

Lecture 6

House Allocation and Kidney Exchange

Aug 18, 2023

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Rohit Vaish

Housing Market

[Shapley and Scarf, *JME* 1974]

Housing Market

[Shapley and Scarf, *JME* 1974]



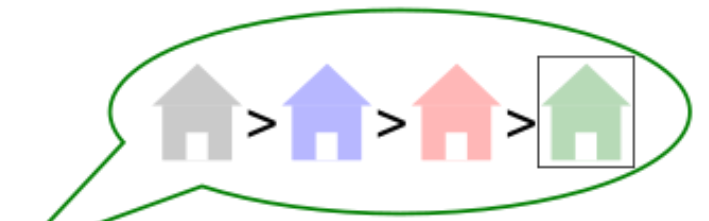
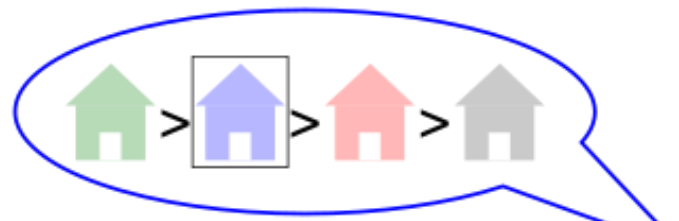
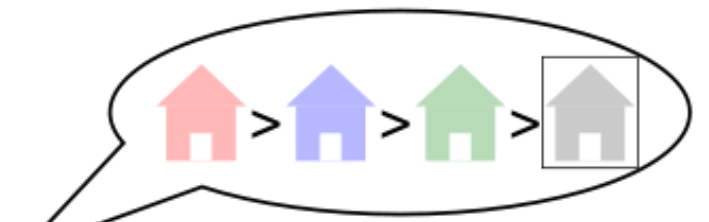
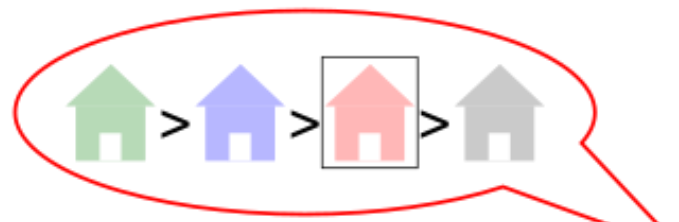
Housing Market

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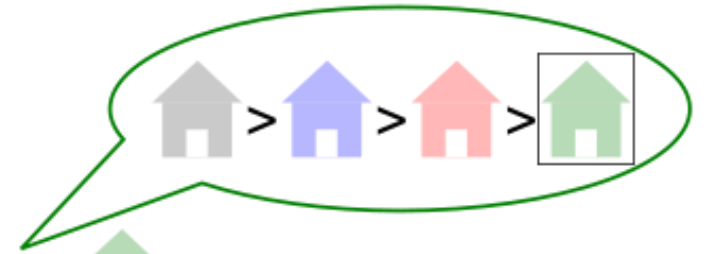
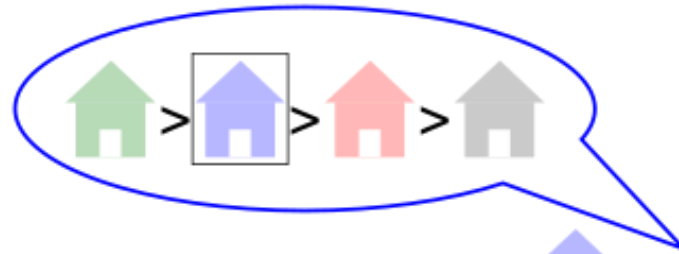
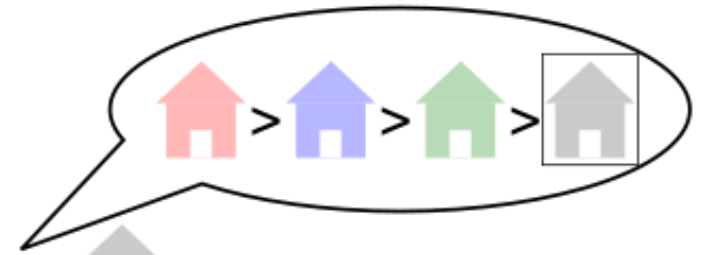
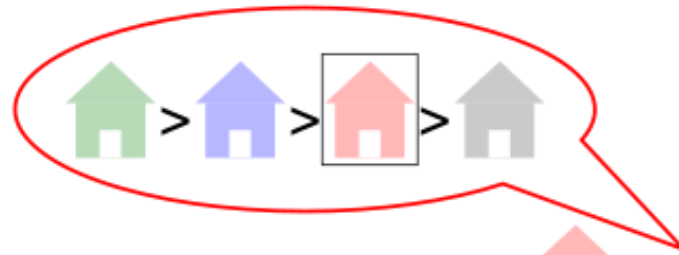
Housing Market

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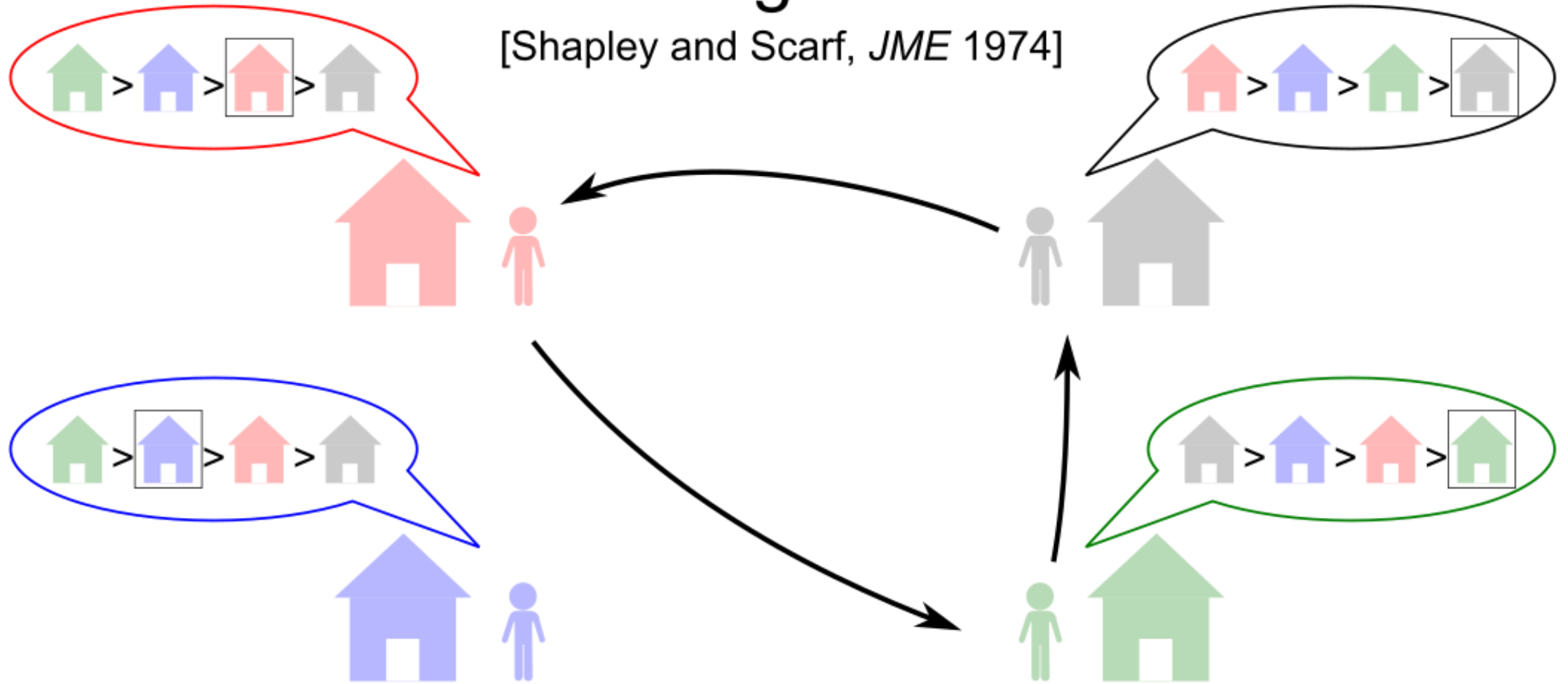
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Can't use money. Only way to make agents happier is via *exchanges*.

Housing Market

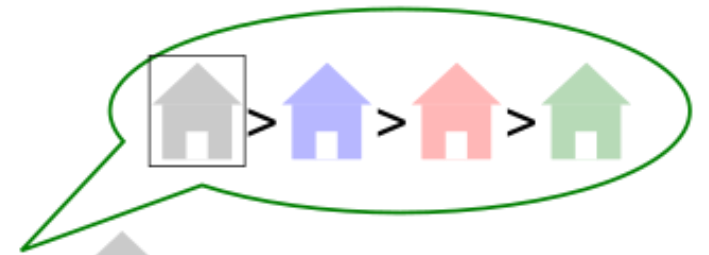
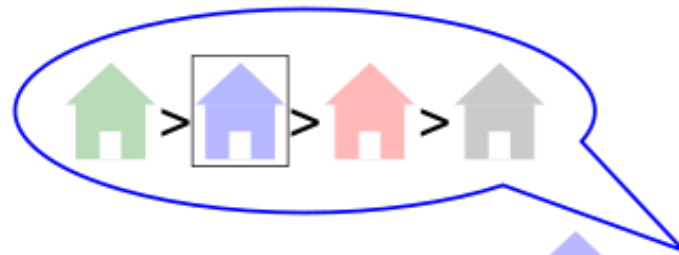
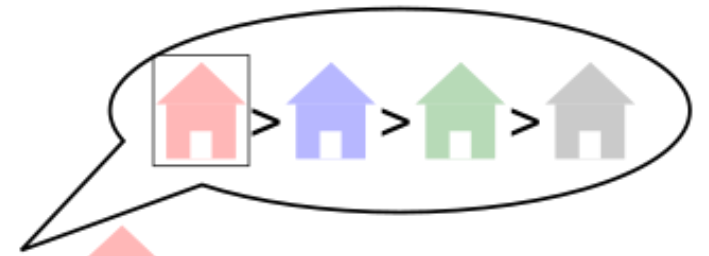
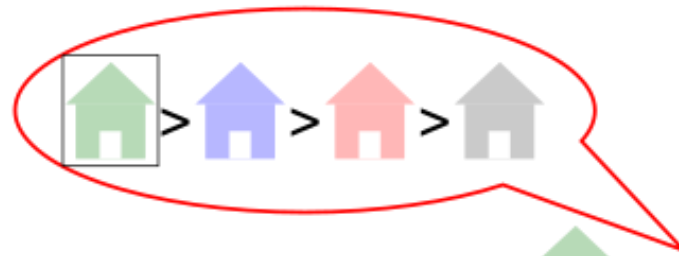
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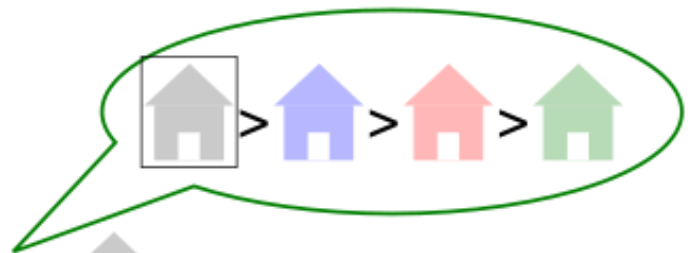
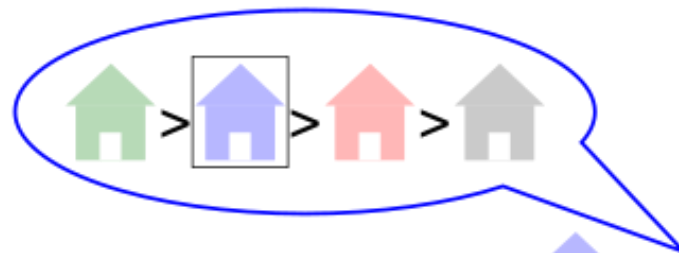
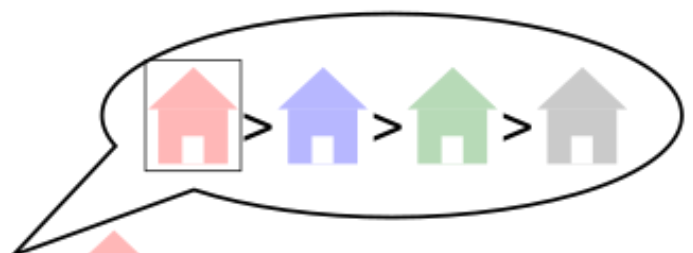
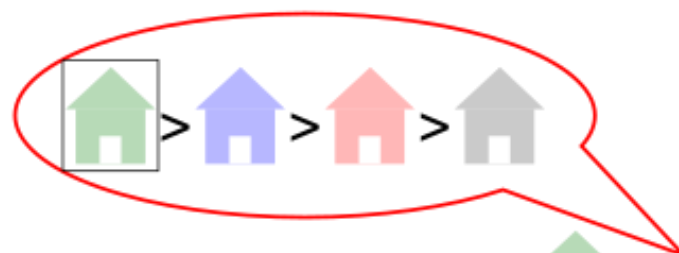
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Housing Market

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Is there a way to exchange houses to make the agents "maximally happy"?

Top-Trading Cycle Algorithm (TTCA)

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

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While there is a remaining agent:

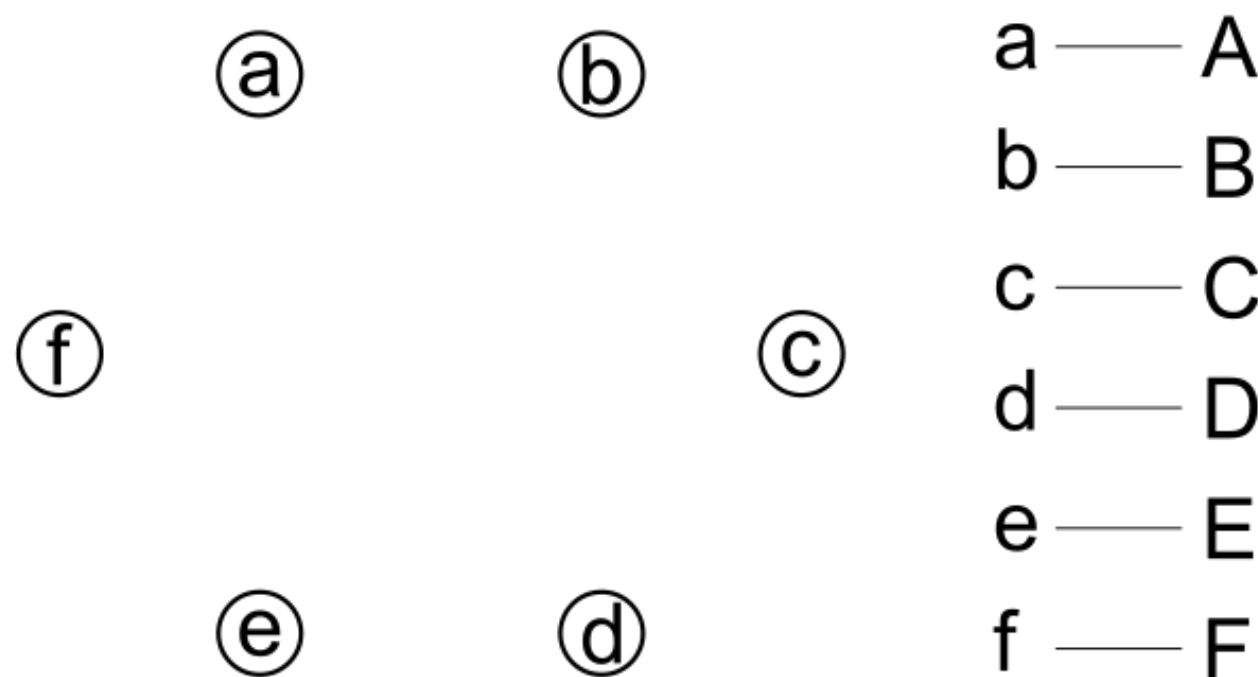
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- There must be a cycle (include self-loops). Do the cyclic exchange.
- Remove the agents and houses involved in the above exchange.

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	a	b	c	d	e	f				
1.	B	F	A	E	C	F	(a)	(b)	a — A	
2.									b — B	
3.				(f)				(c)	c — C	
4.									d — D	
5.									e — E	
6.					(e)			(d)	f — F	

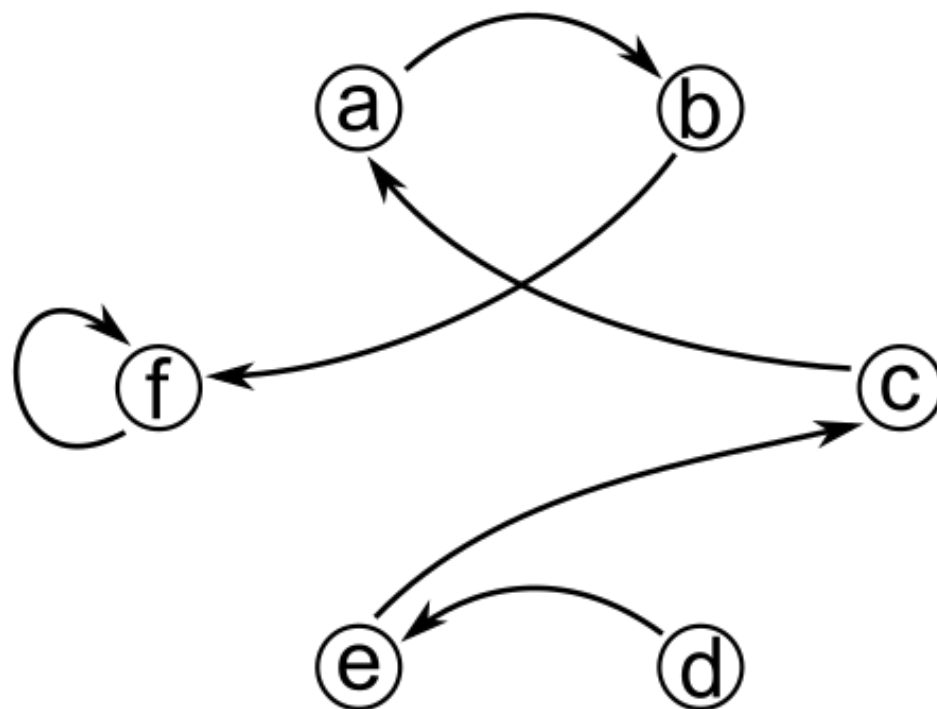
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a	—	A
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c	—	C
d	—	D
e	—	E
f	—	F

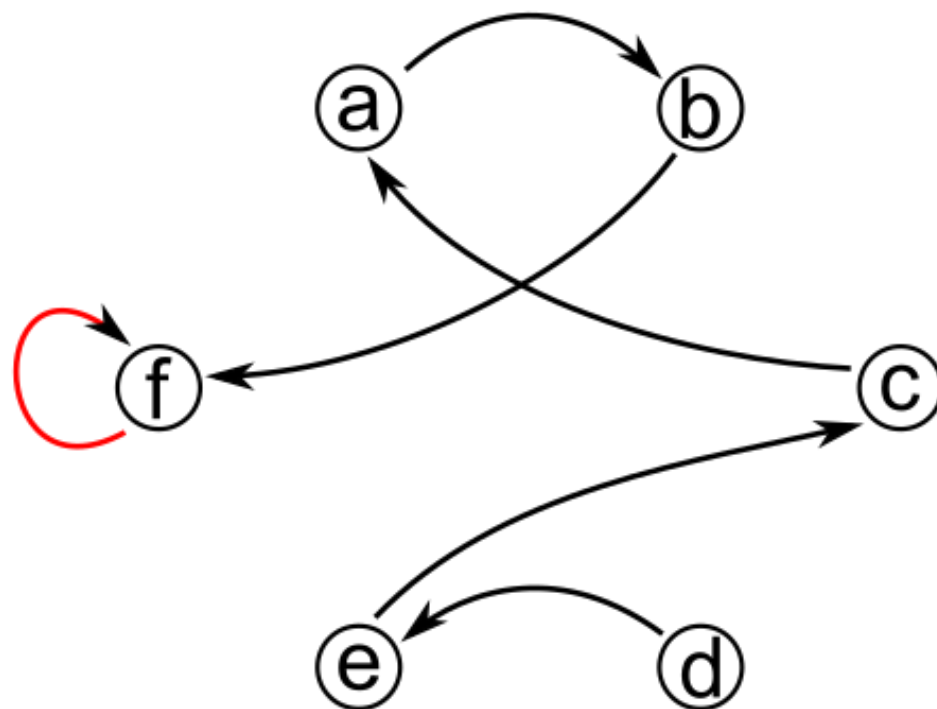
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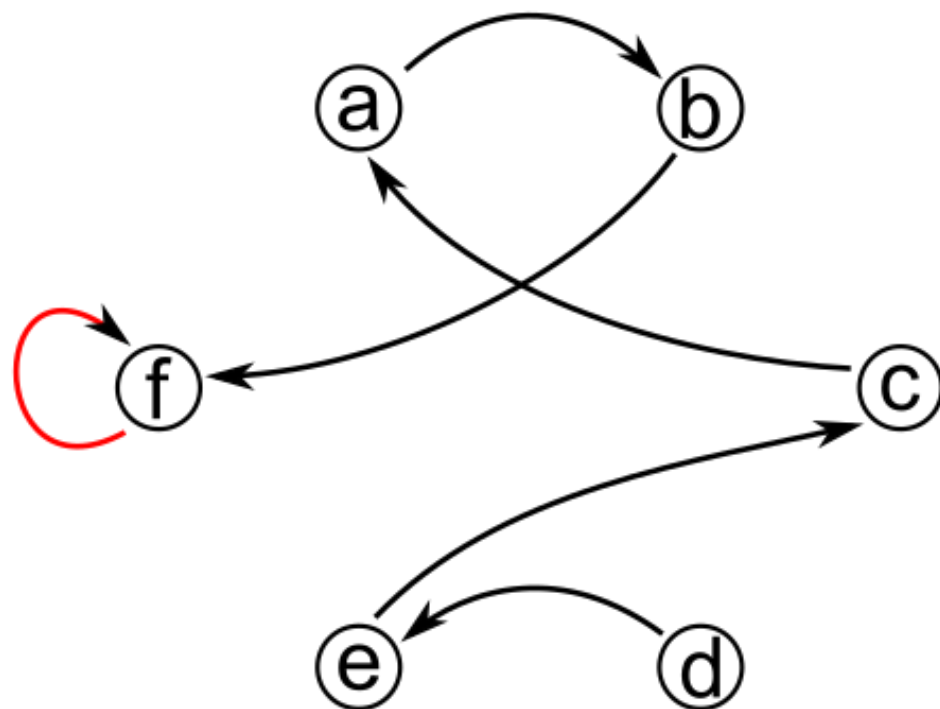
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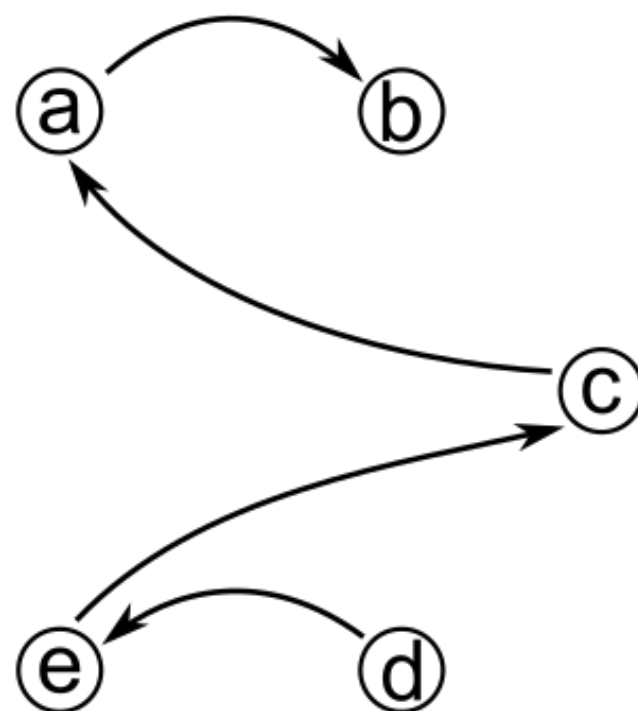
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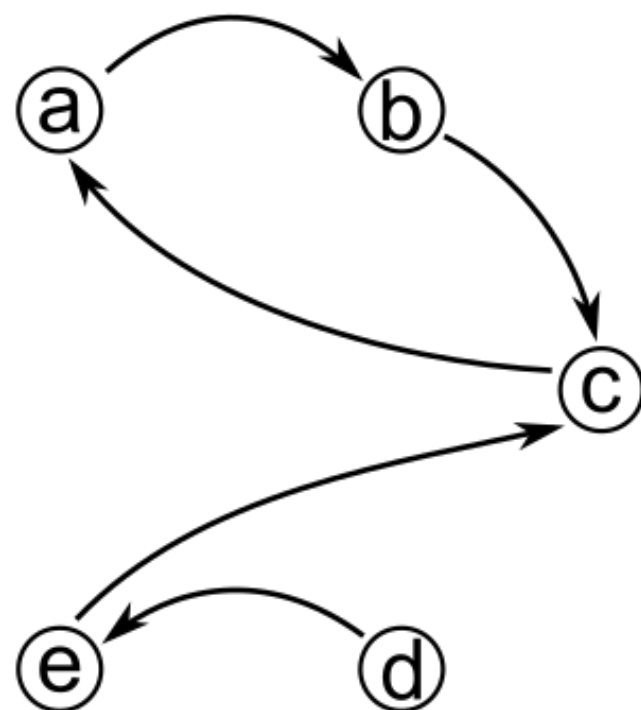
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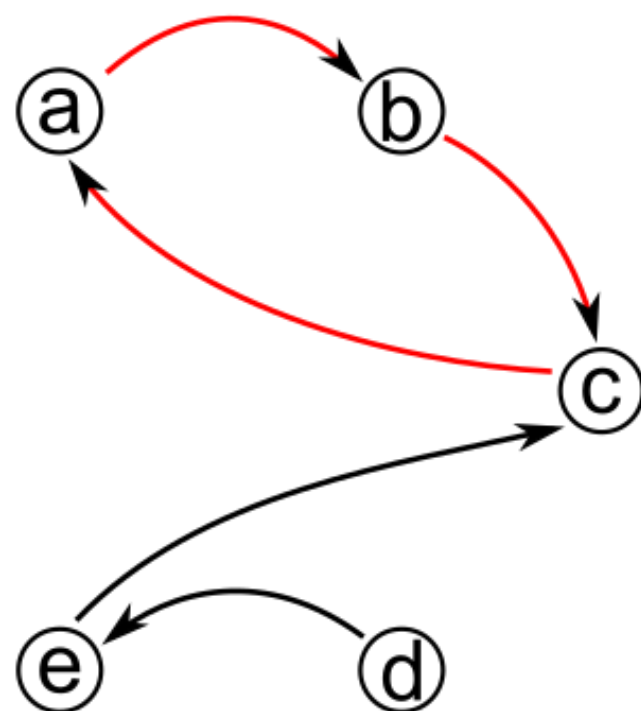
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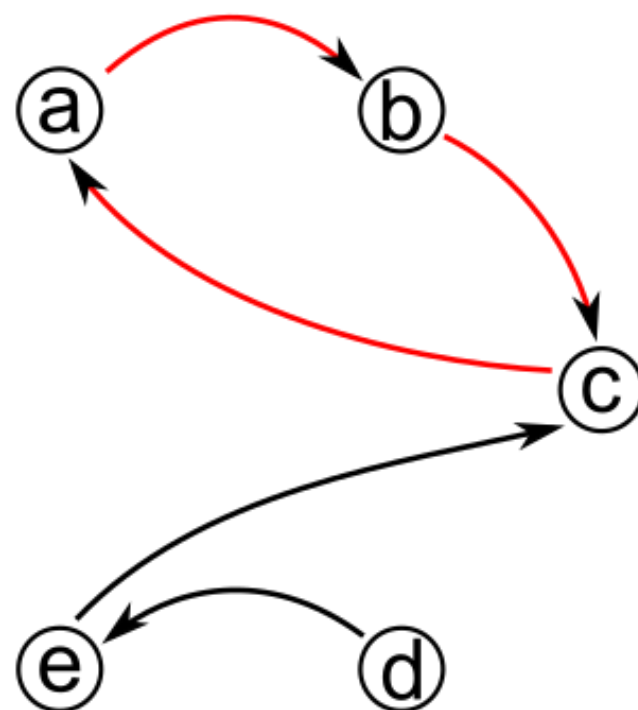
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		d	e
1.	_____	E	
2.			
3.			
4.		D	
5.			
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a	—	B
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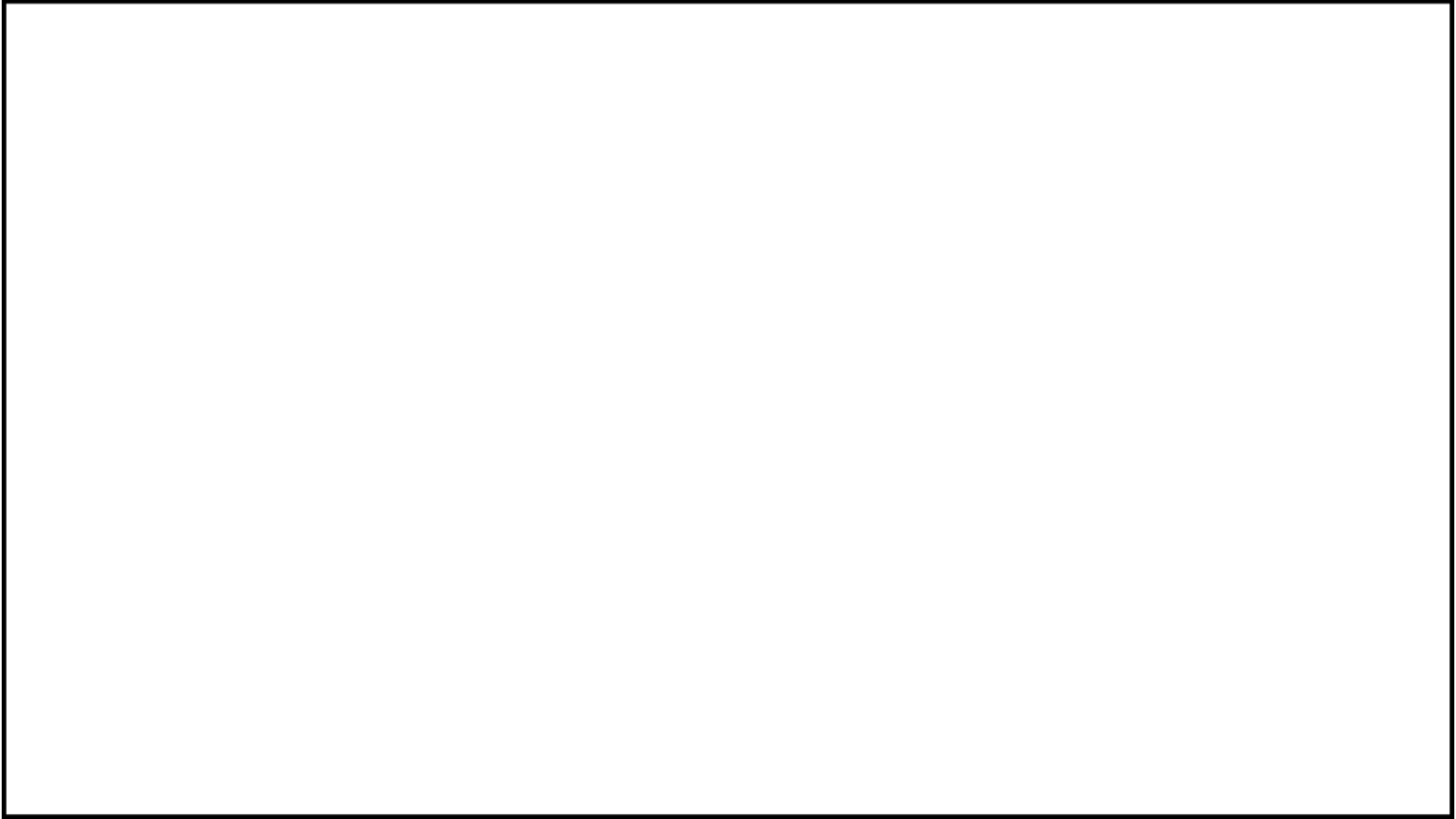
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1.	_____	E	
2.			
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Q1. Is a cyclic exchange always possible?

