COL202: Discrete Mathematical Structures **Spring 2024**

Tutorial Sheet 12

Announced on: Apr 09 (Tue)

1. Based on Sec 17.3 in [LLM17].

   Let A, B, and C be three 3-sided dice. Die A contains the numbers 2, 6, and 7, die B has 1, 5, and 9, and die C has 3, 4, and 8. Upon rolling any die, there is an equal chance of getting one of its three numbers.

   Suppose you and your friend play a game in which each person picks a different die and rolls it (i.e., picks one of its three numbers on that die with equal probability). The person who rolls a larger number wins that roll.

   a) Suppose you pick die A and your friend picks die C. What is the probability that you win?

   b) Suppose you pick die B and your friend picks die A. What is the probability that you win?

   c) Suppose you pick die C and your friend picks die B. What is the probability that you win?

   d) Suppose you pick a die first and then your friend picks one from the remaining dice. Which die would you pick to give yourself a better-than-50% chance of winning (without assuming anything about what your friend will pick)?

2. Let's modify the dice from Problem 1 as follows: Die A now contains the numbers 2, 4, and 9, die B has 1, 6, and 8, and die C has 3, 5, and 7.

   How do the answers to questions (a)-(d) change after this modification?

3. Based on Problem 17.10 in [LLM17].

   Let $A_1, A_2, \ldots, A_n$ be n events. Using the sum rule from counting, show that

   $$\Pr\left(\bigcup_{i=1}^{n} A_i\right) \leqslant \sum_{i=1}^{n} \Pr(A_i).$$

   The above inequality is commonly known as *union bound*.

4. Based on Problem 18.12 in [LLM17].

   Suppose you repeatedly flip a fair coin until you see the sequence `HTT` or `HHT`. What is the probability you see the sequence `HTT` first?

References

[LLM17] Eric Lehman, Tom Leighton, and Albert R Meyer. *Mathematics for Computer Science*. 2017. URL: https://courses.csail.mit.edu/6.042/spring18/mcs.pdf.