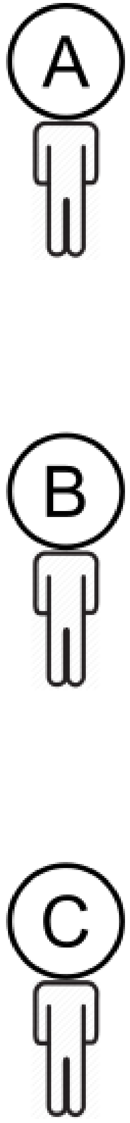
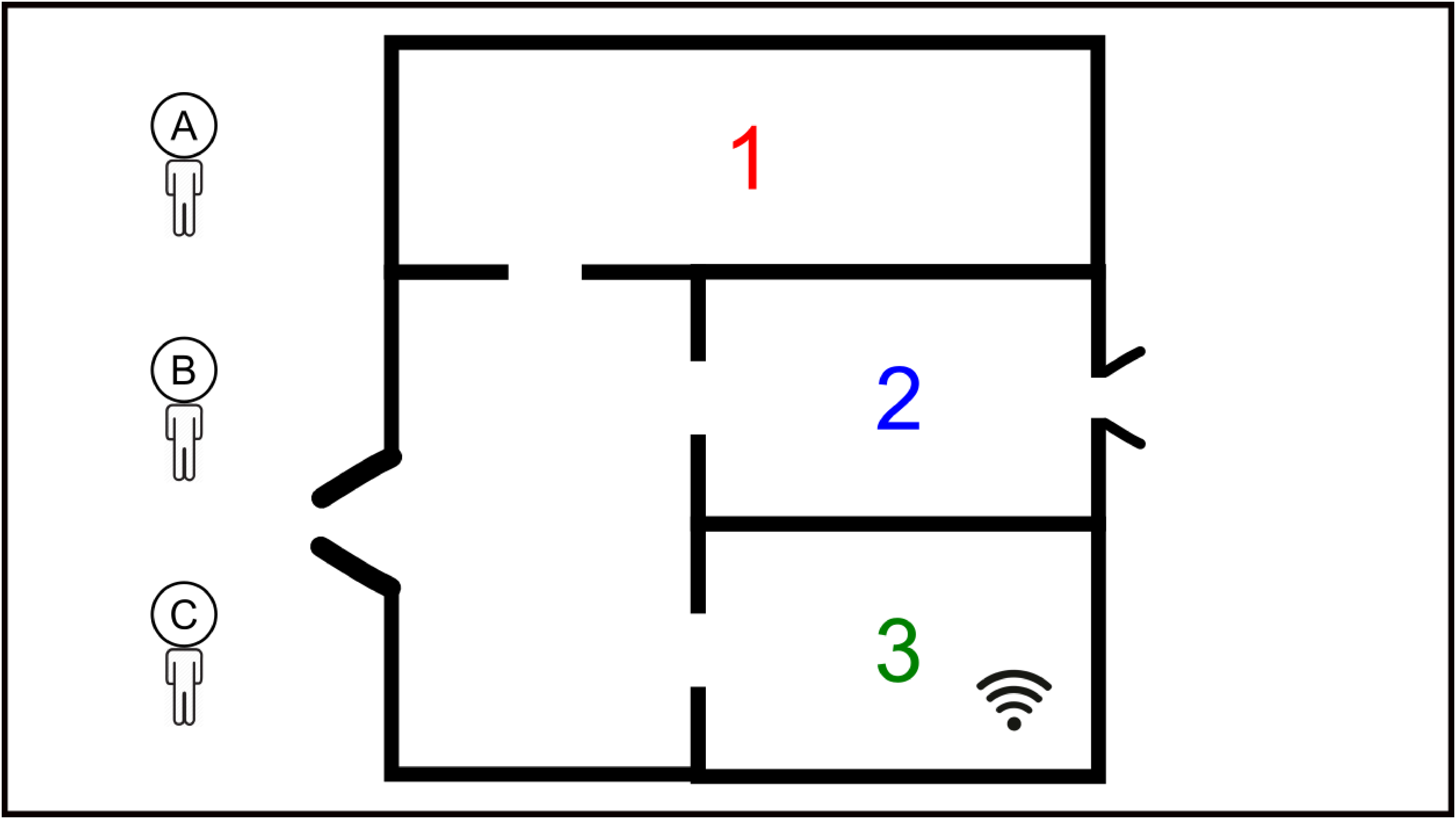
COL866: Special Topics in Algorithms