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Imagine a graph where the *vertices* are the (remaining) agents and the *edge* $i \rightarrow j$ means that agent i 's favorite house is owned by agent j .

At each step, each vertex has an outgoing edge.

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Q2. **Does TTCA always terminate?** Yes.

At least one agent kicked out in each round.

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Q3. **Does TTCA terminate in polynomial time?**

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Q3. **Does TTCA terminate in polynomial time?** Yes.

TTCA is strategyproof

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Let S_k denote the set of agents who are removed in round k of TTCA.

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S_1



S_2



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Each agent in S_1 gets its favorite house and therefore has no incentive to misreport.

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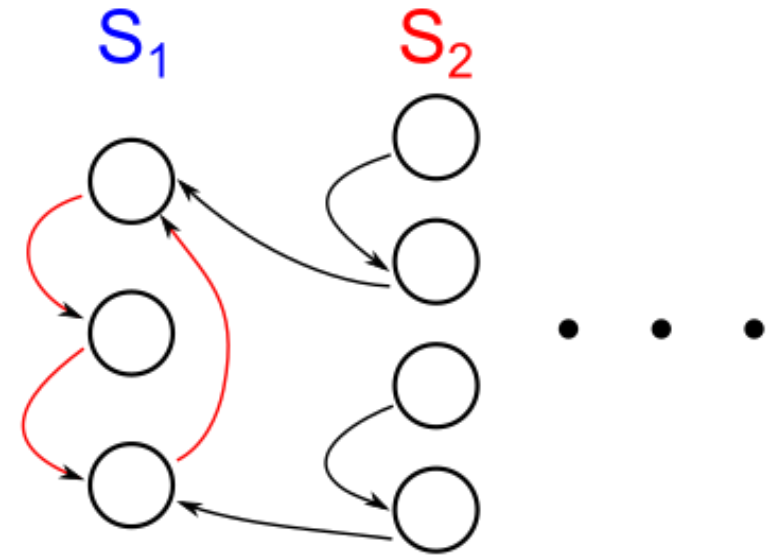
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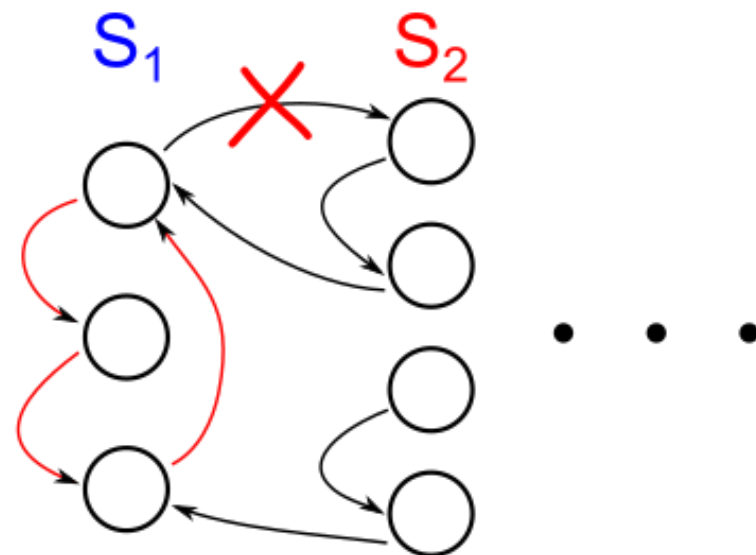
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Subject to that, no agent in S_2 has any incentive to misreport.

- No agent in S_2 can get a house owned by an agent in S_1 since, in round 1, no agent in S_1 points to an agent in S_2 .

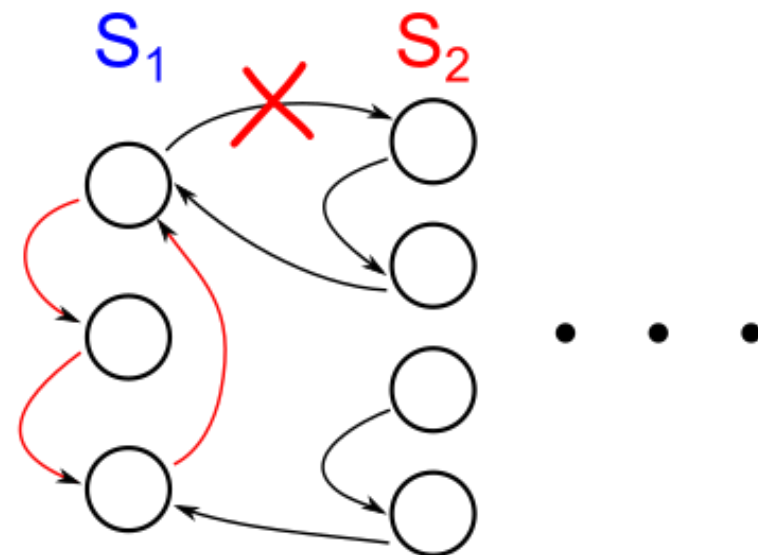


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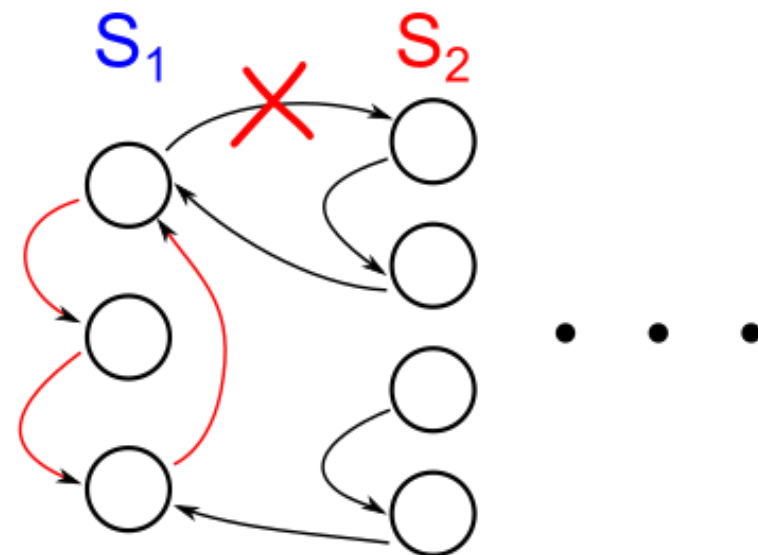
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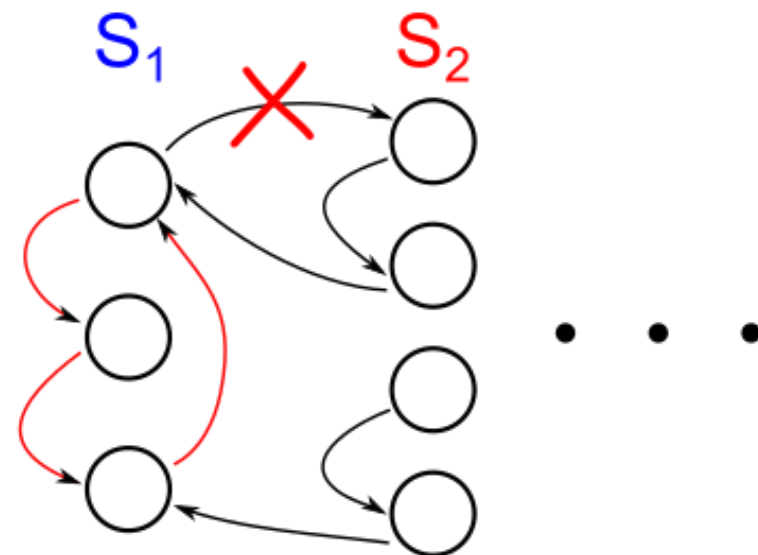
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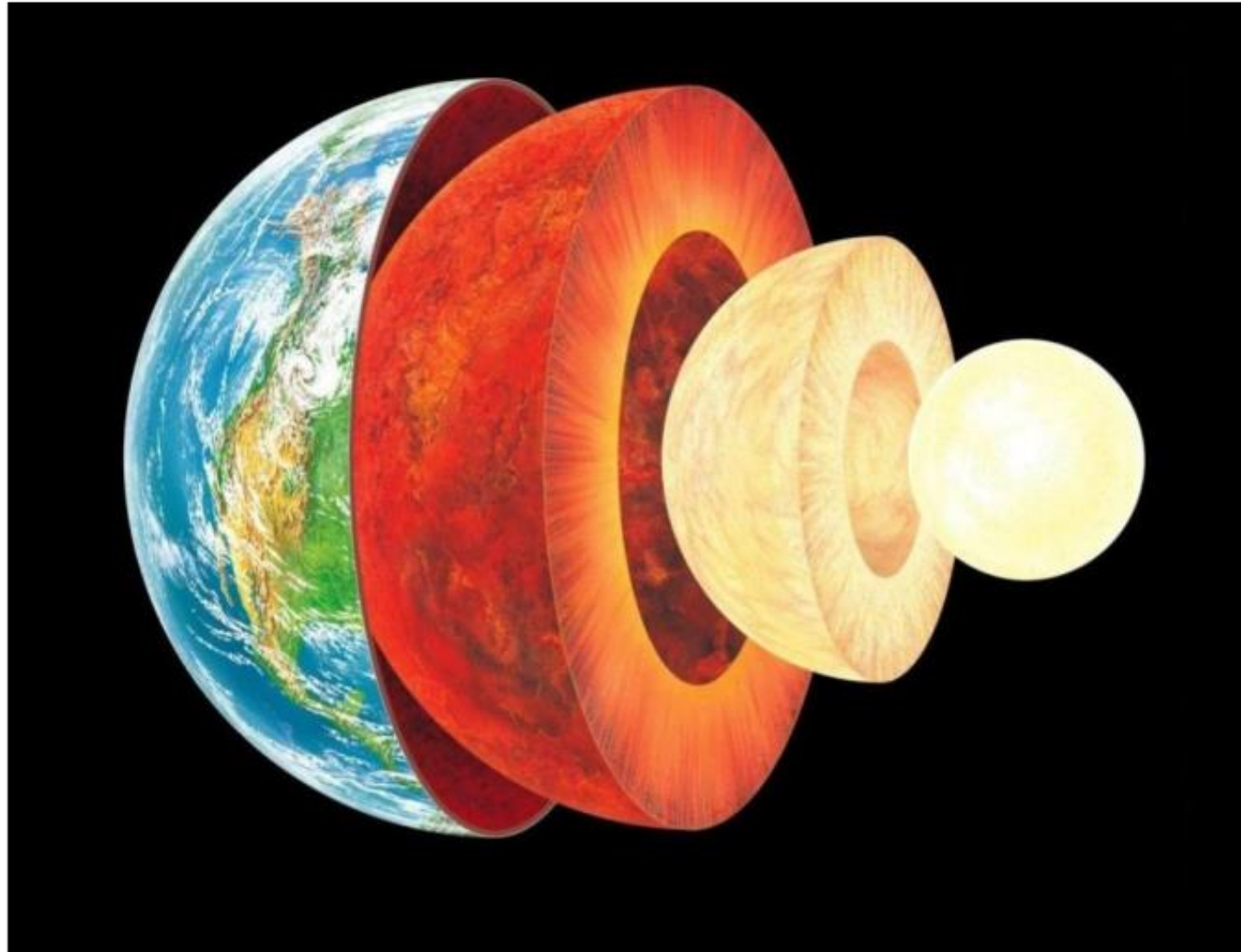


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The Core



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An allocation is in the **core** if no coalition blocks it.

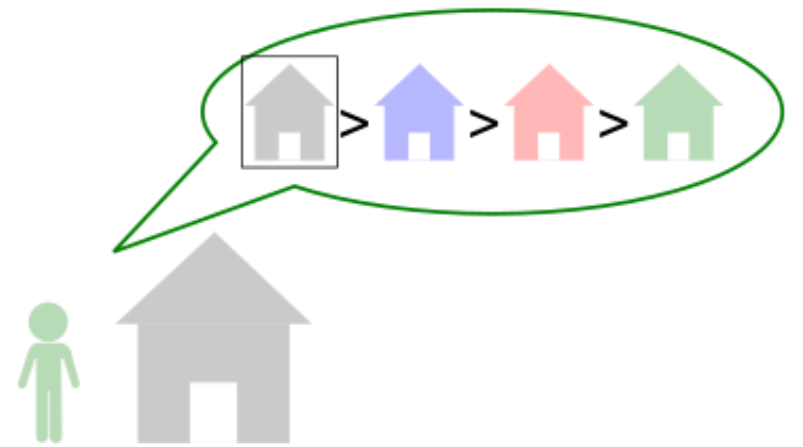
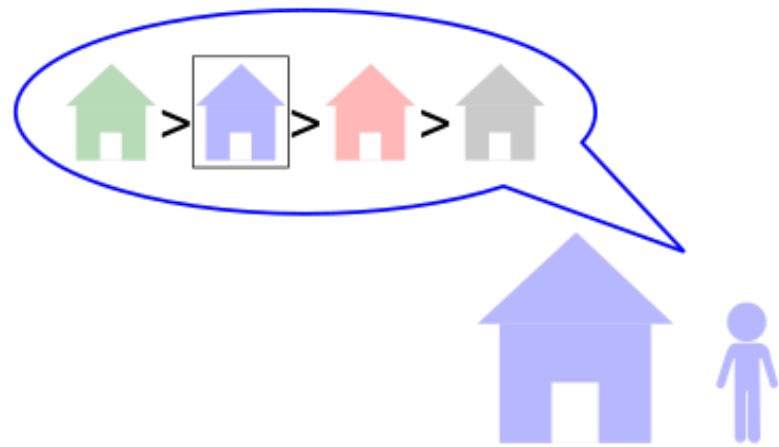
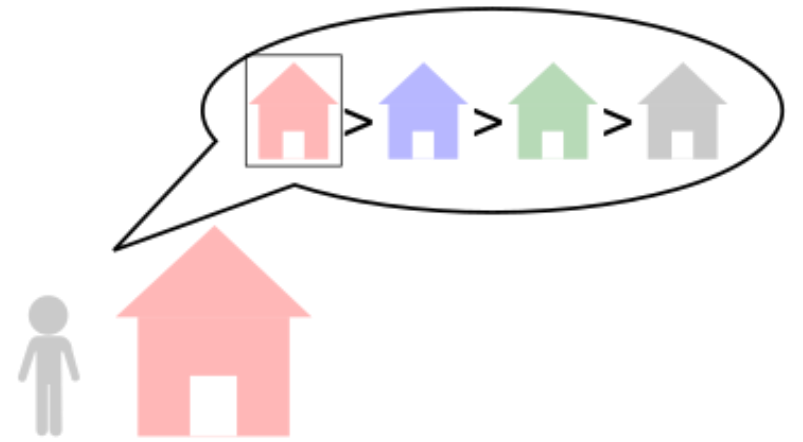
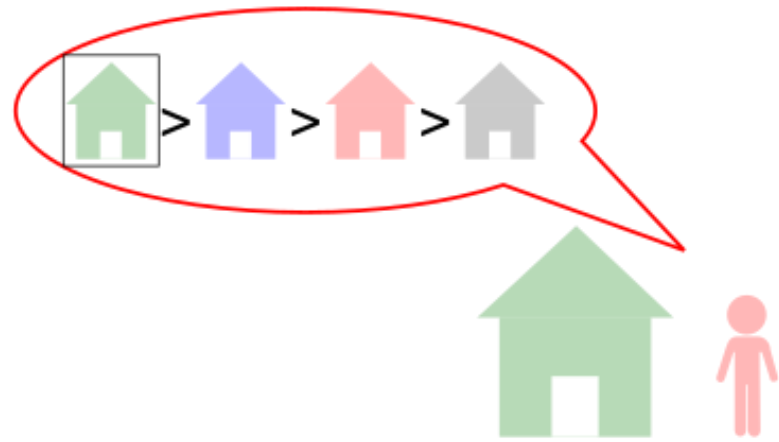
The Core

An allocation is in the **core** if no coalition blocks it.

A coalition of agents **blocks** an allocation **A** if:

they can redistribute their **endowed houses** among themselves such that, compared to **A**, none of them is worse off and at least one of them is strictly better off (i.e., redistributing endowments is a Pareto improvement over **A**).

Example of a Core Allocation



TTCA outcome is in the core.

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

TTCA outcome is in the core.

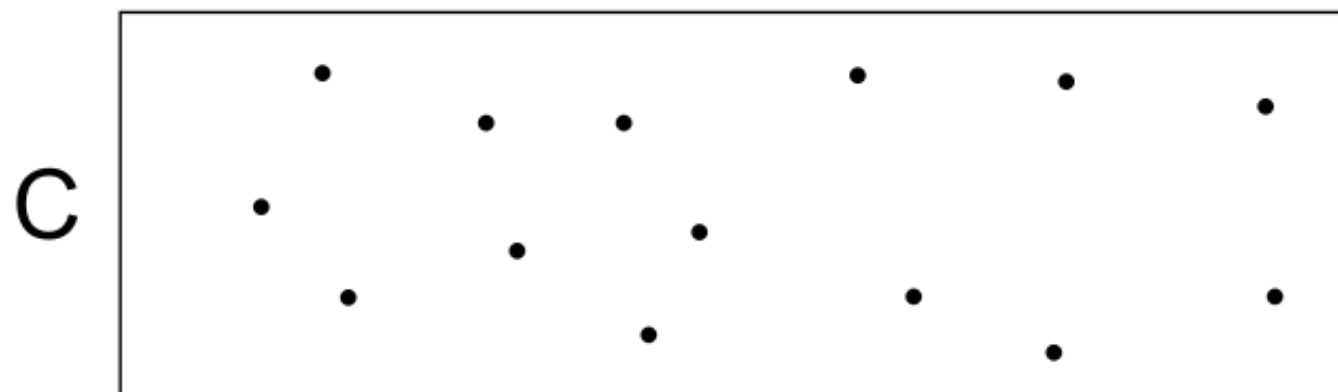
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Suppose not. Then, a coalition C of agents must block the TTCA allocation T . Let R be a redistribution of endowments among agents in C that they find Pareto better than T .

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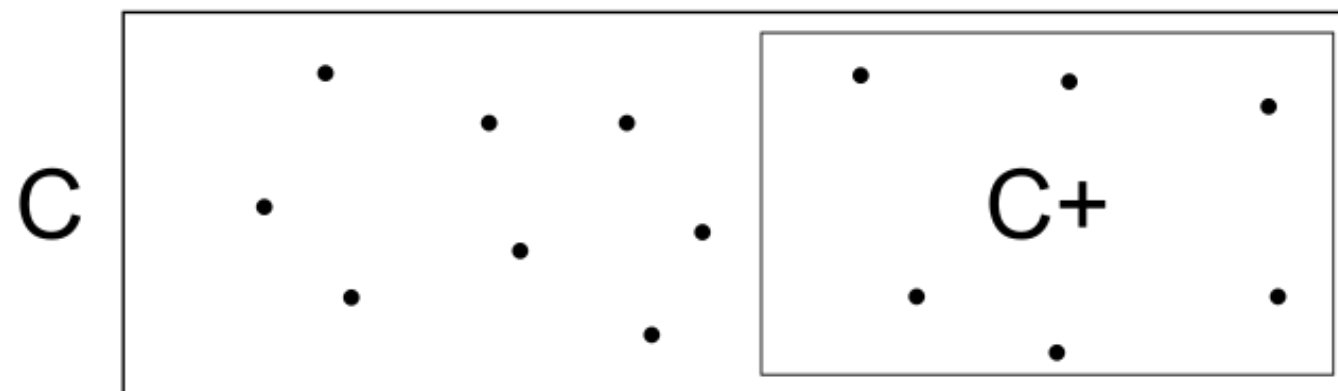


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Let $C^+ \subseteq C$ be the agents in C who *strictly prefer* R over T .

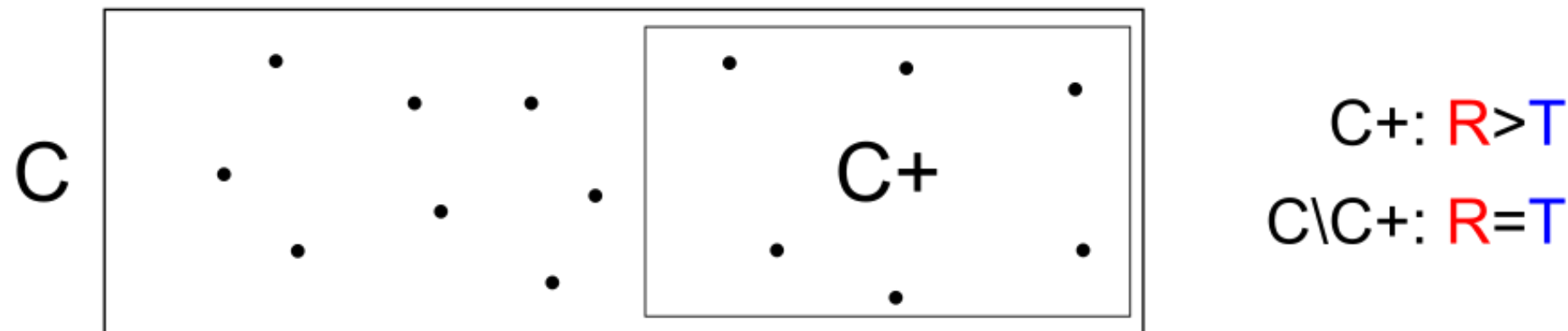


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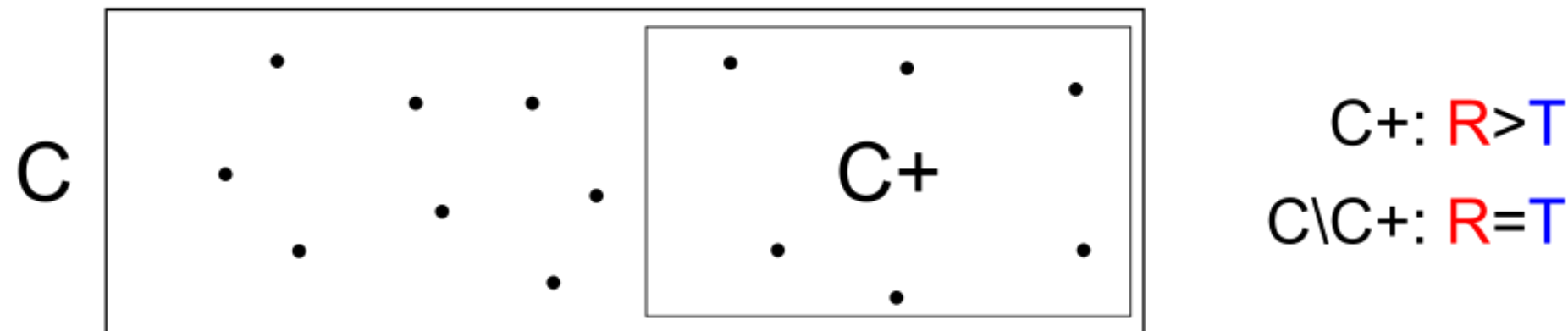


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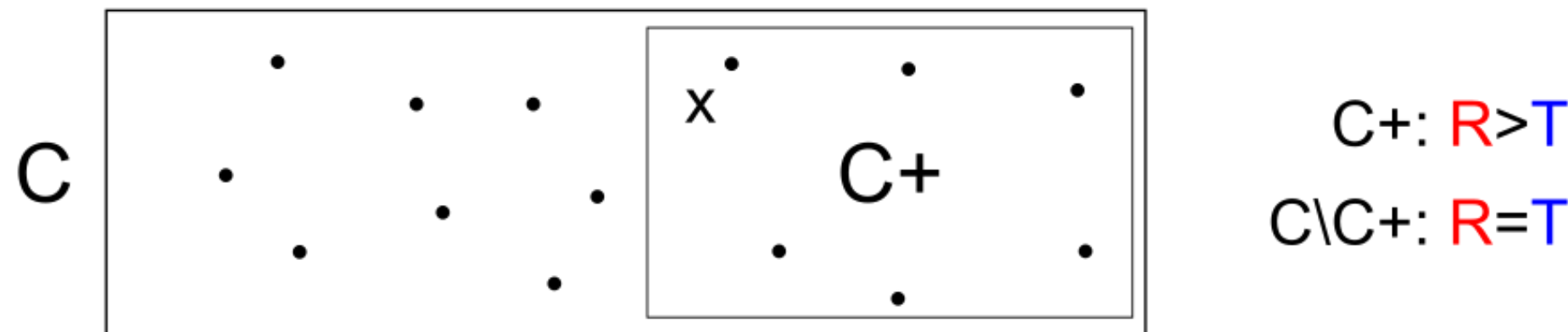


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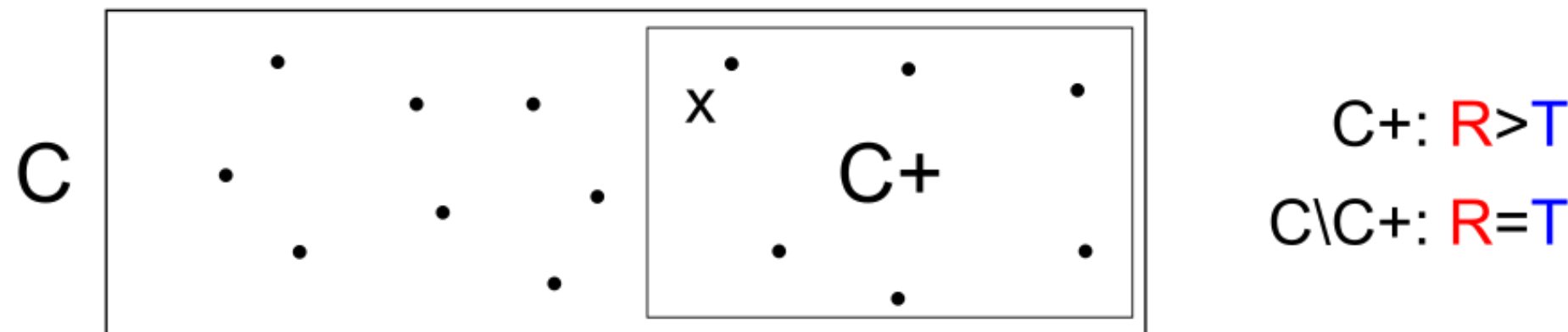


T = TTCA outcome, R = redistribution of endowments within C

TTCA outcome is in the core.

