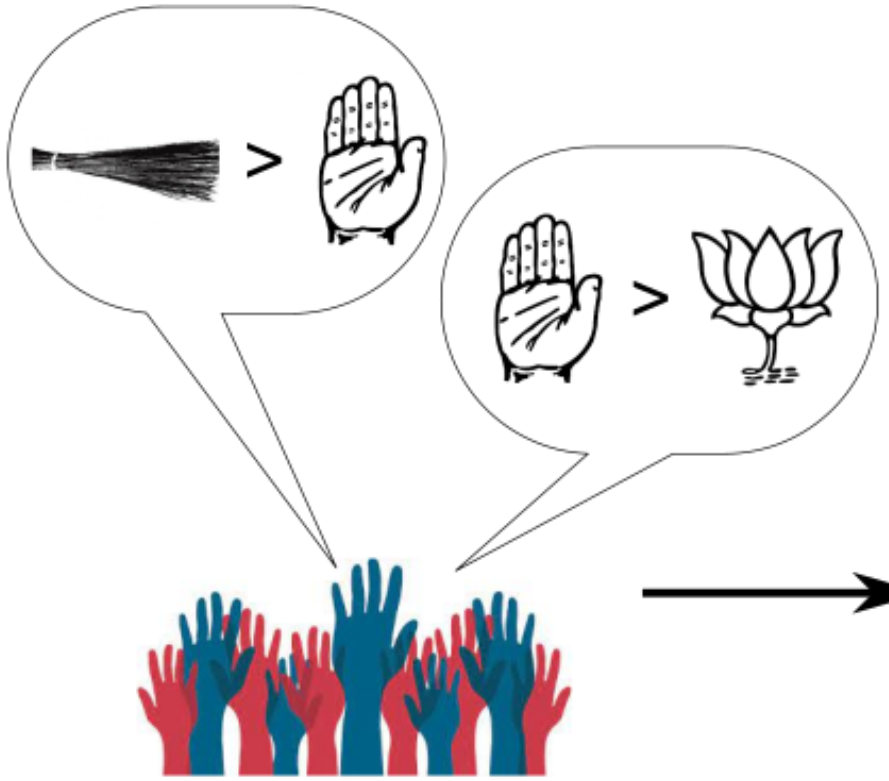
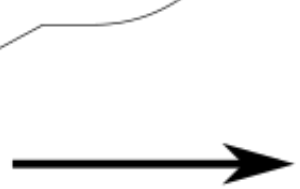
COL866: Special Topics in Algorithms

Lecture 15

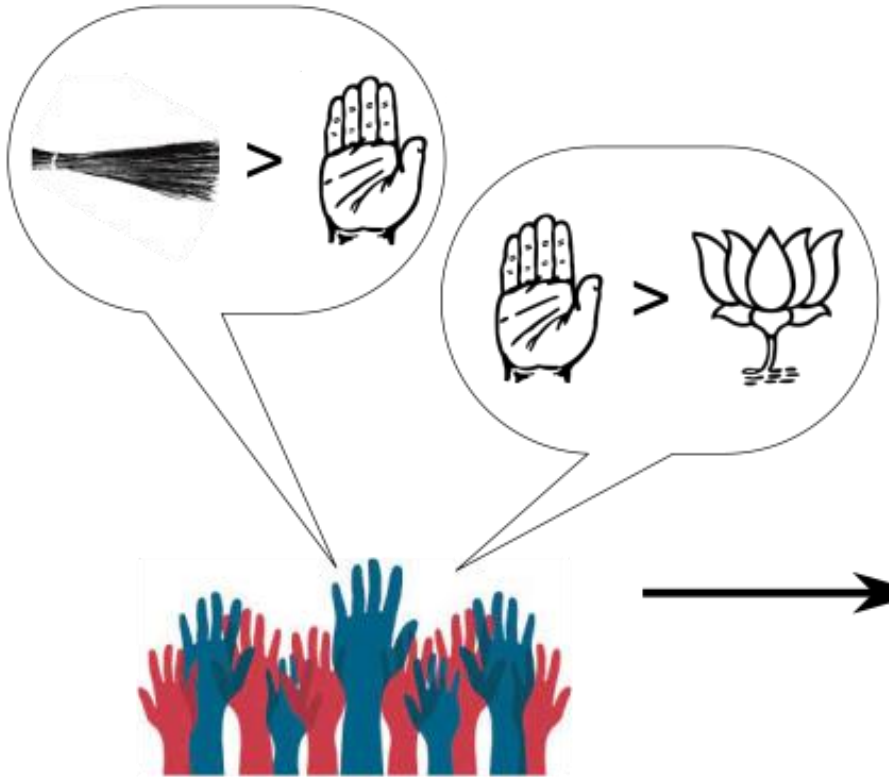
Voting Rules

Oct 10, 2023 | Rohit Vaish





Voting rule



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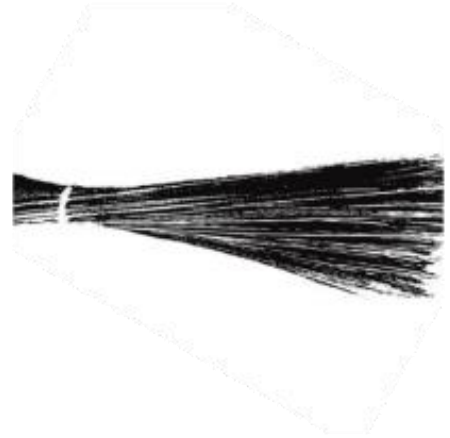

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Voting Setup

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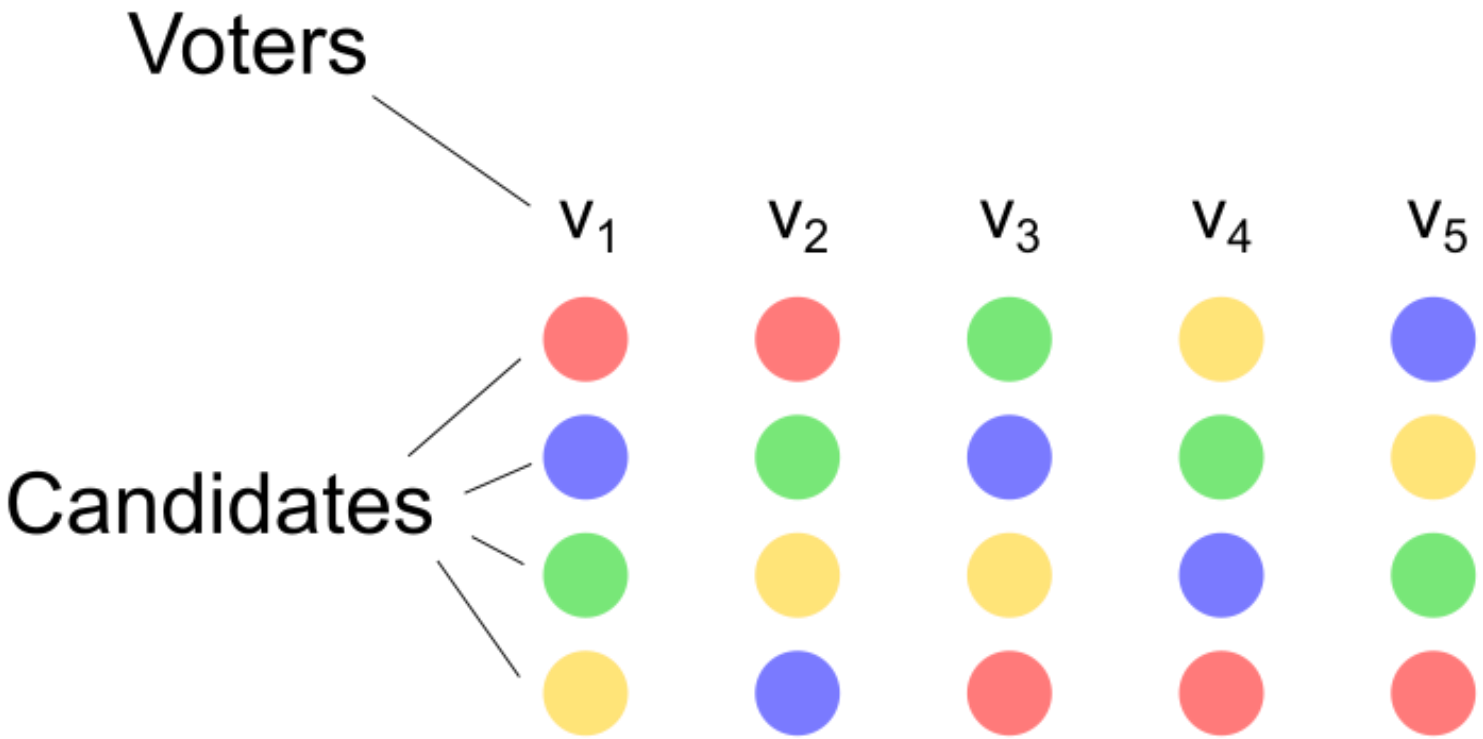

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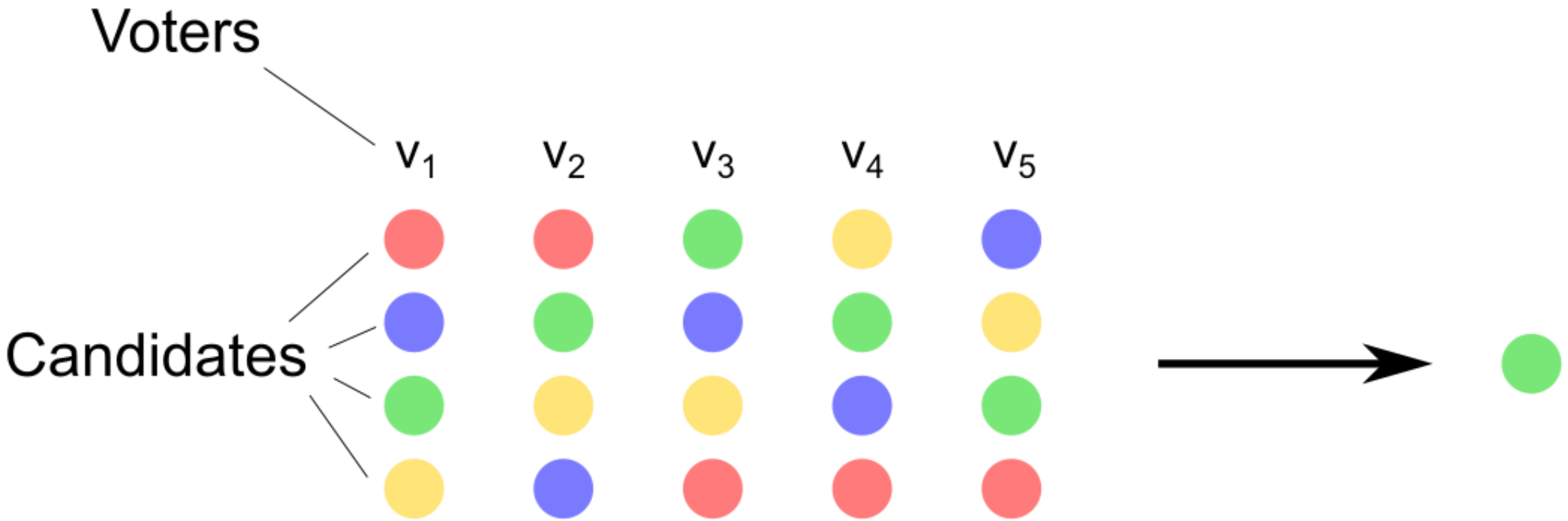
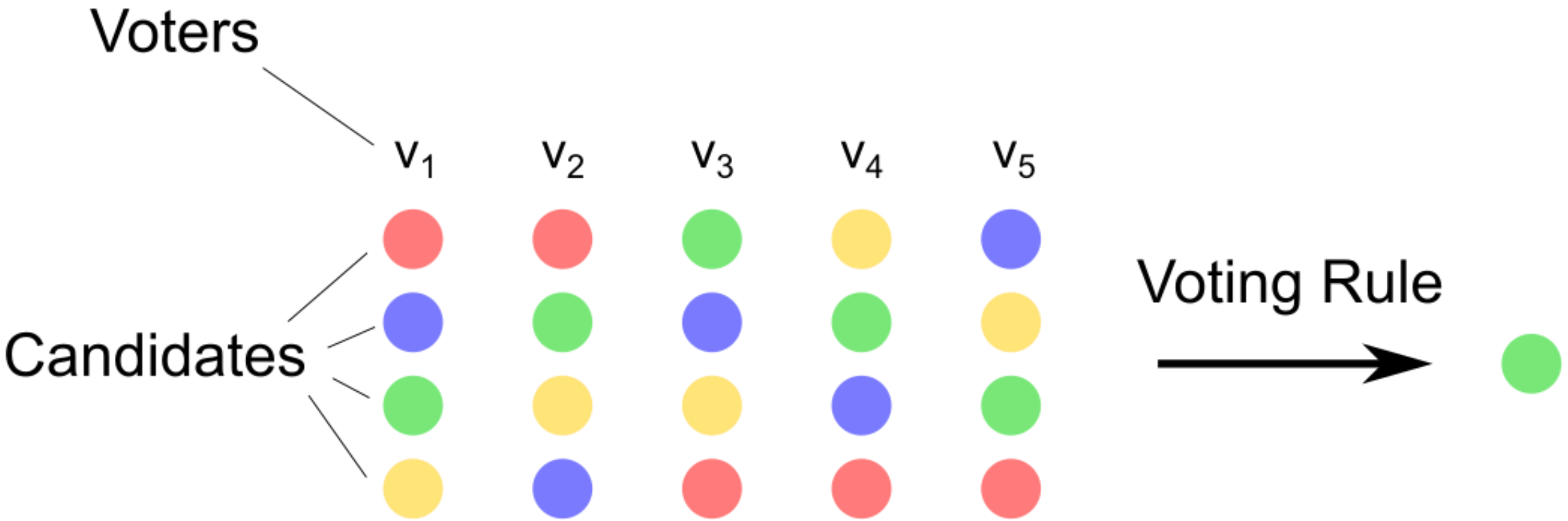


Goal: Pick exactly one winning candidate.

Voting Setup



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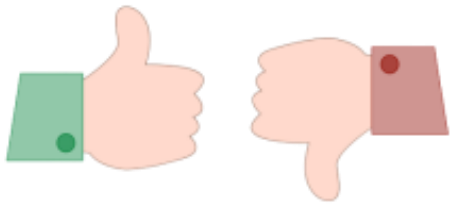

Voting with Two Candidates

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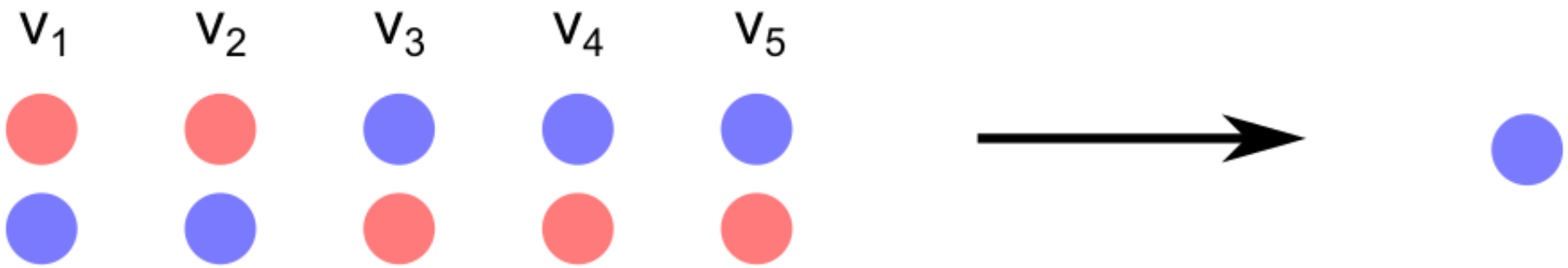
Majority!



Voting with More Than Two Candidates

Voting with More Than Two Candidates

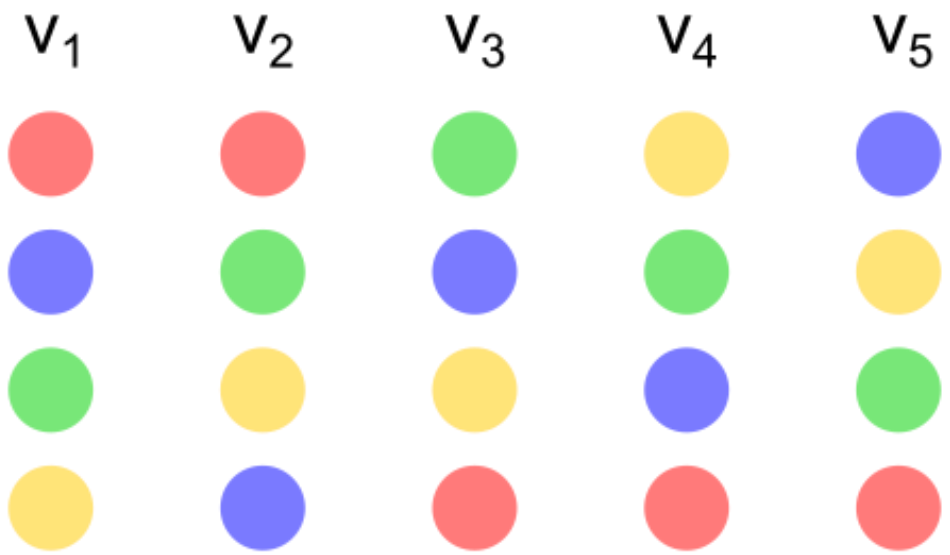
No candidate may have a majority...

1

Plurality

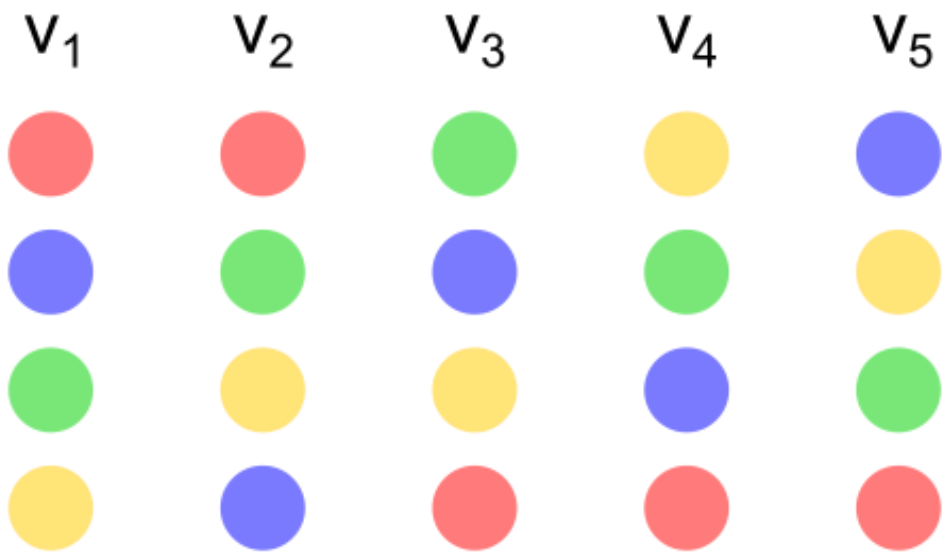
1

Plurality



1

Candidate with the most first-place votes wins



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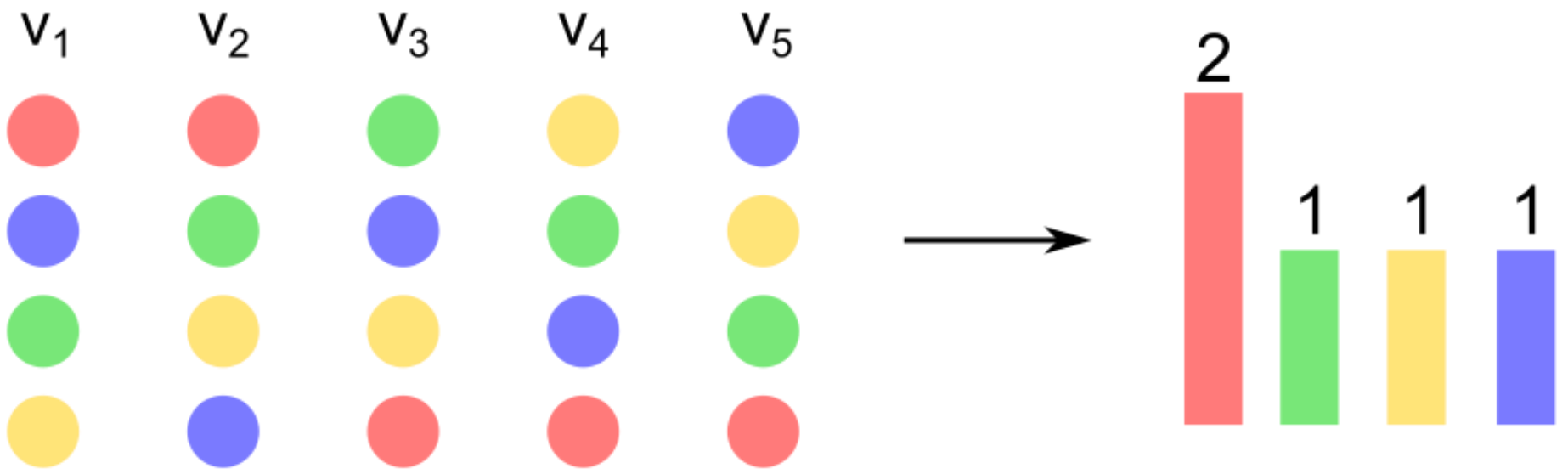


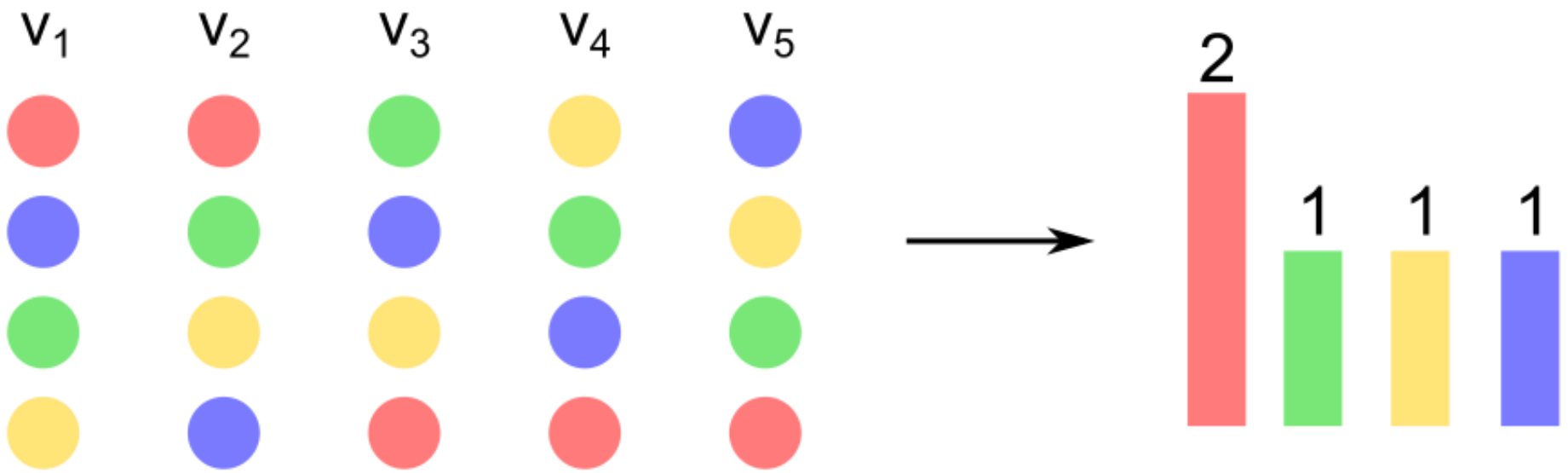
Plurality winner: 

1

Plurality

Candidate with the most first-place votes wins*

*subject to tie-breaking: lexicographic, random, ...