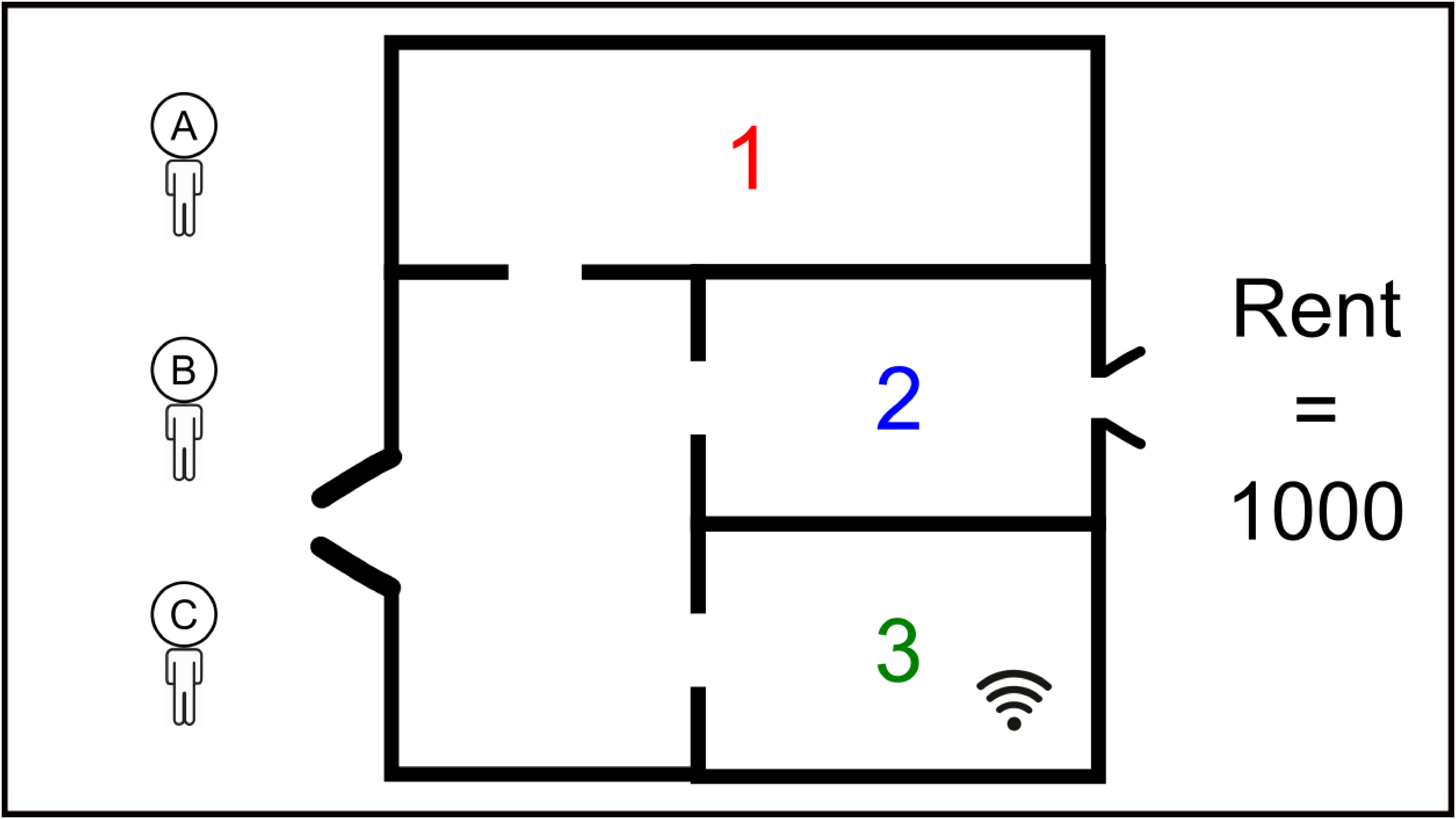
Lecture 13

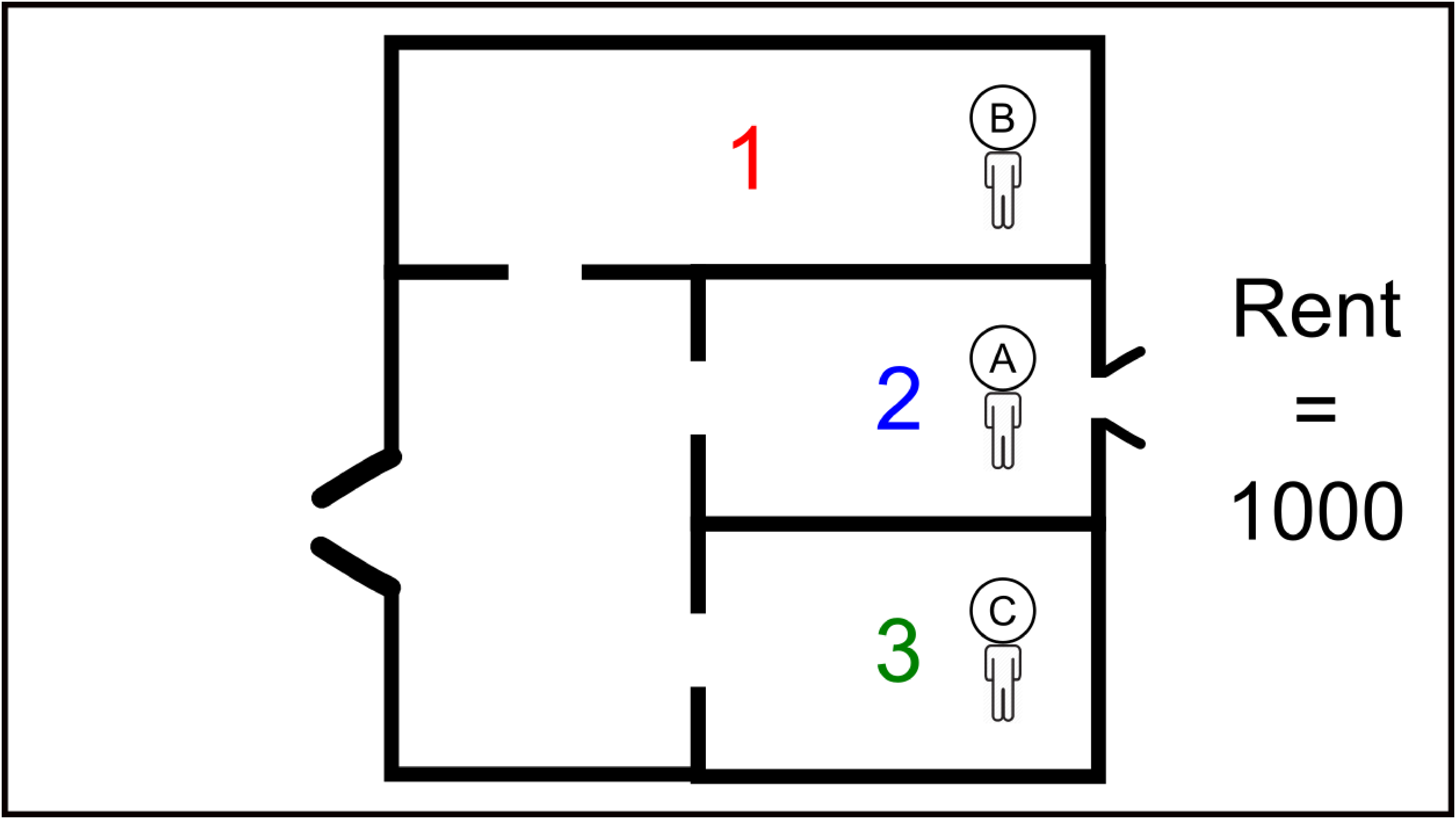
Rent Division

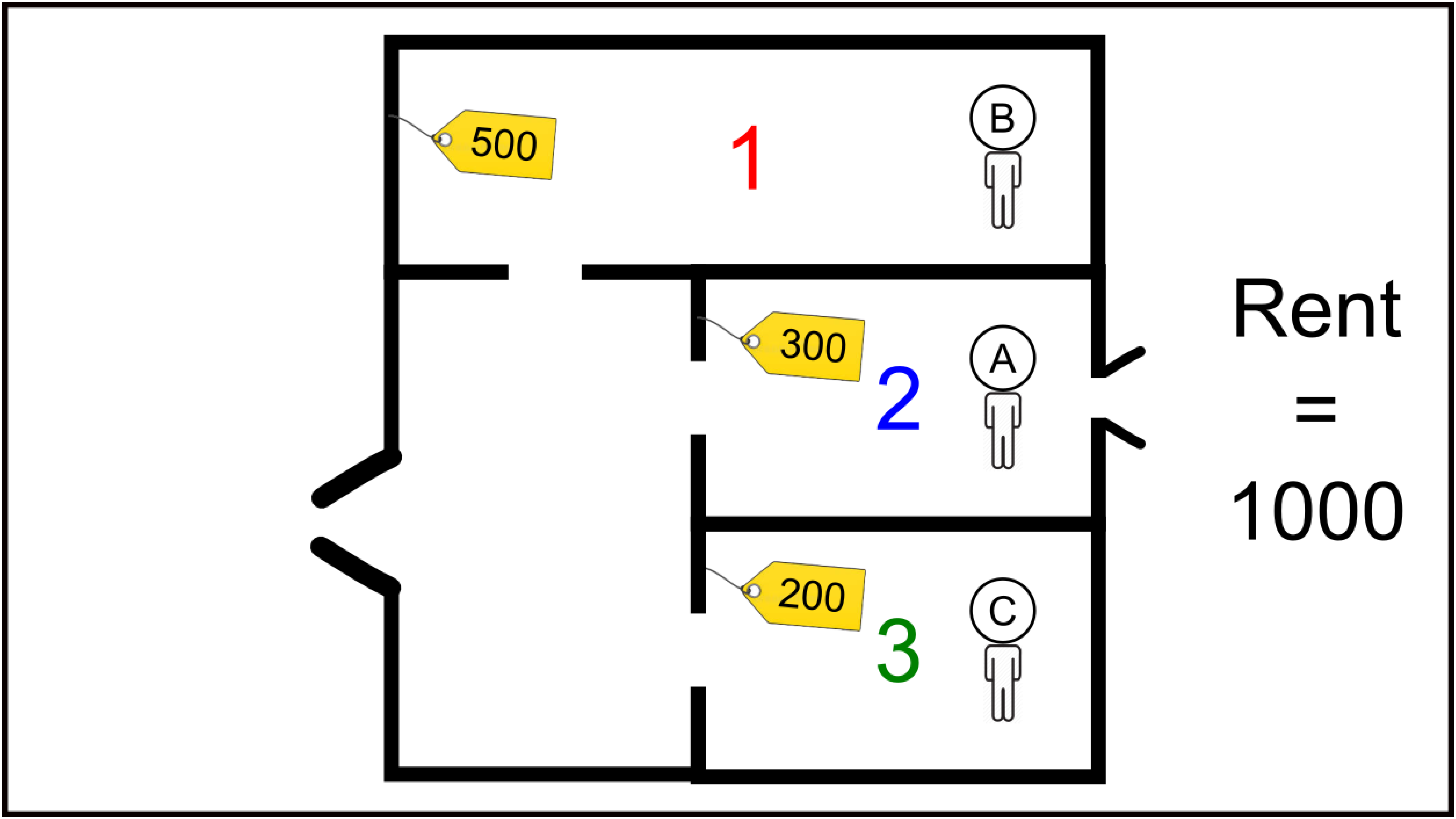
Sept 26, 2023 | Rohit Vaish

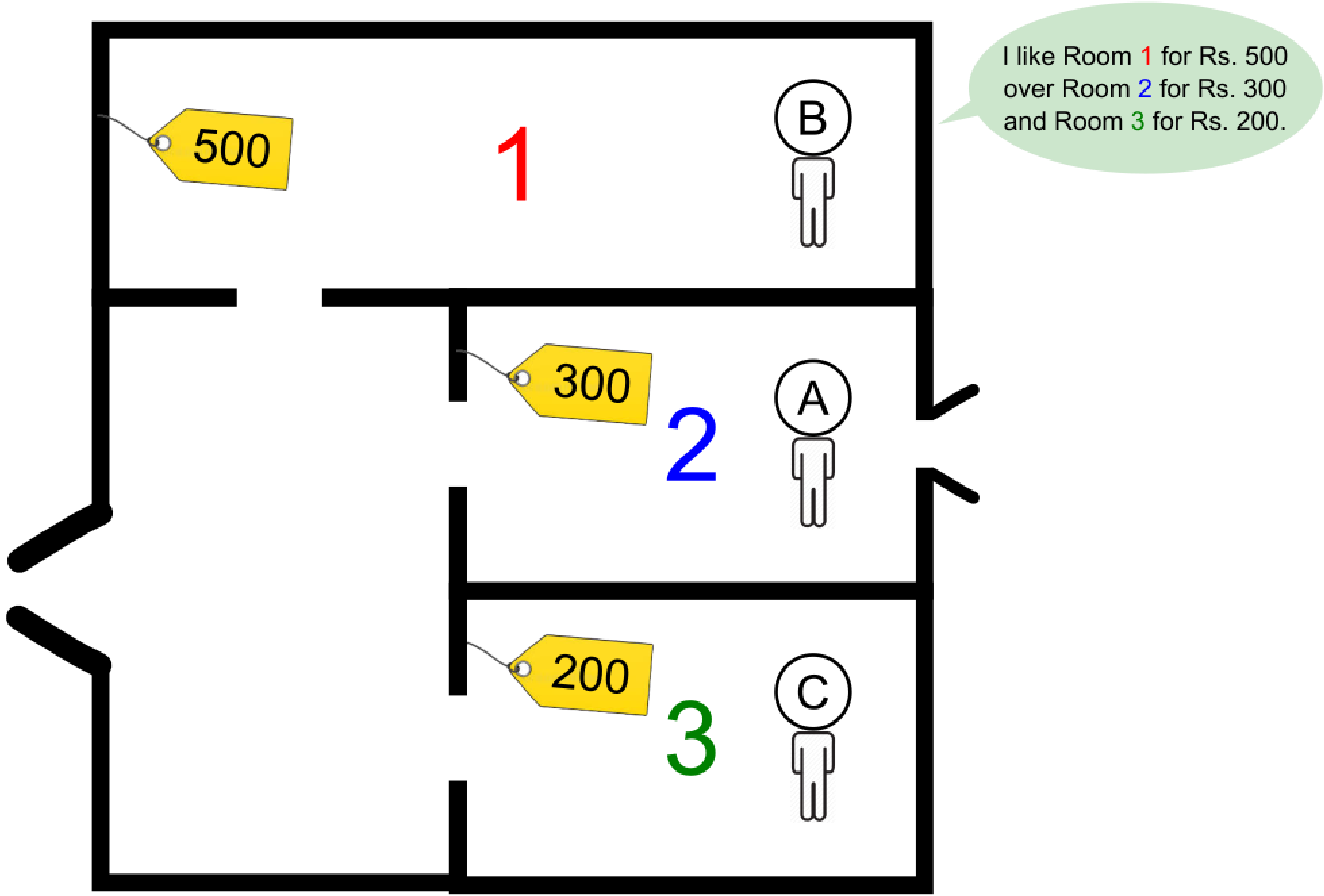

A
B
C

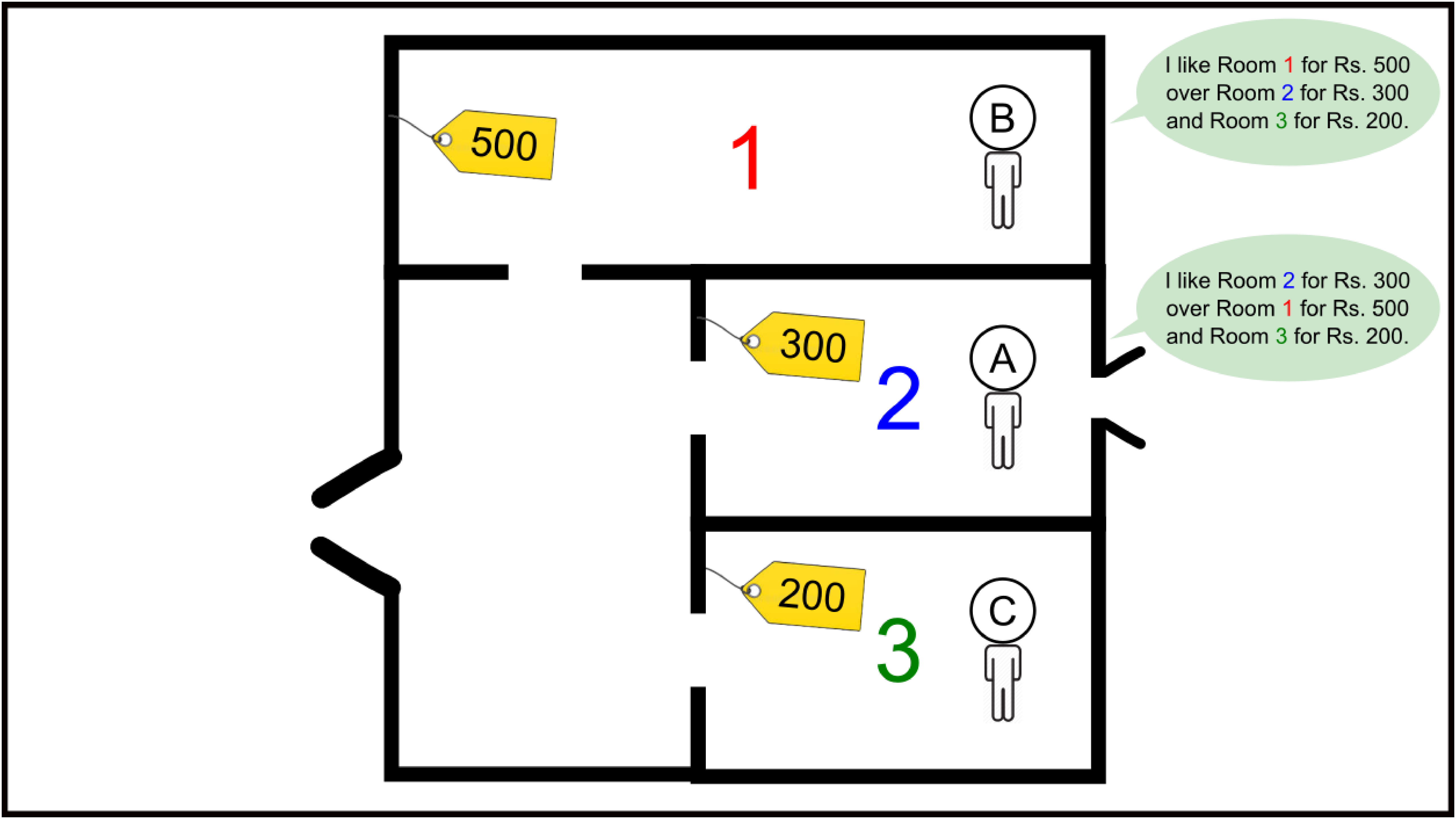

A
B
C
1
2
3

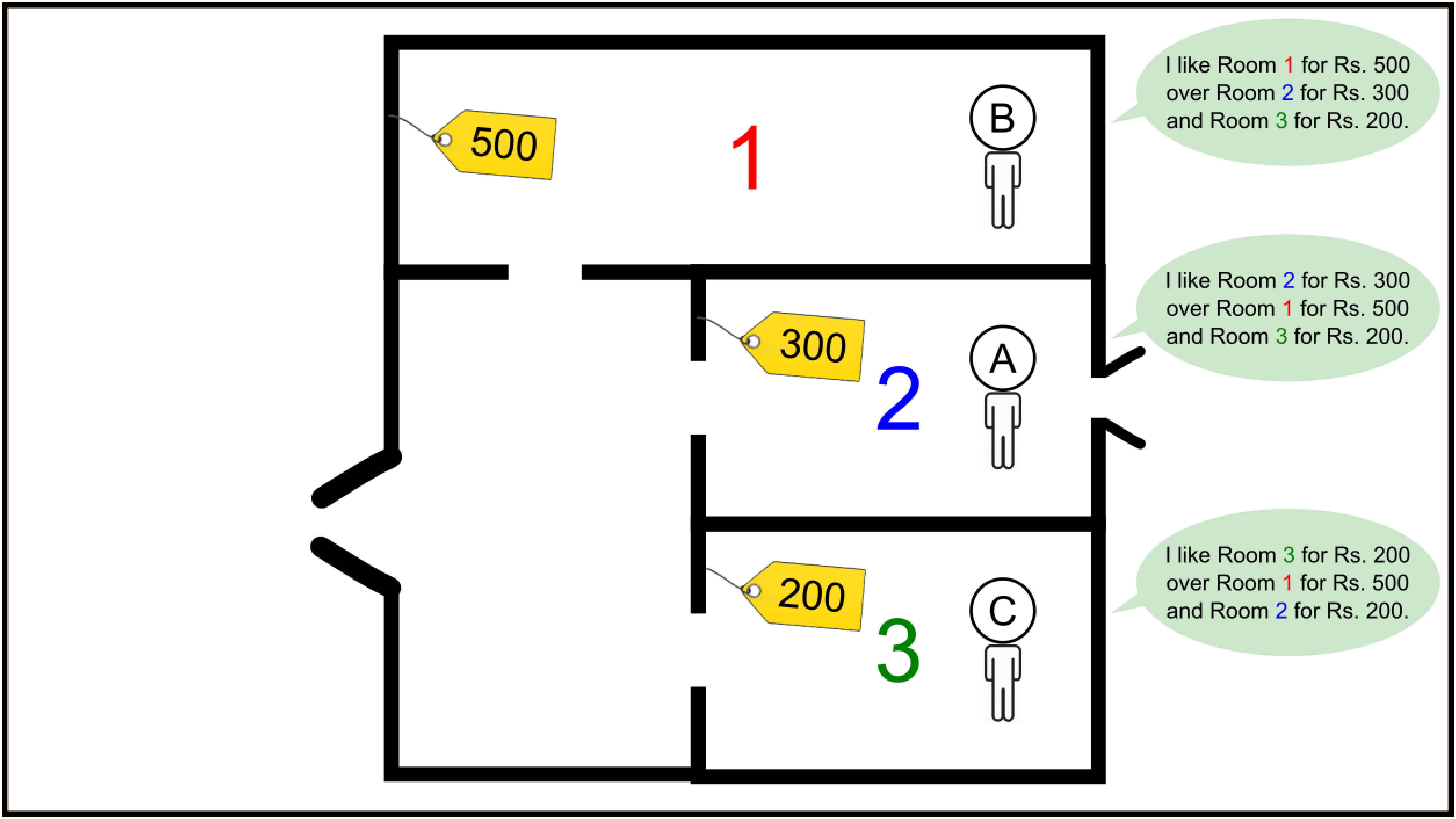

A
B
C
1
2
3
Rent
=
1000

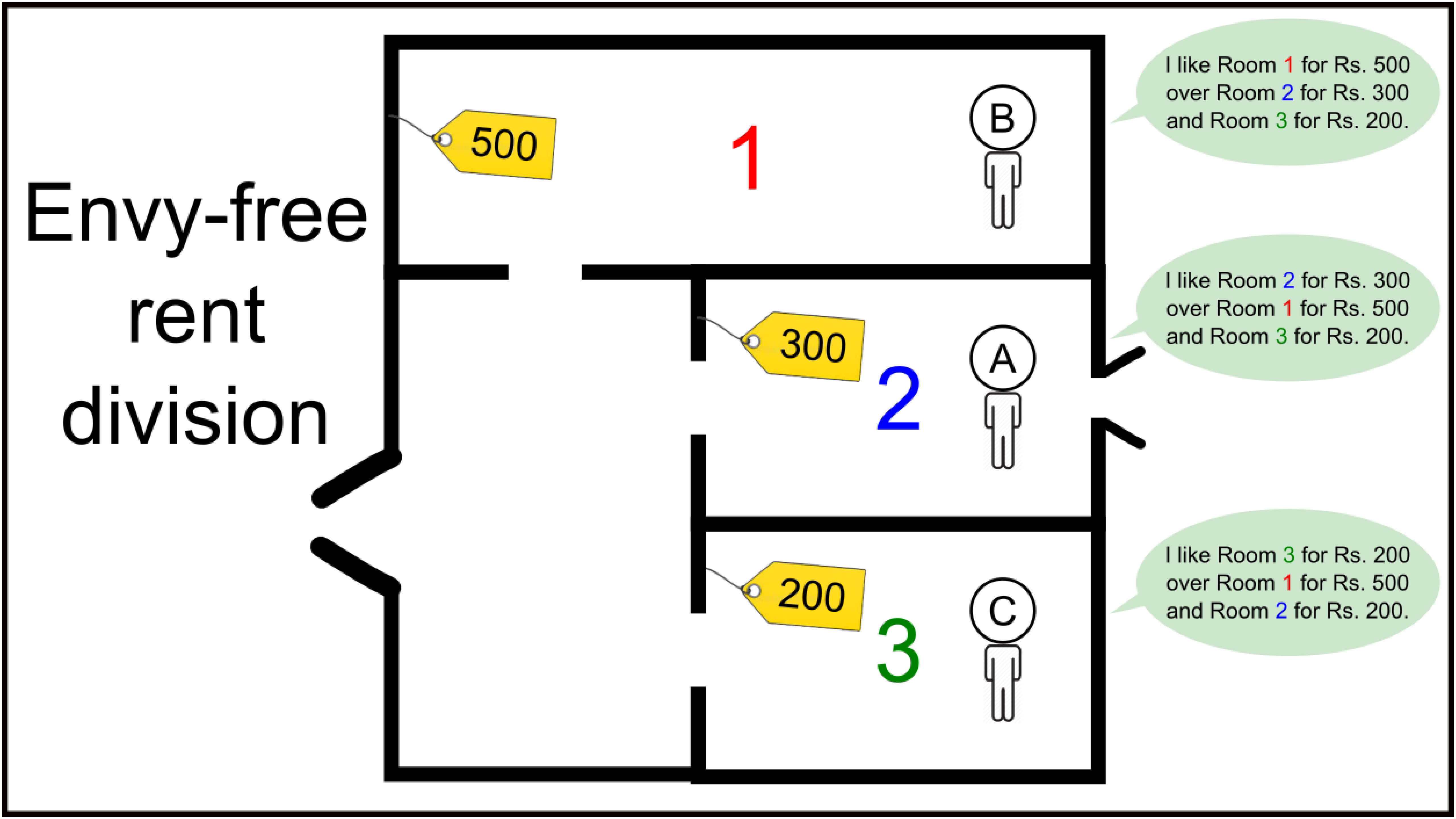

1
B
2
A
3
C
Rent
=
1000


500
1
B
300
2
A
200
3
C
Rent
=
1000


I like Room 1 for Rs. 500
over Room 2 for Rs. 300
and Room 3 for Rs. 200.
500
1
B
300
2
A
200
3
C


500
1
B
I like Room 1 for Rs. 500 over Room 2 for Rs. 300 and Room 3 for Rs. 200.
300
2
A
I like Room 2 for Rs. 300 over Room 1 for Rs. 500 and Room 3 for Rs. 200.
200
3
C


500
1
B
