COL866: Special Topics in Algorithms

Lecture 18

Voting with Structured Preferences

Oct 20, 2023 | Rohit Vaish

Story So Far

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No group-level transitivity

Condorcet paradox

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No "reasonable" voting rules

Gibbard-Satterthwaite and Arrow's theorems

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Manipulation can be easy

Greedy strategy

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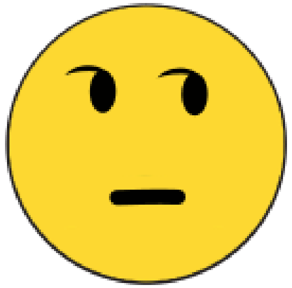

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NP-hardness as a barrier to manipulation

Bartholdi-Tovey-Trick

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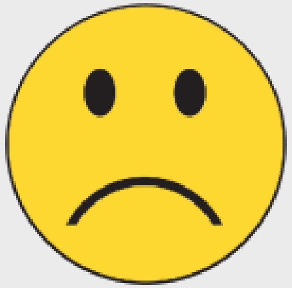

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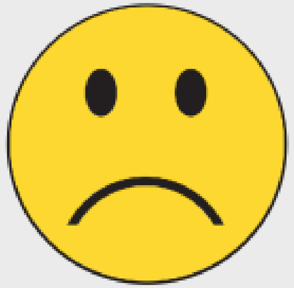

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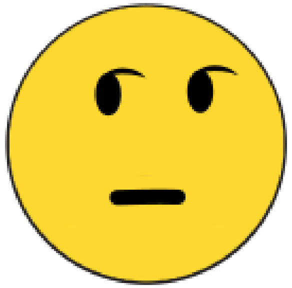

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Single-Peaked Preferences

[Black'48]

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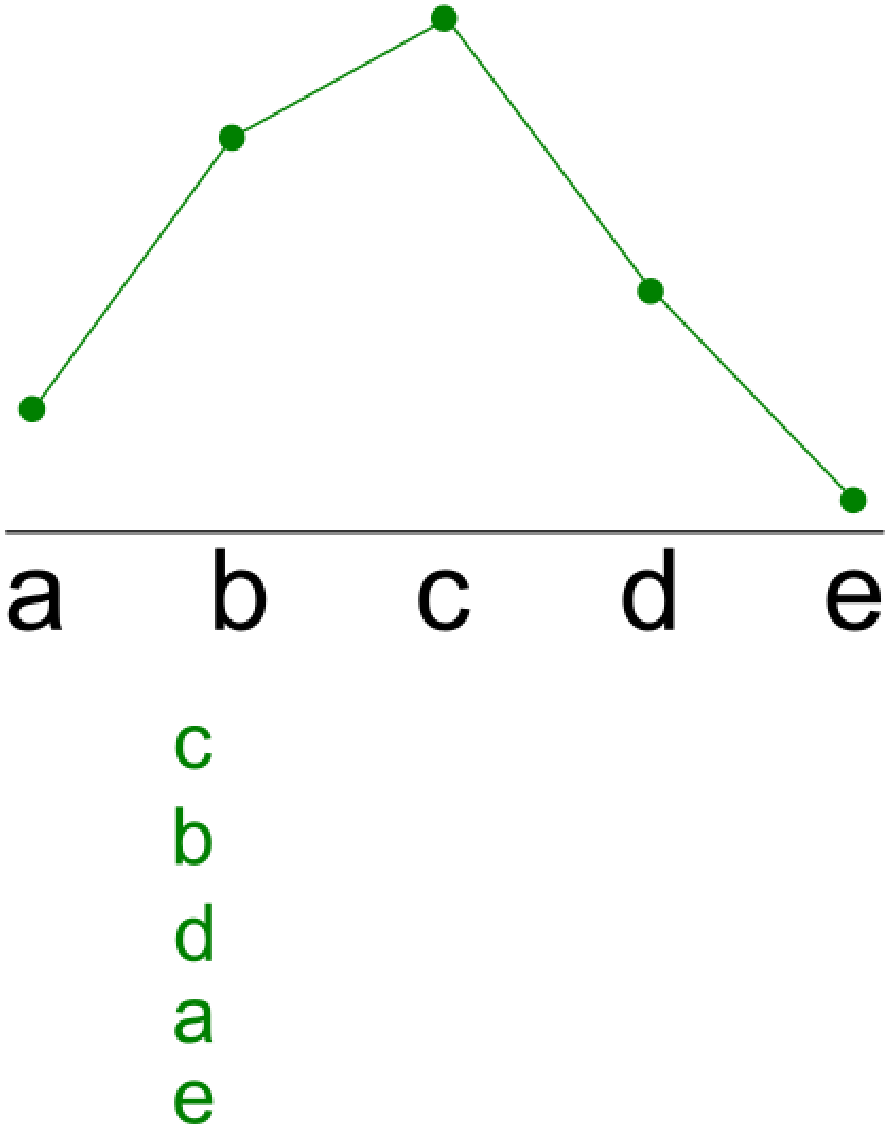

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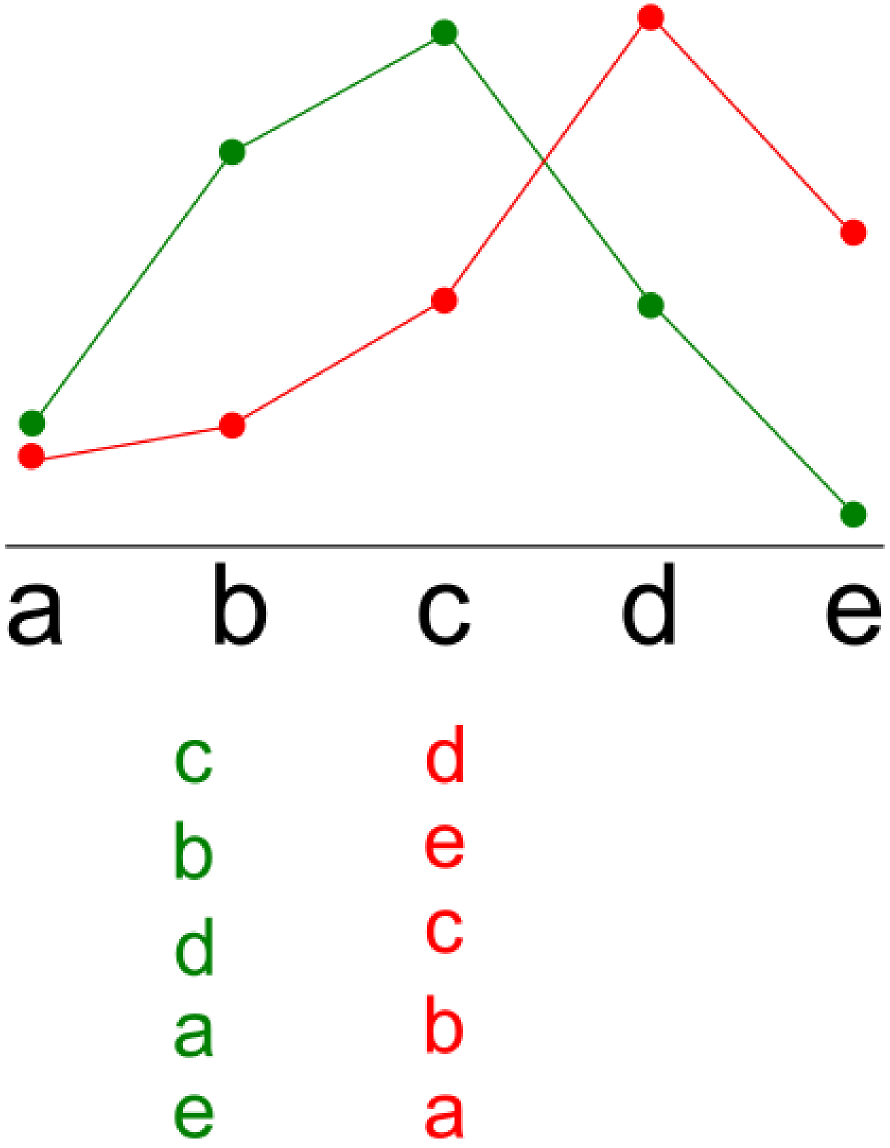

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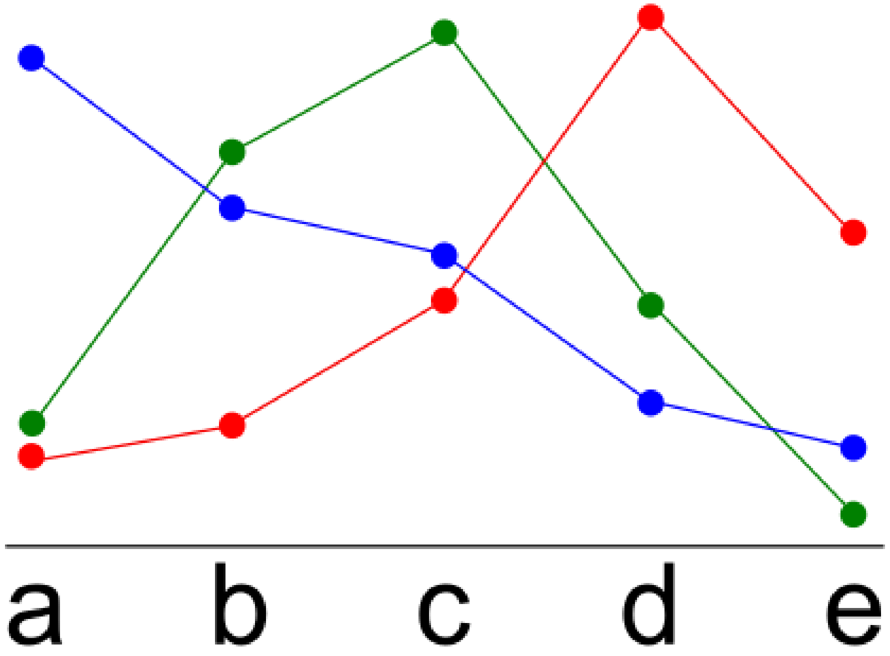

c	d	a
b	e	b
d	c	c
a	b	d
e	a	e

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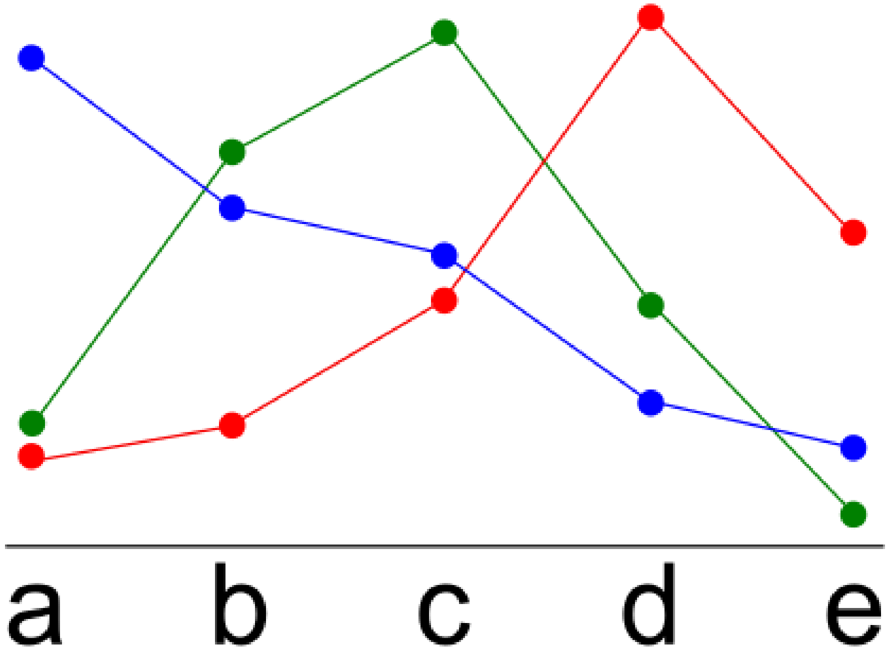

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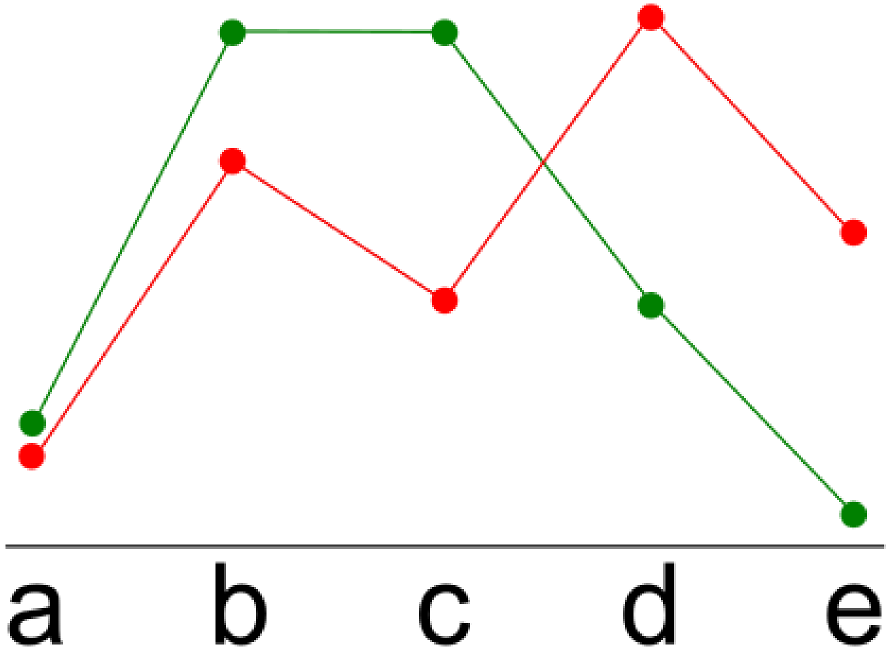

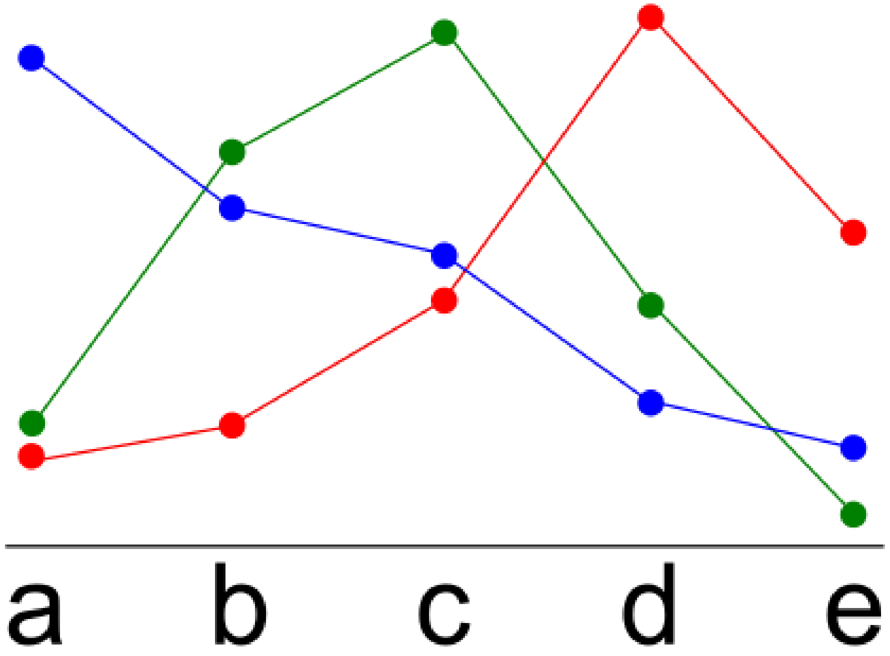

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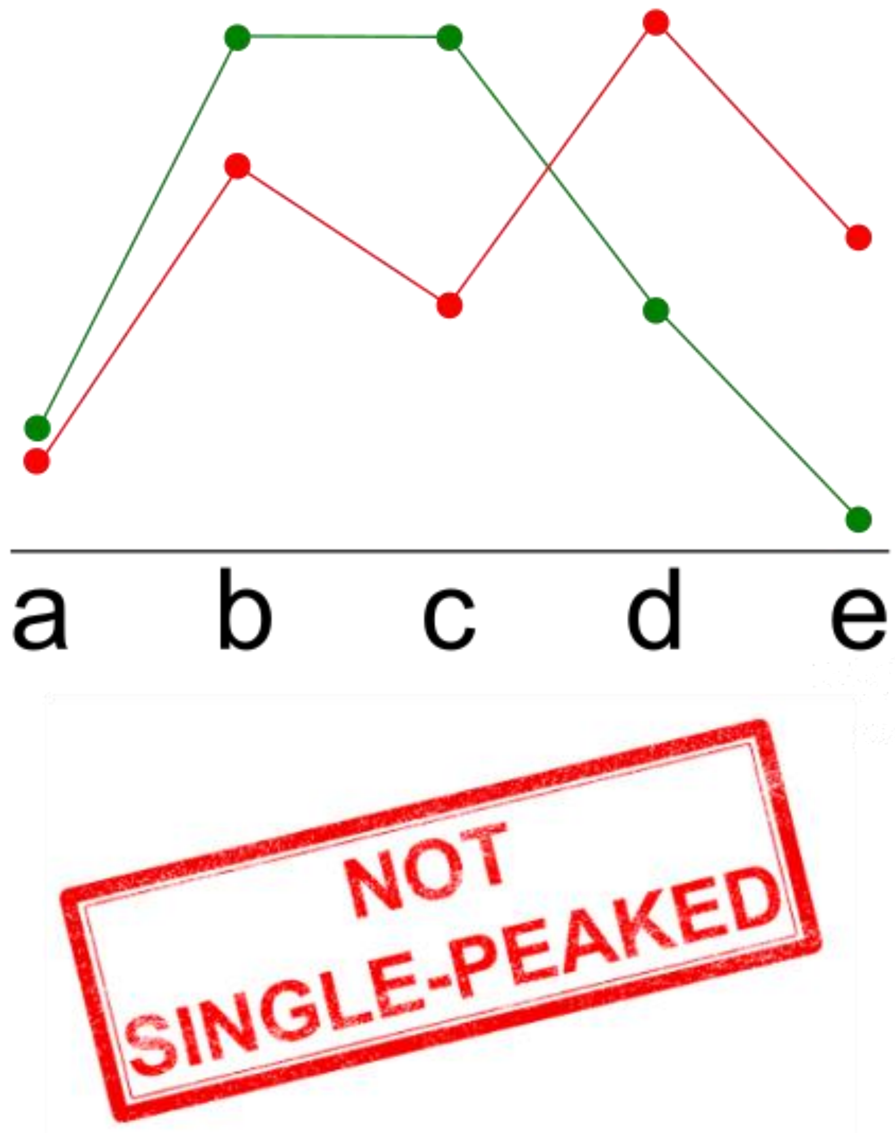

