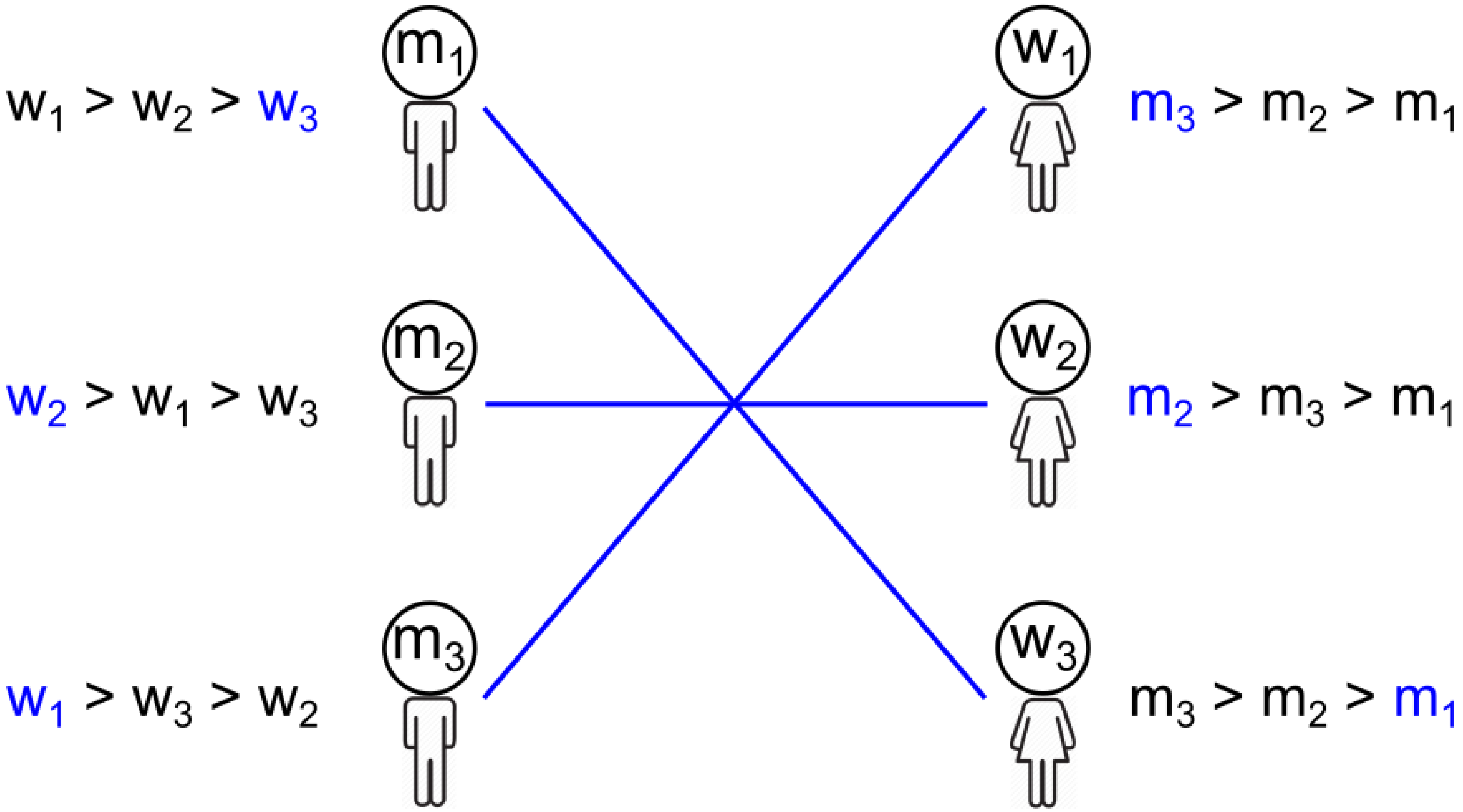


Stable Matching Problem



A matching is stable if there is no blocking pair.

Stable Matching Problem



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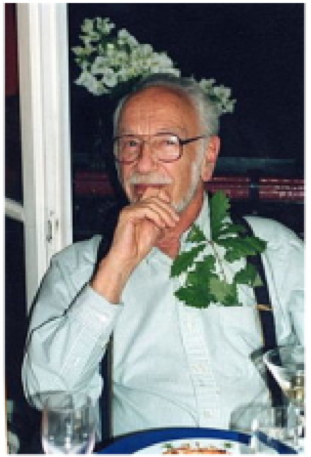


COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

Source: *The American Mathematical Monthly*, Jan., 1962, Vol. 69, No. 1 (Jan., 1962), pp. 9–15

Given any preference profile, a stable matching for that profile always exists and can be computed in polynomial time.

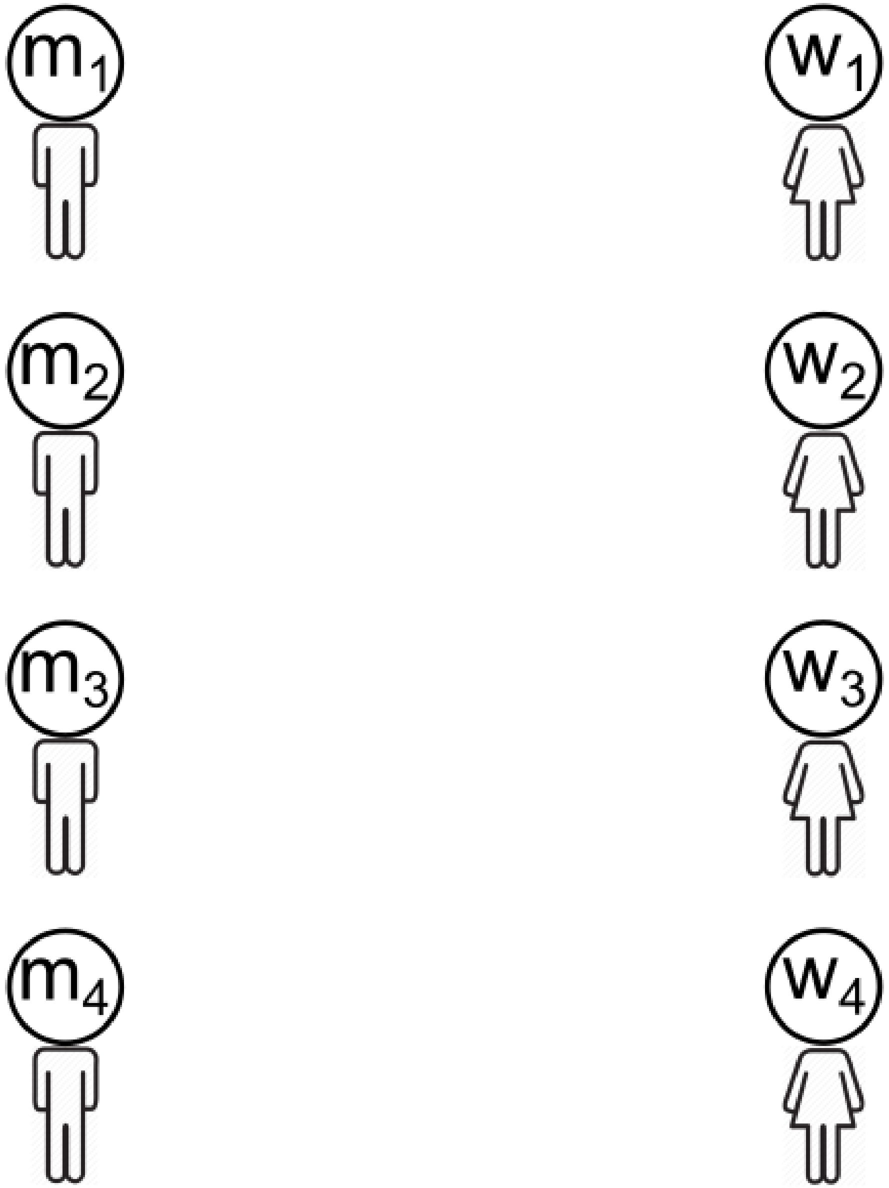
Deferred-Acceptance Algorithm

Deferred-Acceptance Algorithm

$w_1 > w_2 > w_3 > w_4$	m_1	w_1	$m_3 > m_2 > m_1 > m_4$
$w_1 > w_4 > w_2 > w_3$	m_2	w_2	$m_4 > m_2 > m_3 > m_1$
$w_1 > w_2 > w_3 > w_4$	m_3	w_3	$m_3 > m_2 > m_1 > m_4$
$w_1 > w_4 > w_2 > w_3$	m_4	w_4	$m_1 > m_2 > m_3 > m_4$

Deferred-Acceptance Algorithm

Round 1



m_1: $w_1 > w_2 > w_3 > w_4$

m_2: $w_1 > w_4 > w_2 > w_3$

m_3: $w_1 > w_2 > w_3 > w_4$

m_4: $w_1 > w_4 > w_2 > w_3$

w_1: $m_3 > m_2 > m_1 > m_4$

w_2: $m_4 > m_2 > m_3 > m_1$

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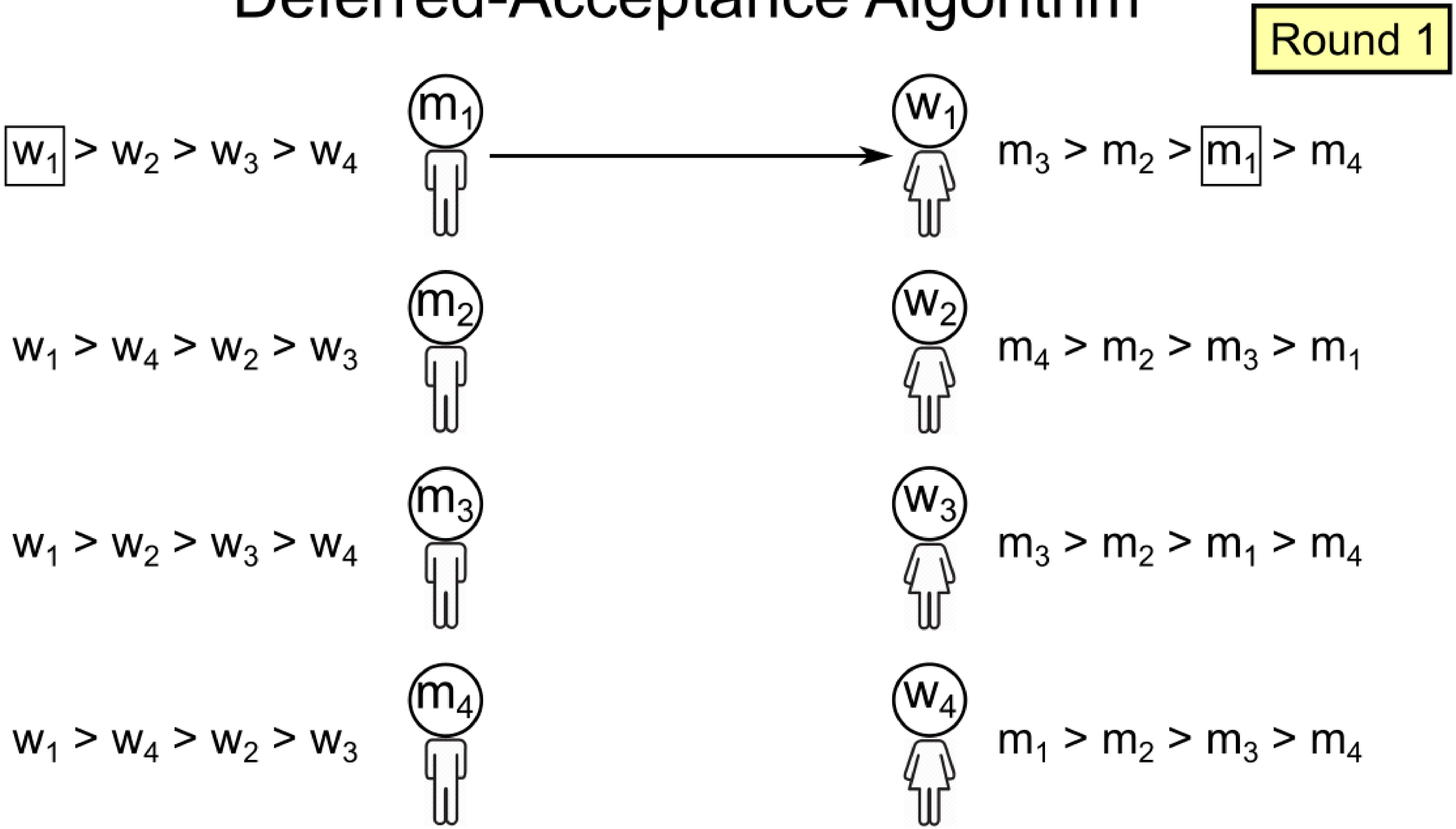