I like Room 1 for Rs. 500 over Room 2 for Rs. 300 and Room 3 for Rs. 200.
300
2
A
I like Room 2 for Rs. 300 over Room 1 for Rs. 500 and Room 3 for Rs. 200.
200
3
C
I like Room 3 for Rs. 200 over Room 1 for Rs. 500 and Room 2 for Rs. 200.



Envy-free rent division
500
1
B
300
2
A
200
3
C
I like Room 1 for Rs. 500 over Room 2 for Rs. 300 and Room 3 for Rs. 200.
I like Room 2 for Rs. 300 over Room 1 for Rs. 500 and Room 3 for Rs. 200.
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Sperner's Lemma

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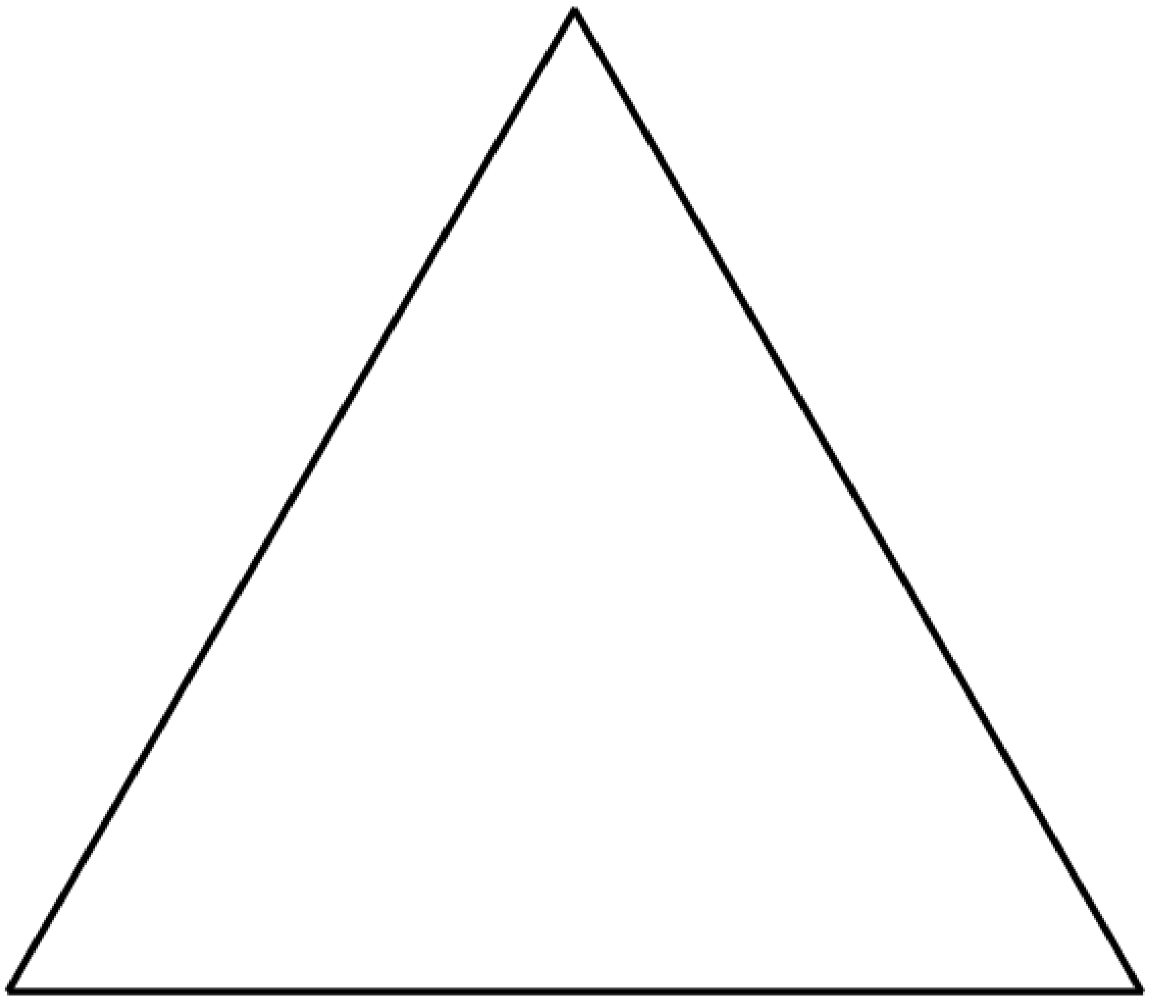

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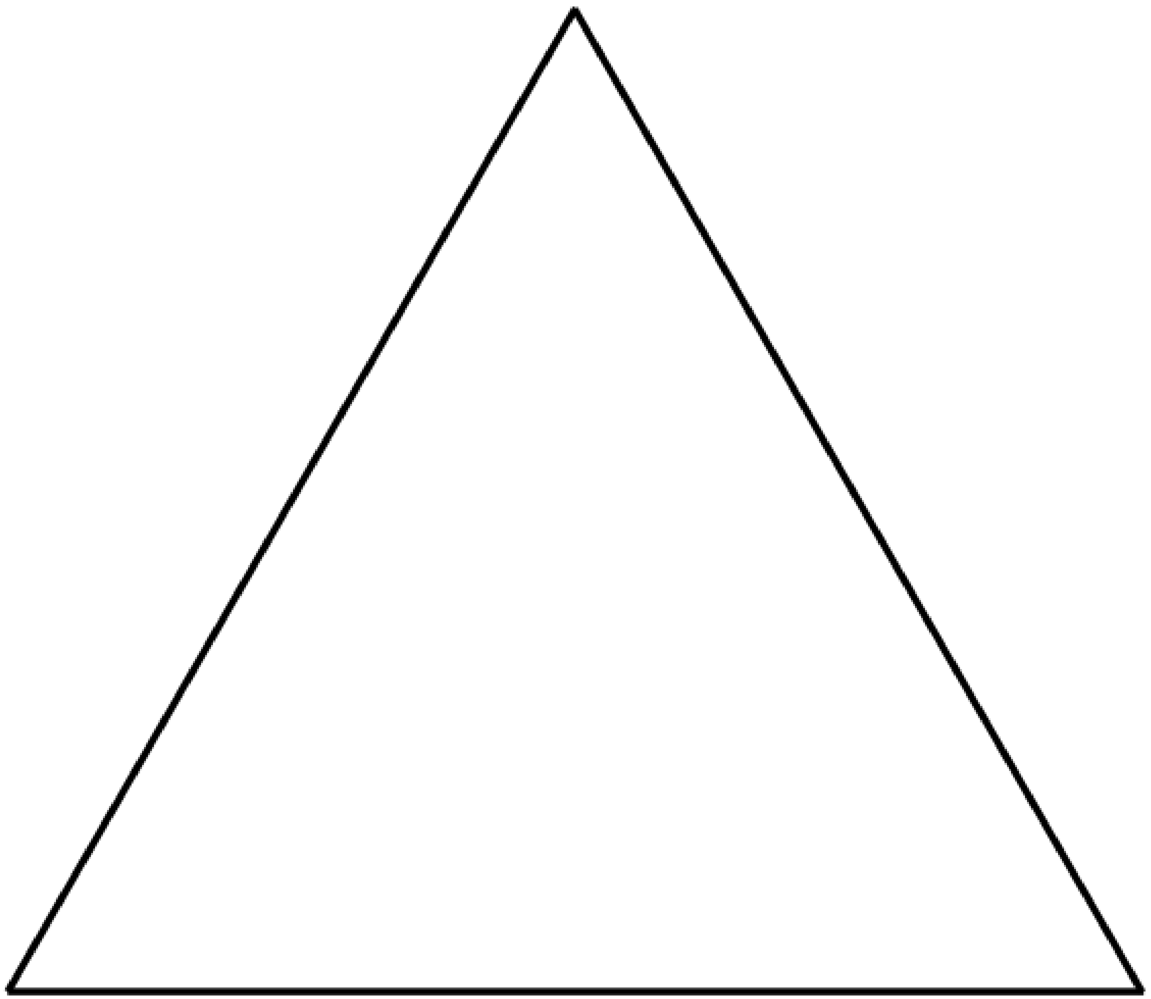
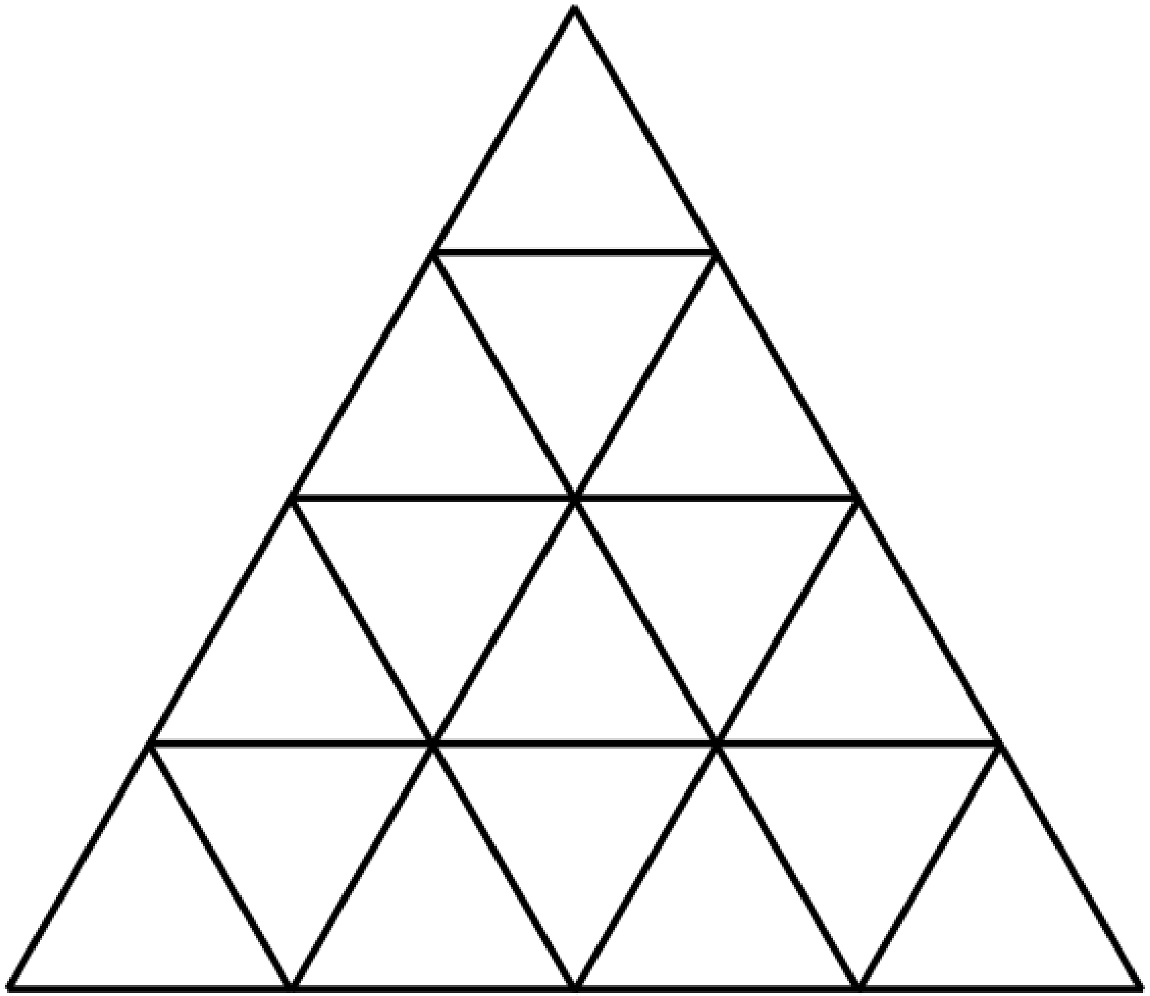