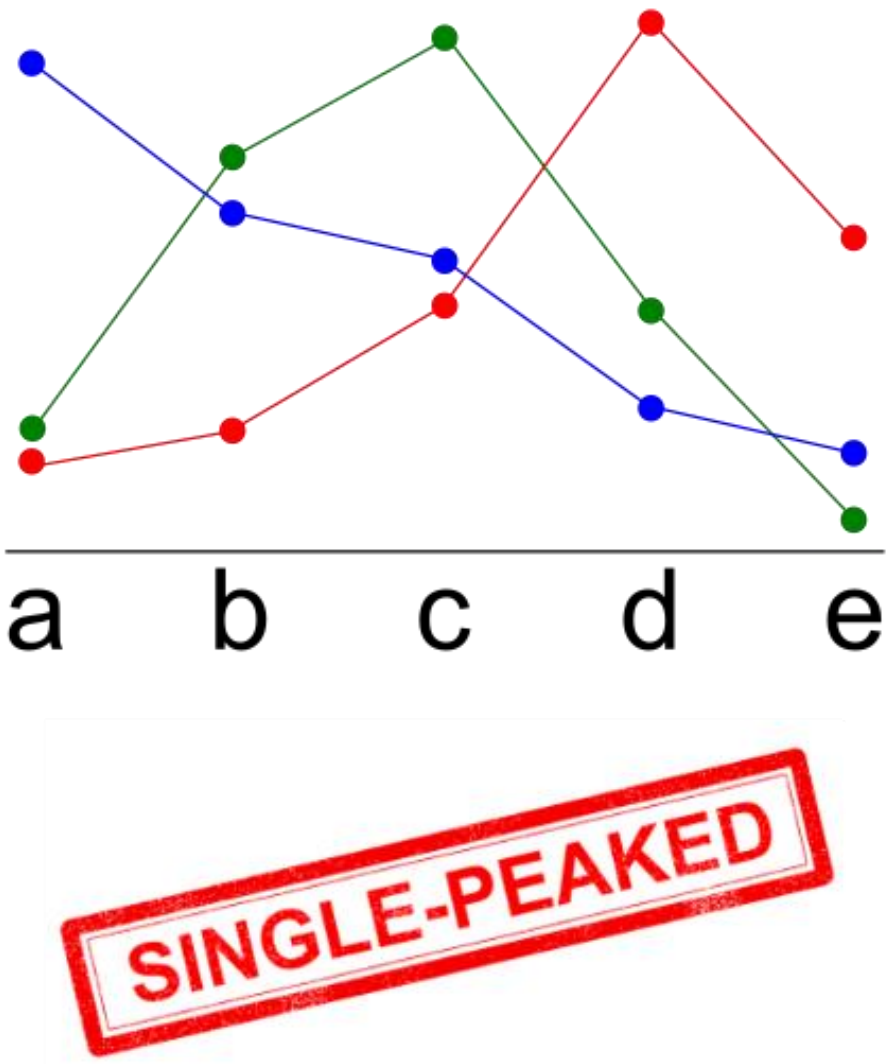

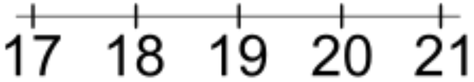
17 18 19 20 21

17 18 19 20 21

5% 10% 15% 20%

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20° 21° 22° 23°

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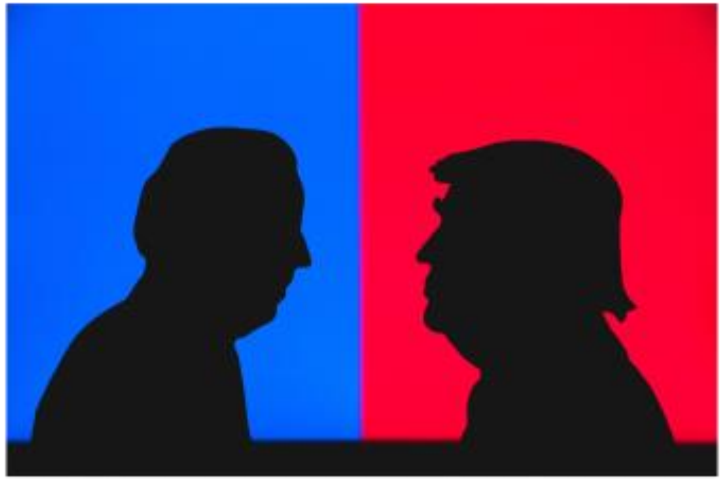

Liberal Conservative

Single-peaked profiles always admit a (weak) Condorcet winner.

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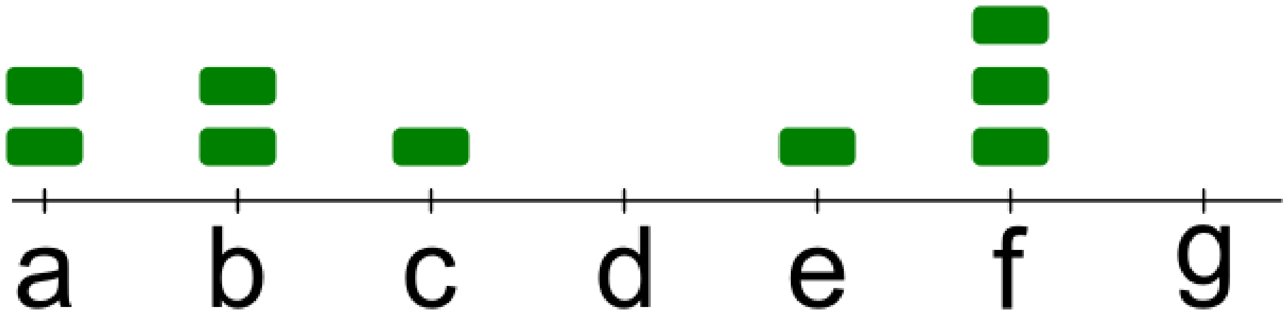
Order the voters according to their top choices



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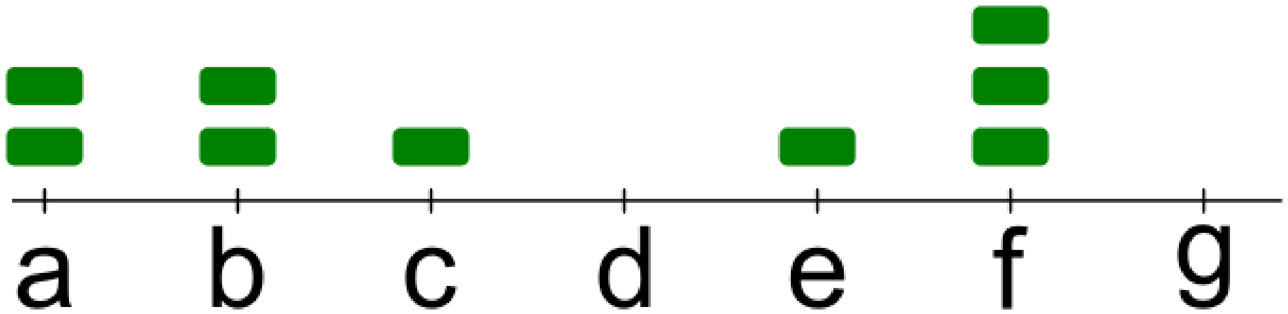

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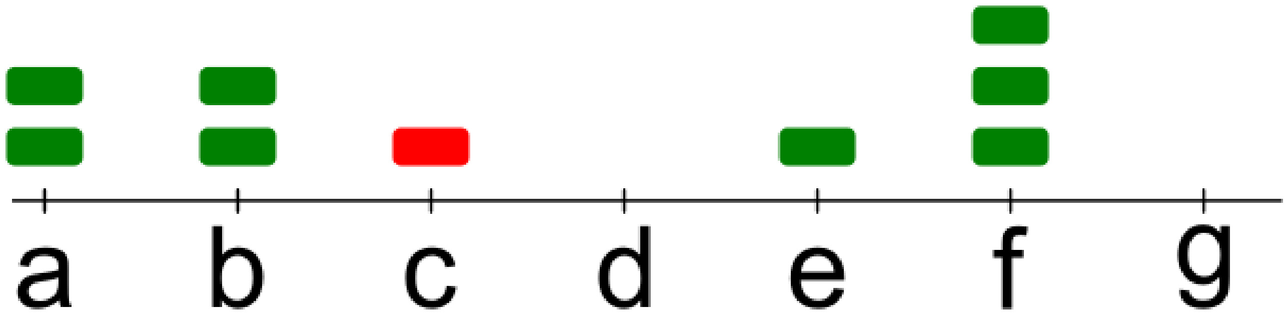

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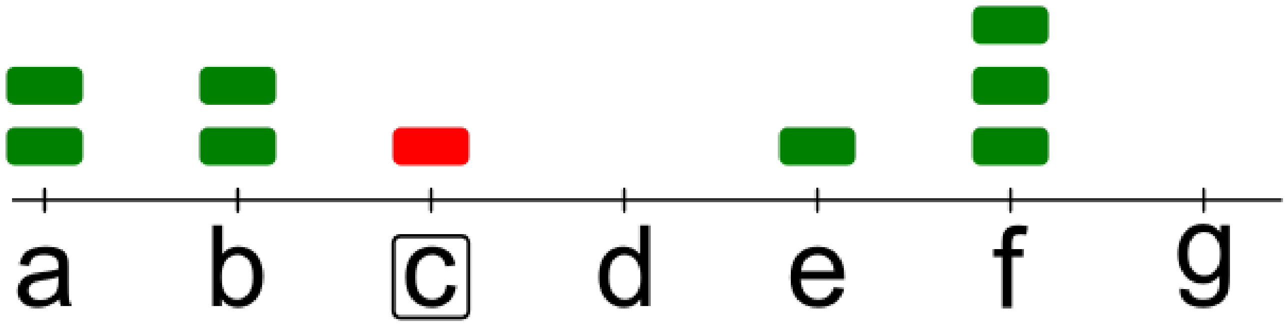

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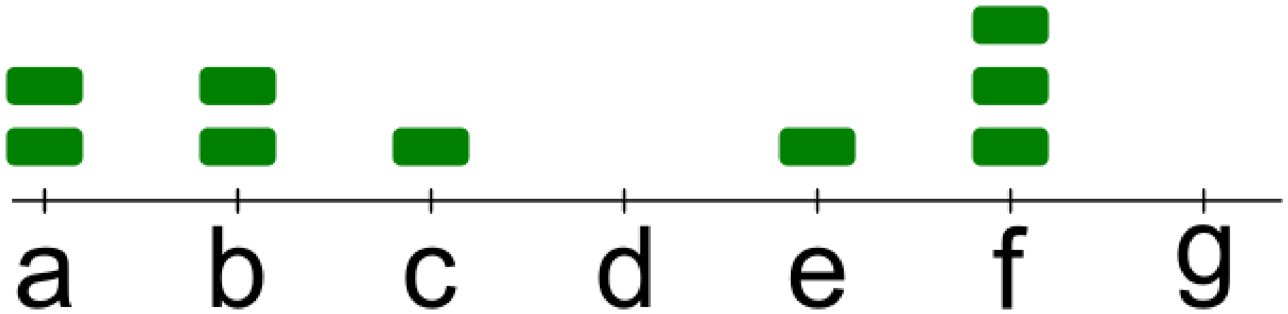

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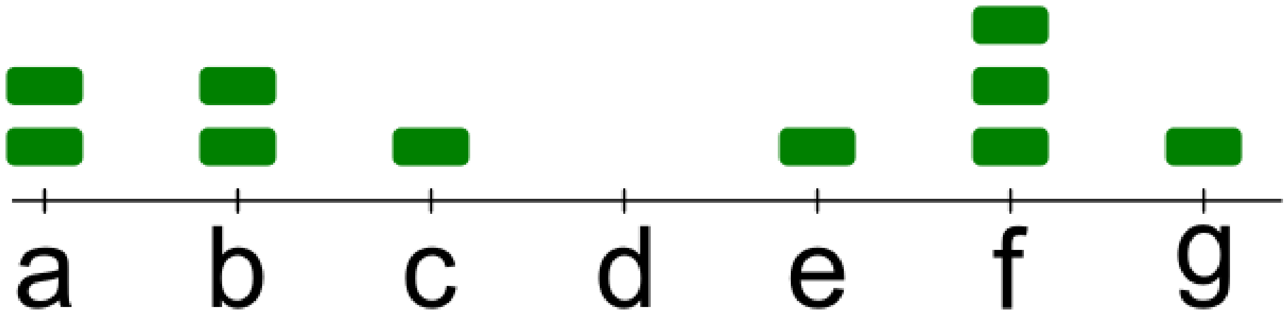

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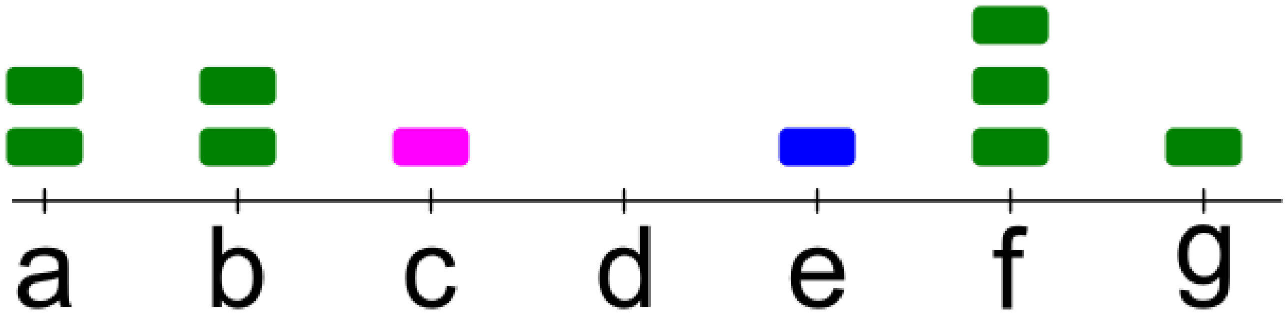

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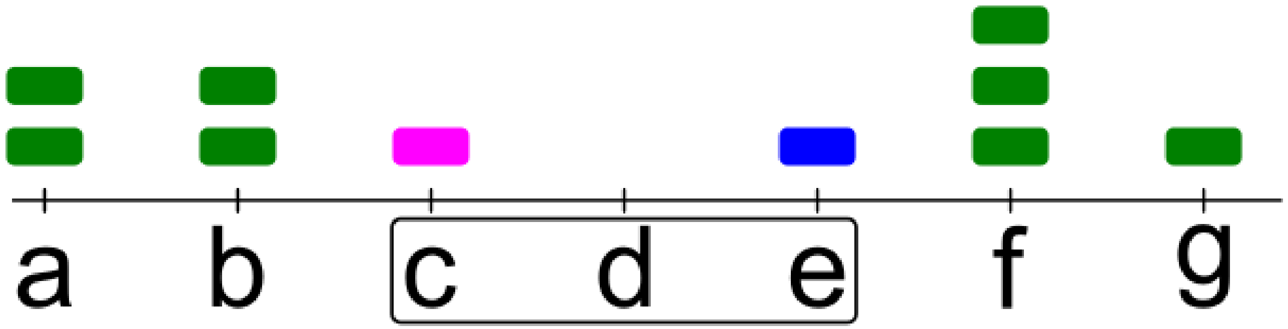

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All candidates between top(v_k) and top(v_{k+1}) are weak Condorcet winners

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Median voter rule

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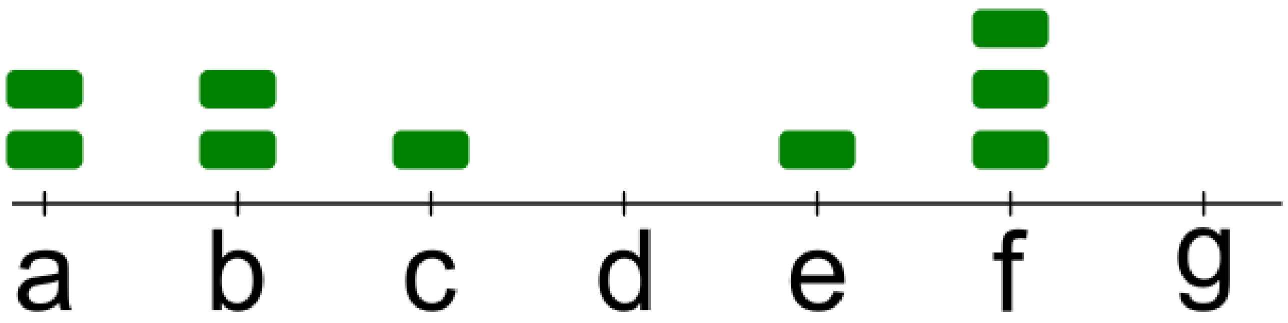