Plurality winner:

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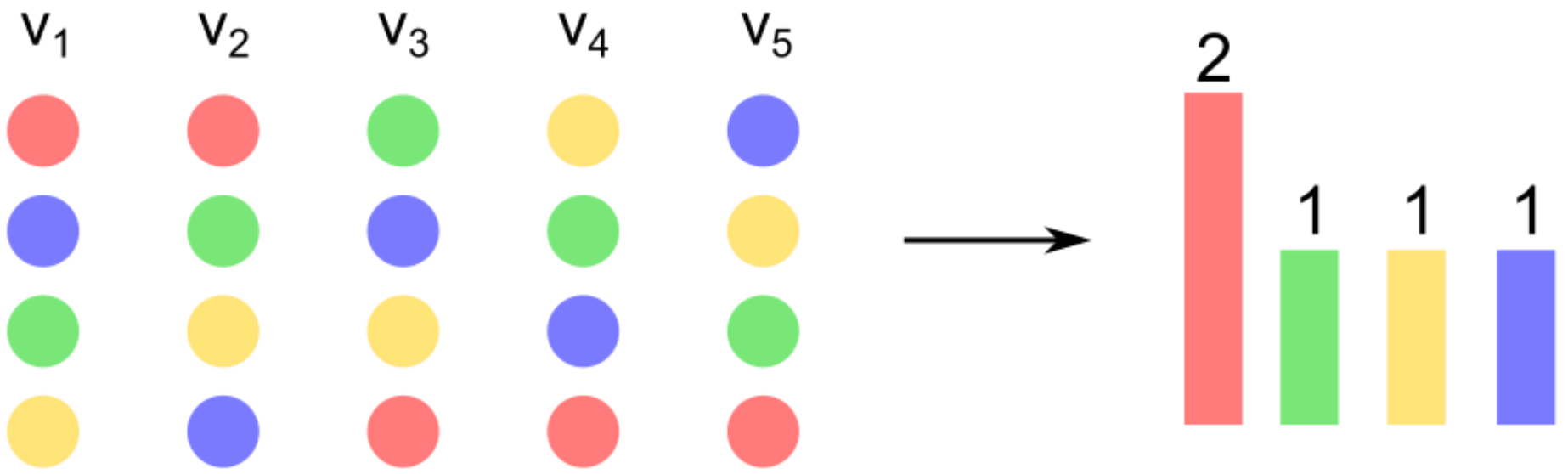
Problem: A majority prefers ● over the Plurality winner.

Plurality



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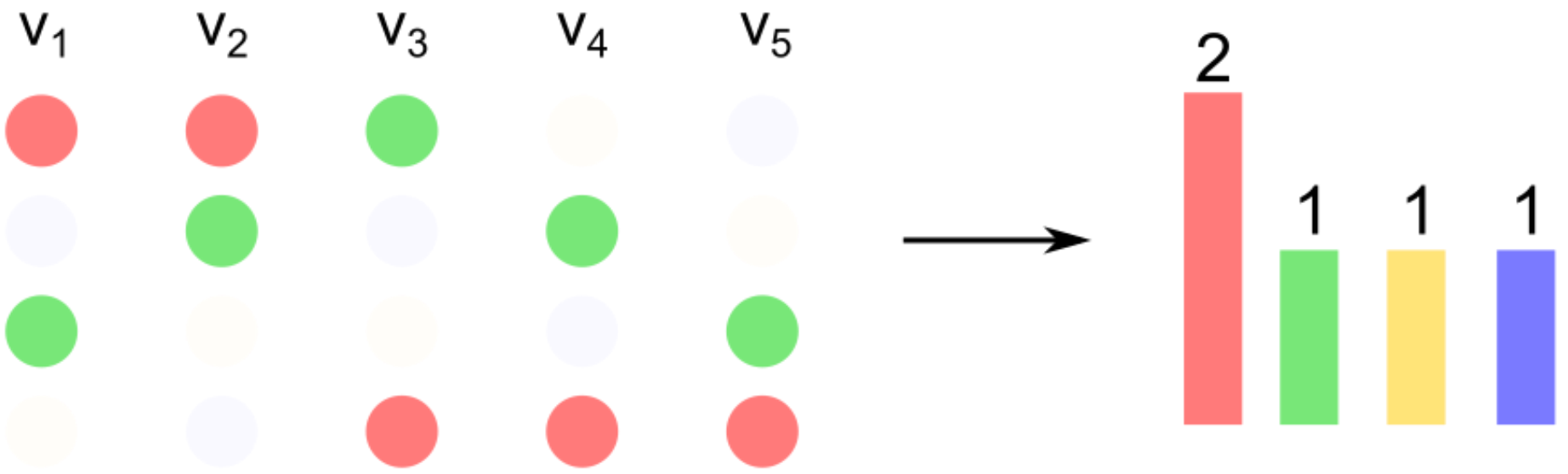

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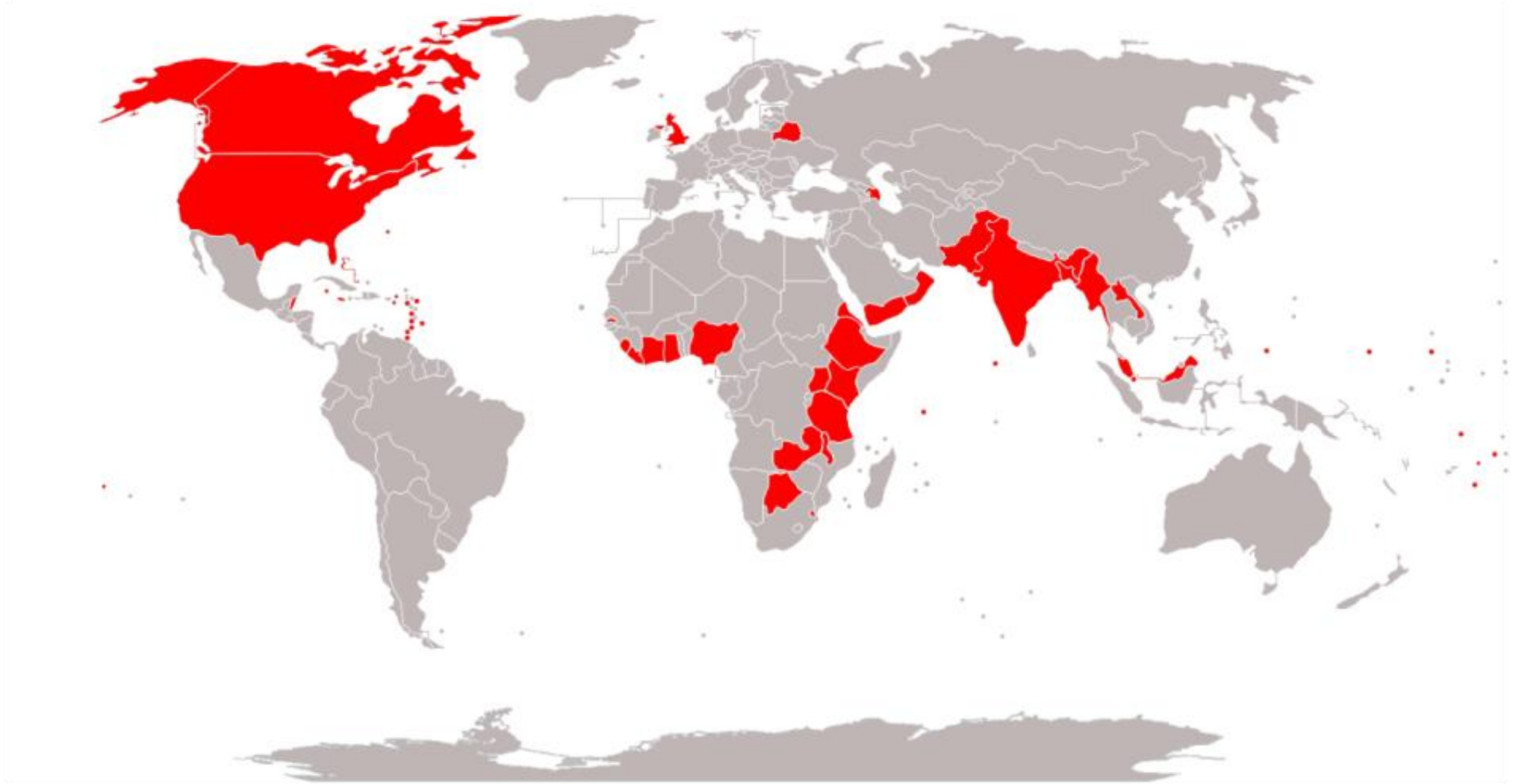

Image Source: Wikipedia article on "Electoral system" (Jan 2022)

2

Borda Count

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Jean-Charles de Borda
(1733-1799)

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Each voter gives its k^{th} ranked candidate m-k points, where m is the number of candidates.

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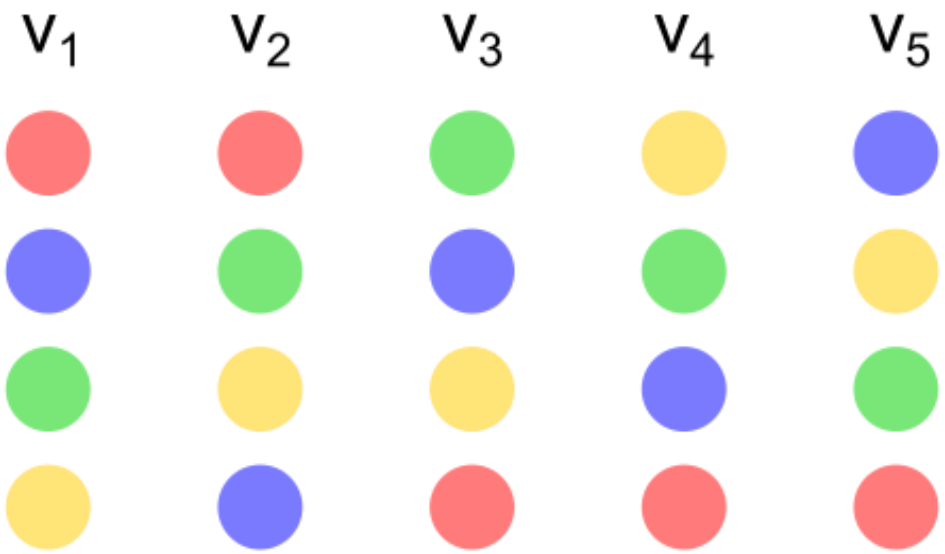

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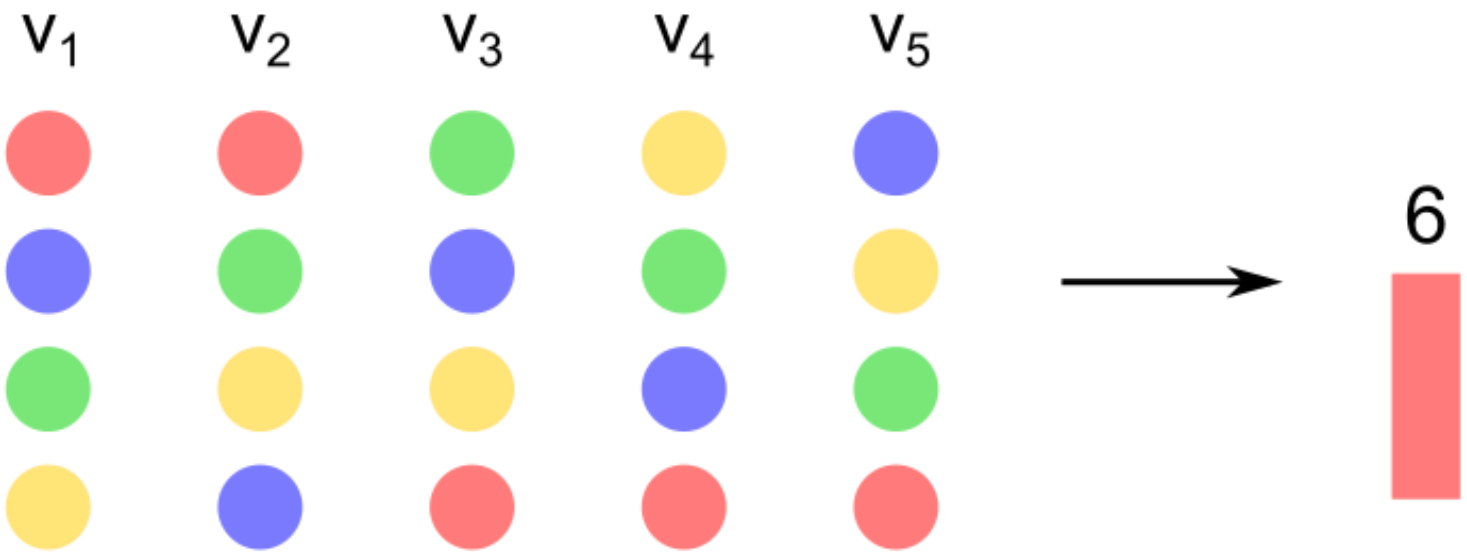

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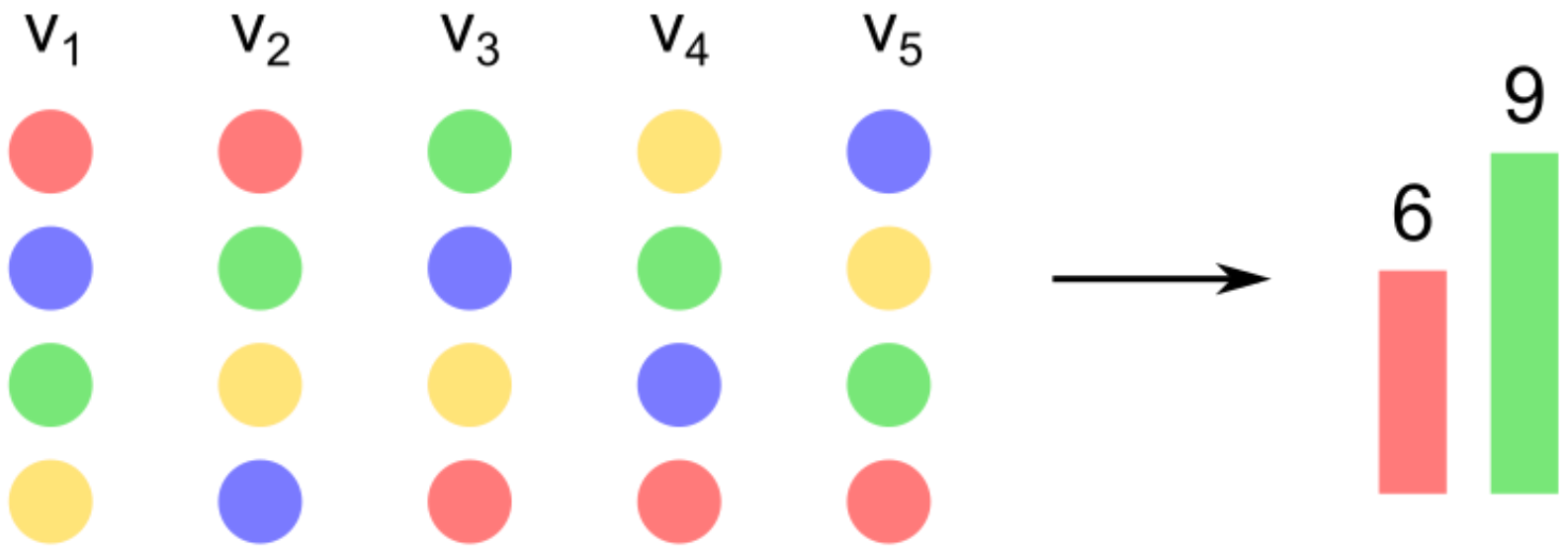

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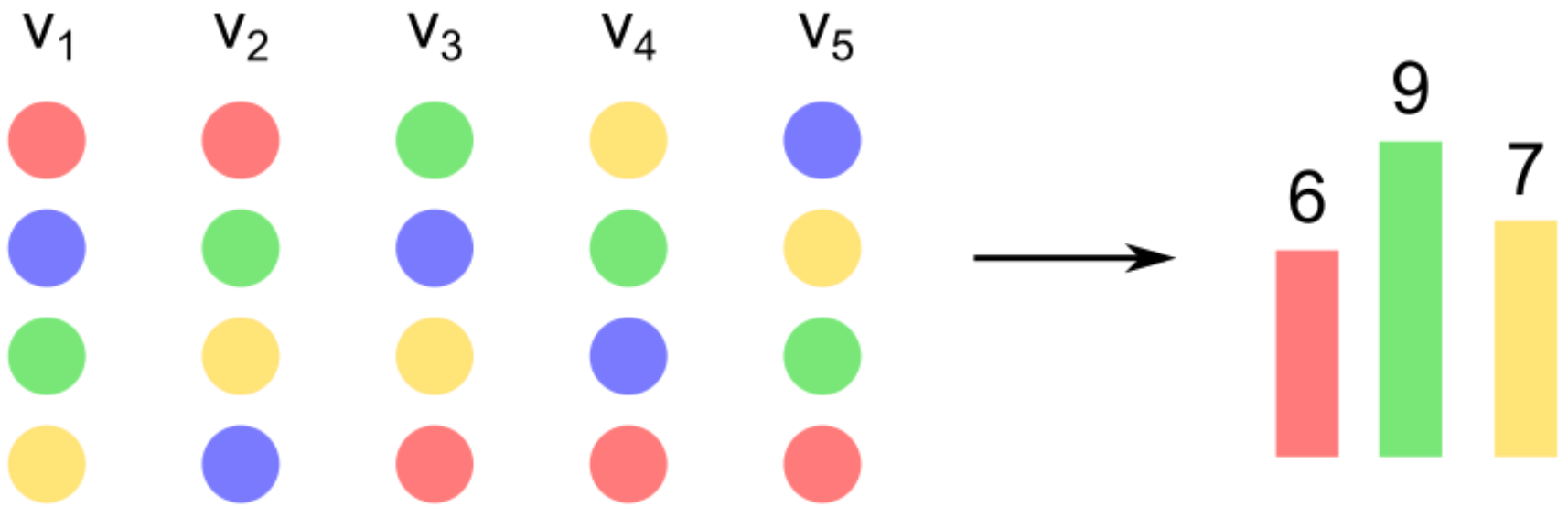

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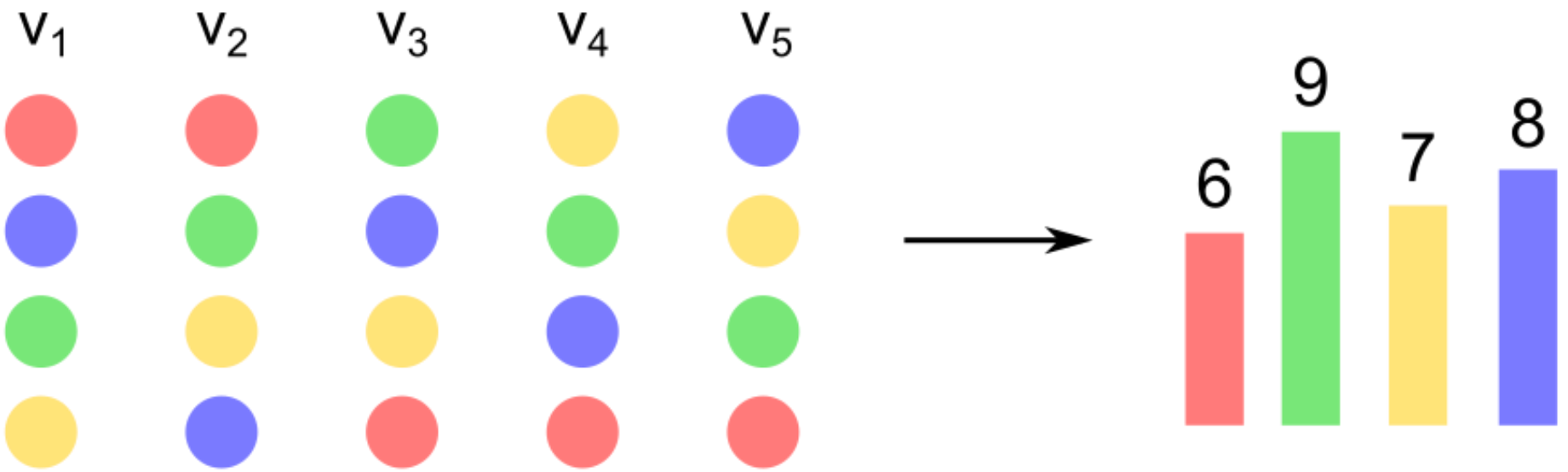

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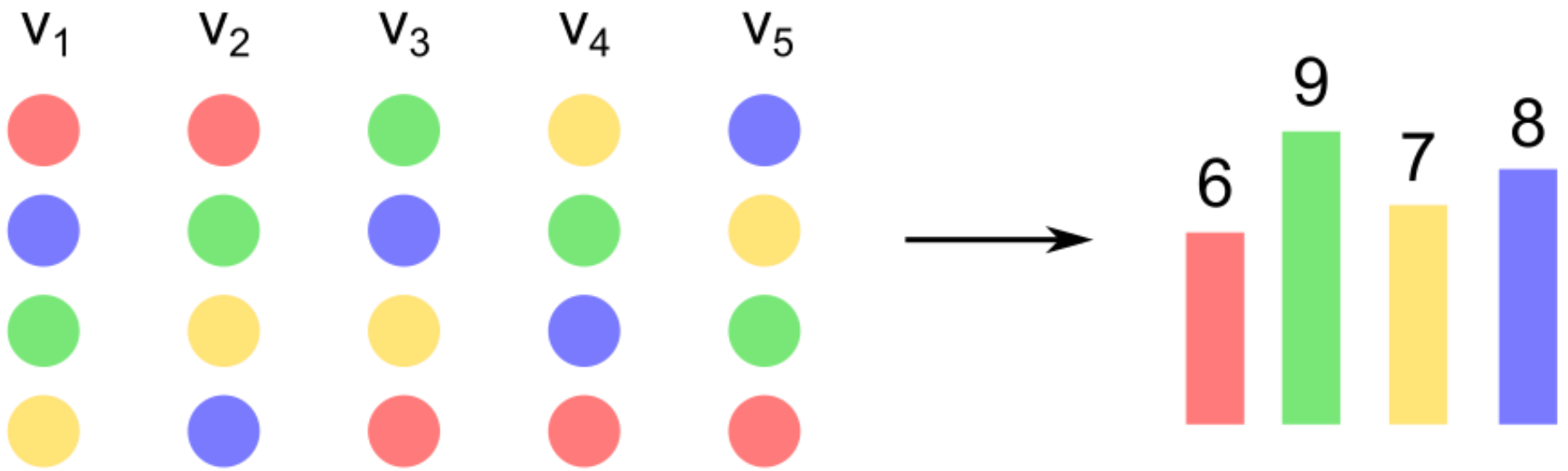

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Problem: Susceptible to strategic voting (manipulation).