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

At the time x is eliminated (round r), all agents in C^+ are still available under TTCA along with their endowed houses.

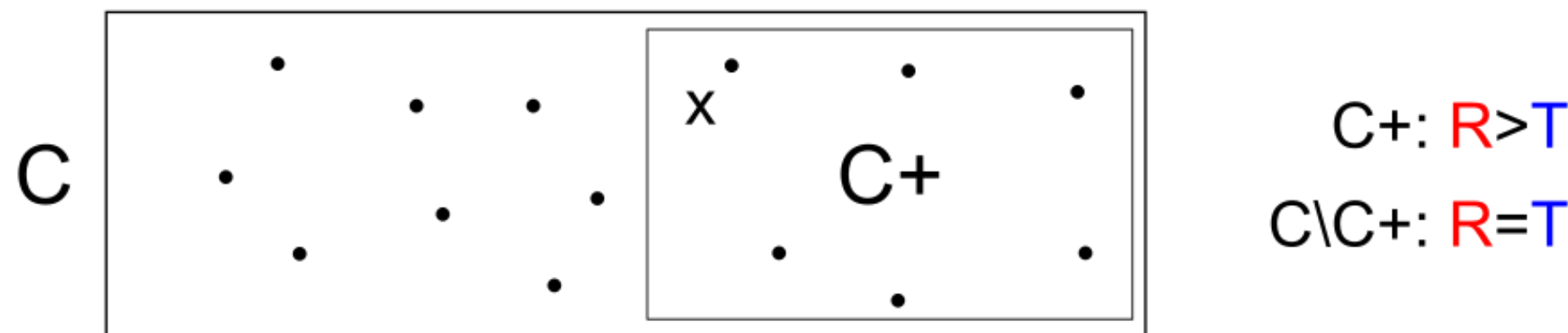


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The house x gets under T is at least as good (according to x 's preference) as the endowed house of any agent in C^+ .

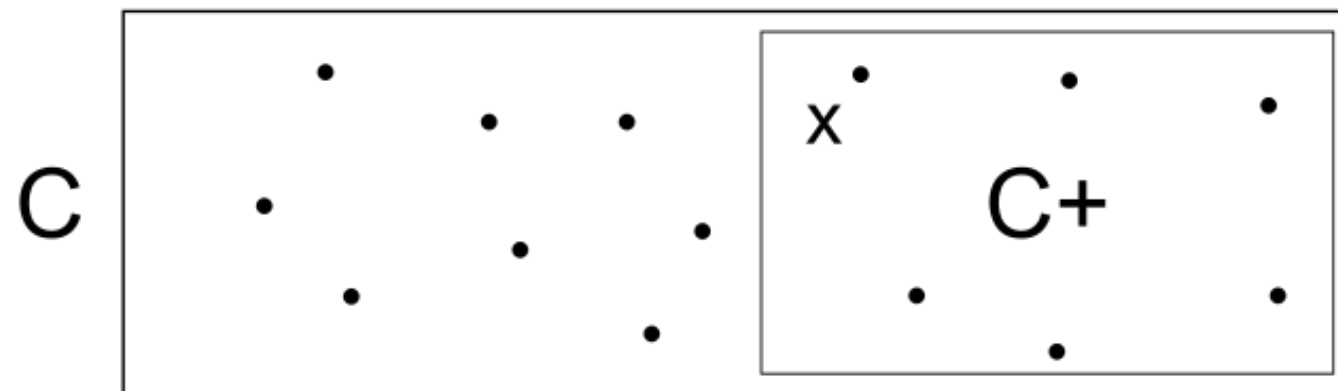


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The house x gets under R is strictly better (according to x 's preference) than the endowed house of any agent in C^+ .



C^+ : $R > T$

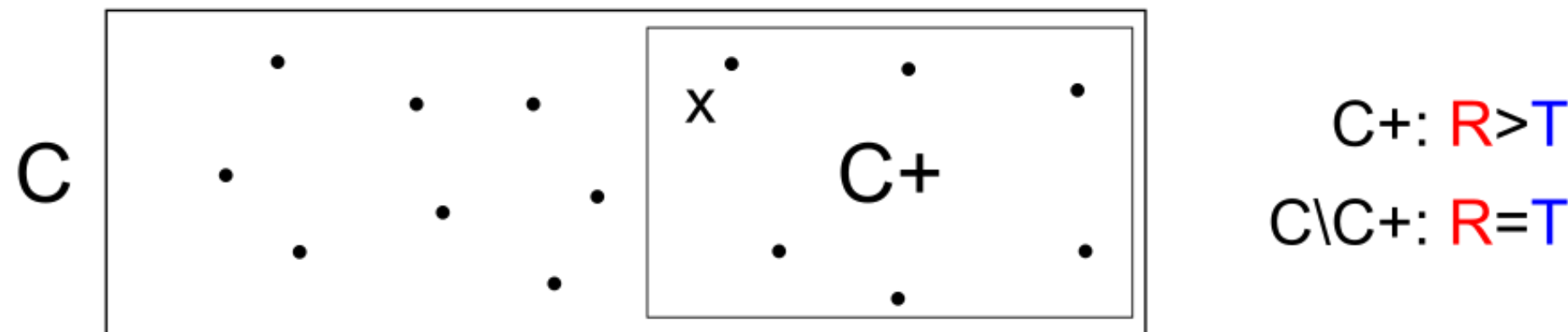
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The house x gets under R must be the endowed house of some agent $y \in C \setminus C^+$.

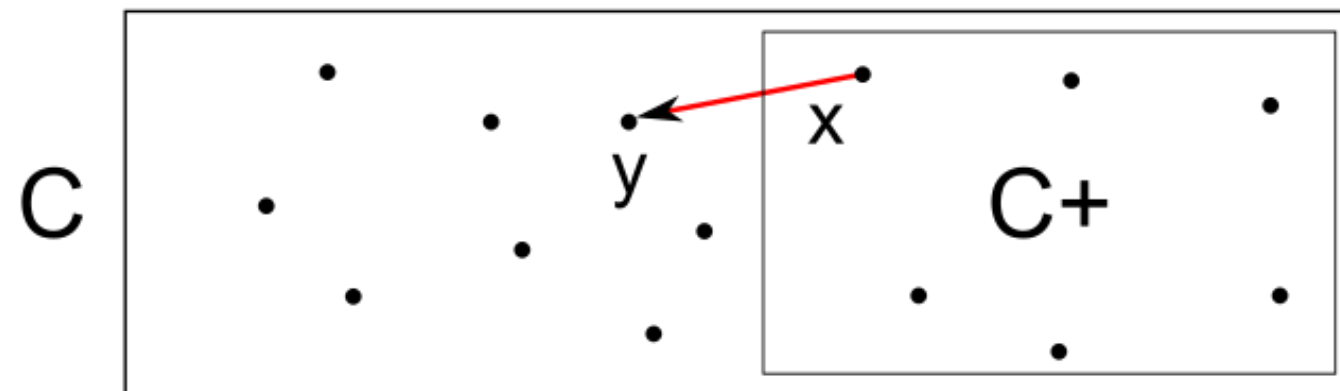


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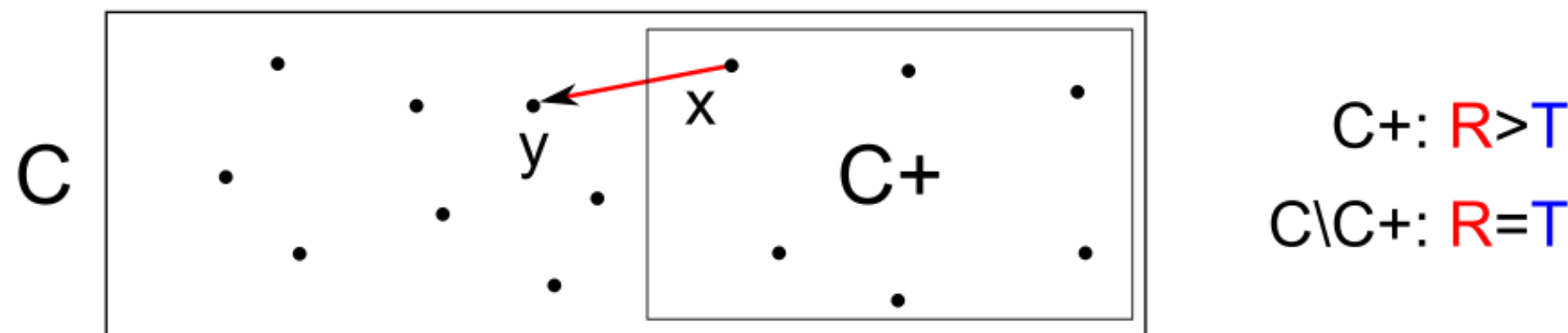
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Then, under TTCA, agent y must have been eliminated in round $r-1$ or earlier (i.e., strictly before x).

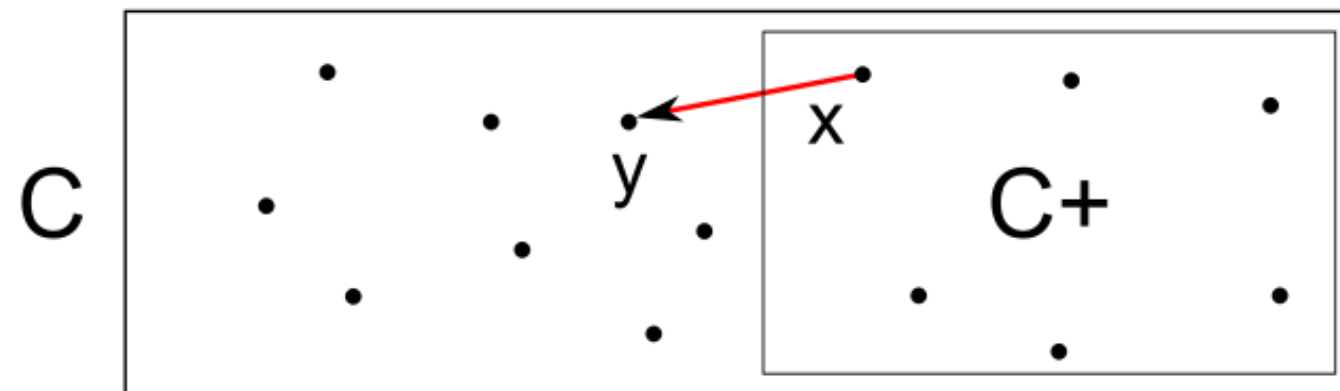


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Agent y cannot get its own endowed house under the TTCA outcome T .



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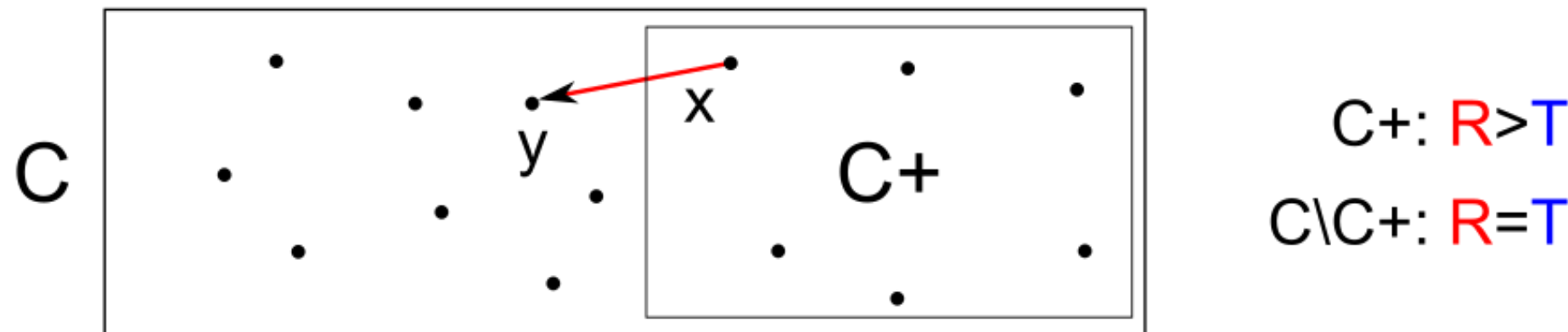
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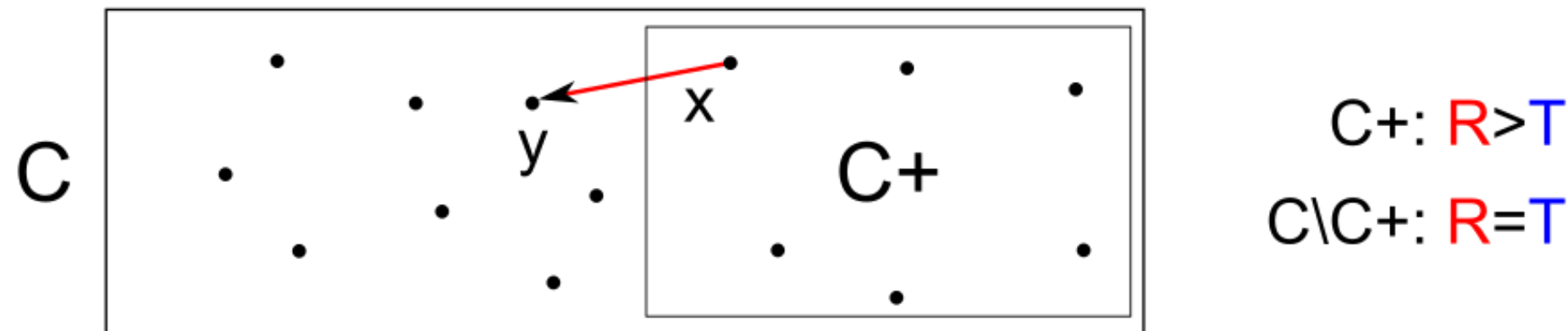
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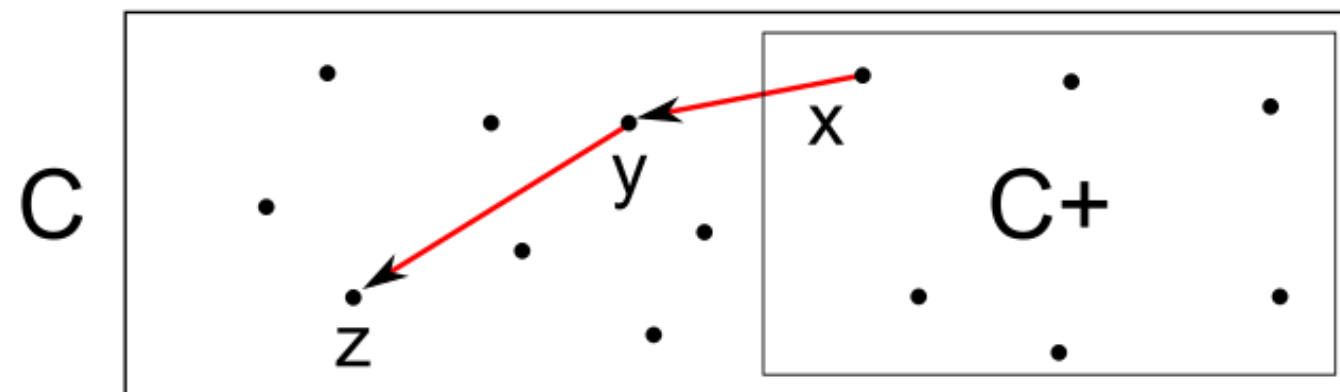
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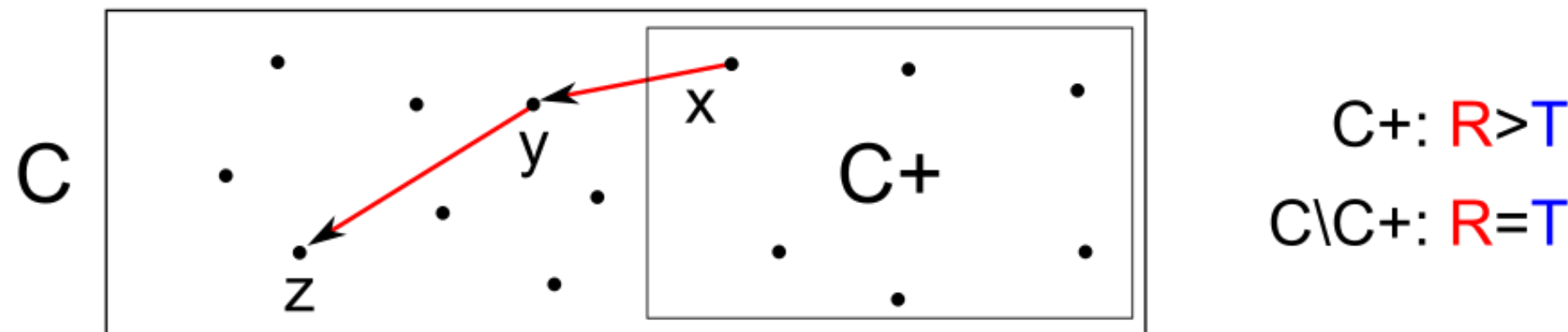
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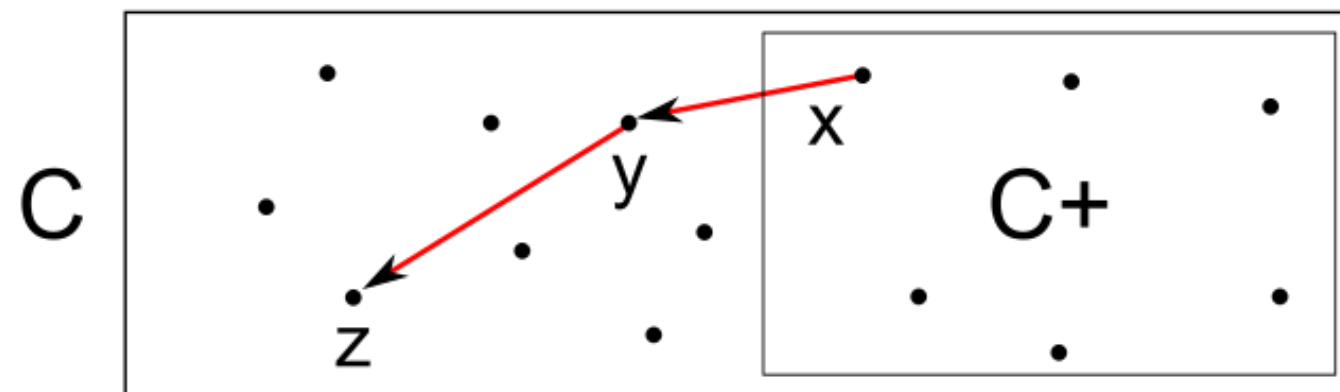
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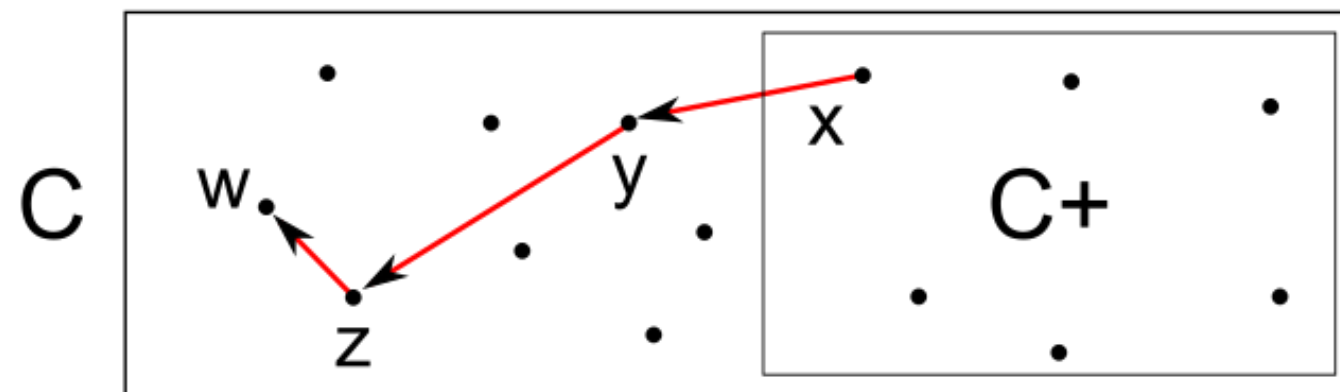
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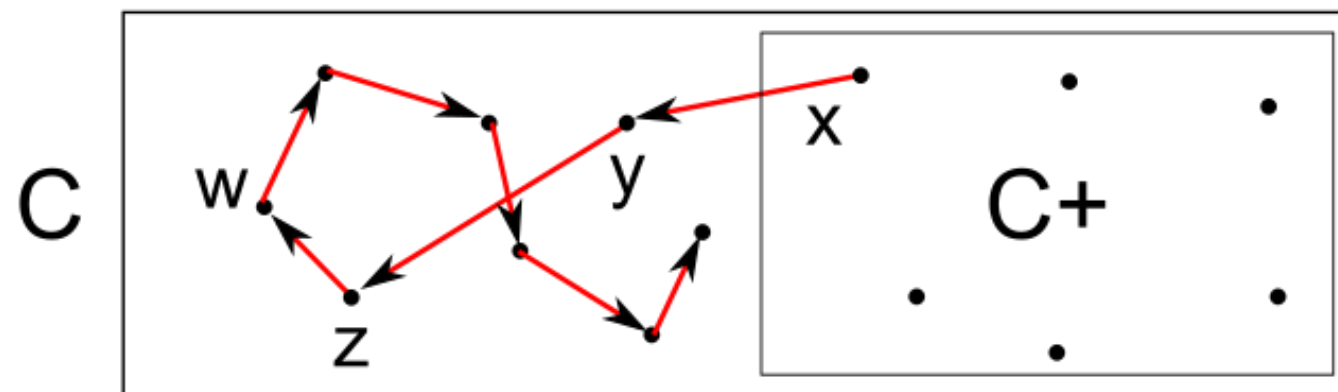
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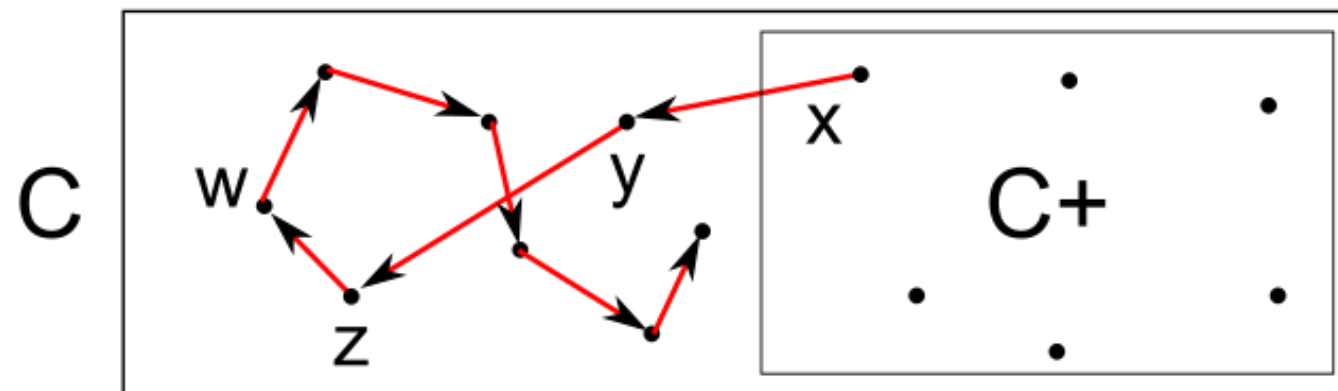
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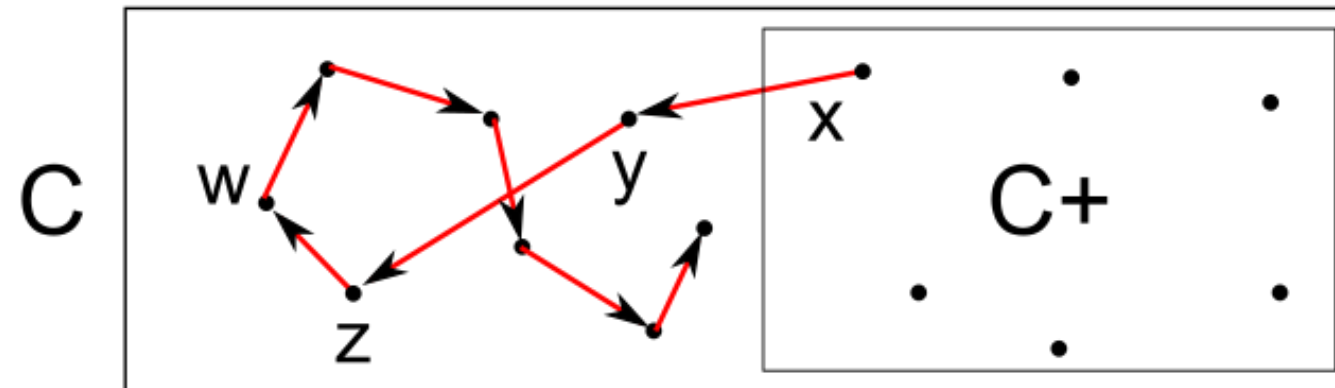
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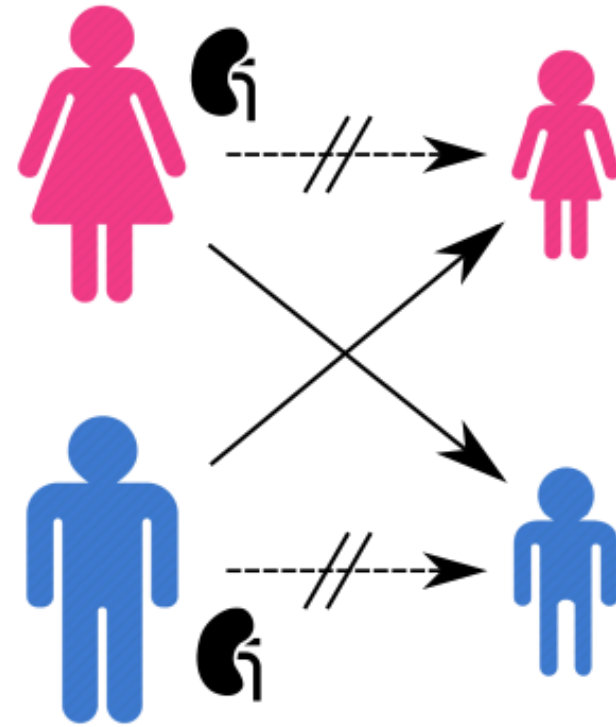


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Exercise: Prove that the core outcome is **unique**.