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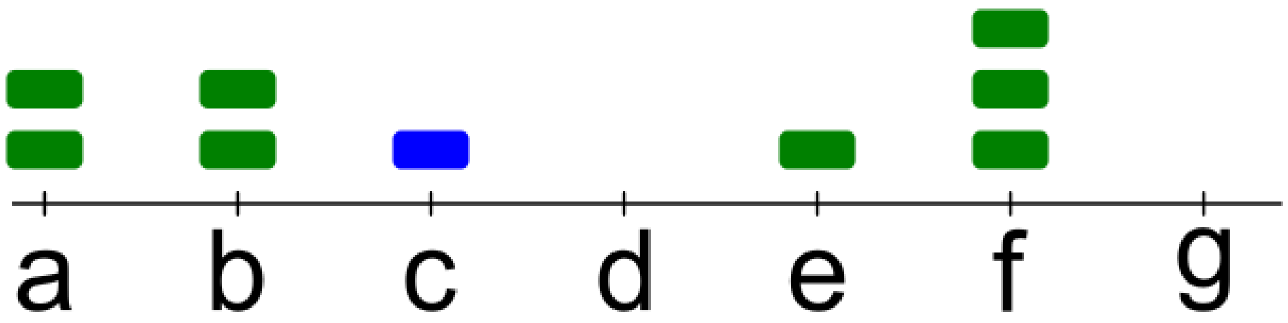

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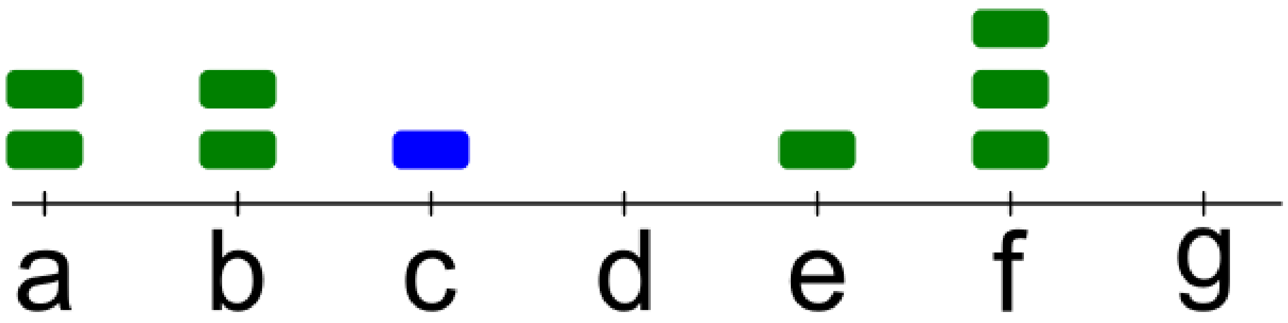

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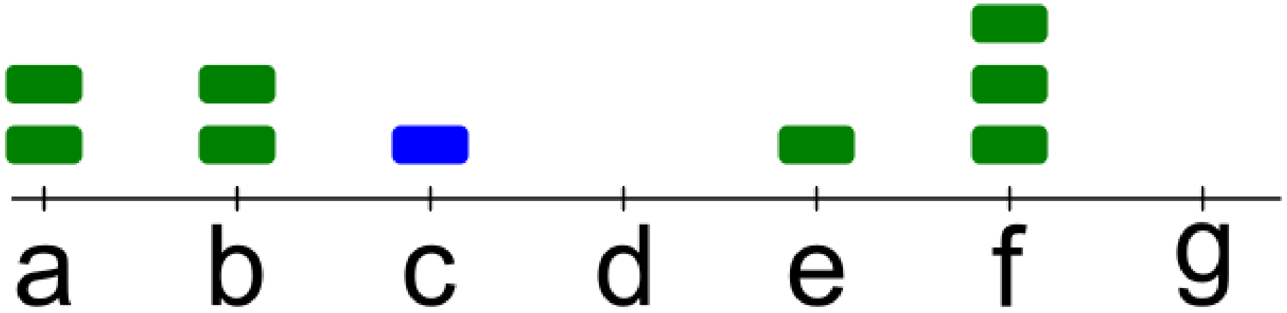

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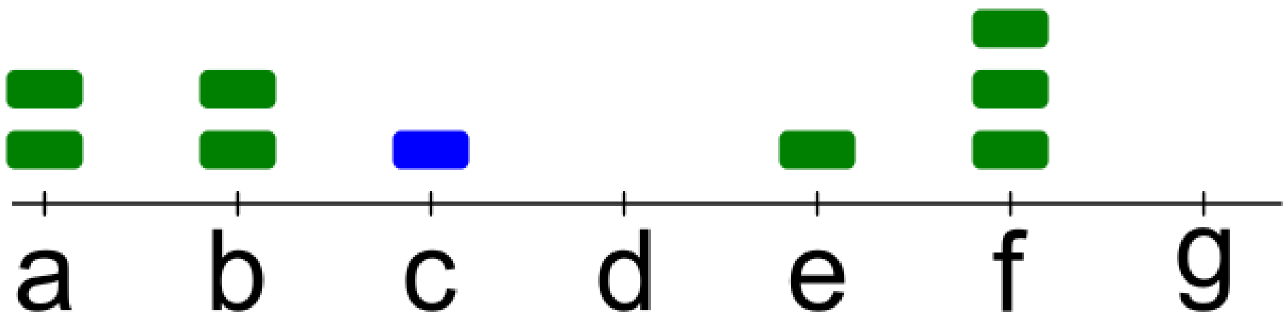

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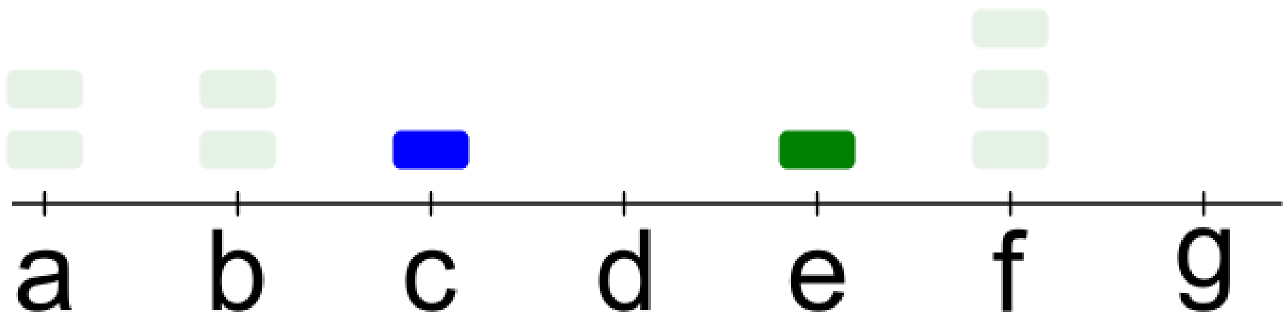

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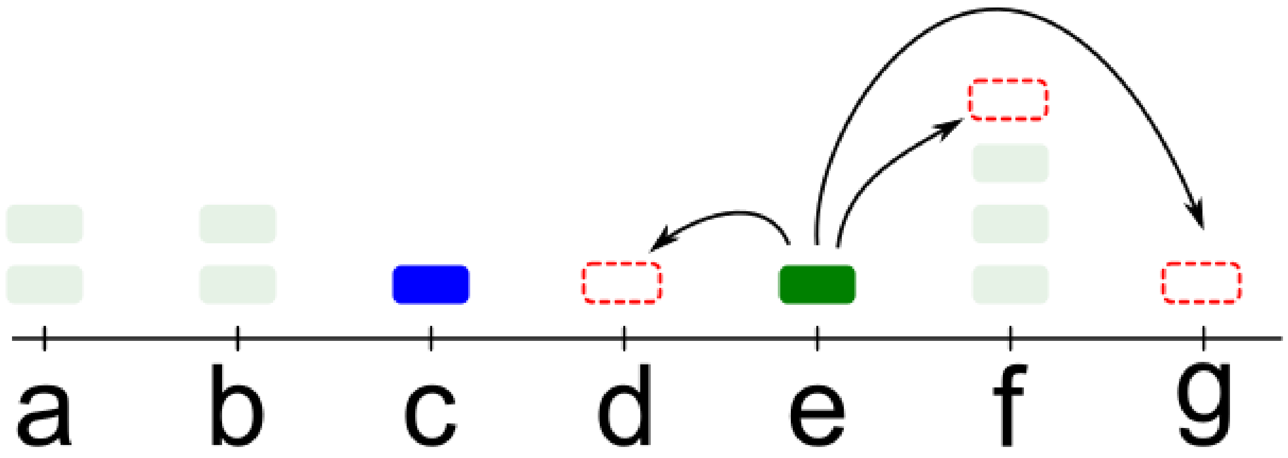

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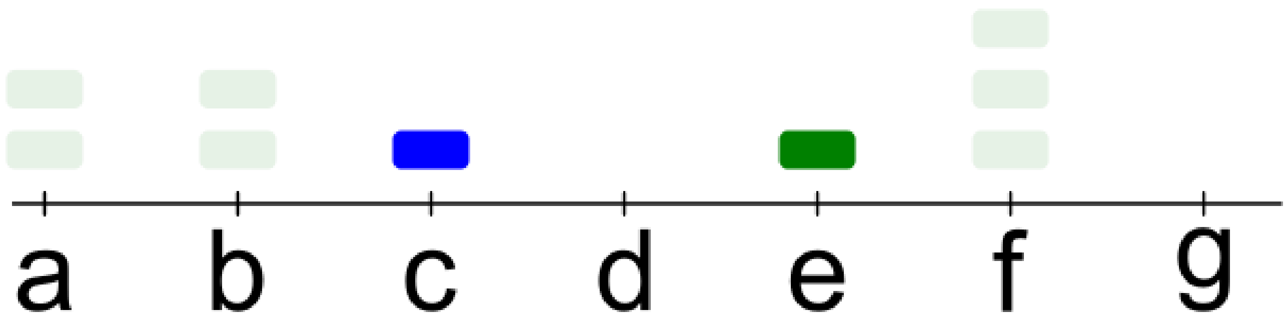

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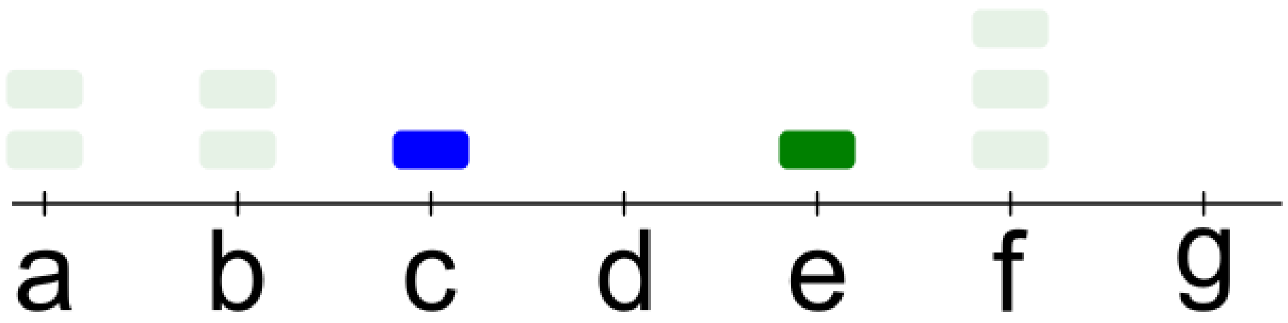

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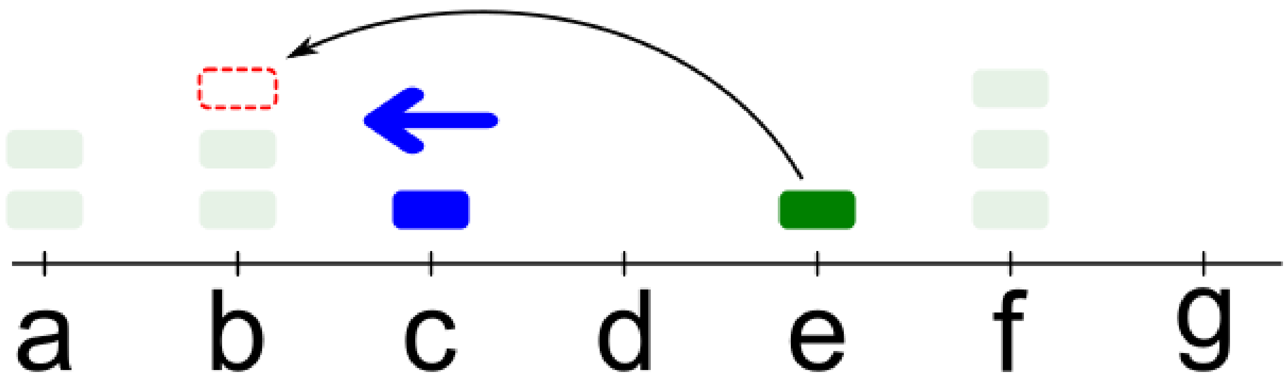

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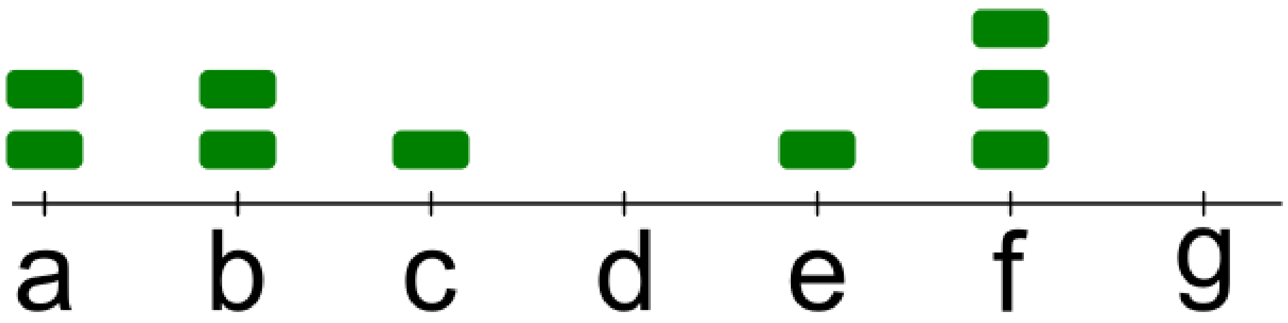

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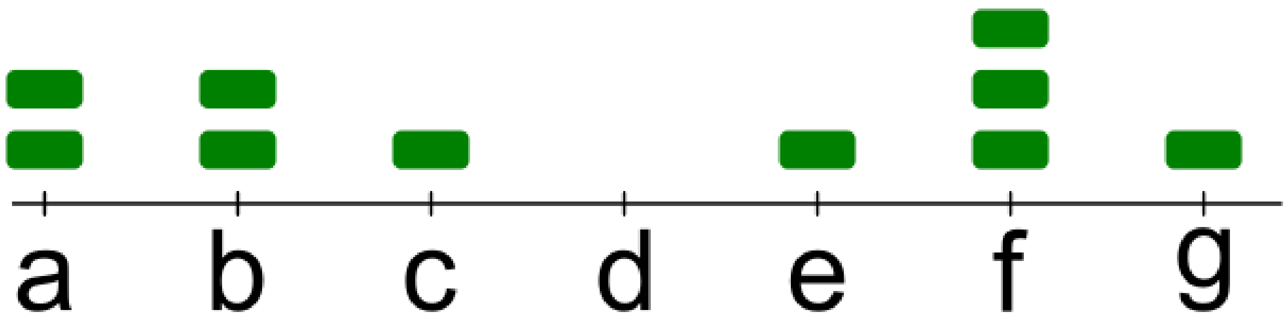

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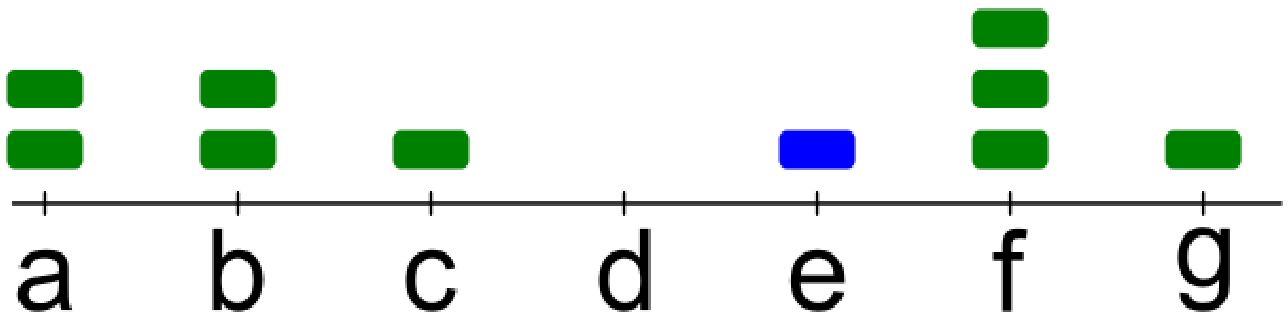

