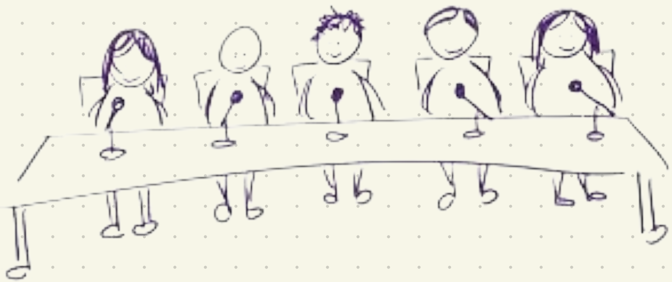
COL 866 : SPECIAL TOPICS IN ALGORITHMS

LECTURE 22

MULTIWINNER VOTING (CONTD.) & WRAP UP

NOV 10, 2023 | ROHIT VAISH

ELECTING A COMMITTEE



VOTING WITH APPROVAL PREFERENCES

* Set of Candidates C
* Set of n voters $\{1, 2, \ldots, n\}$
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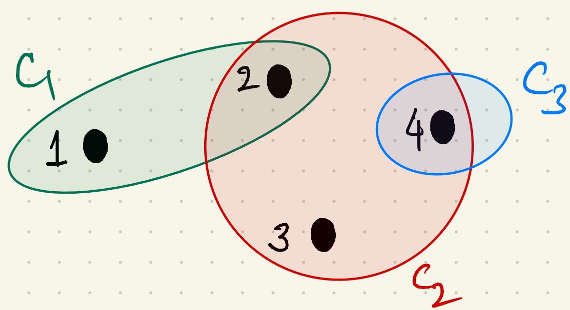

1 : c_1

2 : c_1, c_2

3 : c_2

4 : c_2, c_3

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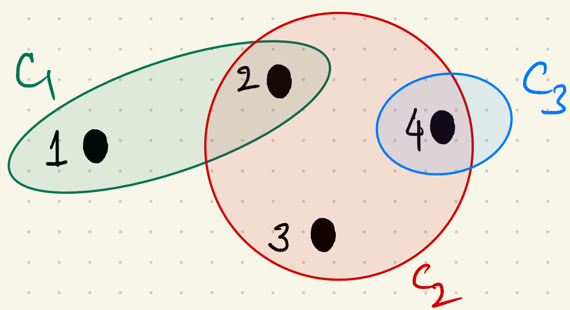

$1 : c_1$

$2 : c_1, c_2$

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$4 : c_2, c_3$

* Goal : Select k candidates (k is given)

VOTING RULES

Thiele methods : Given weights $w_1 \geq w_2 \geq w_3 \geq \cdots \geq 0$

find a size k committee W that maximizes

$$\text{Score}(W) = \sum_{\text{voters } i} w_1 + w_2 + \cdots + w_{|W \cap A_i|}$$

Approval Voting (AV) : $1, 1, 1, \ldots$

Chamberlin Courant (CC) : $1, 0, 0, \ldots$

Proportional Approval Voting (PAV) : $1, \frac{1}{2}, \frac{1}{3}, \ldots$

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utility function $u : \mathbb{N} \to \mathbb{R}$

$u(\ell) := w_1 + w_2 + \cdots + w_\ell$

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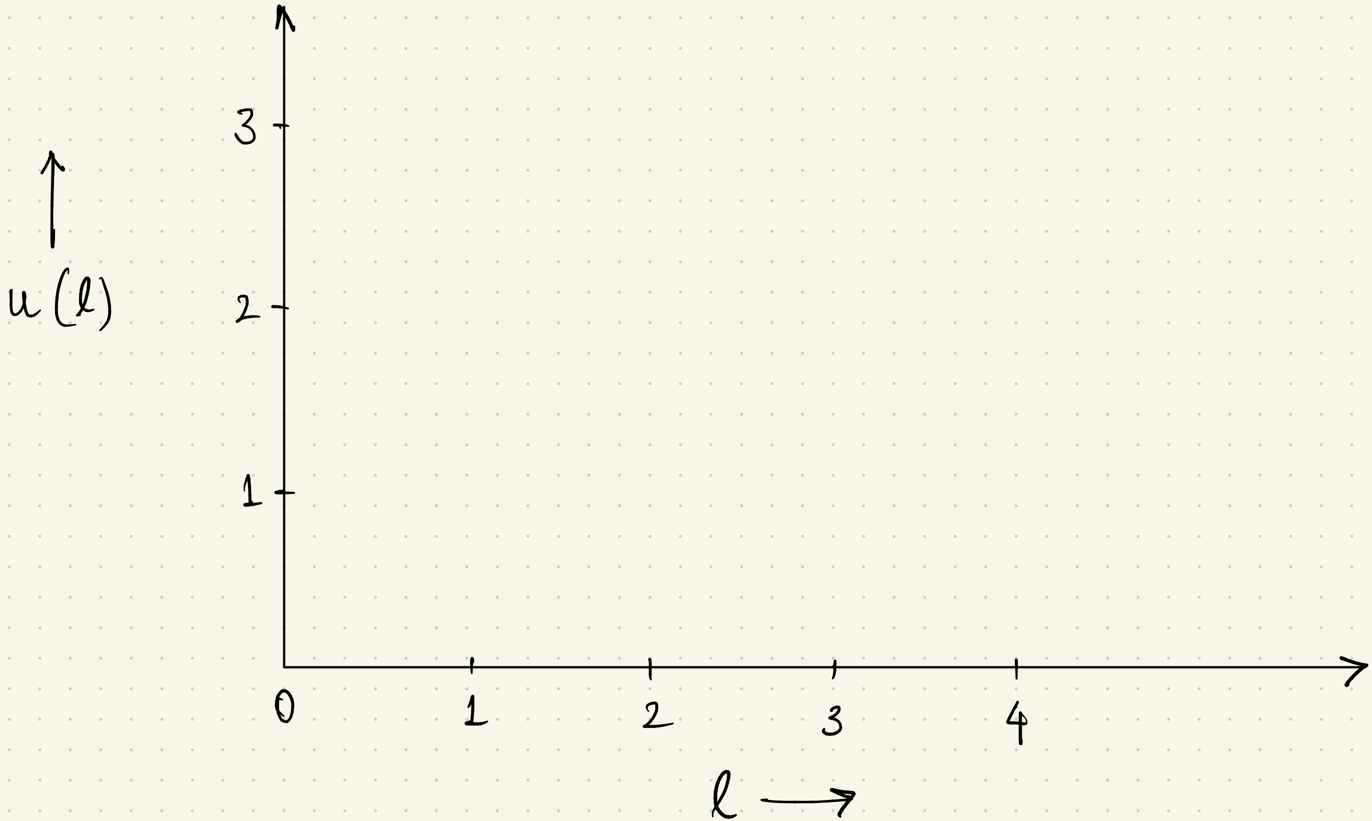

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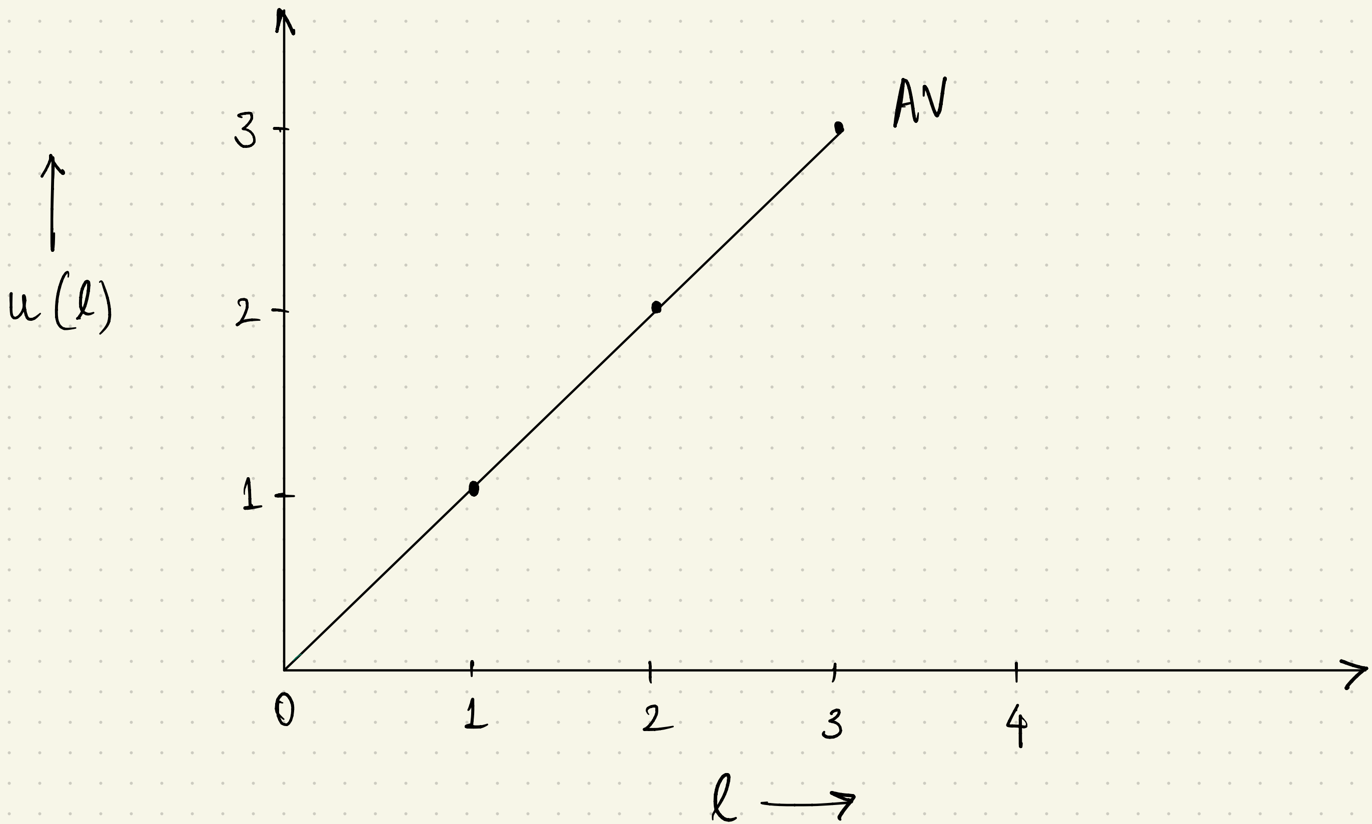

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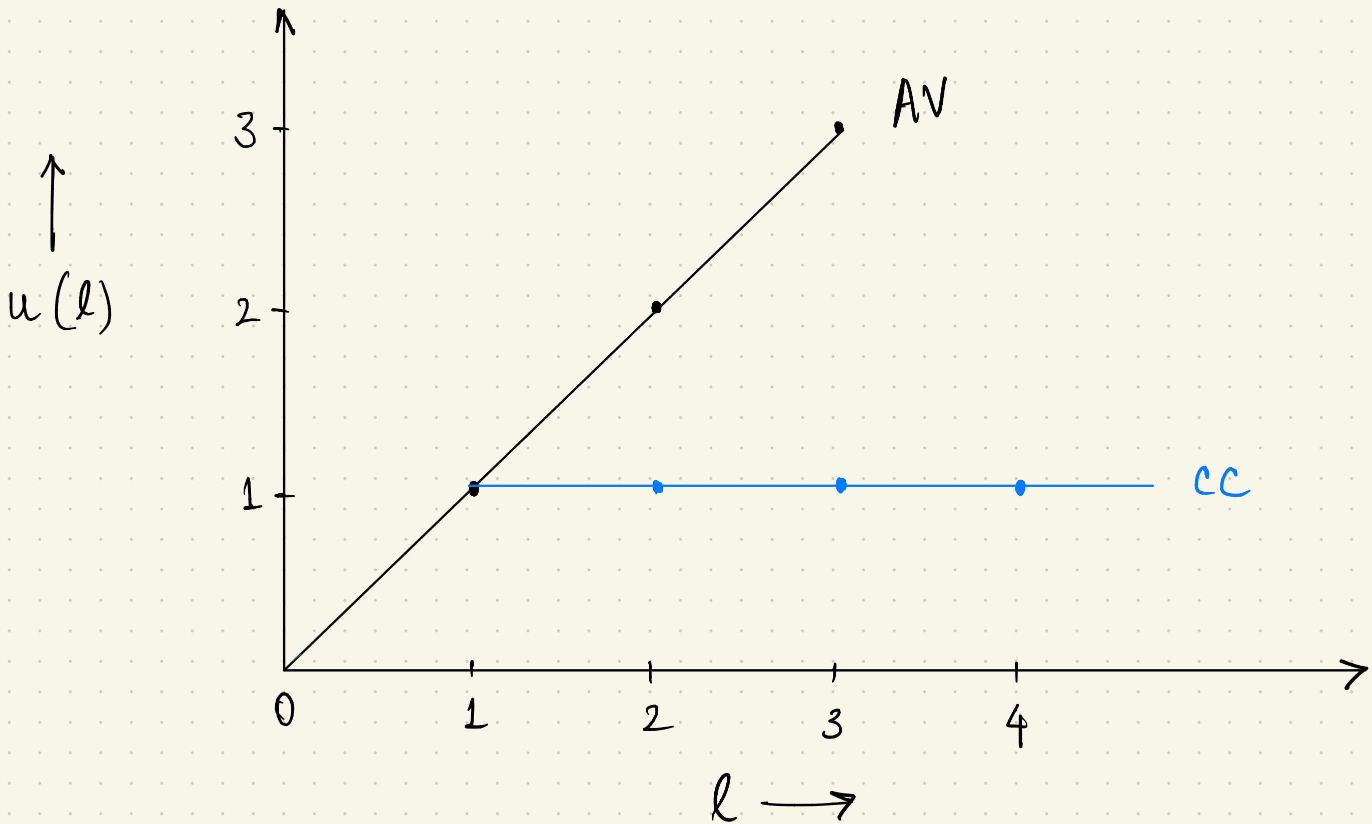