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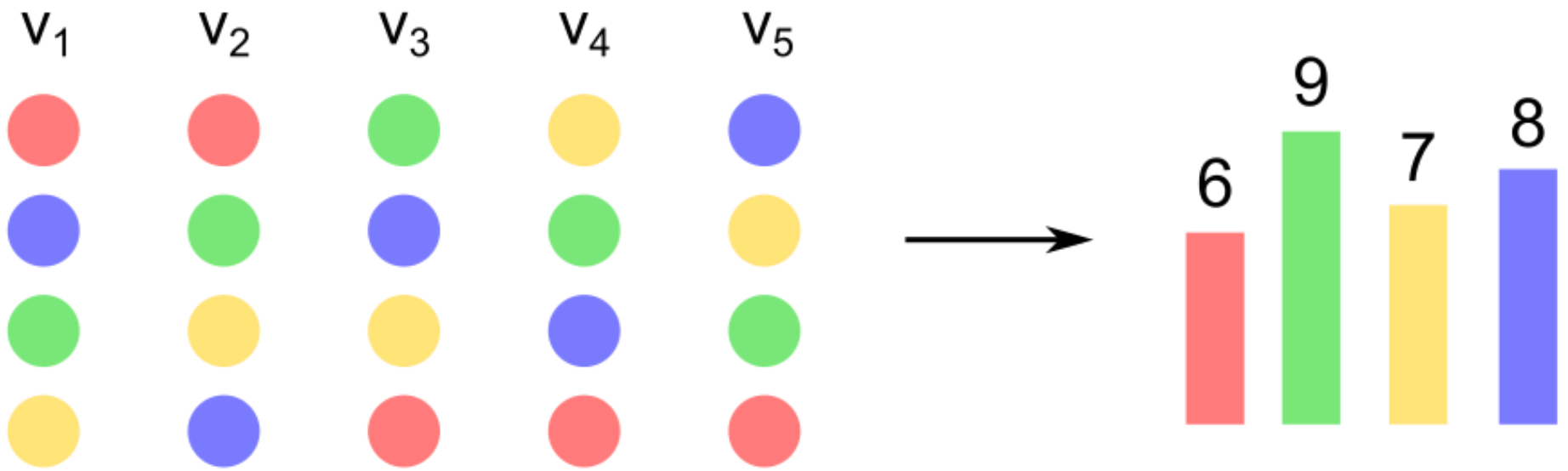

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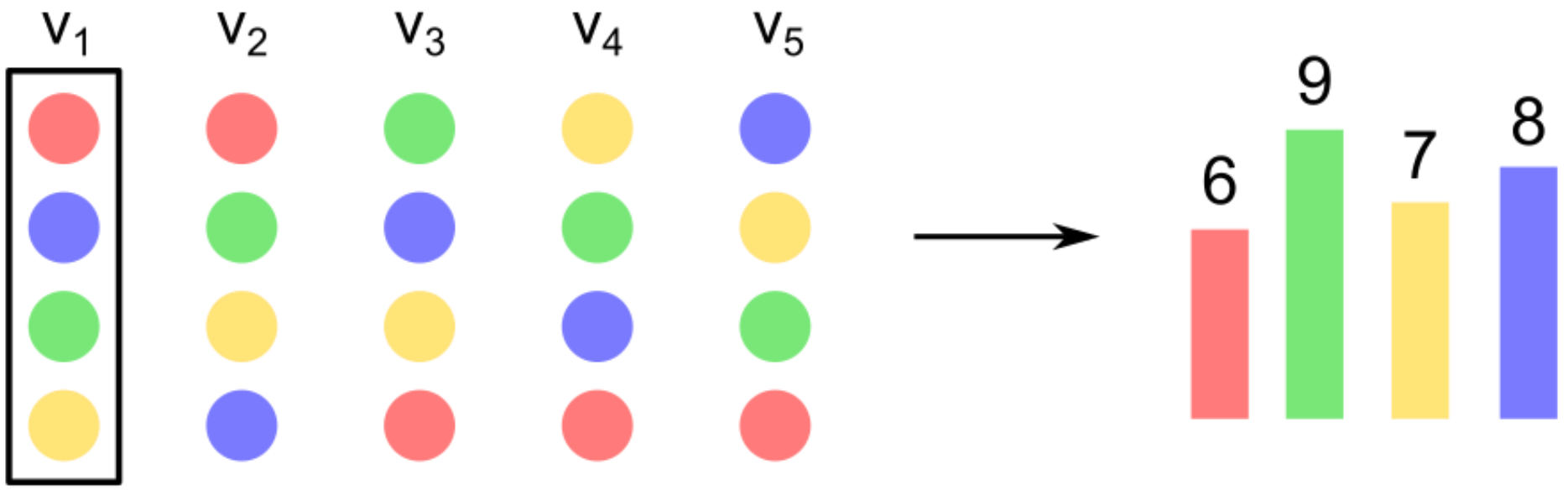

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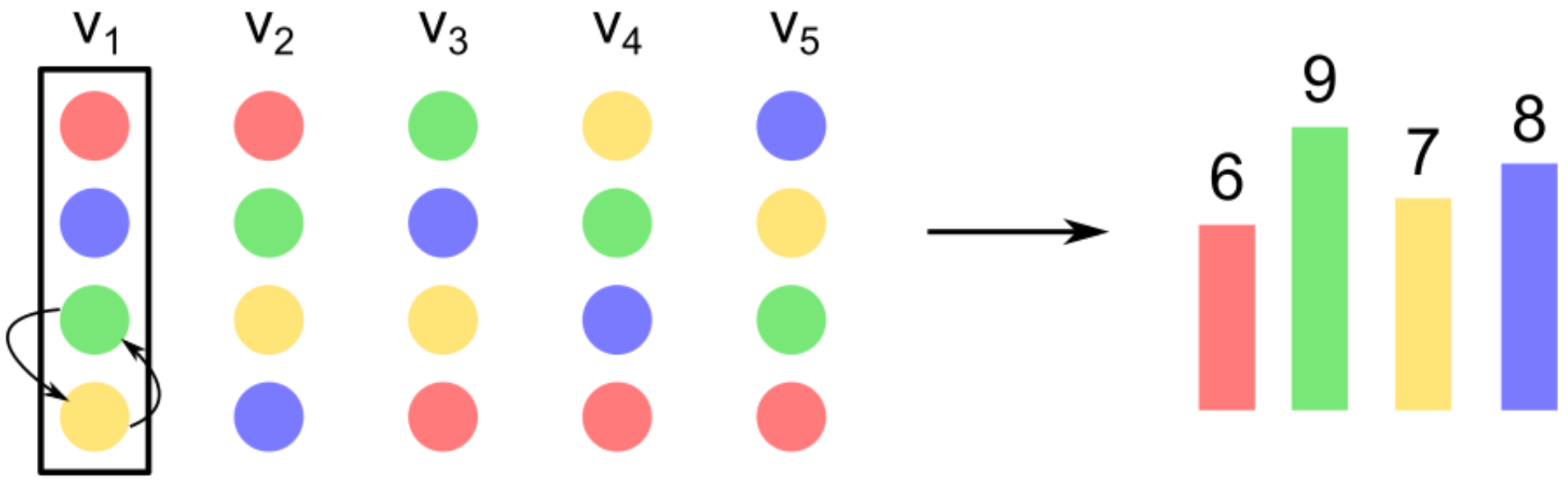

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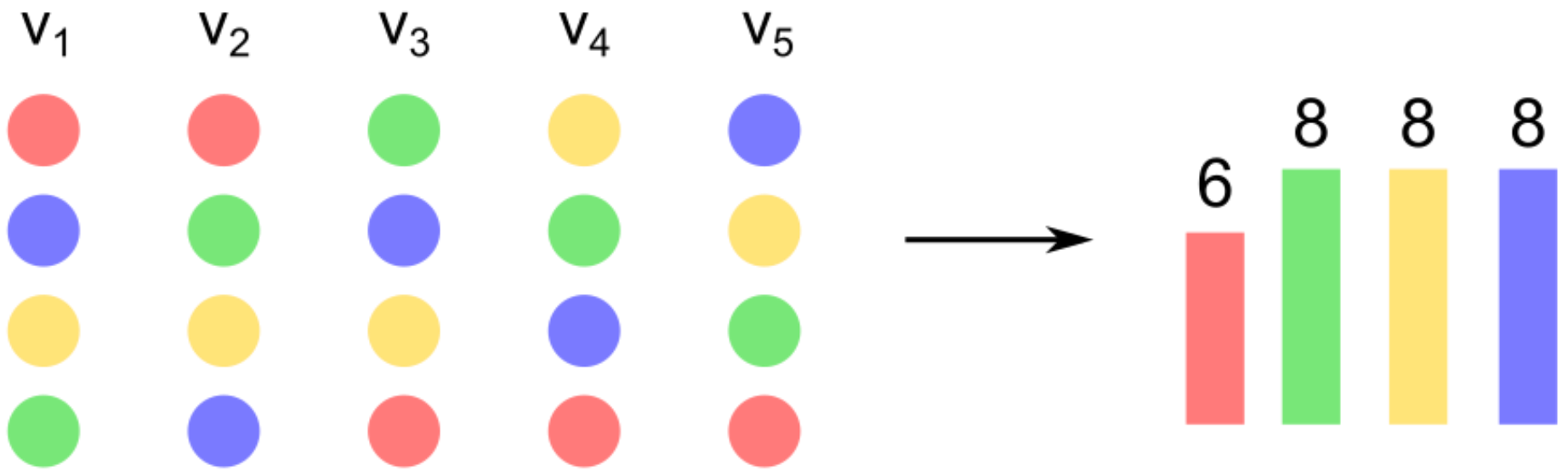

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"My scheme is intended for only honest men."

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Borda Count

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Plurality With Runoff

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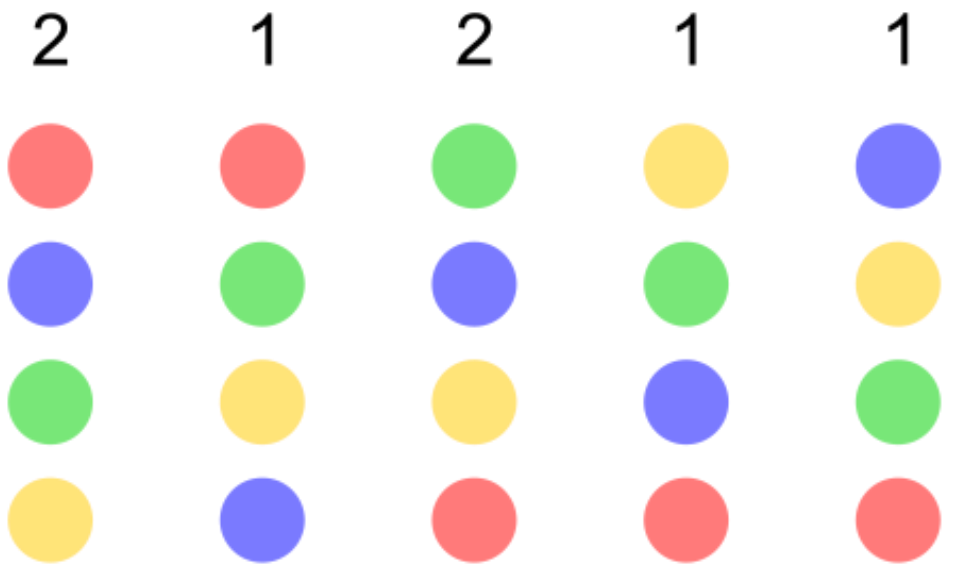

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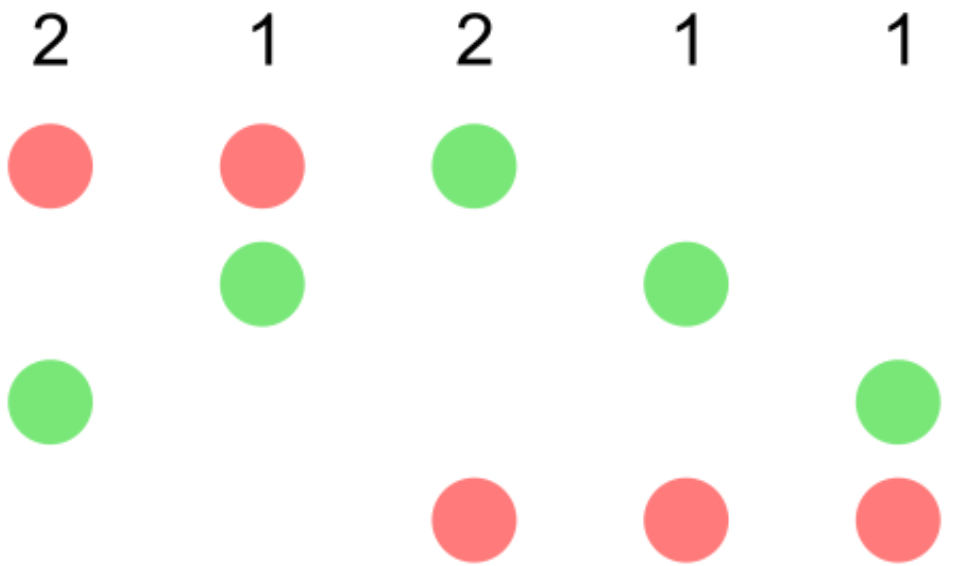

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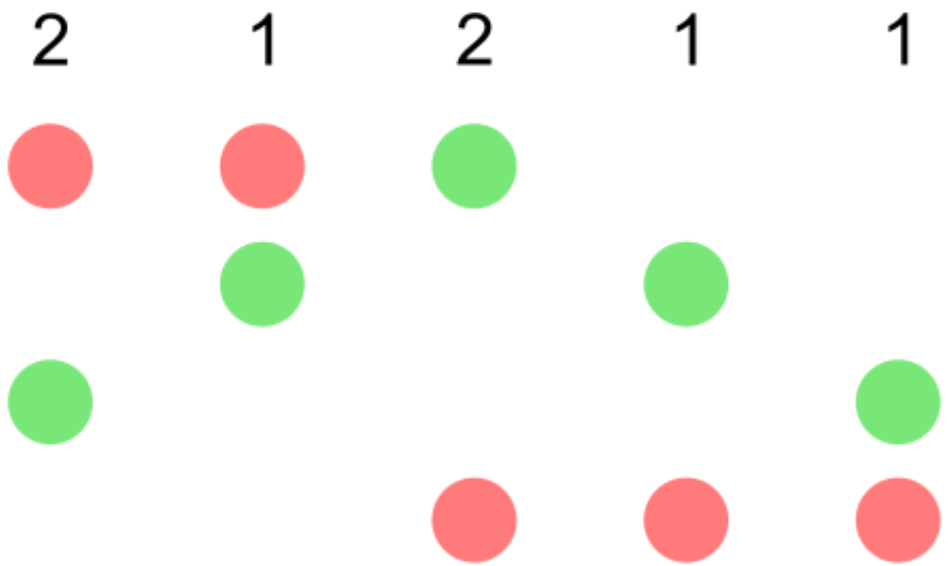

Plurality with runoff winner:

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Single Transferable Vote

(Instant-Runoff)

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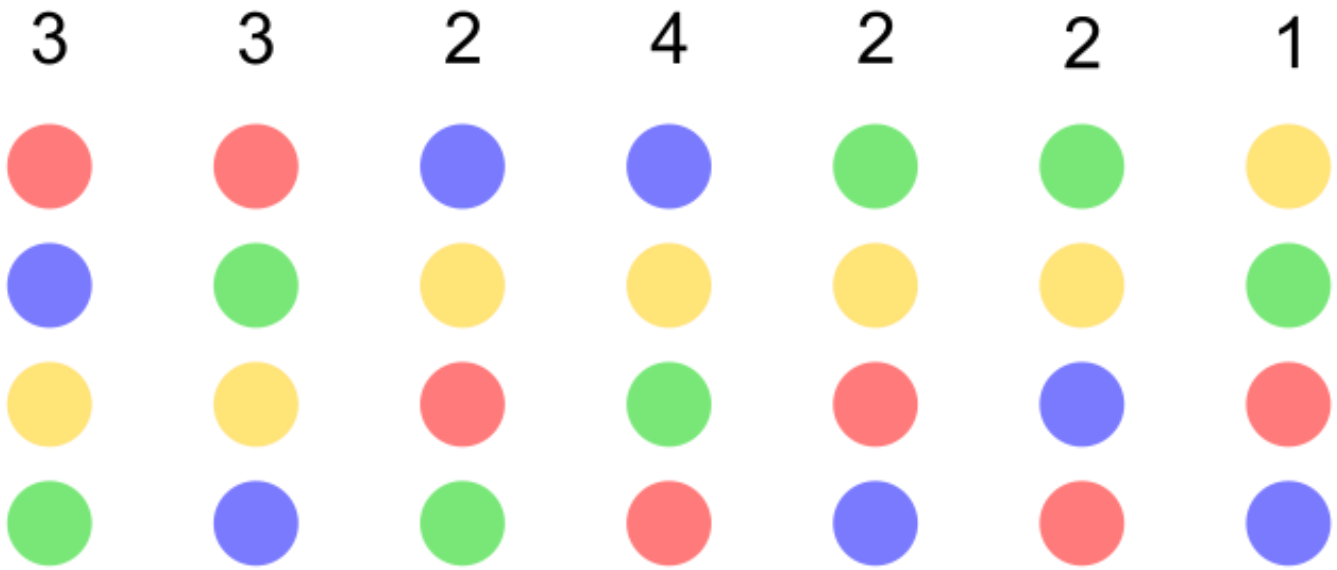

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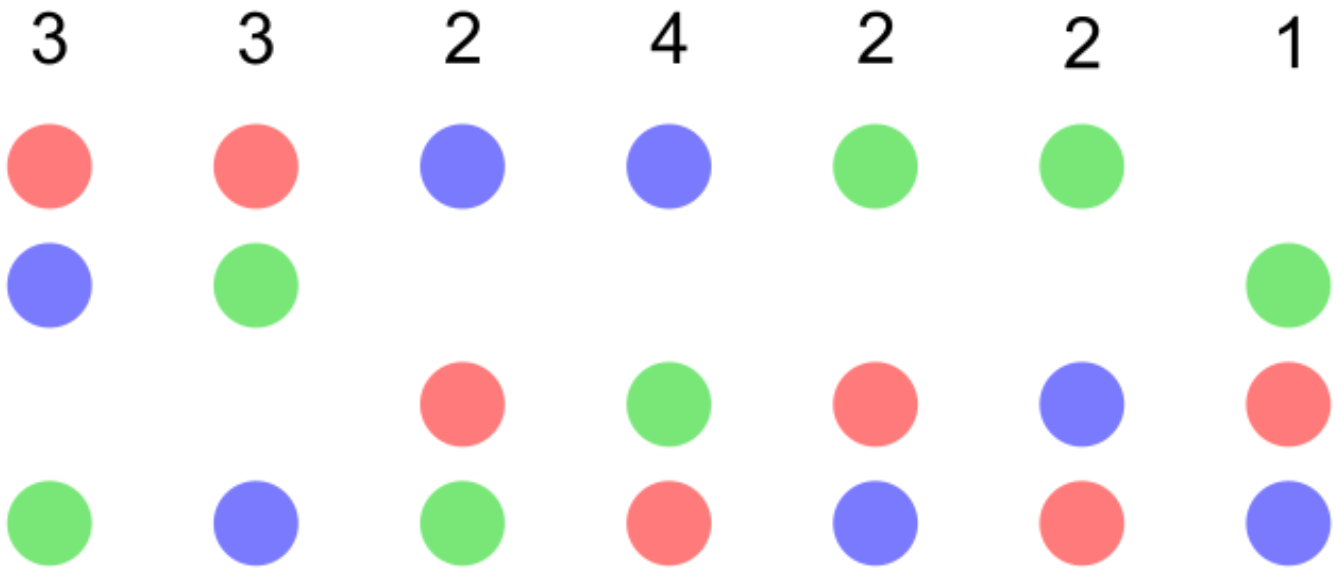

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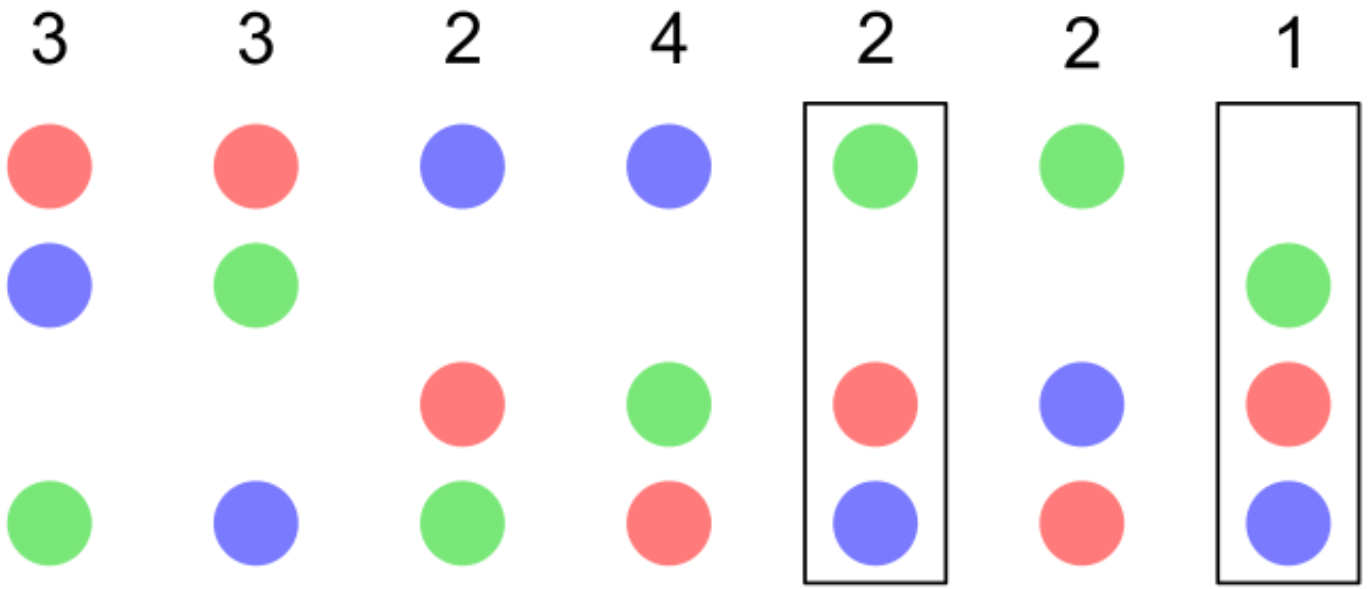

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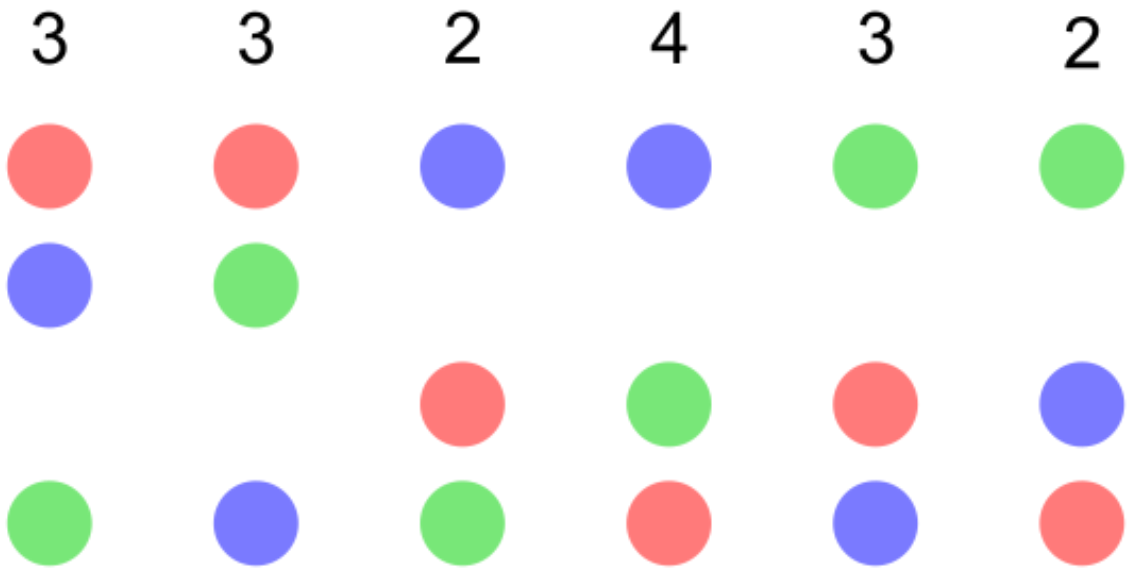