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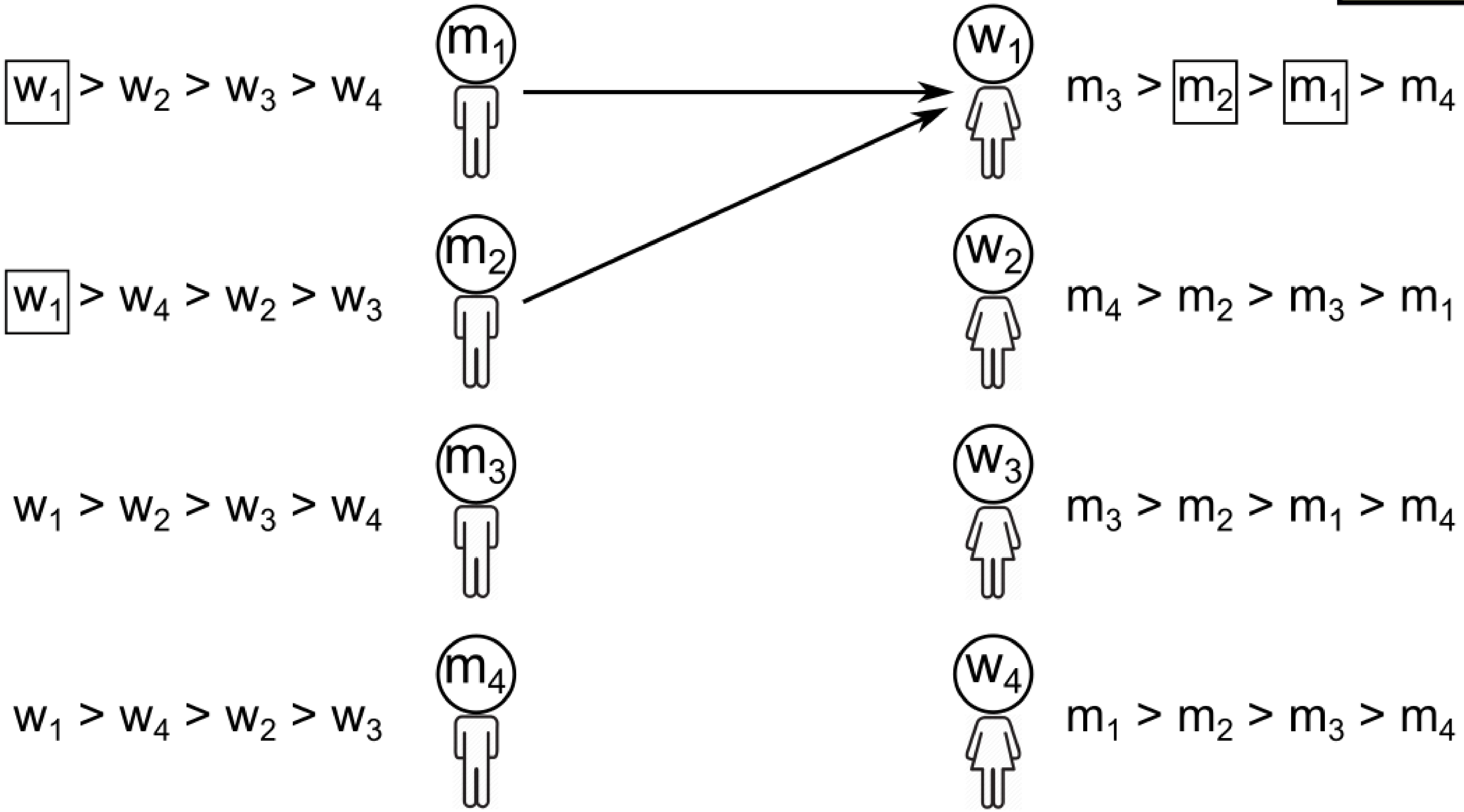
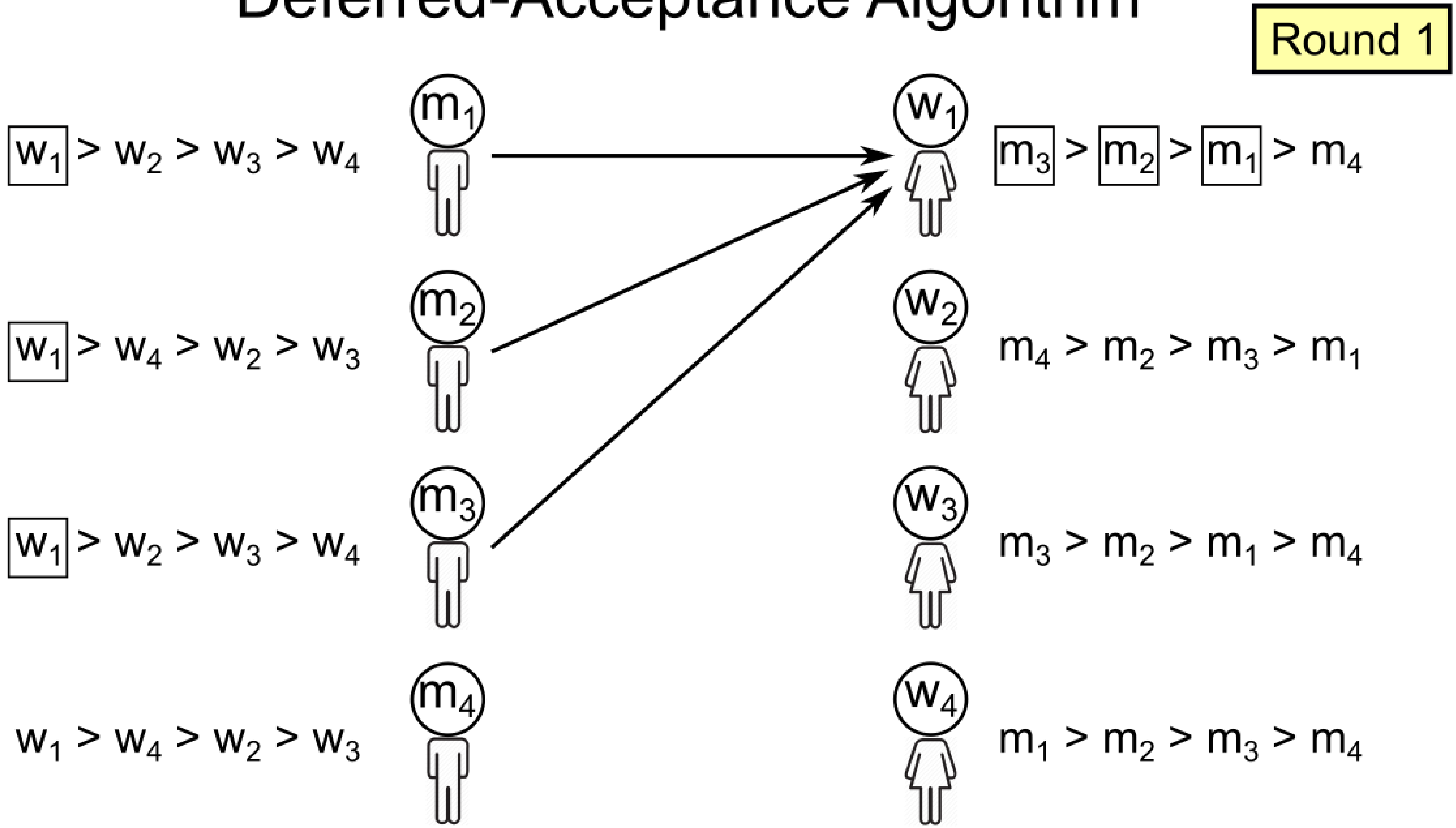


Deferred-Acceptance Algorithm



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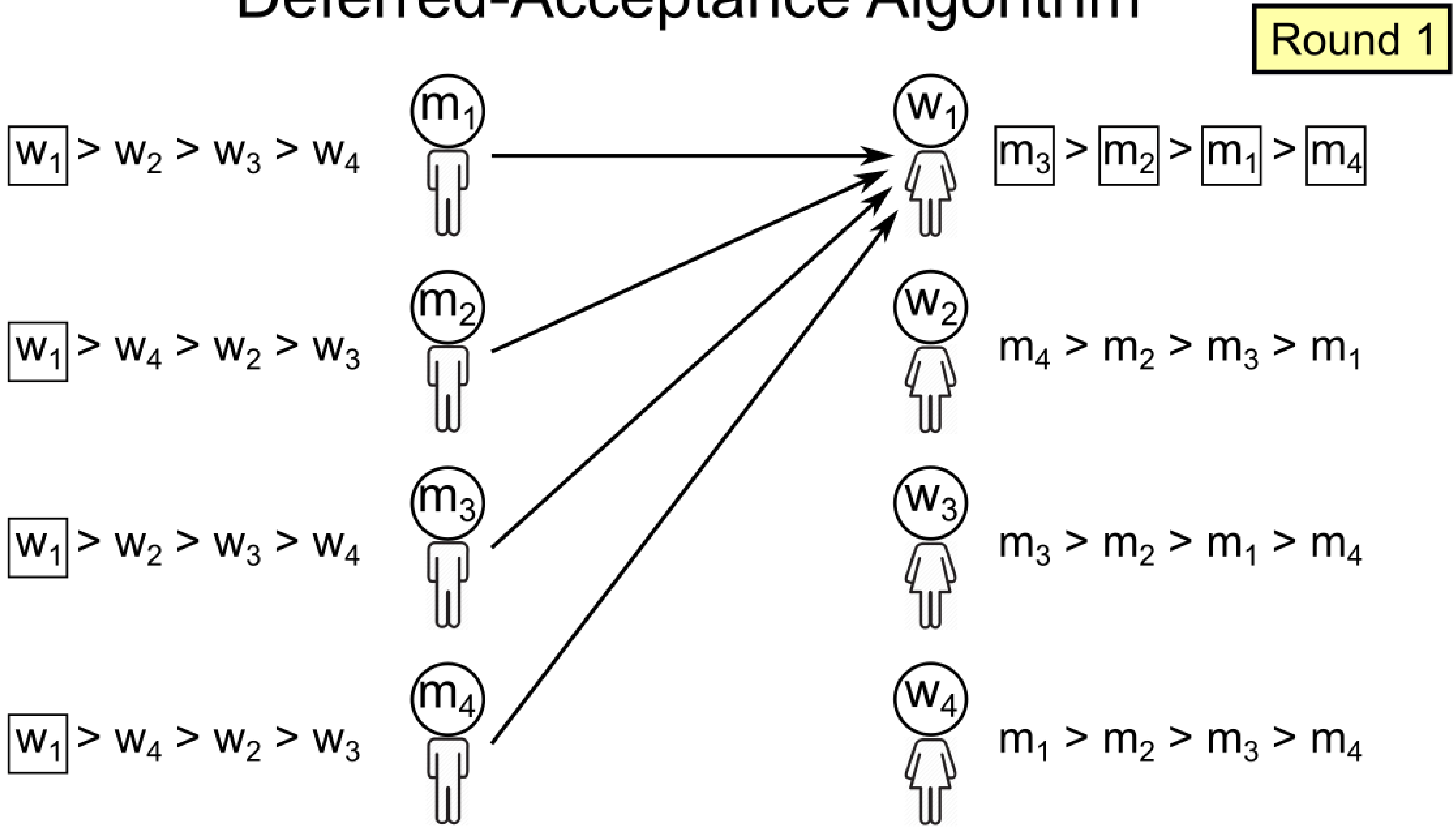

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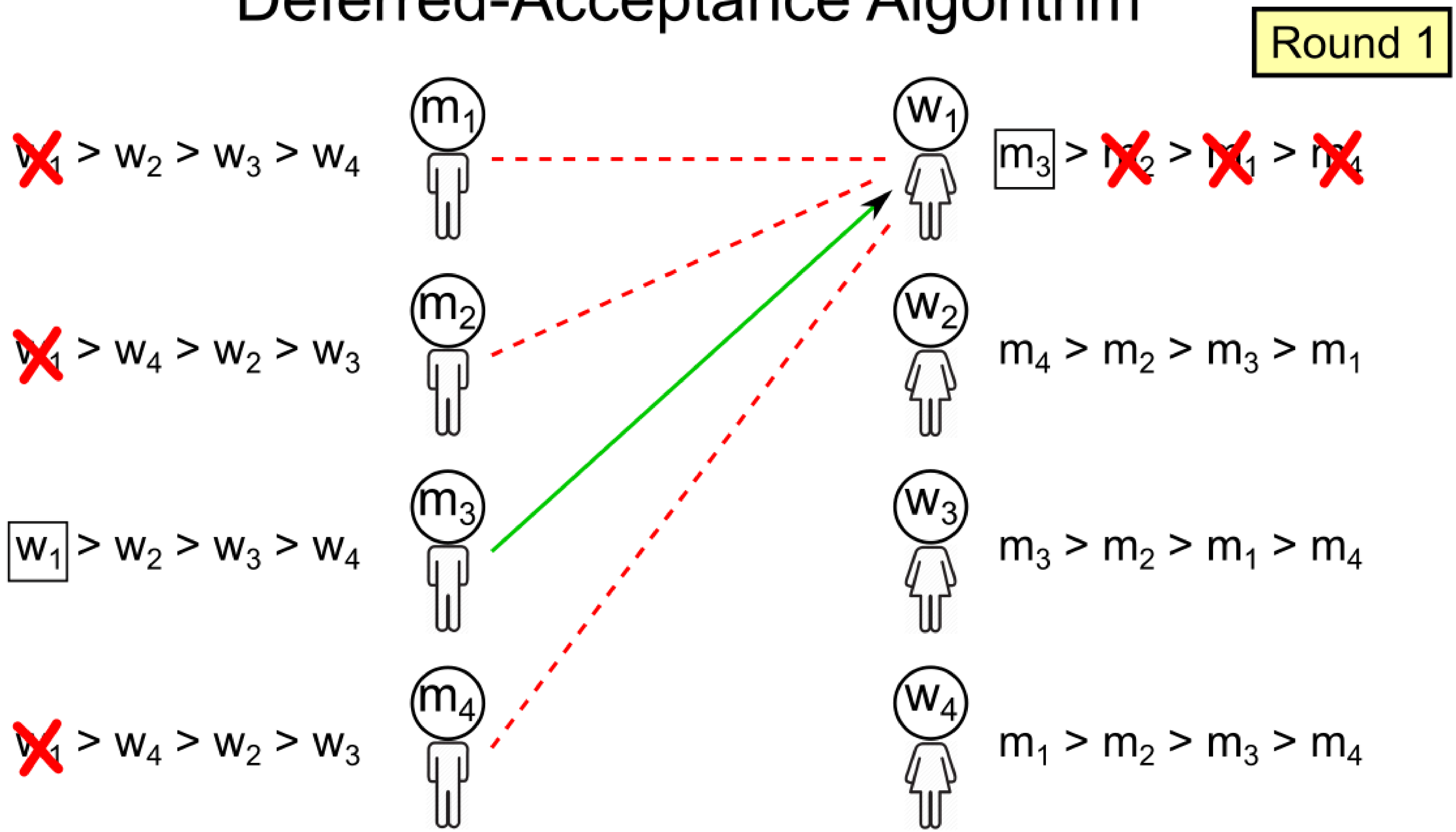