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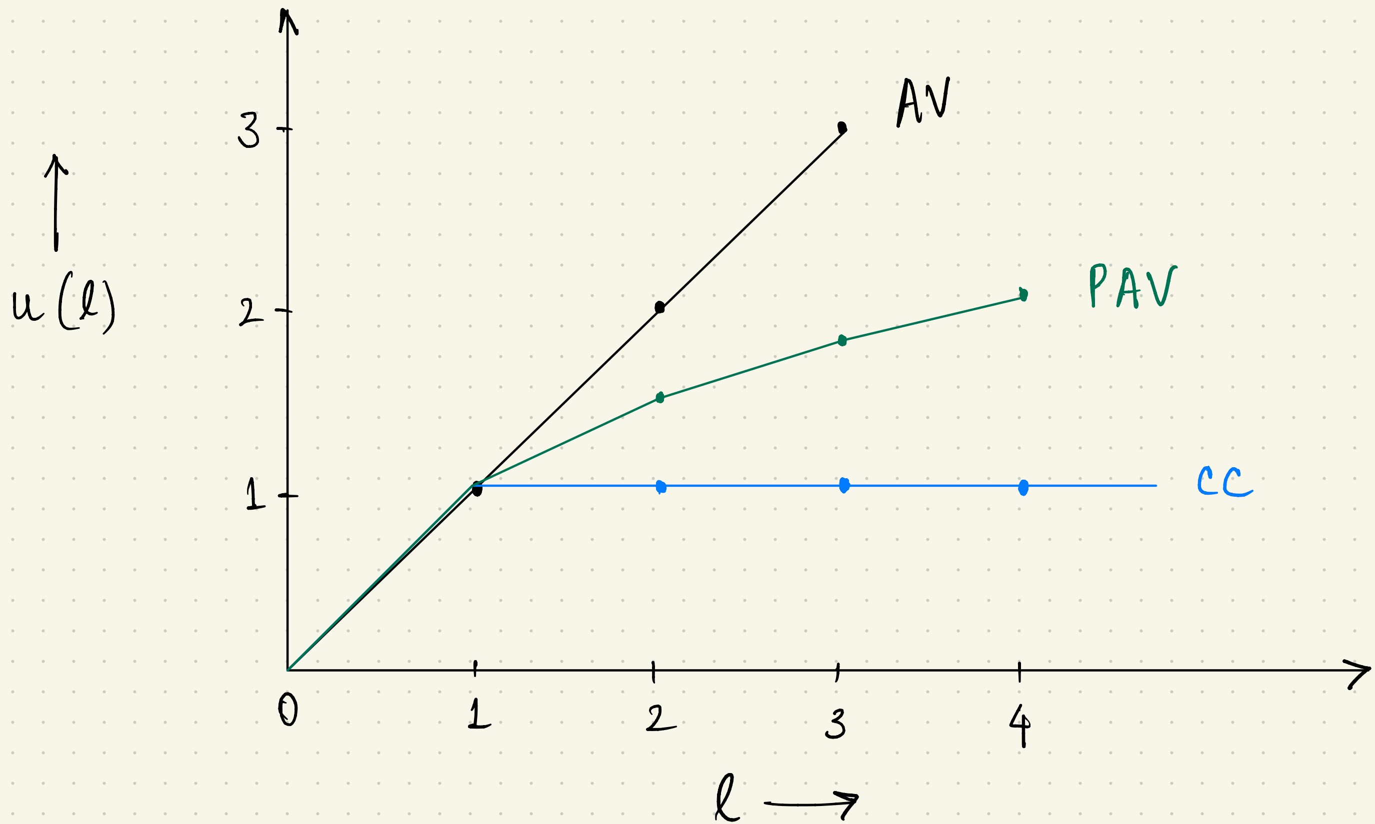
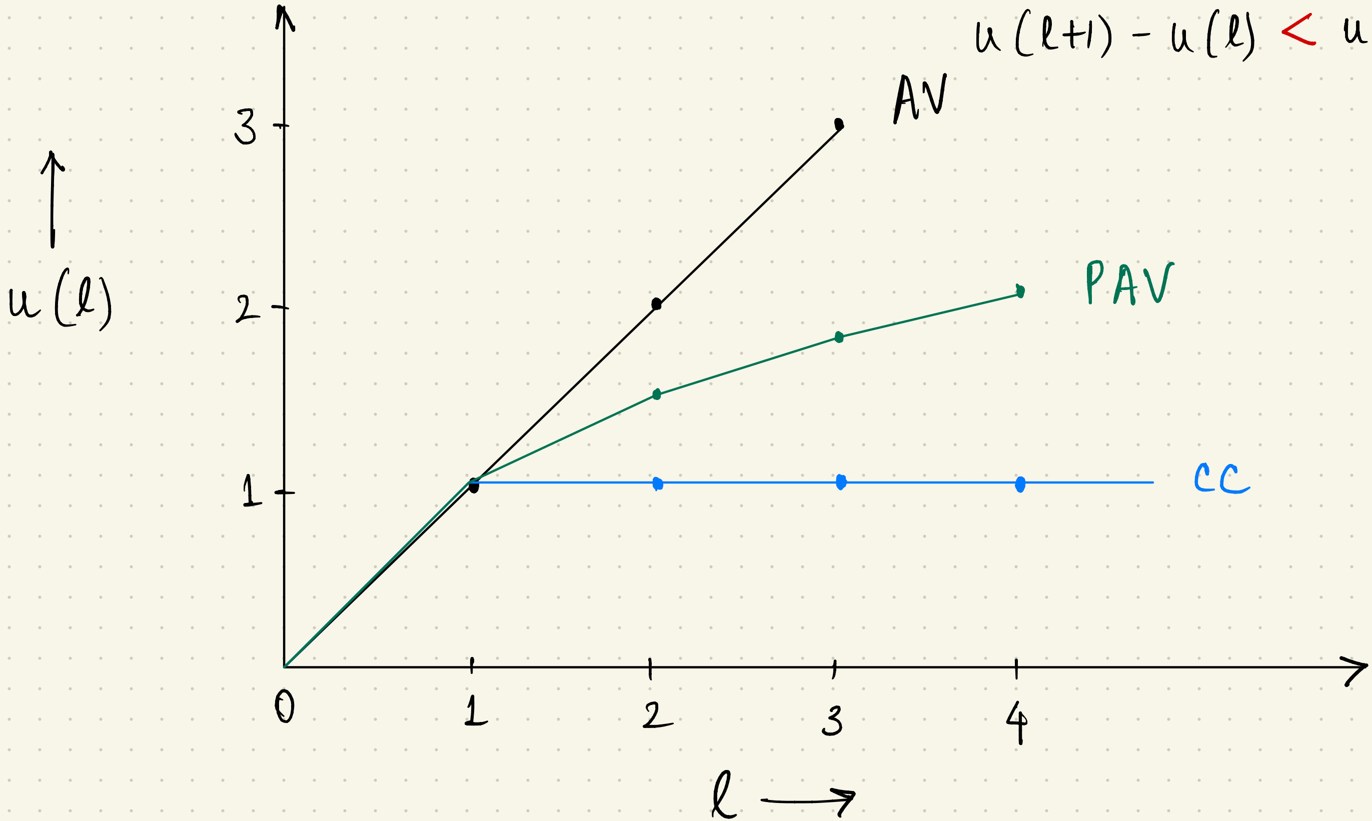


THIELE METHODS

[Skowron, Faliszewski, Lang AIJ 2016]

NP-hard whenever $\exists \ell$ s.t.

$$u(\ell+1) - u(\ell) < u(\ell) - u(\ell-1)$$



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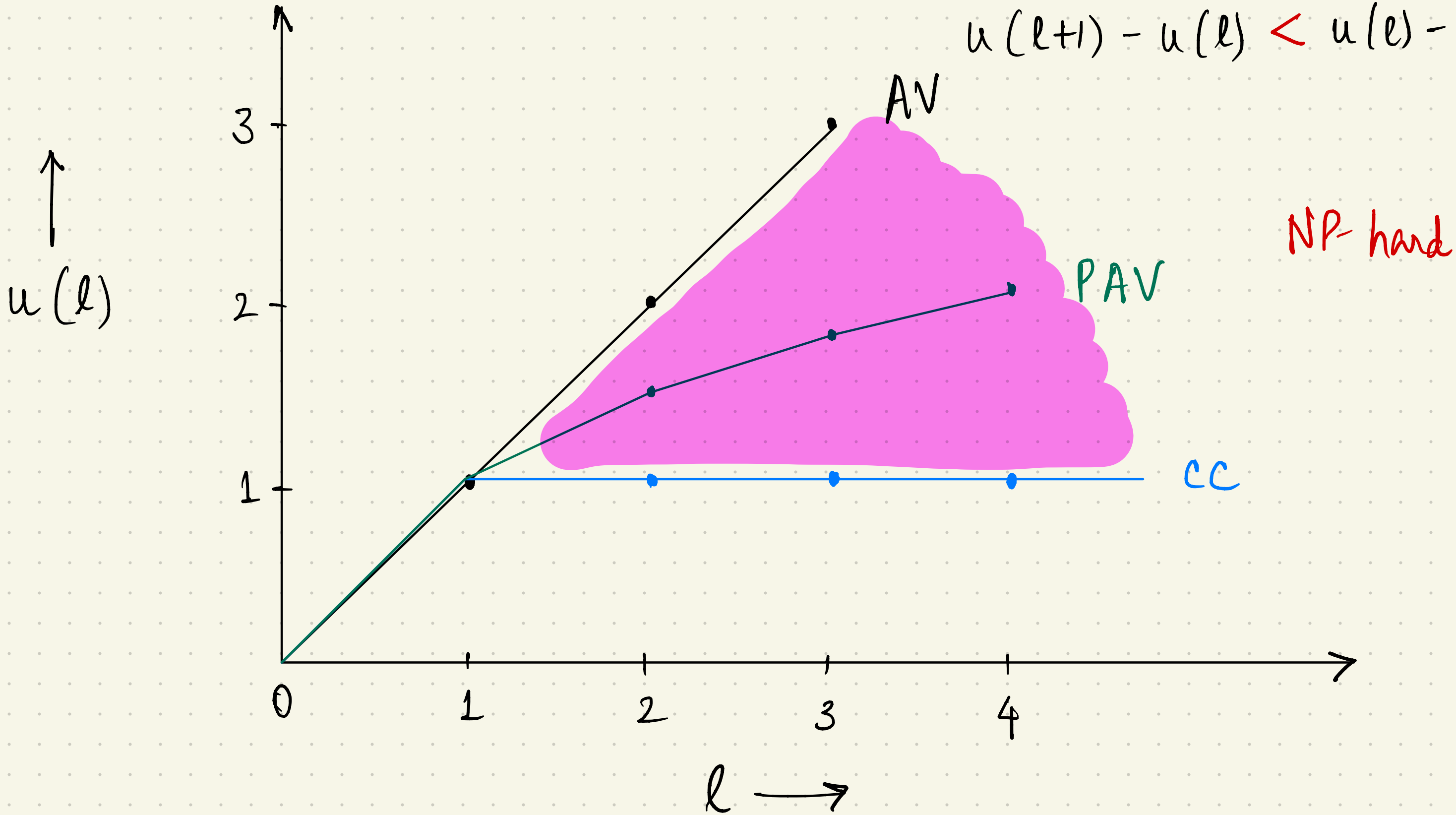
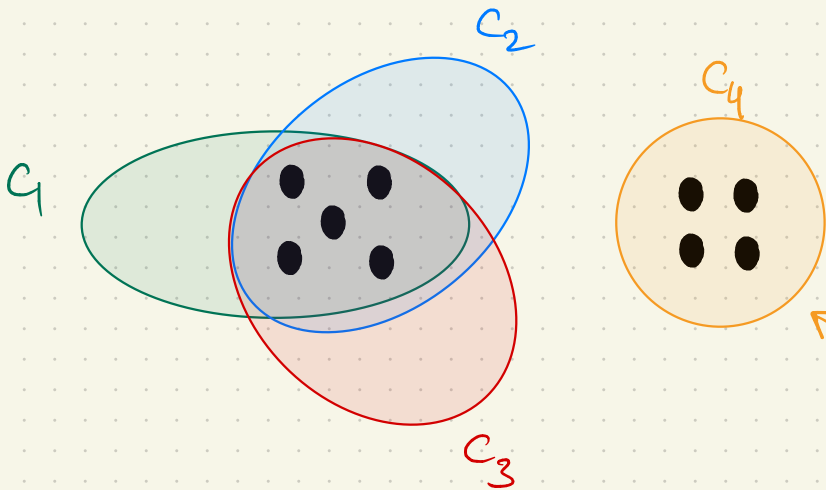
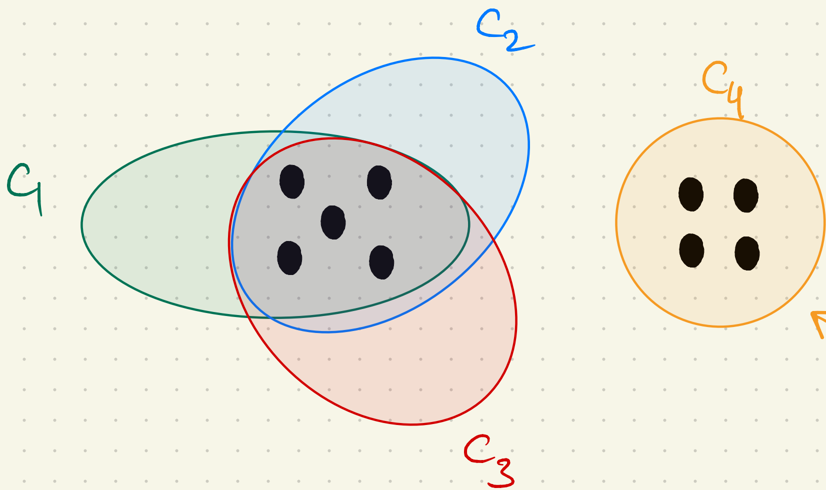