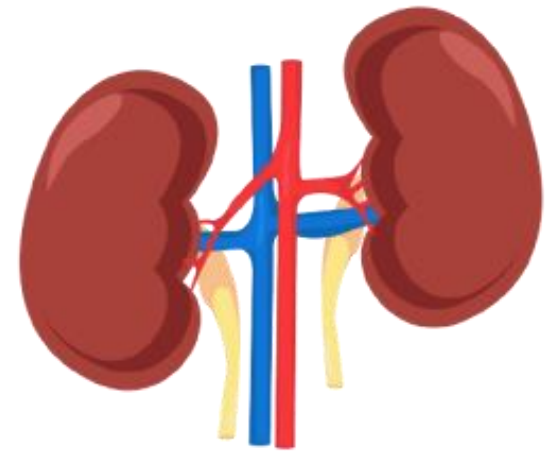
Application of TTCA



Kidney Exchange

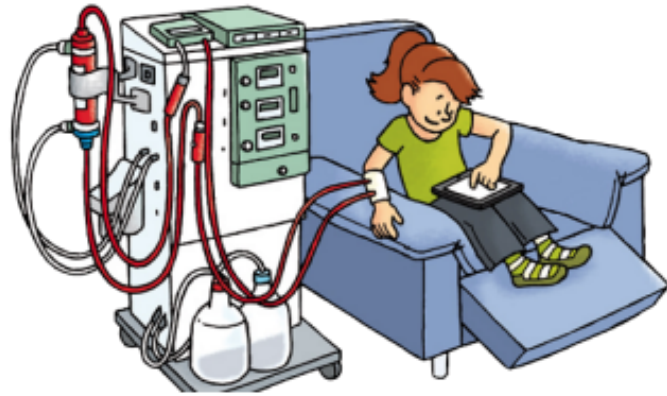
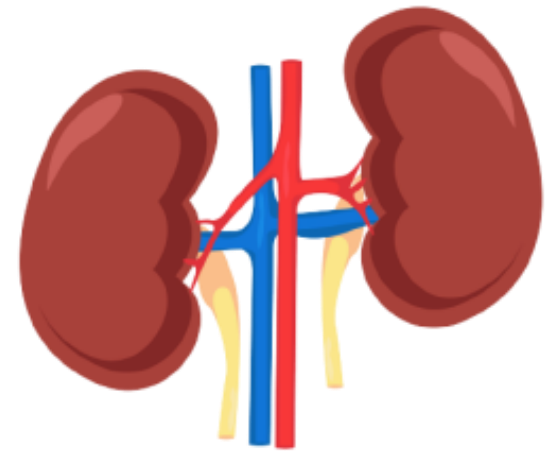
Kidney Exchange

Many people suffer from kidney failure
(waiting list in US: ~90,000; in India: ~200,000).



Kidney Exchange

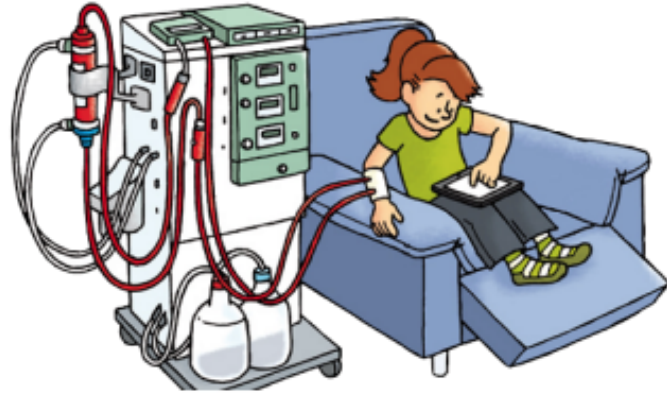
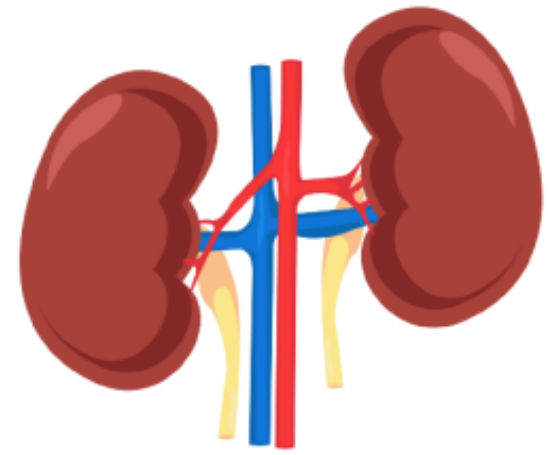
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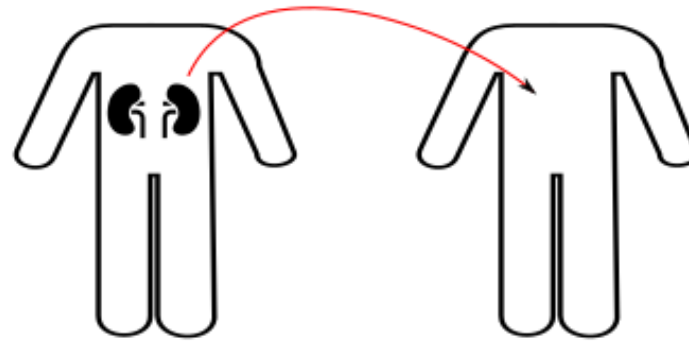
Dialysis

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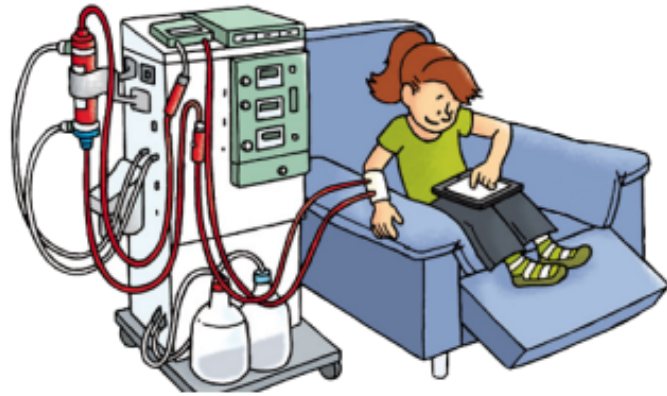
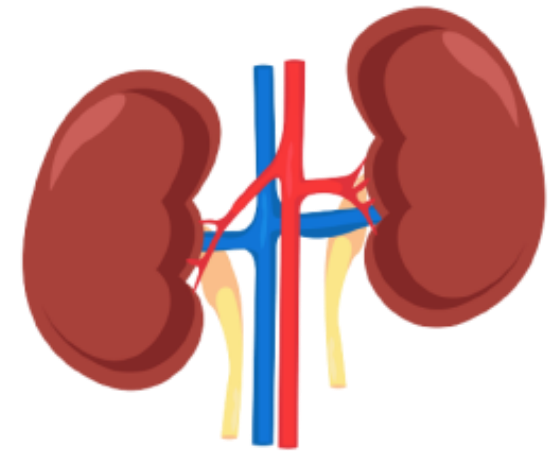
Dialysis



Transplant

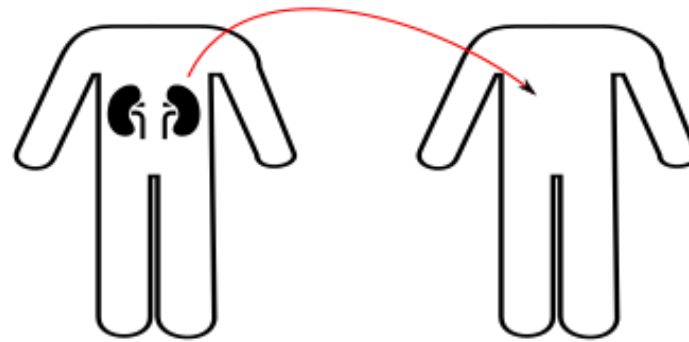
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Dialysis

Poor quality of life
High cost

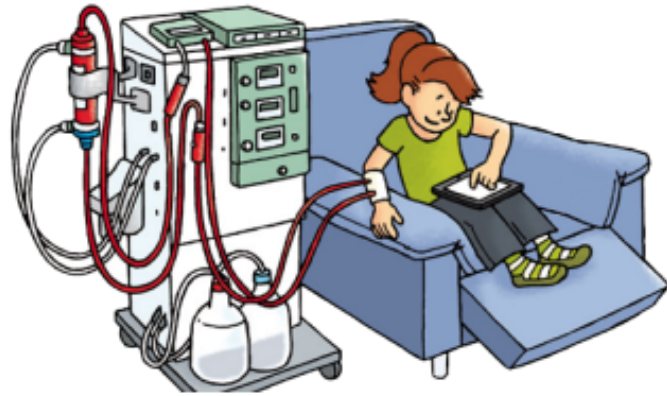
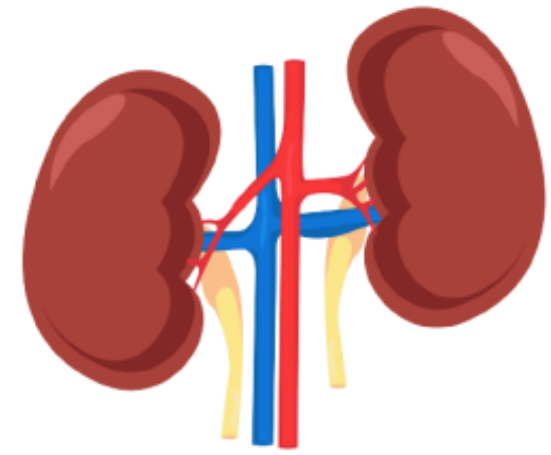


Transplant

Better life quality
Lower cost

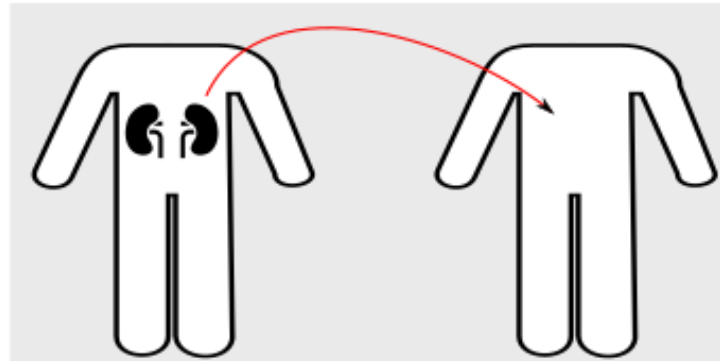
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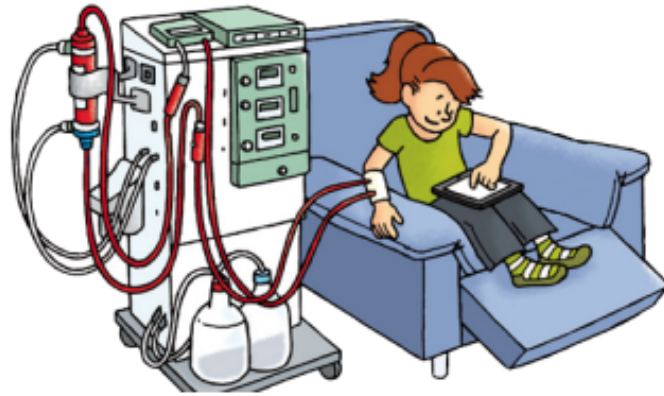
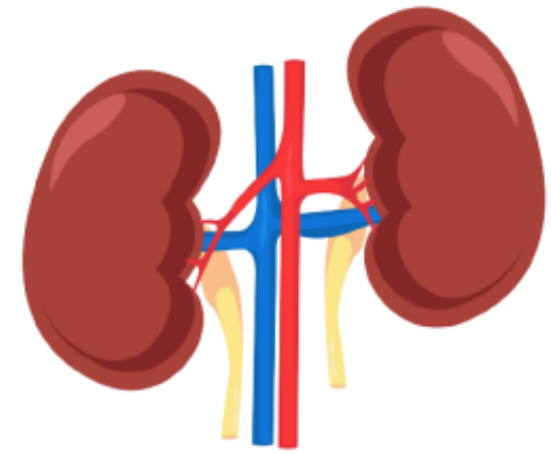
Better life quality
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- From a **deceased donor**

- From a **living donor**

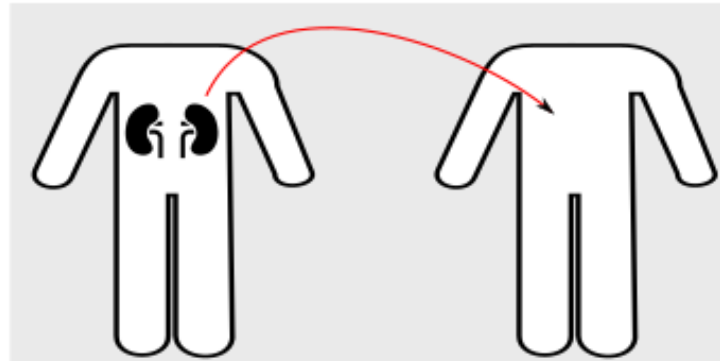
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Dialysis

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Transplant

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- From a **living donor**

E.g., first-degree relatives,
spouse etc.

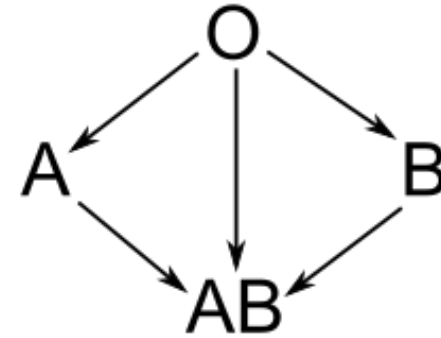
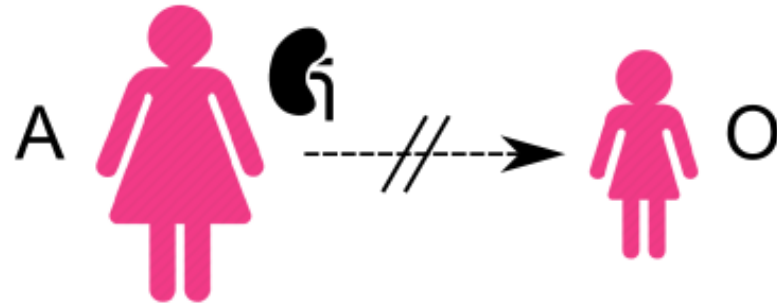
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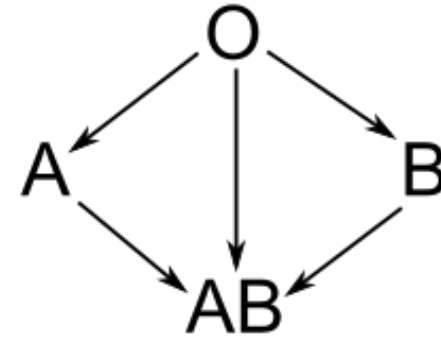
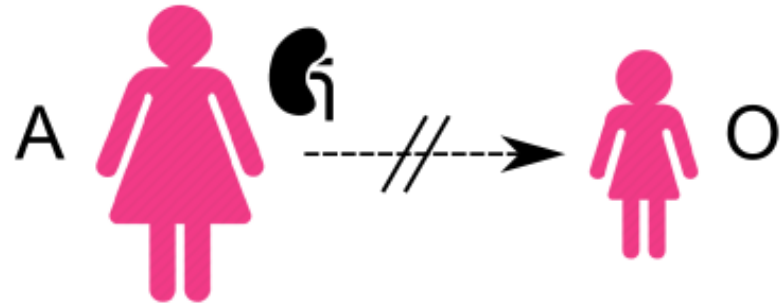
Blood-type incompatibility



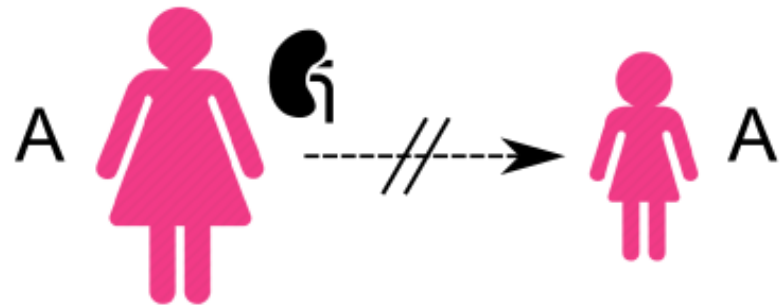
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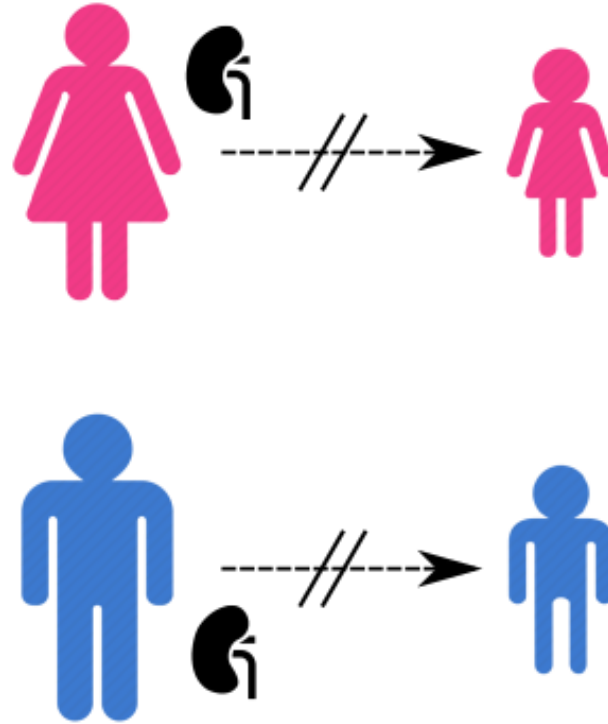
Tissue-type incompatibility



Sensitized patients with antibodies that will attack a foreign tissue (often due to prior exposure during blood transfusion, pregnancy, previous transplants).

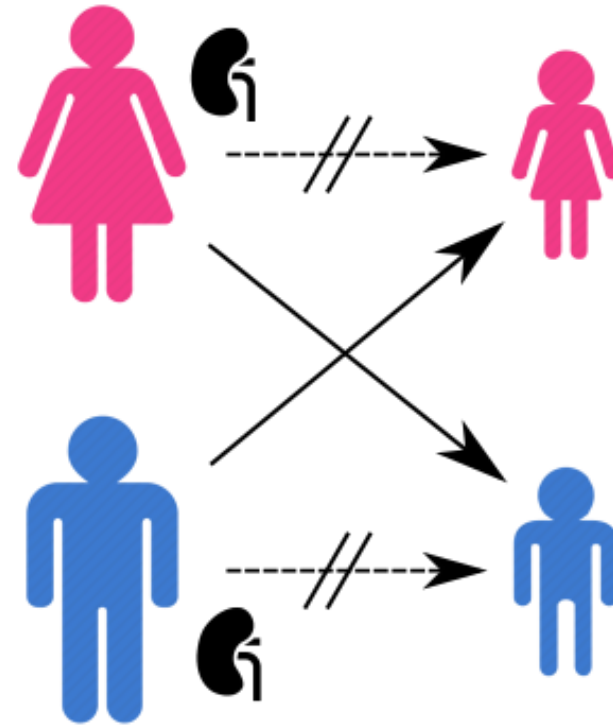
Kidney Exchange

[Rapaport, *Transpl. Proc.* 1986; Ross et al. *NEJM* 1997]

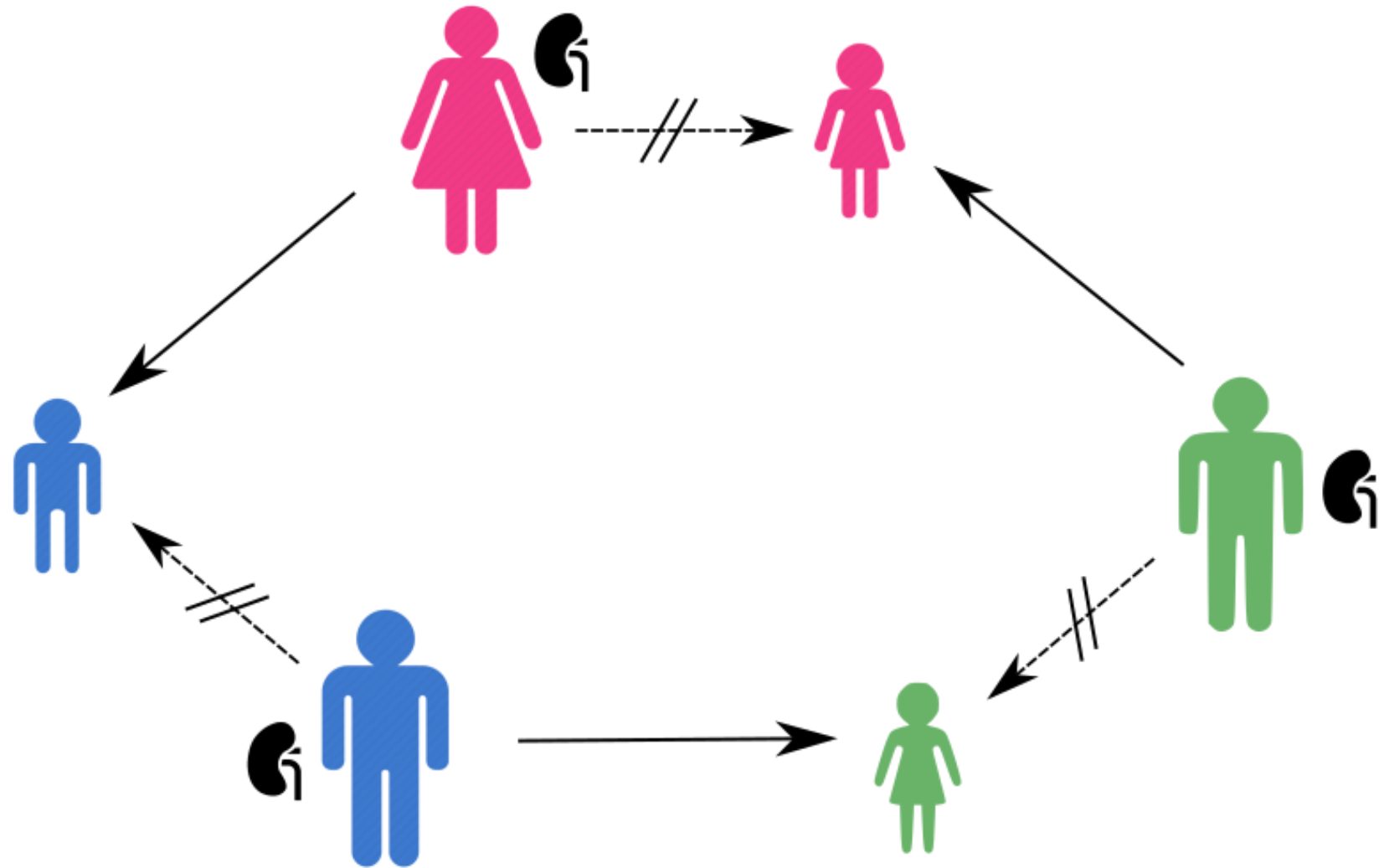


Kidney Exchange

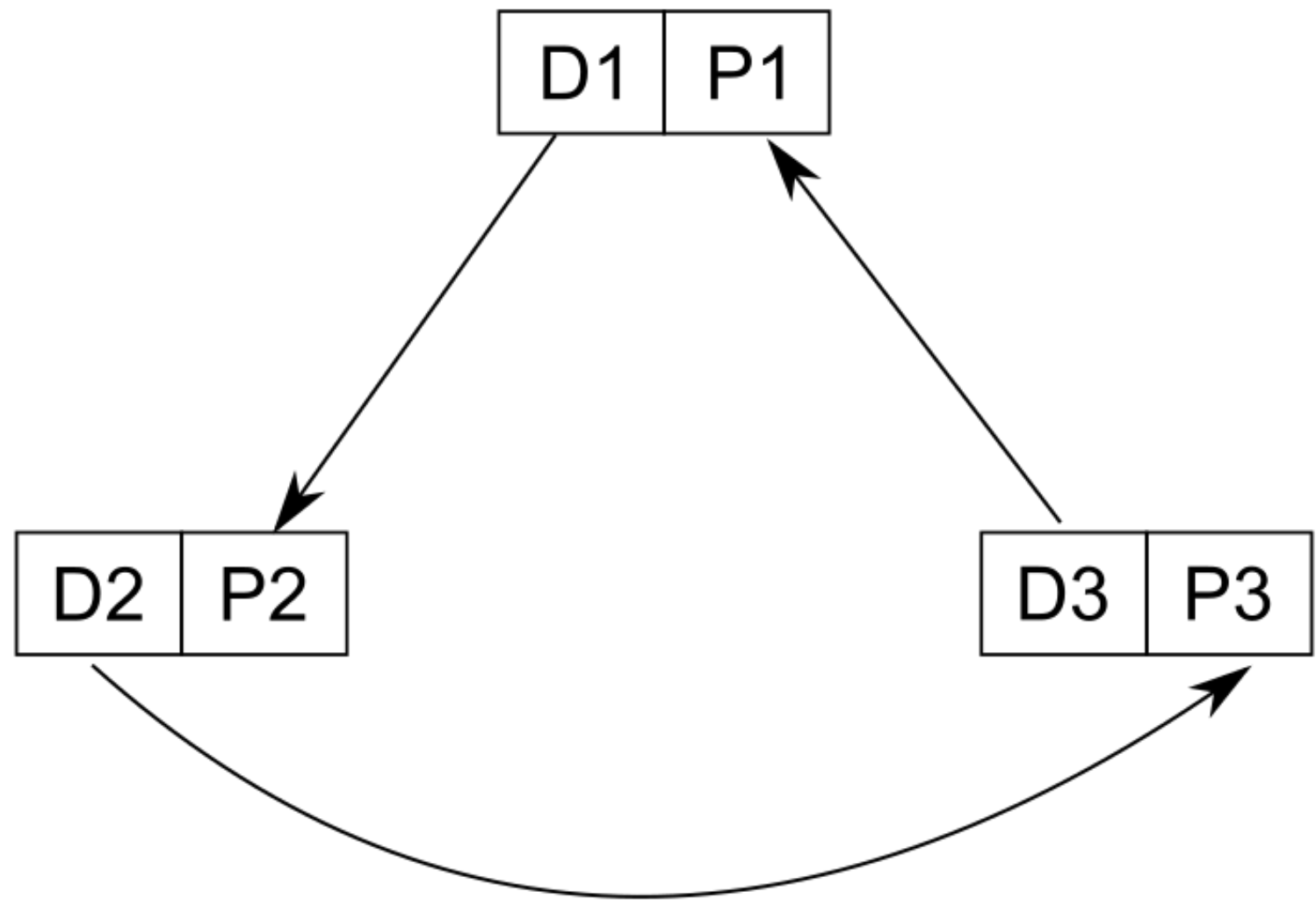
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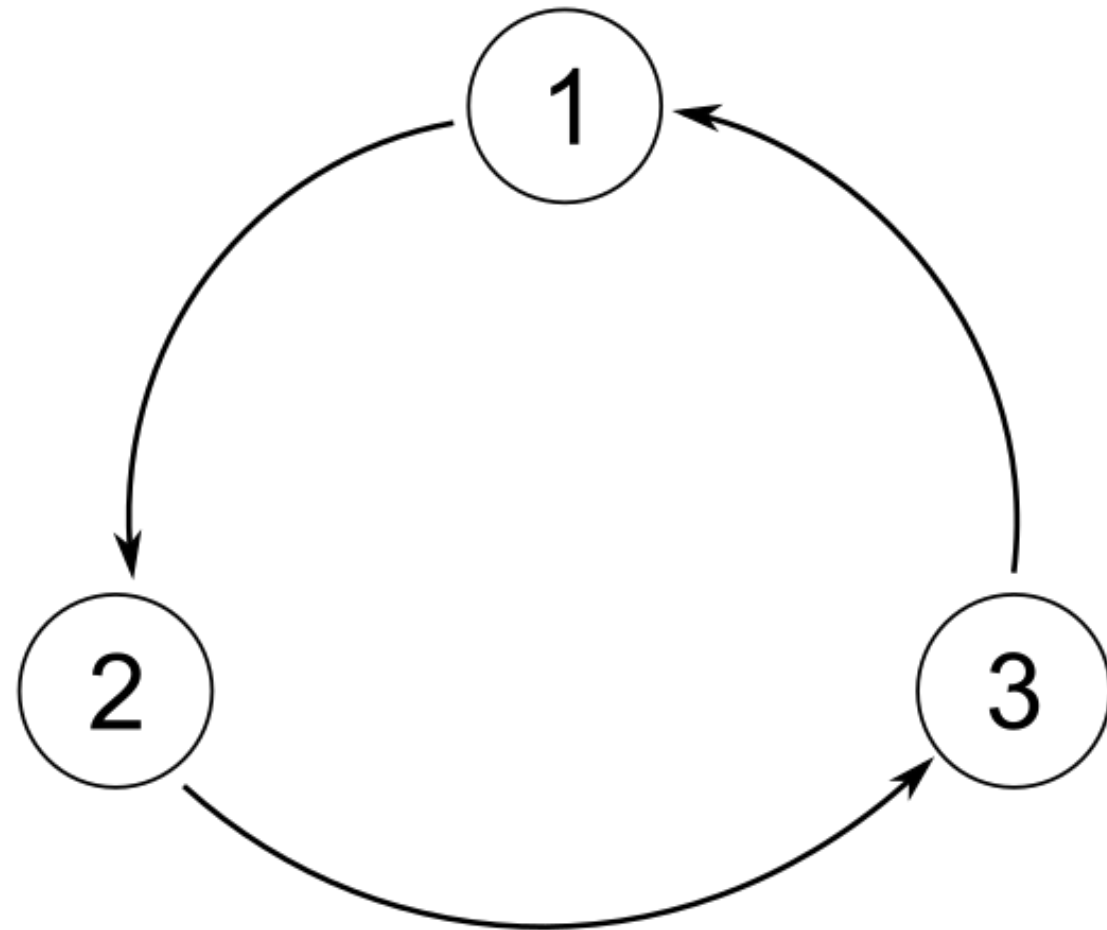
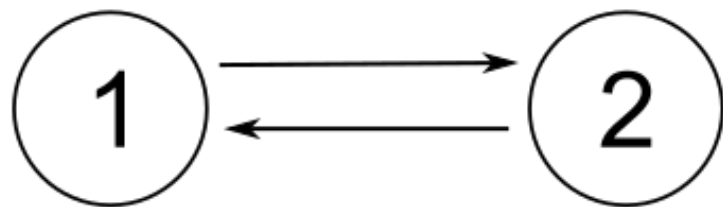
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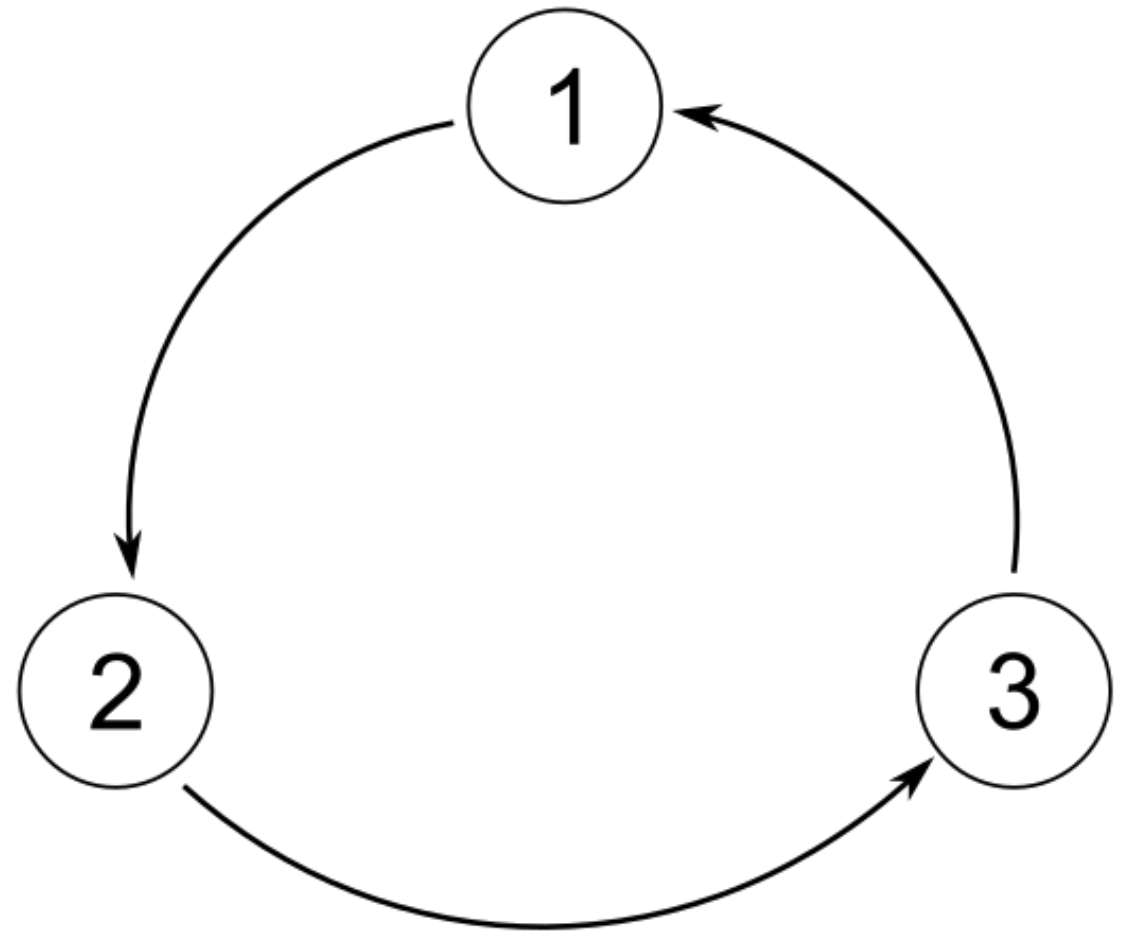
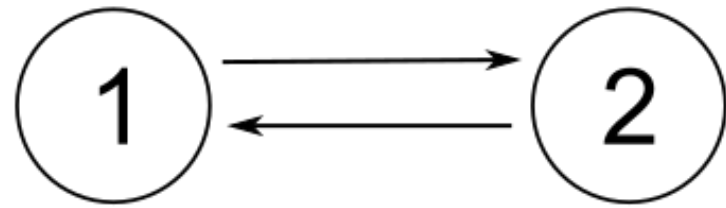
Kidney Exchange