1. Triangulation into *elementary* triangles

Sperner's Lemma



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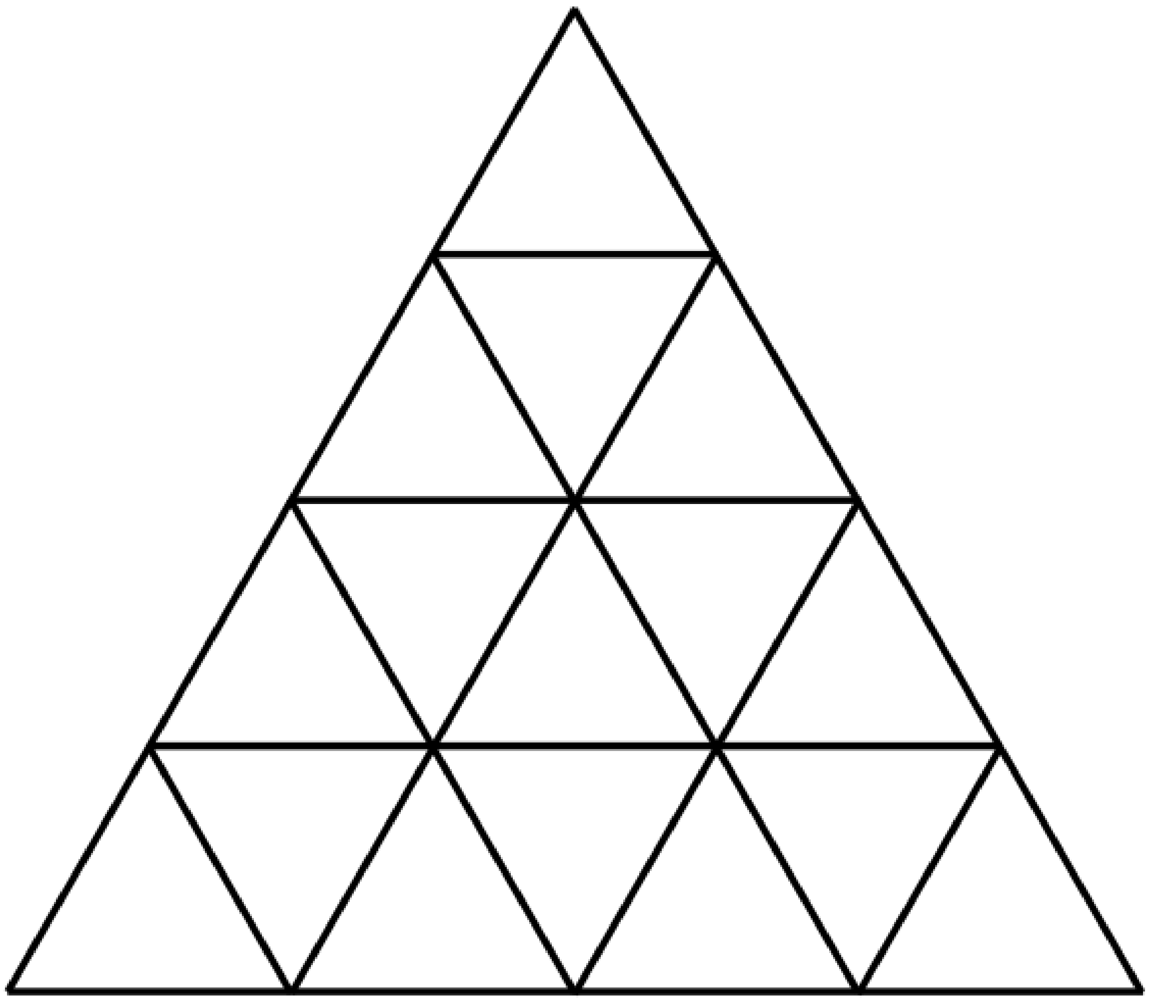
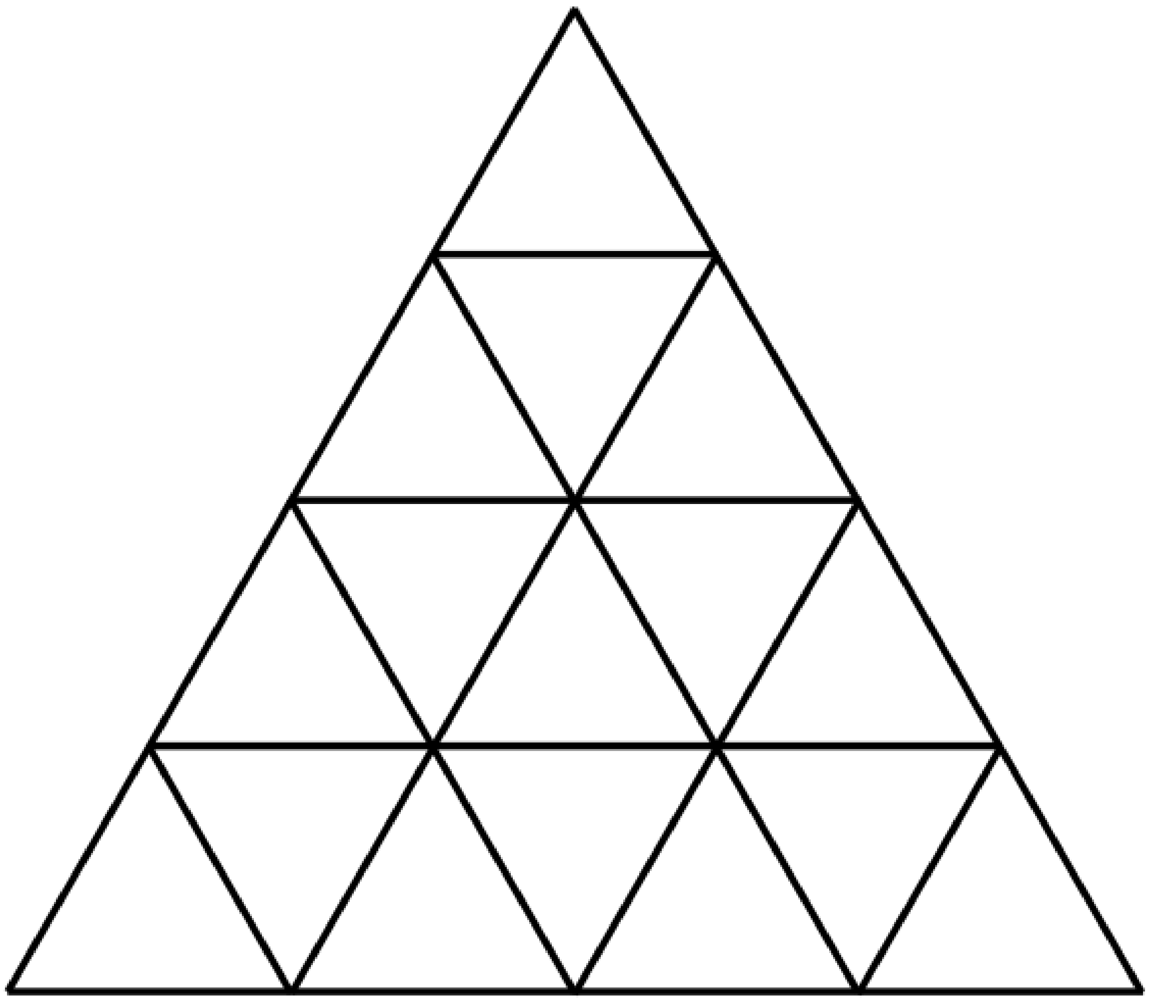


1. Triangulation into *elementary* triangles
2. Sperner labeling

Sperner's Lemma



1. Triangulation into *elementary* triangles

2. Sperner labeling

   - Main vertices have distinct labels

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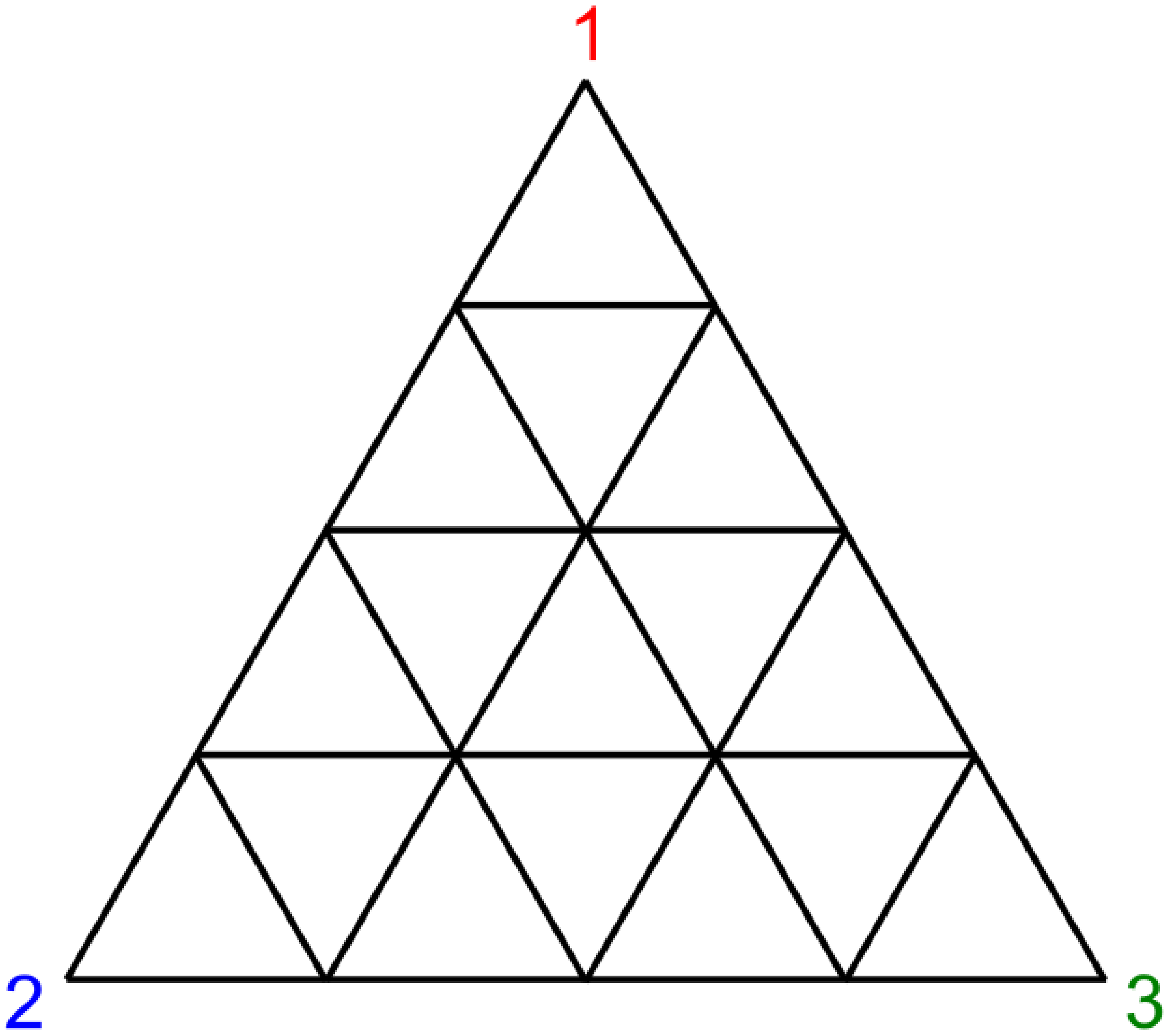