$k=3$: AV picks $\{c_1, c_2, c_3\}$



Not represented

WHEN IS A COMMITTEE "REPRESENTATIVE"?



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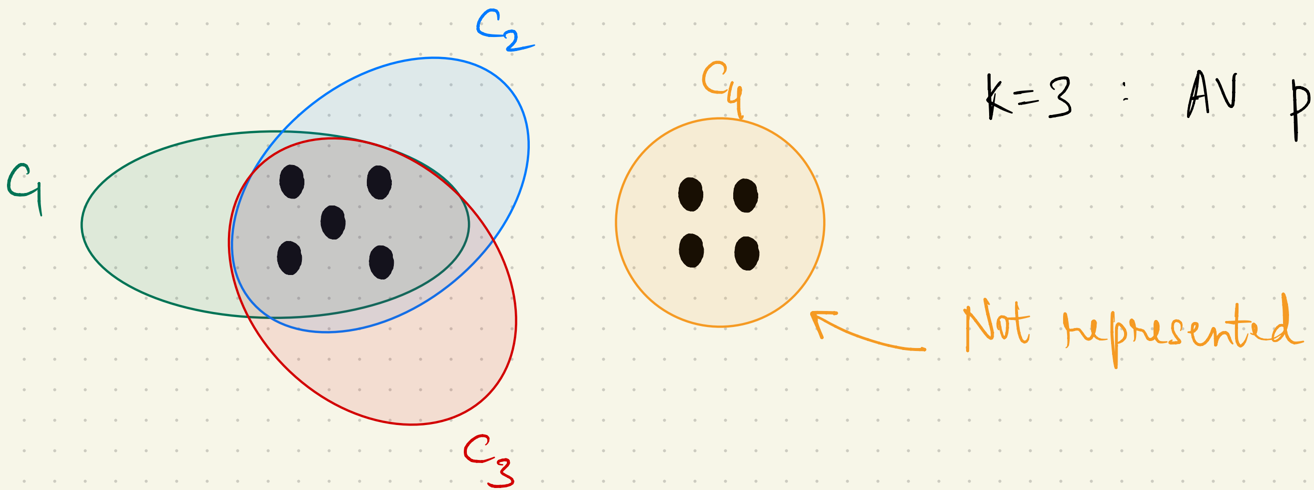

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Any large and cohesive set of voters deserves representation

WHEN IS A COMMITTEE "REPRESENTATIVE"?

	Large	Cohesive	Representation
Strong justified repr^n			
Justified representation			
Extended justified repr^n			

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Extended justified repr^n	$\lvert S\rvert \geq \ell\frac{n}{k}$	$\left\lvert \bigcap_{i\in S} A_i \right\rvert \geq \ell$	$\exists\ i \in S$ s.t. $\lvert W \cap A_i \rvert \geq \ell$

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	Existence	Computation	Verification
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Extended justified repr^n	Always exists (PAV)	Poly time (Local search PAV)	NP-hard

PLAN FOR TODAY

* More voting rules : Sequential PAV , Phragmén's rule
* More representation axioms : Proportional Justified Repr^n , Core
* Other properties : Strategyproofness

SEQUENTIAL PAV

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* $W \leftarrow \emptyset$
* while $|W| < k$

  find candidate c that maximizes PAV score of $W \cup \{c\}$

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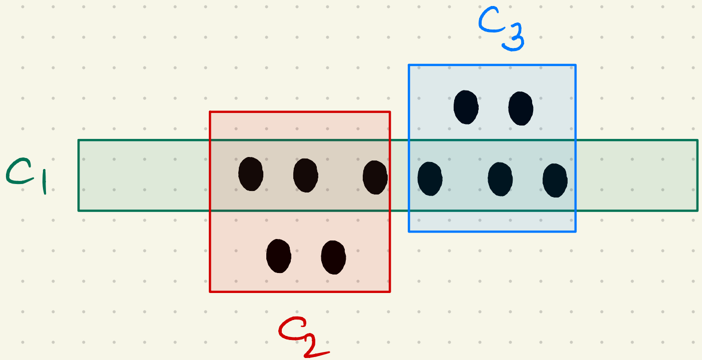

$k = 2$

PAV :

Seq. PAV :

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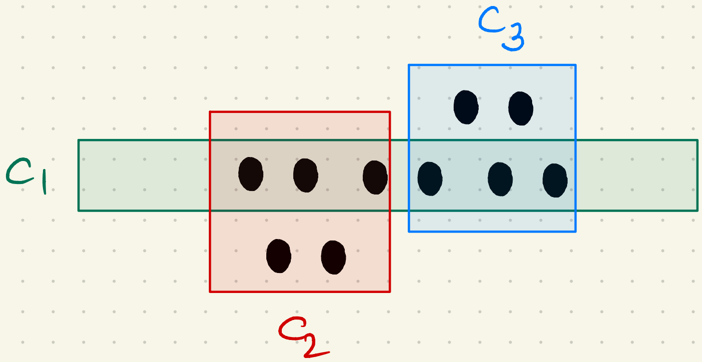

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PAV : $\{c_2, c_3\}$

Seq. PAV : $\{c_1, c_2\}$ or $\{c_1, c_3\}$

SEQUENTIAL PAV

Thm: PAV score of seq. PAV committee $\geq \left(1 - \frac{1}{e}\right)$ opt. PAV score

$\approx 63\%$

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[Sánchez-Fernández, Elkind, Lackner, Fernández, Fisteus, Val, Skowron AAAI 2017]

Seq. PAV satisfies JR for $k \leq 5$ and fails JR for $k \geq 6$.

IS EJR TOO PERMISSIVE?

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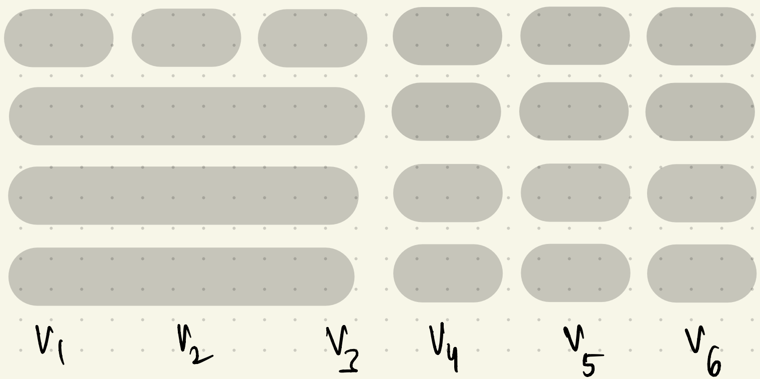
$k = 12$



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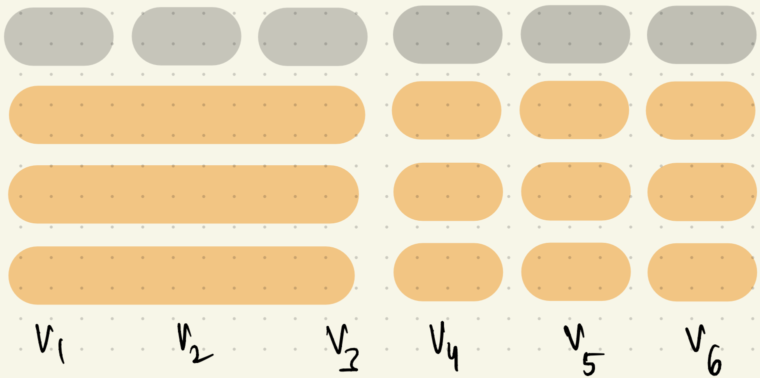

PAV committee

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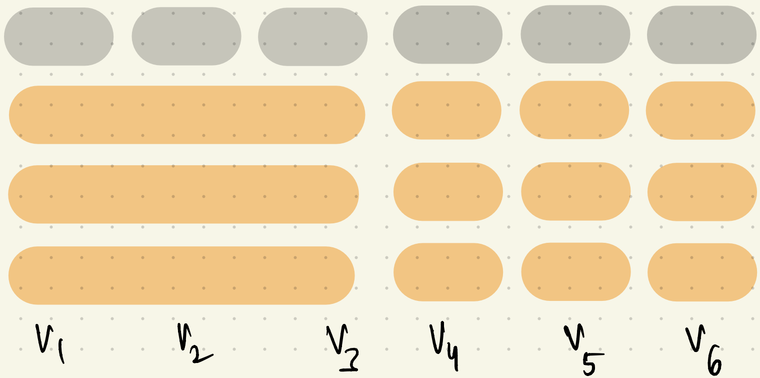

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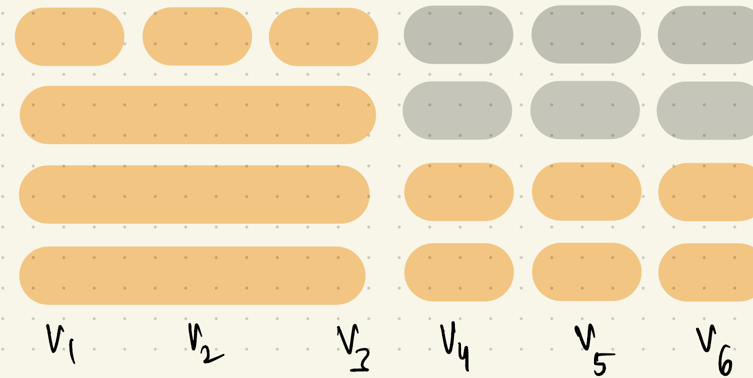