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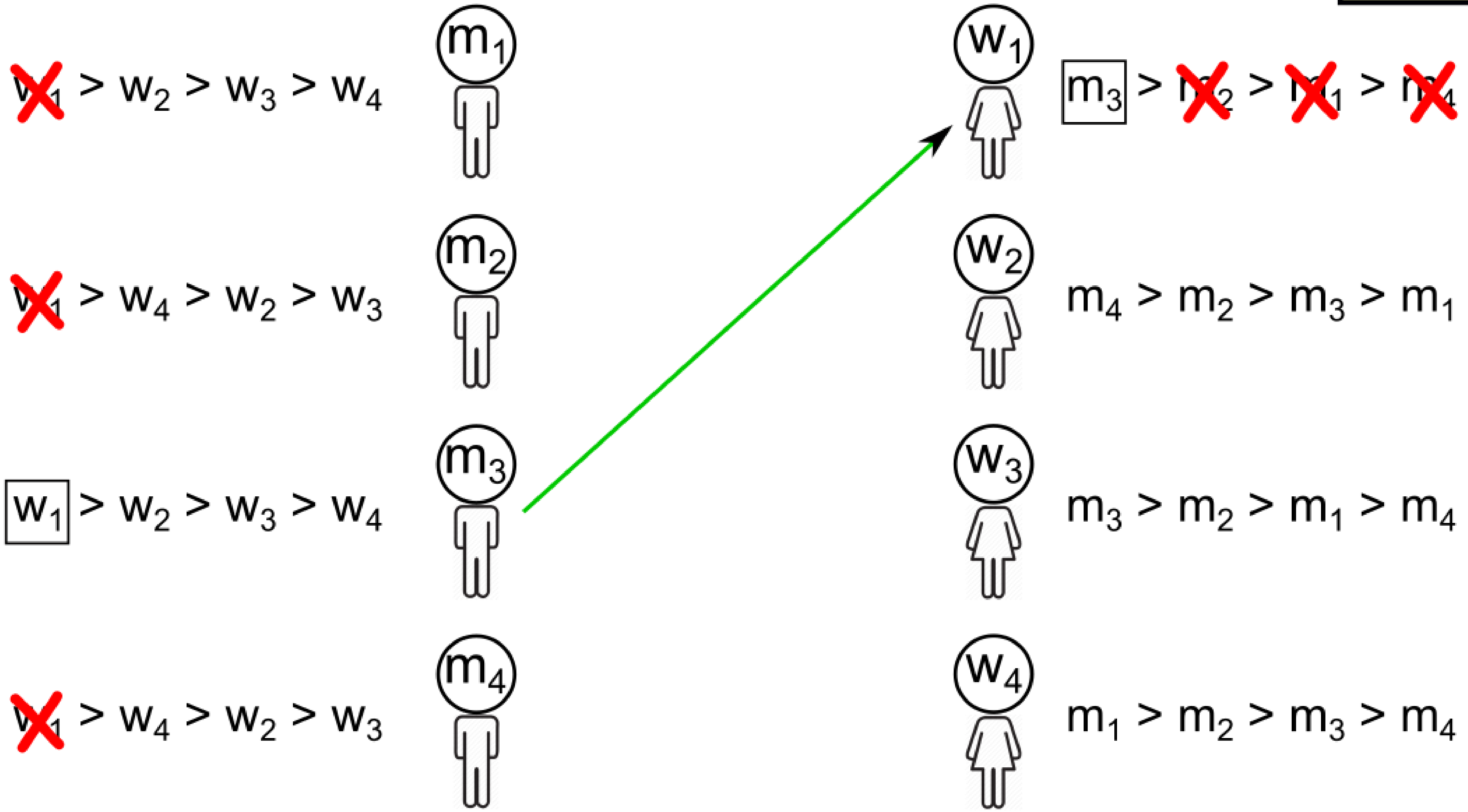

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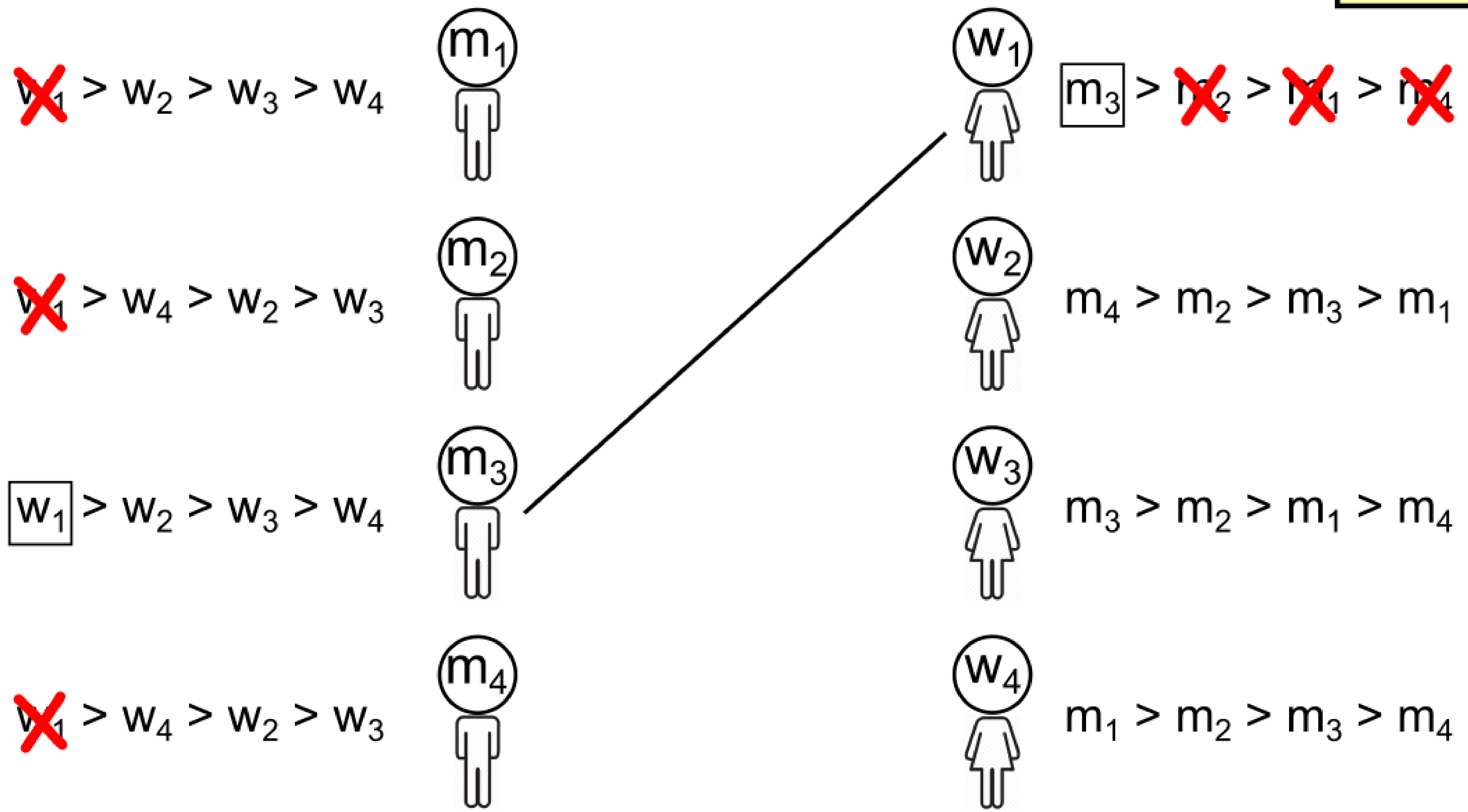