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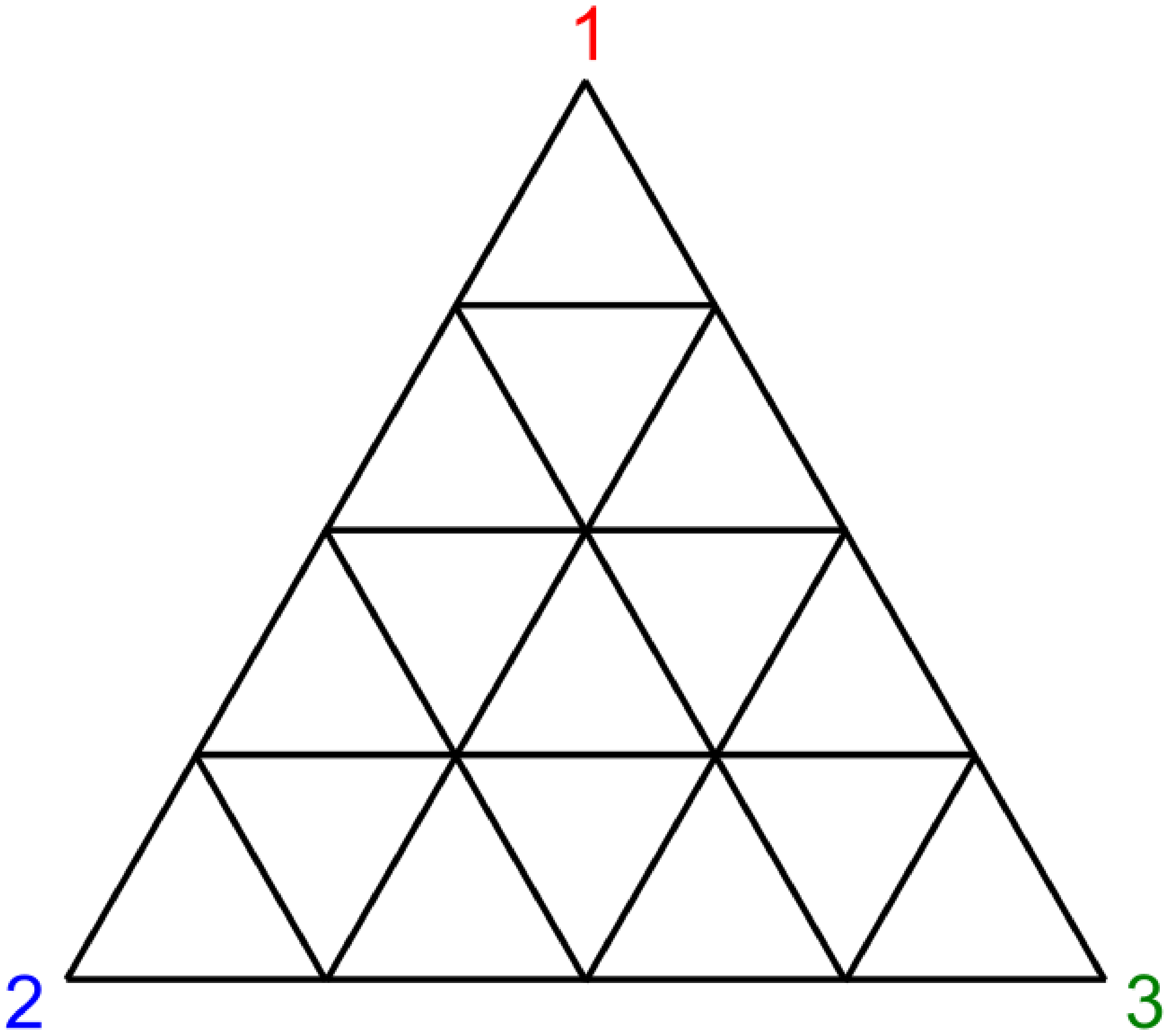

1. Triangulation into *elementary* triangles

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   - Main vertices have distinct labels
   - Boundary vertices inherit the labels of the adjacent main vertices

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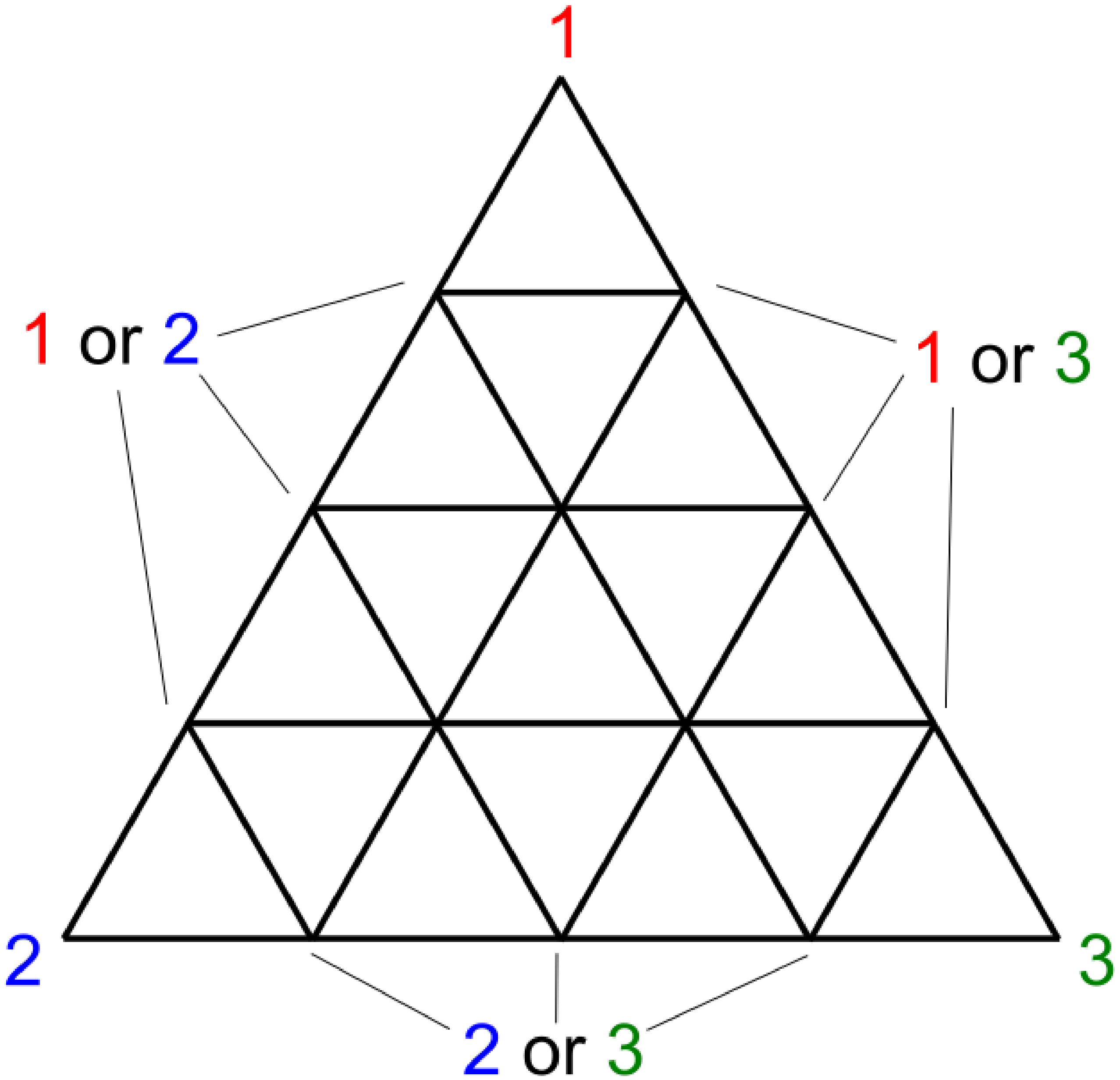

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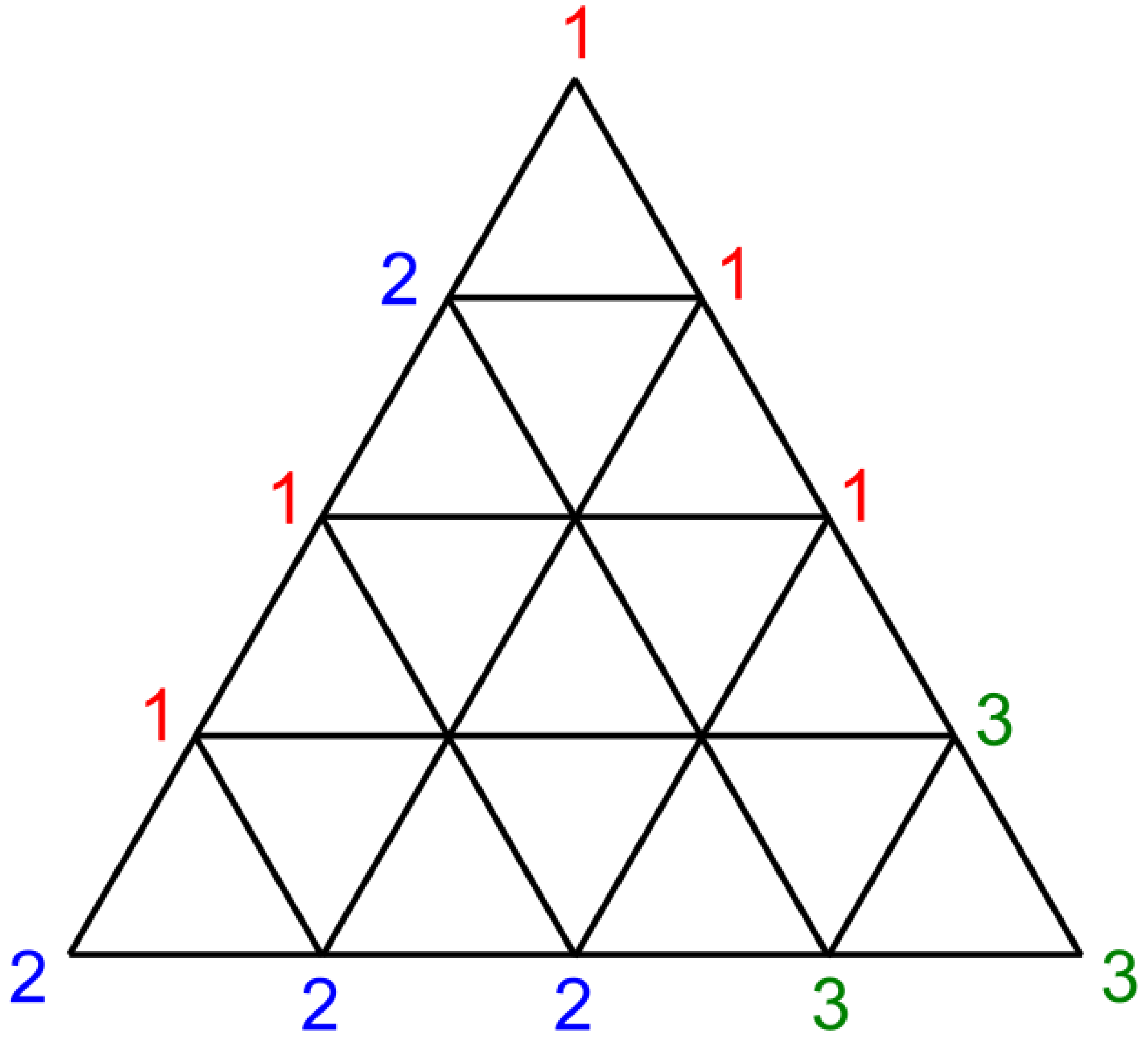

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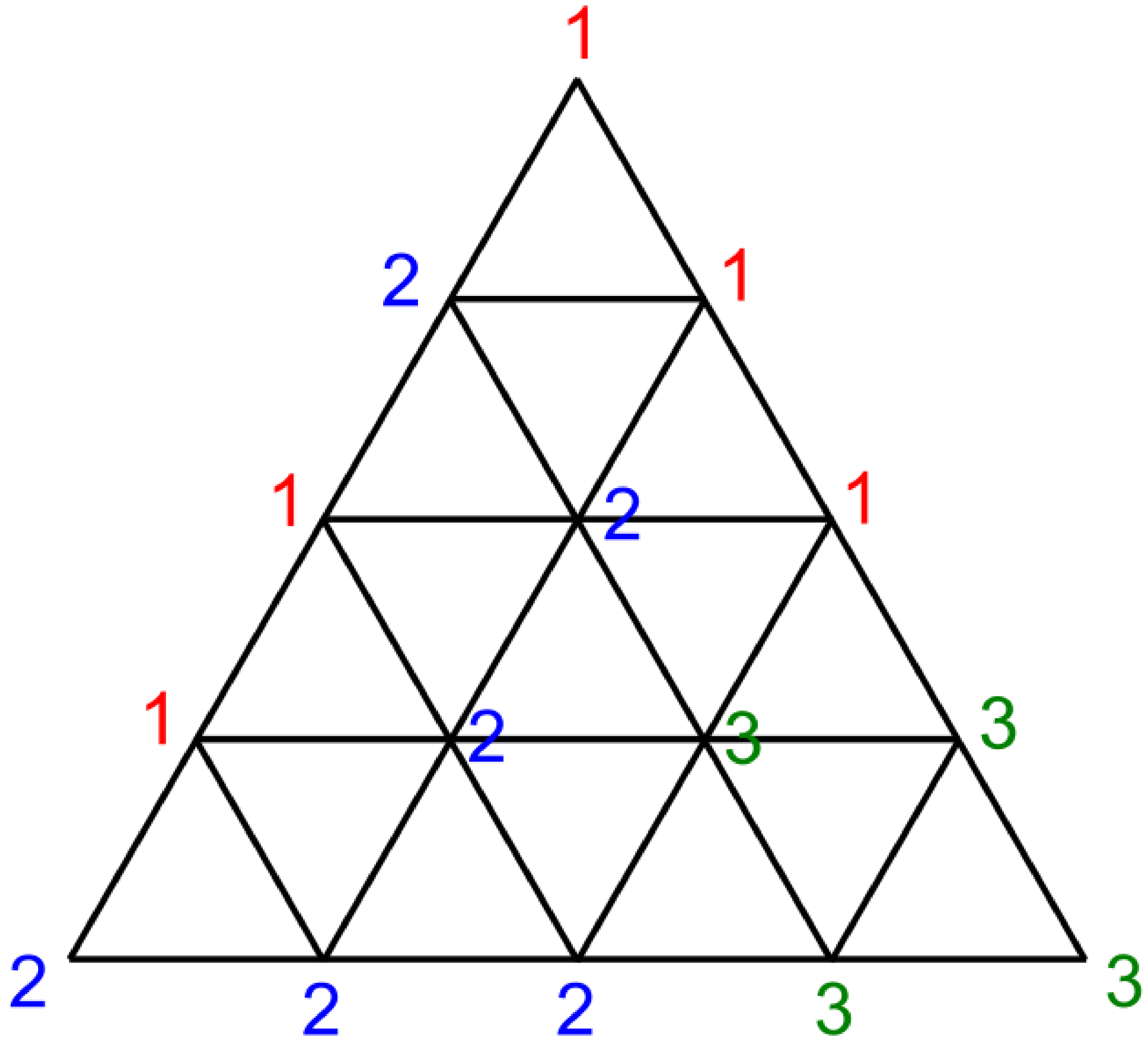