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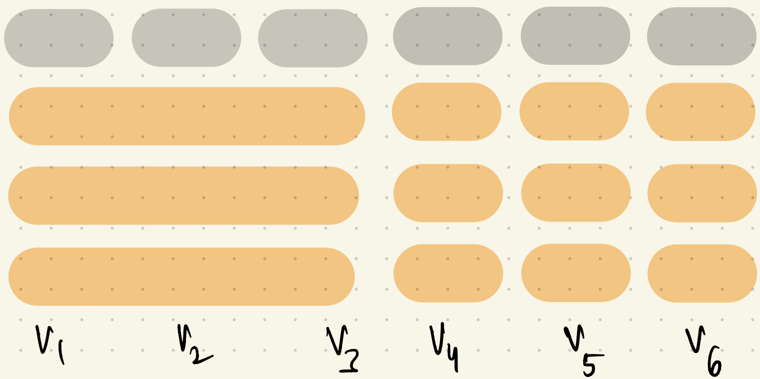

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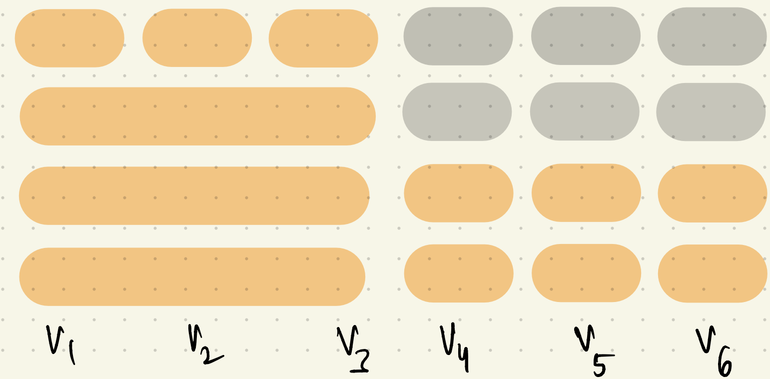

"Fairer" representation

Half of voter population deserves half of the seats.

CORE

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A group of voters S with $|S| \geq \ell \cdot \frac{n}{k}$ blocks committee W if there is a subset $T \subseteq C$ with $|T| = \ell$ such that

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Committee W is in the core if it is not blocked.

CORE

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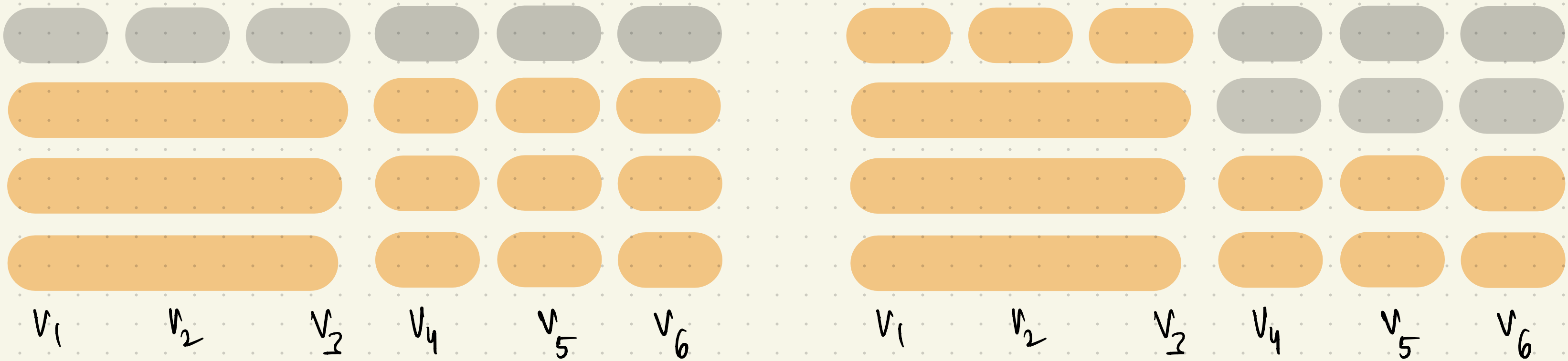

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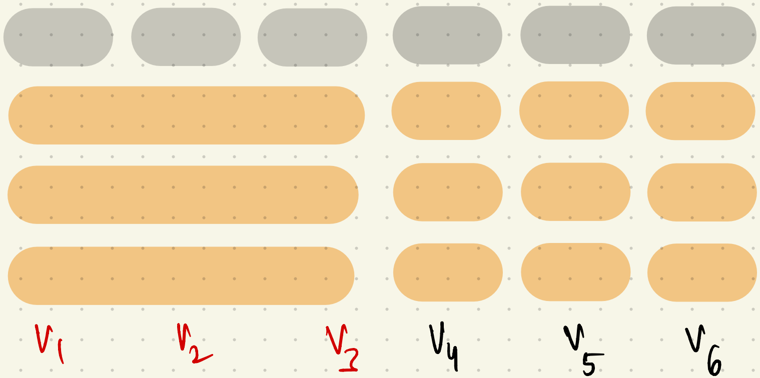
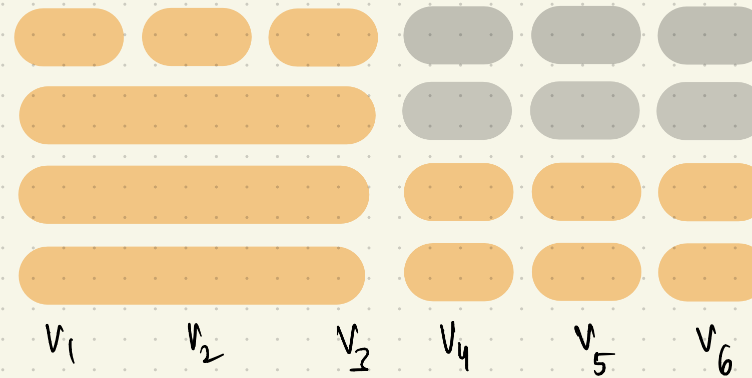


Not Core



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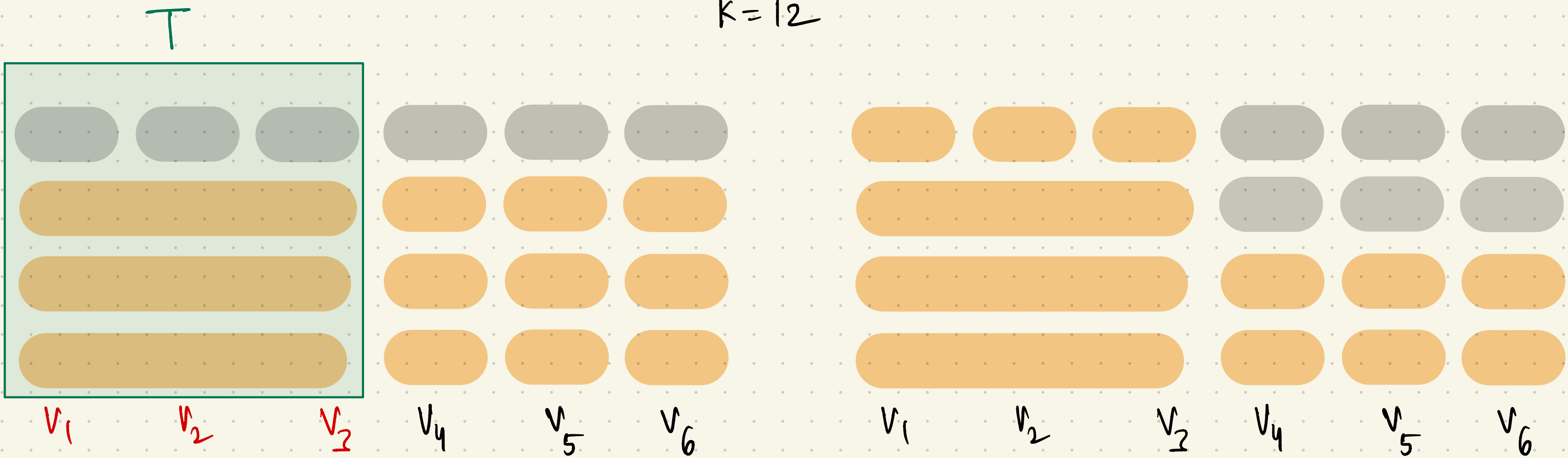

Not Core

Core

$S = \{ v_1, v_2, v_3 \}$

$$|S| \geq \overset{\ell}{6} \cdot \frac{\overset{n}{6}}{\underset{k}{12}}$$

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Observe: Core $\Rightarrow$ EJR.

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Why? Voter violating EJR constitute a blocking coalition with $T \subseteq \bigcap_{i \in S} A_i$

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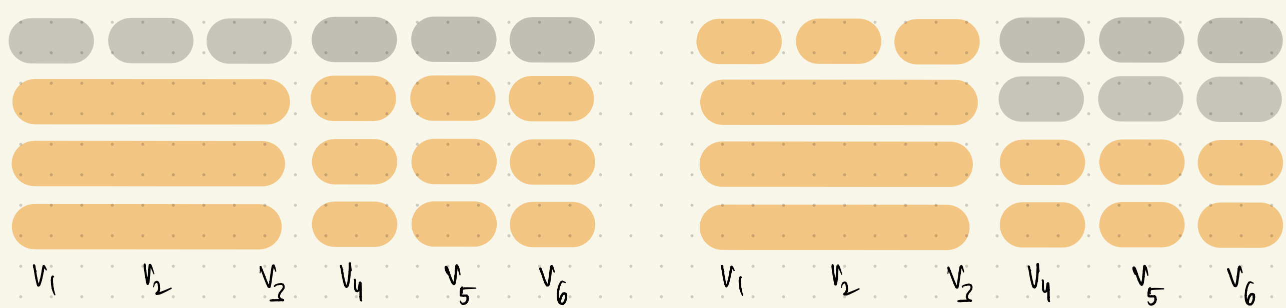
Core v/s egalitarian (Pigou-Dalton)



PICTURE SO FAR

$$\begin{tikzcd} & \text{Core} \arrow[d] \\ & \text{EJR} \arrow[d] \\ \text{SJR} \arrow[r] & \text{JR} \end{tikzcd}$$

Core: Existence open!

EJR: Exists + Poly time, hard to verify

SJR: May not exist

JR: Exists + Poly time, easy to verify

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Idea: Cost sharing / load balancing

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* Stop after k candidates are elected.

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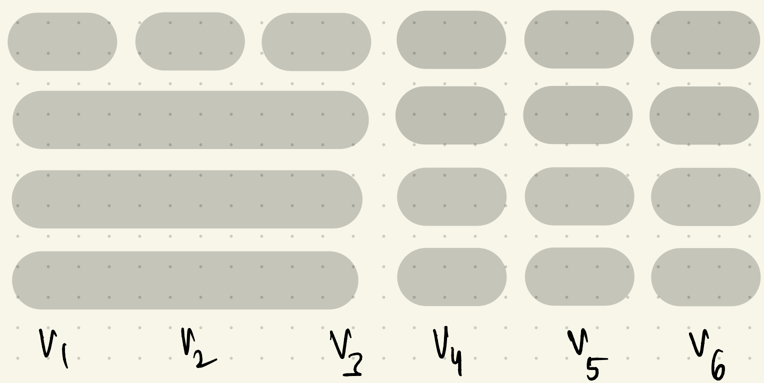