Kidney Exchange



Kidney Exchange



Intuitively, having more and more of such willing-but-incompatible pairs will create more opportunities for an exchange.

History [\[edit \]](#)

Year	Event
1986	KPD first suggested by Rapaport ^[14]
1991	First KPD program started in South Korea by Dr. Park ^[8]
1999	First European KPD transplants performed in Switzerland ^[8]
2000	First KPD transplants performed in USA at the Rhode Island Hospital ^[8]
2001	Hopkins completes KPD transplants and begins first KPD program in USA ^[15]
2004	Dutch established first national KPD program ^[8]
2007	NEAD chain started by APD utilizing first bridge donor ^[16]
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA ^[17]
2008	National Kidney Registry organizes its first transplants on Valentine's Day ^[18]
2009	Hopkins leads first 16 patient multicenter Domino Chain ^{[19][20]}
2010	UNOS organizes its first KPD transplants ^[8]
2012	National Kidney Registry completes largest chain involving 60 participants ^[6]
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Early Proposals for Kidney Exchange

Proposal 1

Using TTCA



Proposal 2

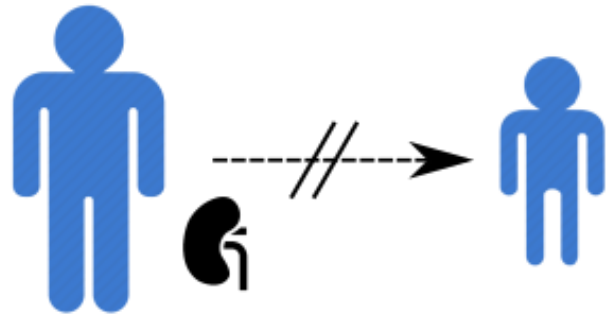
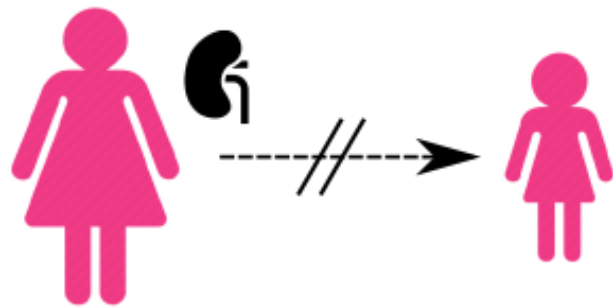


Proposal 1: Kidney Exchange using TTCA

[Roth, Sönmez, and Ünver, *QJE* 2004]

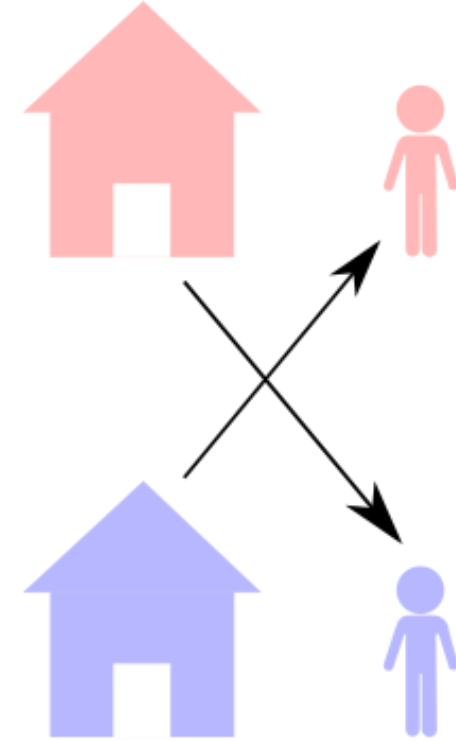
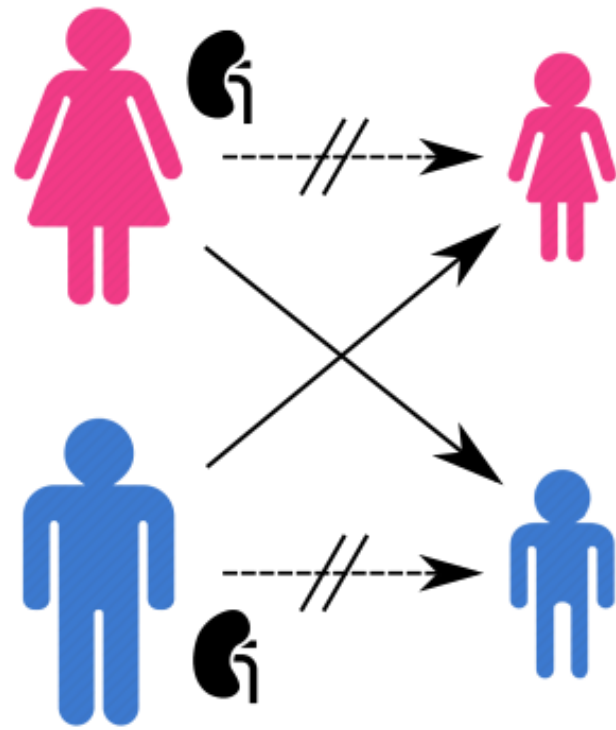
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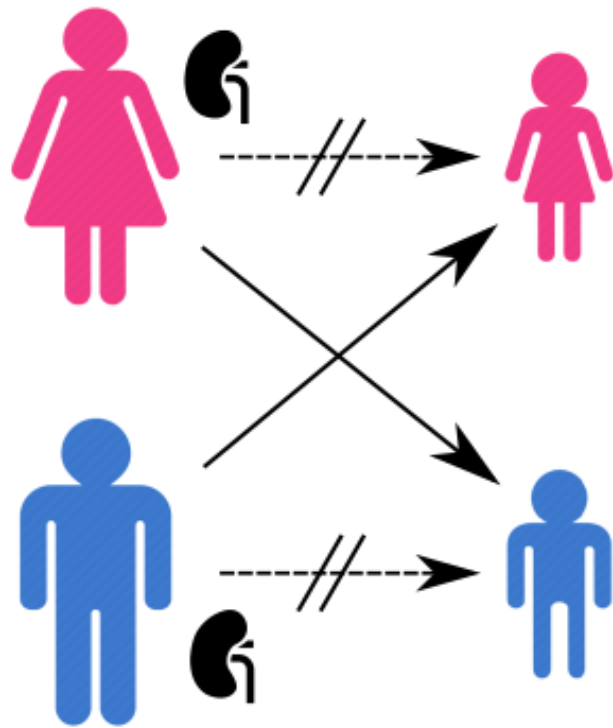
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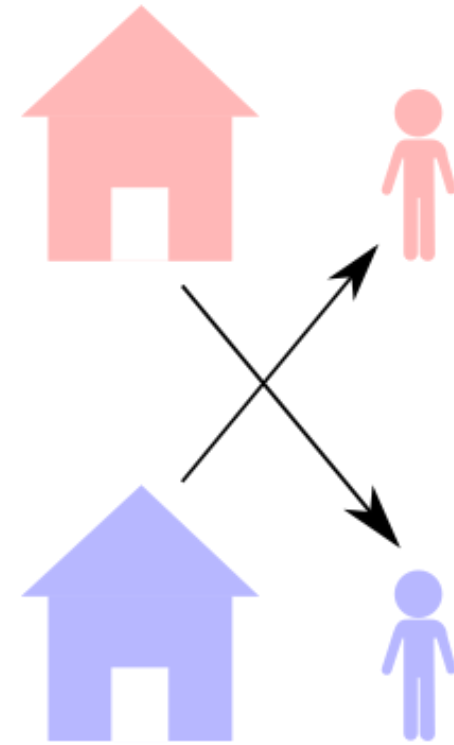


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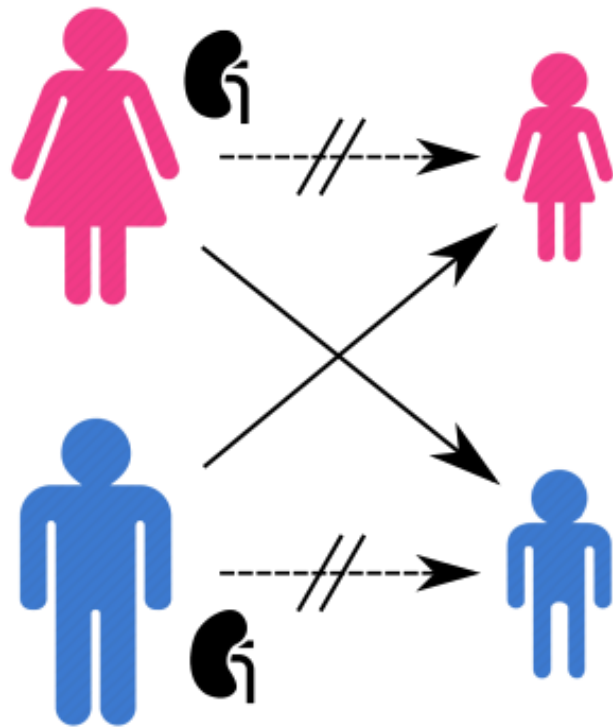
Each patient has a strict ranking of donors.



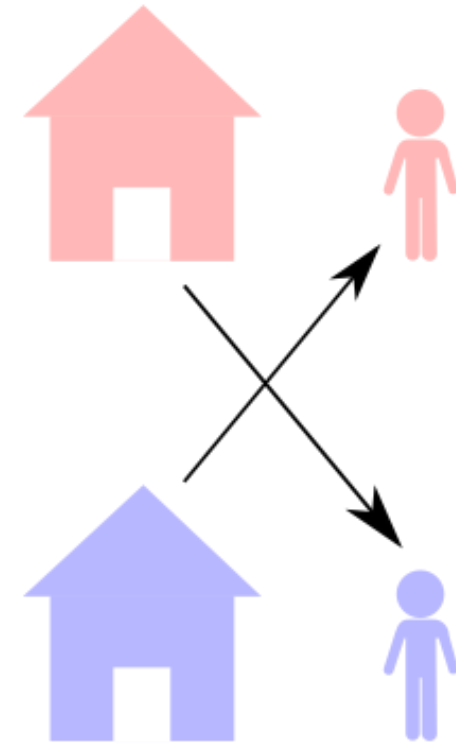
Each agent has a strict ranking of houses.

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Each patient has a
strict ranking of donors.
(medically determined)



Each agent has a
strict ranking of houses.
(idiosyncratic)

Advantages of Kidney Exchange using TTCA

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No group of patients can strictly improve by reallocating donors among themselves.



Every patient is (weakly) better off than before.



Polynomial time.

Limitations of Kidney Exchange using TTCA

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In practice, there may be:



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3

4



2



1

5



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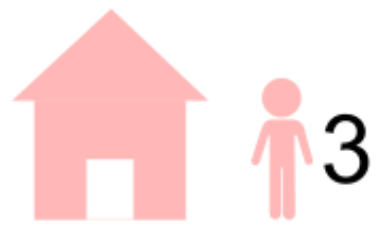
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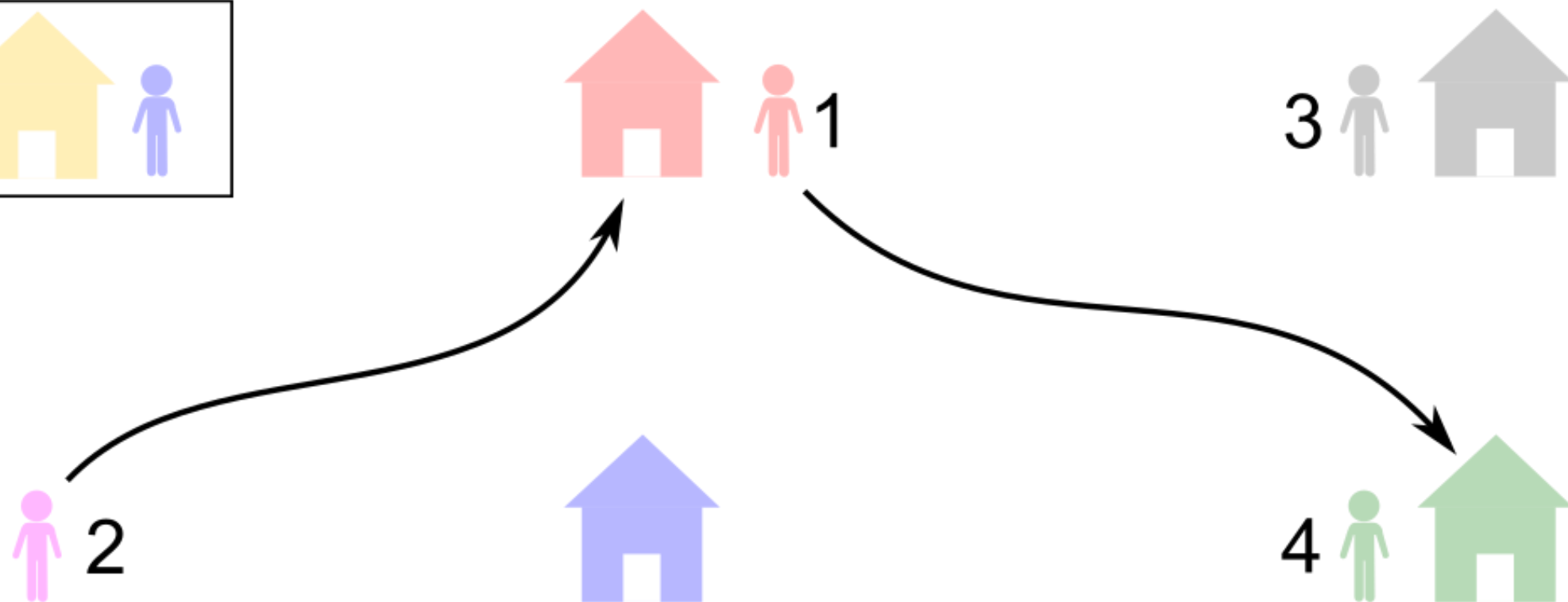
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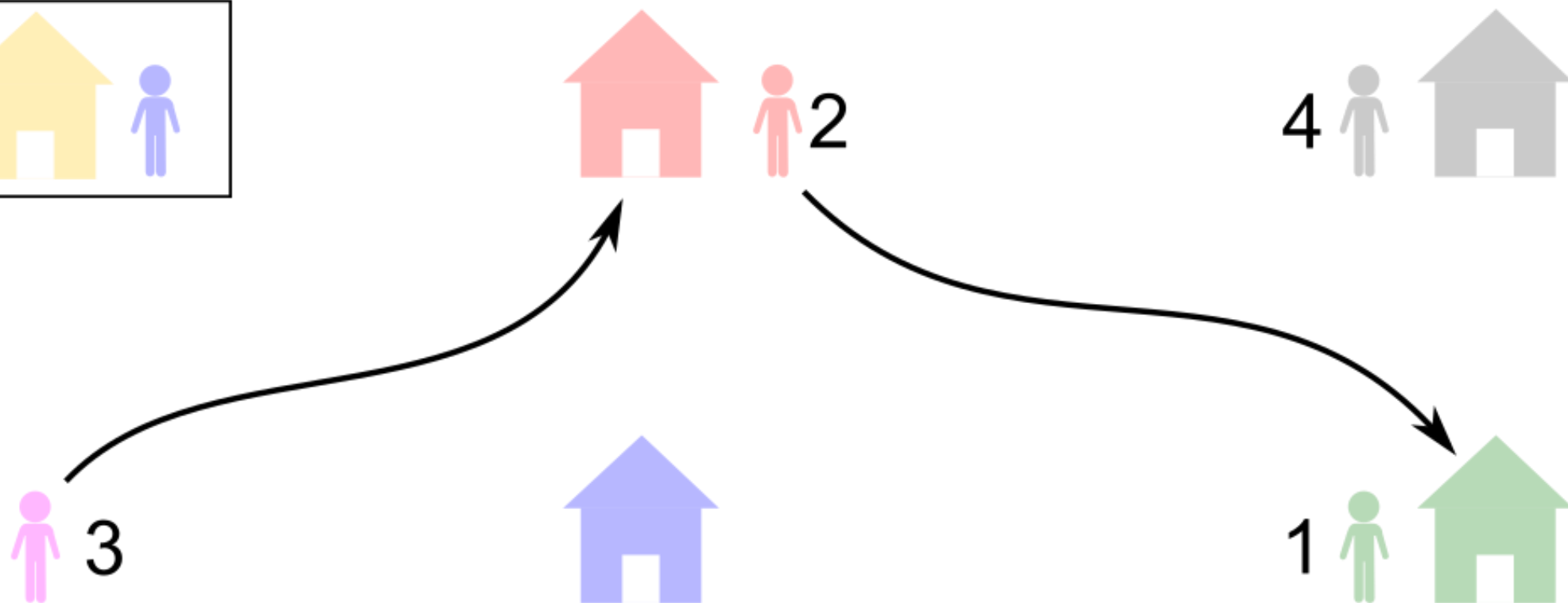
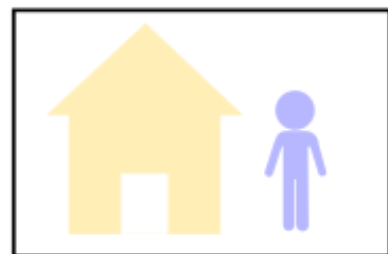
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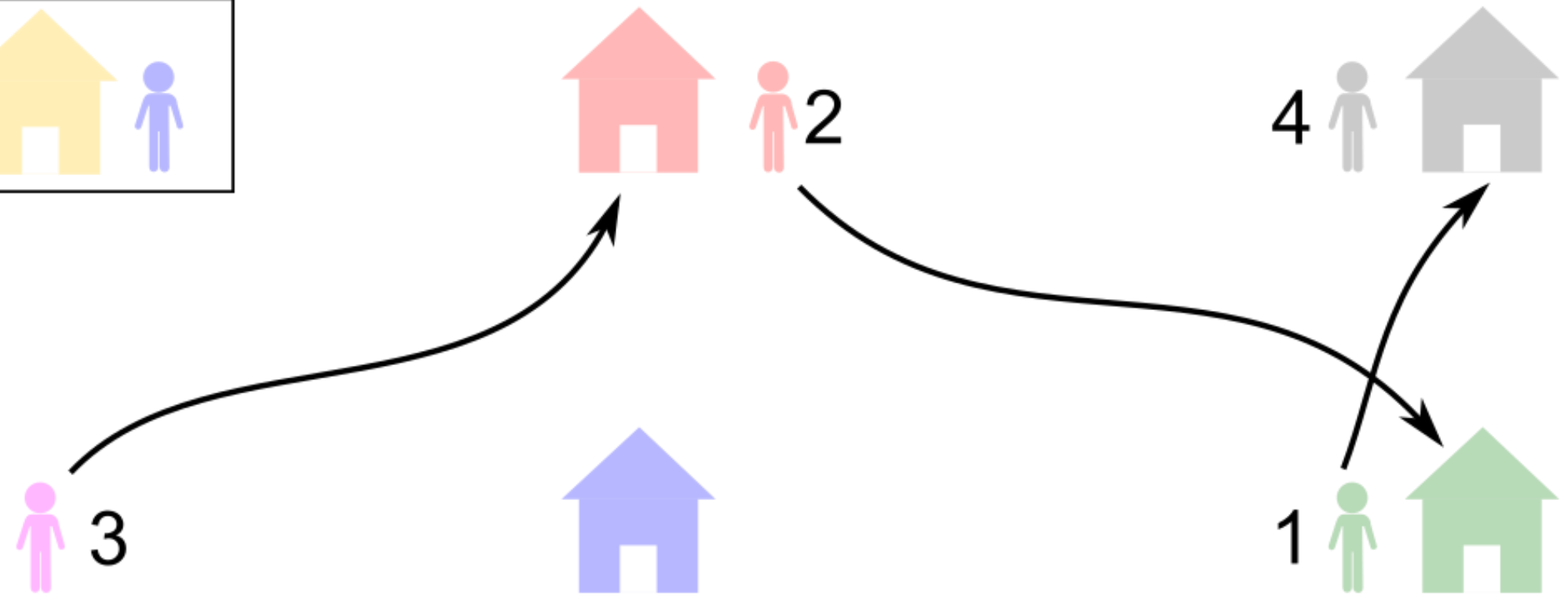
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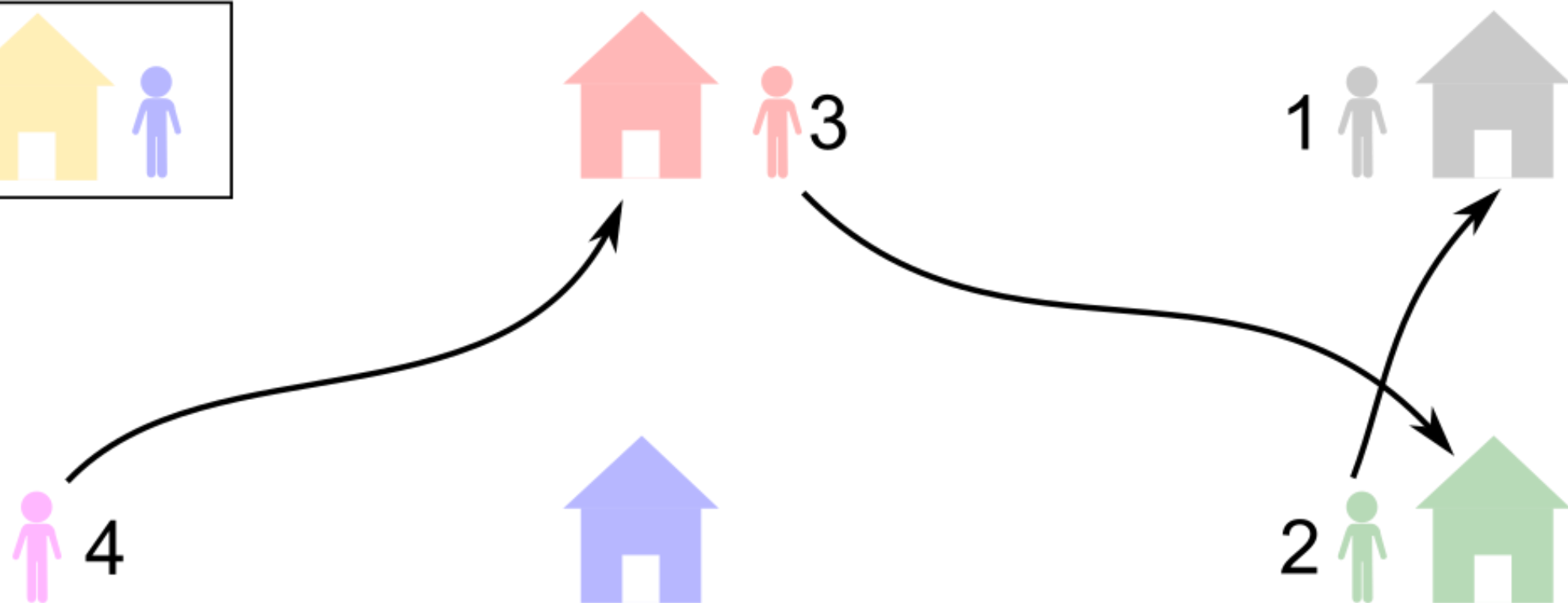
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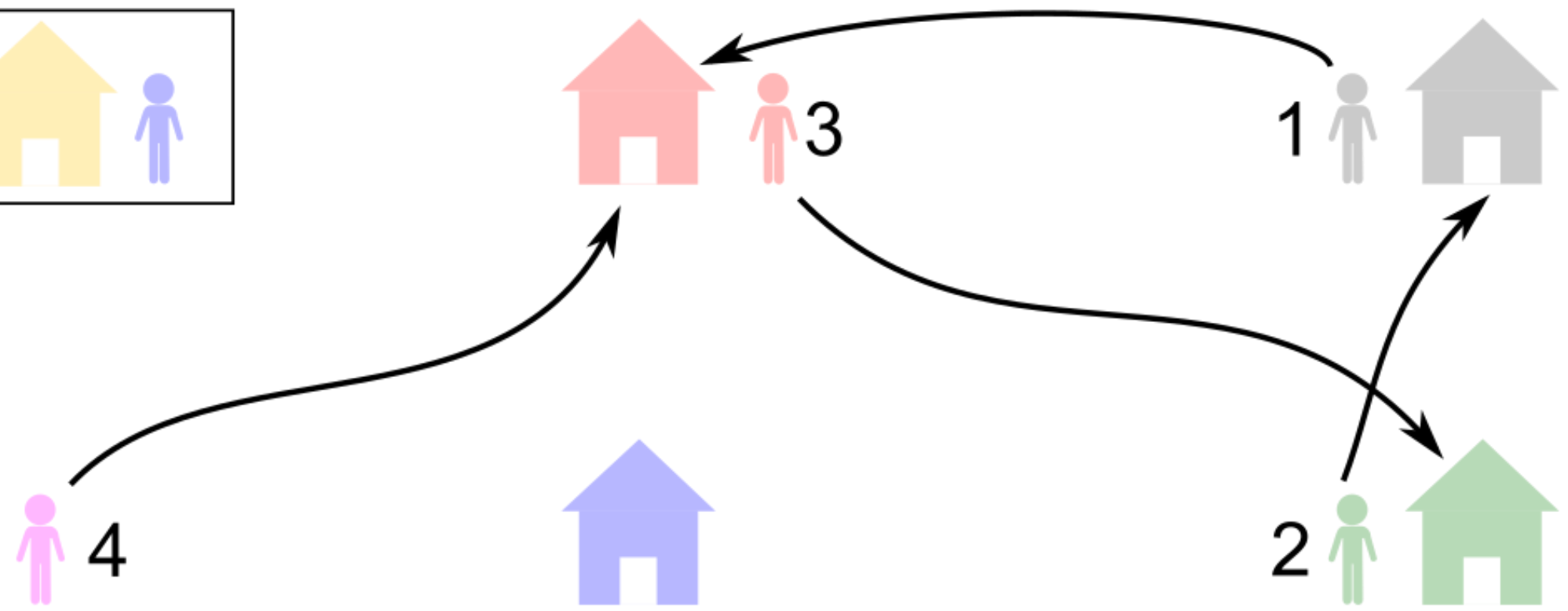
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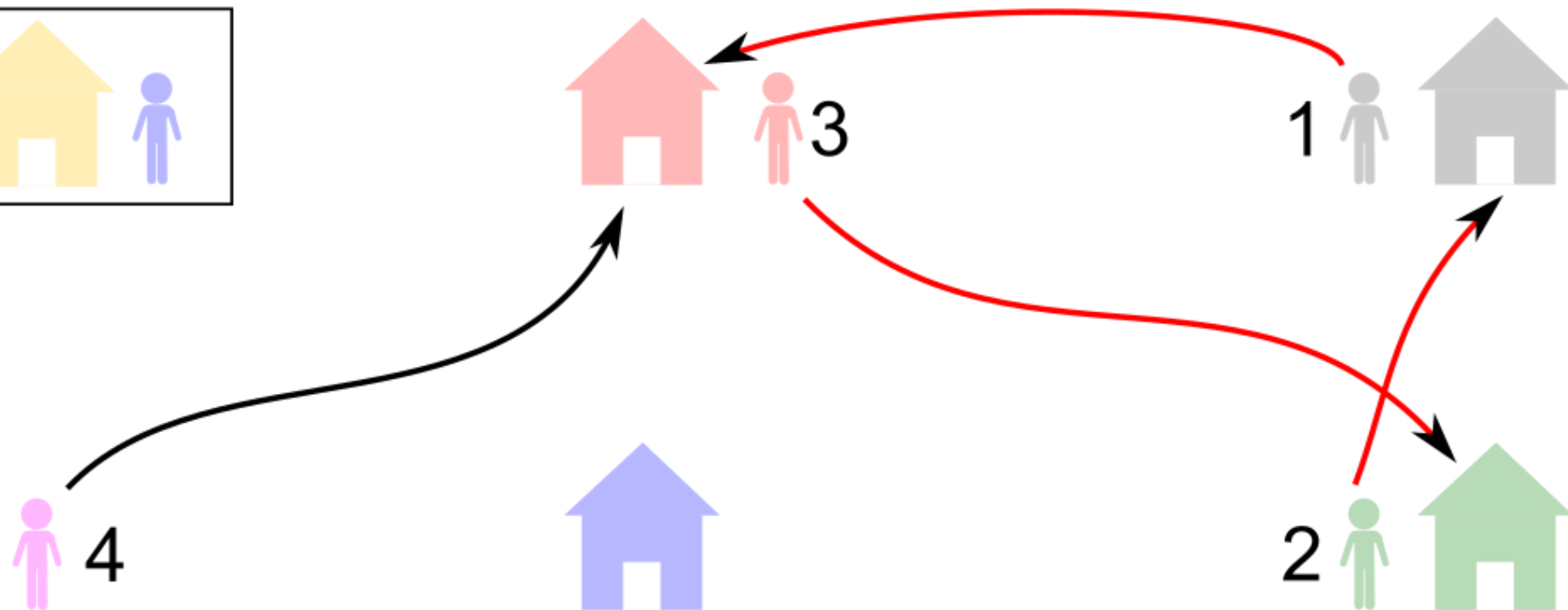
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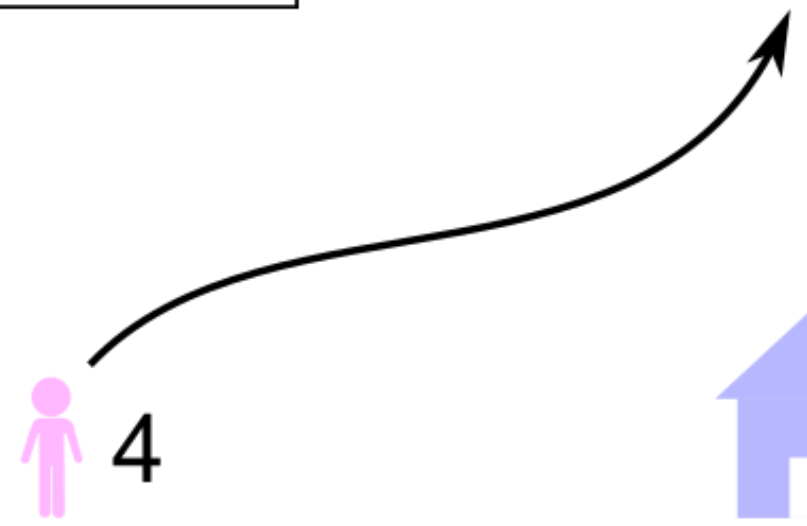
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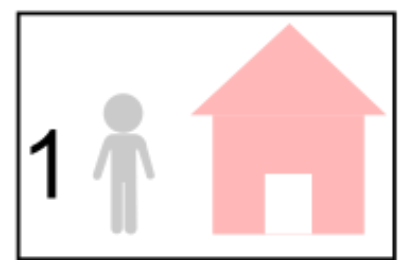
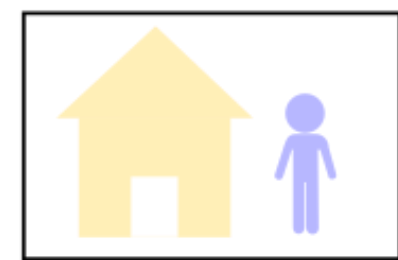
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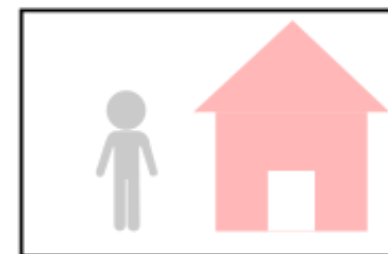
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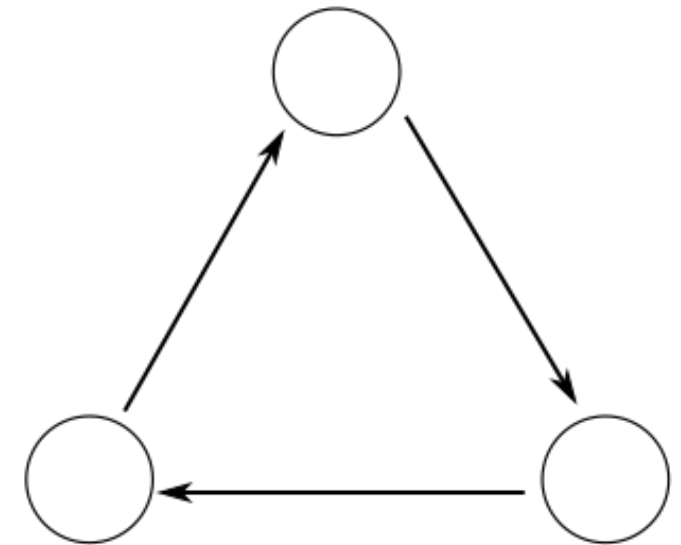
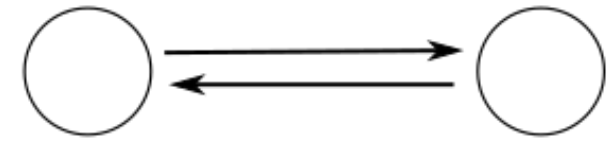
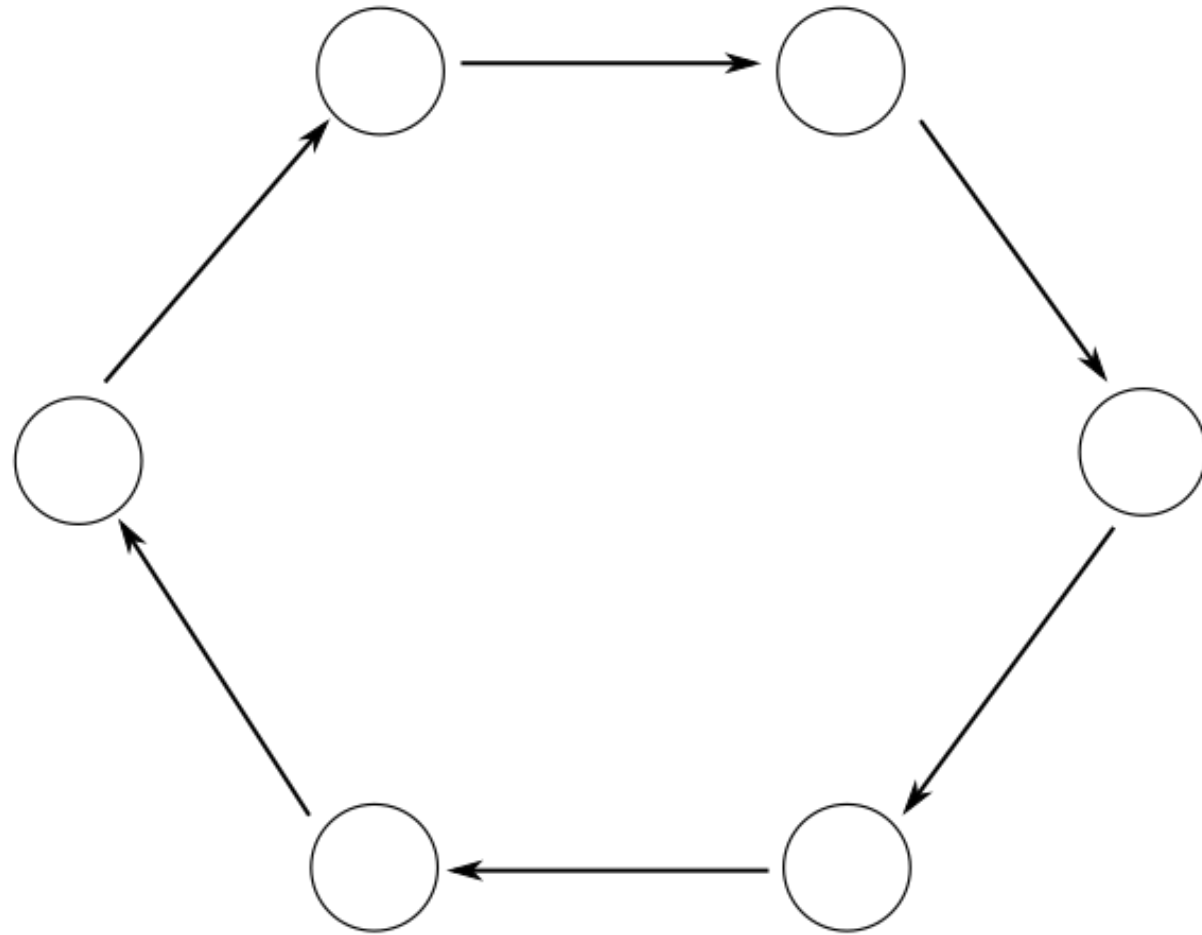


Long cycles can lead to **logistical problems**.

Limitations of Kidney Exchange using TTCA



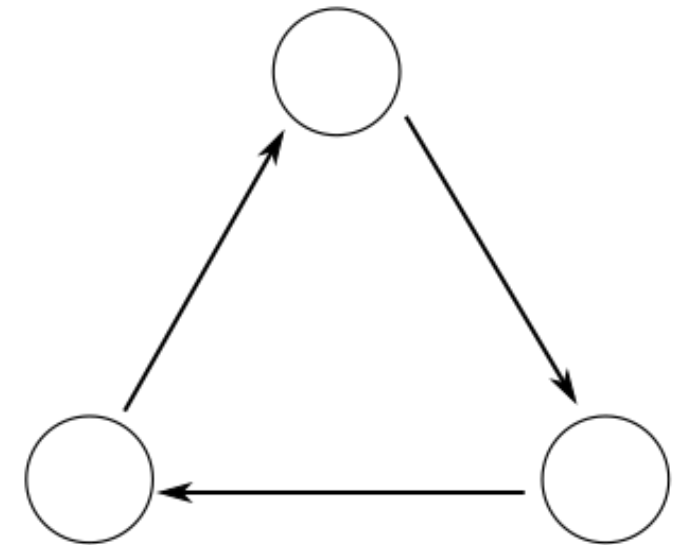
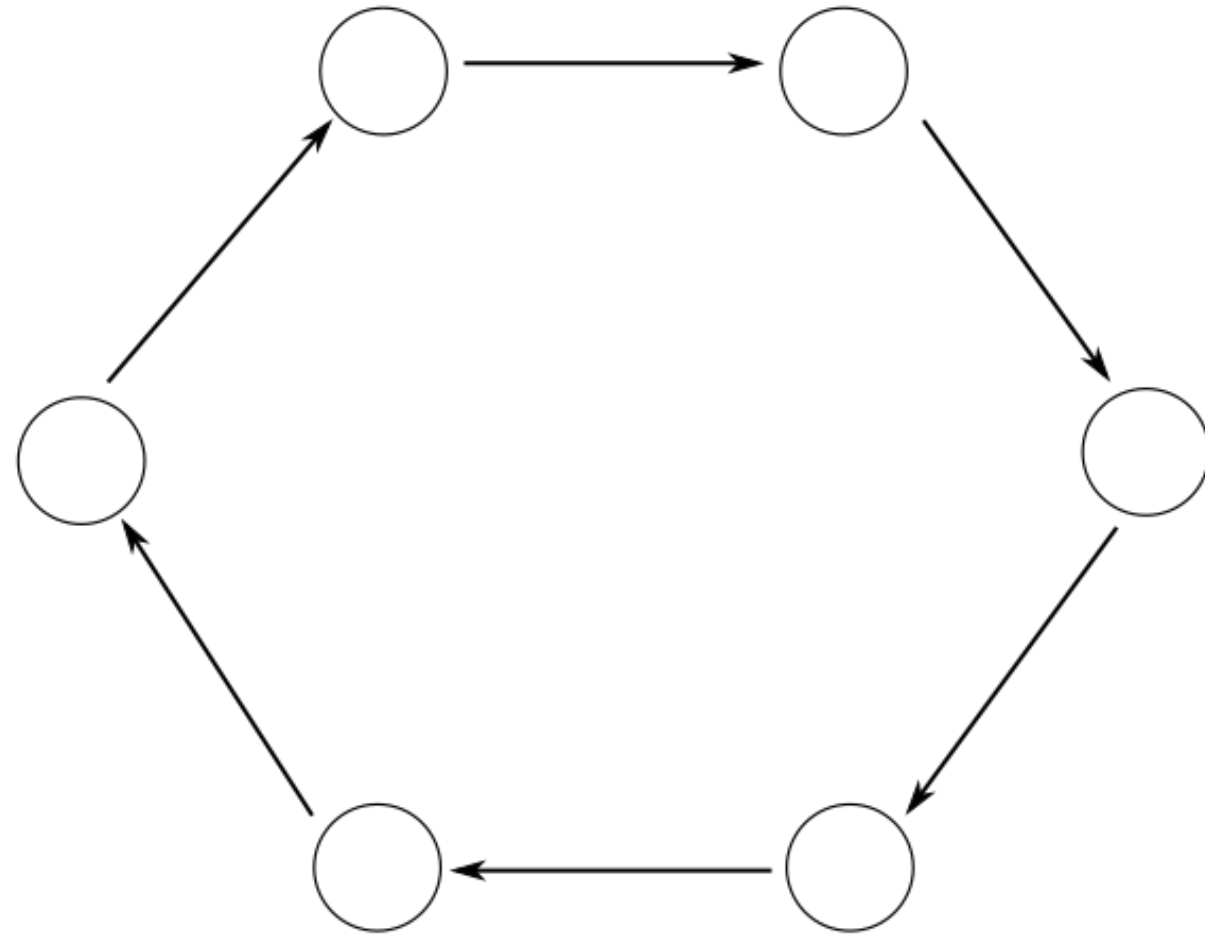
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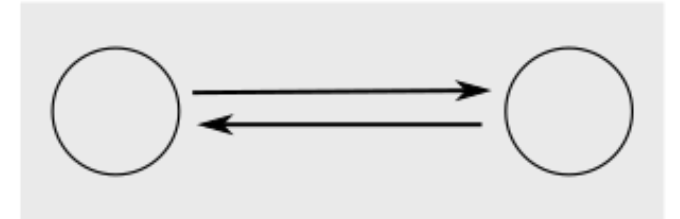
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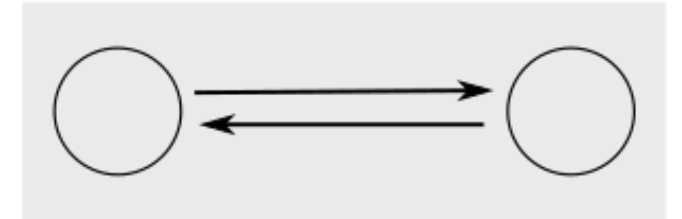
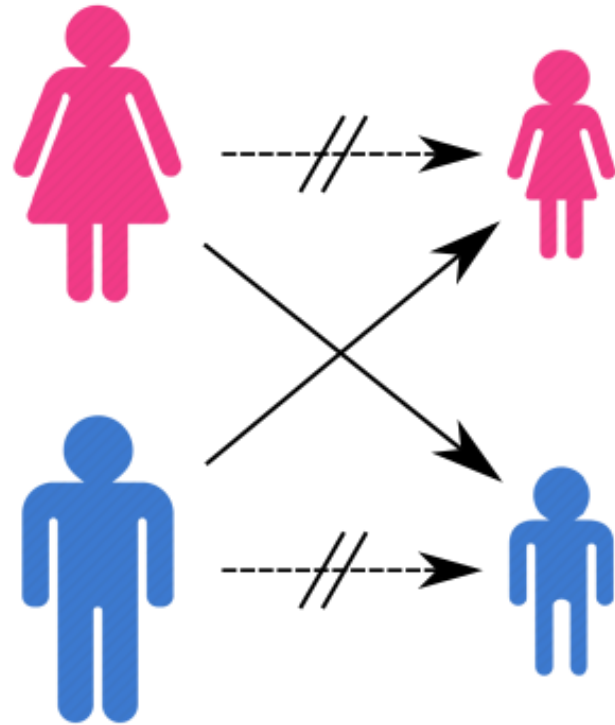
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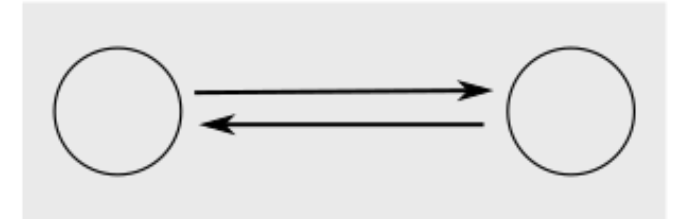
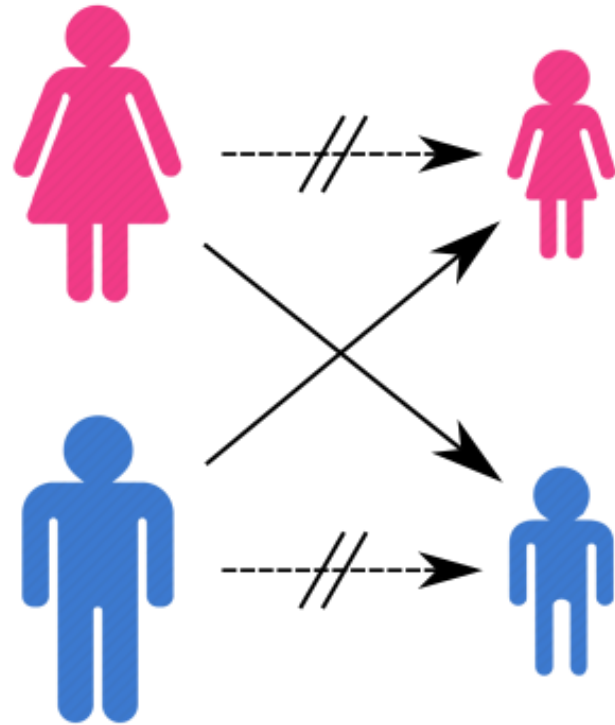
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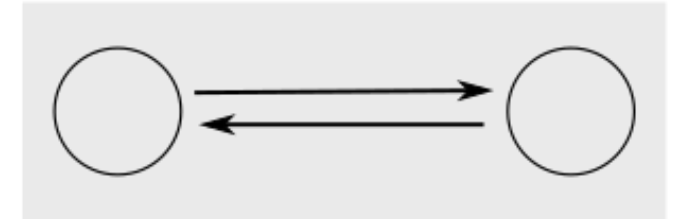
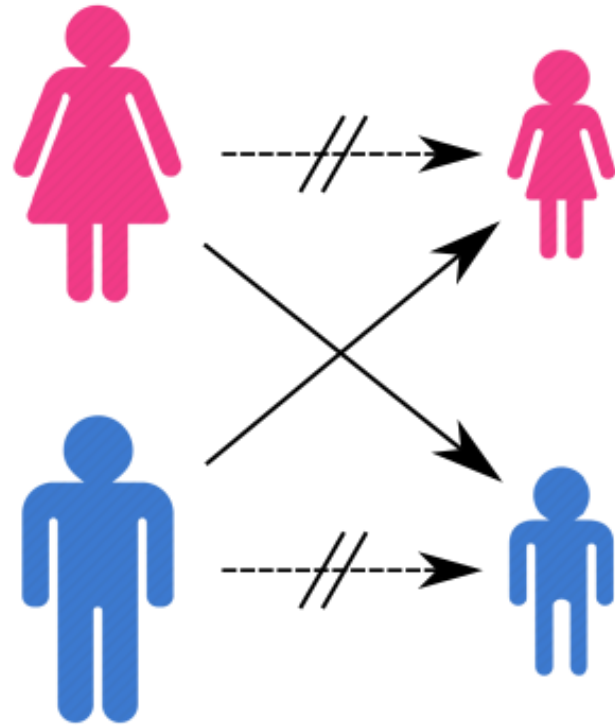


Four surgeries

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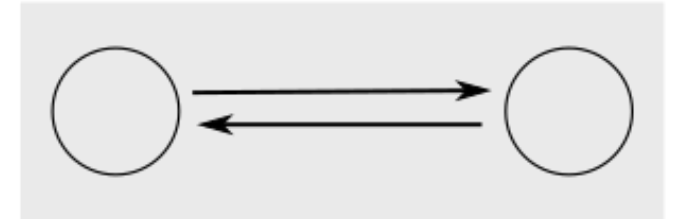
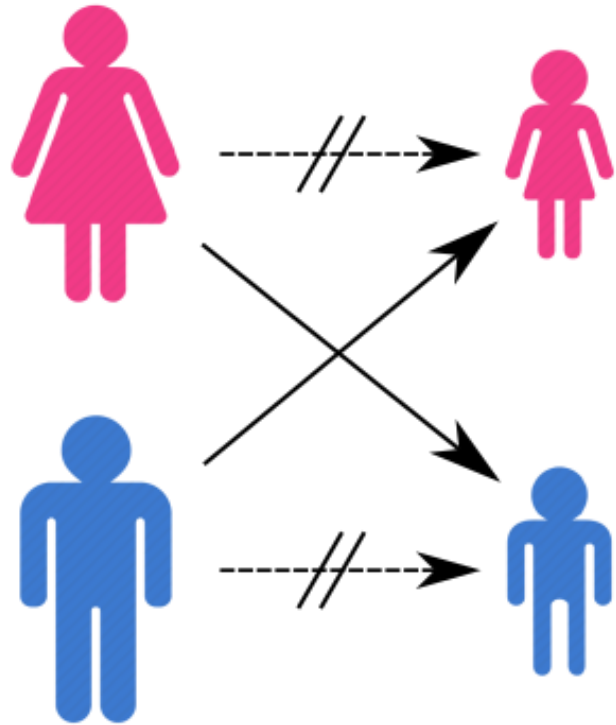


Four **simultaneous** surgeries

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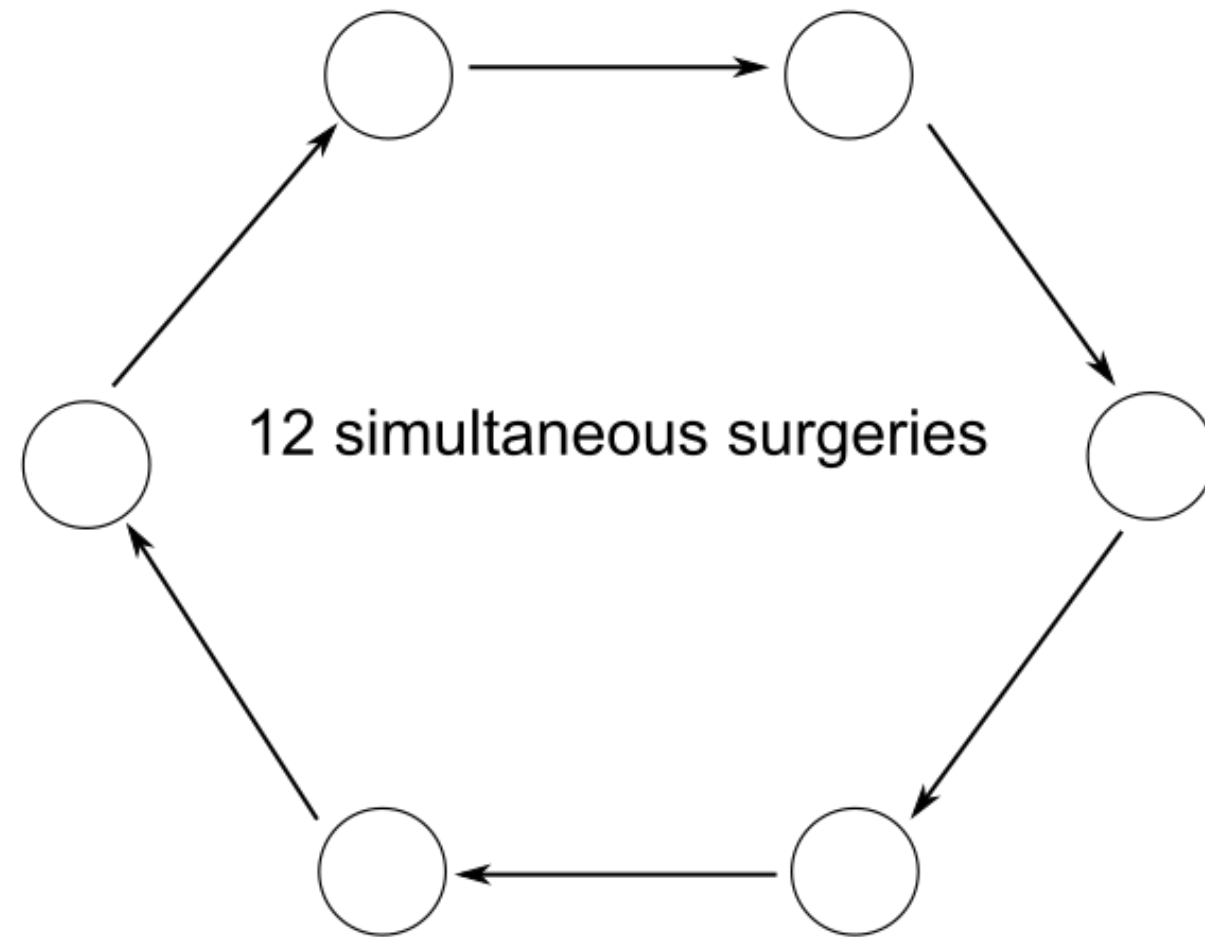
Cannot risk doing