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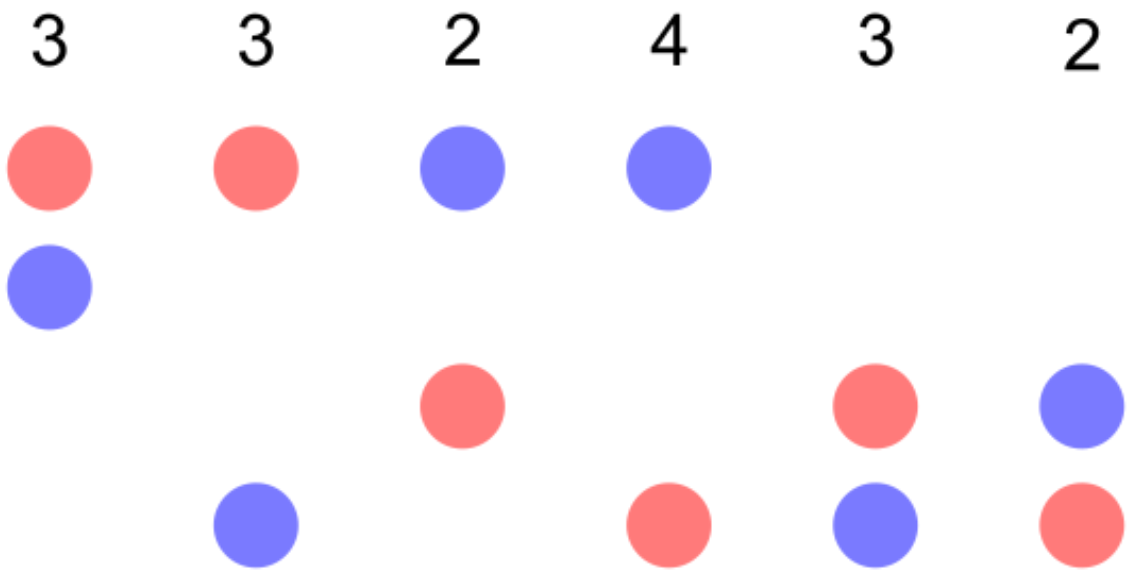

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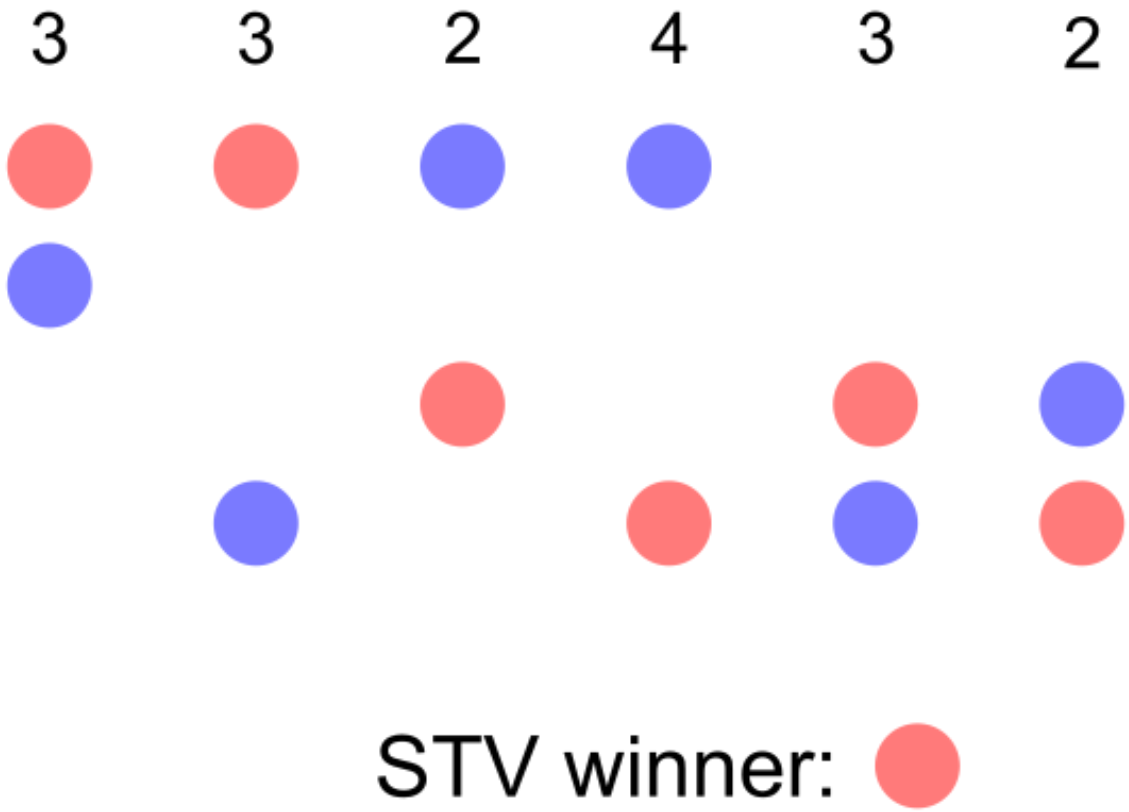

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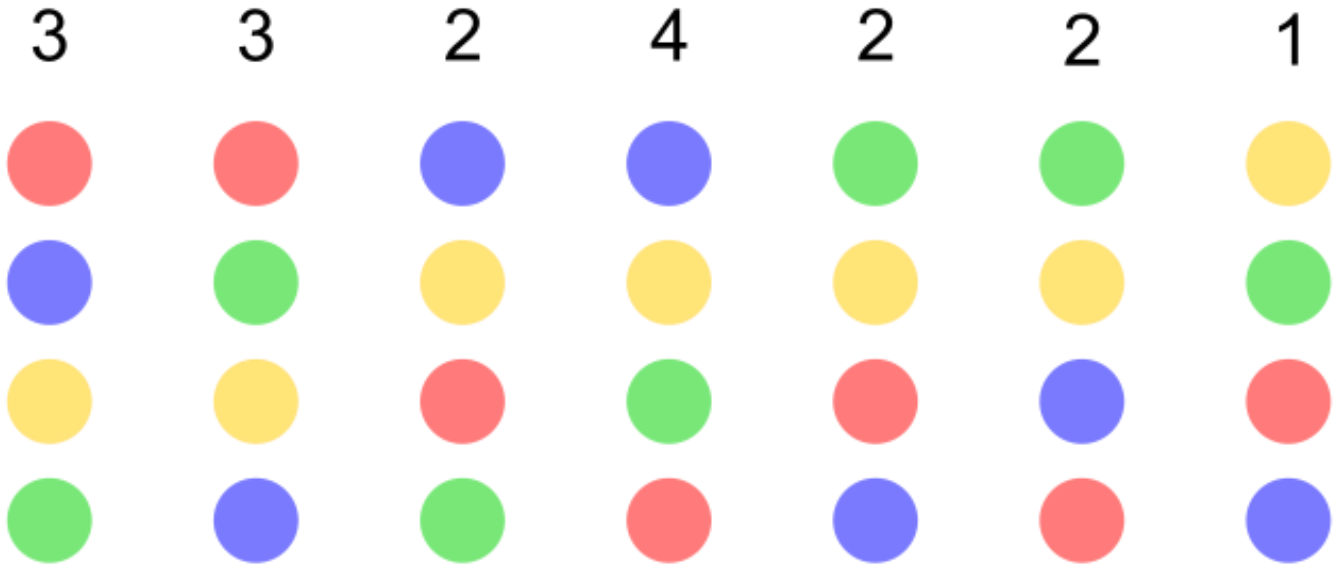

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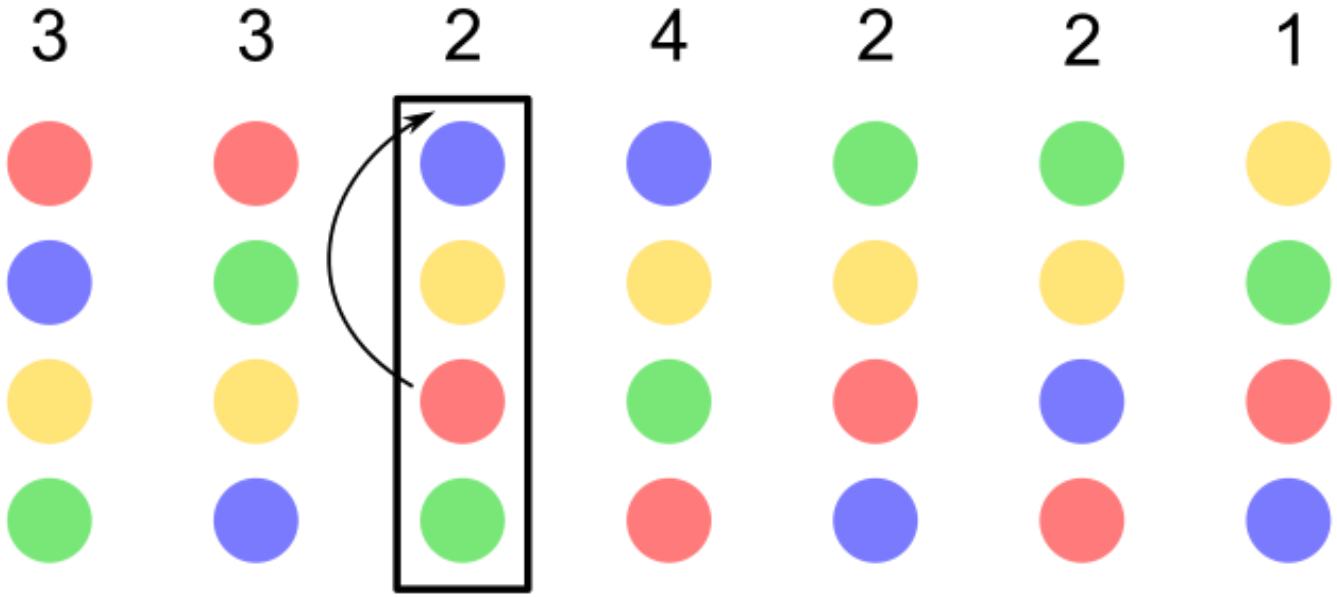

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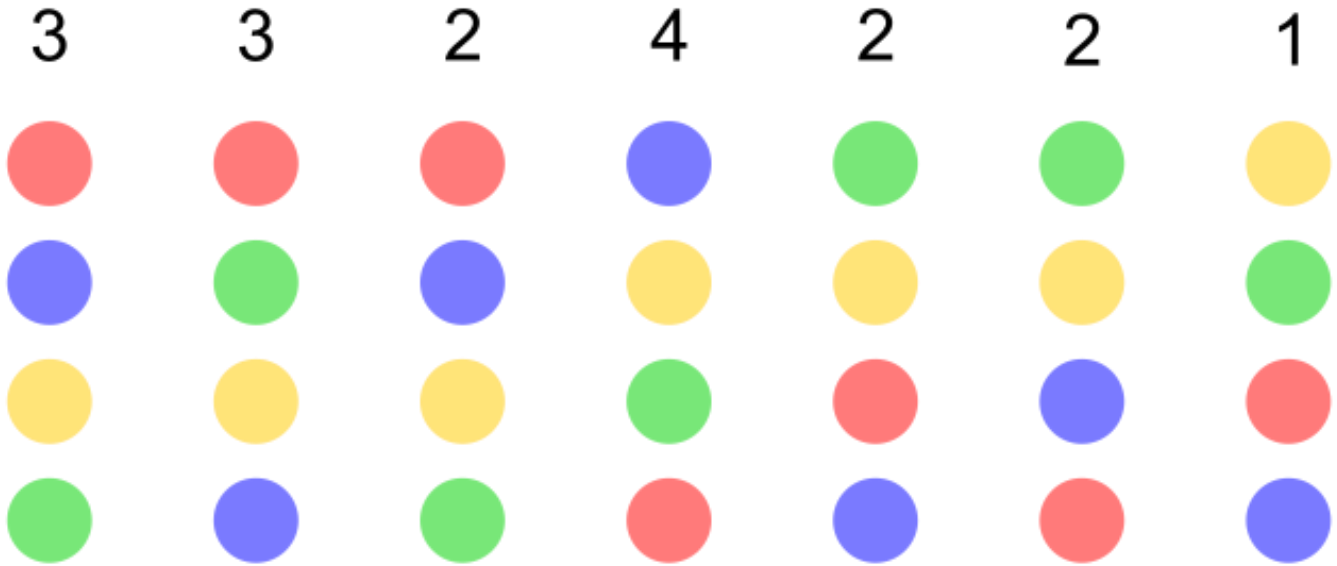

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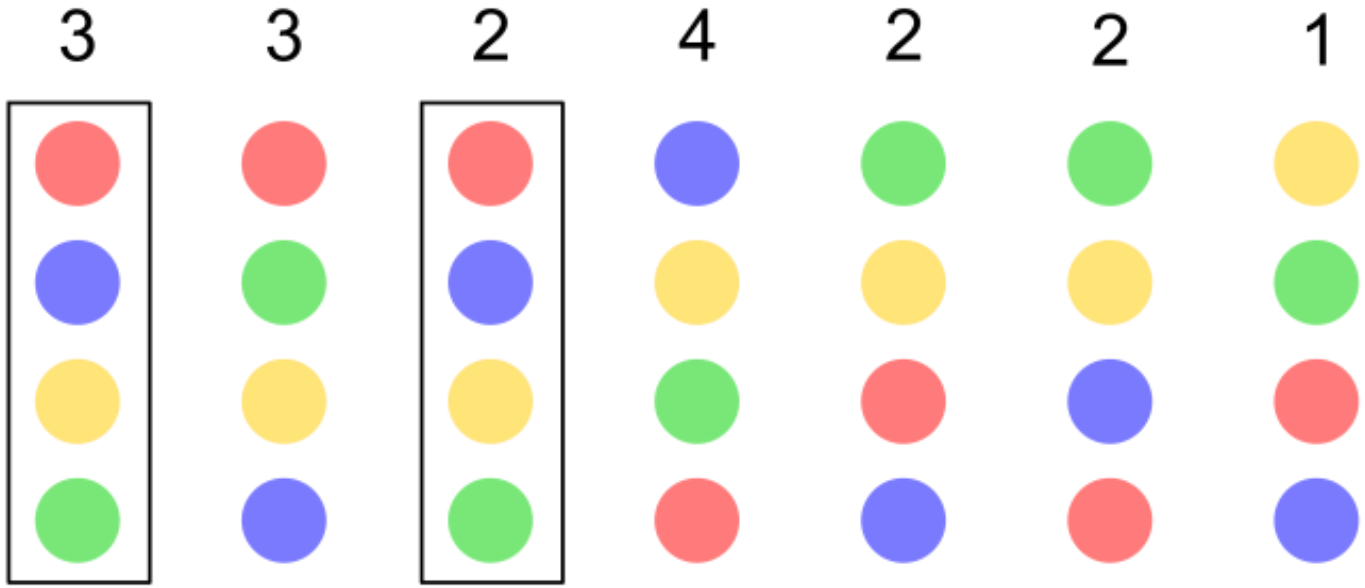

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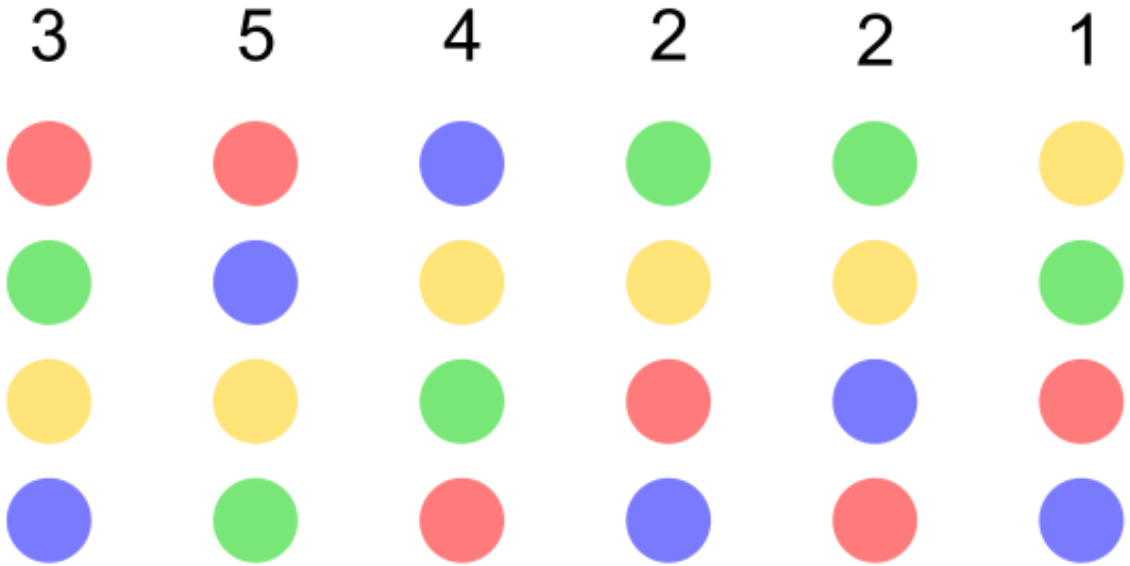

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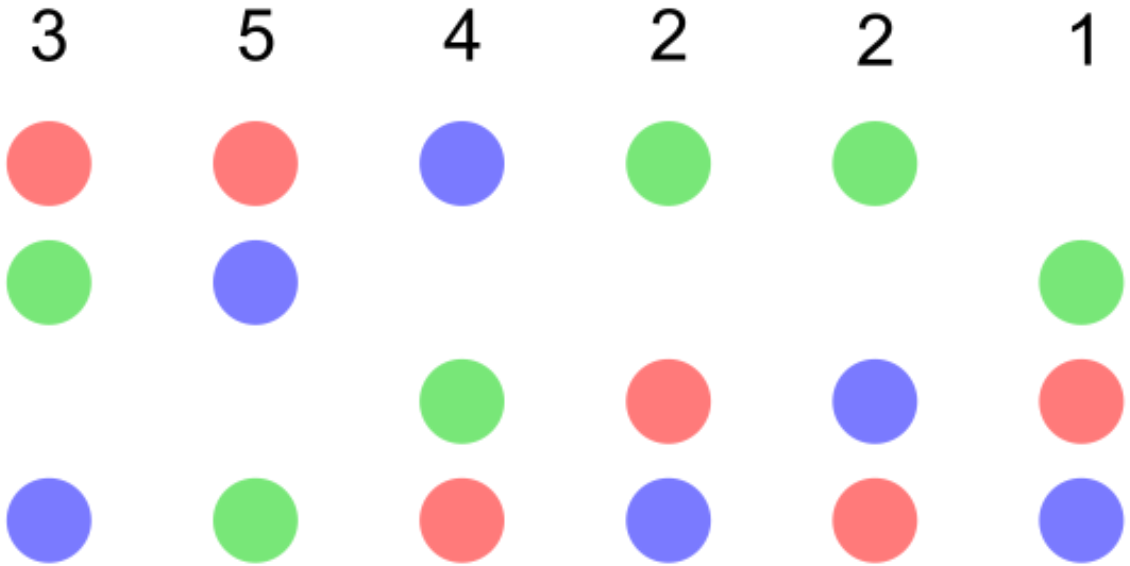

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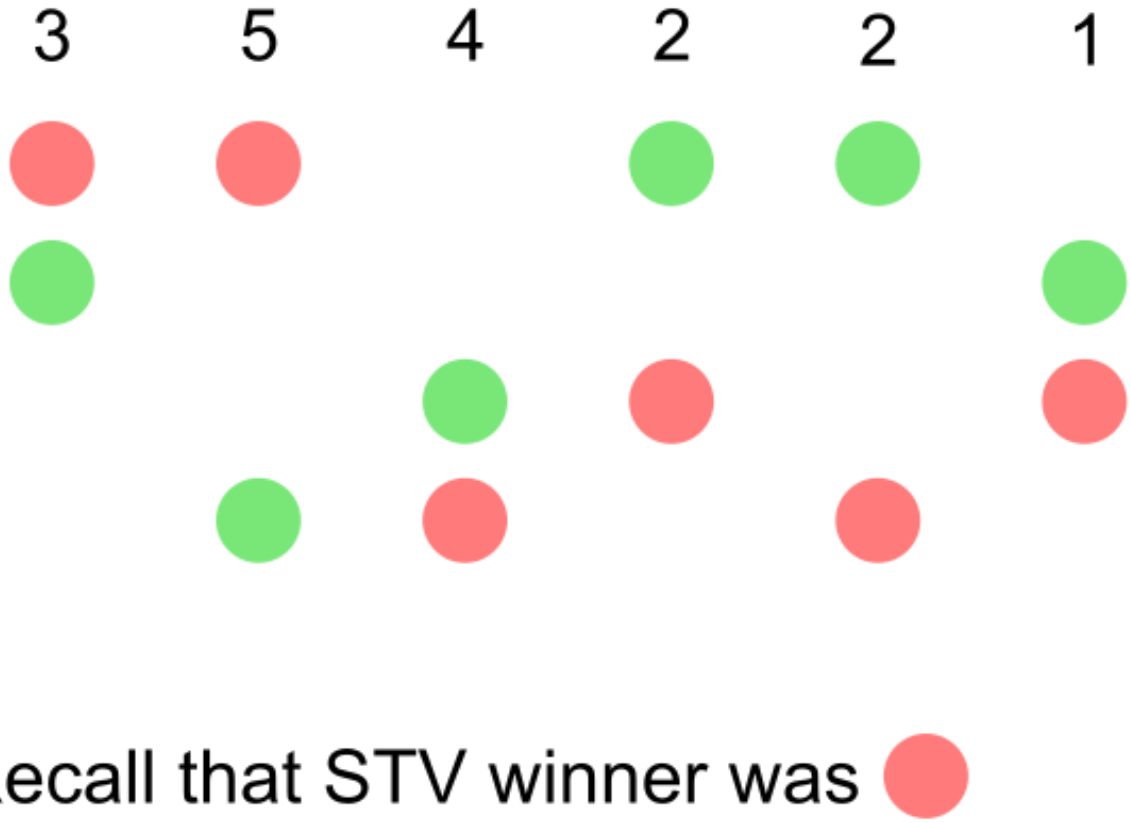

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2016 host city election ballots results [edit]

City	NOC	Round 1	Round 2	Round 3
Rio de Janeiro	Brazil (COB)	26	**46**	**66**
Madrid	Spain (COE)	**28**	29	32
Tokyo	Japan (JOC)	22	20	—
Chicago	United States (USOC)	18	—	—

Venue
Bella Center **121st IOC Session** October 2, 2009 Copenhagen

Vote details	Round 1	Round 2	Round 3
Eligible members	95	97	99
Participants	94	96	98
Abstentions	0	1	0
Valid ballots	94	95	98

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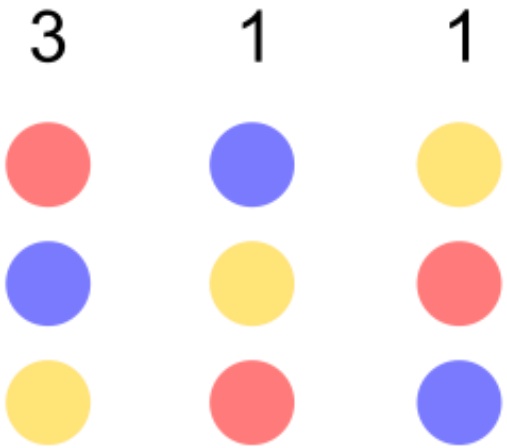

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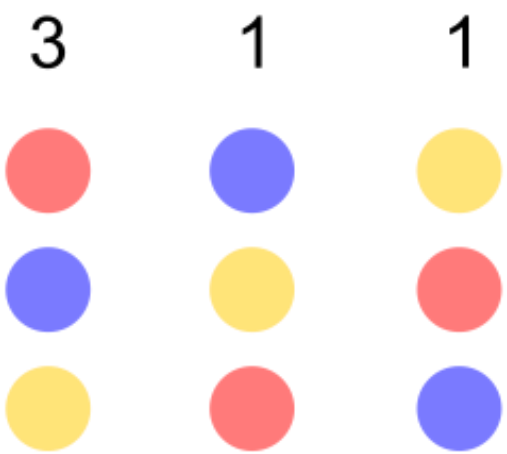

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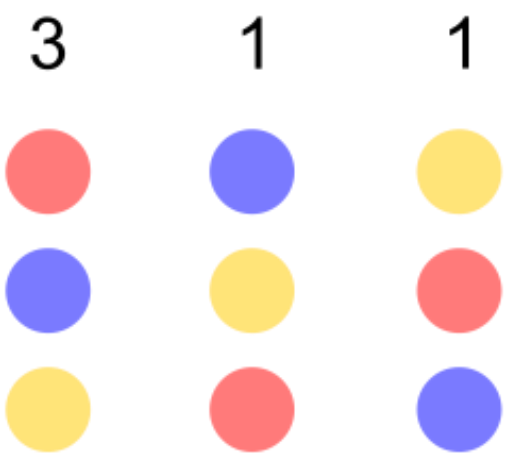