$n = 6$

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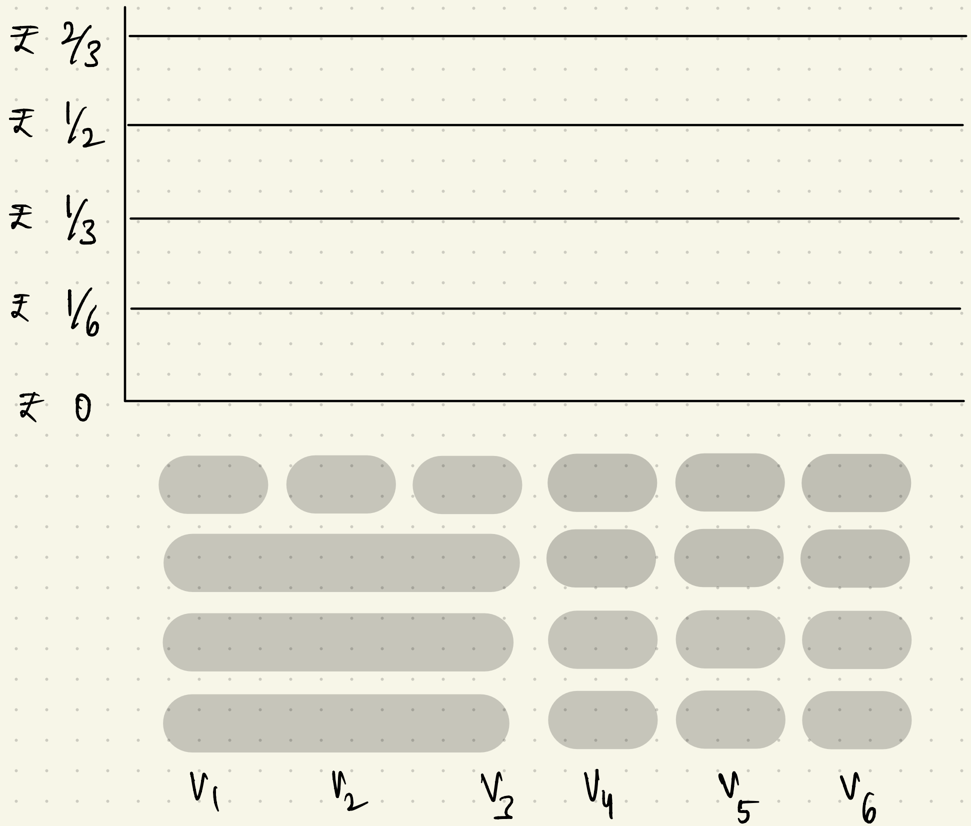

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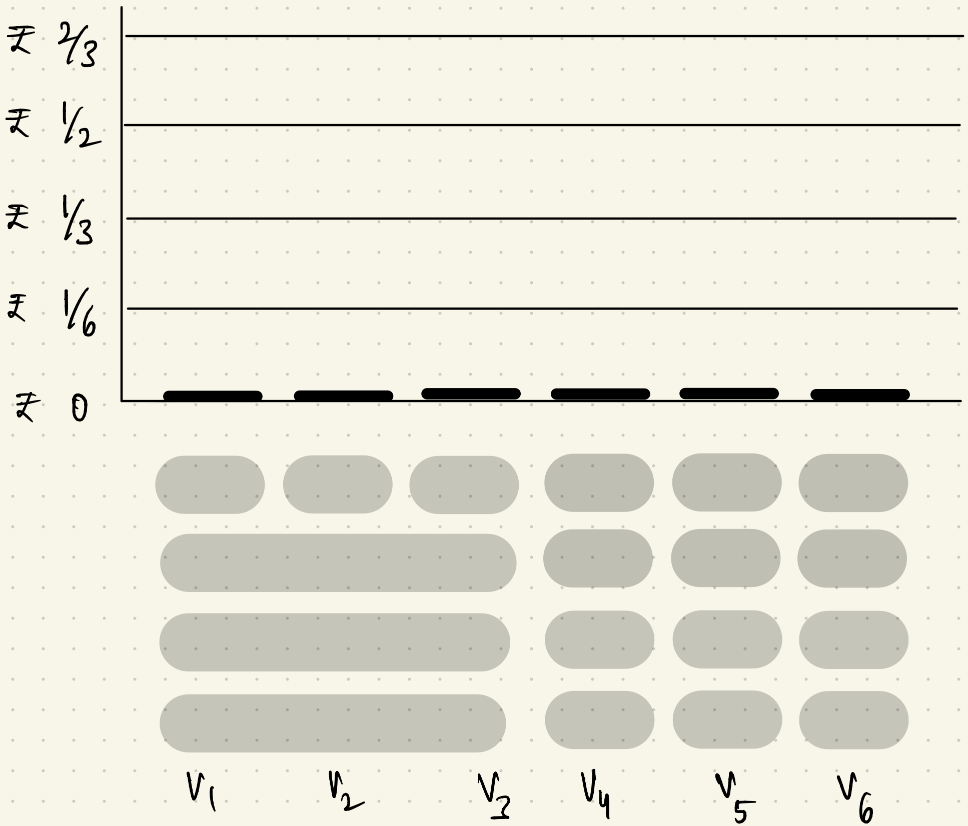

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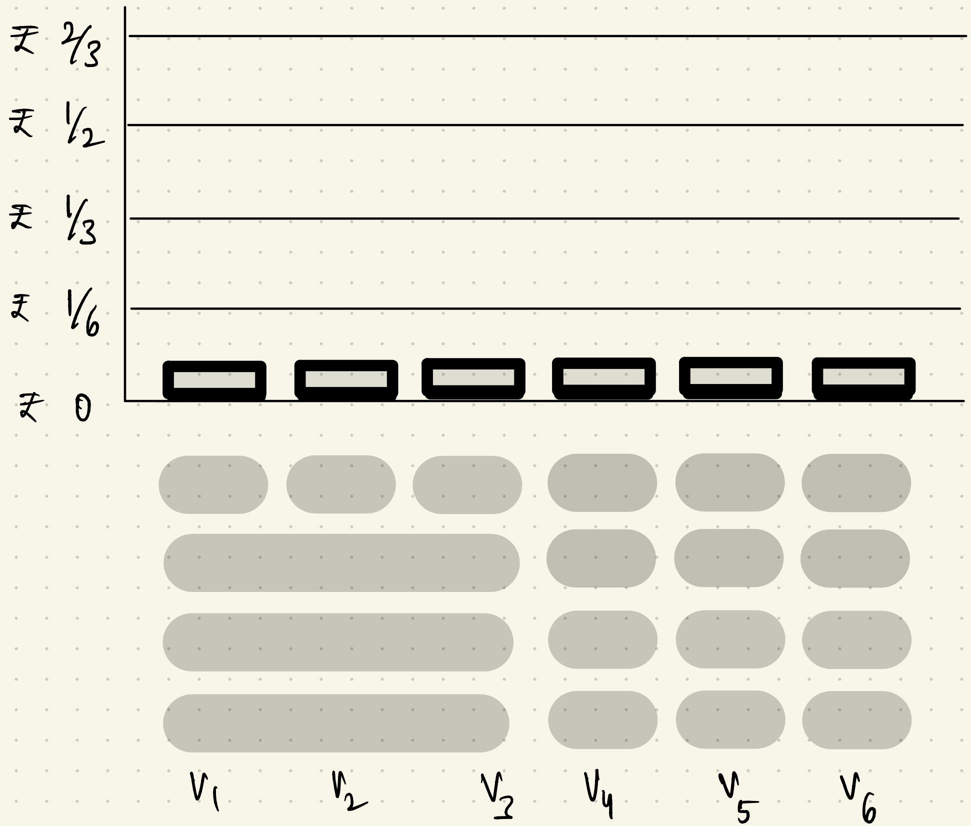

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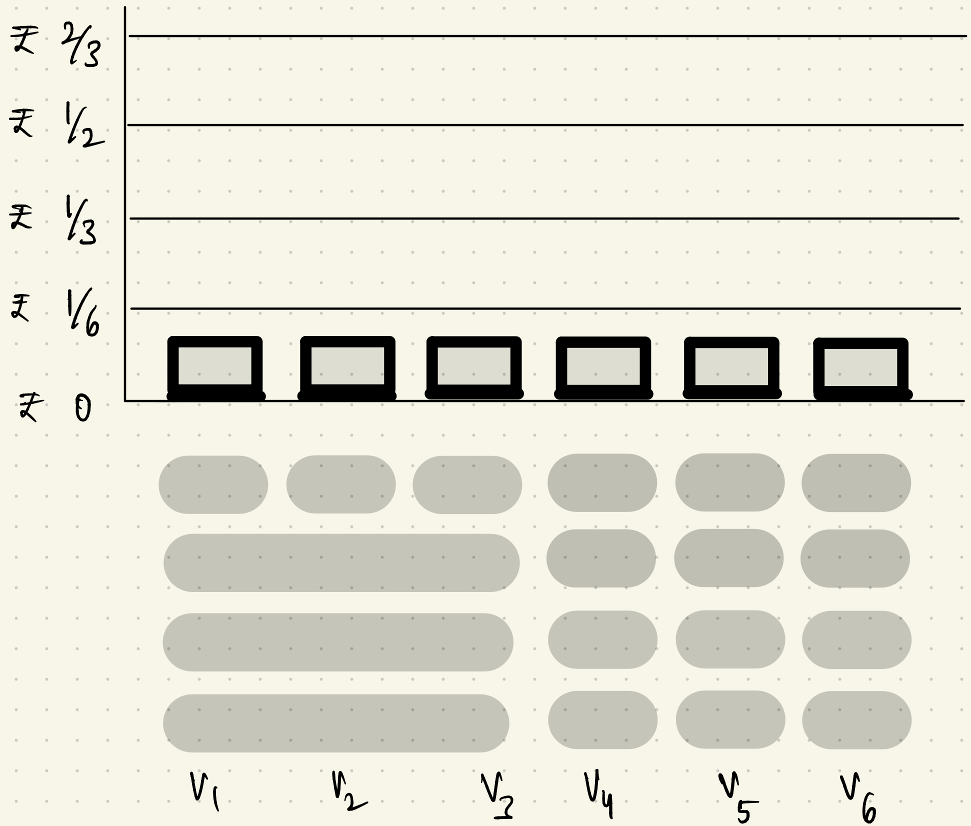

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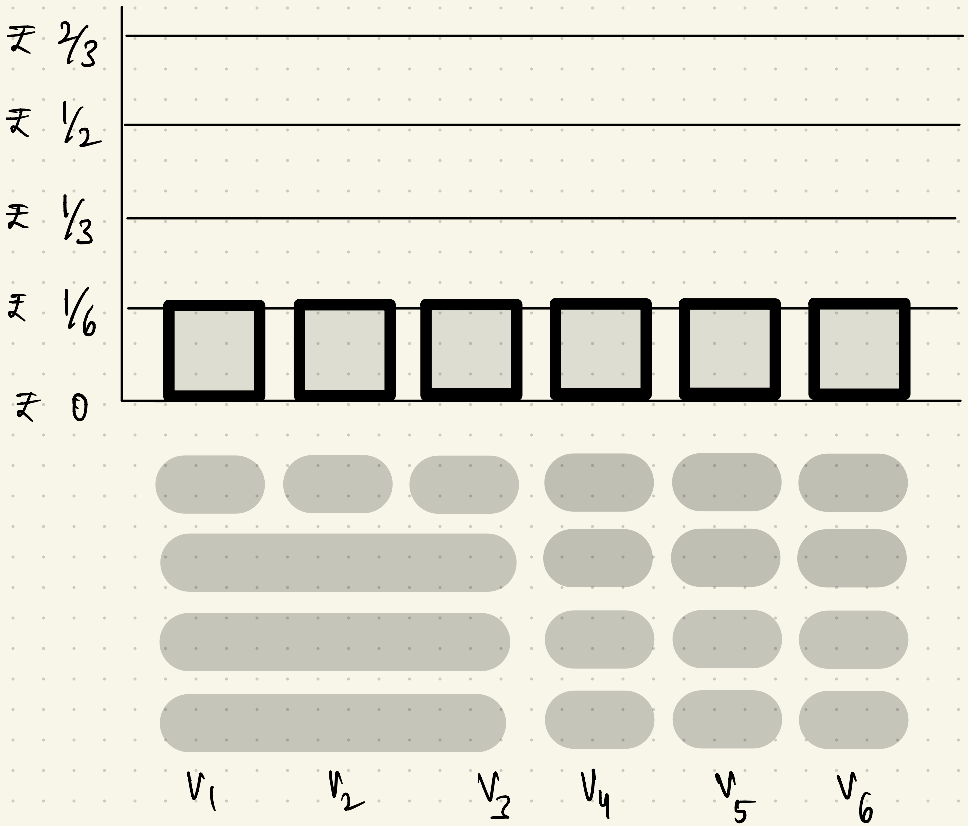