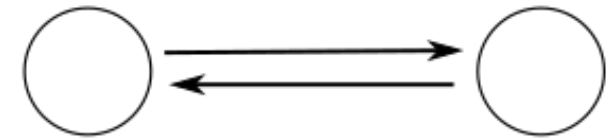
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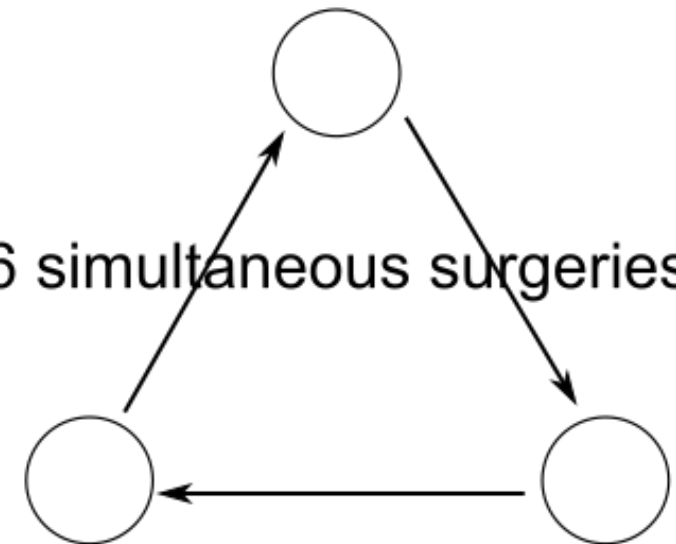
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6 simultaneous surgeries



Limitations of Kidney Exchange using TTCA

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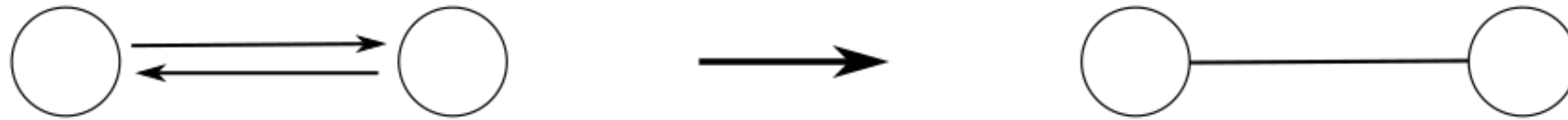
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Early Proposals for Kidney Exchange

Proposal 1

Using TTCA



Proposal 2



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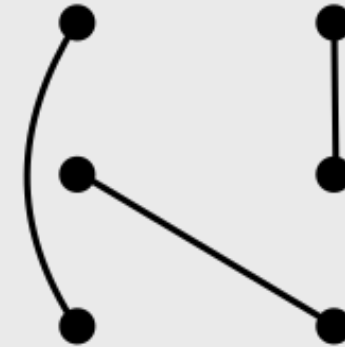
Proposal 1

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Proposal 2

Using Matchings

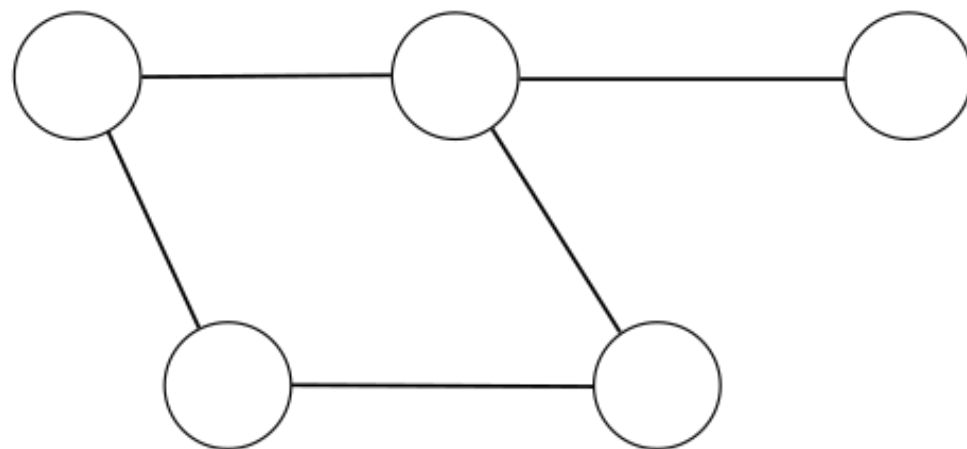


Proposal 2: Kidney Exchange using Matchings

[Roth, Sönmez, and Ünver, *JET* 2005]

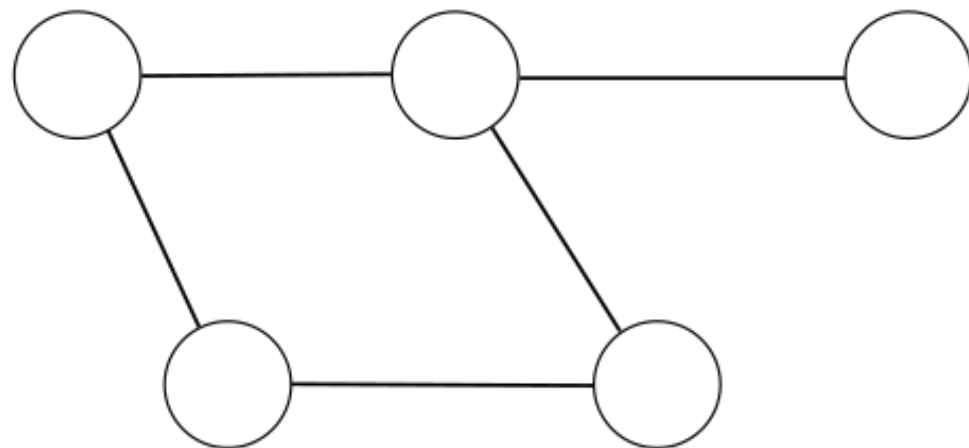
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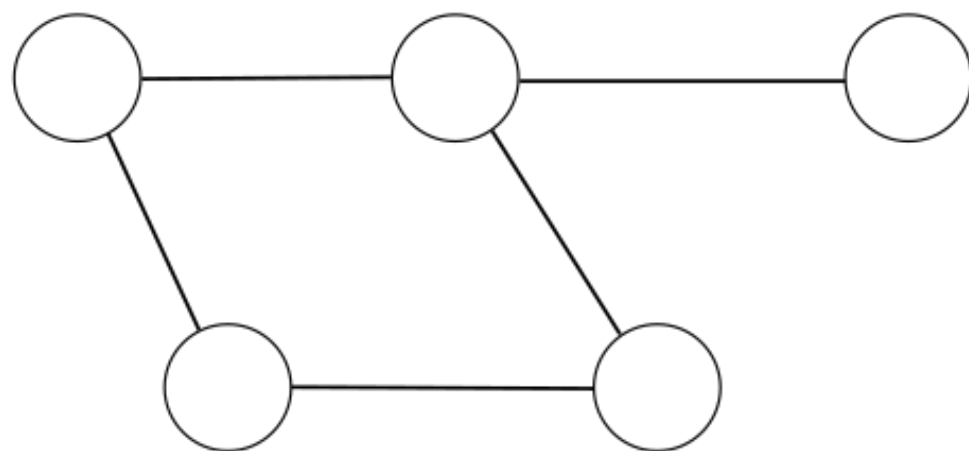
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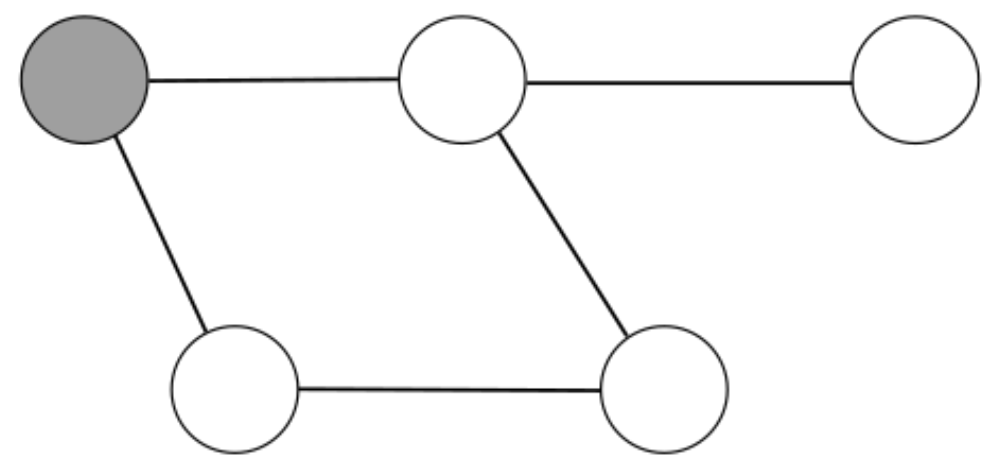


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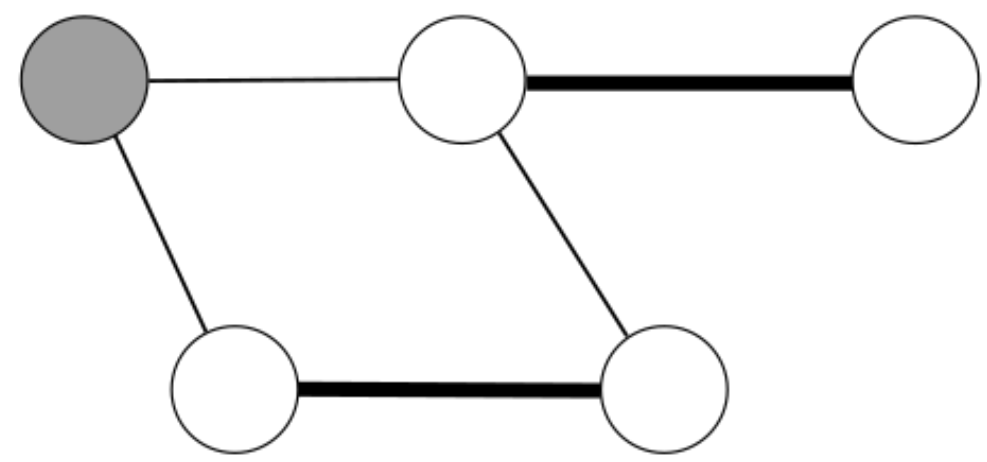


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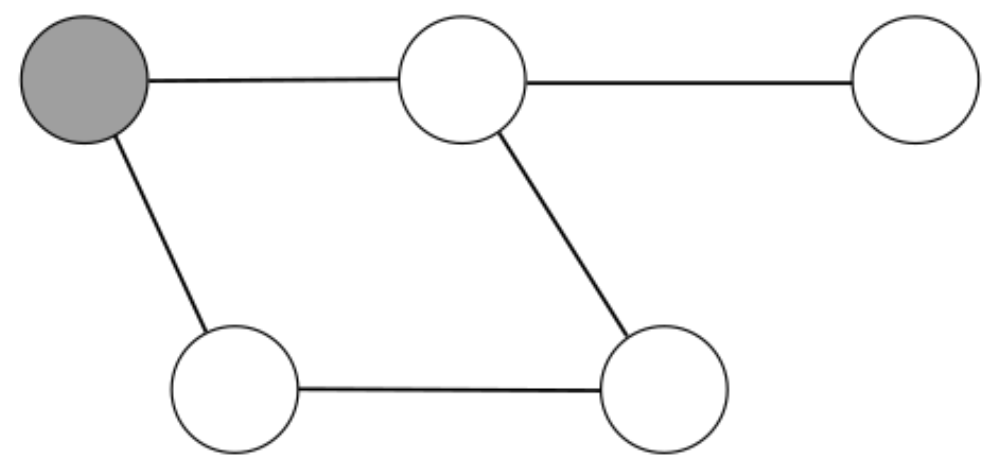


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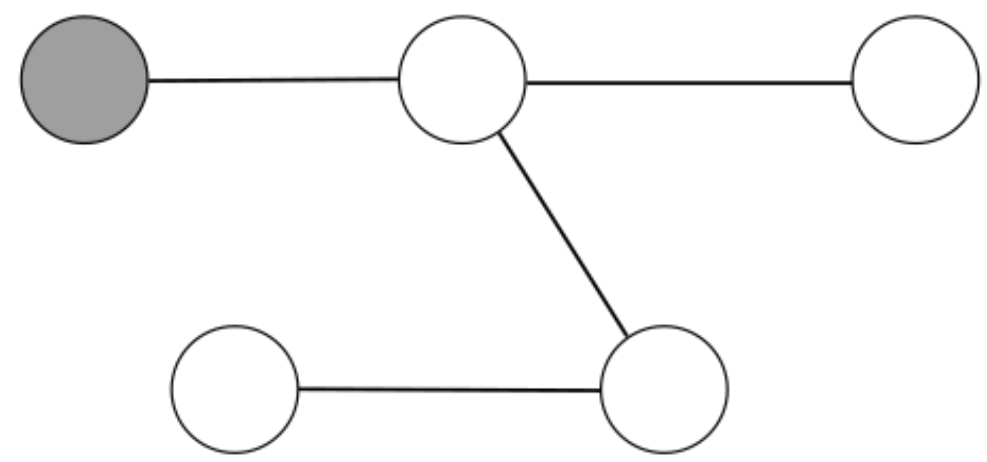


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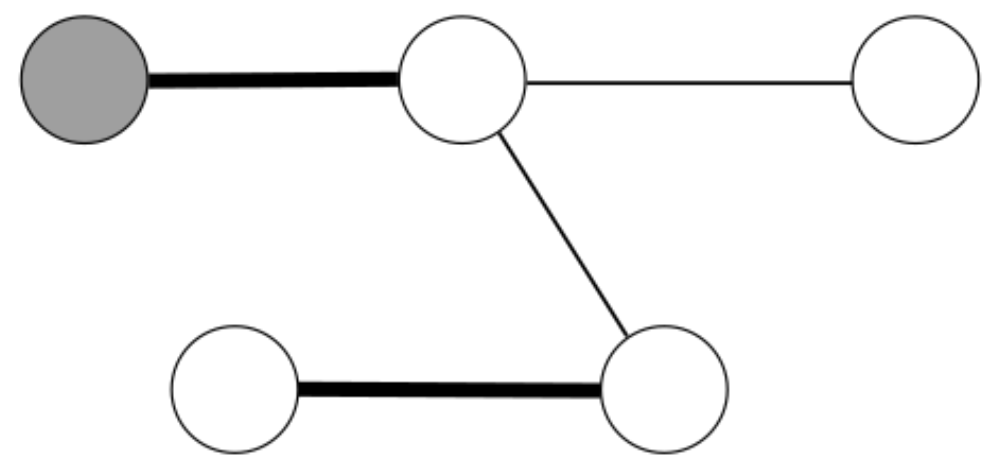


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Polynomial time
(uses Edmonds-Gallai decomposition from Graph Theory)

Limitations of Kidney Exchange using Matchings

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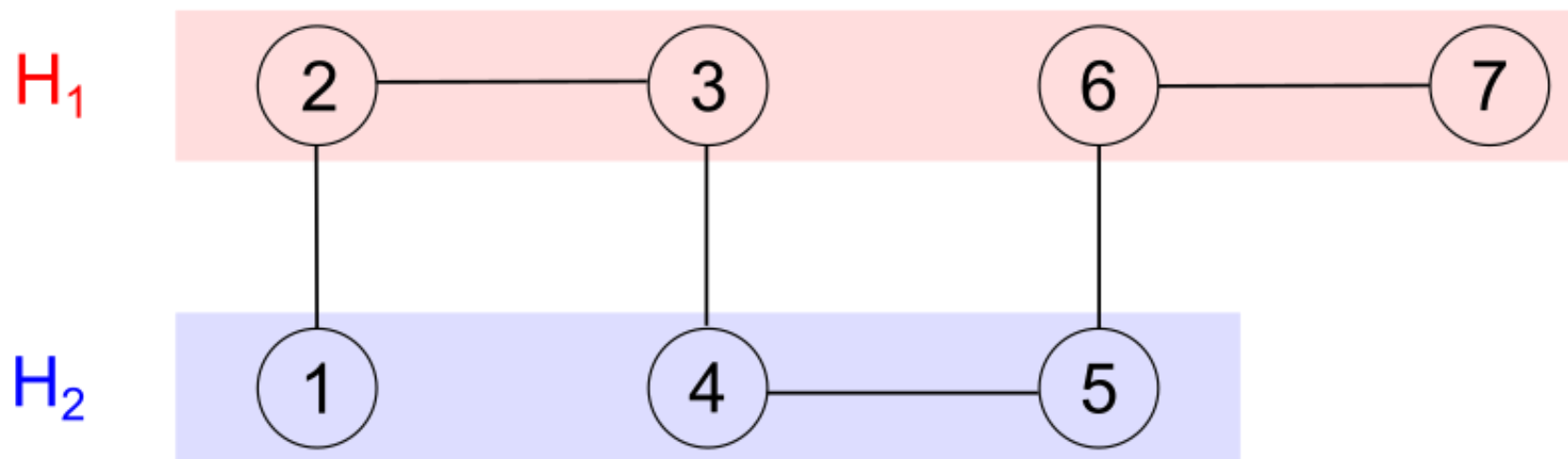
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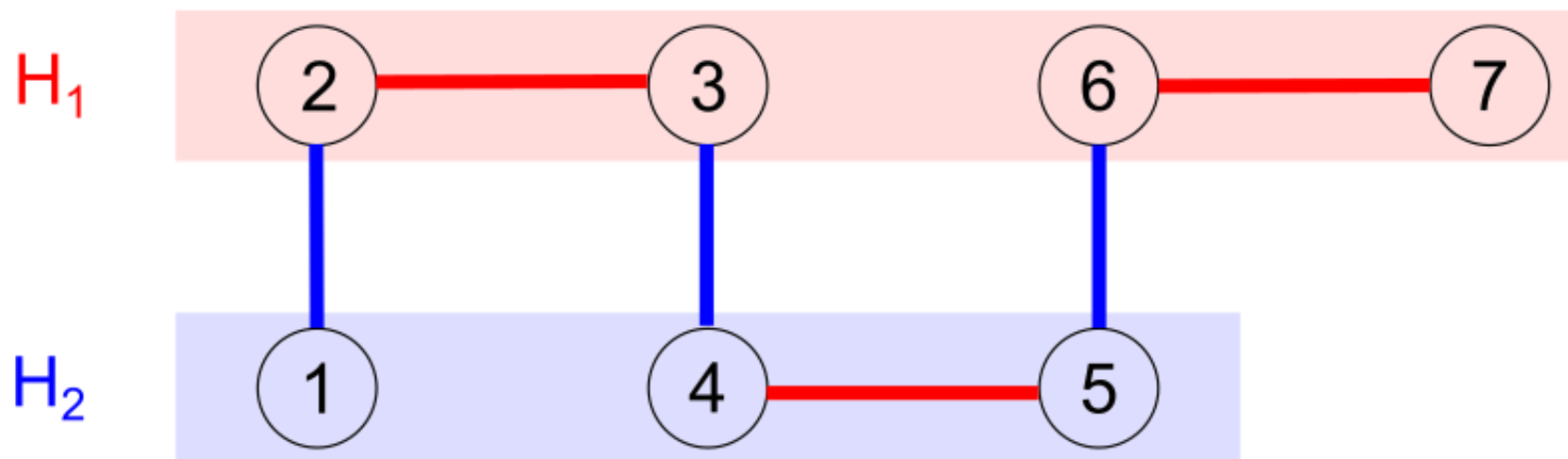
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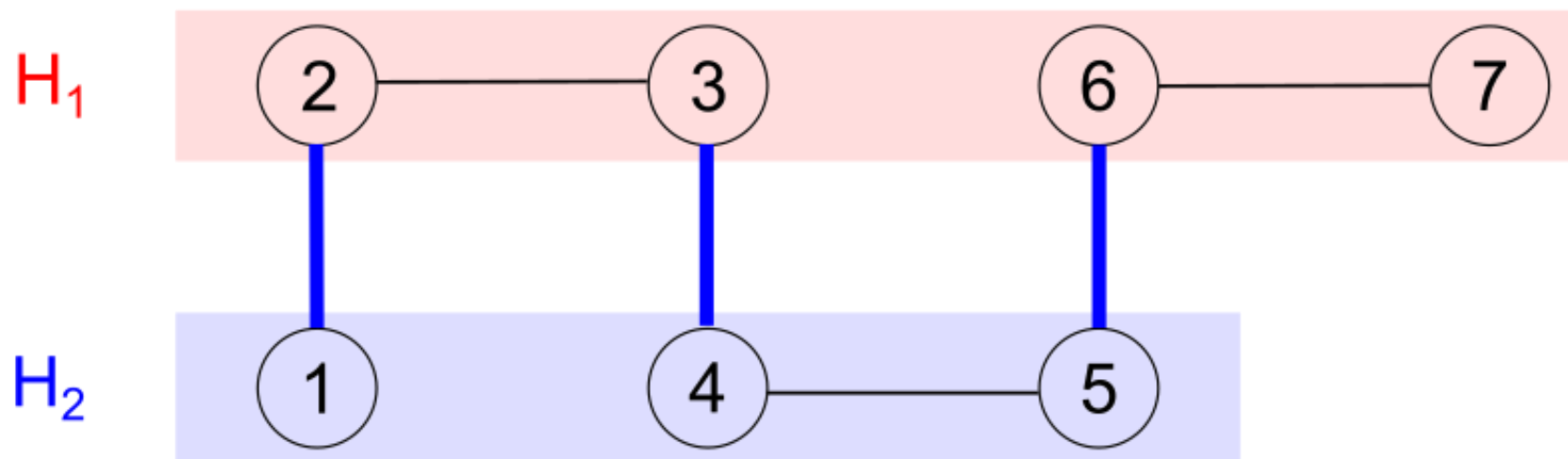
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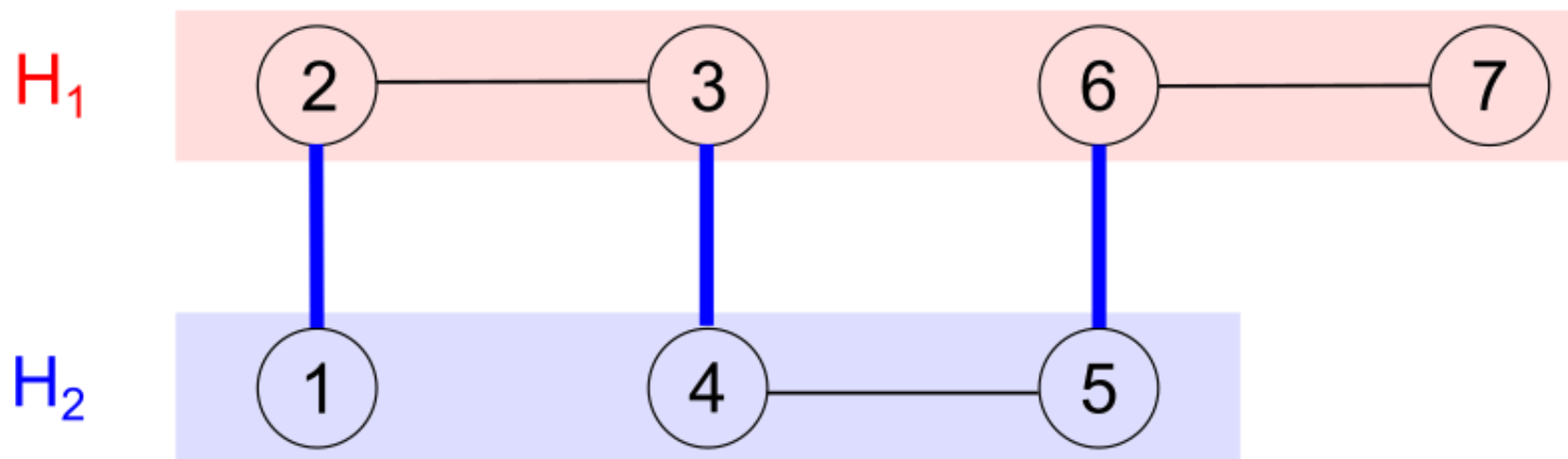
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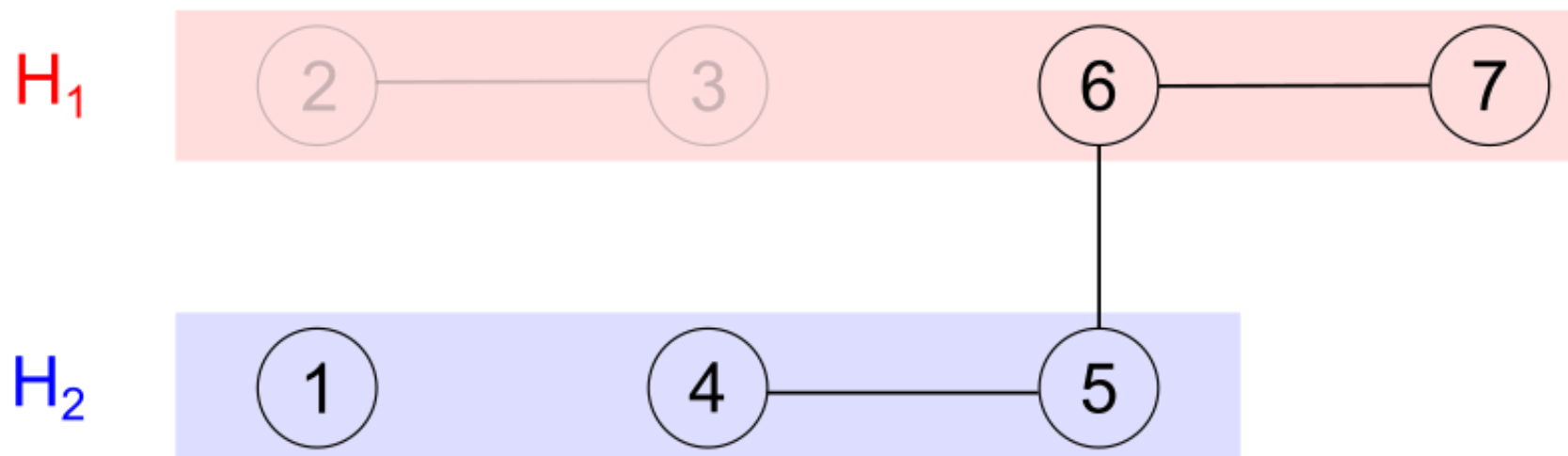