Deferred-Acceptance Algorithm

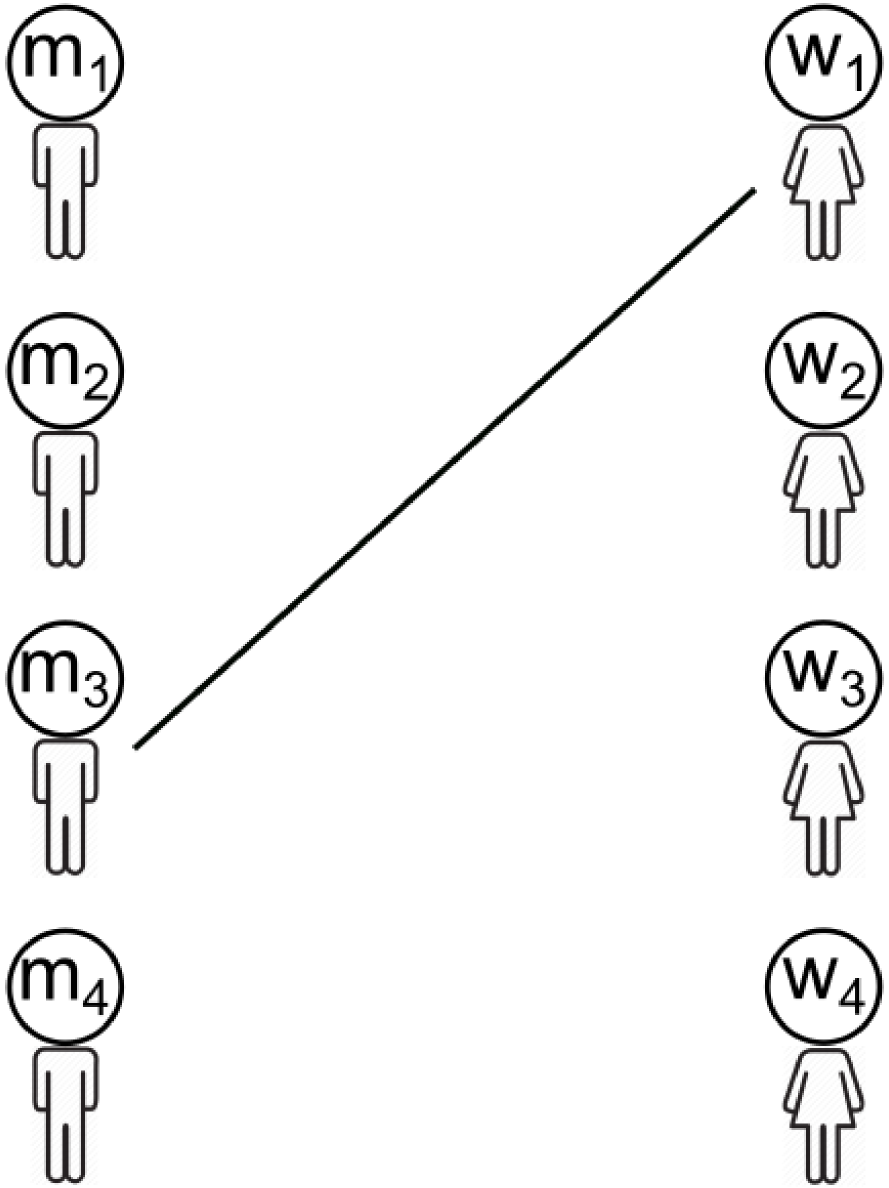
Round 2

m_1: ~~w_1~~ $> w_2 > w_3 > w_4$

m_2: ~~w_1~~ $> w_4 > w_2 > w_3$

m_3: [w_1] $> w_2 > w_3 > w_4$

m_4: ~~w_1~~ $> w_4 > w_2 > w_3$



w_1: [m_3] $>$ ~~m_2~~ $>$ ~~m_1~~ $>$ ~~m_4~~

w_2: $m_4 > m_2 > m_3 > m_1$

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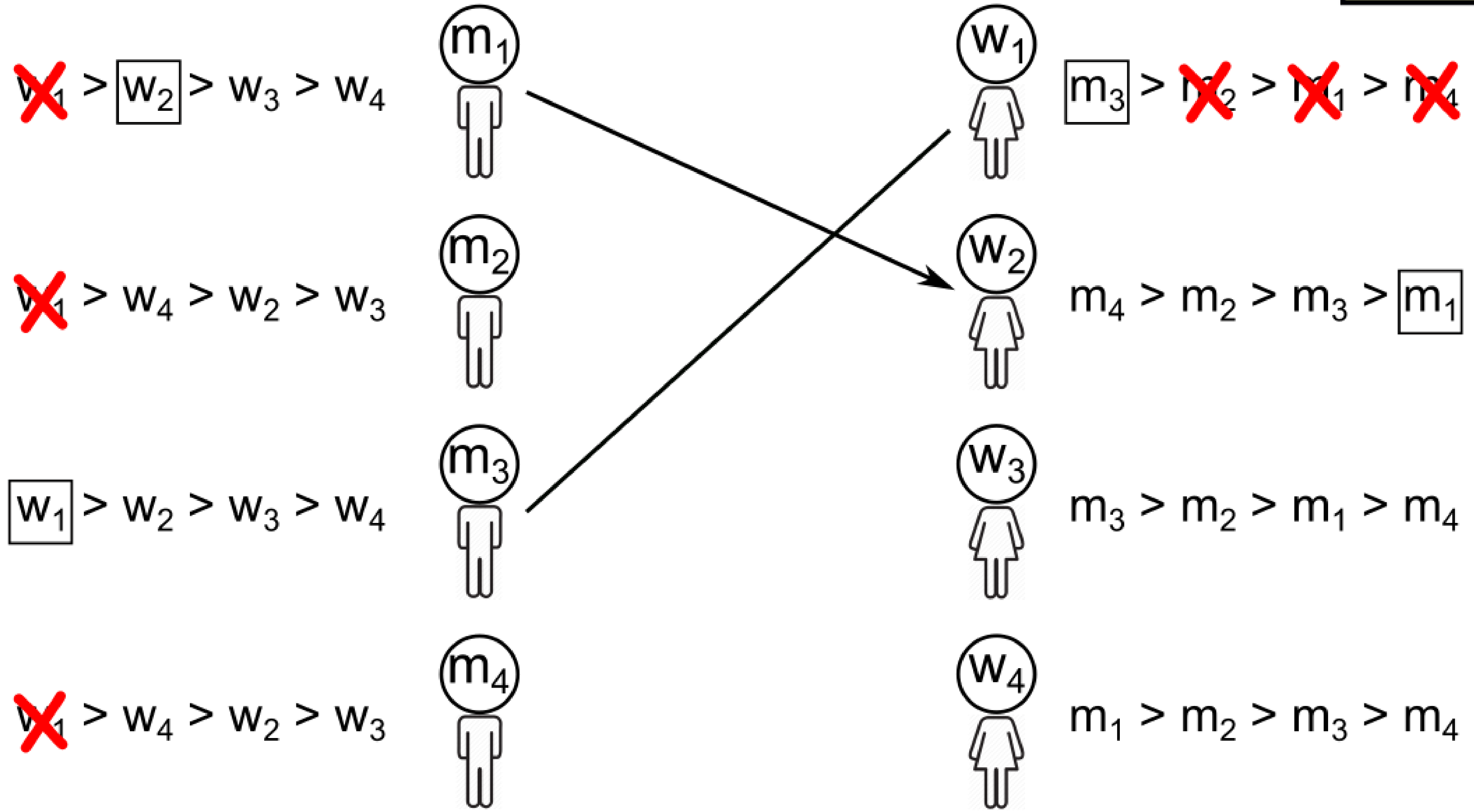
Round 2



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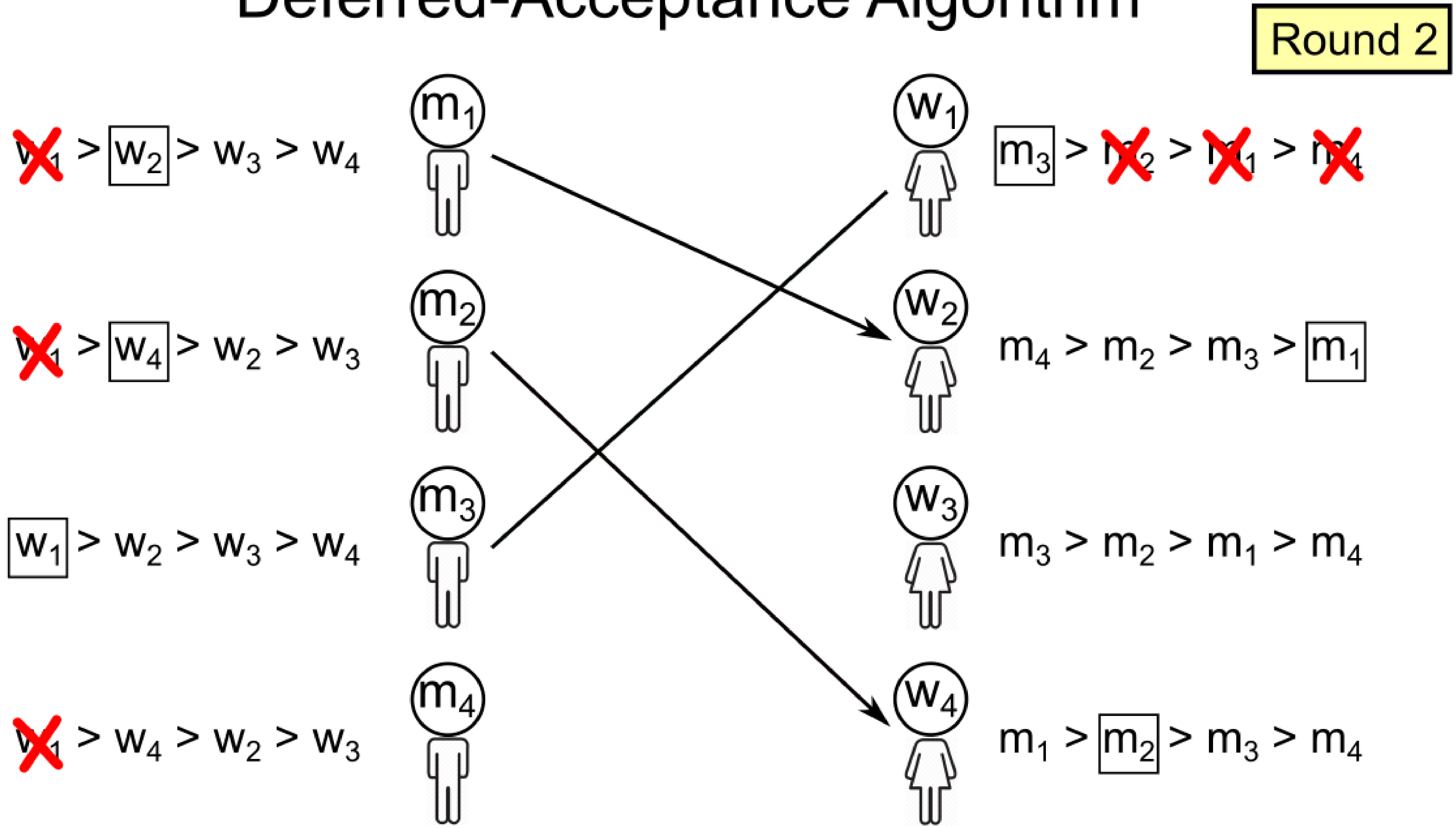

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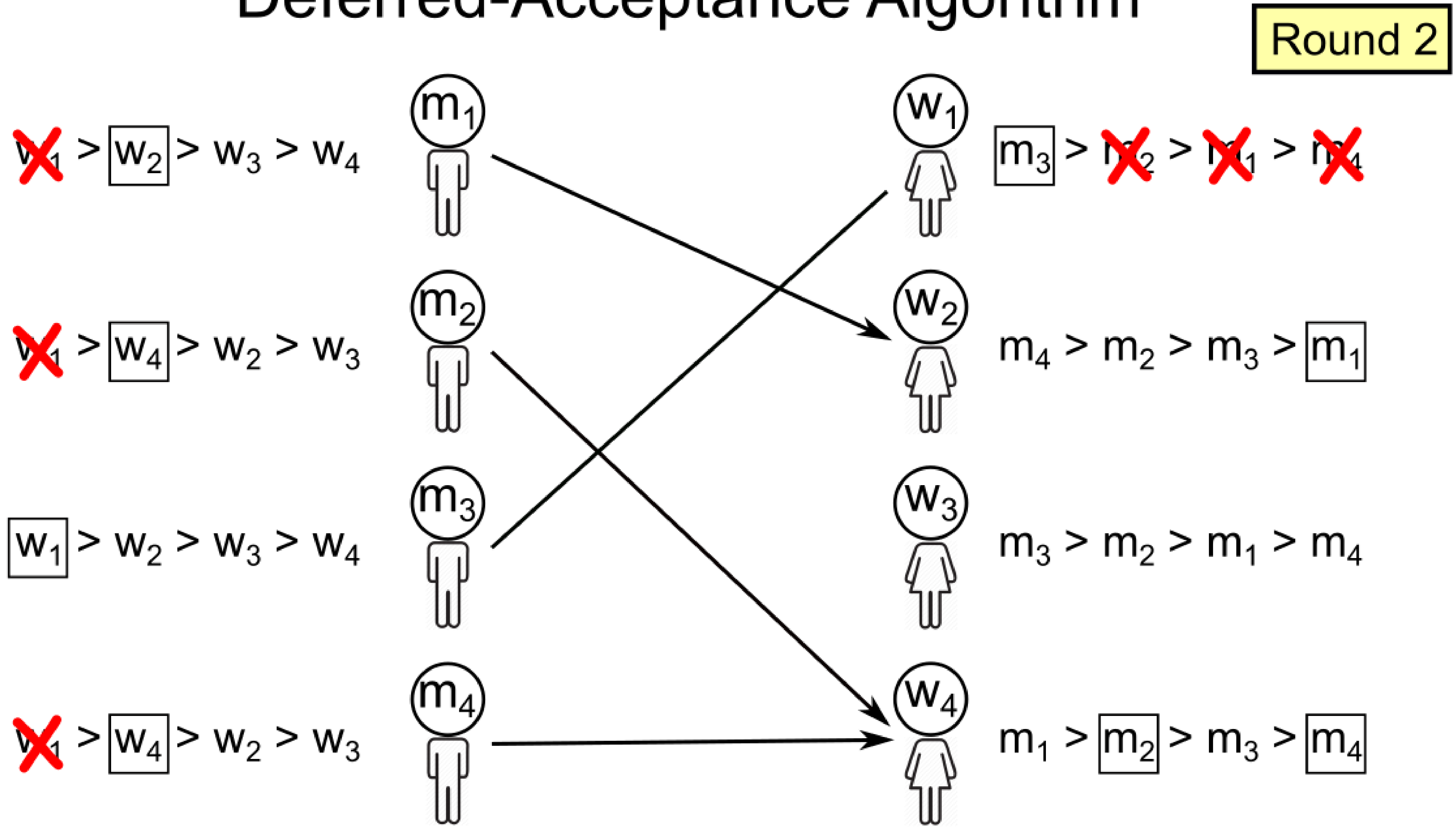

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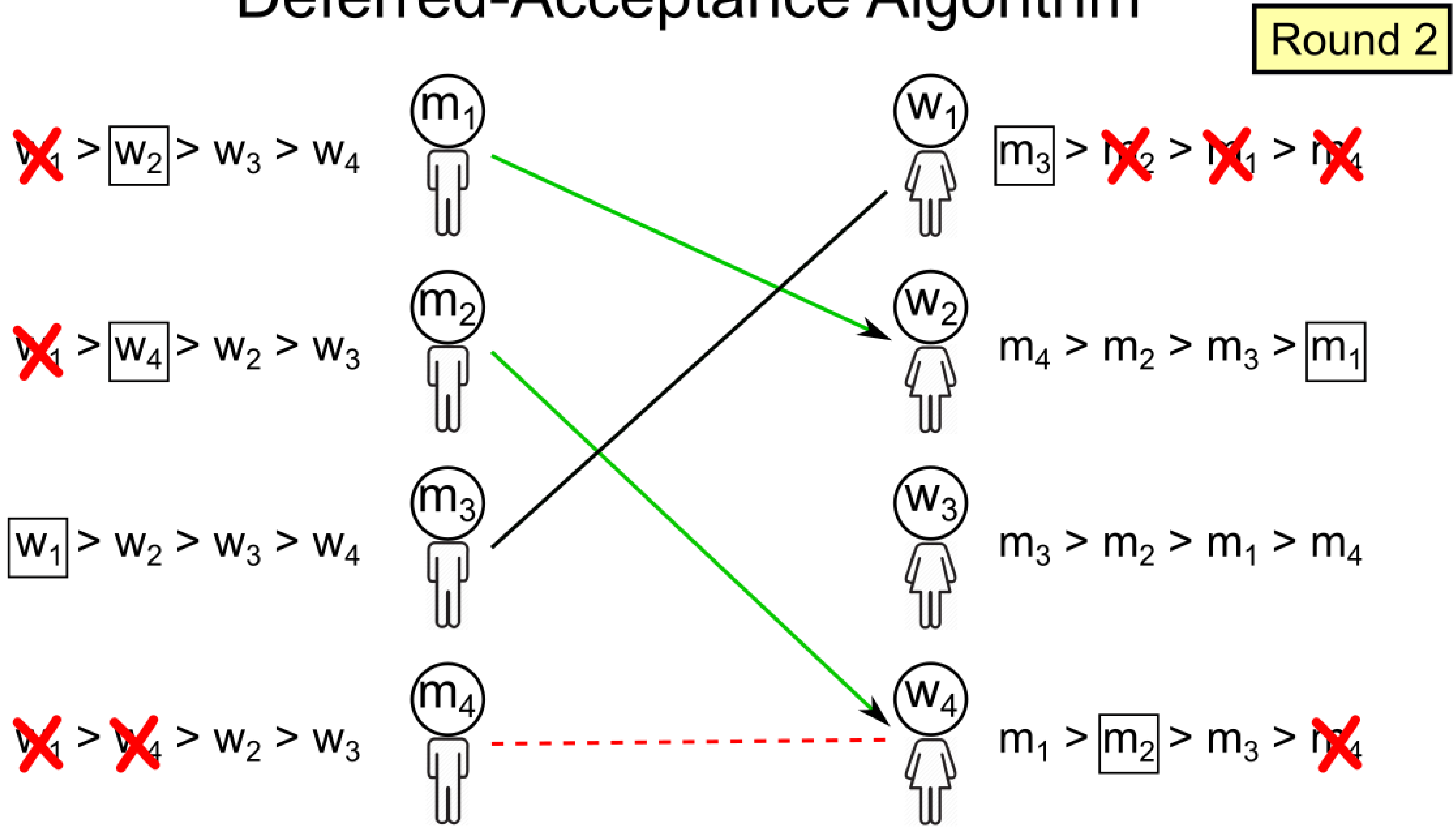