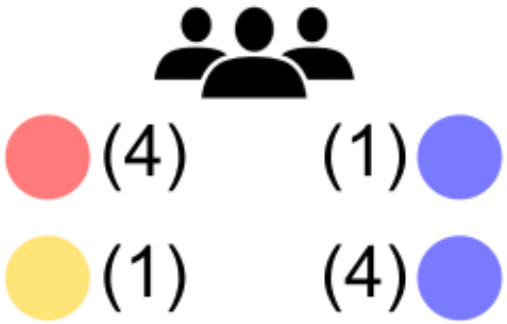

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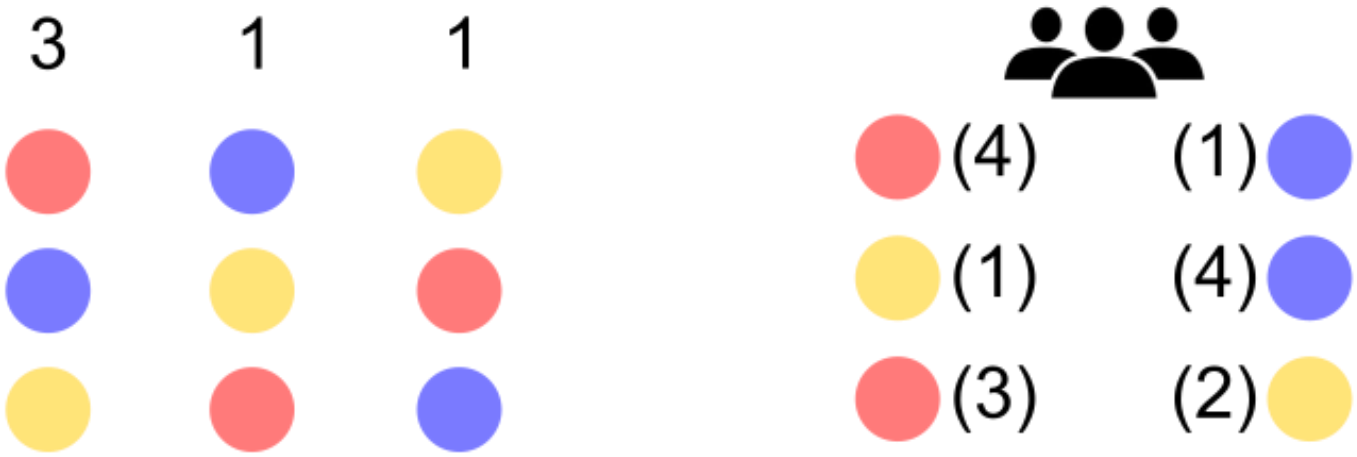

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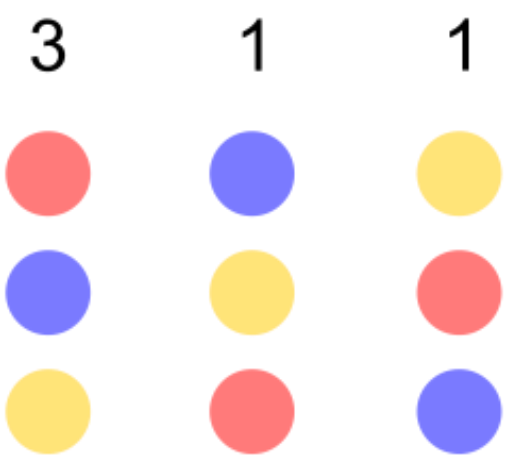
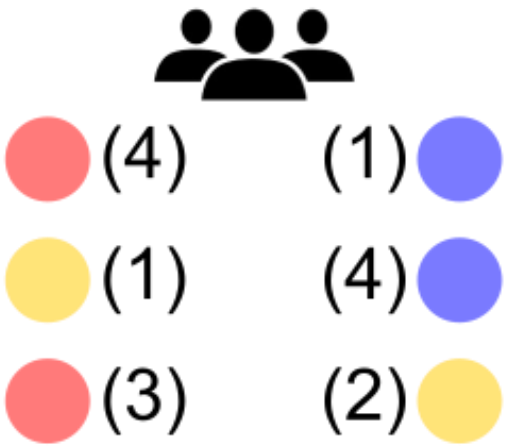
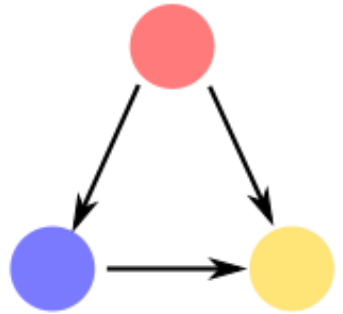




Condorcet winner



Condorcet Criterion

Problem: A Condorcet winner may not exist

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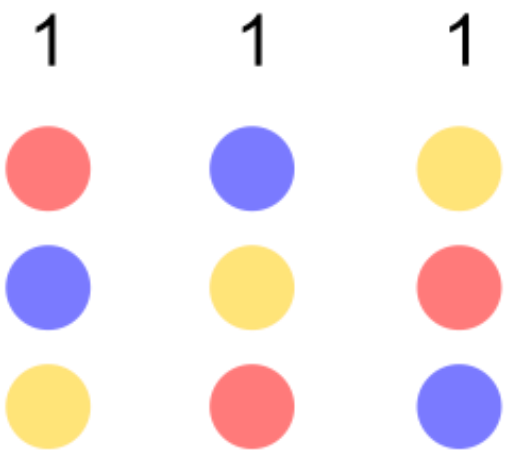
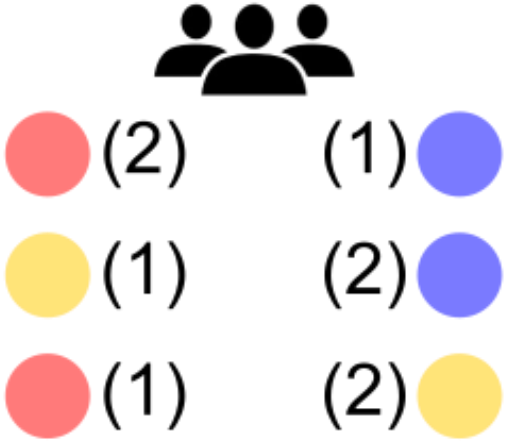
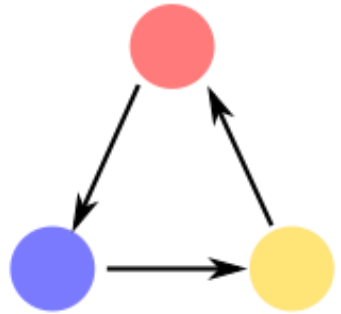




Condorcet cycle



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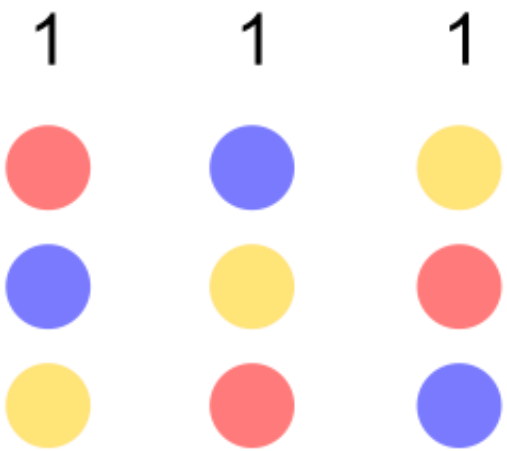

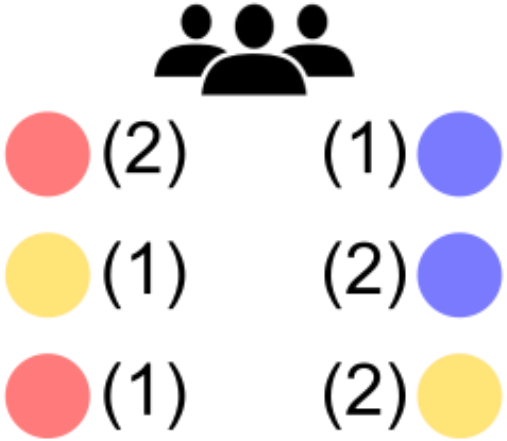

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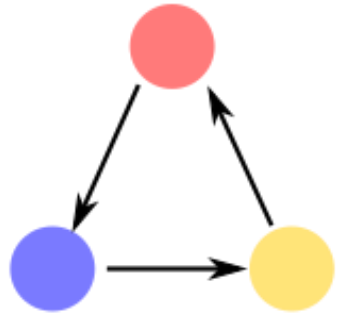

Condorcet paradox

Transitivity of individual preferences $\not\Rightarrow$ Transitivity of societal preferences

5

Copeland

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For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie

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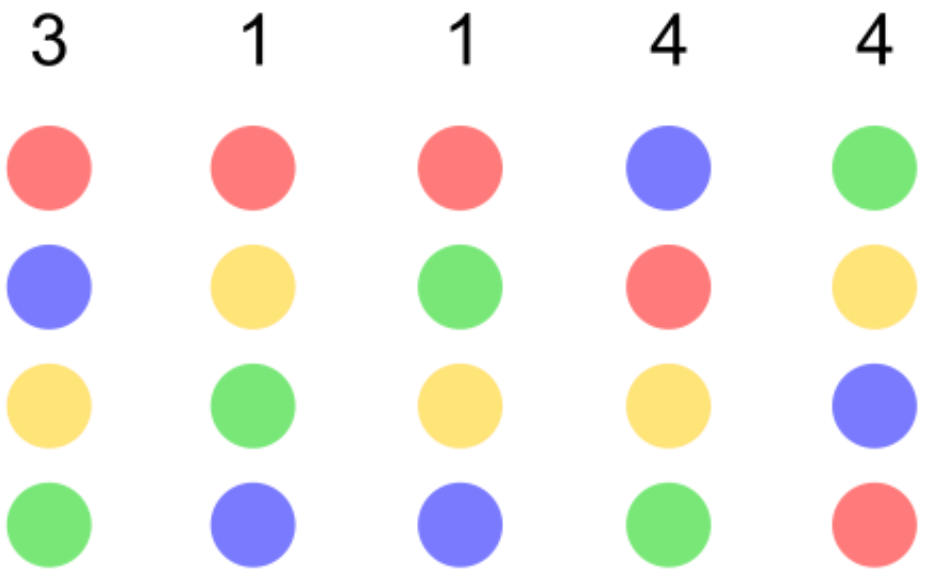

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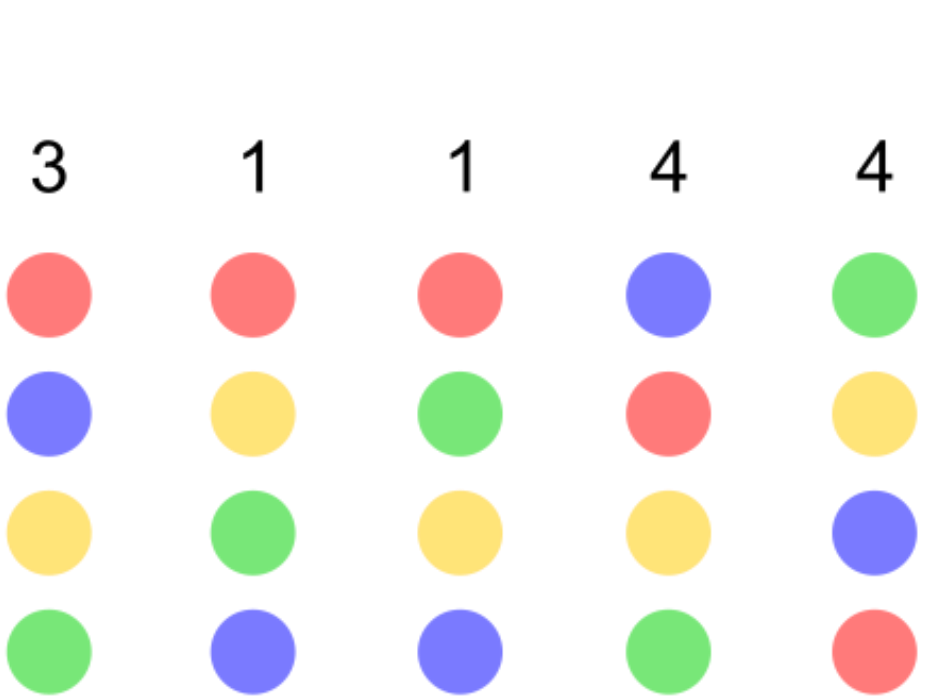

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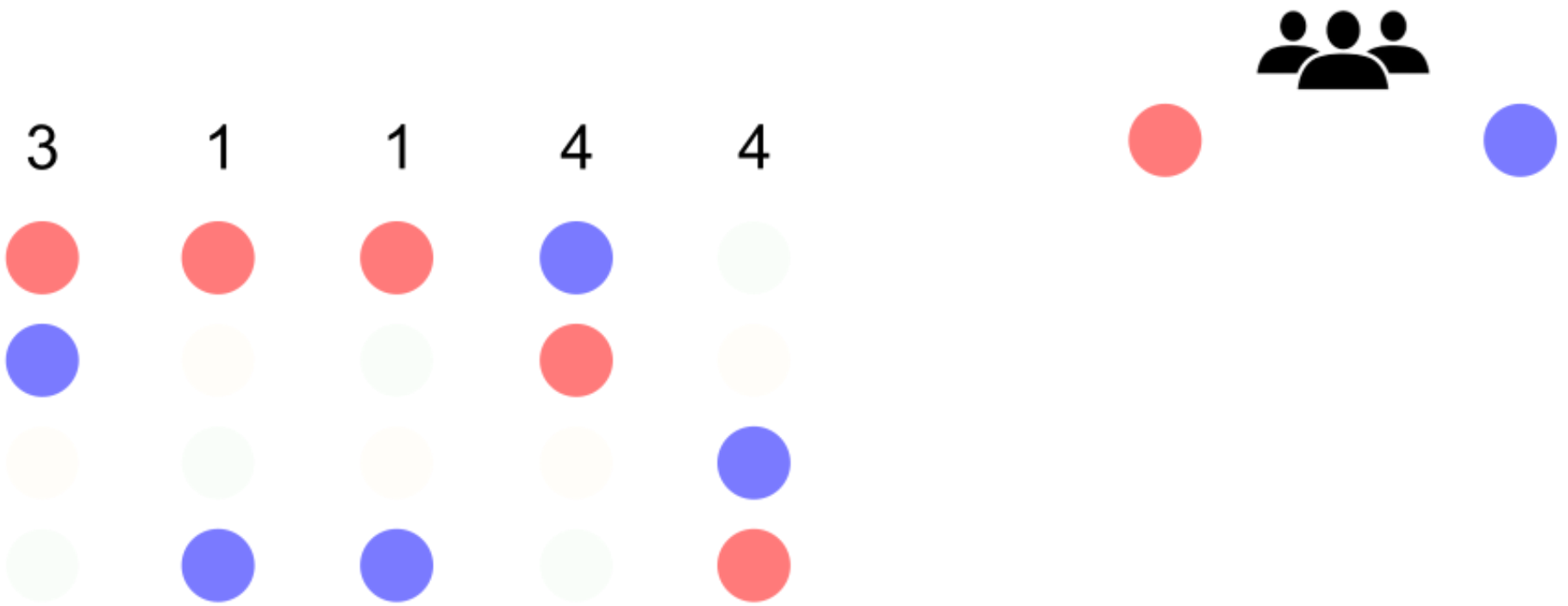

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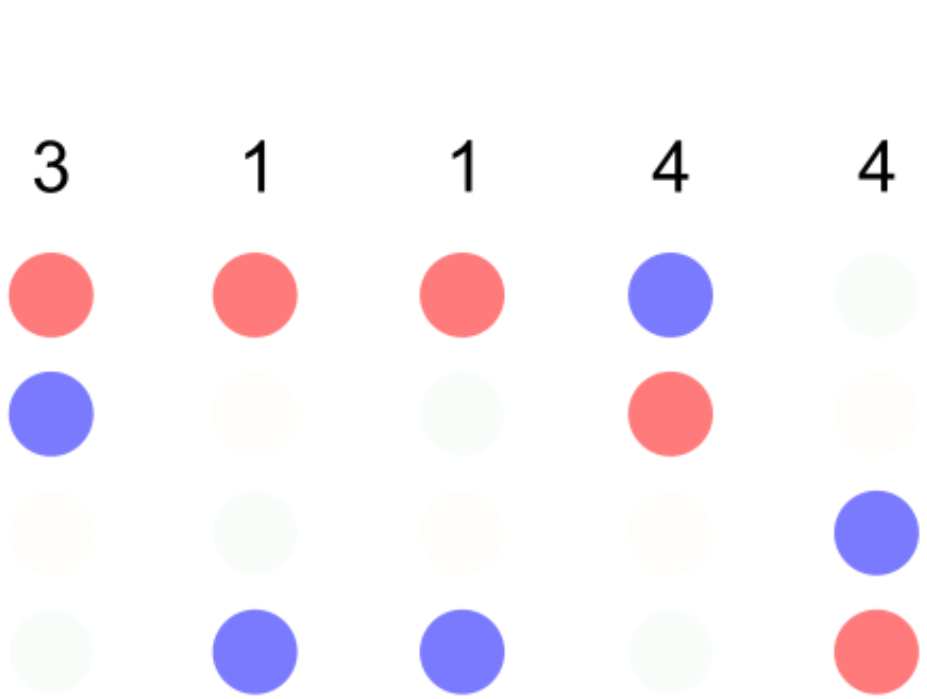

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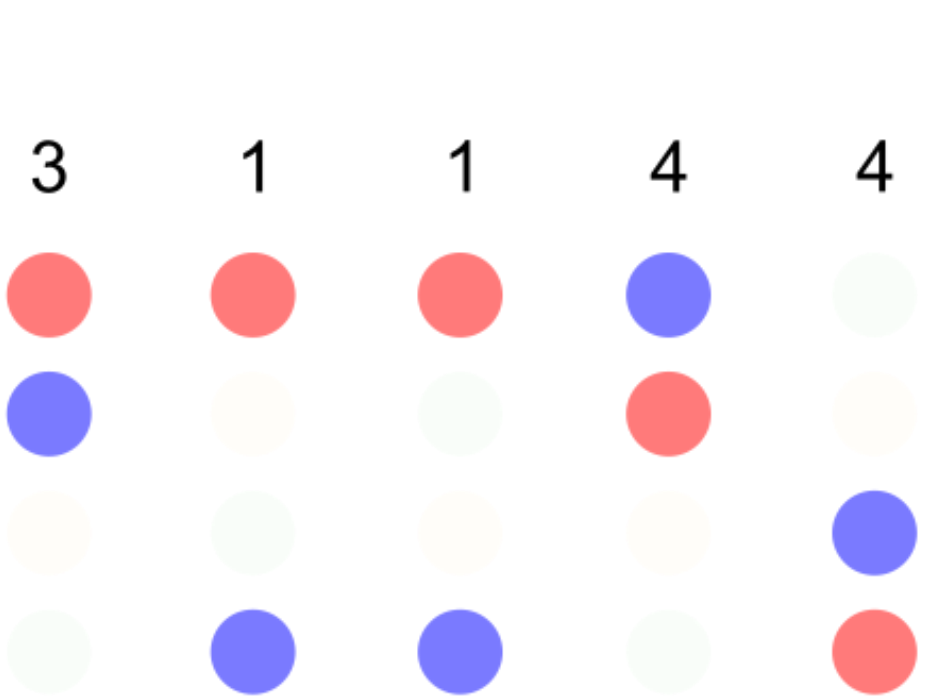

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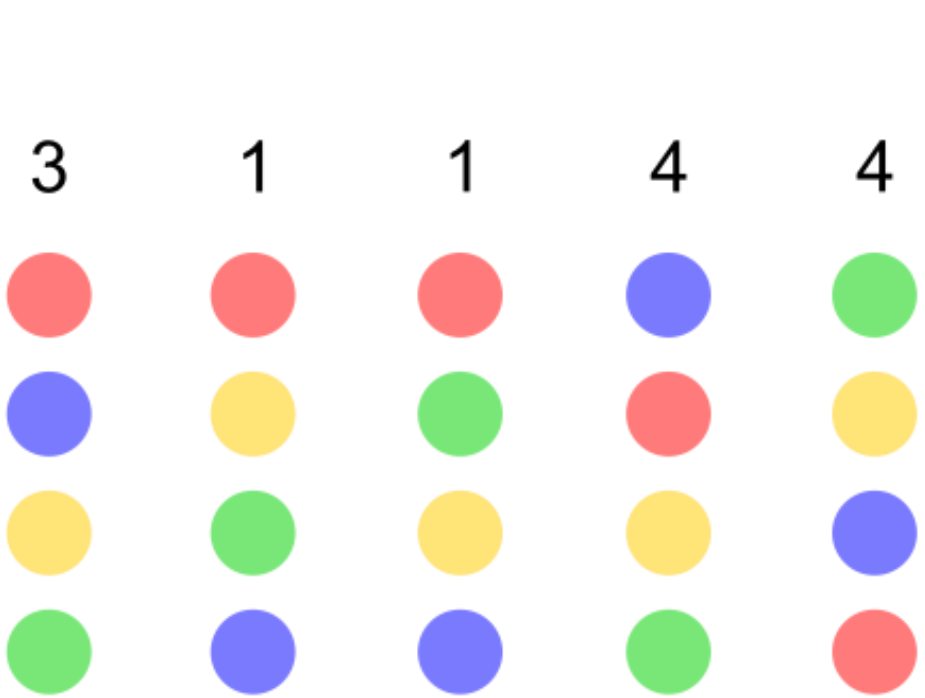

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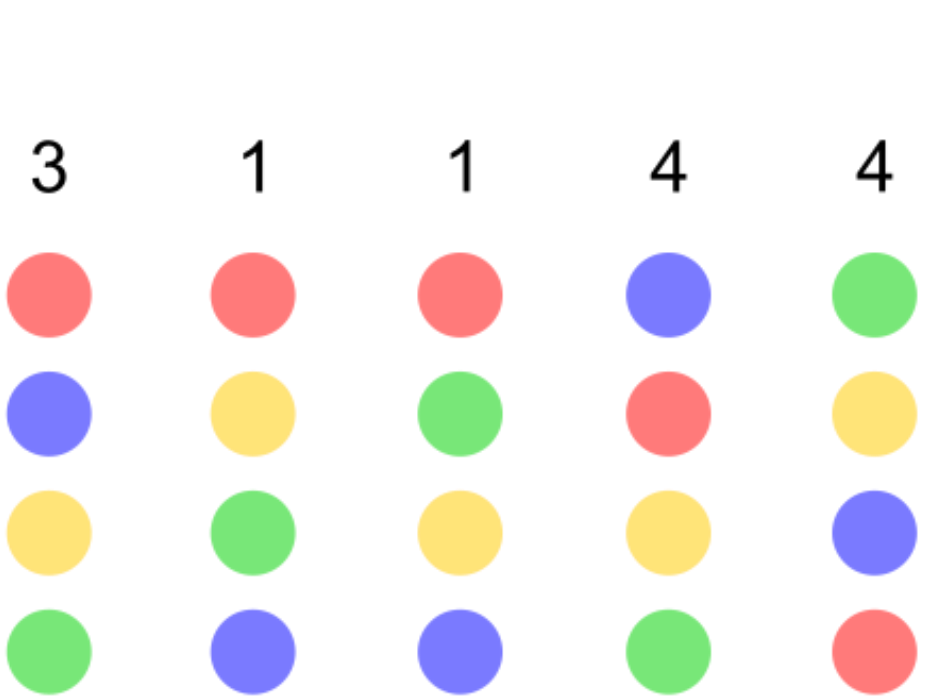