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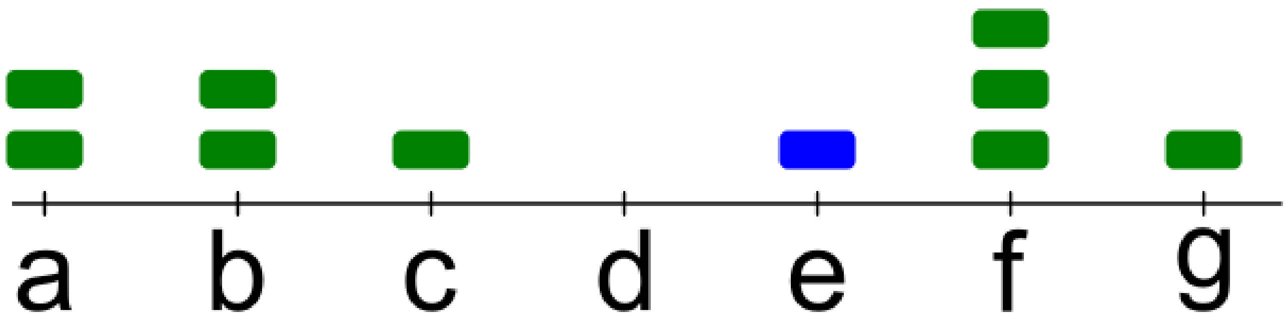

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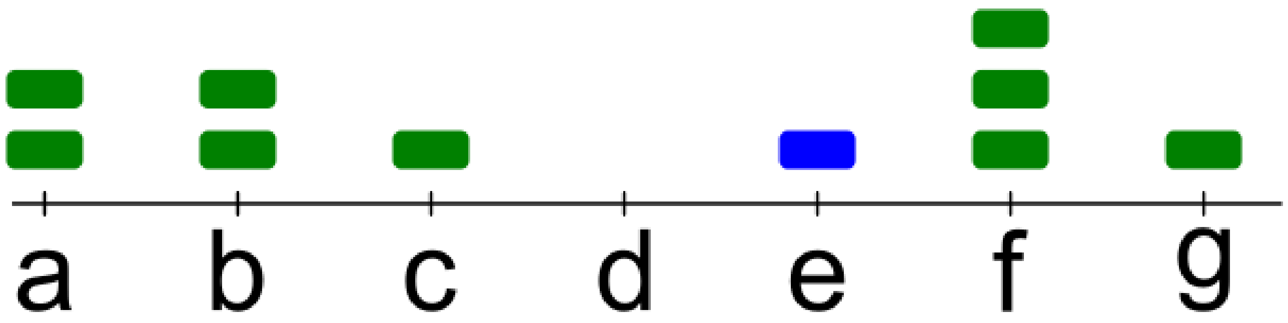

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w.r.t. a given axis

w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. a given axis

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a b c d e

Recognizing Single-Peaked Prefs w.r.t. a given axis

a b c d e

b
c
d
a
e

Recognizing Single-Peaked Prefs w.r.t. a given axis

a b c d e

b
c
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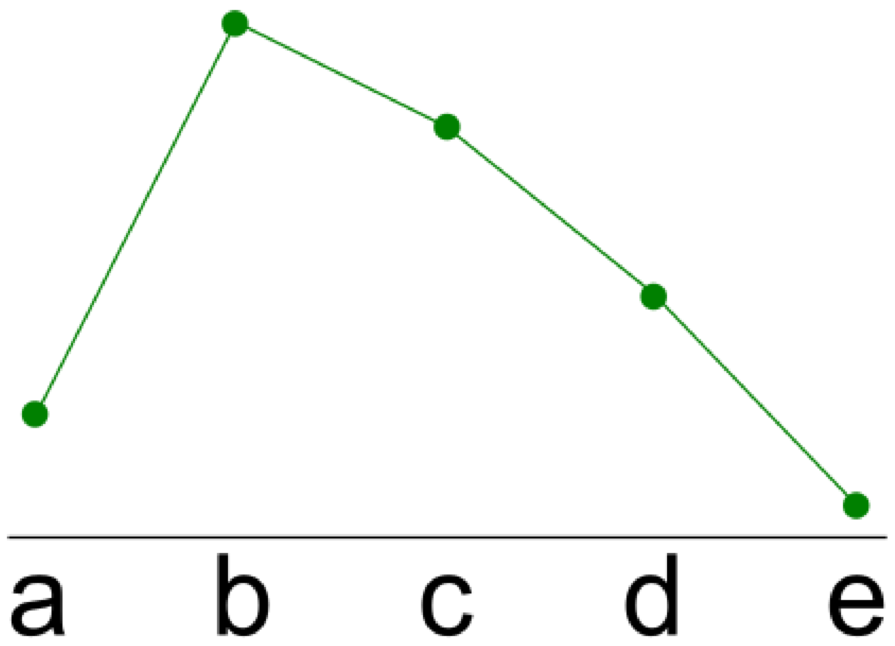

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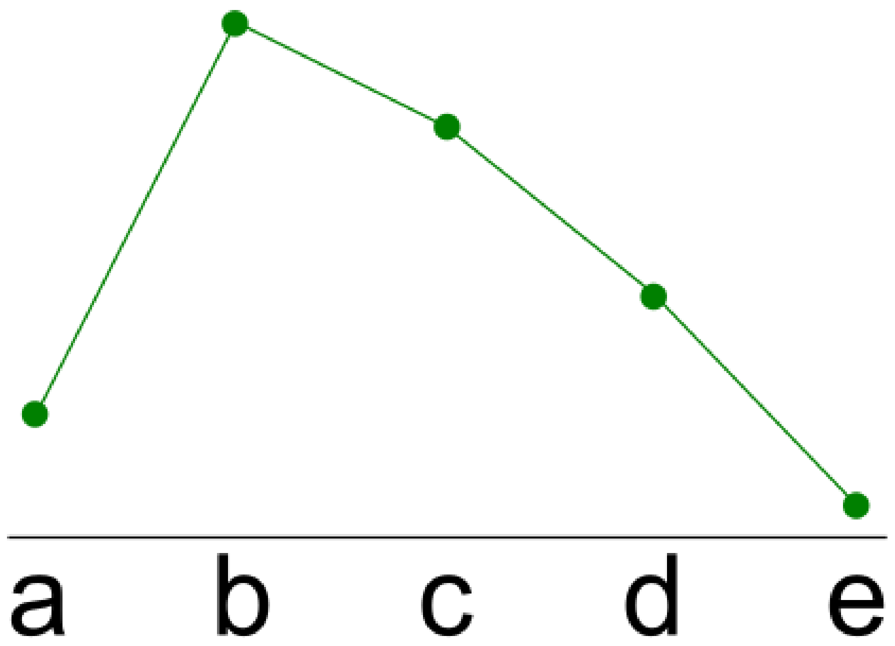

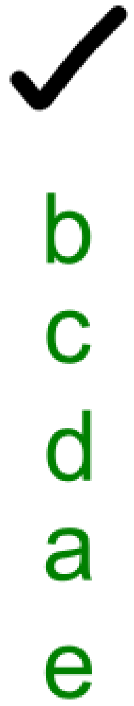

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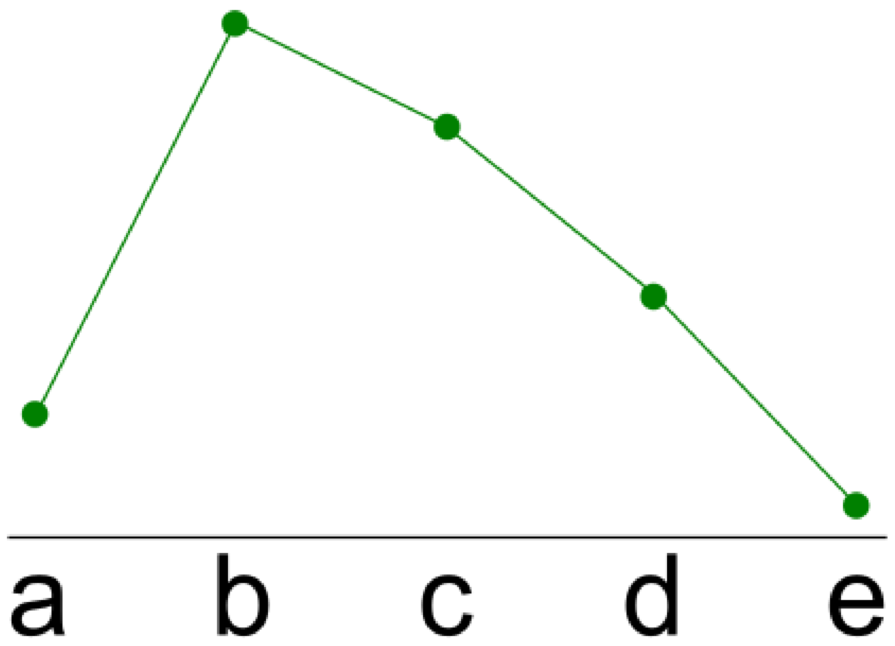

✓

b	d
c	e
d	c
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e	a

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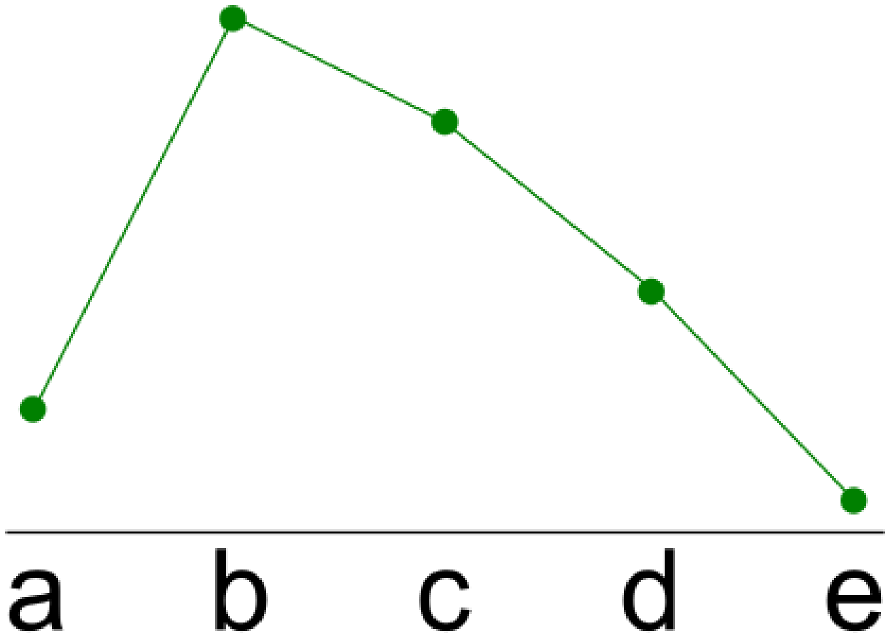

✓

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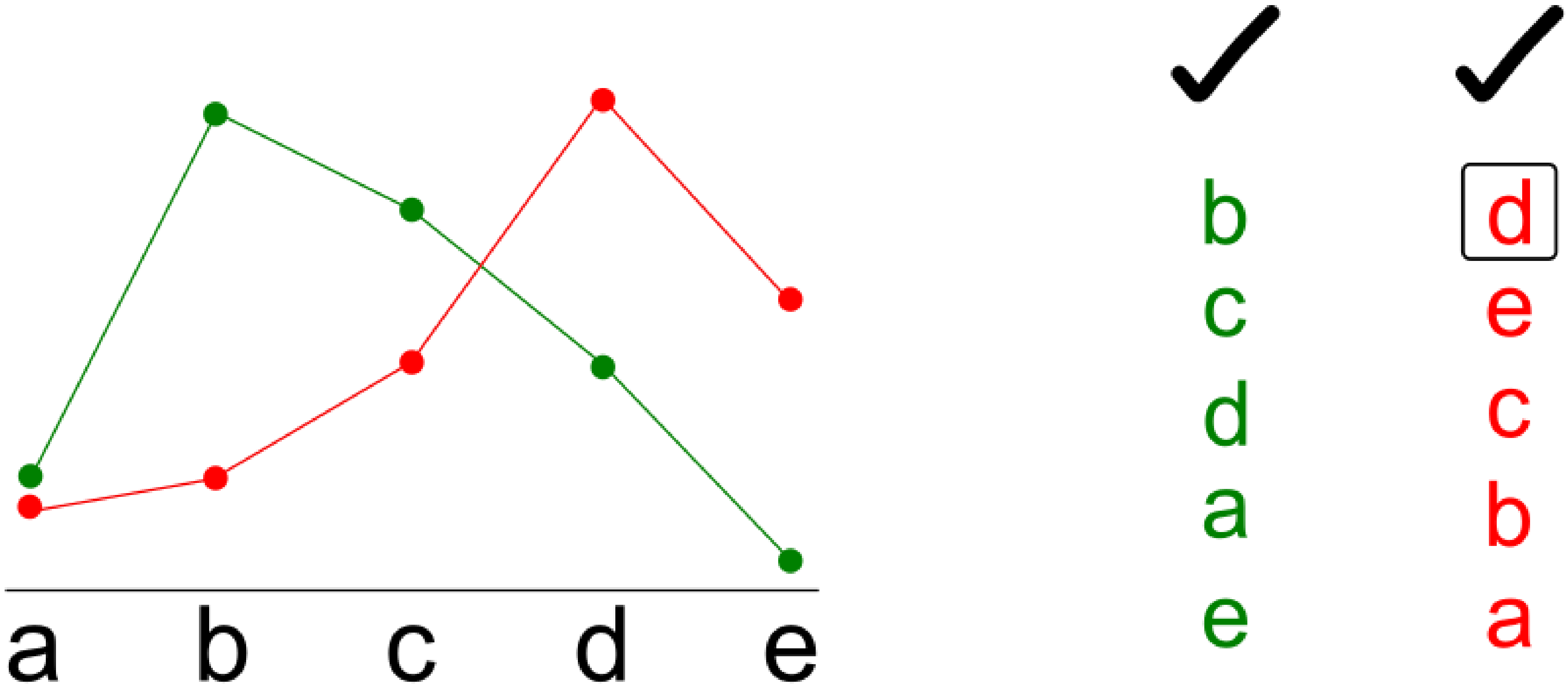

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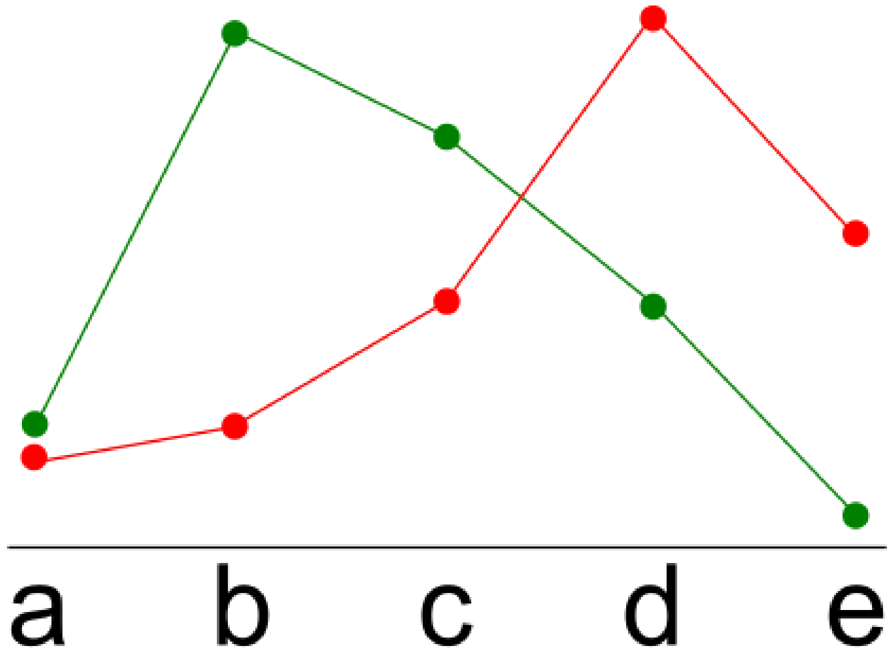

✓	✓	
b	d	a
c	e	b
d	c	c
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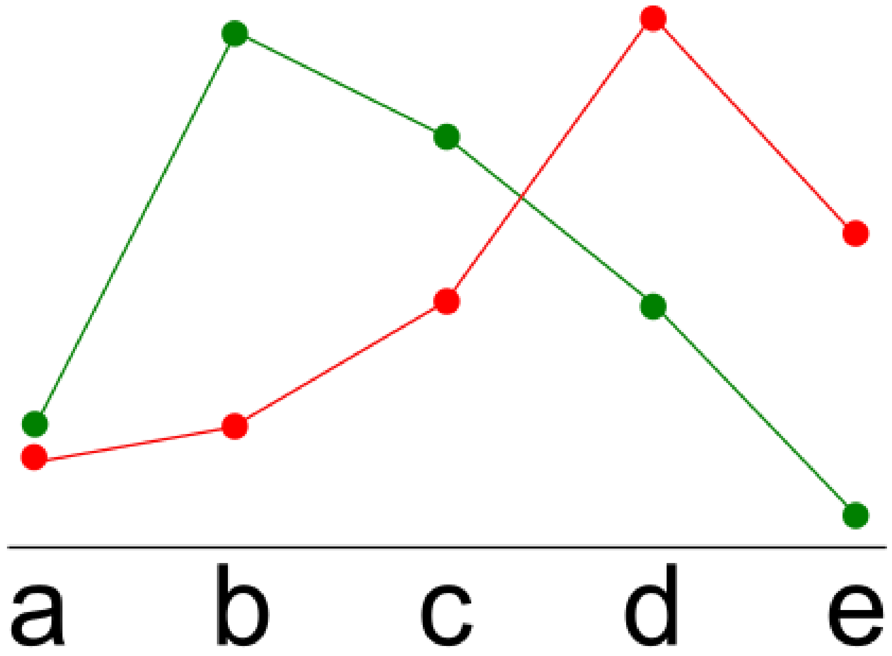
Recognizing Single-Peaked Prefs w.r.t. a given axis