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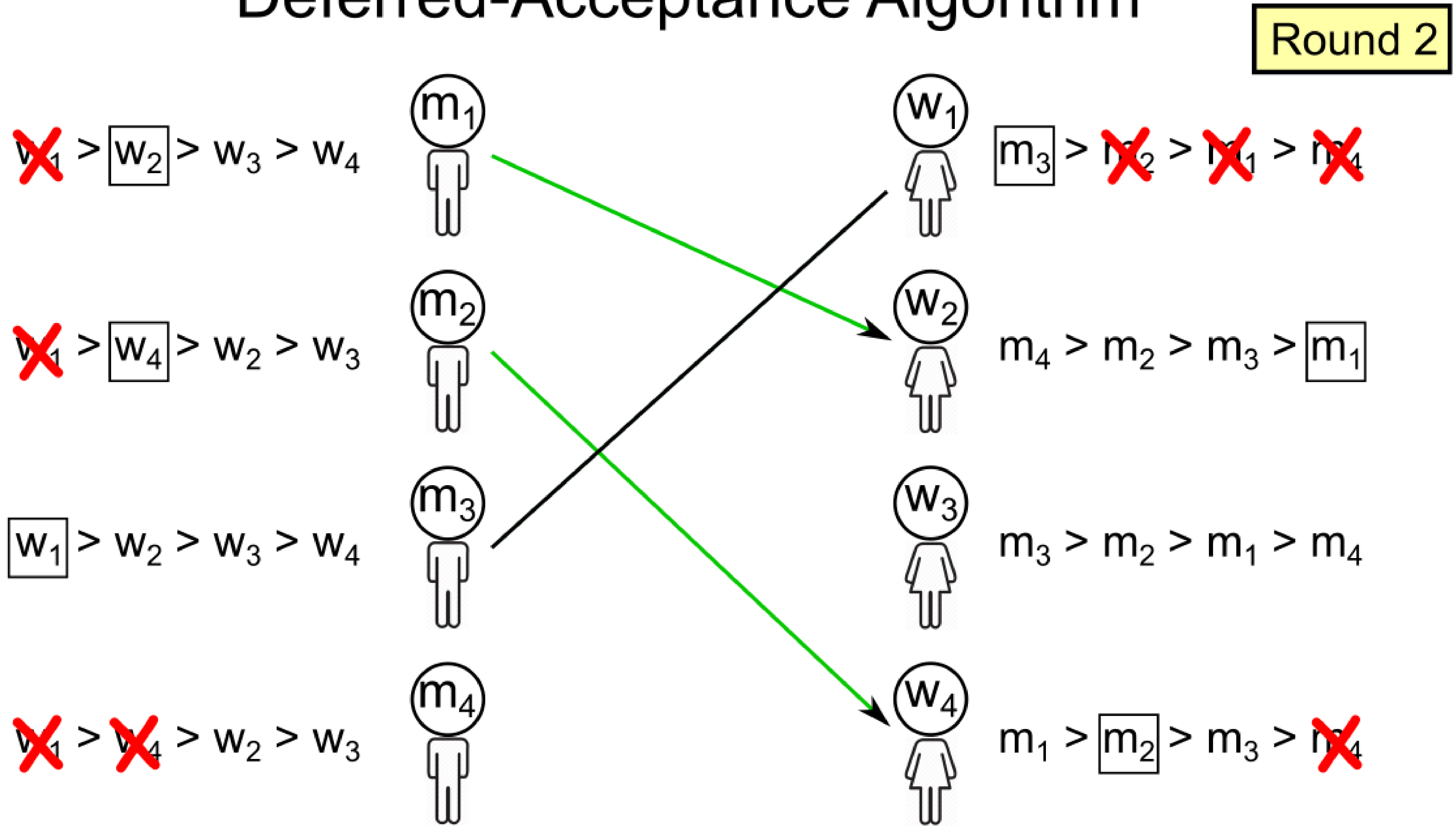

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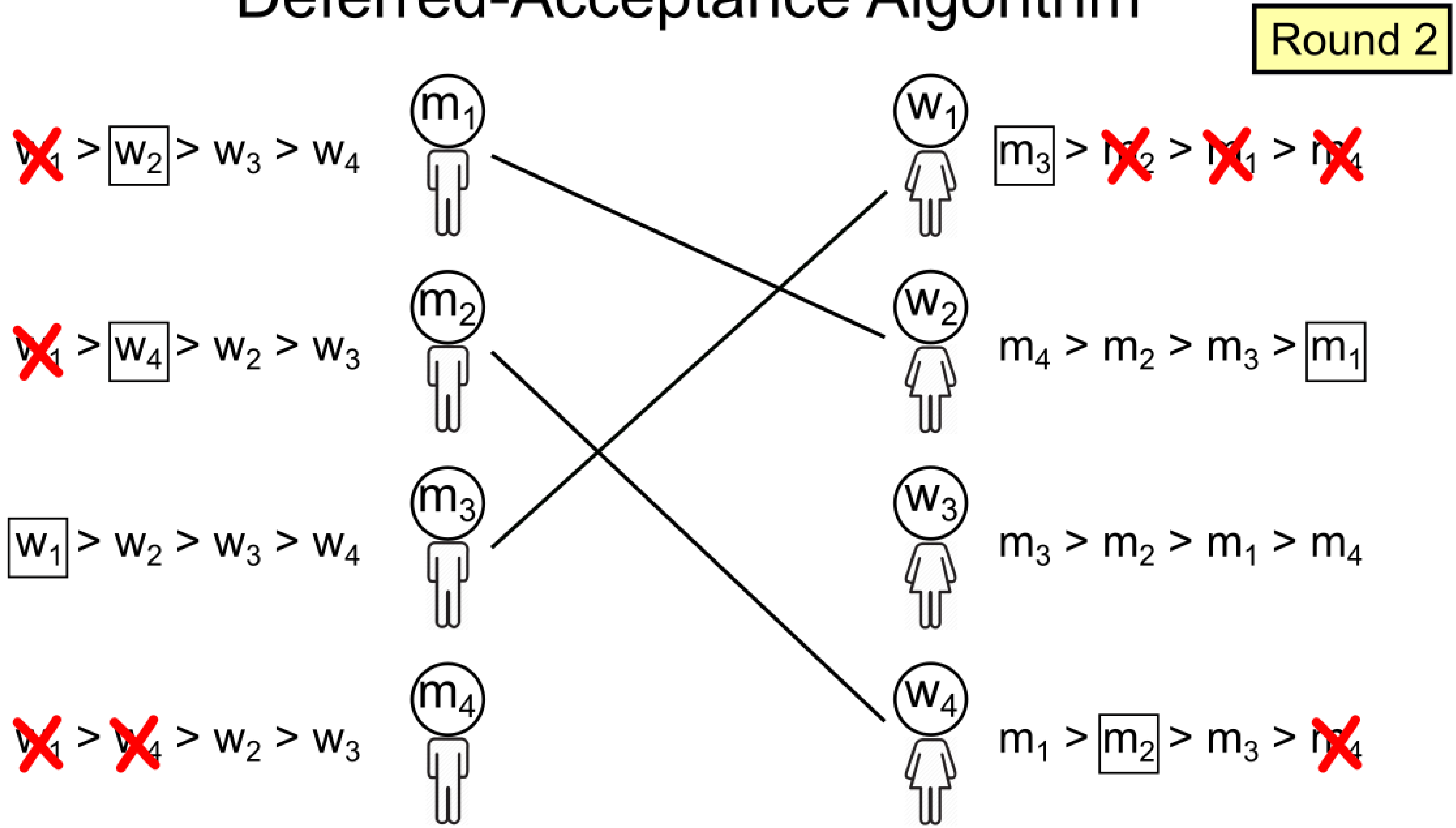
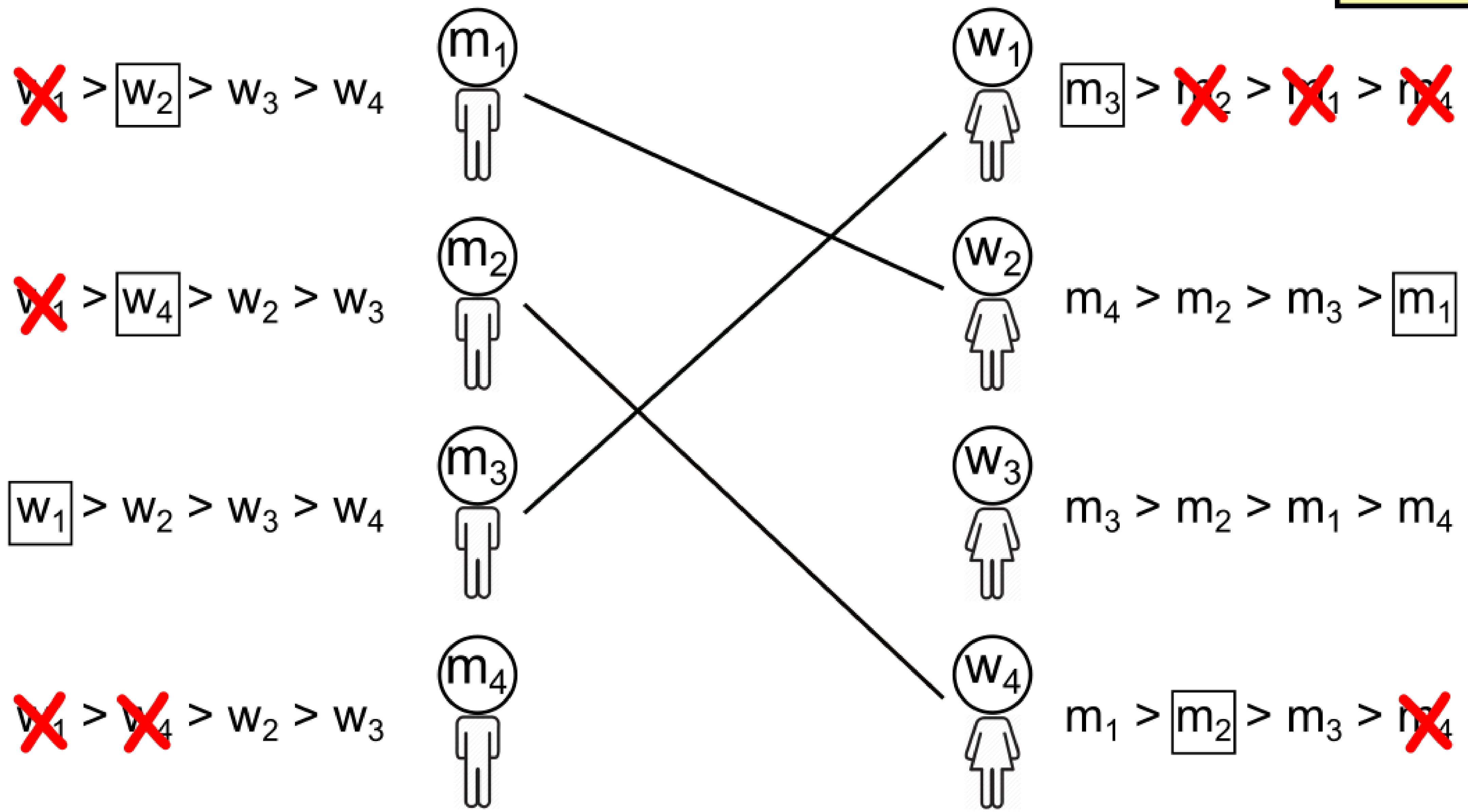


