COL 202: DISCRETE MATHEMATICAL STRUCTURES

LECTURE 1

INTRODUCTION TO DISCRETE MATH

JAN 02, 2024 | ROHIT VAISH

SLIDES AVAILABLE AT

https://rohitvaish.in/Teaching/2024-Spring

WHAT THIS COURSE IS ABOUT

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DISCRETE MATHEMATICS

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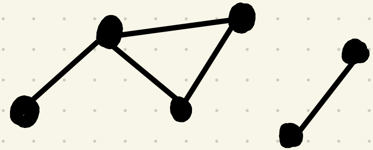
Numbers

1, 2, 3, ...

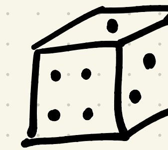
Graphs / Networks



Codes

0110101

Events



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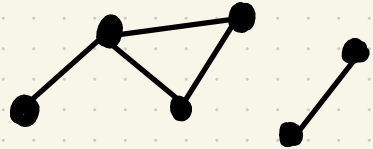

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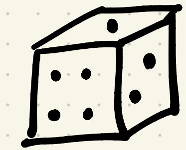

DISCRETE MATHEMATICS

A collection of formulas and equations?

A systematic and rigorous way of thinking?

Something invented by mean university professors?

Most mathematicians at one time or another have probably found themselves in the position of trying to refute the notion that they are people with "a head for figures," or that they "know a lot of formulas." At such times it may be convenient to have an illustration at hand to show that mathematics need not be concerned with figures, either numerical or geometrical. For this purpose we recommend the statement and proof of our Theorem 1. The argument is carried out not in mathematical symbols but in ordinary English; there are no obscure or technical terms. Knowledge of calculus is not presupposed. In fact, one hardly needs to know how to count. Yet any mathematician will immediately recognize the argument as mathematical, while people without mathematical training will probably find difficulty in following the argument, though not because of unfamiliarity with the subject matter.

What, then, to raise the old question once more, is mathematics? The answer, it appears, is that any argument which is carried out with sufficient precision is mathematical, and the reason that your friends and ours cannot understand mathematics is not because they have no head for figures, but because they are unable to achieve the degree of concentration required to follow a moderately involved sequence of inferences. This observation will hardly be news to those engaged in the teaching of mathematics, but it may not be so readily accepted by people outside of the profession. For them the foregoing may serve as a useful illustration.

SOURCE: "COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE"
D. GALE AND L. SHAPLEY AMER. MATH. MONTHLY 1962

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A method of establishing truth.

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A method of establishing truth.

Legal

Evidence

Scientific

Experiments / Observations

Social

Trusted authority

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Mathematical

This course

PROPOSITION

AXIOMS

LOGICAL DEDUCTIONS

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A statement that is either TRUE or FALSE

AXIOMS

Assumptions / Propositions that are "accepted" as TRUE

LOGICAL DEDUCTIONS

A collection of rules for proving new propositions using previously known ones.

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Prove that $5+5+5=15$

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We know that:

$$5+5=10$$
$$10+5=15$$

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Rule: for all x, y, z

$$x+y+z=(x+y)+z$$

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$$\begin{aligned} 5+5+5 &= (5+5)+5 \\ &= 10+5 \\ &= 15 \end{aligned}$$

EXAMPLES OF PROPOSITION

* Jan 02, 2024 is a Tuesday

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* Jan 02, 2024 is a Tuesday
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* Every even number is a prime
* For every integer n, $n^2 + 2n + 1 = (n+1)^2$

Non-examples

* $x + 2 = 5$
* It will rain tomorrow.

Ex: $\forall n \in \mathbb{N}$ $n^2 + n + 41$ is a prime

Ex: $\forall n \in \mathbb{N}$ $\underbrace{n^2 + n + 41 \text{ is a prime}}$

for all

$\mathbb{N} = \{1, 2, 3, \ldots\}$

PREDICATE

A proposition whose truth depends on the value of variable (n).

Ex: $\forall n \in \mathbb{N}$ $n^2 + n + 41$ is a prime

Is this proposition true?

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n	$n^2 + n + 41$	Prime?
1	43	✓

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n	n^2+n+41	Prime?
1	43	✓
2	47	✓

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n	$n^2 + n + 41$	Prime?
1	43	✓
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⋮	⋮	⋮
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1	43	✓
2	47	✓
3	53	✓
$\vdots$	$\vdots$	$\vdots$
39	1601	✓
40	$1681 = 41 \times 41$	✗

"there exists"

Correct: $\exists\, n \in \mathbb{N}$ such that n^2+n+41 is not prime.

Ex: $313(x^3+y^3)=z^3$ has no positive integer solutions

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Is this proposition true?

Ex: $313(x^3+y^3)=z^3$ has no positive integer solutions

Is this proposition true?