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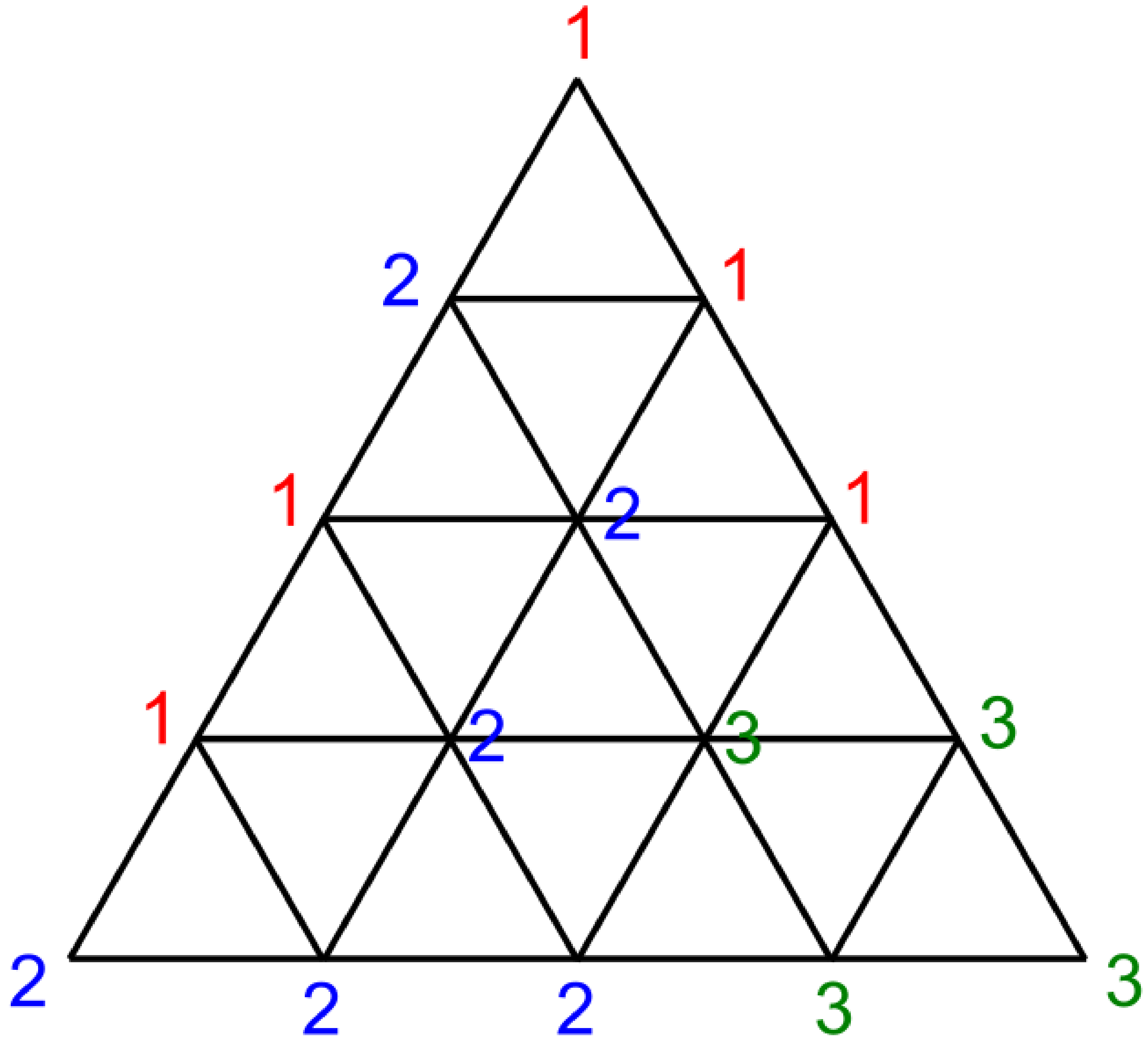

1. Triangulation into *elementary* triangles

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- Main vertices have distinct labels
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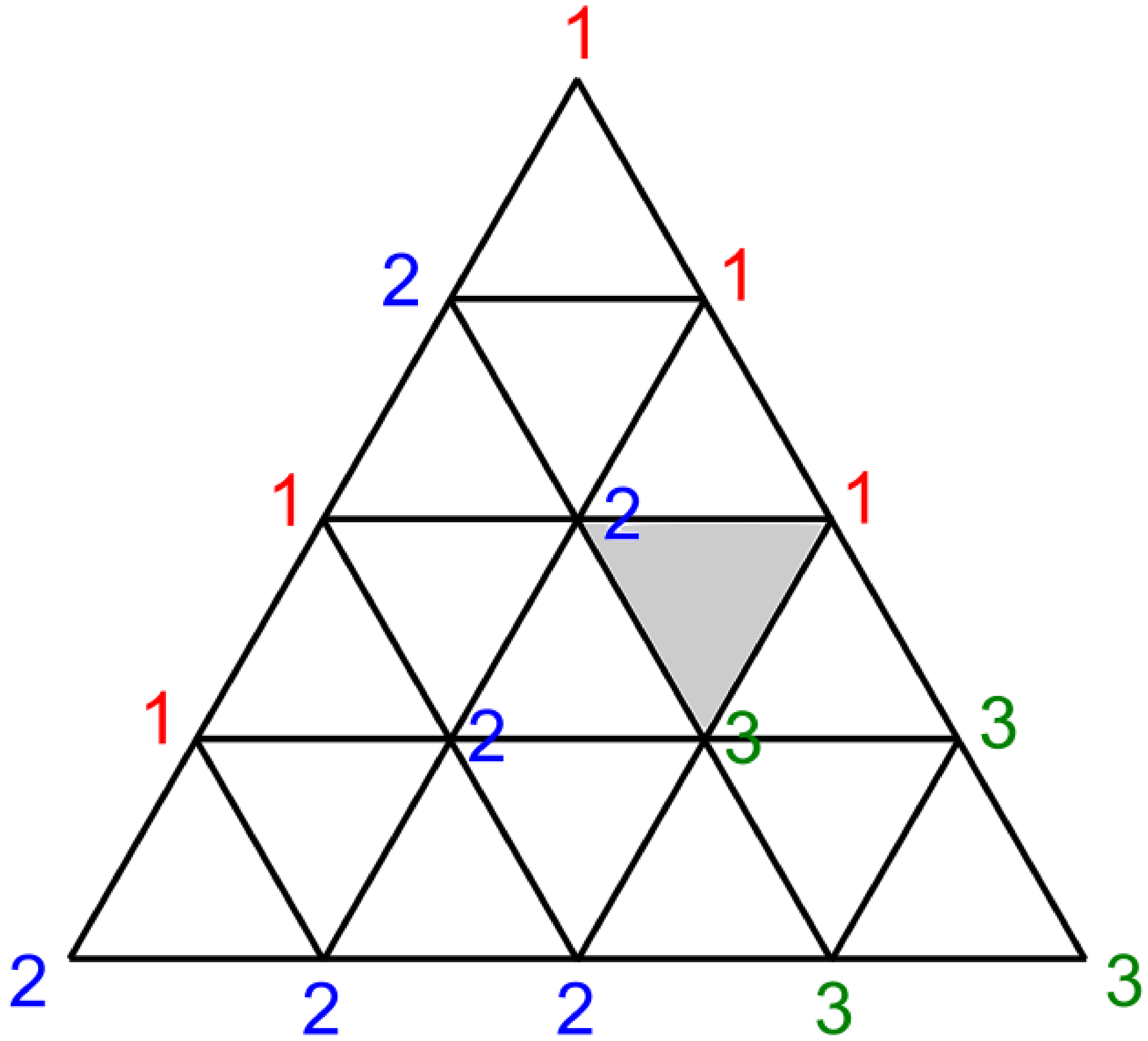
[Sperner, 1928]

Any Sperner labeled triangulation has at least one fully labeled elementary triangle.

Sperner's Lemma



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Proof Sketch of Sperner's Lemma

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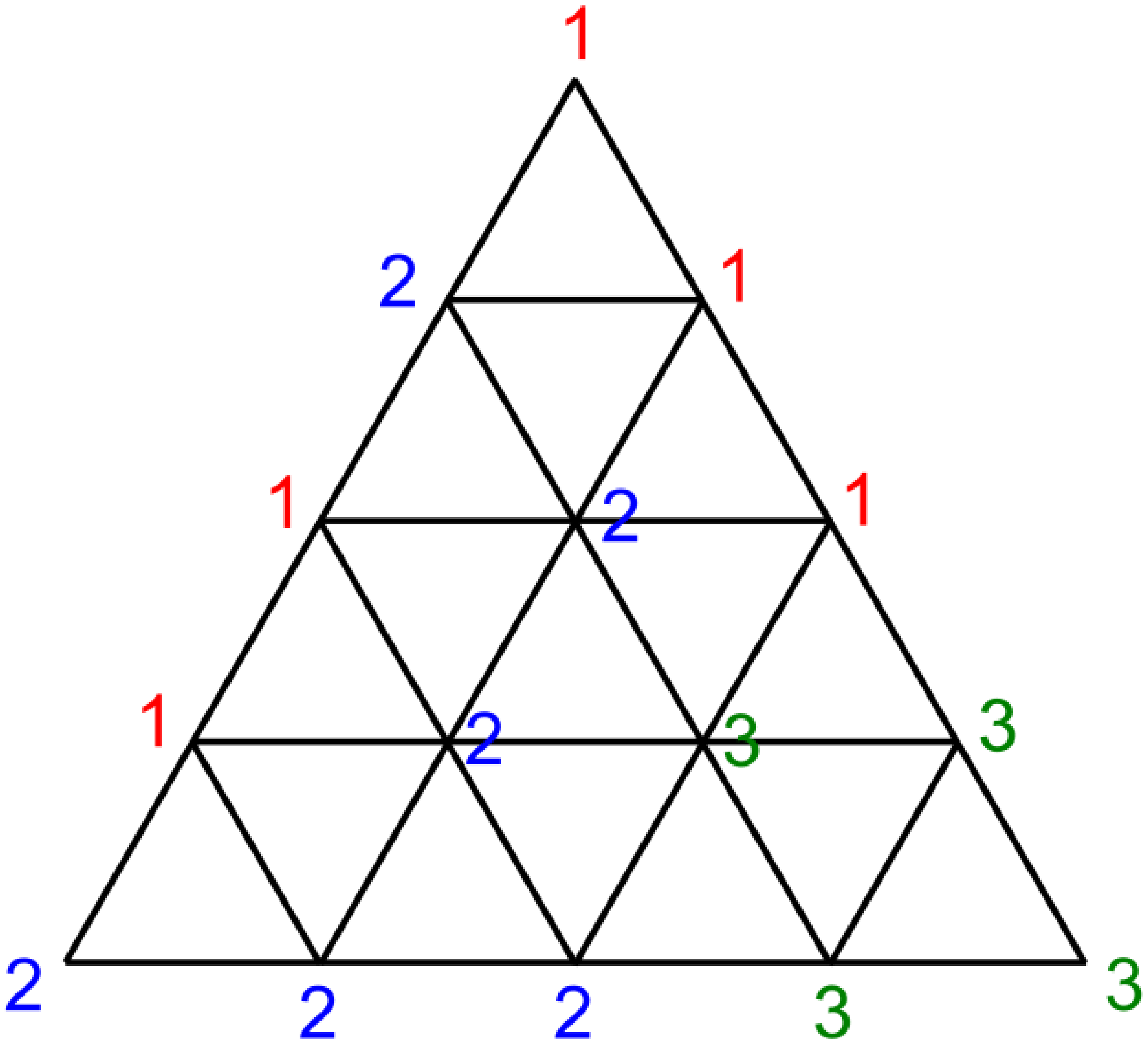