Deferred-Acceptance Algorithm

Round 3



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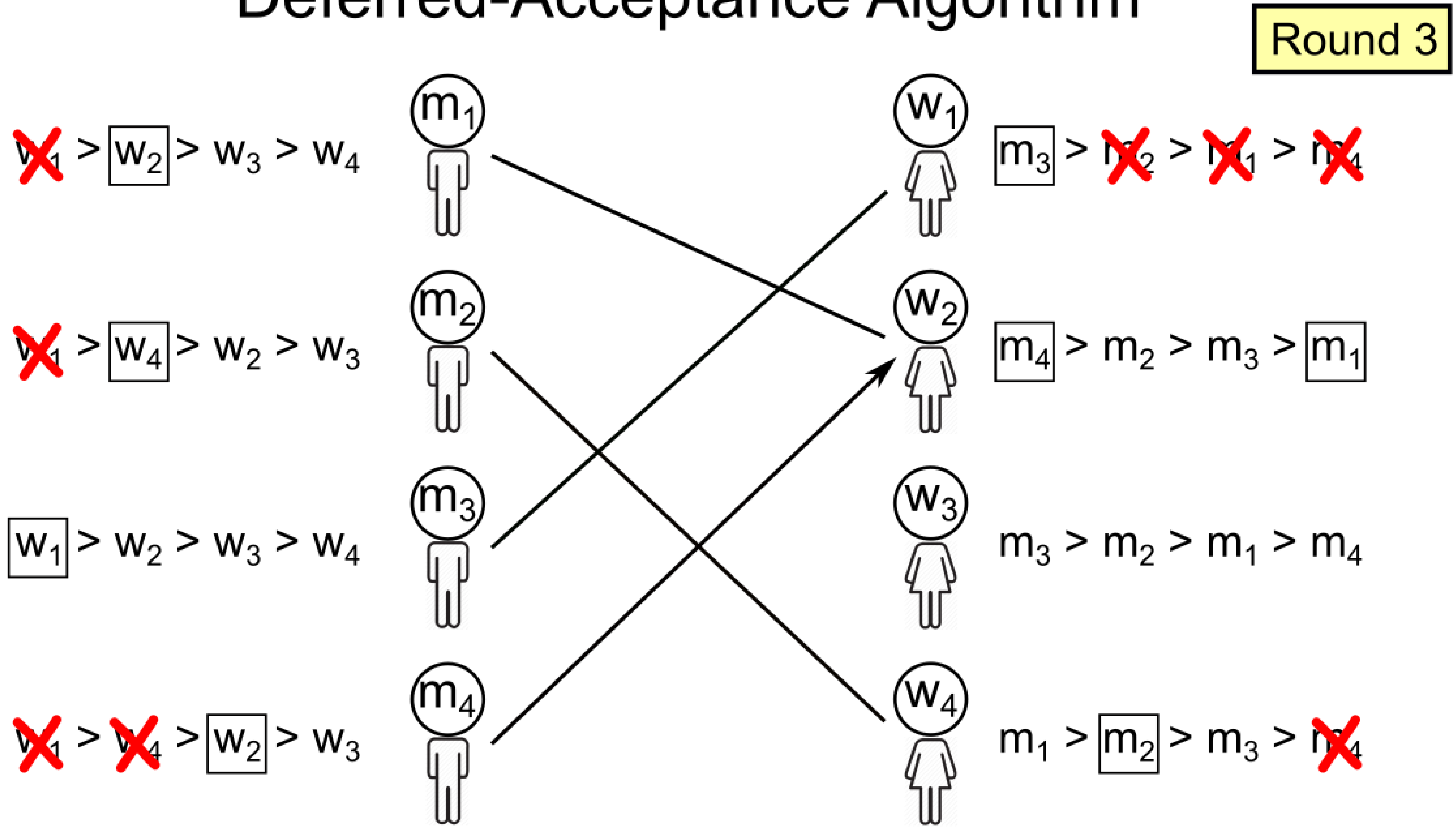

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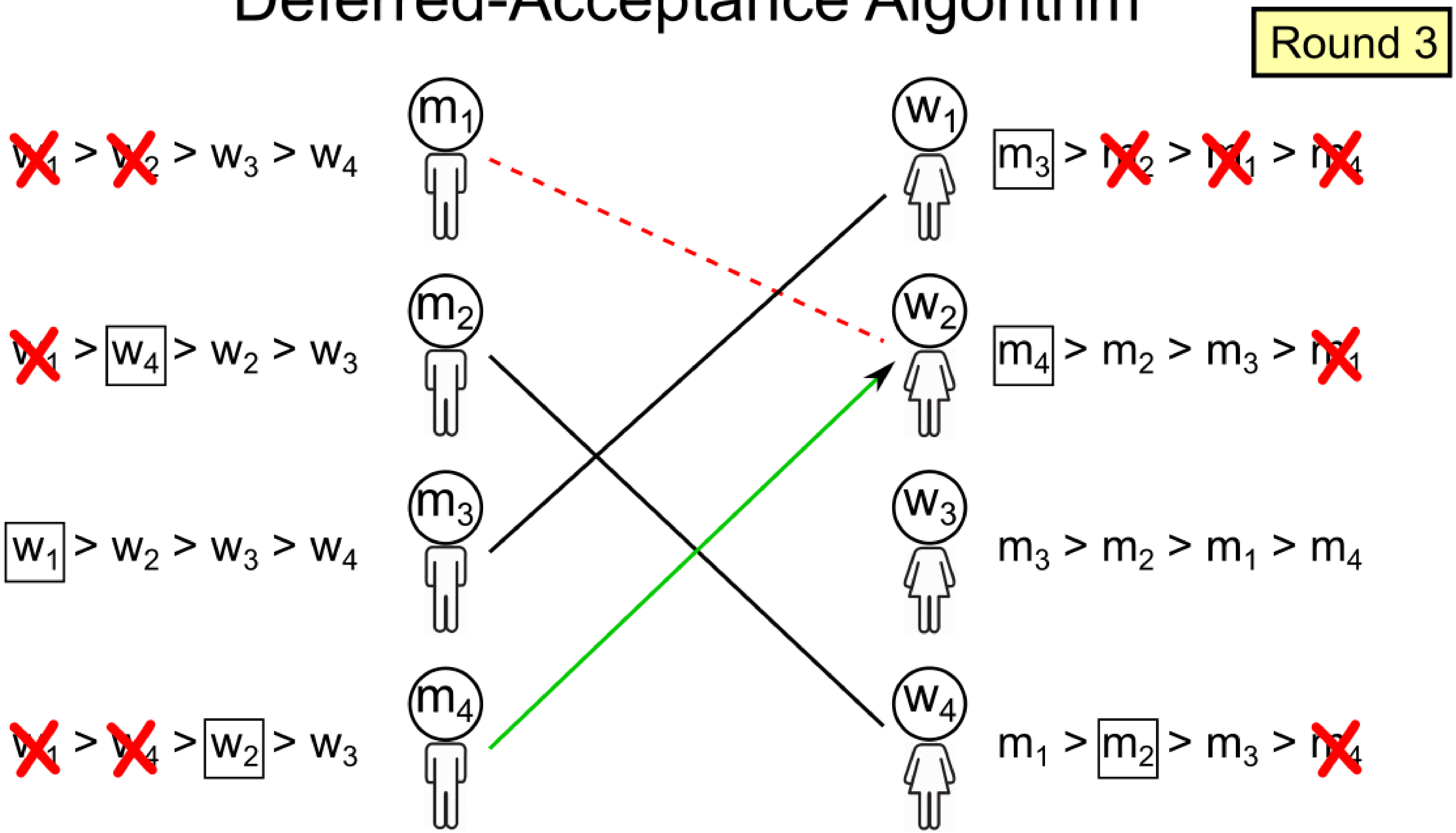

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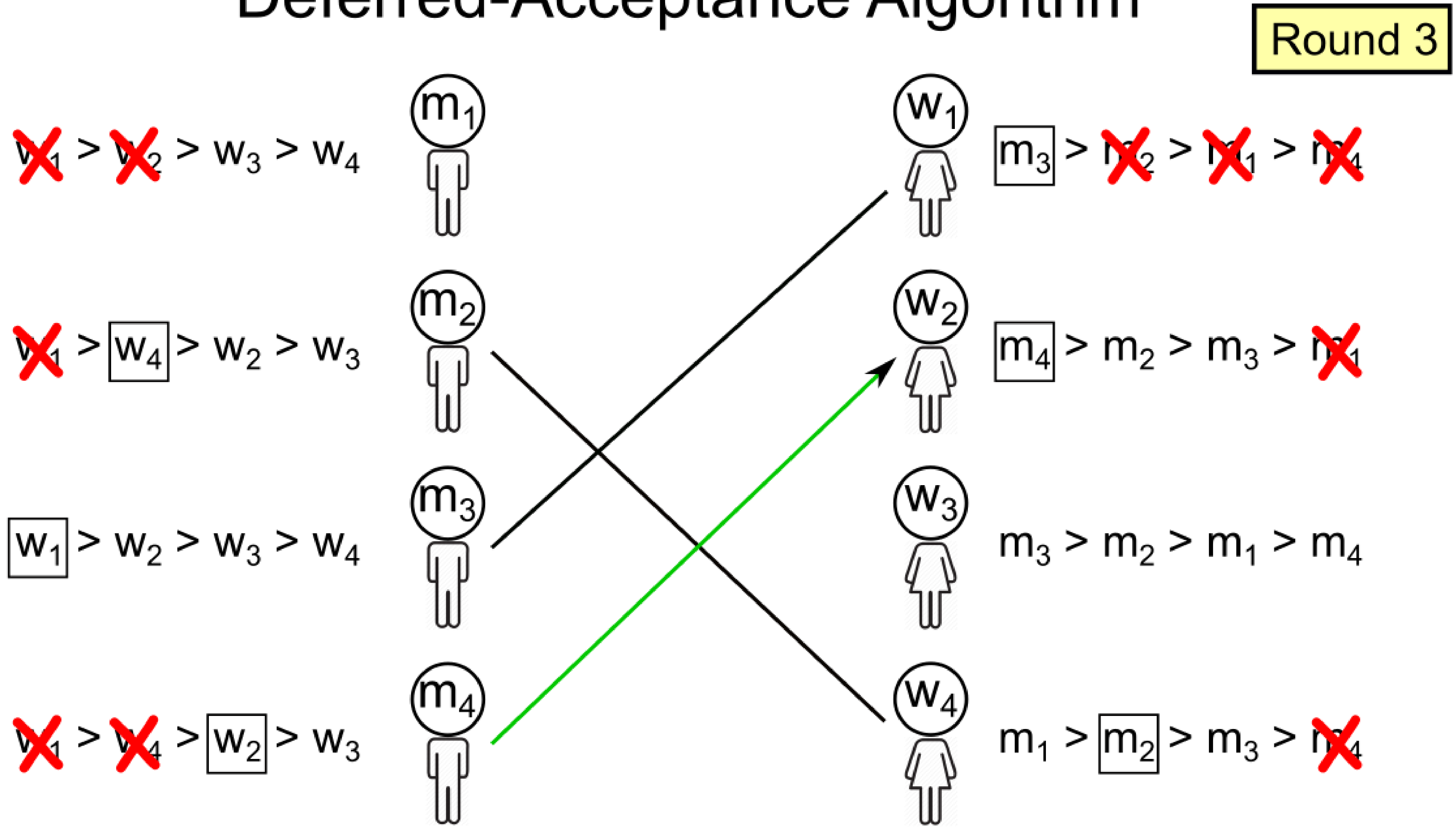