No! But the smallest positive x, y, z satisfying this equation have over a 1000 digits each.

Ex: Four Color Theorem

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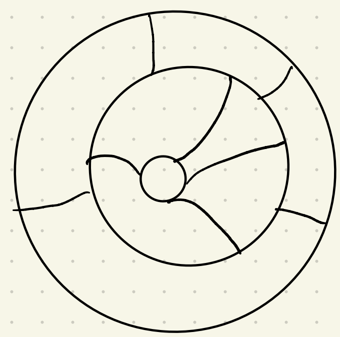

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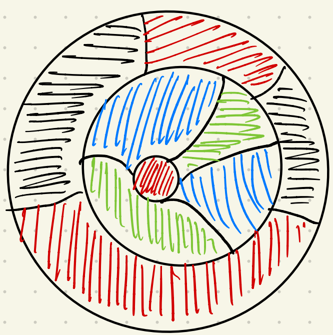

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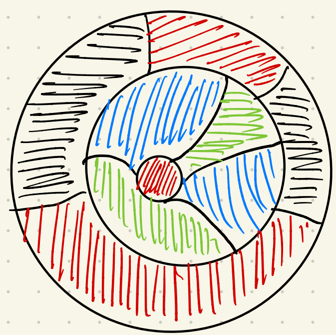

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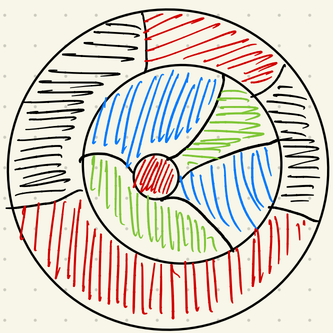

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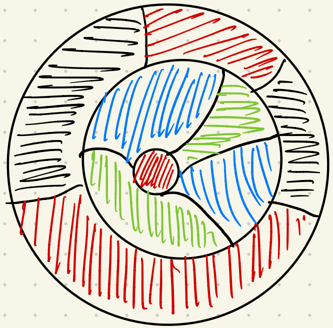

Computer-assisted proof
↳ extensive case analysis

Unsatisfying to mathematicians at the time.

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GOLDBACH'S CONJECTURE (1742)

A major open problem in number theory.

COURSE LOGISTICS

INSTRUCTOR : ROHIT VAISH (Please call me Rohit)

Lectures on Tue/Wed/Fri 10-11 AM LH 316

TAs : JATIN YADAV

SURBHI RAJPUT

AKSHAY PRATAP SINGH

SOUMIL AGGARWAL

Tutorials on Mon/Tue/Thurs/Fri 1-2 PM LH 615

+ Weekly office hours (ask your TA!)

Check your tutorial group on the website

EVALUATION

~ 1 hr	MINOR	16 %	
~ 2 hrs	MAJOR	24 %	
~ 40 mins	QUIZZES	36 %	[12% × best 3 out of 4]
~ 10 mins	TUTORIALS	24 %	[3% × best 8 out of 12]

ATTENDANCE Not monitored by me (but might be required by the institute)

AUDITS Not allowed

EXTRA CREDIT In-class exercises

RE-EVALUATION

NO re-minor, re-major, make-up quizzes or tutorials

MINOR
MAJOR

Adjusted score for minor : $\frac{1}{2}(\underset{12\%}{\text{Quiz 1}} + \underset{12\%}{\text{Quiz 2}})$

Adjusted score for major : $\frac{1}{3}(\underset{16\%}{\text{Minor}} + \underset{36\%}{\text{3 best Quizzes}})$

Only after prior approval from the instructor

QUIZZES
TUTORIALS

No score adjustment

TUTORIALS

* Once (almost) every week
* First ~ 10 mins: Quiz 1 problem from weekly problem set

  announced by Thursday of previous week

  announced by TA at the start of the tutorial
* Remaining time: Discussion (Ask questions!)
* Each tutorial submission is worth 3% (best eight).
* Ungraded problems ≠ Not relevant

REGRADING

* Gradescope (for tutorials, quizzes, exams)

* "Frivolous" requests ⟶ −2% from overall score for the course for the first one and subsequently in powers-of-two −4%, −8%, −16%, ...

If in doubt, talk to your group's TA.

PLAGIARISM

* Any kind, at any stage : -50% for the first offense

  F subsequently

* Suggestions :

  - Acknowledge your sources (books, articles, websites, In-person or online discussions)
  - Discussion of tutorial problems is welcome, but you should write the solution in your own words
  - If in doubt, talk to your TA or me.

COMMUNICATION

* **Most Important!**

* Please reach out to me or TAs about ANY issues at any time. Do not wait until the majors.

* We want you to enjoy learning about discrete math and do well in the course.

All the best!