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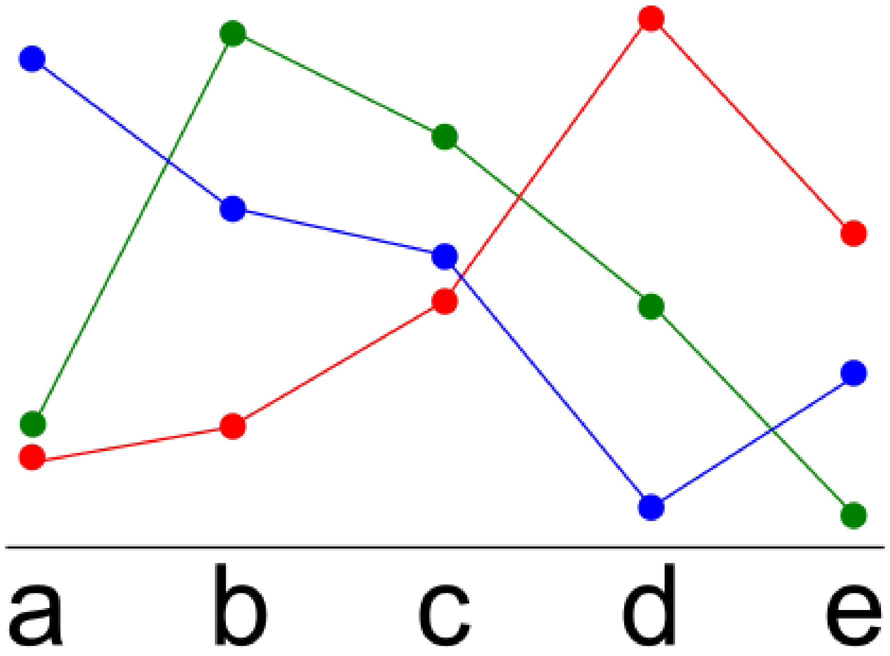

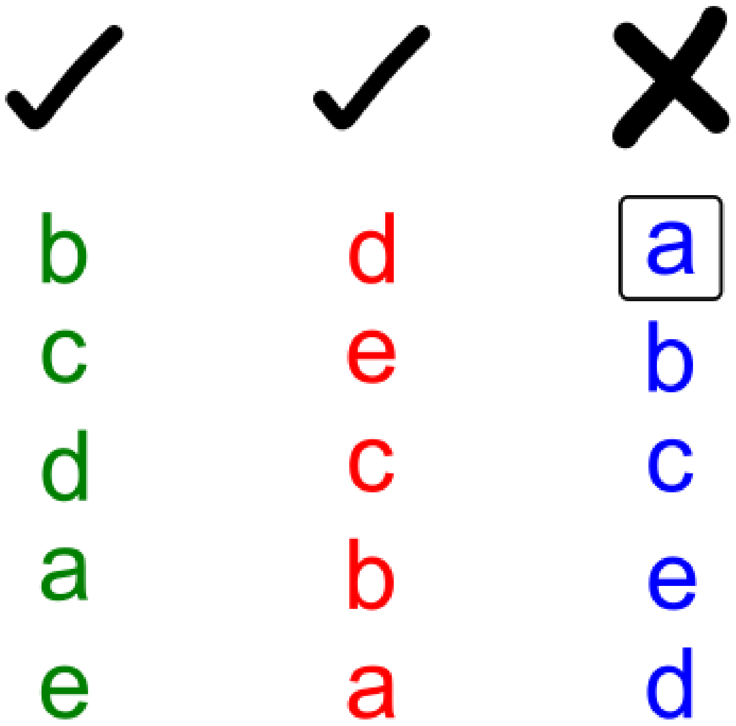

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

For this, let us discuss an equivalent definition of single-peaked preferences.

Contiguous Segments Property

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A preference profile satisfies contiguous segments property w.r.t. <
if, for each vote and for every k, the set of top-k candidates
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a b c d e

b
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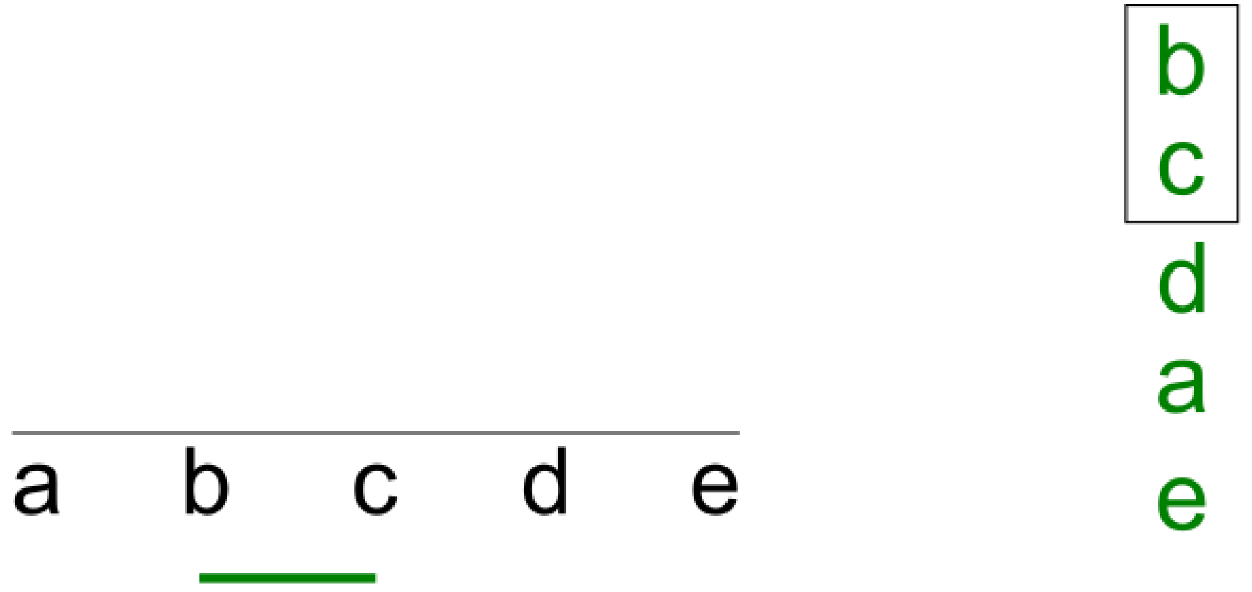

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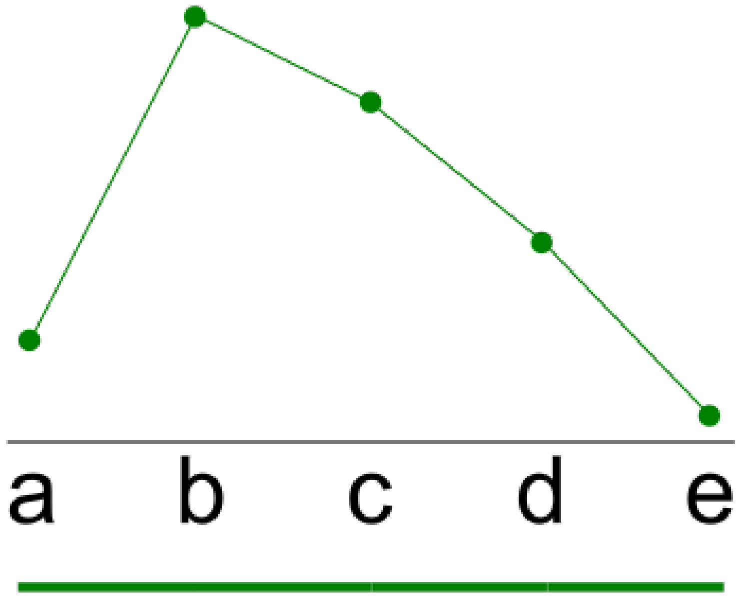

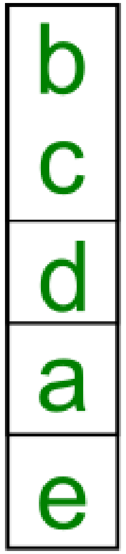

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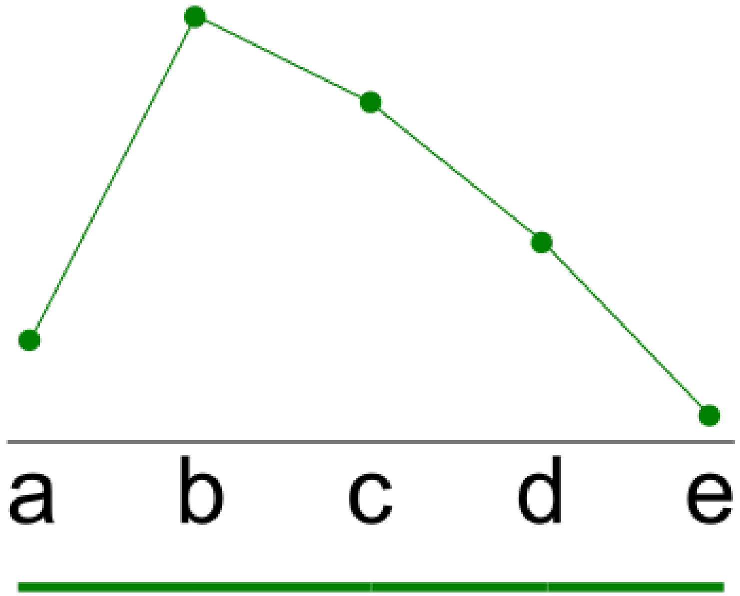

b	d
c	e
d	c
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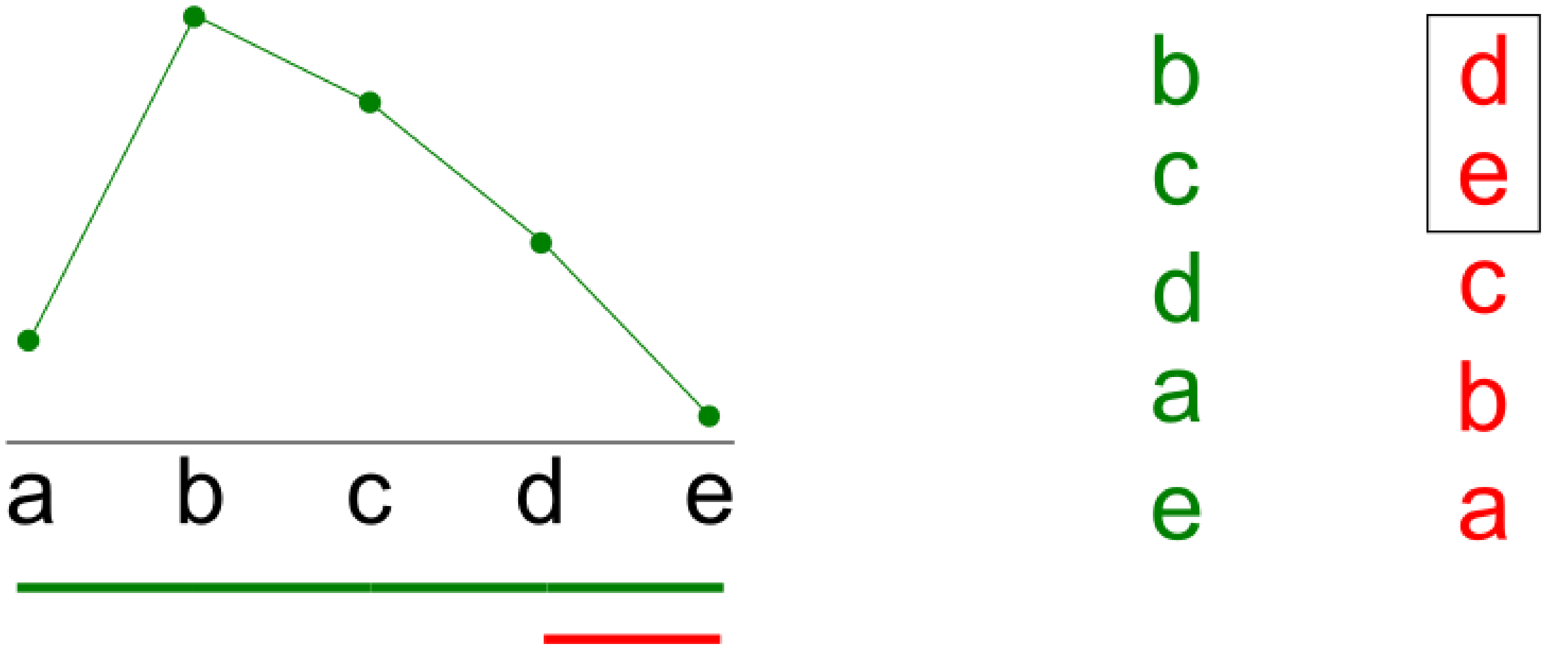

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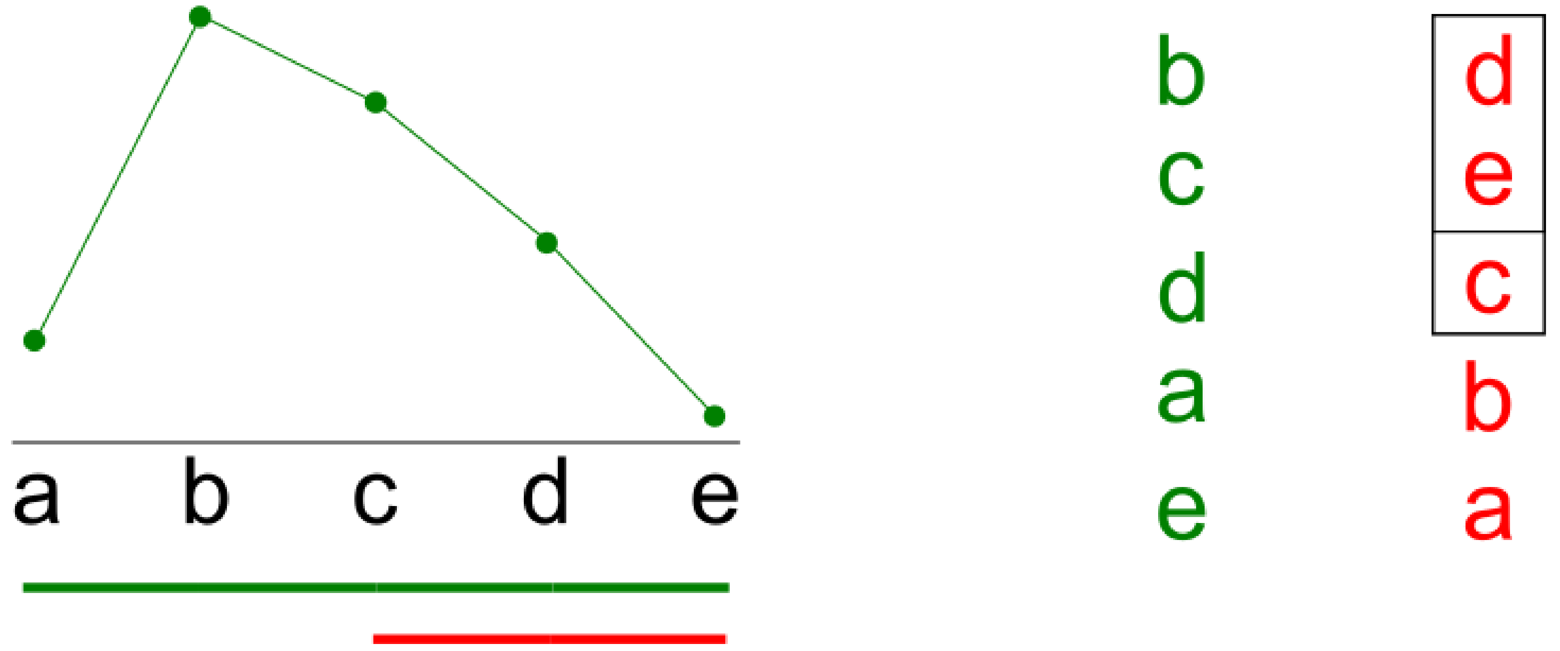

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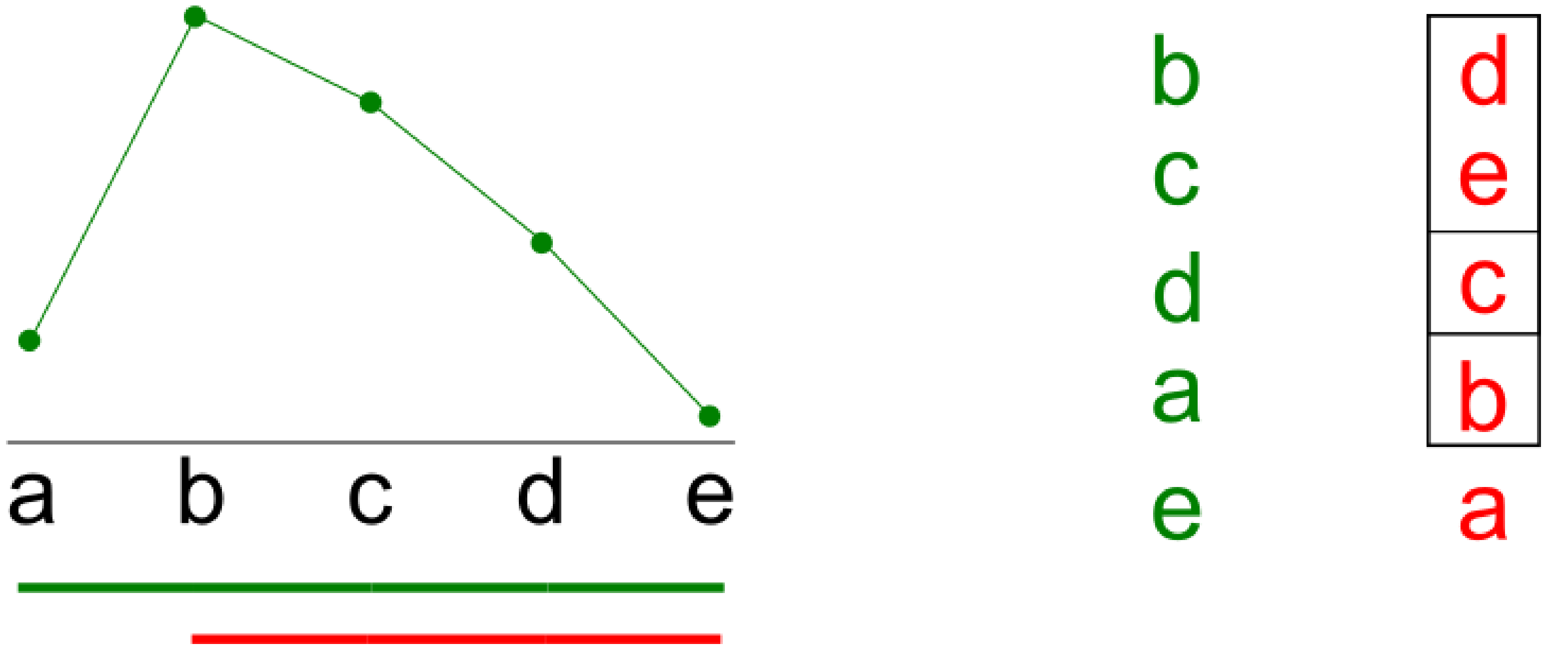

