

## Tutorial Sheet 10

Announced on: Mar 27 (Wed)

1. Based on Problem 22.1 in [LLM17].

The running time of an algorithm  $A$  is described by the recurrence  $T(n) = 7 \cdot T(n/2) + n^2$ . A competing algorithm  $B$  has a running time of  $T(n) = a \cdot T(n/4) + n^2$ . For what values of  $a$  is  $A$  asymptotically faster than  $B$ ? How does your answer change if the running time of  $B$  is modified to  $T(n) = aT(n/4) + n^4$ ?

2. In the Tower of Hanoi puzzle, suppose our goal is to transfer all  $n$  disks from peg 1 to peg 3, but we cannot move a disk directly between pegs 1 and 3. Each move of a disk must be a move involving peg 2. As usual, we cannot place a disk on top of a smaller disk.
  - a) Find a recurrence relation for the number of moves required to solve the puzzle for  $n$  disks with this added restriction.
  - b) Solve the recurrence derived in part (a).

3. Based on Problem 22.2 in [LLM17].

Use the Akra-Bazzi formula to find  $\Theta(\cdot)$  asymptotic bounds for the following divide and-conquer recurrence:

$$T(n) = 3T(\lfloor n/3 \rfloor) + n,$$

where  $T(0) = 0$ .

4. Based on Problem 16.15 in [LLM17].

Solve the linear recurrence given below:

$$T(n) = 2T(n-1) + 2T(n-2) \text{ for } n > 1,$$

where  $T(1) = 1$  and  $T(0) = 0$ .

5. Based on Problem 15.3, 15.6, and 15.33 in [LLM17].

- a) How many functions are there in total from set  $A$  to set  $B$  if  $|A| = 3$  and  $|B| = 7$ ?
- b) How many of the billion numbers in the integer interval  $[1, 10^9]$  contain the digit 1 in their decimal representation?
- c) There is a robot that steps between integer positions in 3-dimensional space. Each step of the robot increments one coordinate and leaves the other two unchanged. How many paths can the robot follow going from the origin  $(0, 0, 0)$  to  $(3, 4, 5)$ ?

6. **[Bonus problem: Not to be included in quiz.]** Show that for any  $n \in \mathbb{N}$ , any sequence of  $n^2 + 1$  distinct real numbers contains a subsequence of  $n + 1$  numbers that is either strictly increasing or strictly decreasing.

## References

- [LLM17] Eric Lehman, Tom Leighton, and Albert R Meyer. *Mathematics for Computer Science*. 2017. URL: <https://courses.csail.mit.edu/6.042/spring18/mcs.pdf>.