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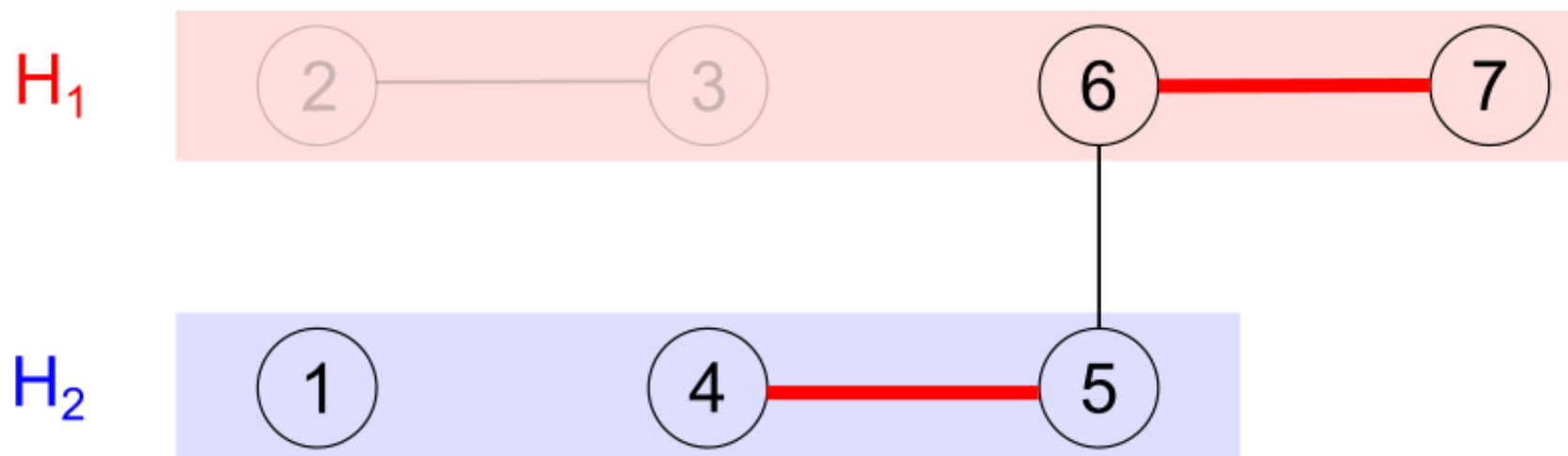


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History [\[edit \]](#)

Year	Event
1986	KPD first suggested by Rapaport ^[14]
1991	First KPD program started in South Korea by Dr. Park ^[8]
1999	First European KPD transplants performed in Switzerland ^[8]
2000	First KPD transplants performed in USA at the Rhode Island Hospital ^[8]
2001	Hopkins completes KPD transplants and begins first KPD program in USA ^[15]
2004	Dutch established first national KPD program ^[8]
2007	NEAD chain started by APD utilizing first bridge donor ^[16]
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA ^[17]
2008	National Kidney Registry organizes its first transplants on Valentine's Day ^[18]
2009	Hopkins leads first 16 patient multicenter Domino Chain ^{[19][20]}
2010	UNOS organizes its first KPD transplants ^[8]
2012	National Kidney Registry completes largest chain involving 60 participants ^[6]
2014	National Kidney Registry sets new record with 70 participant chain ^[7]
2017	First Academic Paper published on "Chronological Incompatibility" outlining the ADP Voucher Program" ^[21]

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1986	KPD first suggested by Rapaport ^[14]
1991	First KPD program started in South Korea by Dr. Park ^[8]
1999	First European KPD transplants performed in Switzerland ^[8]
2000	First KPD transplants performed in USA at the Rhode Island Hospital ^[8]
2001	Hopkins completes KPD transplants and begins first KPD program in USA ^[15]
2004	Dutch established first national KPD program ^[8]
2007	NEAD chain started by APD utilizing first bridge donor ^[16]
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA ^[17]
2008	National Kidney Registry organizes its first transplants on Valentine's Day ^[18]
2009	Hopkins leads first 16 patient multicenter Domino Chain ^{[19][20]}
2010	UNOS organizes its first KPD transplants ^[8]
2012	National Kidney Registry completes largest chain involving 60 participants ^[6]
2014	National Kidney Registry sets new record with 70 participant chain ^[7]
2017	First Academic Paper published on "Chronological Incompatibility" outlining the ADP Voucher Program" ^[21]

Altruistic Donors and Chains

Altruistic Donors and Chains

Donors who want to donate an organ without a designated recipient
(also known as a *non-directed* or *Good Samaritan* donor)

Altruistic Donors and Chains

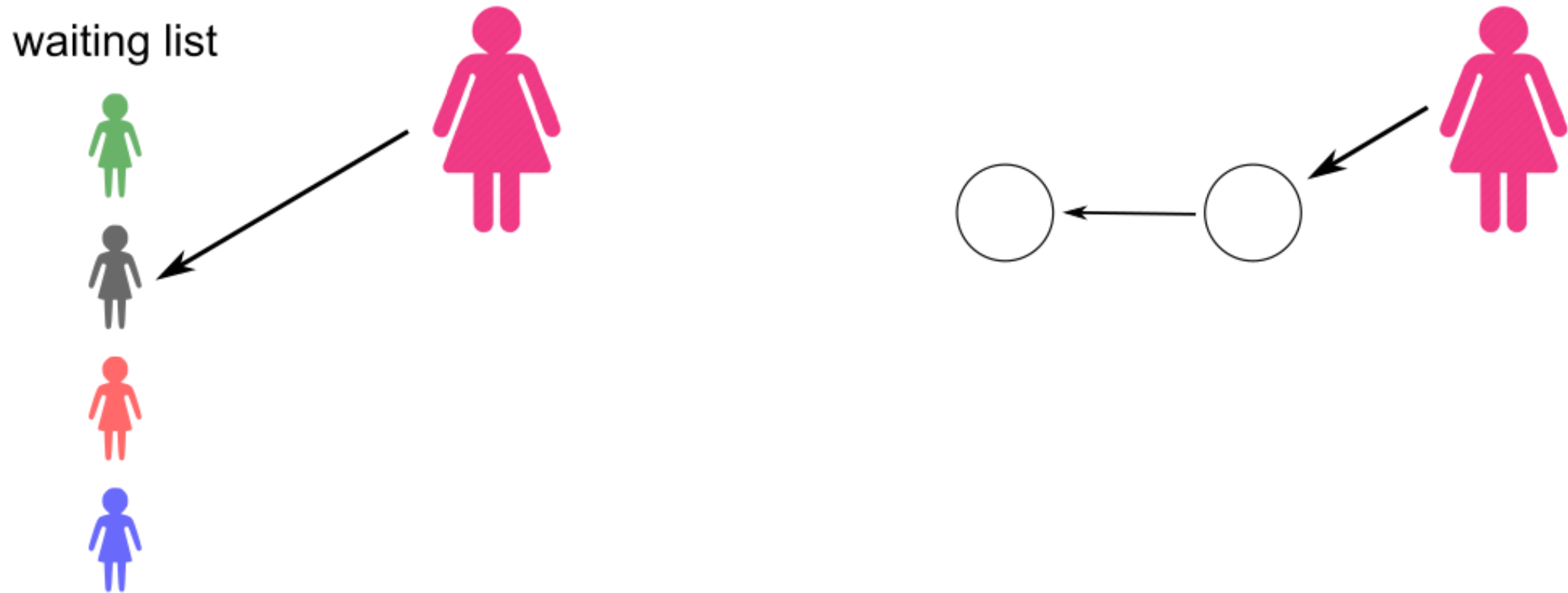
Donors who want to donate an organ without a designated recipient
(also known as a *non-directed* or *Good Samaritan* donor)

waiting list



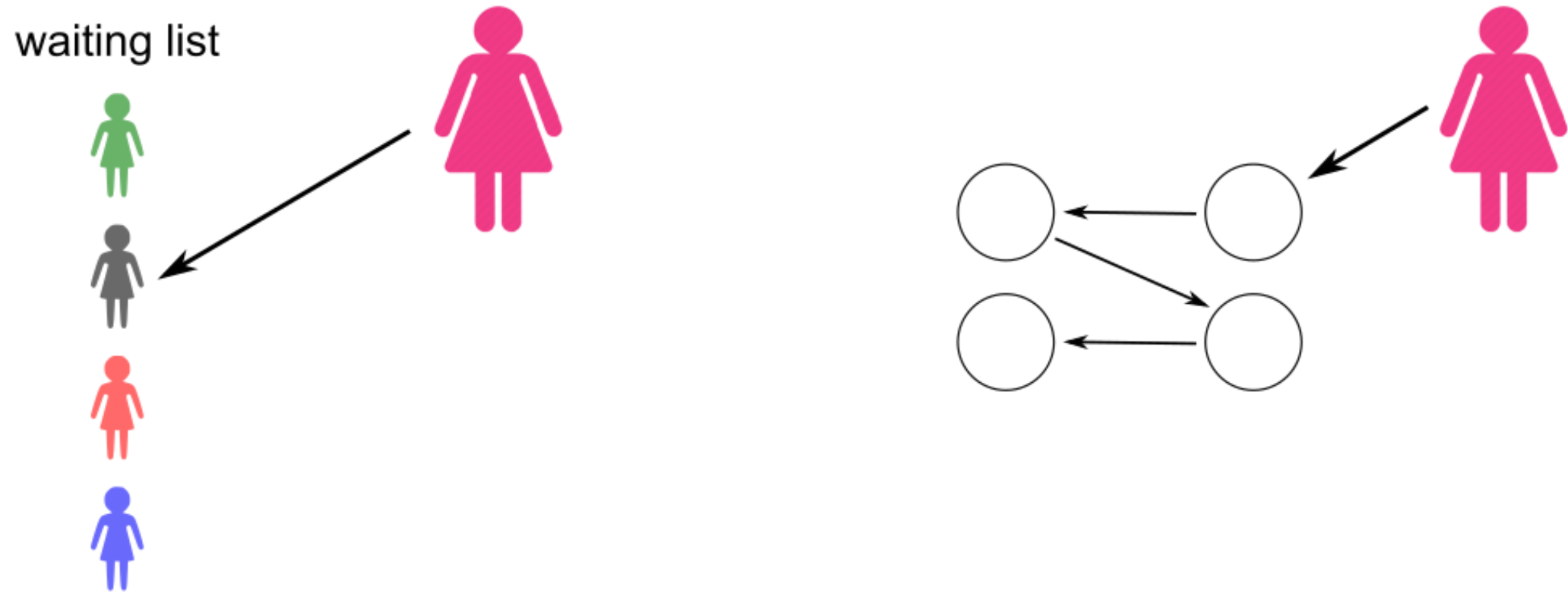
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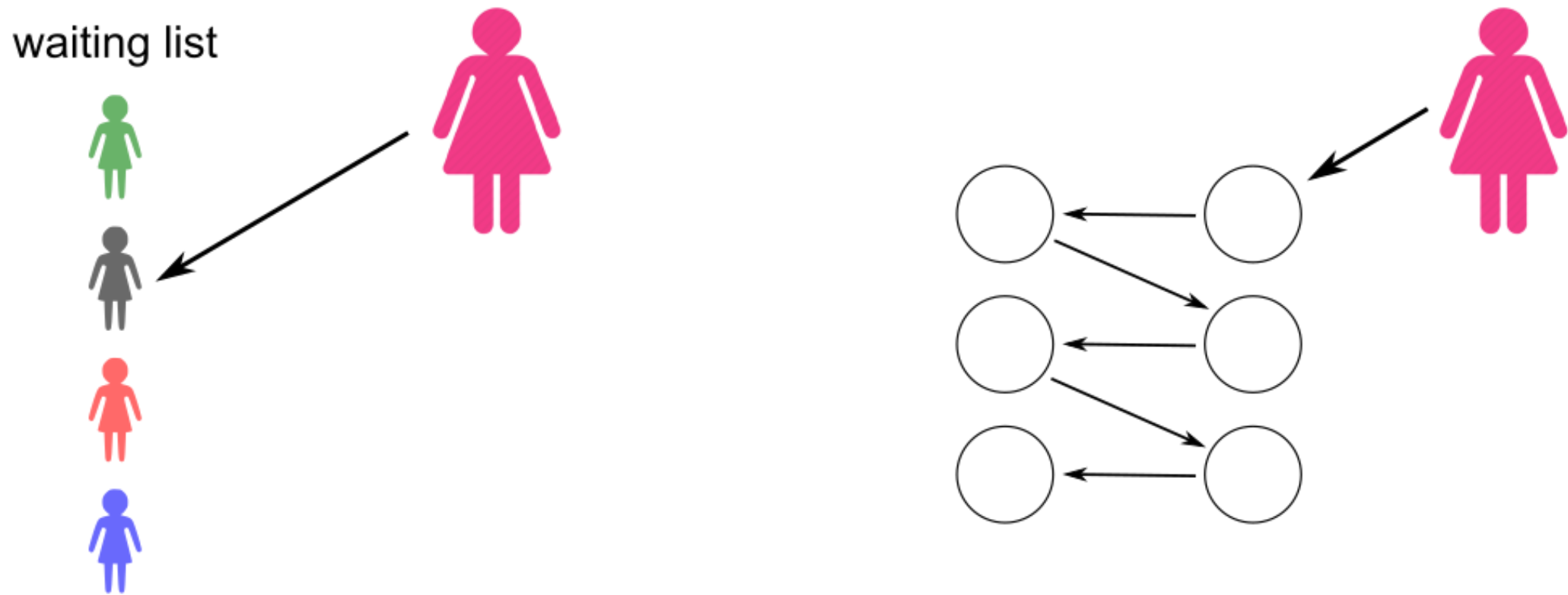
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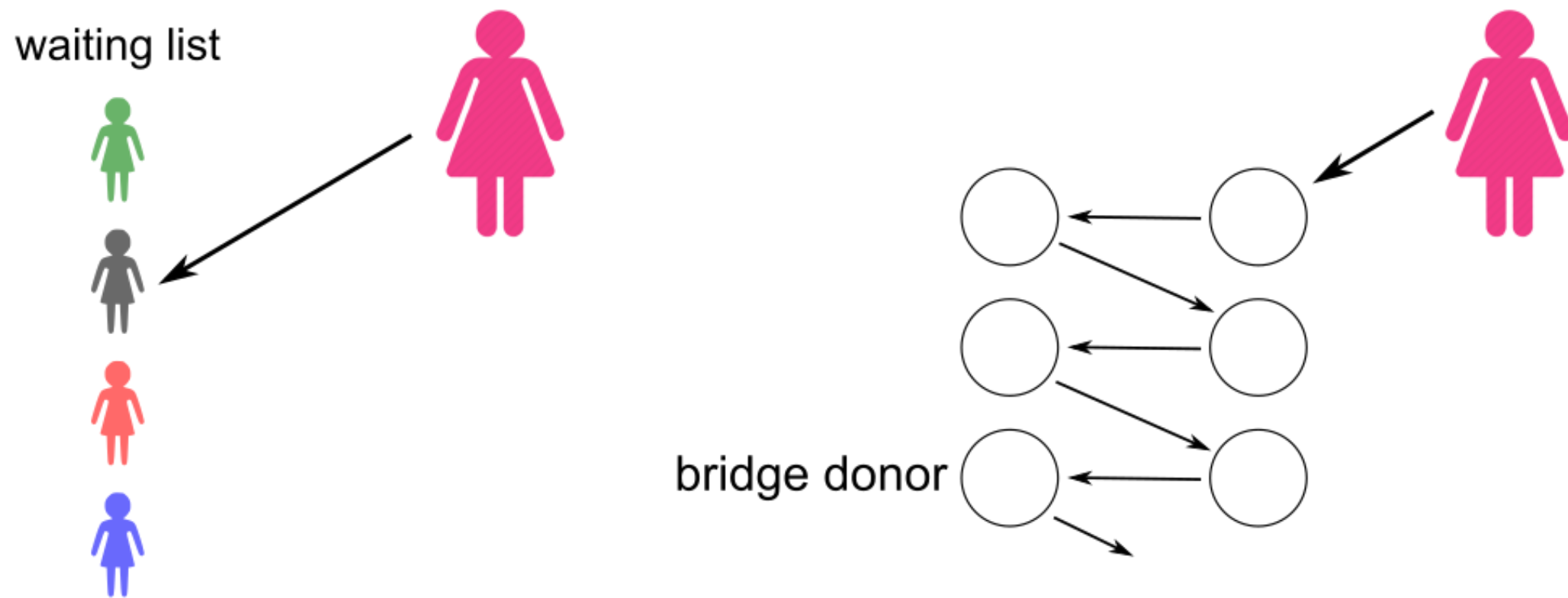
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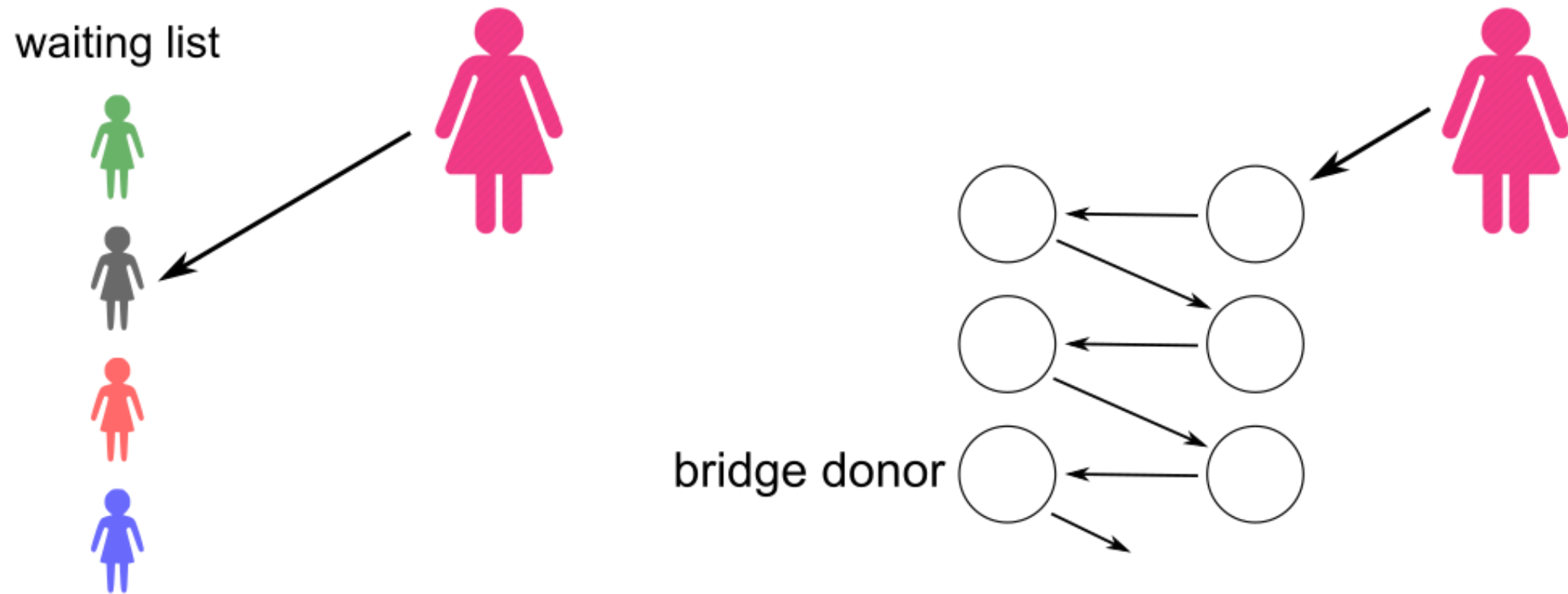
Altruistic Donors and Chains

Donors who want to donate an organ without a designated recipient
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Altruistic Donors and Chains

Donors who want to donate an organ without a designated recipient
(also known as a *non-directed* or *Good Samaritan* donor)



Transplants in an altruistic donor chain can be done **non-simultaneously**.

THE KIDNEY CHAIN

How a single organ donation changed 20 lives and created the longest-running transplant chain



MATT JONES, 30
Petoskey, Mich.
First donor

donated to



BARBARA BUNNELL, 56
Phoenix

married to



RON BUNNELL, 56
Phoenix

donated to



ANGELA HECKMAN, 34
Toledo, Ohio

daughter of



LAURIE SARVO, 54
Toledo, Ohio

donated to



REYNALDO ESPINOZA, 59
Germantown, Md.

donated to



CLAUDIA ALAS, 32
Germantown, Md.

daughter of



JEAN STAYLOR, 53
Charleston, S.C.

donated to



RAYMOND STAYLOR, 53
Charleston, S.C.

married to



AVA ROBY, 54
Marysville, Ohio

donated to



GEORGE LEONER, 51
Chillicothe, Ohio

brother of



LINDA JANISIESKI, 42
Miamisburg, Ohio

donated to



CECILIA JANISIESKI, 71
Huber Heights, Ohio

mother of



ANONYMOUS RECIPIENT

married to



ANONYMOUS DONOR

donated to



BILL CORAM, 55
Lincolnton, N.C.

friend of



TIM SHAIN, 43
Lincolnton, N.C.

donated to



LINLEY BLENKENSOPP, 51
Patchogue, N.Y.

brother of



KURT BLENKENSOPP, 41
Patchogue, N.Y.

donated to



KATHERINE MCKINNEY, 62
Toledo, Ohio

mother of



HELEENA MCKINNEY, 29
Cincinnati
Donor-in-waiting

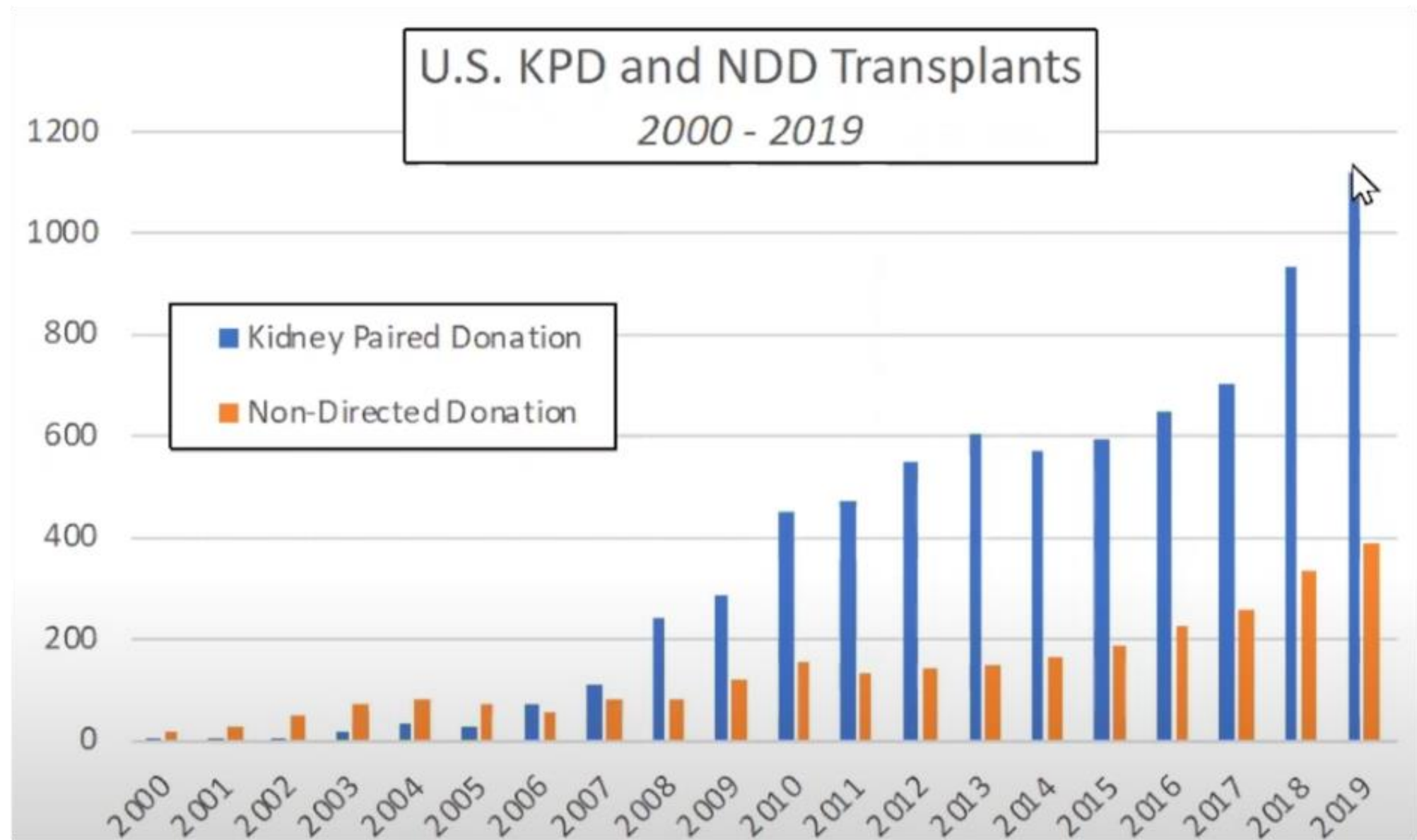


Figure source: Talk by Alvin Roth on "*Kidney Exchange: An Operations Perspective*" hosted by INFORMS Health Applications Society; June 2021

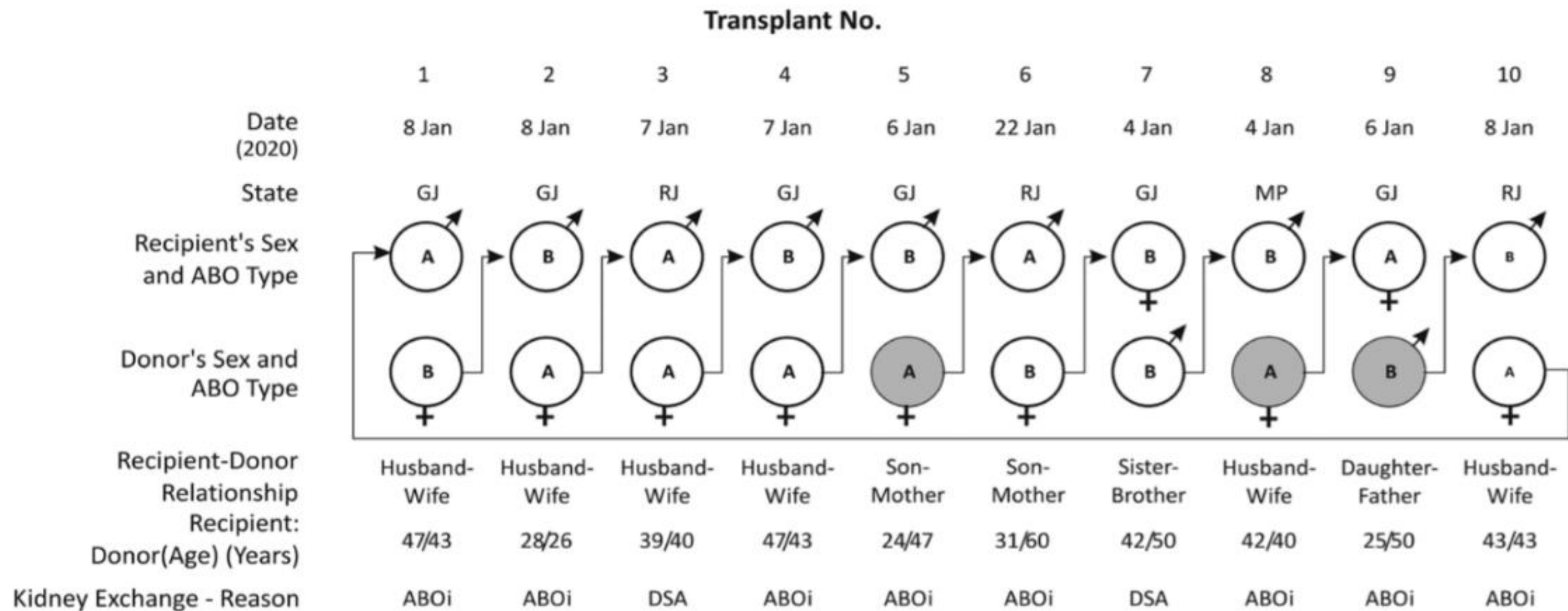
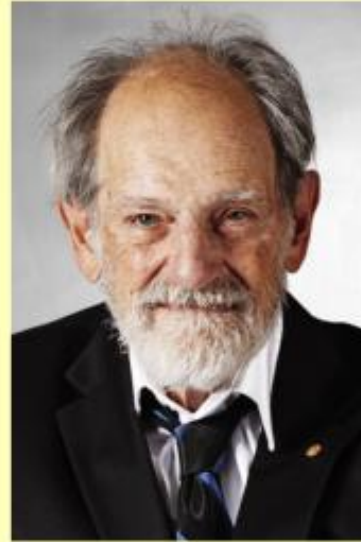


FIGURE 1. A nonsimultaneous 10-way kidney exchange cycle. GJ, Gujarat; MP, Madhya Pradesh; RJ, Rajasthan.

Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012



Alvin E. Roth



Lloyd S. Shapley

*"for the theory of stable allocations
and the practice of market design."*

Challenges in Kidney Exchange

Economic: Incentives of hospitals vs maximizing overall welfare.

Computational: Finding a maximum matching using 3-cycles is NP-hard.

Informational: Patients and donors arrive sequentially.

Logistical: Long cycles require many simultaneous surgeries.

Legal: Many countries do not permit altruistic donation.

Ethical: Is it OK to buy and sell kidneys?

Reminders

Assignment 1: Due Aug 21

Next Time

Cake cutting



Quiz

Quiz

Prove that TTCA outcome is Pareto optimal.

(An outcome A is **Pareto optimal** if there is *no other* outcome B such that everyone is at least as happy under B as they are under A and someone is strictly happier.)

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