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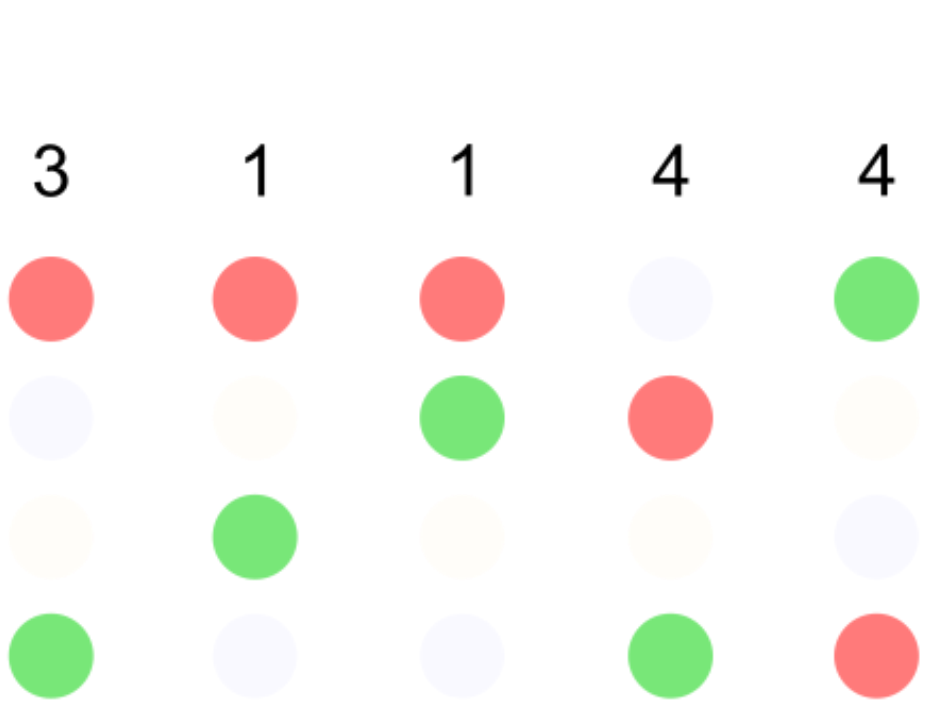

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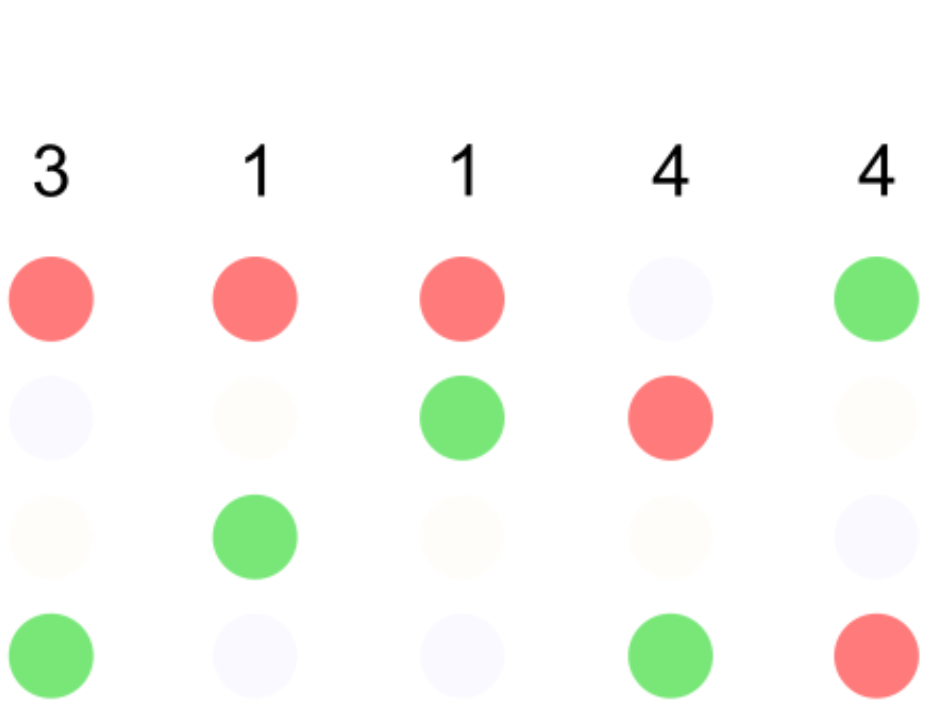

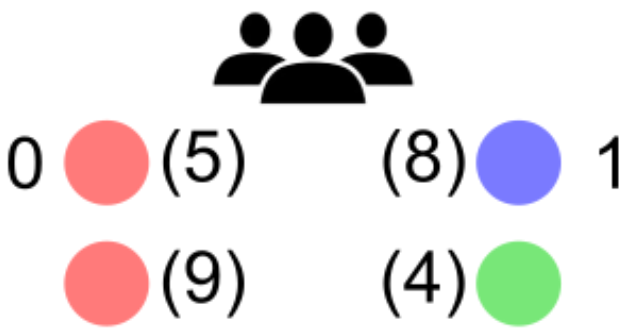

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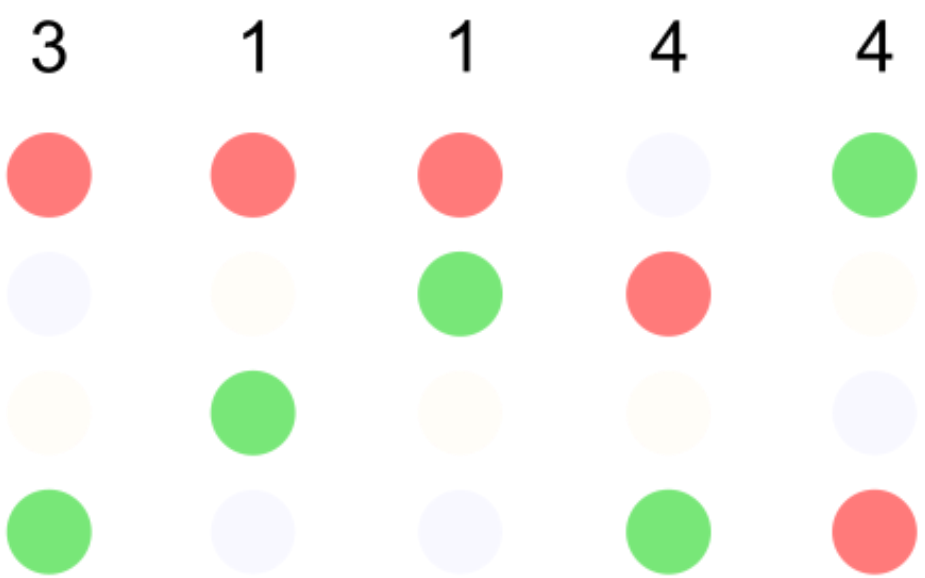

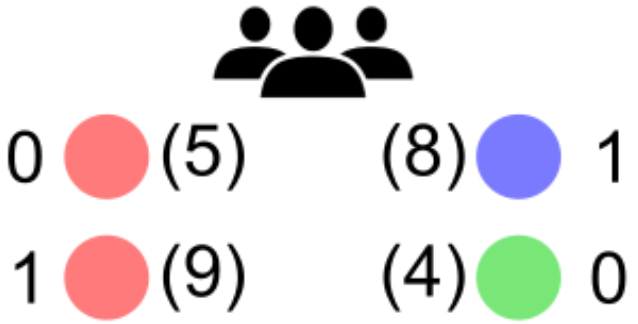

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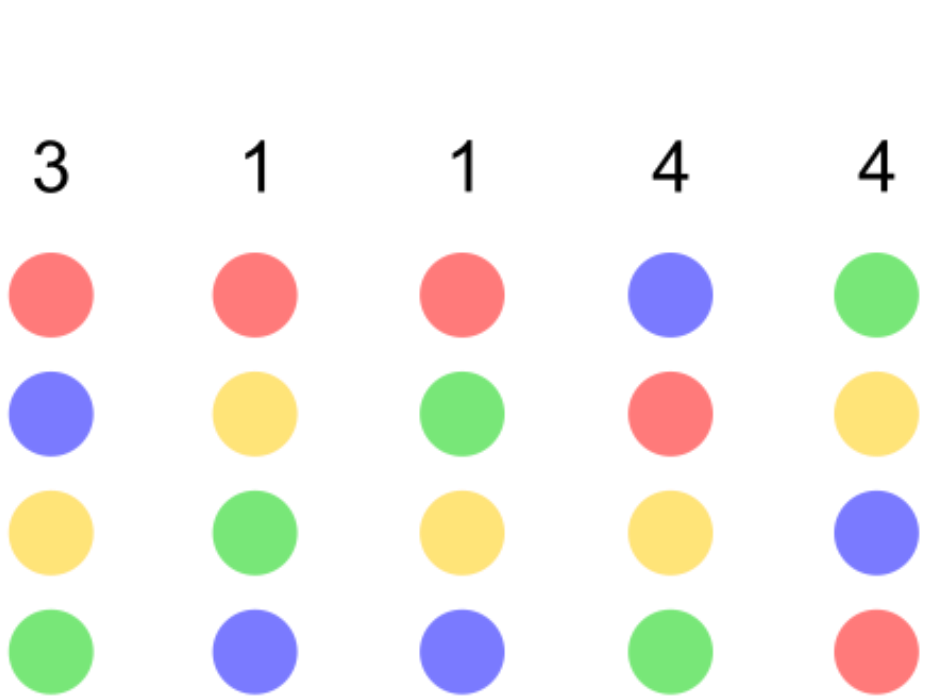

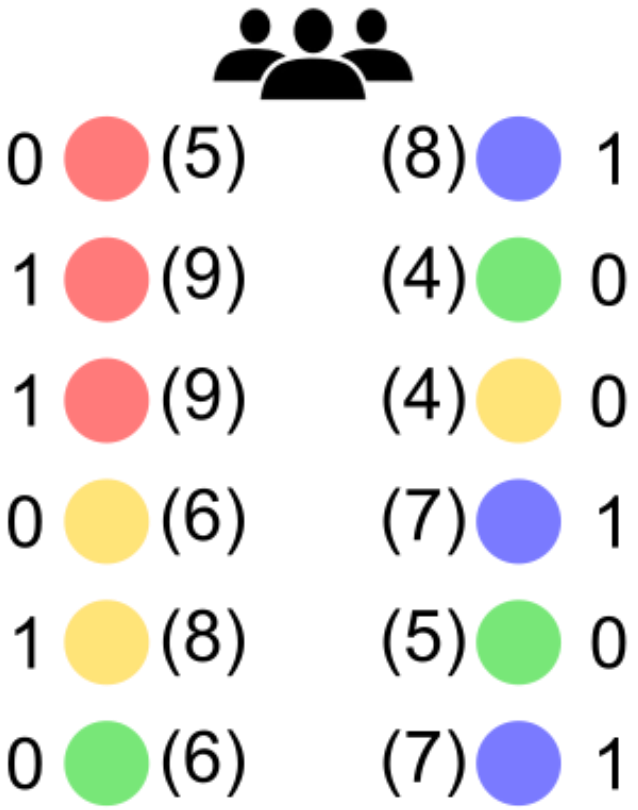

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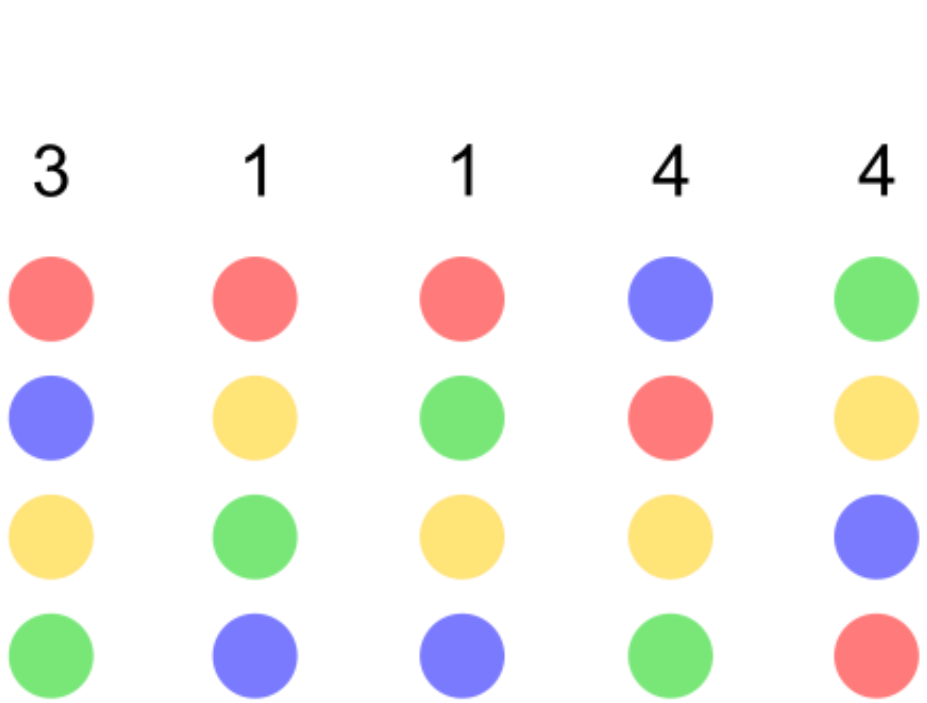

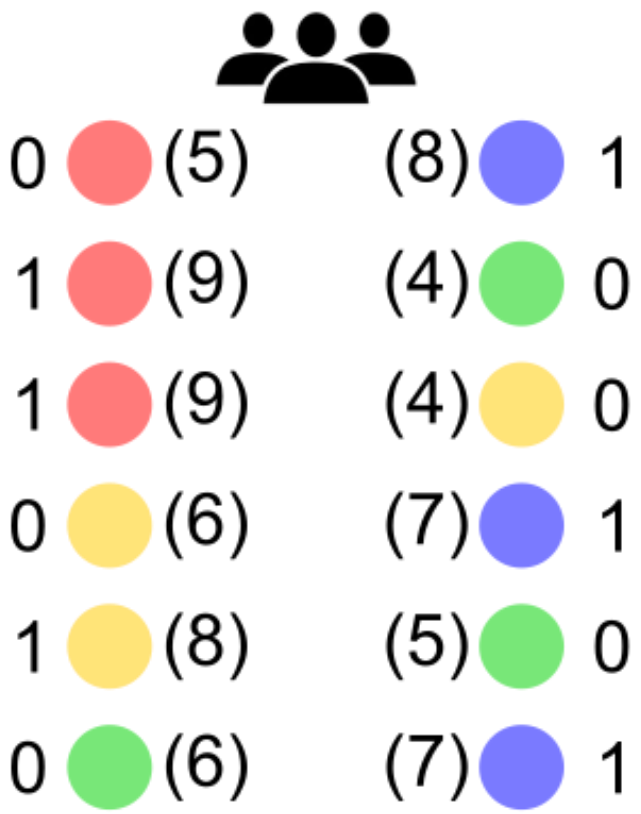

Copeland winner:

5

Copeland

LaLiga

BUNDESLIGA

ICC WOMEN'S CRICKET WORLD CUP NEW ZEALAND 2022

Premier League

INDIAN PREMIER LEAGUE ®

5

Problem: Voters are sometimes better off not voting

Copeland

5

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Copeland winner:

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Schulze

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Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

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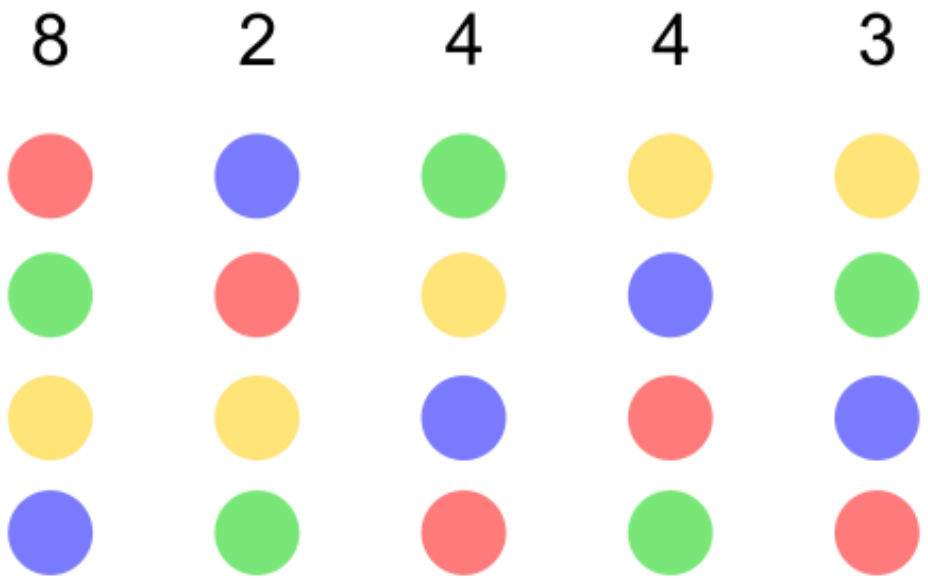

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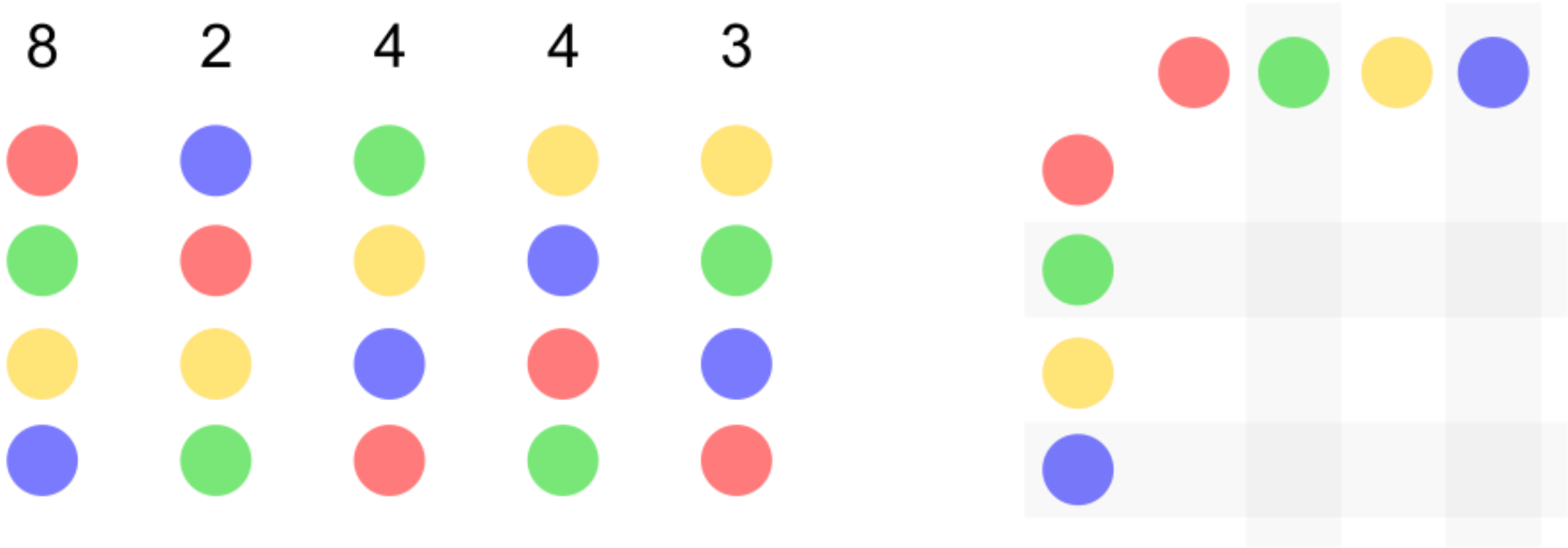

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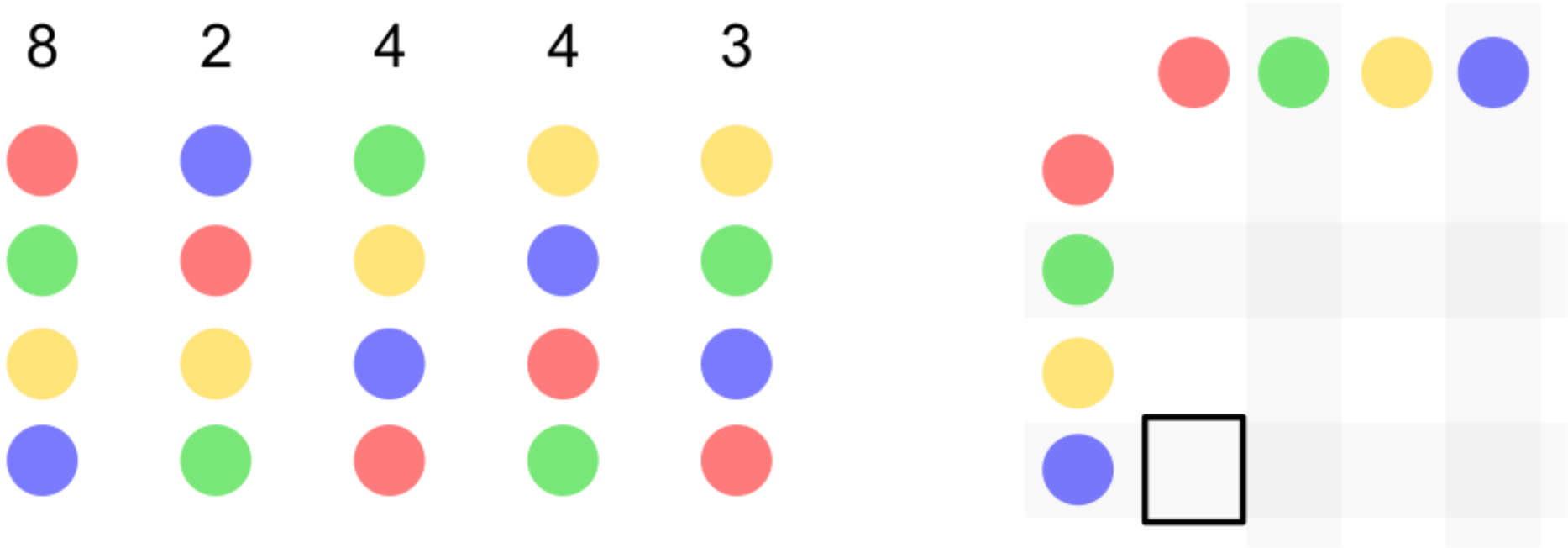

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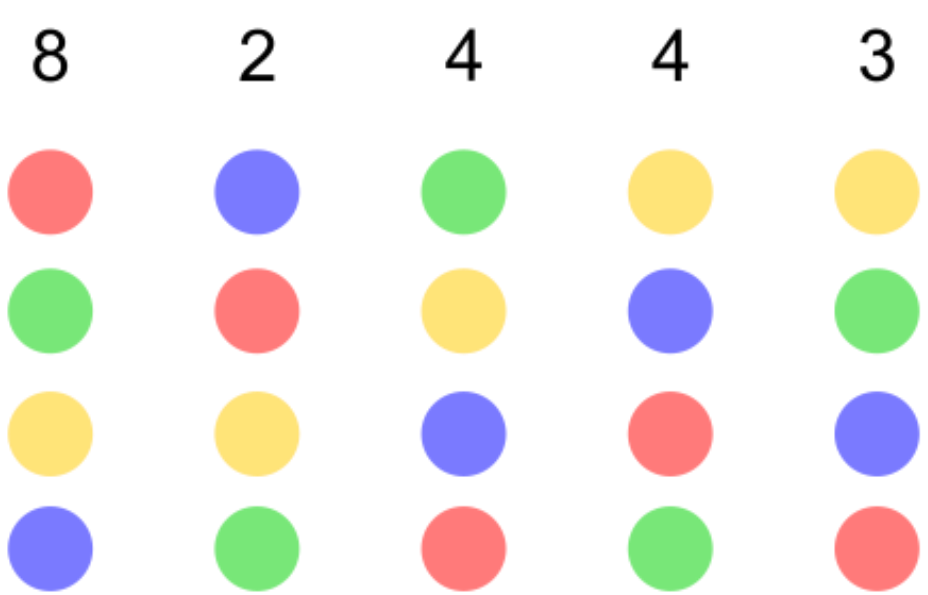