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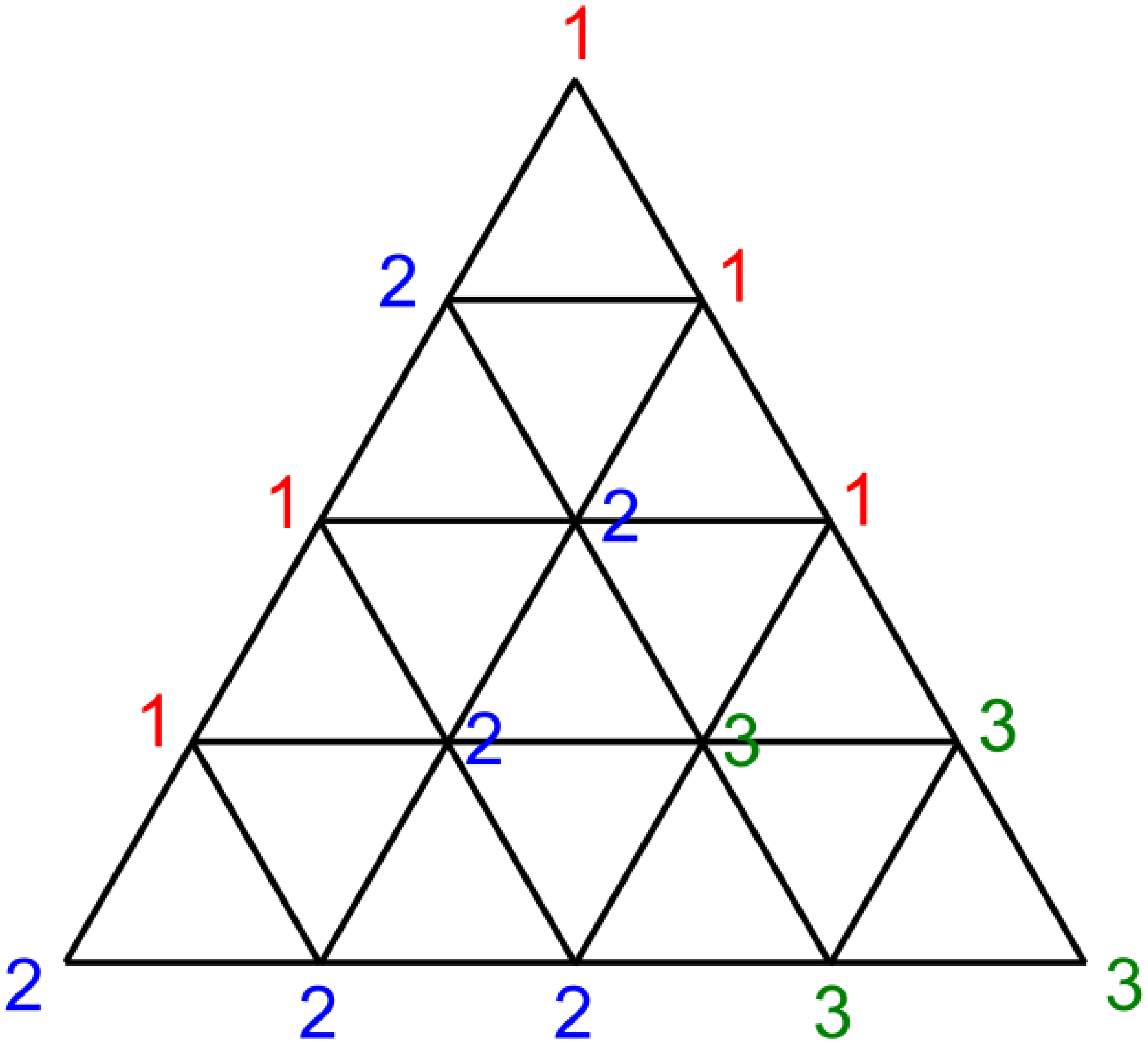

Call any 1——2 edge a "door".

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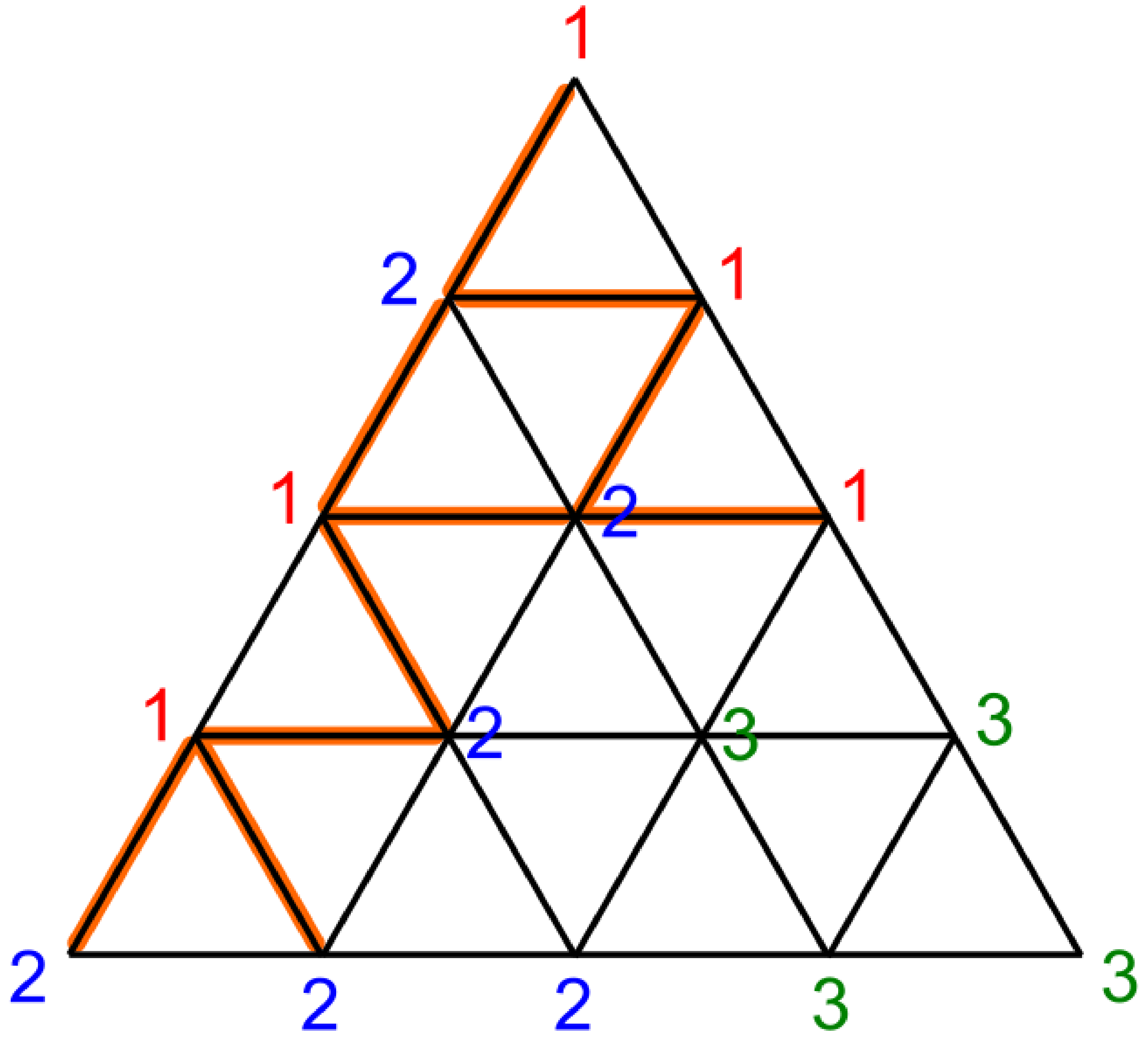

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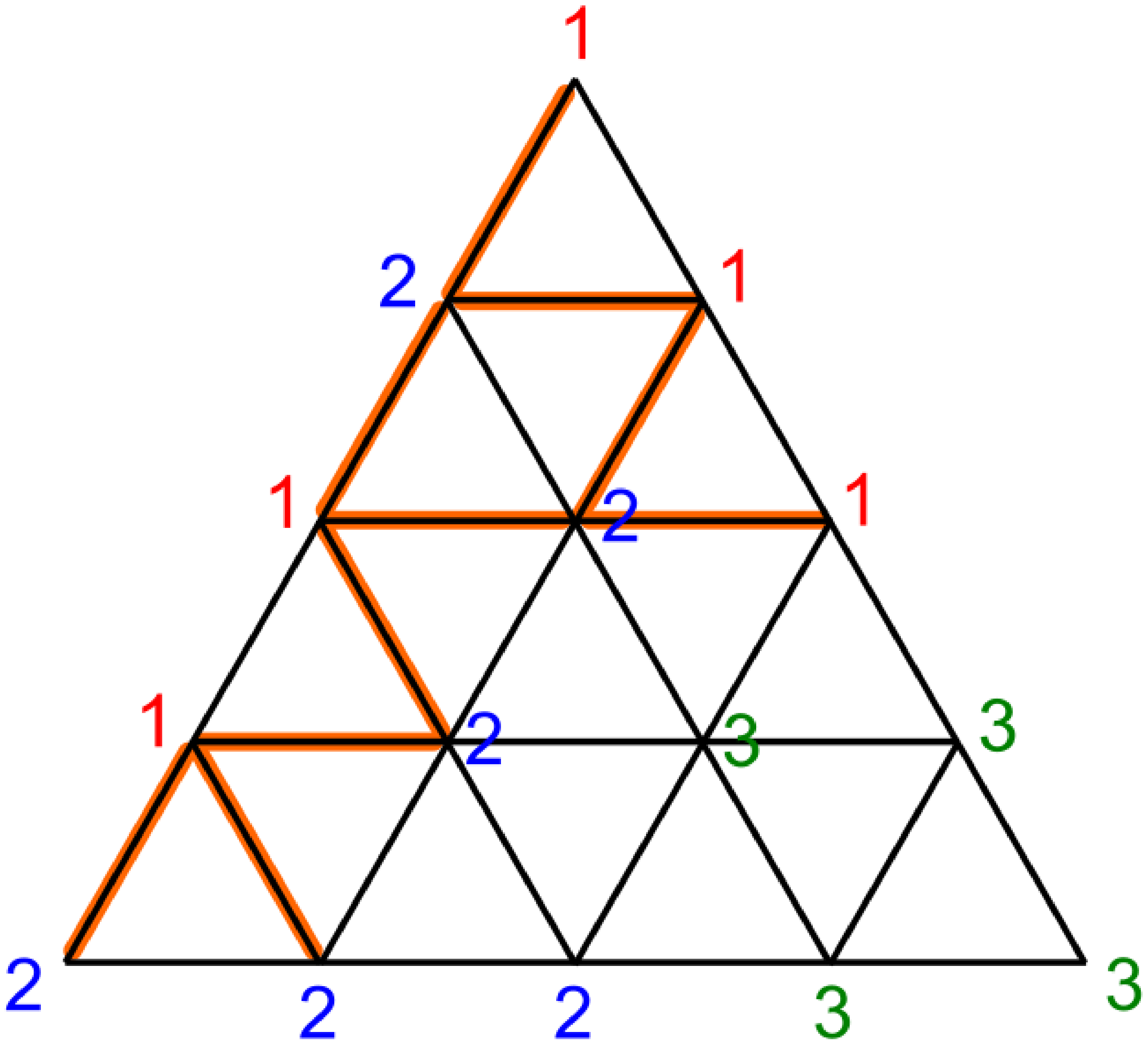

Call any 1——2 edge a "door".

Call any elementary triangle a "room".

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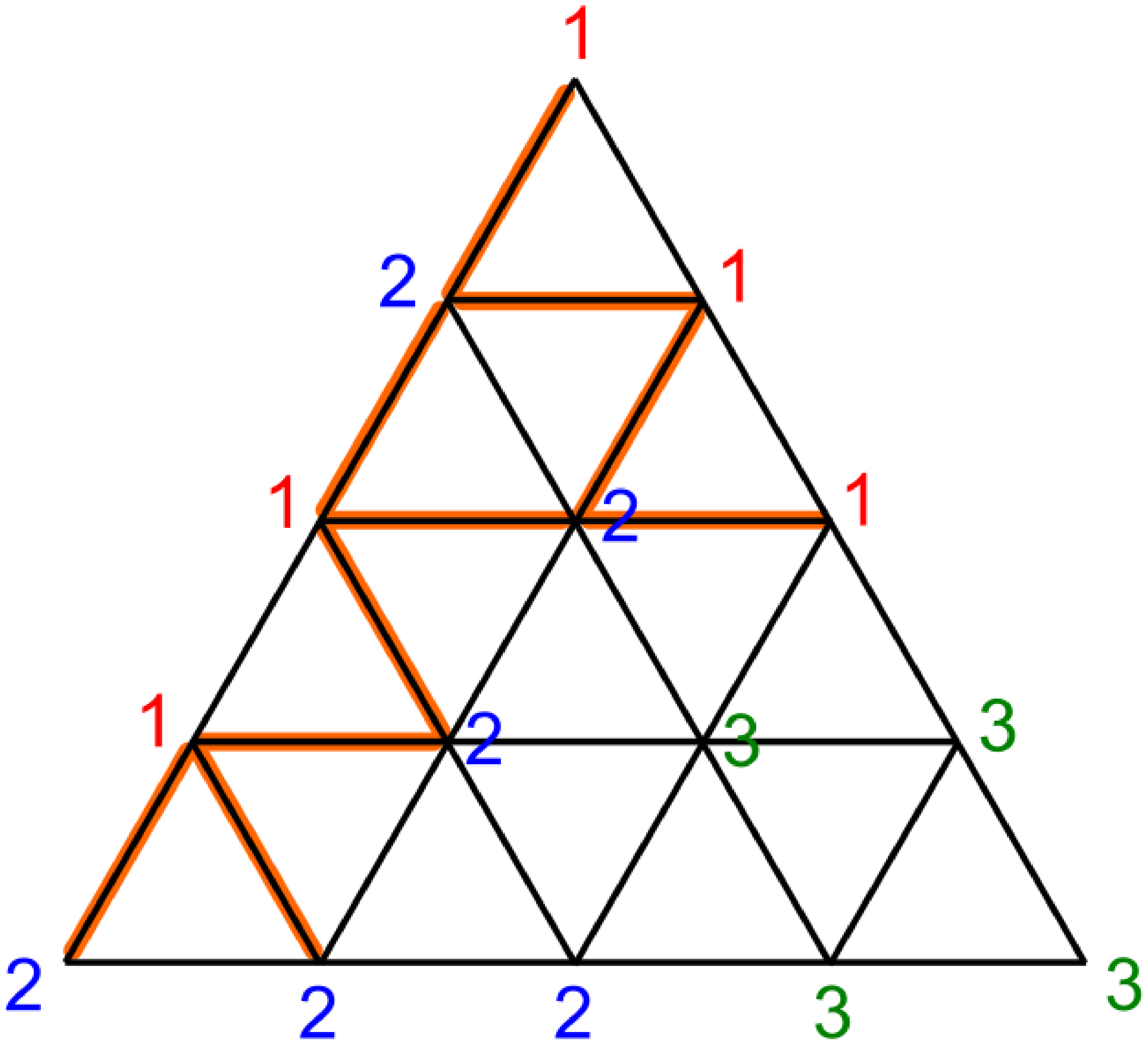

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Observe:

- No. of doors on the boundary is odd.