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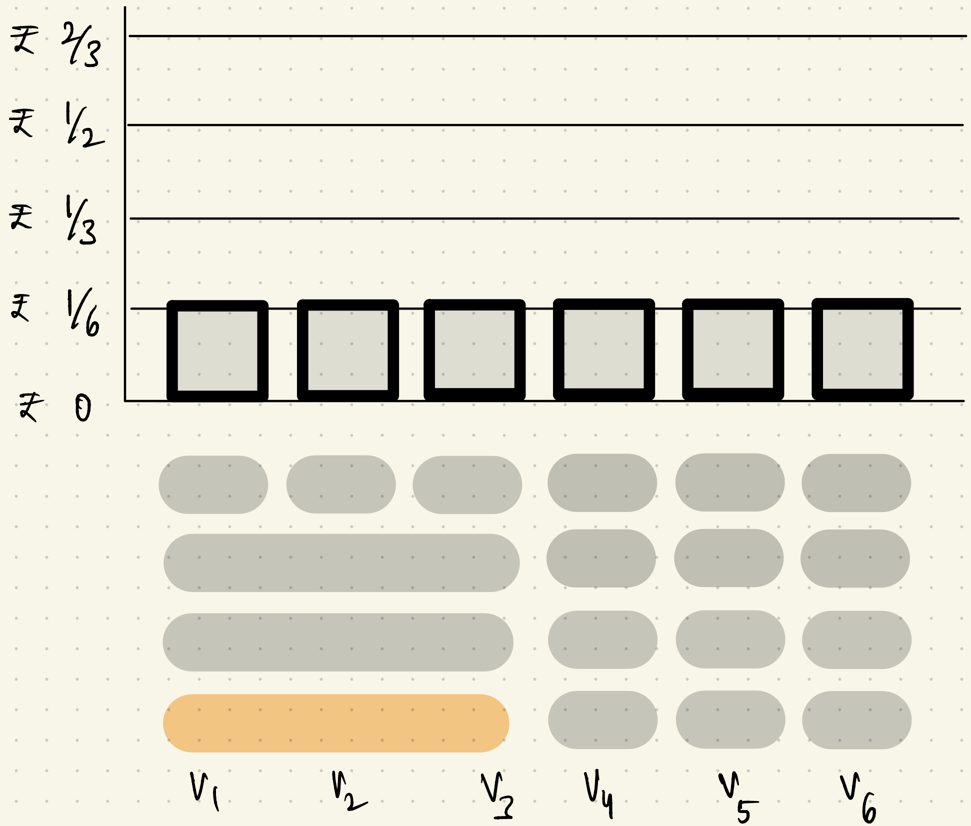

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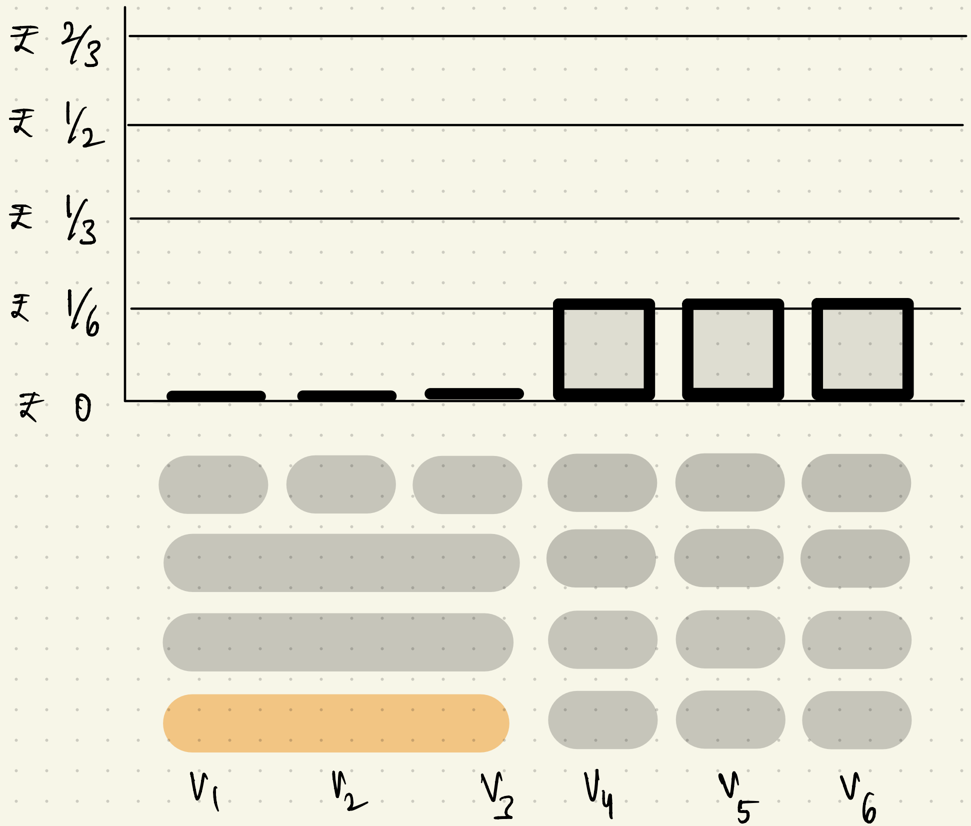

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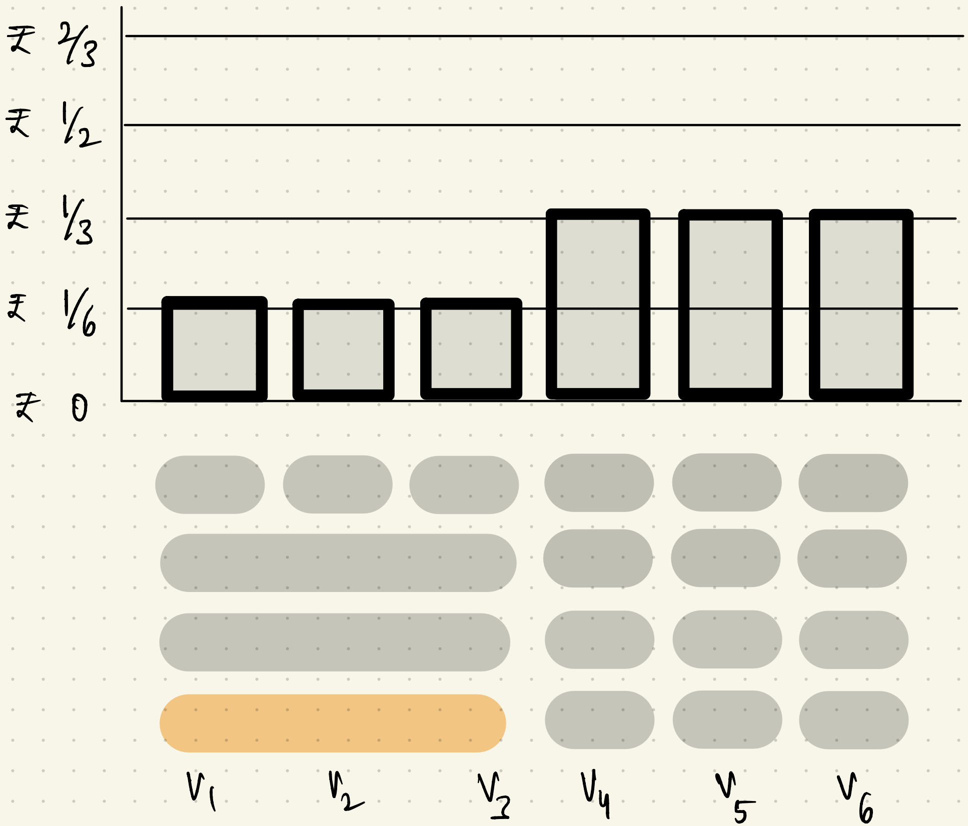
Phragmén's rule



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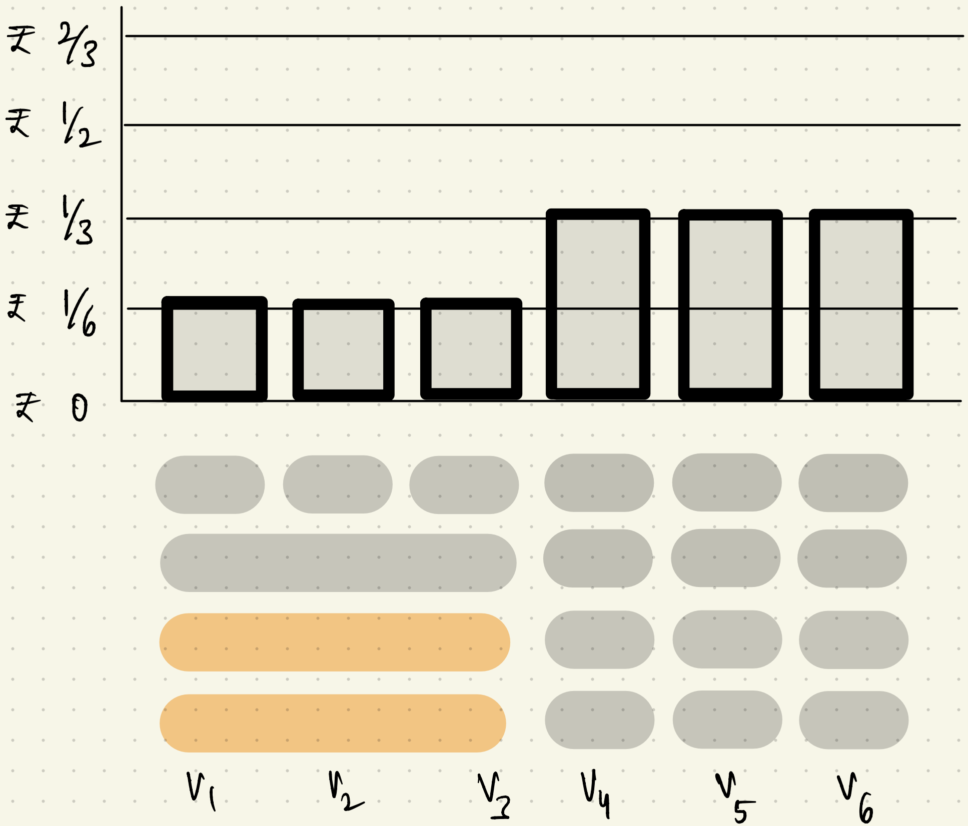

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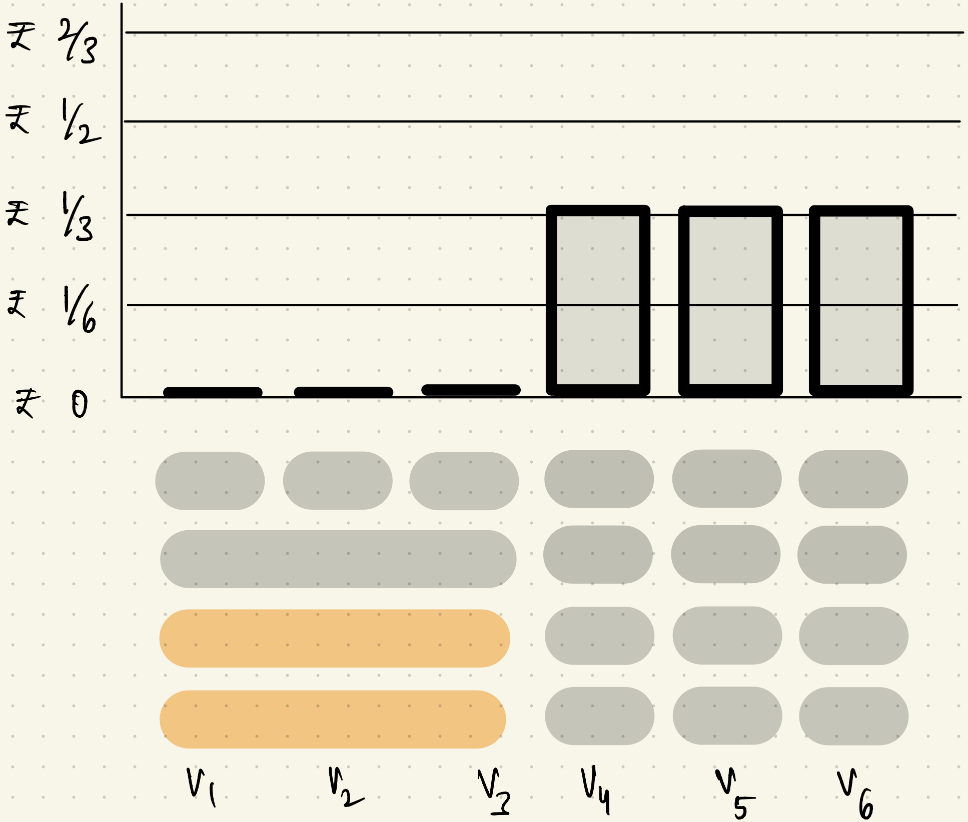