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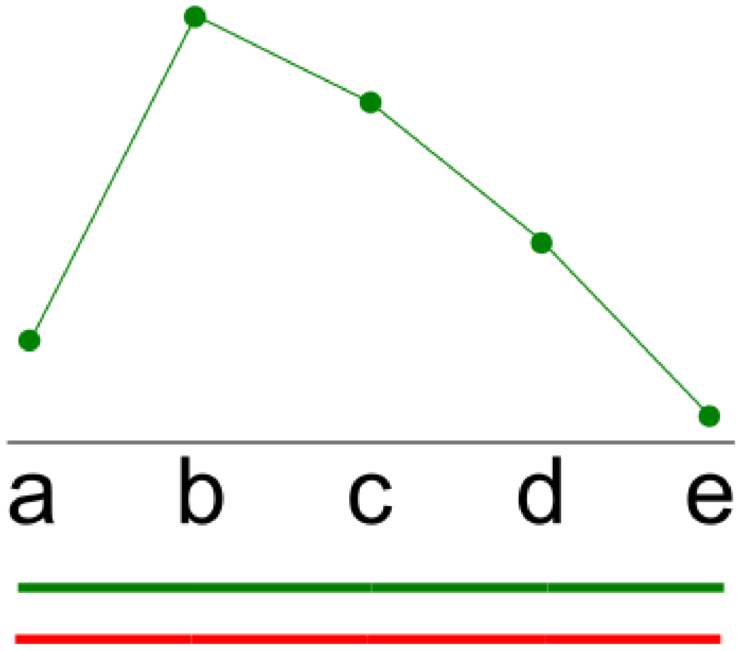

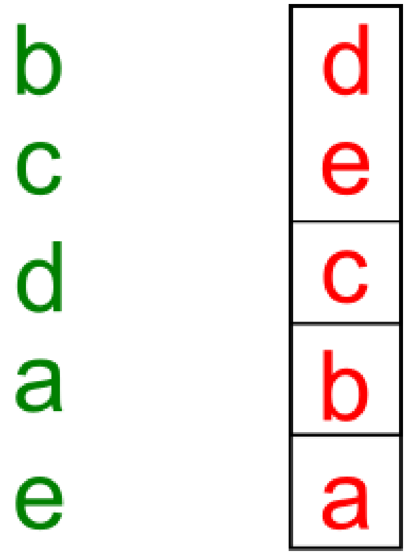

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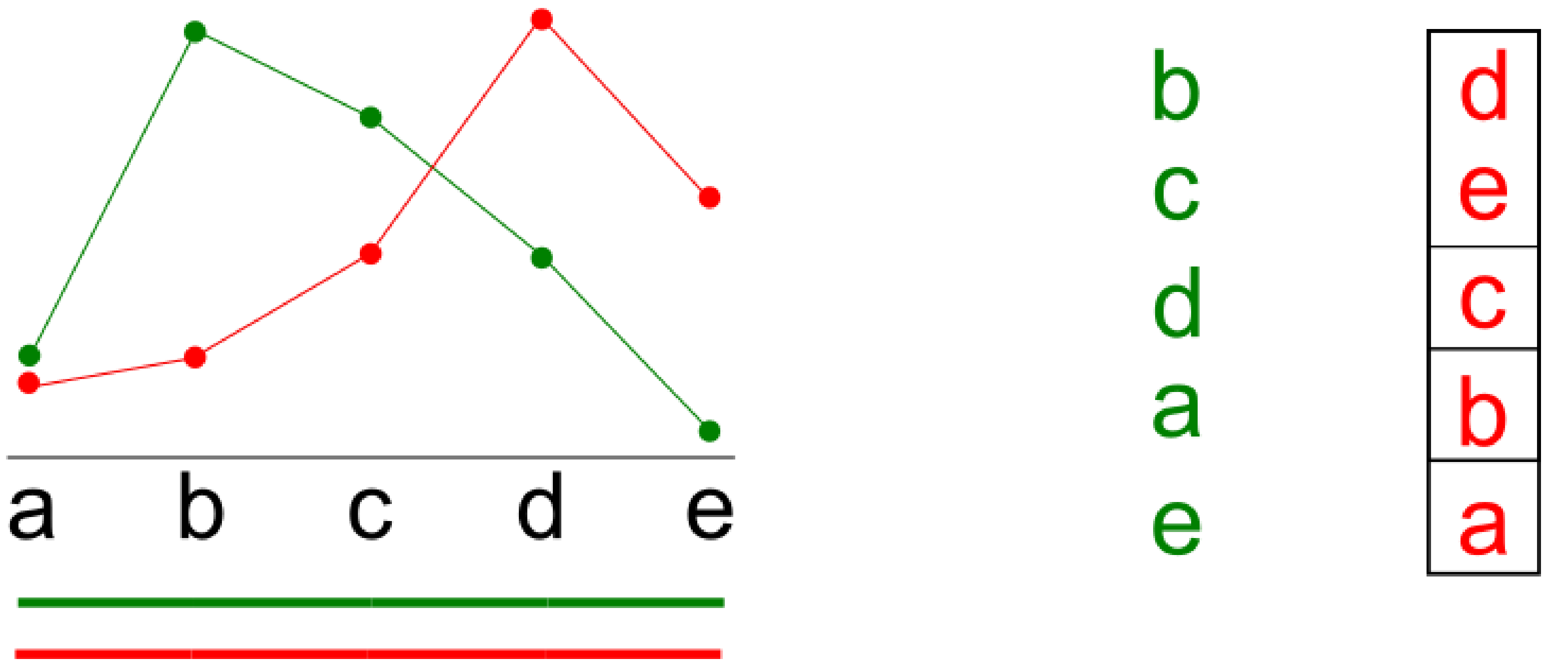

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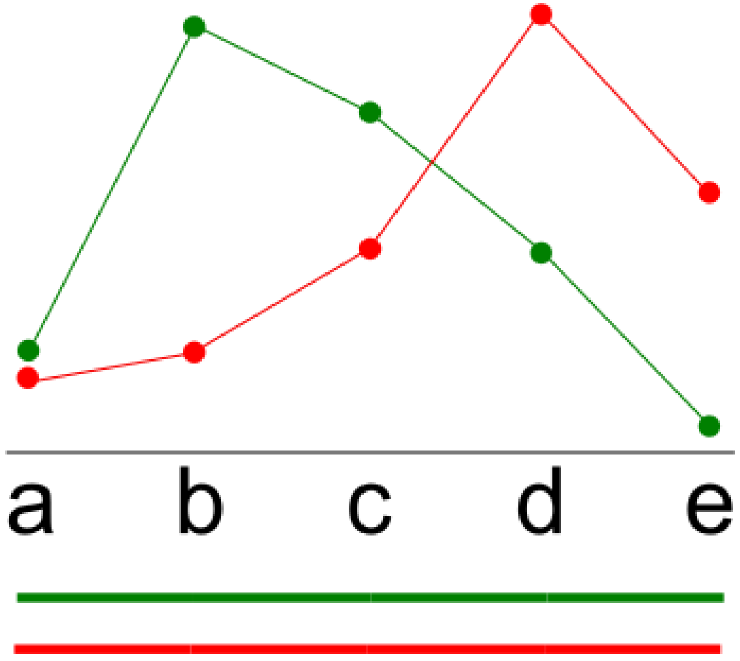

b	d	a
c	e	b
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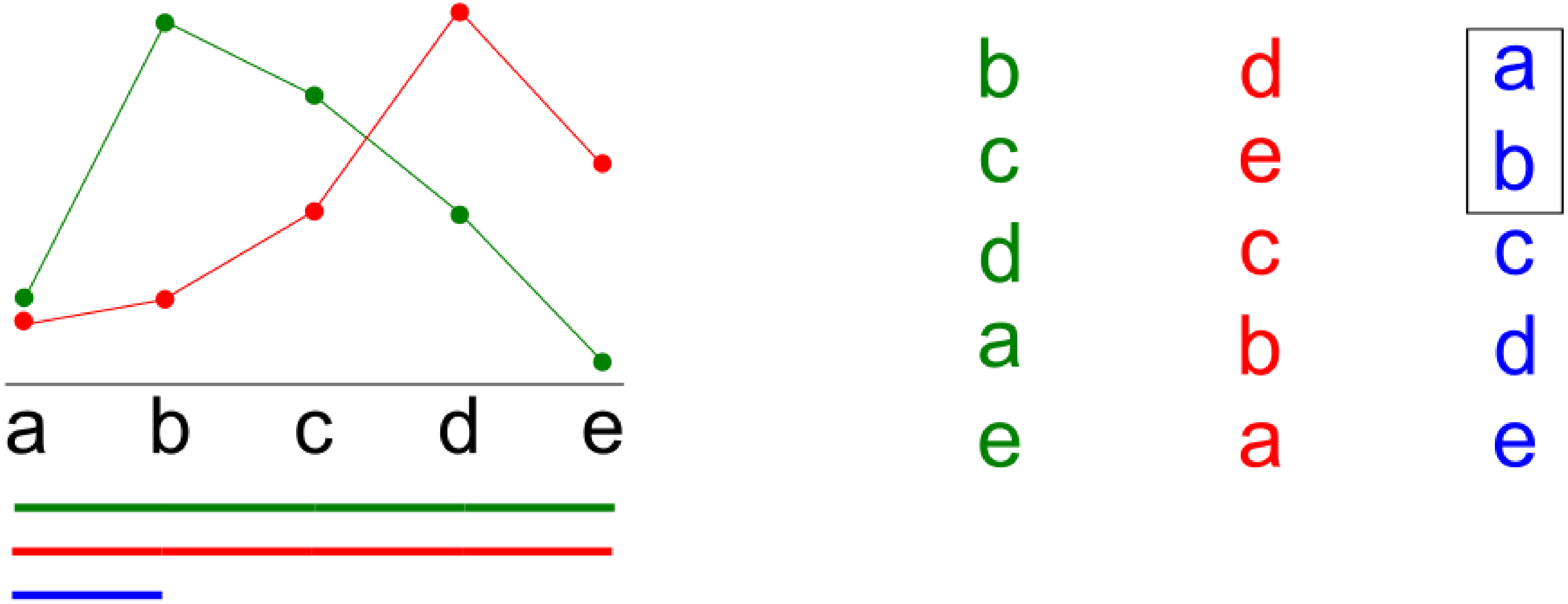

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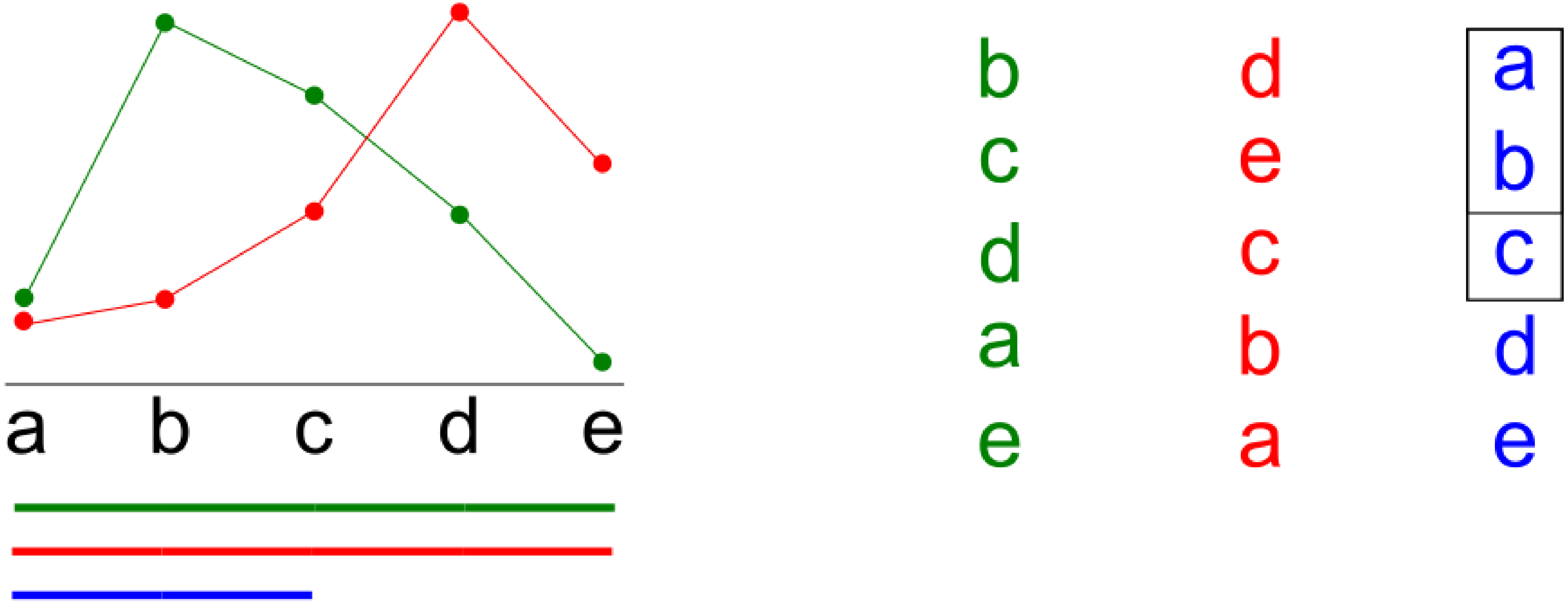

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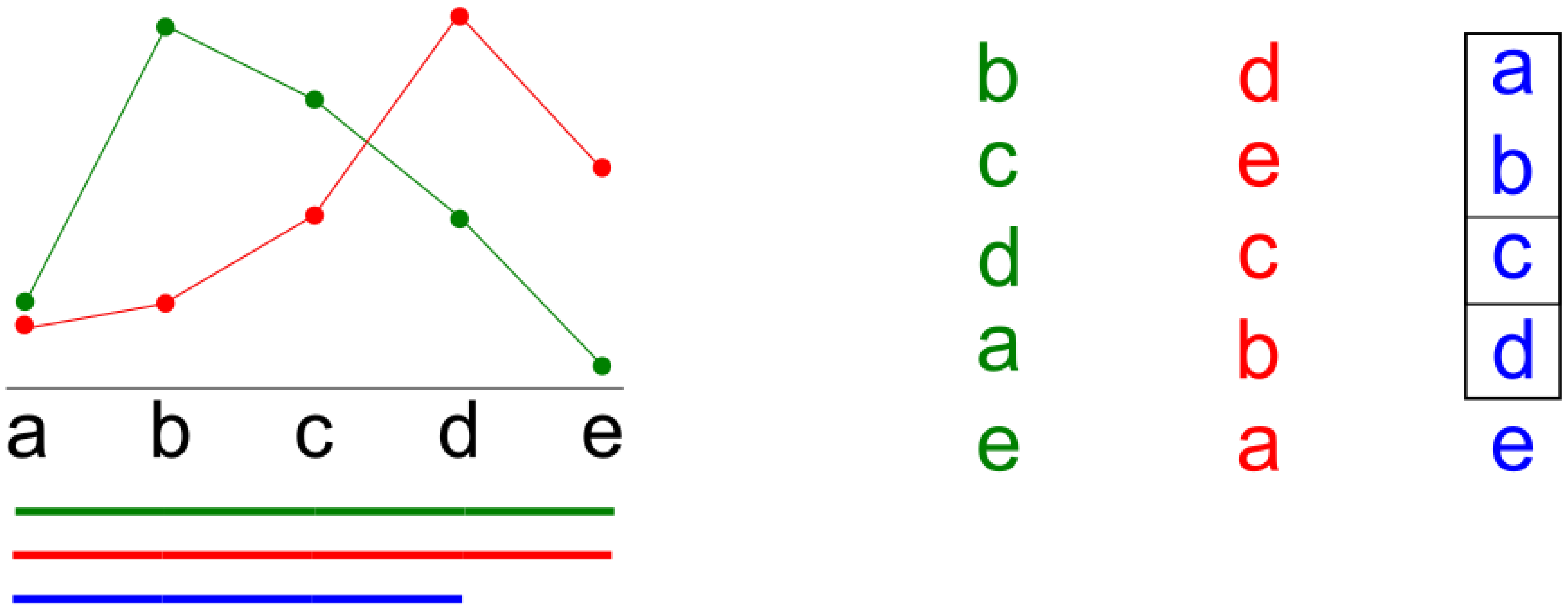