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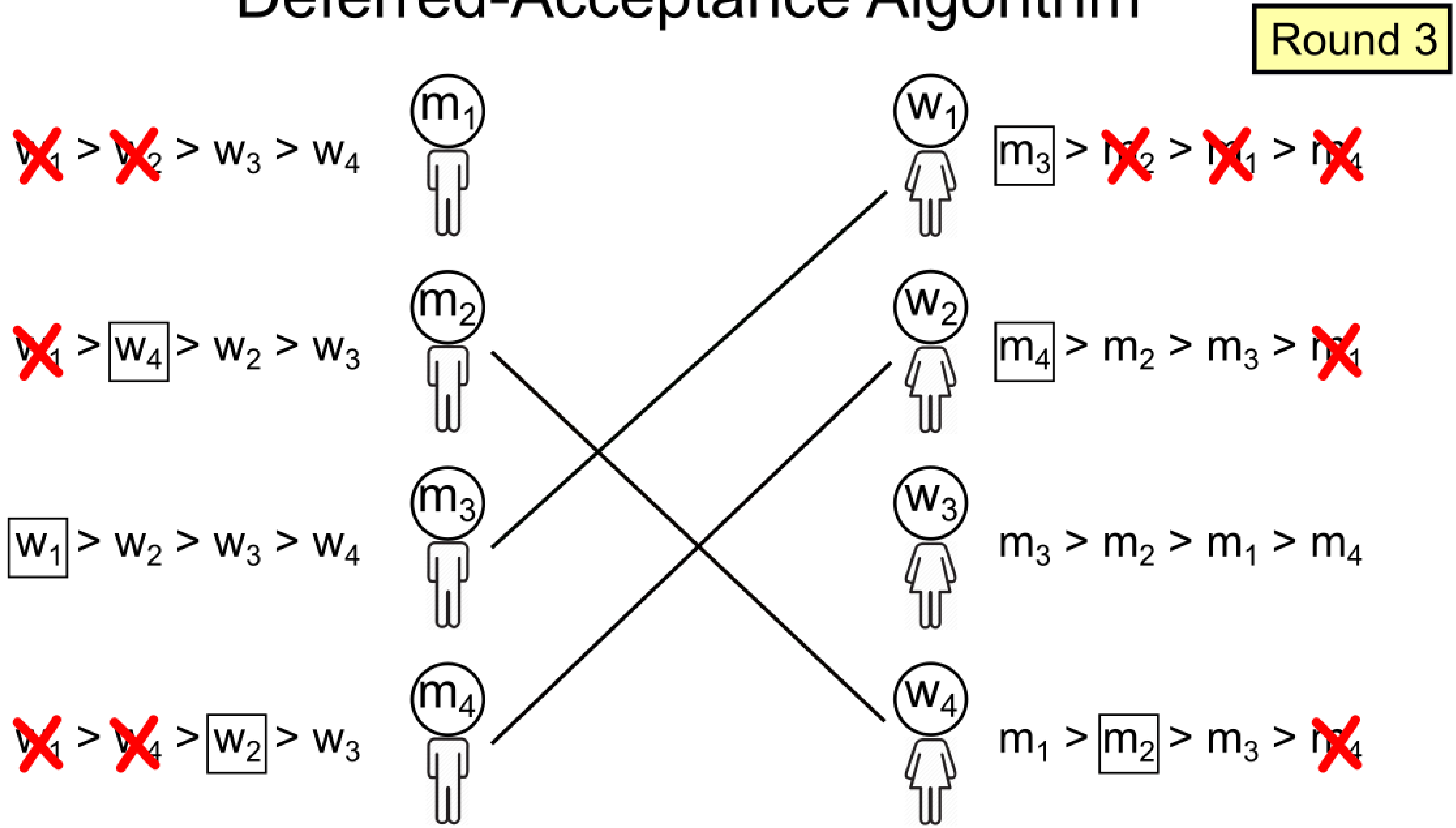
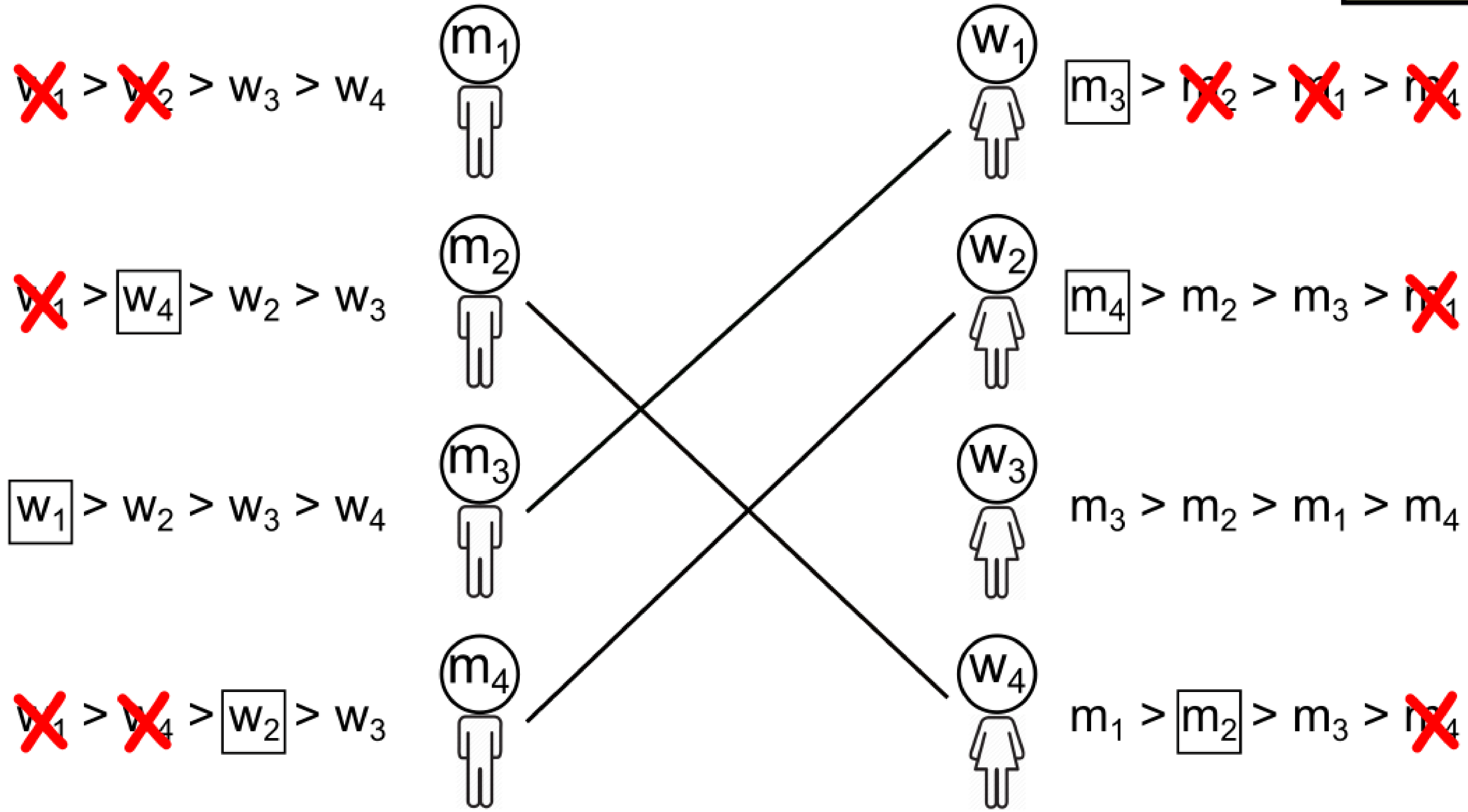


Deferred-Acceptance Algorithm

Round 4



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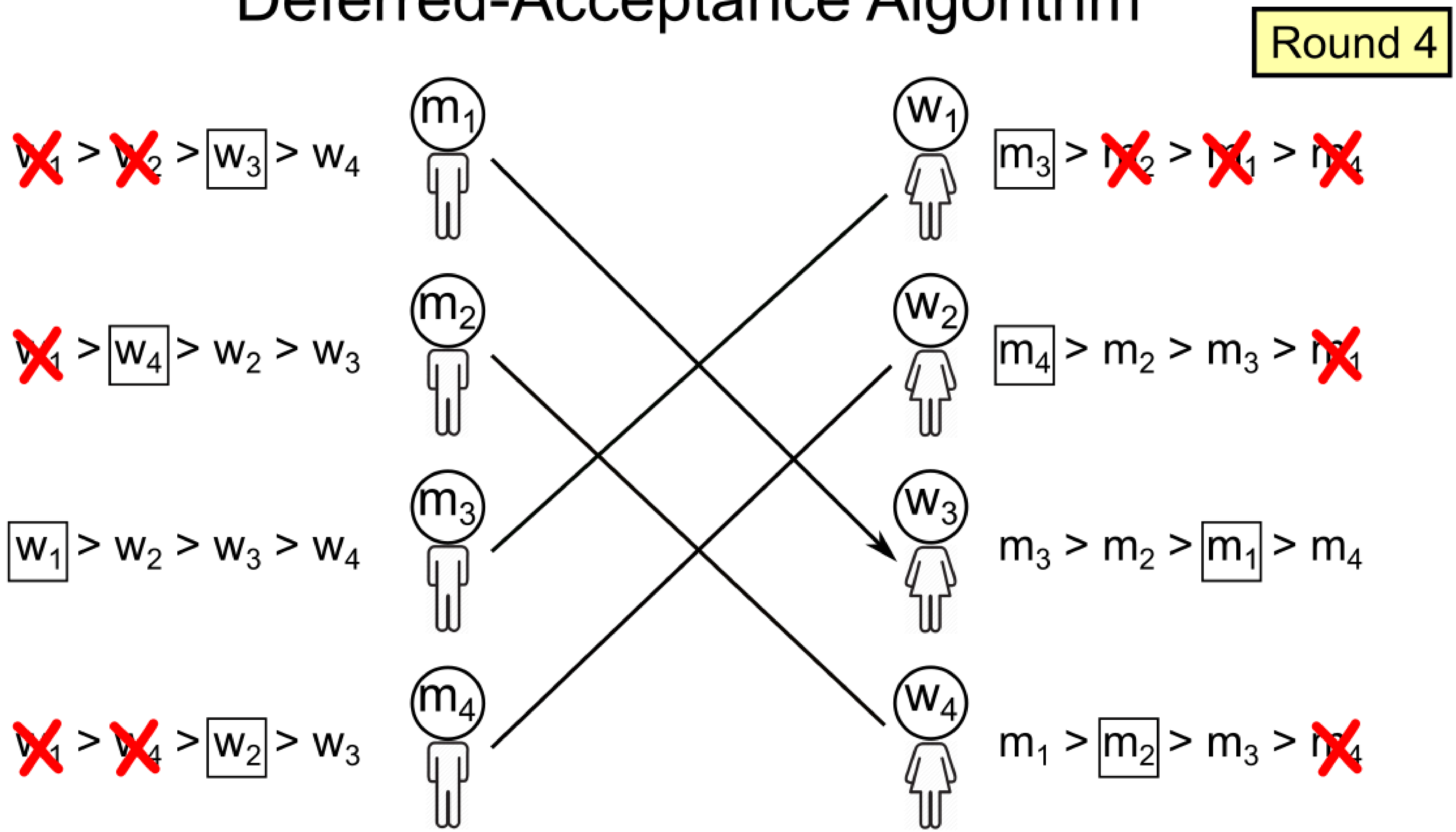

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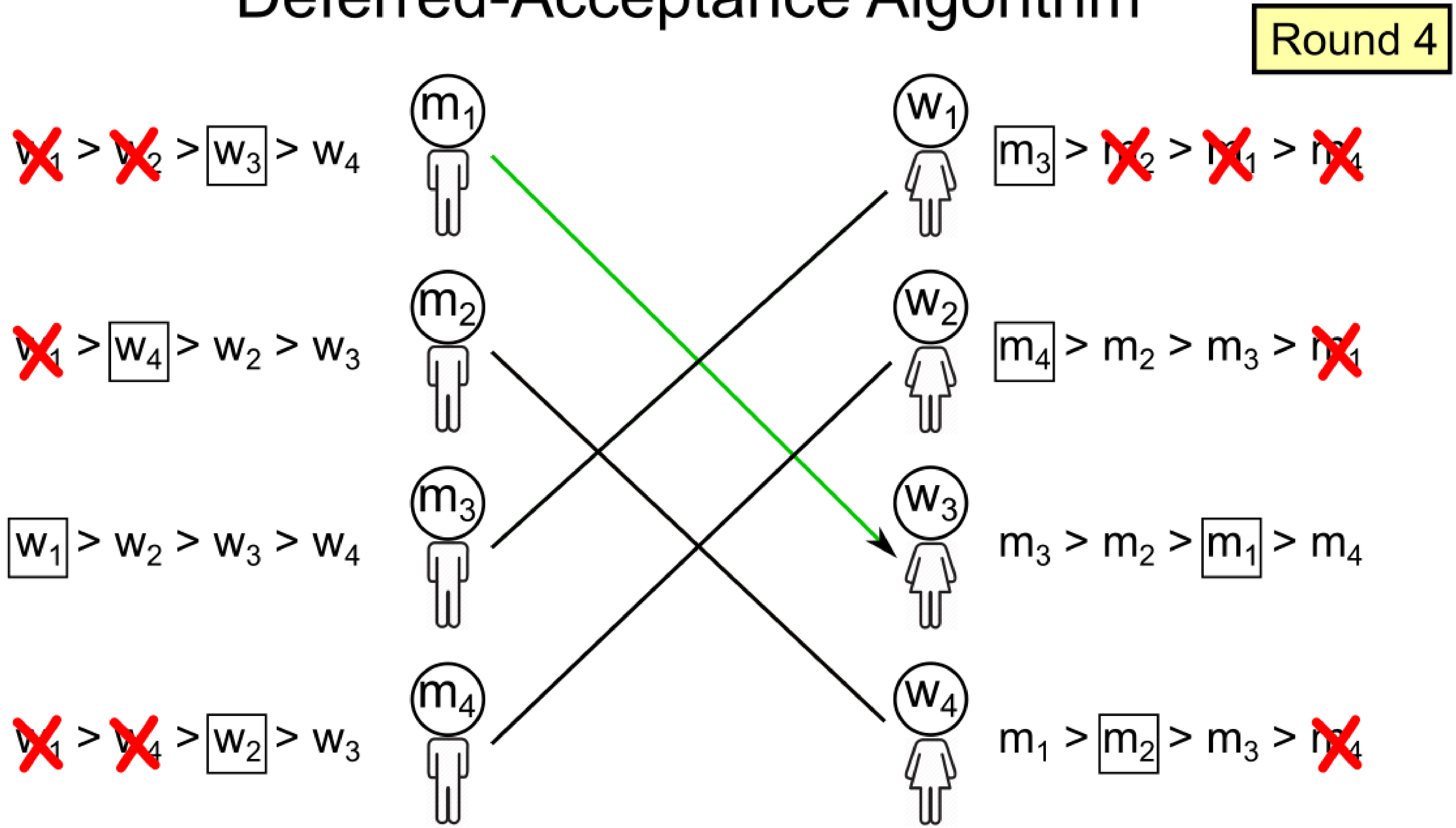