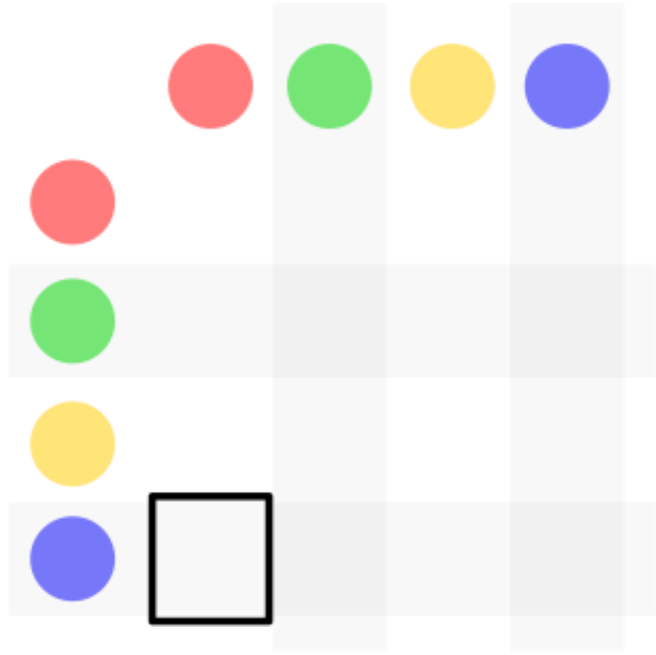

(2+4+4+3) - 8

#voters who think >

#voters who think >

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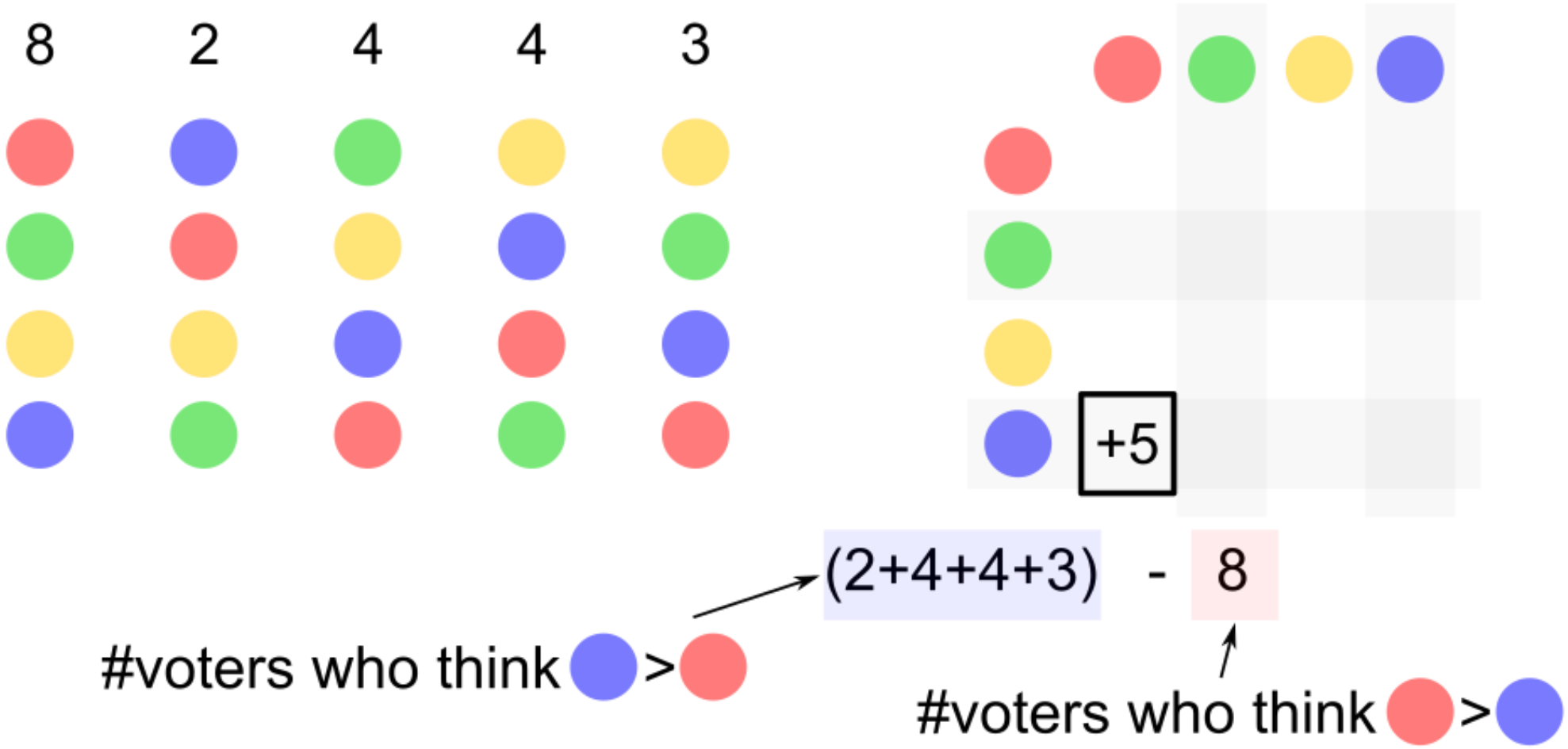
Schulze



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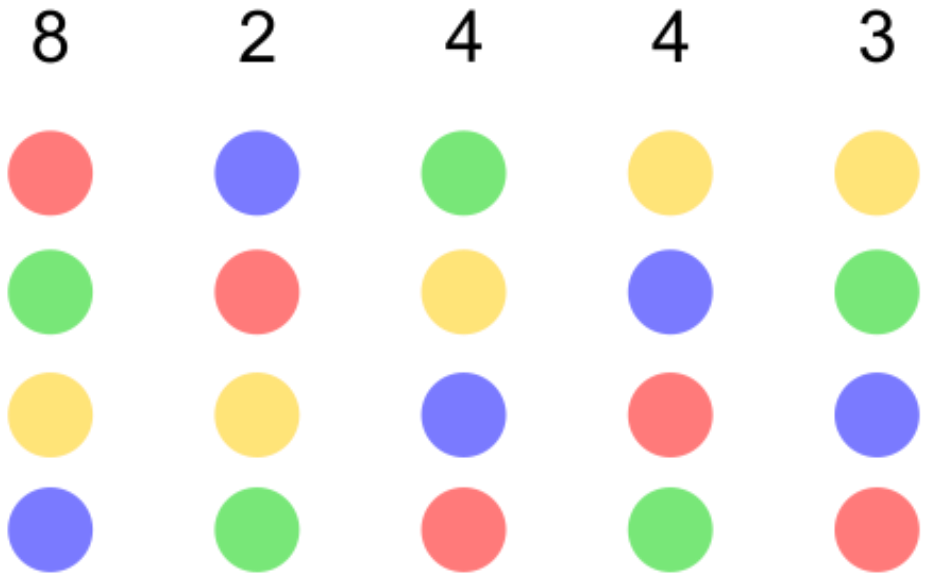

	0	+7	-1	-5
	-7	0	+3	+9
	+1	-3	0	+17
	+5	-9	-17	0

#voters who think >

(2+4+4+3) - 8

#voters who think >

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Schulze

8	2	4	4	3

	0	+7	-1	-5
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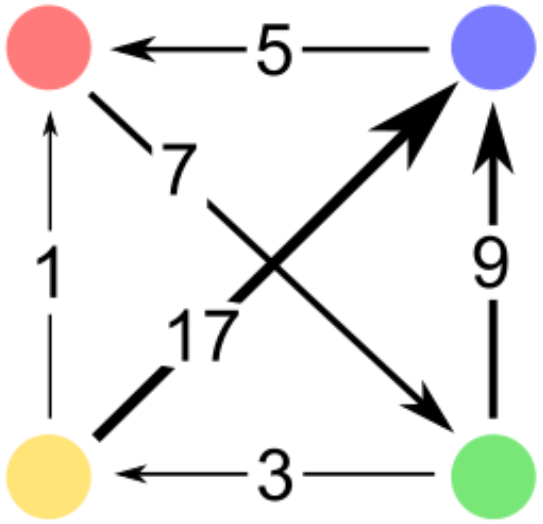

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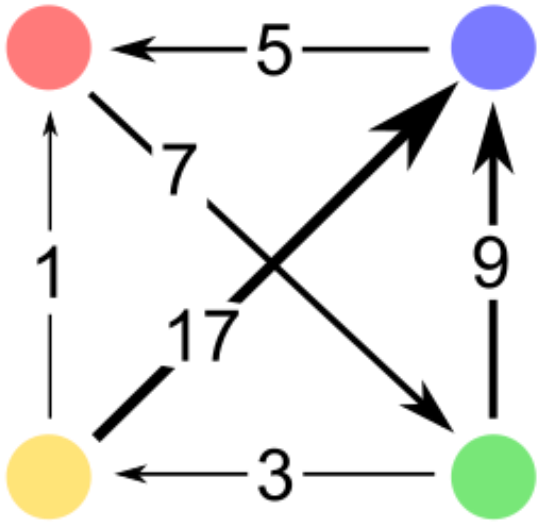

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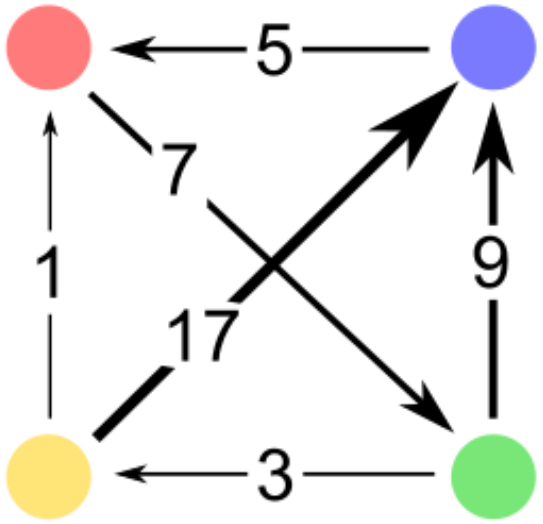

The strength of a path/chain is the weight of the weakest link in it

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Schulze

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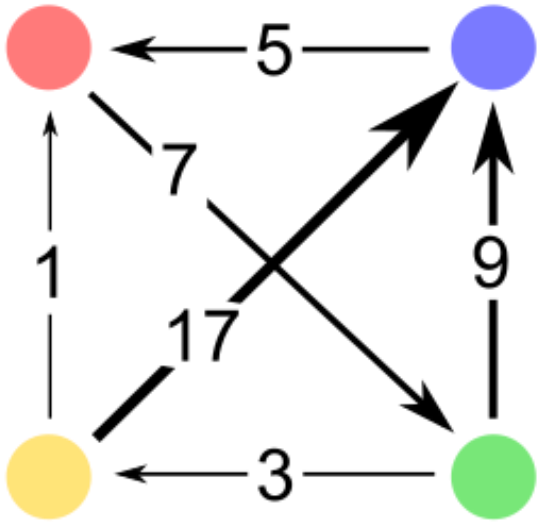

The strength of a path/chain is the weight of the weakest link in it

What's the strongest path from  to  ?

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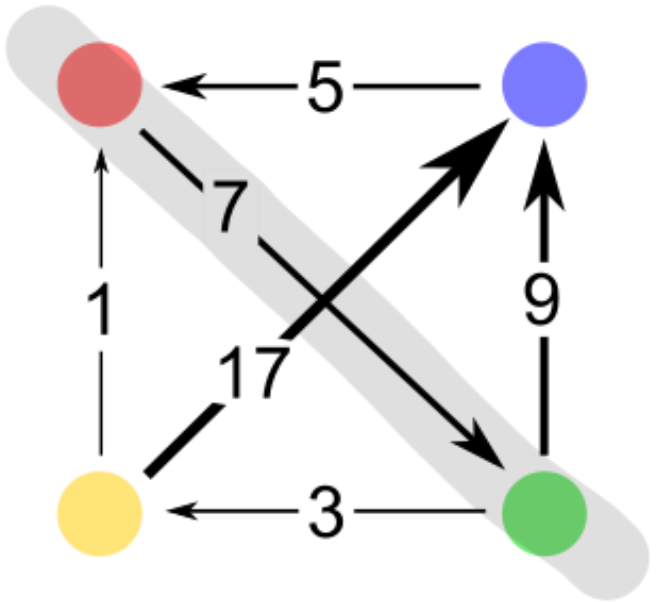

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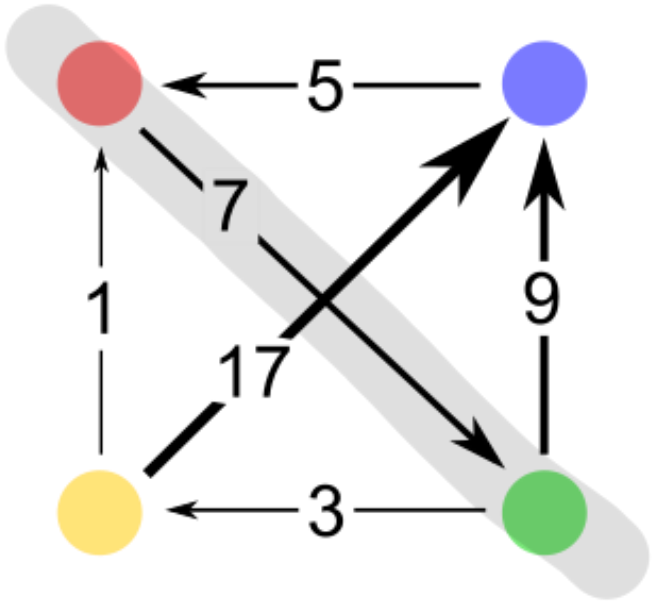

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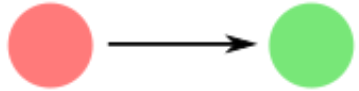

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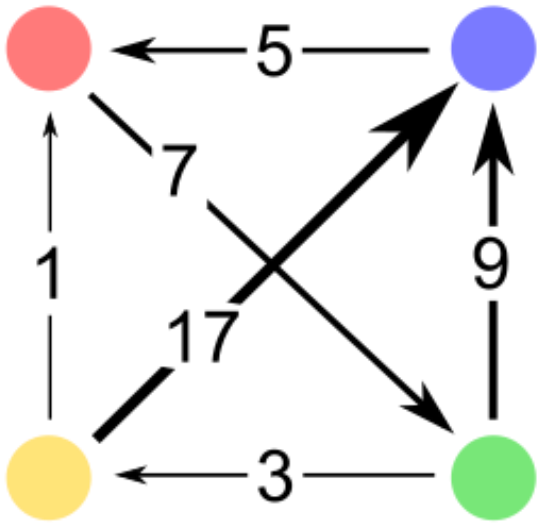

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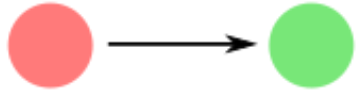

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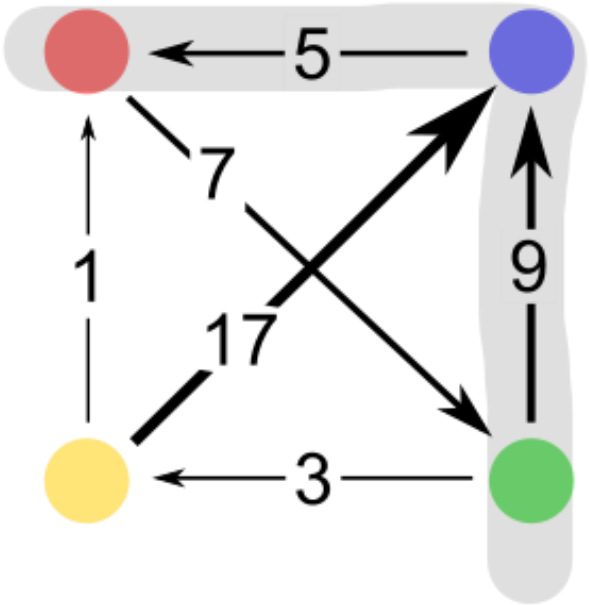

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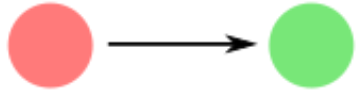

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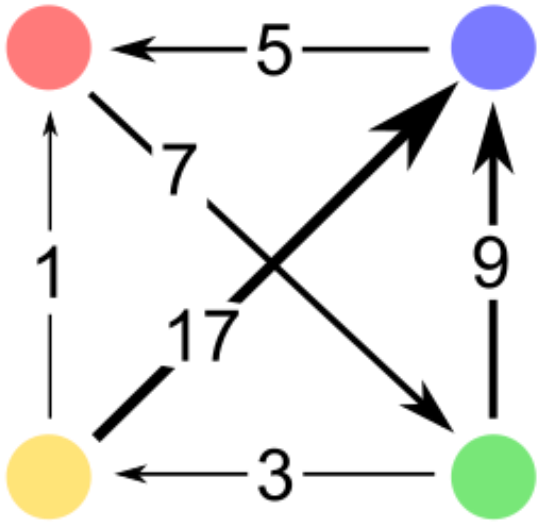

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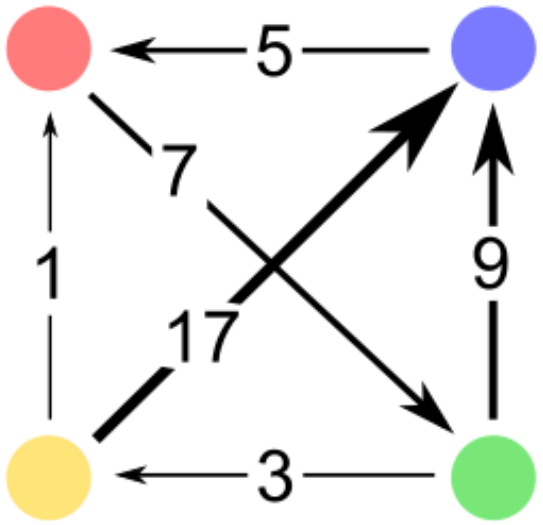

ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
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is *stronger* than
the *strongest* path from ⓑ to ⓐ

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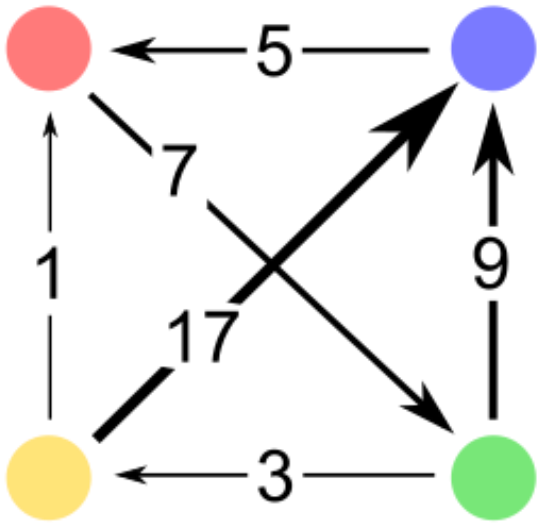

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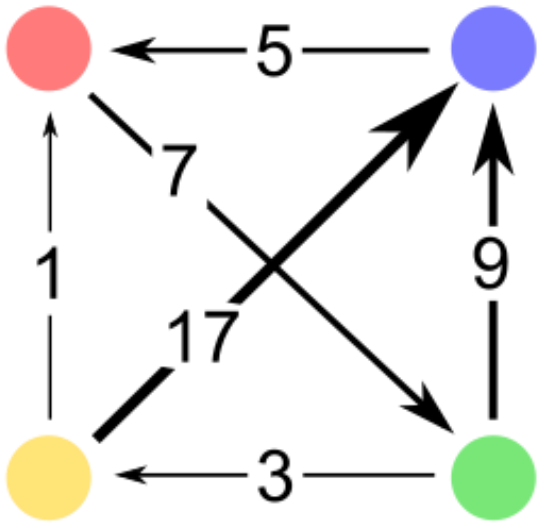
Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
the *strongest* path from Ⓐ to Ⓑ
is *stronger* than
the *strongest* path from Ⓑ to Ⓐ

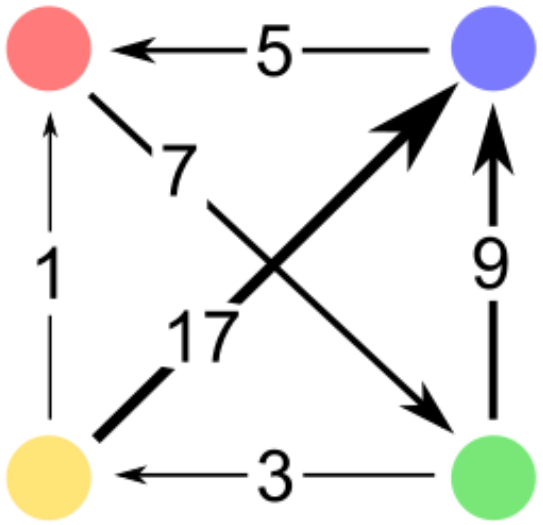
● >> ●

the strongest path from ● to ● has strength 7

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



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the strongest path from ● to ● has strength 5

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Schulze

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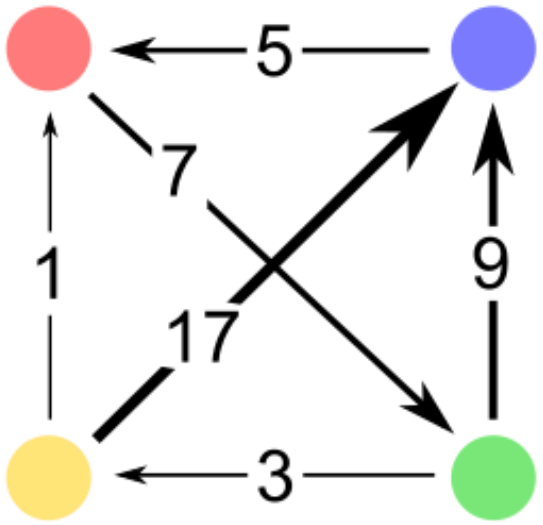

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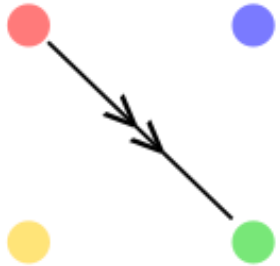

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Schulze

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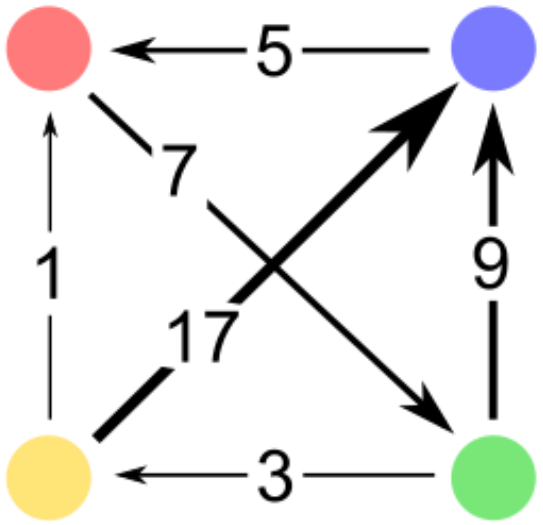

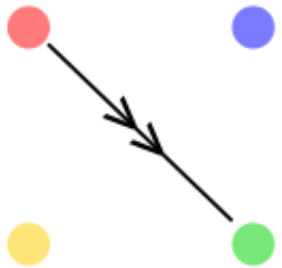

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Schulze

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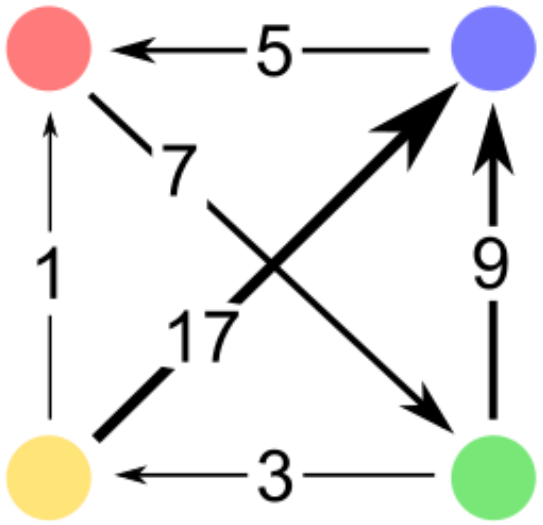