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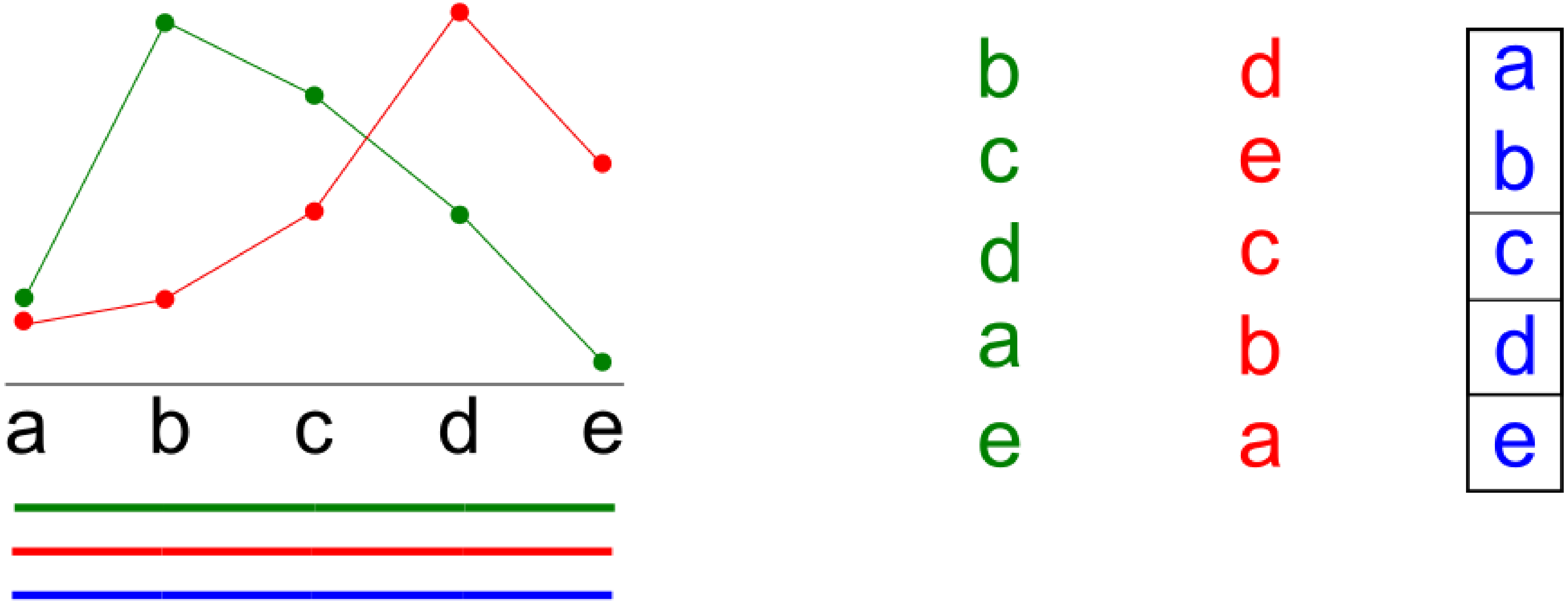

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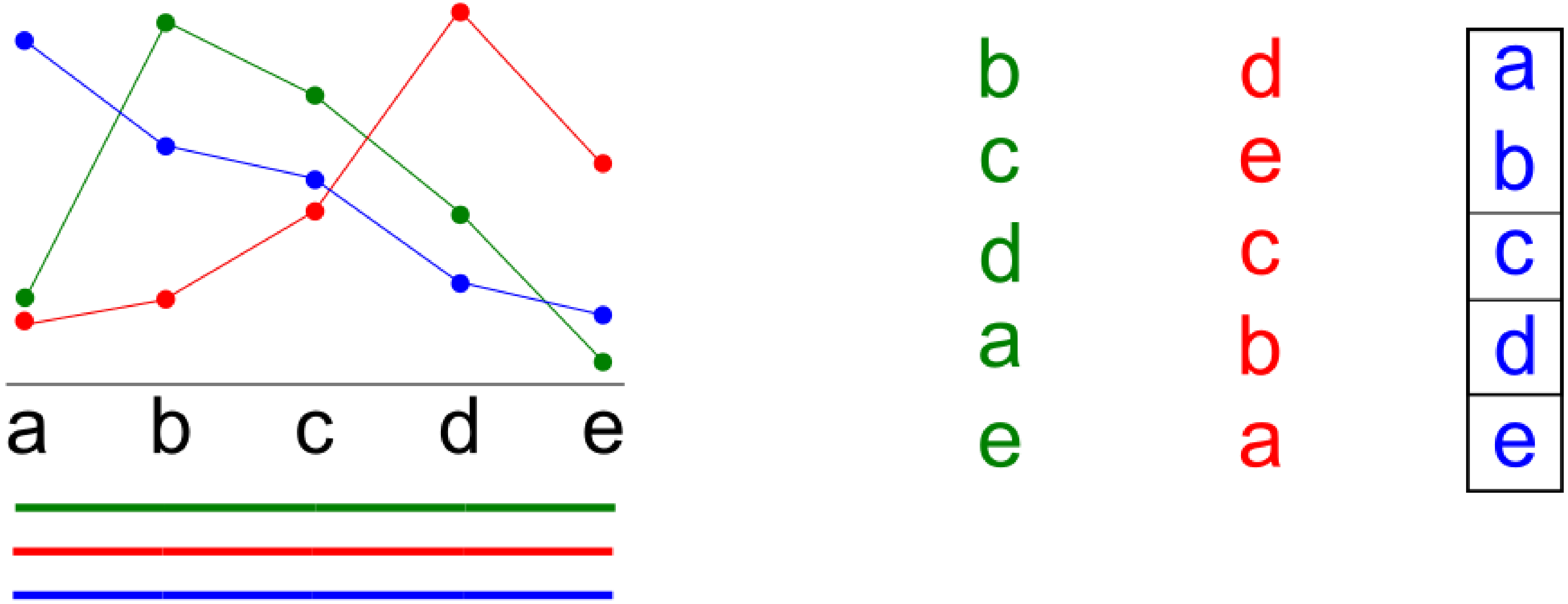

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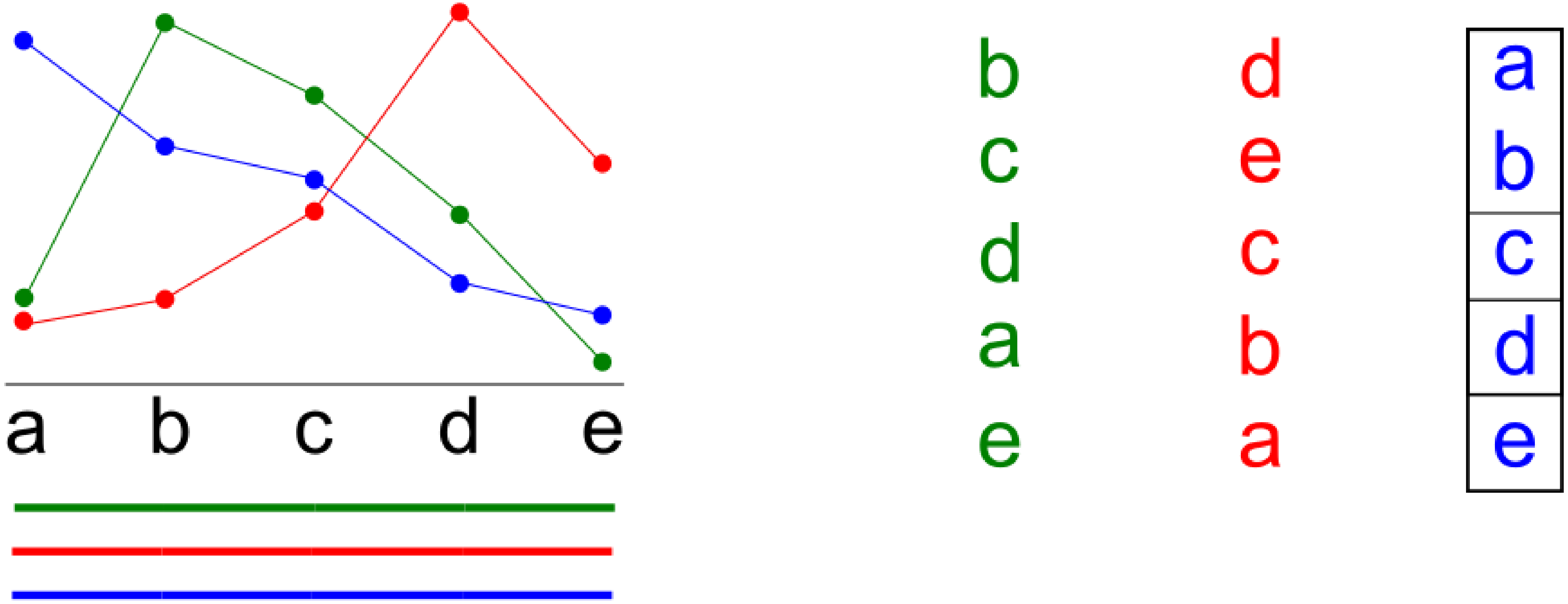

Contiguous segments (w.r.t. <) ⇔ Single-peaked (w.r.t. <)

Contiguous Segments ⇒ Single-Peaked

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Suppose the contiguous segments property holds w.r.t. the axis $<$.

Contiguous Segments ⇒ Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis <.

Suppose, for contradiction, that for some pair of candidates x,y and some voter v, top(v) < x < y but v prefers y over x.

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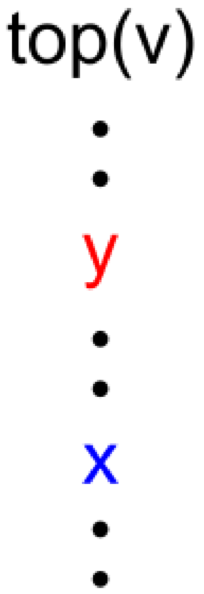

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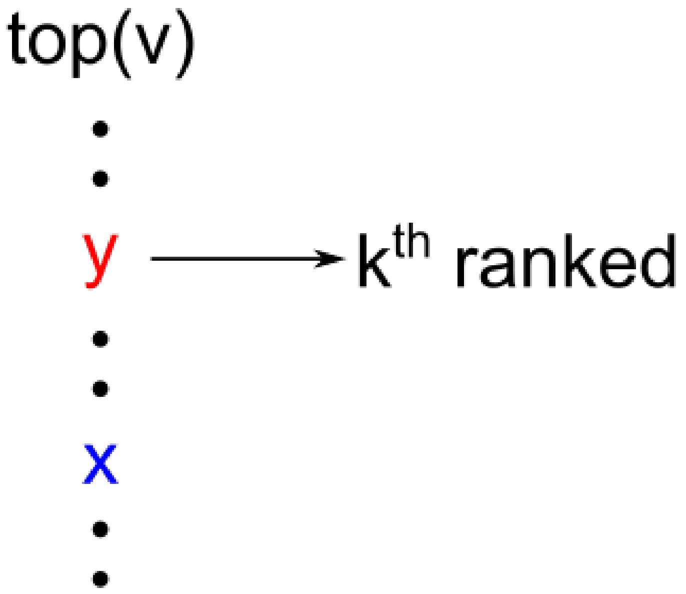

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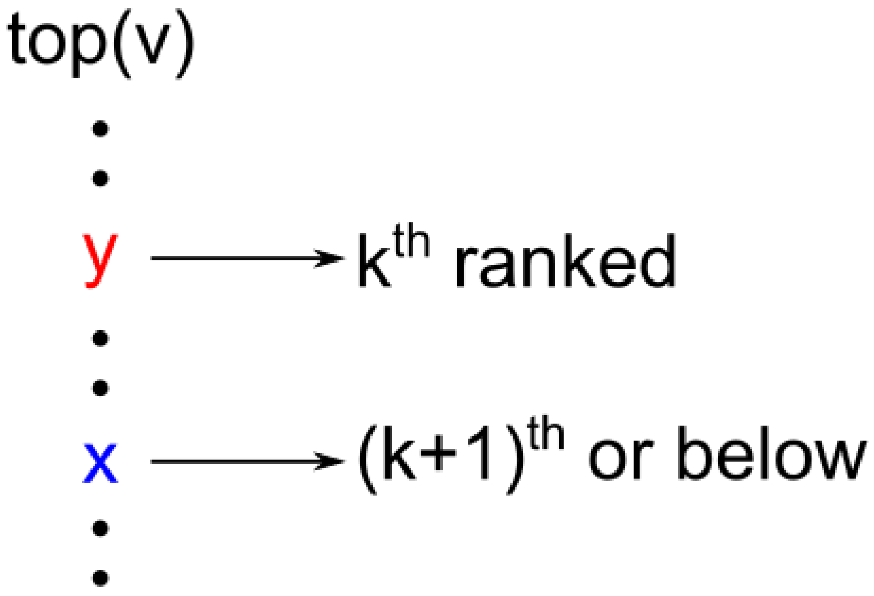

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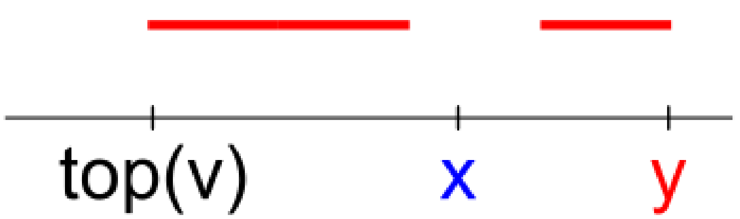

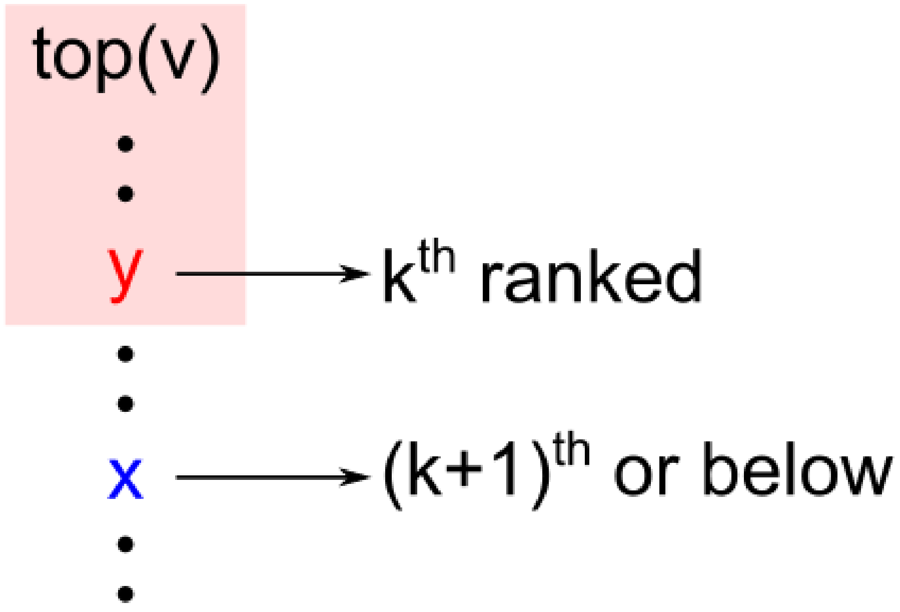

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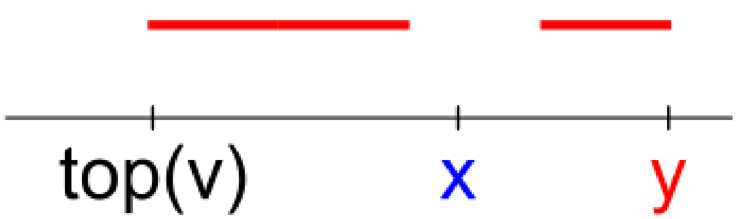
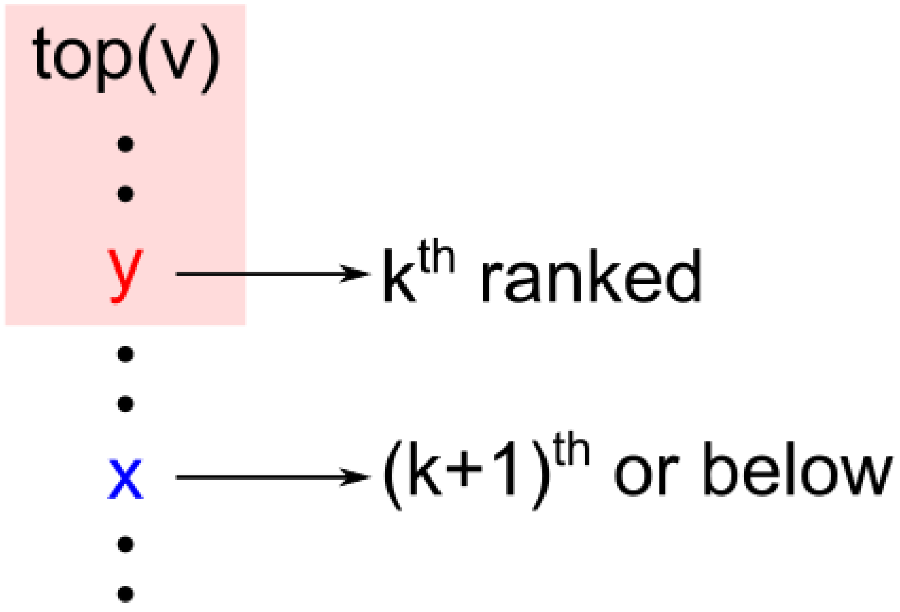


x disconnects the top-k segment



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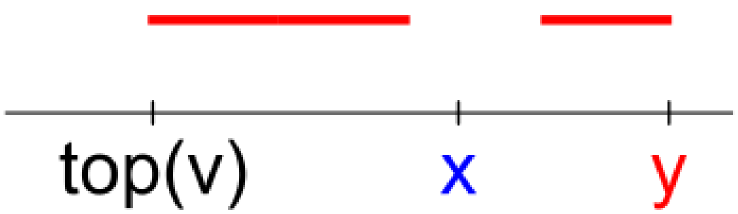