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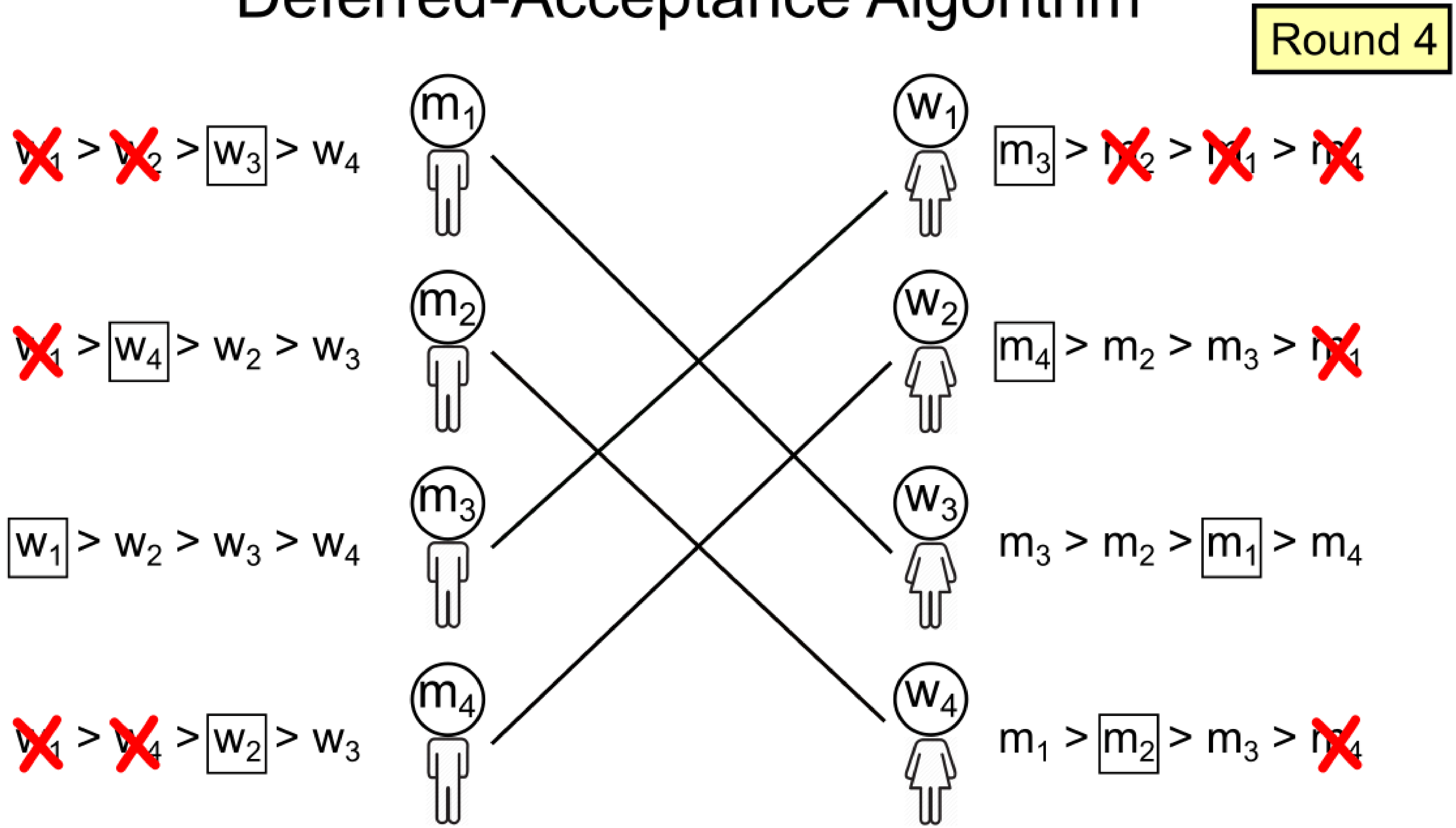

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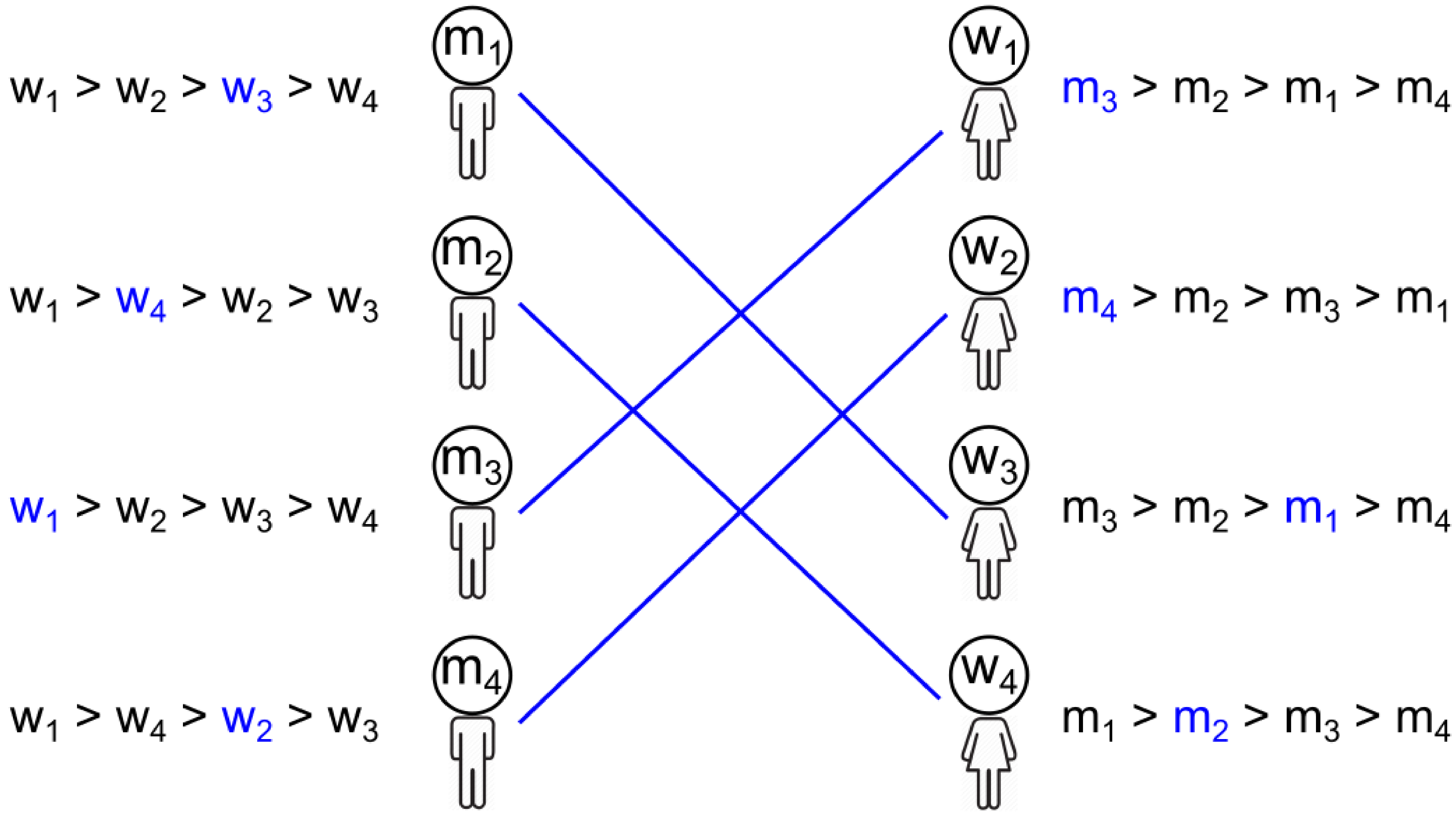

1

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Deferred-acceptance algorithm terminates in polynomial time.

2

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2

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Once tentatively matched, a woman never becomes unmatched.

3

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3

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Women only "trade up" during the DA algorithm.

Applications


HOSPITAL


UNHCR

Trends in Computational Social Choice

CHAPTER 18

Applications of Matching Models under Preferences

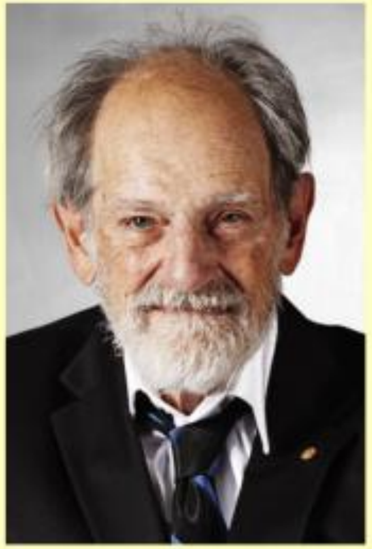
Péter Biró

18.1 Introduction

Matching problems under preferences have been studied widely in mathematics, computer science and economics, starting with the seminal paper by Gale and Shapley (1962). A comprehensive survey on this topic was published also in Chapter 14 of the Handbook of Computational Social Choice (Klaus et al., 2016), and for the interested reader we recommend consulting the following four comprehensive books on the computational (Gusfield and Irving, 1989; Manlove, 2013) and game-theoretical, market design aspects (Roth and Sotomayor, 1990; Roth, 2015) of this topic. In this chapter our goal is to give a general overview of the related applications.

Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012

Alvin E. Roth



Lloyd S. Shapley

"for the theory of stable allocations and the practice of market design."

Logistics

Logistics

- *Course website*: https://rohitvaish.in/Teaching/2025-Spring/

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Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)

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Preferably in LaTeX
(~6 problems, ~*ten days*)

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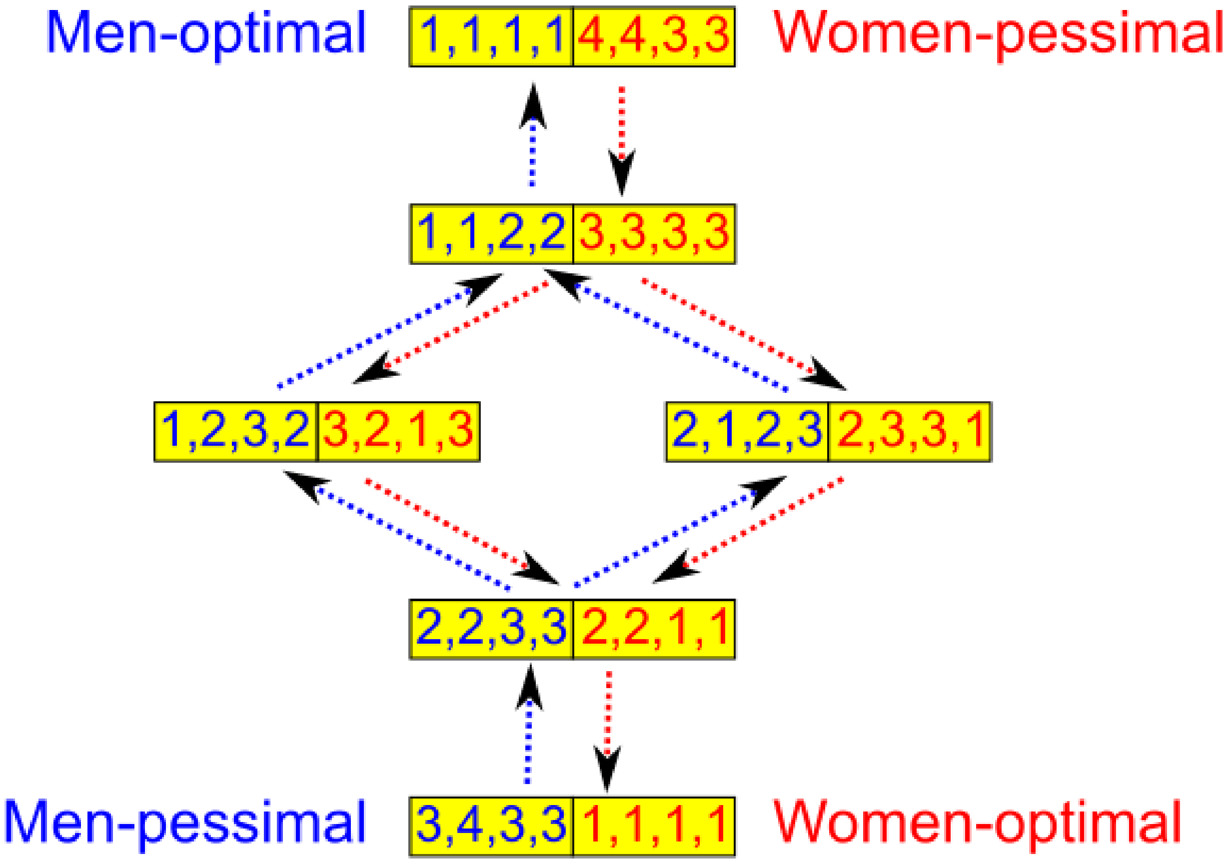
Next Time

Structure
of
Stable Matchings



References

- Stability and the Deferred Acceptance Algorithm

David Gale and Lloyd Shapley
"College Admissions and the Stability of Marriage"
American Mathematical Monthly, 69(1), 1962 pg 9-15