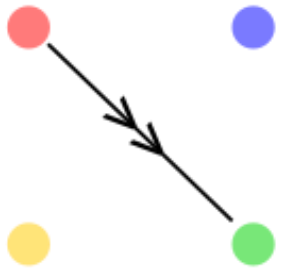

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Schulze

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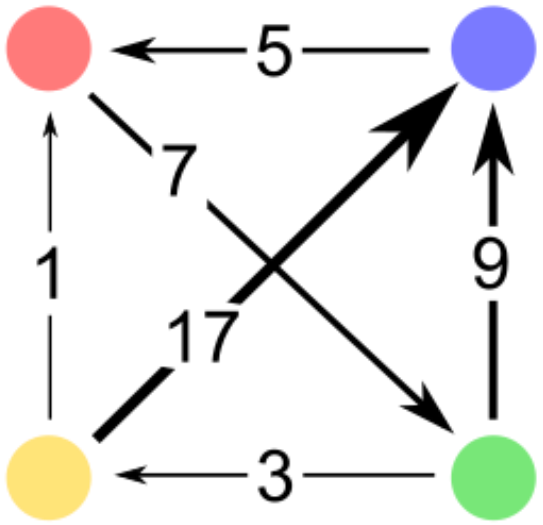

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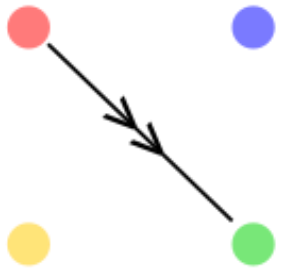
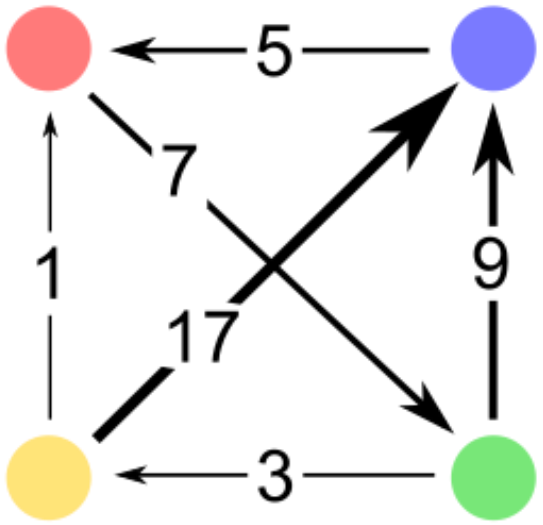


the strongest path from  to  has strength 7

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



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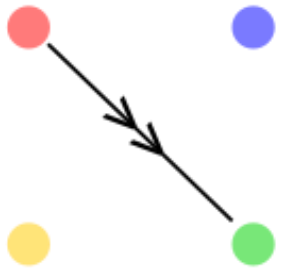


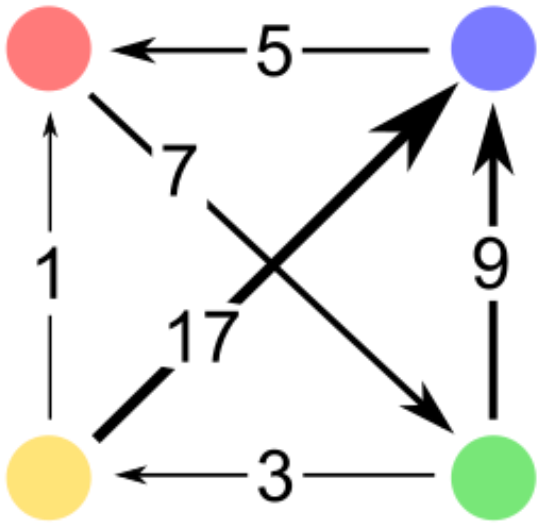
the strongest path from  to  has strength 7

the strongest path from  to  has strength 5

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



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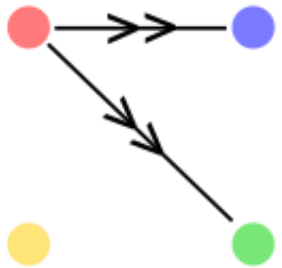


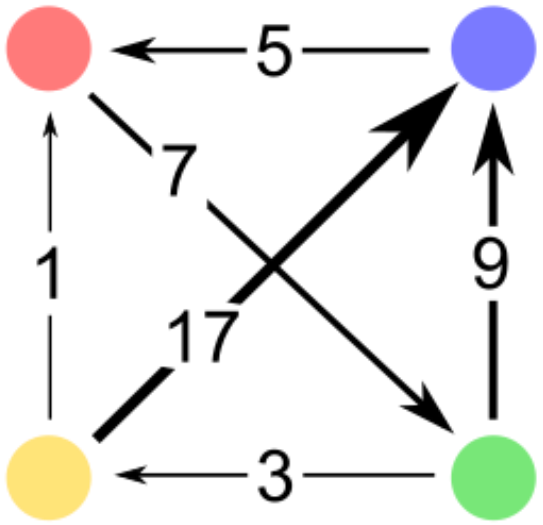
the strongest path from  to  has strength 7

the strongest path from  to  has strength 5

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



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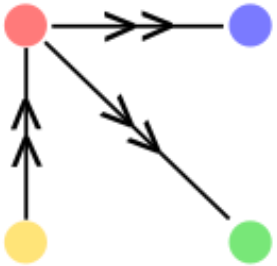


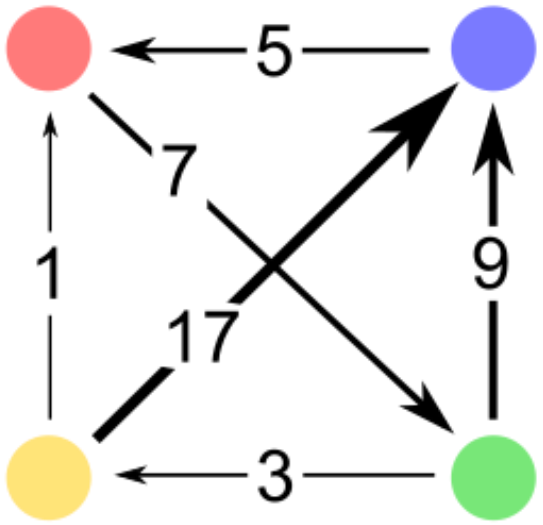
the strongest path from ● to ● has strength 5

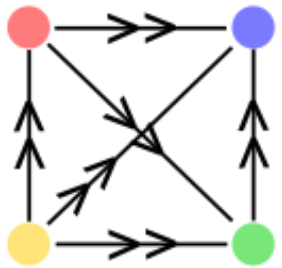
the strongest path from ● to ● has strength 3

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph





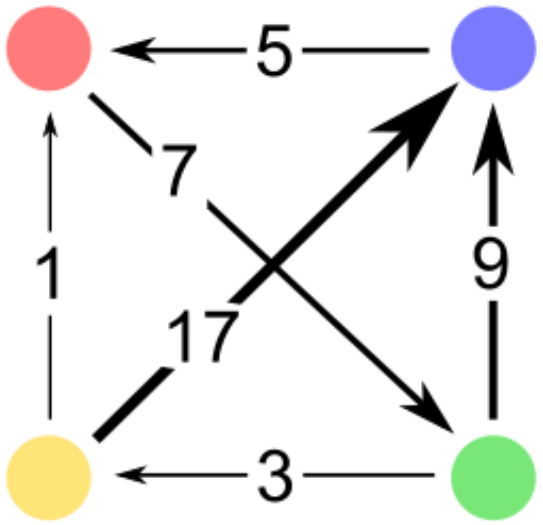
ⓐ>>ⓑ (ⓐ"chain beats"ⓑ) if
the *strongest* path from ⓐ to ⓑ
is *stronger* than
the *strongest* path from ⓑ to ⓐ

● >> all others

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
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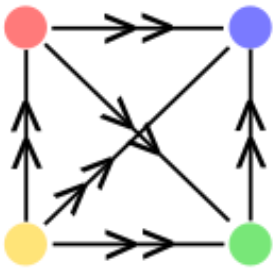


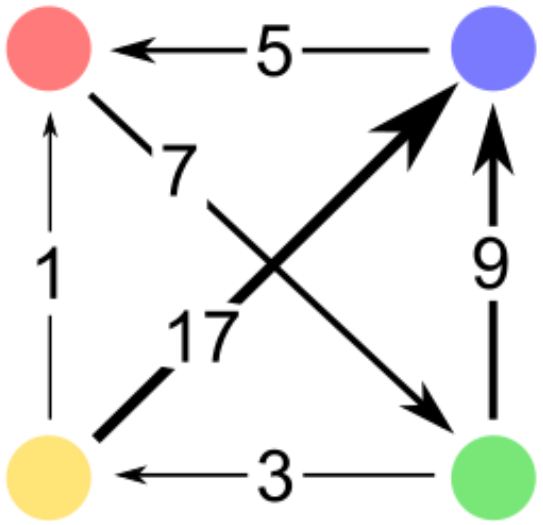
● >> all others

Schulze winner: ●

6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



Ⓐ>>Ⓑ (Ⓐ"chain beats"Ⓑ) if
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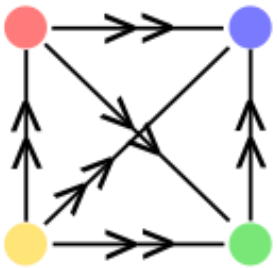
is *stronger* than
the *strongest* path from Ⓑ to Ⓐ

A Schulze winner always exists!



The "chain beats" relation is transitive
Ⓐ>>Ⓑ and Ⓑ>>Ⓒ, then Ⓐ>>Ⓒ

6

Schulze


ubuntu


debian
GNU/Linux


gentoo linux™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Which Voting Rule is the Best?

Which Voting Rule is the Best?

Voting Power in Practice Summer Workshop

Assessing Alternative Voting Procedures

Sponsored by The Leverhulme Trust

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30 July – 2 August 2010

22 voting theorists

"What is the best voting rule for your town to use to elect the mayor?"

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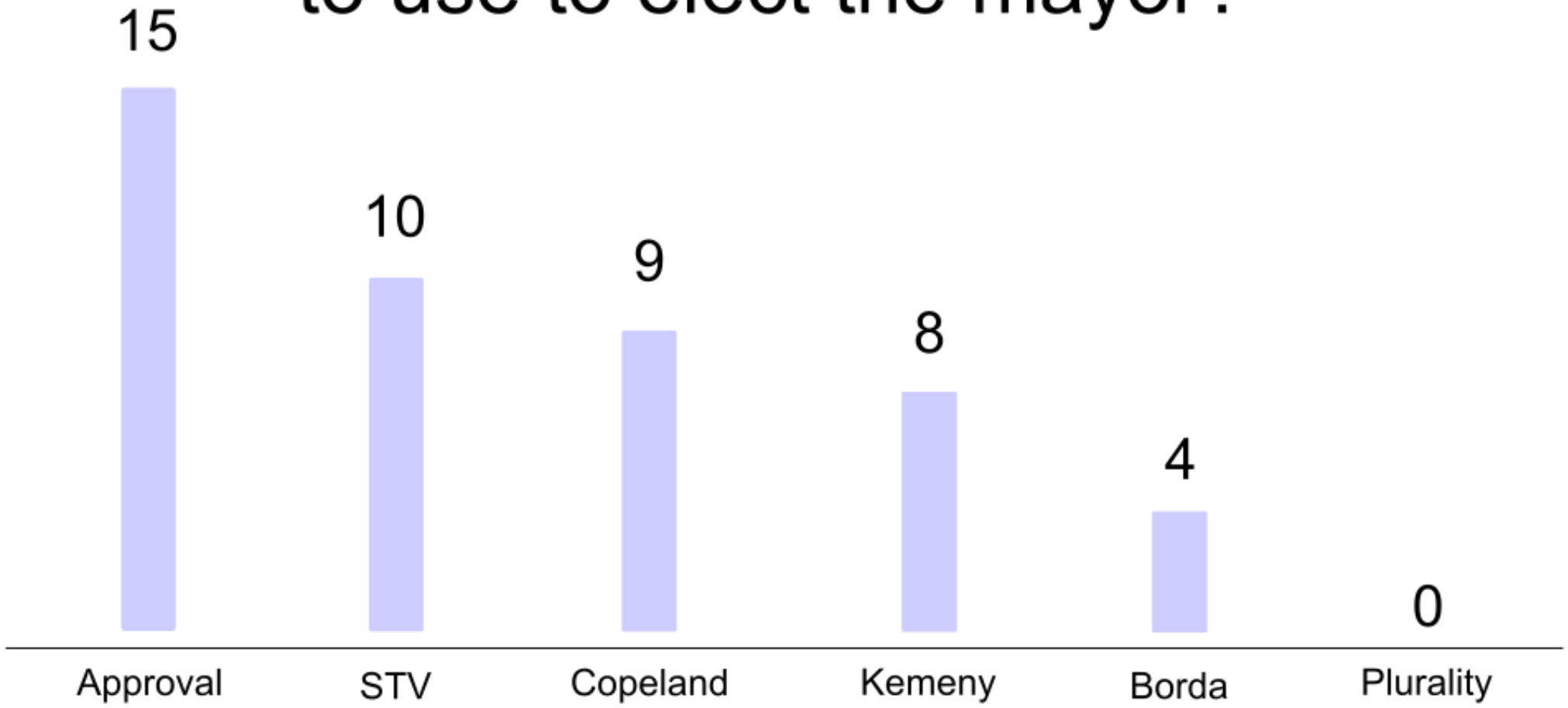
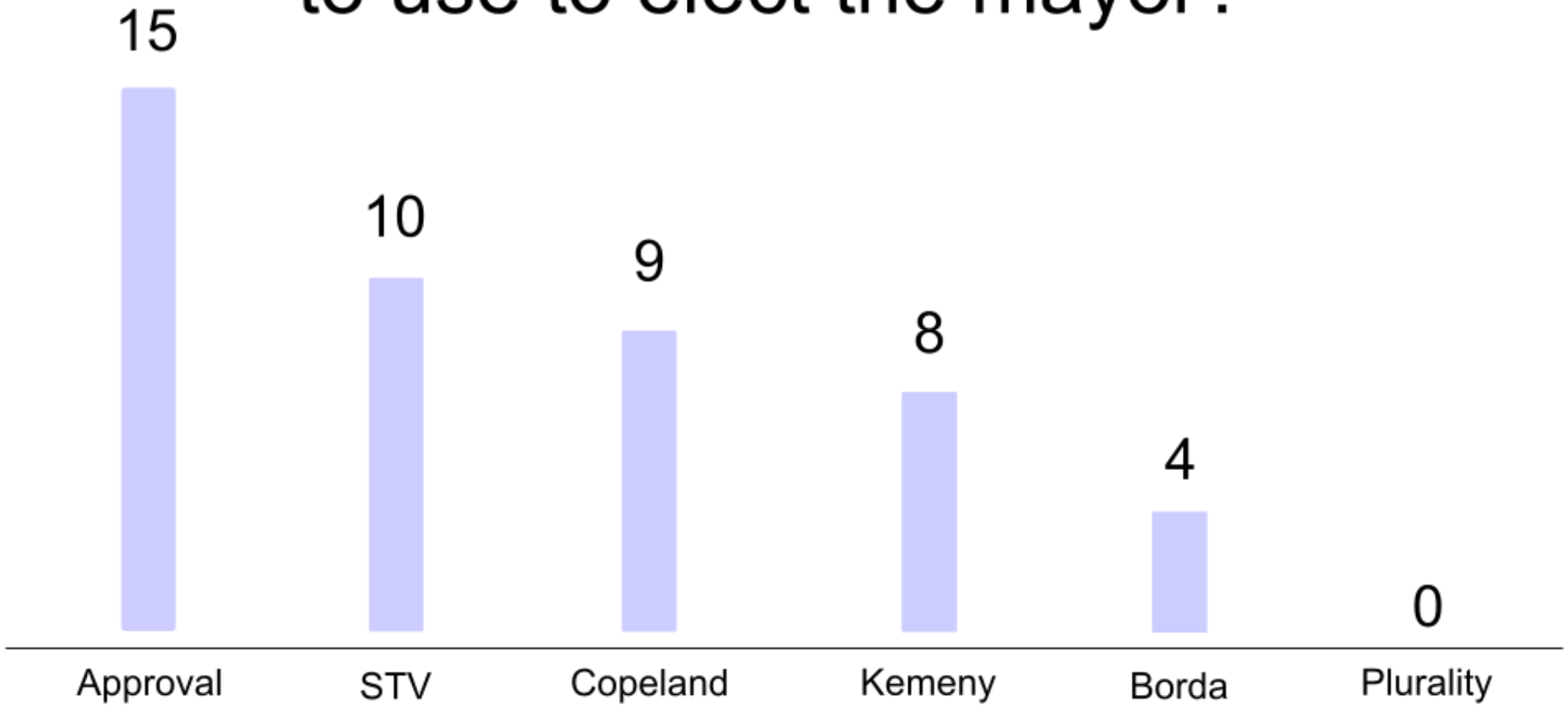
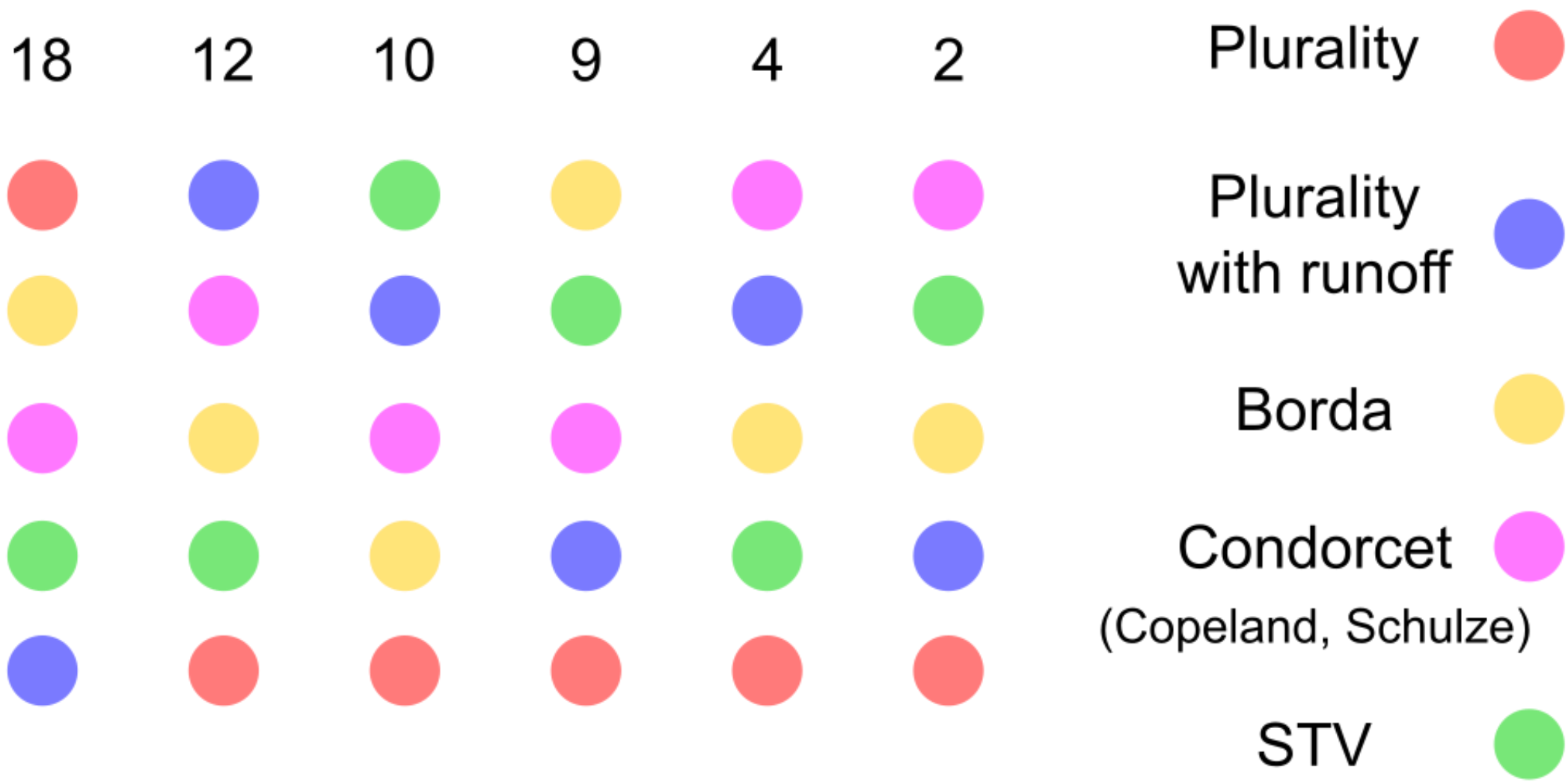


"What is the best voting rule for your town to use to elect the mayor?"

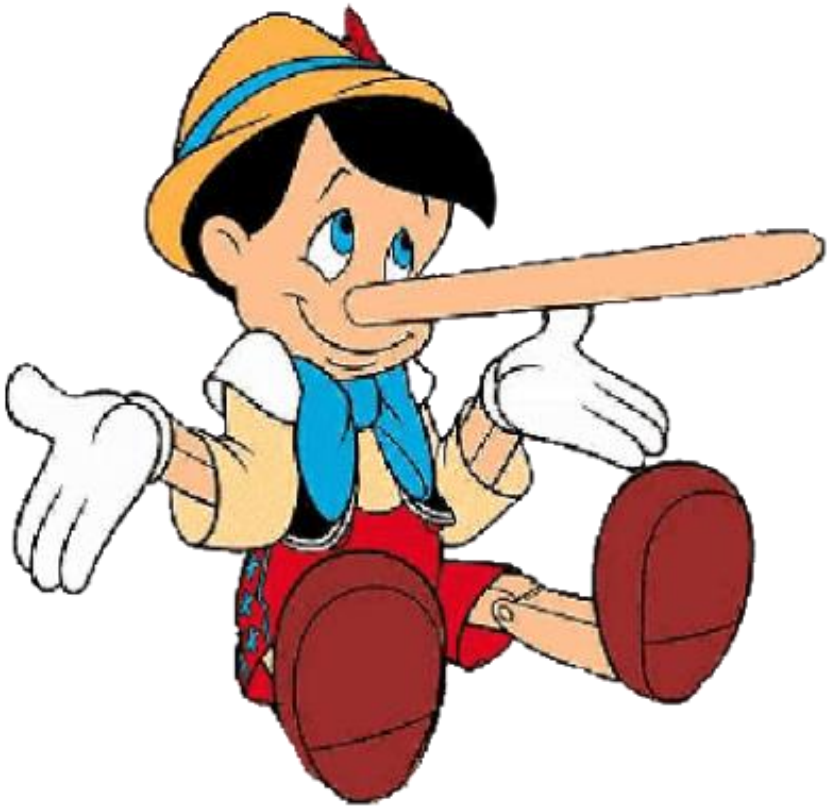


Votes were counted via approval voting.


18
12
10
9
4
2
Plurality
Plurality
with runoff
Borda
Condorcet
(Copeland, Schulze)
STV

Next Time

Manipulation in voting



Quiz

Quiz

Suggest a voting rule you think is "reasonable"
(other than the ones we discussed today)
and justify why it is reasonable.

References

- “Disagreement between voting rules” example: http://www.ams.org/publicoutreach/feature-column/fcarc-voting-decision
- The “chain beats” terminology in the description of Schulze rule was borrowed from Hubert Bray’s explanatory video: https://www.youtube.com/watch?v=_HVeN0GnnuA
- The Schulze rule example is from the paper “The Schulze Method of Voting” by Markus Schulze: https://arxiv.org/abs/1804.02973
- (Possibly) strategic voting in the selection of Olympics host: https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic_2_Plurality_Runoff.pdf

References

- Non-monotonicity of STV:

  Gideon Doron and Richard Kronick
  Single Transferable Vote: An Example of a Perverse Social Choice Function
  American Journal of Political Science, Vol. 21, No. 2 (May, 1977), pg 303-311
  https://www.jstor.org/stable/2110496

- “Which voting rule is the best” poll
  https://hal.archives-ouvertes.fr/hal-00609810/document