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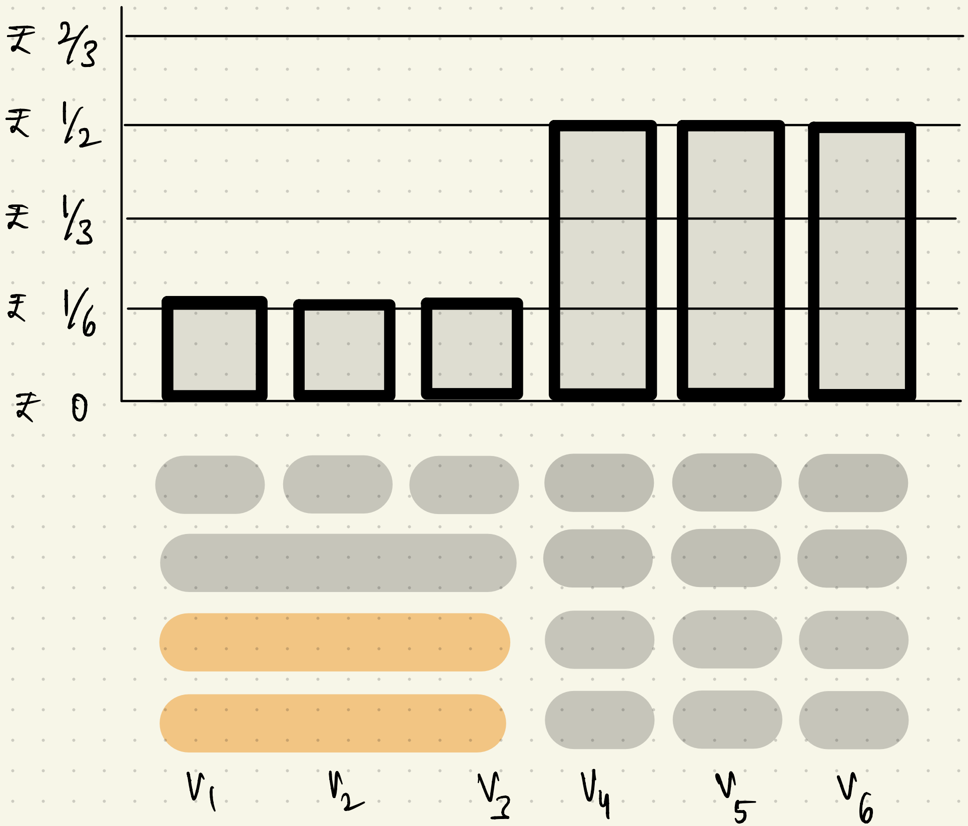

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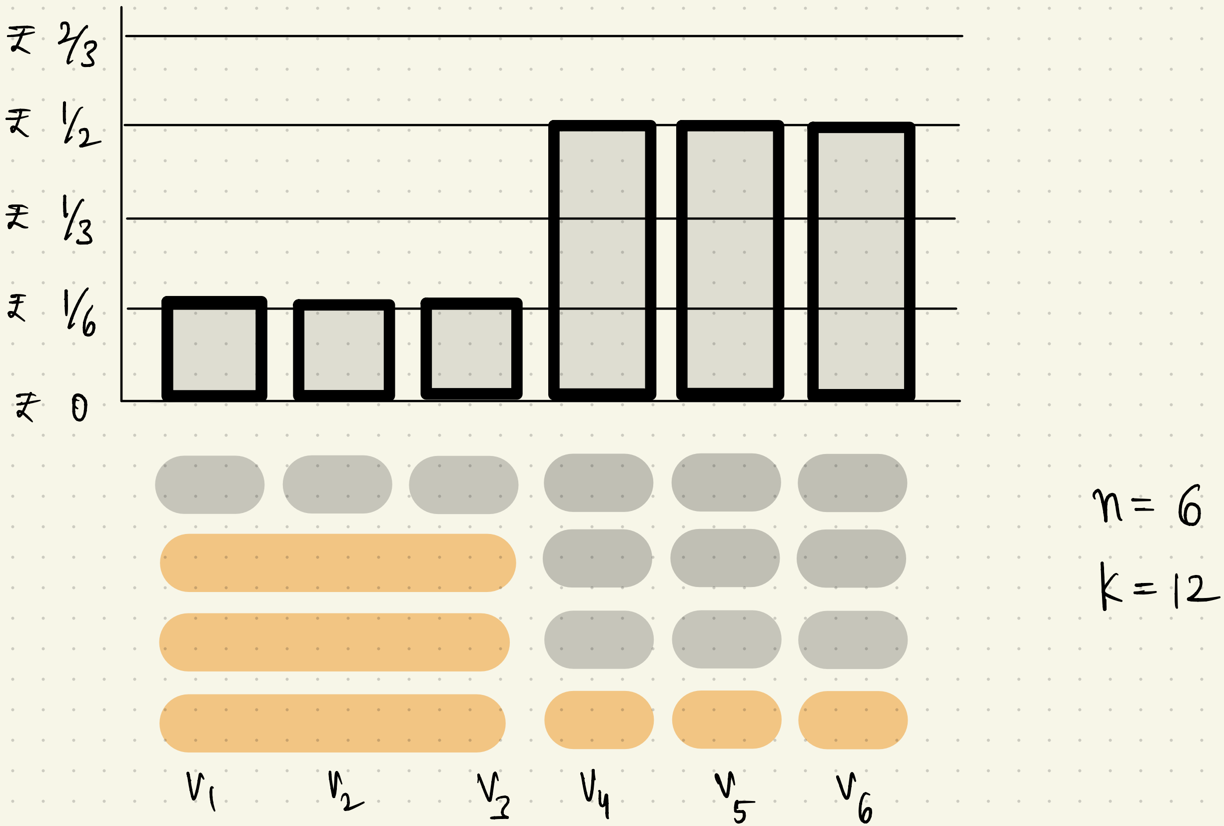

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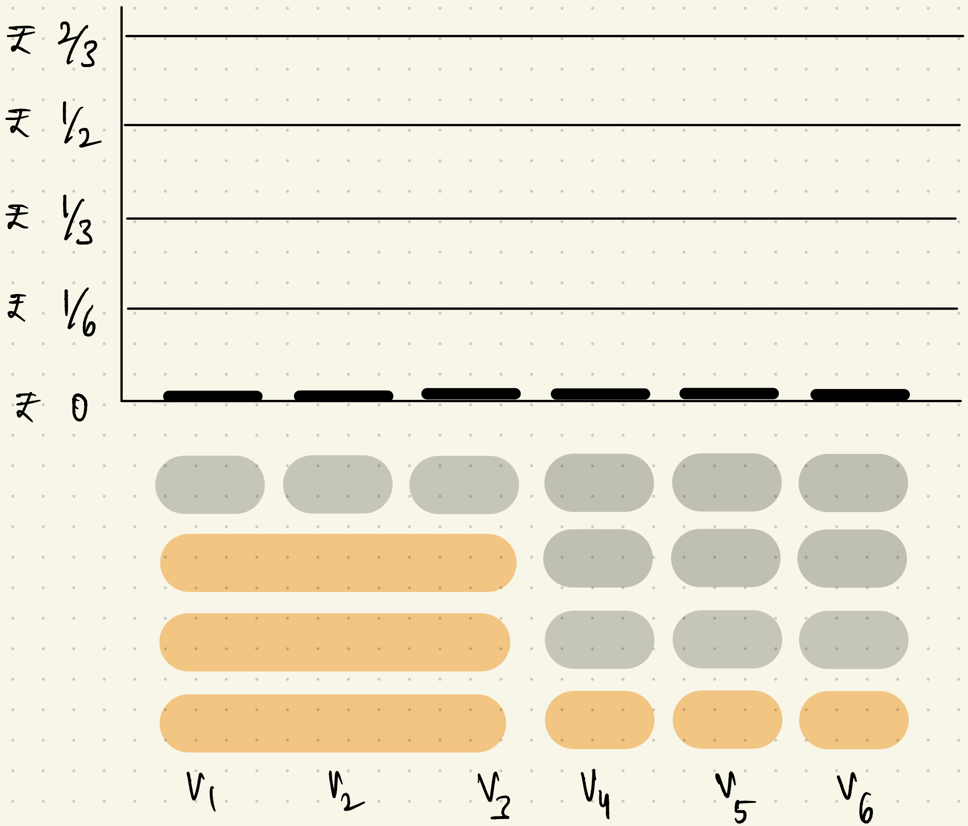

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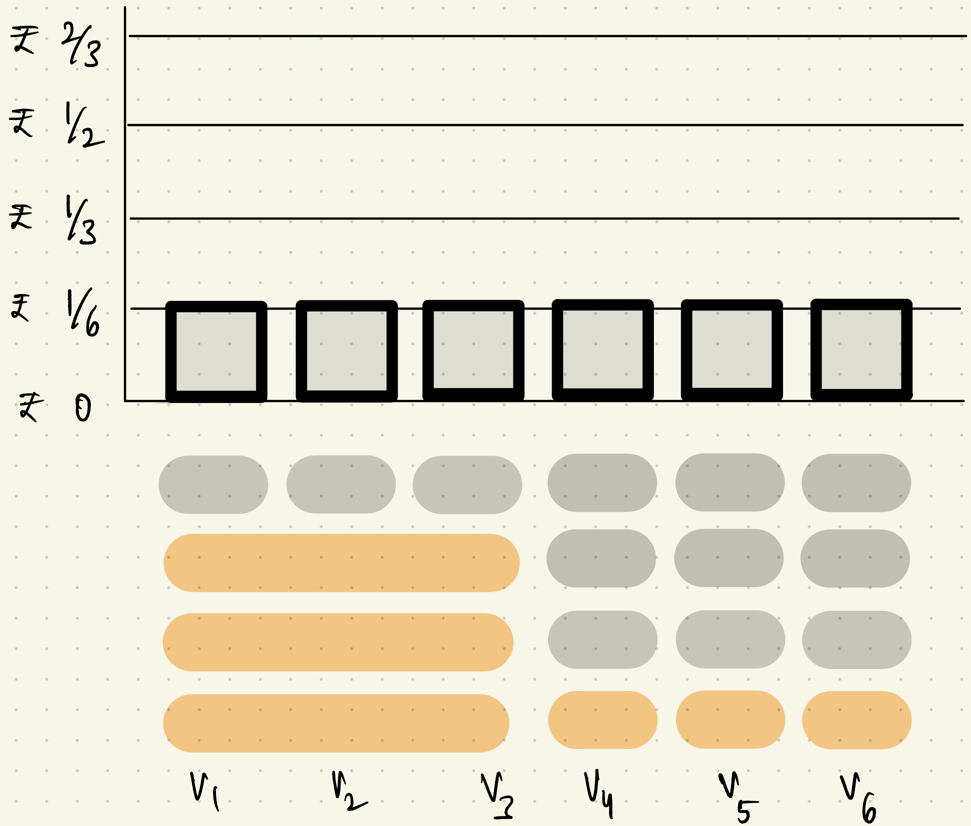

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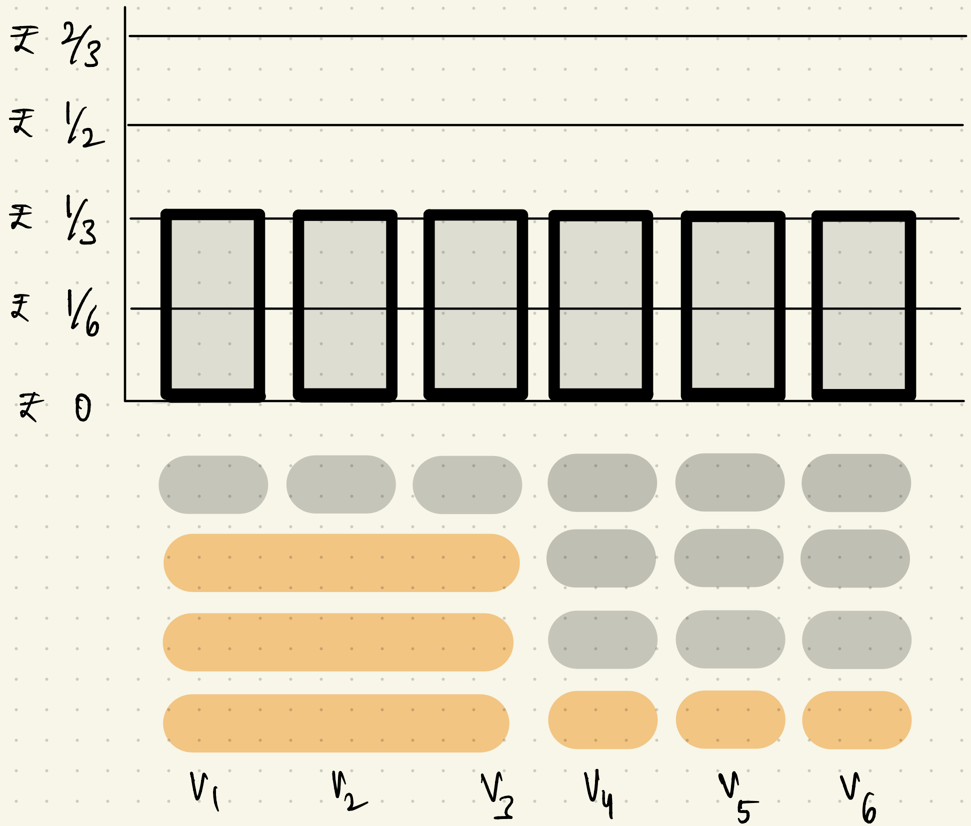