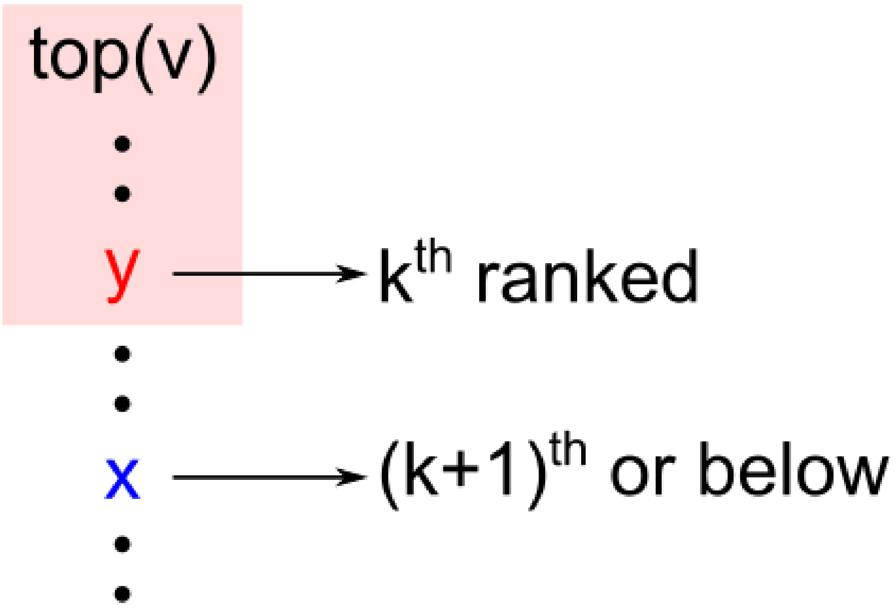

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Then, for any $x < y < z$, a voter v will never rank y below x and z.
("**no valleys**" property)

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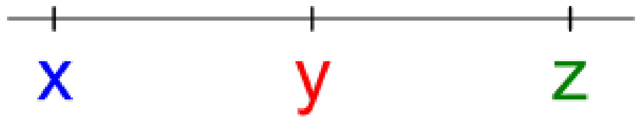

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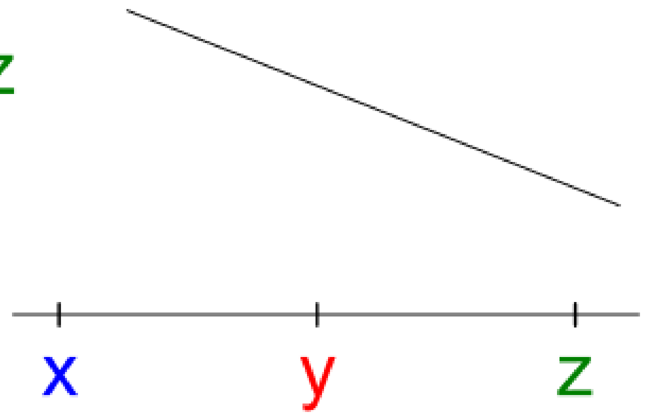

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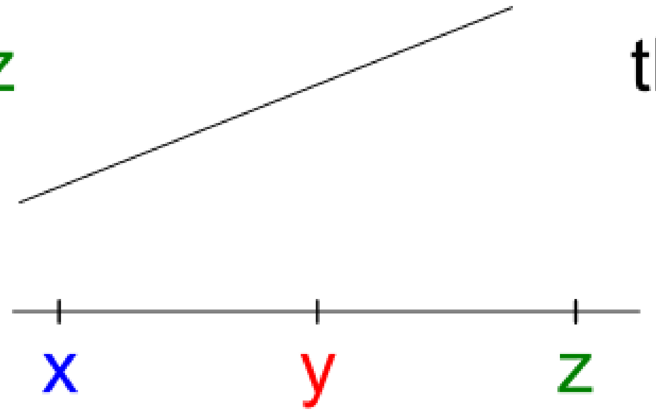

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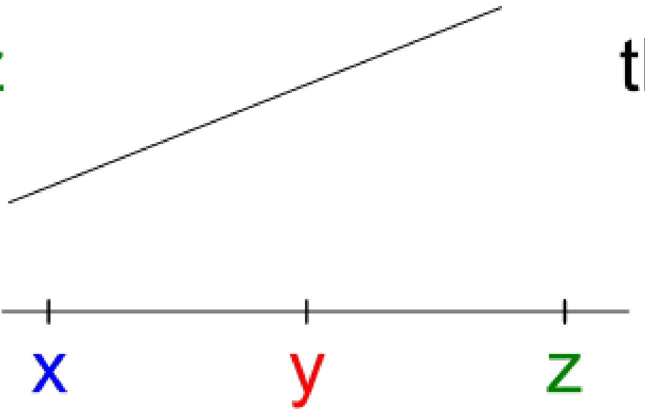

Thus, single peaked (w.r.t. $<$) ⇒ no valleys (w.r.t. $<$).

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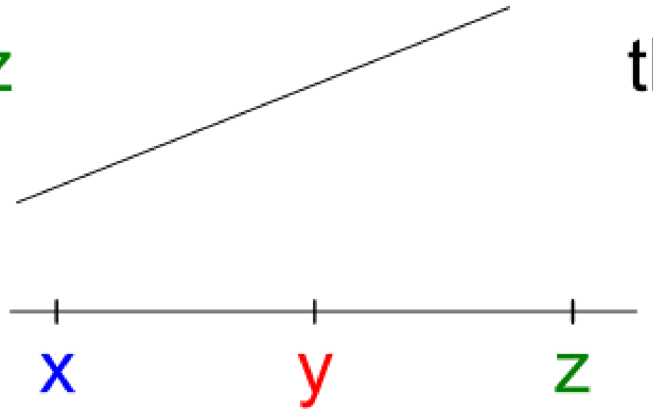

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Let us now show that no valleys (w.r.t. <) ⇒ contiguous segments (w.r.t. <).

Single-Peaked $\Rightarrow$ Contiguous Segments

Single-Peaked ⇒ Contiguous Segments

Suppose, for contradiction, that for some voter v, the set of top k candidates is not contiguous (w.r.t. <).

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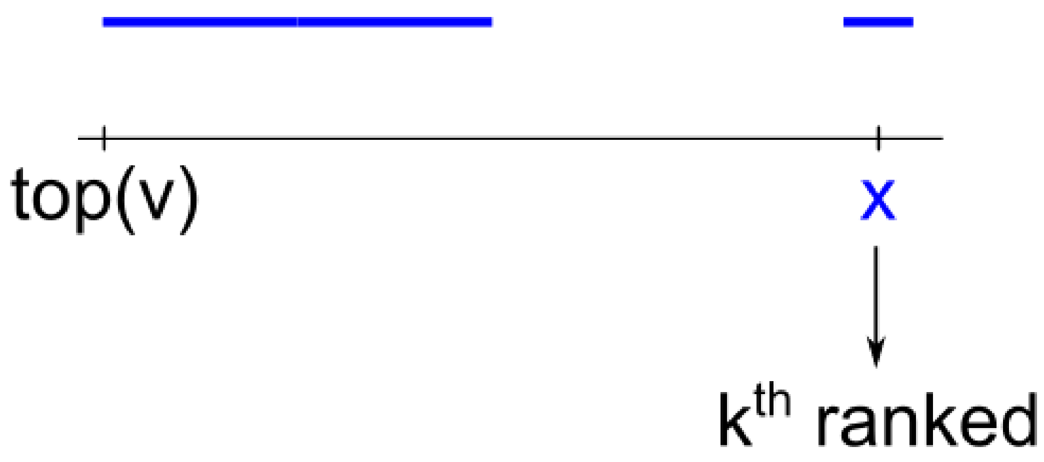

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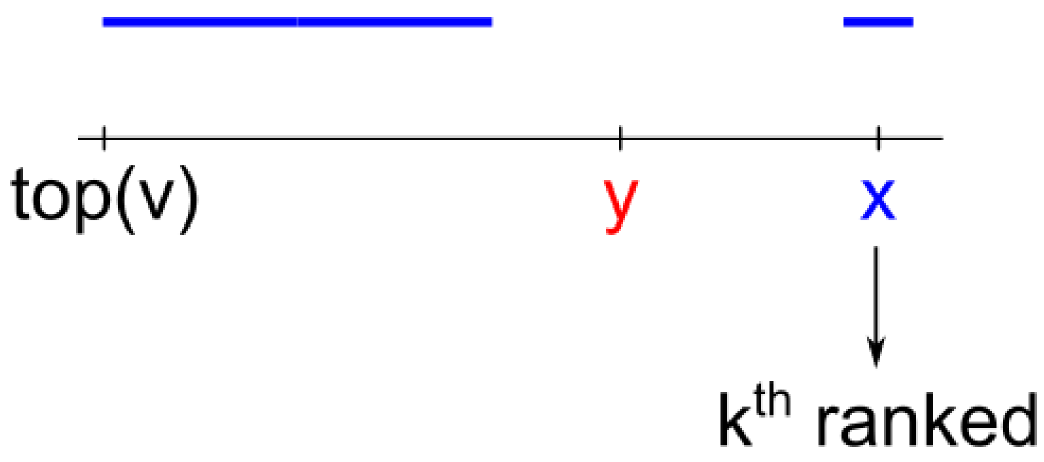

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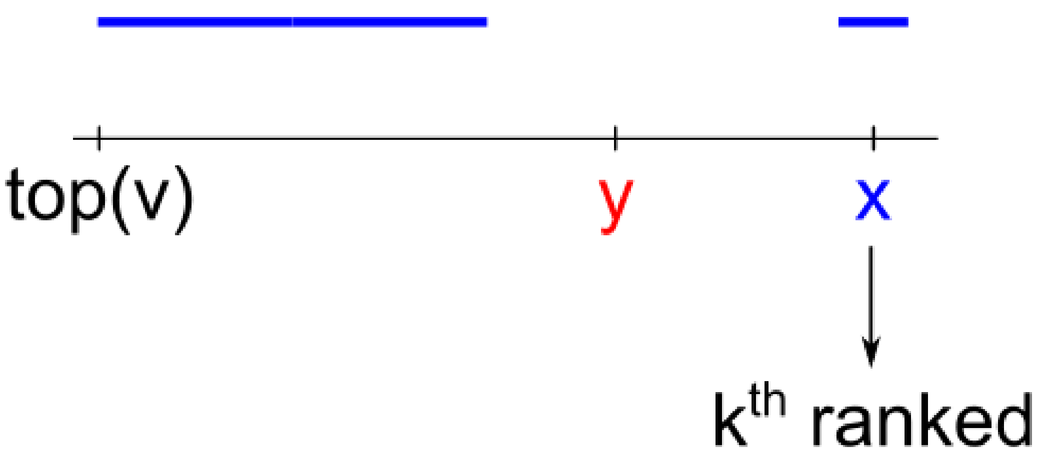

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Then, top(v), y and x constitute a valley.

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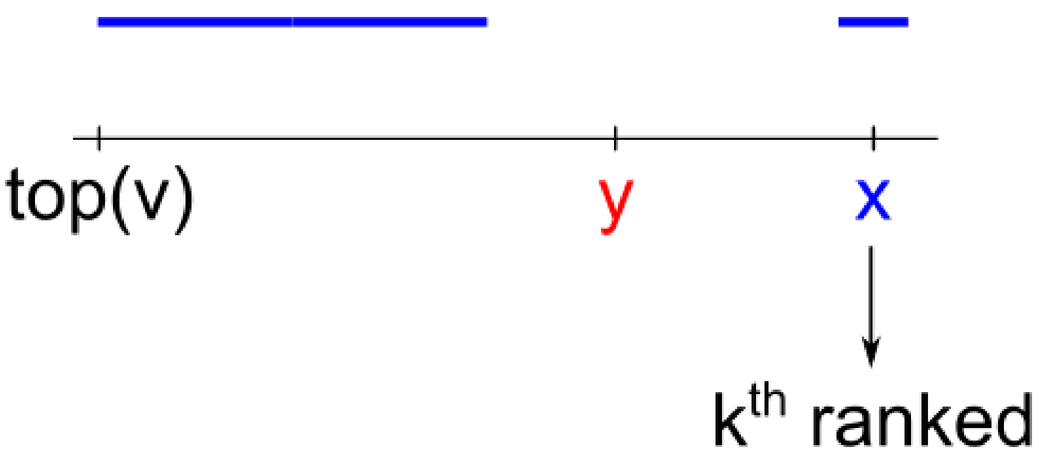

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Recognizing Single-Peaked Prefs w.r.t. some axis

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We will use the contiguous segments property
to design an algorithm
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But, before that, another digression.

Consecutive 1's Problem

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Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

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[Booth and Leuker, JCSS 1976]

The consecutive 1's problem can be solved in polynomial time.

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
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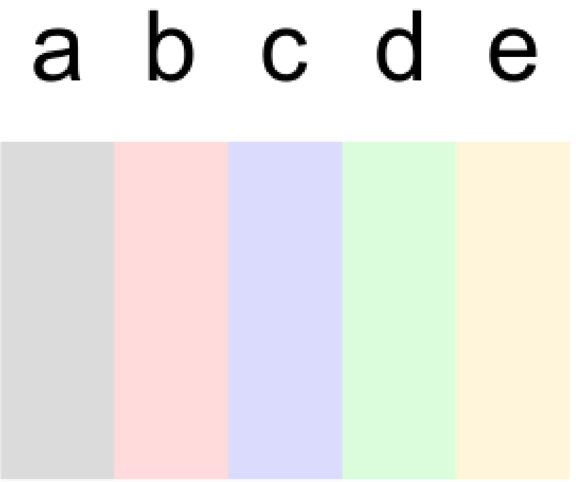

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a	d	b
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e	b	e

a	b	c	d	e
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Recognizing Single-Peaked Prefs w.r.t. some axis

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d	c	c
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c	e	a
d	c	c
b	a	d
e	b	e

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0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1

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c	e	a
d	c	c
b	a	d
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a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

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c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
its prefix matrix satisfies consecutive 1's property.

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
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Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

b	a	c	d	e
0	1	1	0	0
0	1	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
0	1	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if
its prefix matrix satisfies consecutive 1's property.

Do single-peaked preferences occur in real world?

https://www.preflib.org/

PrefLib: A Library for Preferences

Data ▾ | Elections | Matchings | Ratings | Search | About

PrefLib

PrefLib is a reference library of preference data and links assembled by *Nicholas Mattei*, *Toby Walsh* and lately *Simon Rey*. This site and library is proudly supported by the *Algorithmic Decision Theory group* at *Data61* and the *The COMSOC Group at the University of Amsterdam*.

We want to provide a comprehensive resource for the multiple research communities that deal with preferences, including computational social choice, recommender systems, data mining, machine learning, and combinatorial optimization, to name just a few.

For more information on PrefLib and some helpful tips on using it, please see Nick's Tutorial *Slides and Code* from *EXPLORE 2014*. Check out the *data type* page to learn more about the kind of data we provide.

Please see the *about* page for information about the site, contacting us, and our citation policy. We rely on the support of the community in order to grow the usefulness of this site. To contribute, please contact *Nicholas Mattei* at: nsmattei{at}gmail or *Simon Rey* at: s.j.rey{at}uva{dot}nl.

In Brief

We currently host:

- 11 types of data
- 38 datasets
- 3668 data files
- More than 3.37 Gb. of data

Other Links

Here are some links that you might find relevant as well.

- *DEMOCRATIX:* A Declarative Approach to Winner Determination

CHAPTER 15

A PrefLib.org Retrospective: Lessons Learned and New Directions

Nicholas Mattei and Toby Walsh

Trends in Computational Social Choice

Realism. Perhaps the key motivating factor behind assembling PrefLib was a desire to have realistic data. Many of the models studied in classical social choice seem to be chosen because they *seem* reasonable or were explicitly chosen for mathematical expediency. Perhaps nothing is more of an exemplar here than the fact that out of over 300 profiles containing strict, complete preference relations, absolutely none are single-peaked, a common profile restriction that has been called "natural" or "well-motivated" numerous times since its introduction by Black (1948). Collecting data has helped us to quantify what is reasonable. Now we have to start using the data.

Next Time

Rank Aggregation

Quiz

Quiz

Given an axis with n candidates,
what is the maximum no. of distinct single-peaked votes with respect to that axis?

References

- Single-peaked preferences in theory:

  Duncan Black
  "*On the Rationale of Group Decision-Making*"
  Journal of Political Economy, Feb 1948, 56(1), pg 23-34
  https://www.journals.uchicago.edu/doi/10.1086/256633

- Single-peaked preferences in the real-world:

  Nicholas Mattei and Toby Walsh
  "*A PREFLIB.ORG Retrospective: Lessons Learned and New Directions*"
  Chapter 15 in Trends in Computational Social Choice
  https://research.illc.uva.nl/COST-IC1205/BookDocs/TrendsCOMSOC.pdf

References

- Lecture by Edith Elkind on restricted preference domains:
  https://www.youtube.com/watch?v=q8vc8Znoev0
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- Strategyproof voting rules using “phantom” voters:

  Hervé Moulin
  “*On Strategyproofness and Single-Peakedness*”
  Public Choice, 35(4), 1980, pp. 437-455
  https://www.jstor.org/stable/30023824

References

- Recognizing single-peaked preferences:

  John Bartholdi III and Michael A. Trick
  "*Stable Matching with Preferences Derived from a Psychological Model*"
  Operations Research Letters, 5(4), 1986, pp. 165-169
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- Consecutive 1's Problem:

  Kellogg S. Booth and George S. Lueker
  "*Testing for the Consecutive One Property, Interval Graphs, and Graph Planarity using PQ Tree Algorithms*"
  JCSS, 13(3), 1976, pp. 335-379
  https://www.sciencedirect.com/science/article/pii/S0022000076800451