$n = 6$

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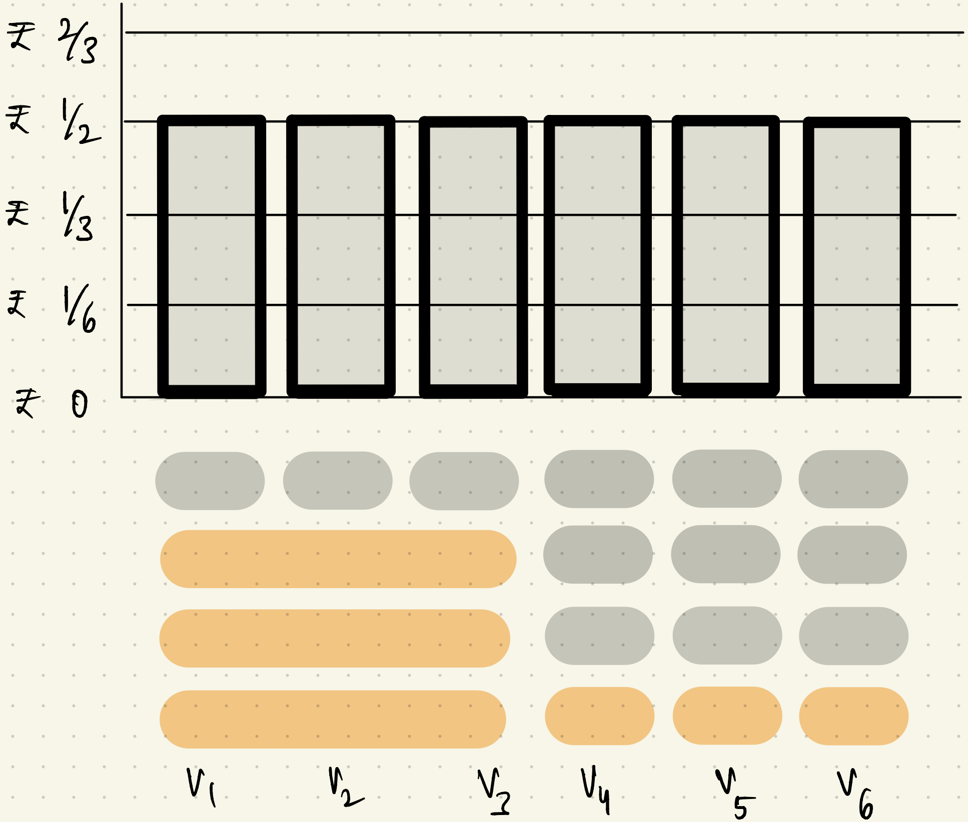

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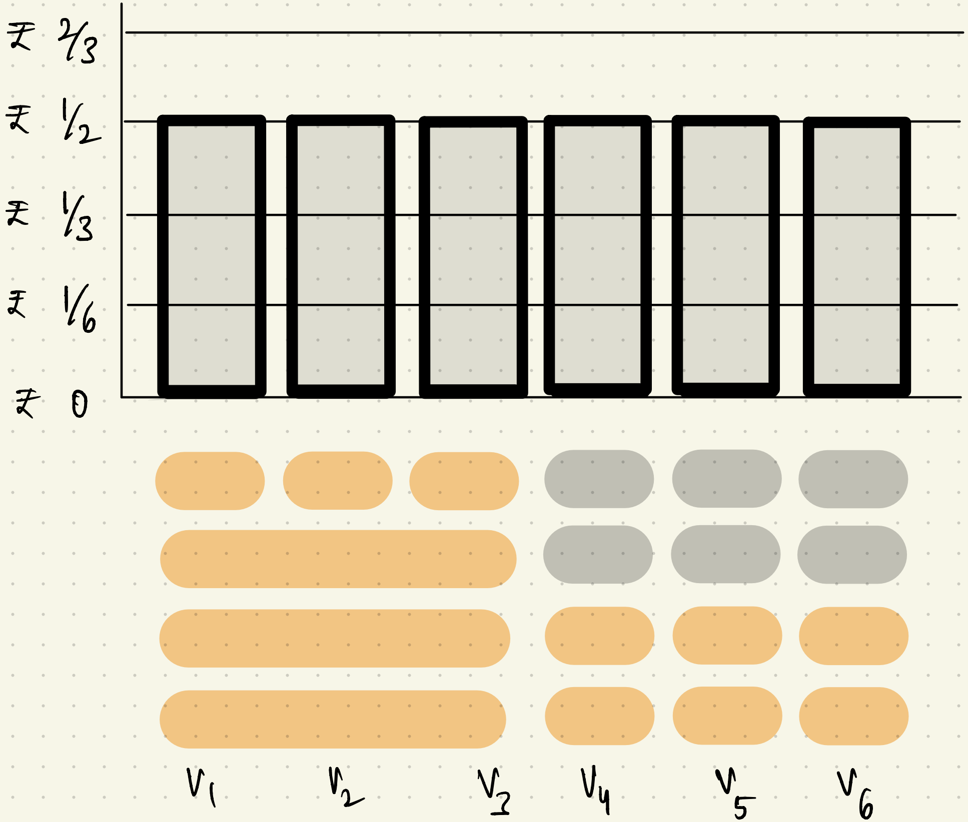

$n = 6$

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PHRAGMÉN'S RULE

[Brill, Janson, Freeman, Lackner Math. Prog. 2023]

Violates EJR

But satisfies a weaker notion: PJR

PROPORTIONAL JUSTIFIED REPRESENTATION

A committee W provides proportional justified representation (PJR) if

for every $\ell \in \{1, 2, \ldots\}$

for every set S of voters s.t. $|S| \geq \ell \cdot \frac{n}{k}$ and $\left|\bigcap_{i \in S} A_i\right| \geq \ell$

then $\left|W \cap \bigcup_{i \in S} A_i\right| \geq \ell$

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ℓ- large $\qquad$ ℓ- cohesive

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weaker than EJR

PROPORTIONAL JUSTIFIED REPRESENTATION

A committee W provides proportional justified representation (PJR) if

for every $\ell \in \{1, 2, \ldots\}$ $\quad \ell = 1$: JR

for every set S of voters s.t. $|S| \geq \frac{\ell \cdot n}{k}$ and $\left|\bigcap_{i \in S} A_i\right| \geq \ell$

ℓ-large $\quad$ ℓ-cohesive

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weaker than EJR

PHRAGMÉN SATISFIES PJR

PJR : $\forall \ell \geqslant 1$, $\forall S$ s.t. $|S| \geqslant \ell \cdot \frac{n}{k}$ and $\left|\bigcap_{i \in S} A_i\right| \geqslant \ell$, $\left|W \cap \bigcup_{i \in S} A_i\right| \geqslant \ell$

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Proof : Each elected candidate costs $\mathcal{L}\ \frac{n}{k}$.

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Uniform rate of top up of bank accounts

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<table>
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Phragmen cannot terminate with $\left|W \cap \bigcup_{i \in S} A_i\right| \leq \ell - 1$

Why ?

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Why? S can afford a candidate.

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PICTURE SO FAR

Core Existence open!

EJR Exists + Poly time, hard to verify

SJR

May not exist

PJR Exists + Poly time, hard to verify

JR Exists + Poly time, easy to verify

STRATEGYPROOFNESS

more approved candidates
wrt true prefs

No voter can derive a higher utility by misreporting its approved set of candidates.

STRATEGYPROOFNESS

PAV is not strategyproof.

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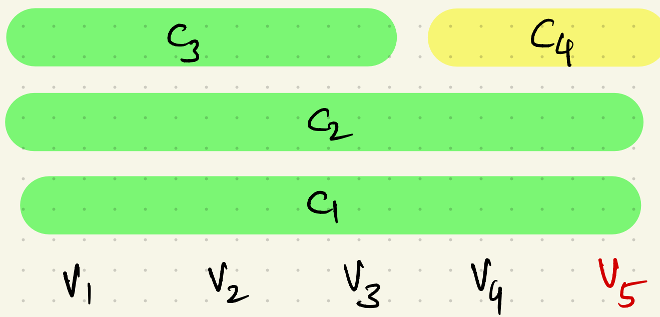
PAV is not strategyproof.

$k = 3$



STRATEGYPROOFNESS

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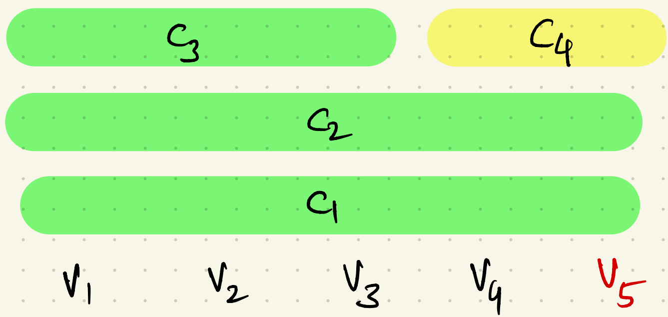

PAV selects $\{c_1, c_2, c_3\}$

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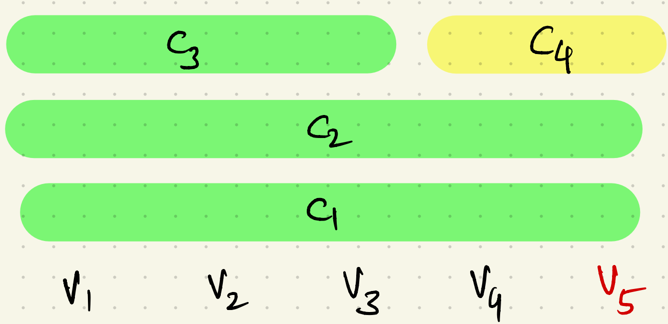

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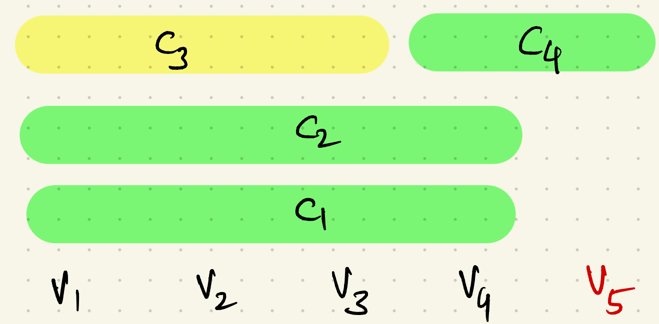

PAV selects $\{c_1, c_2, c_4\}$

STRATEGYPROOFNESS

[Peters , AAMAS 2018]

Thm: No voting rule is strategyproof and satisfies JR.

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Proof: By SAT solving.

STRATEGYPROOFNESS

[Peters, AAMAS 2018]

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Proof: By SAT solving.

* Approval voting is strategyproof but fails JR.

COURSE RECAP

* Interplay of computation and incentives

Matching

Fair Division

Voting

COURSE RECAP

* Interplay of computation and incentives

Matching | **Fair Division** | **Voting**

* Deferred-acceptance
* Stable lattice
* Stability v/s truthfulness
* Fair matchings
* Top-trading cycles
  and kidney exchange

COURSE RECAP

* Interplay of computation and incentives

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Fair Division

* Cake and rent division
* EF1 and envy cycles
* Nash social welfare
* Top-trading envy cycles
* Best of both worlds

Voting

COURSE RECAP

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Fair Division

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* Best of both worlds

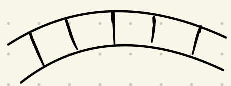
Voting

* Rules, rules, rules!
* Manipulation is inevitable
* ... but CS saves the day!
* Structure helps
* Axioms → Quantitative

TAKEAWAYS

* Collective decision-making is everywhere!

* Algorithms affect and are affected by users' incentives

* Computation  Economics

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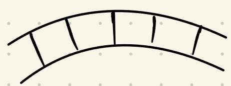
* Computation  Economics

Thanks!