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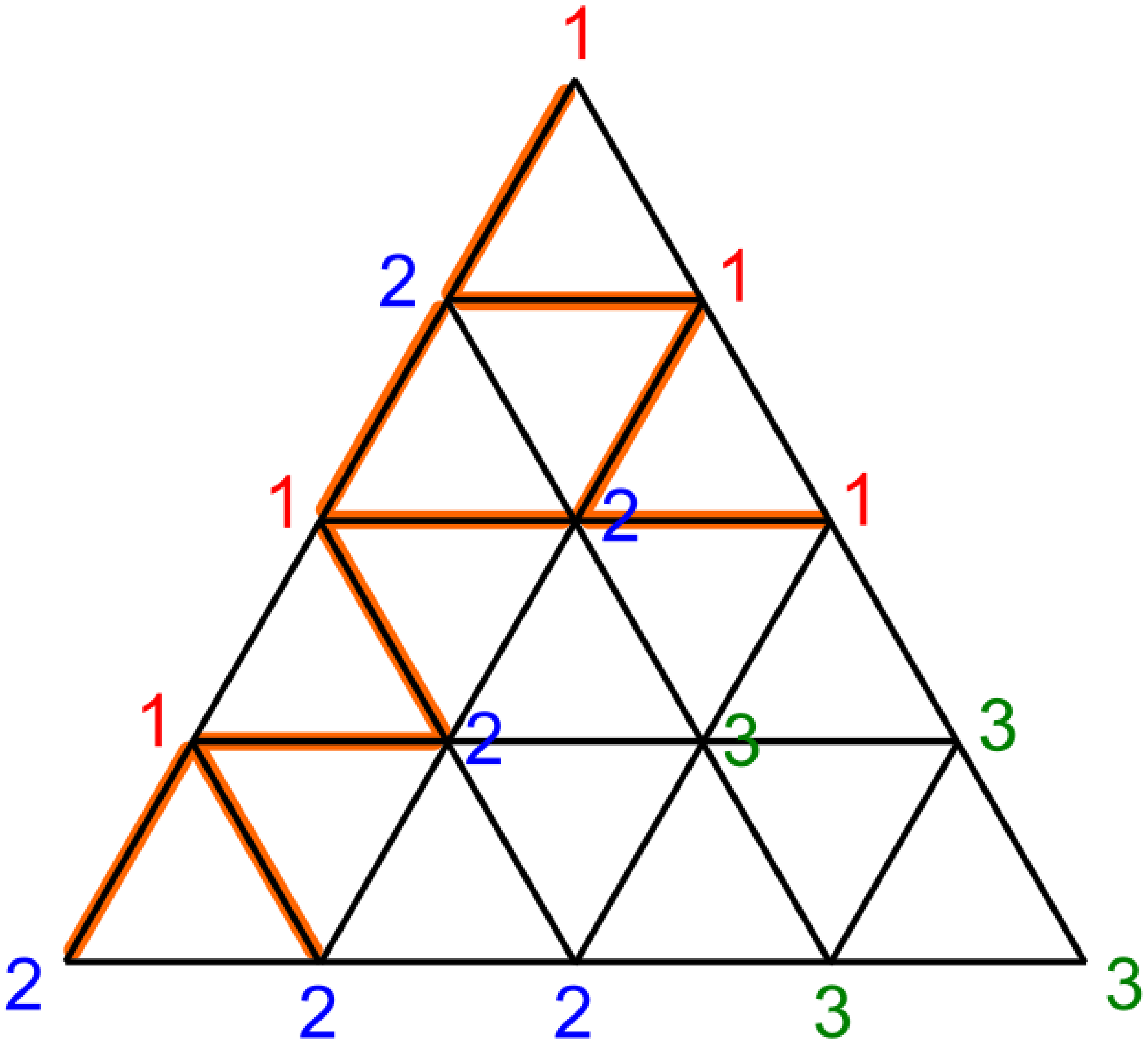

Call any 1——2 edge a "door".

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Observe:

- No. of doors on the boundary is odd.
- A room can have zero, one, or two doors.

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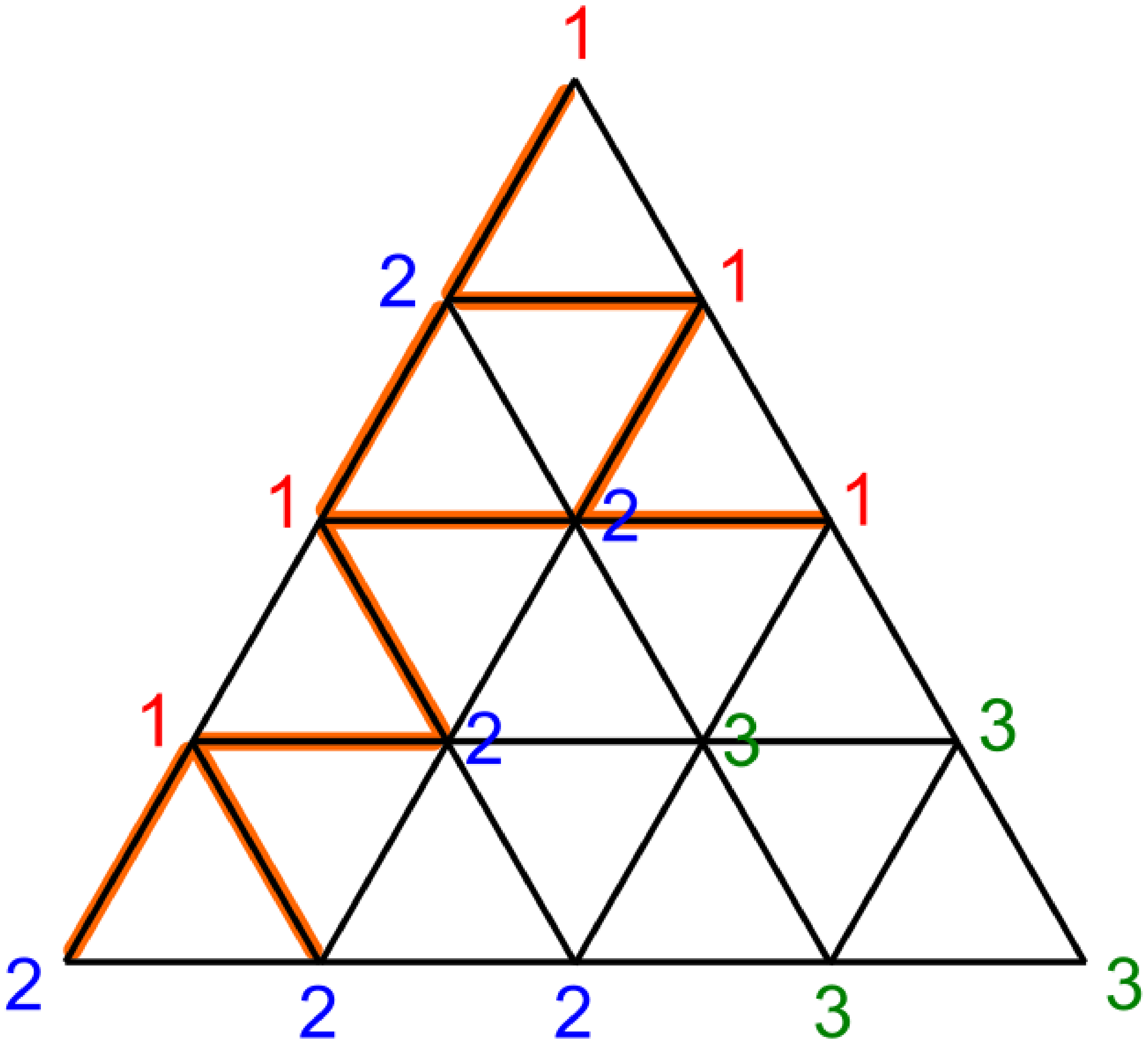

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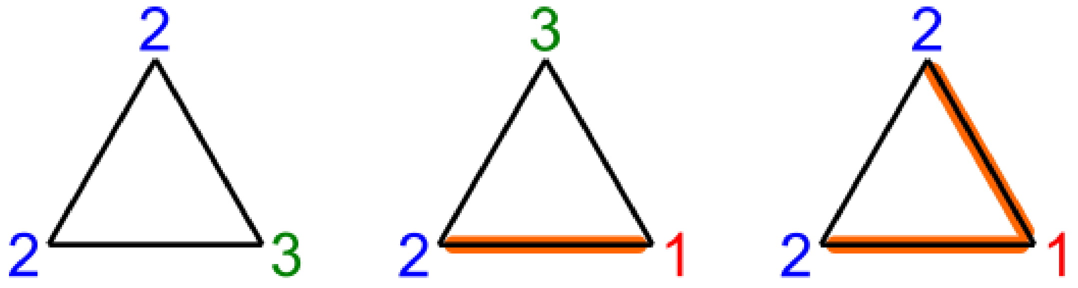

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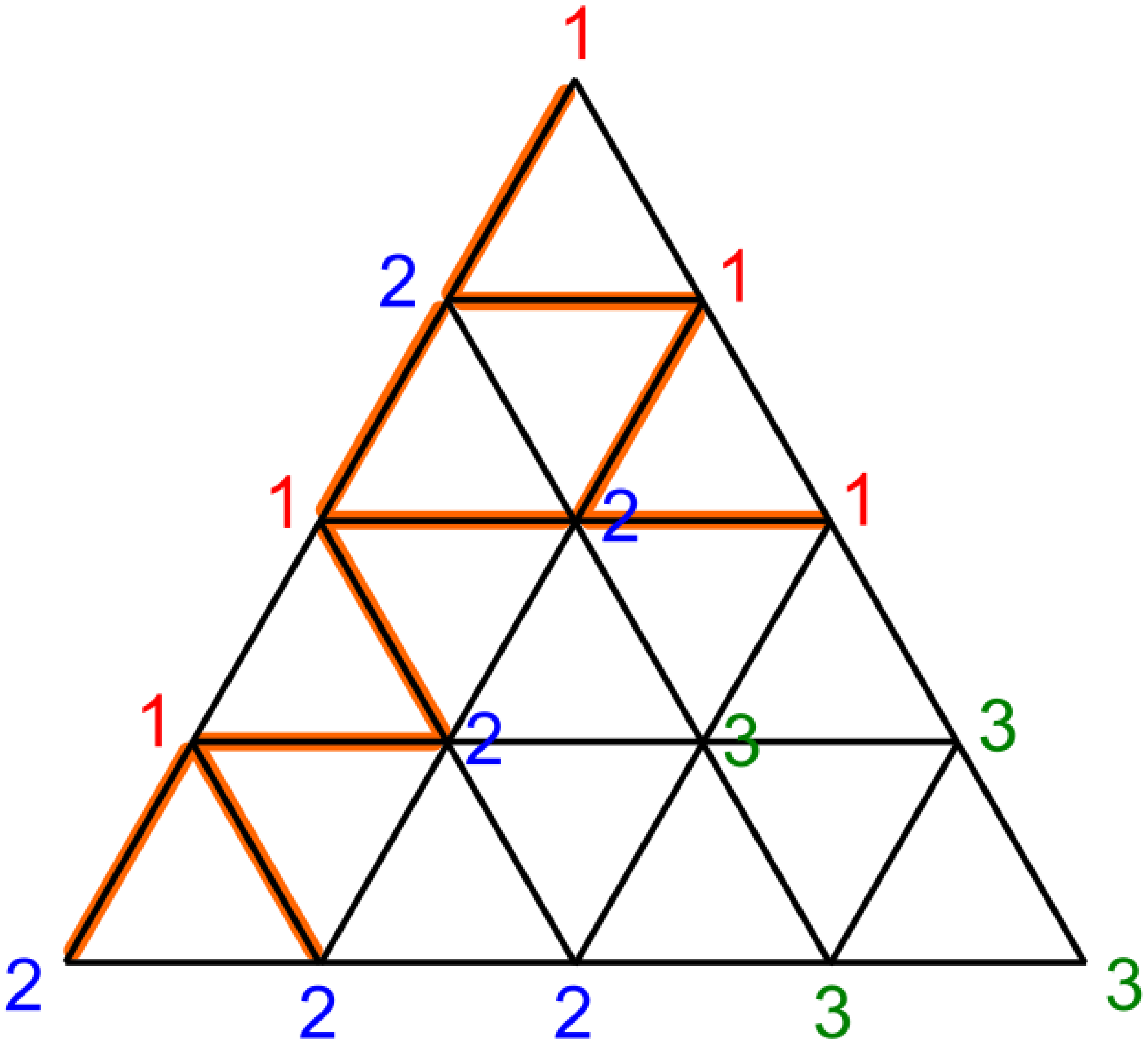

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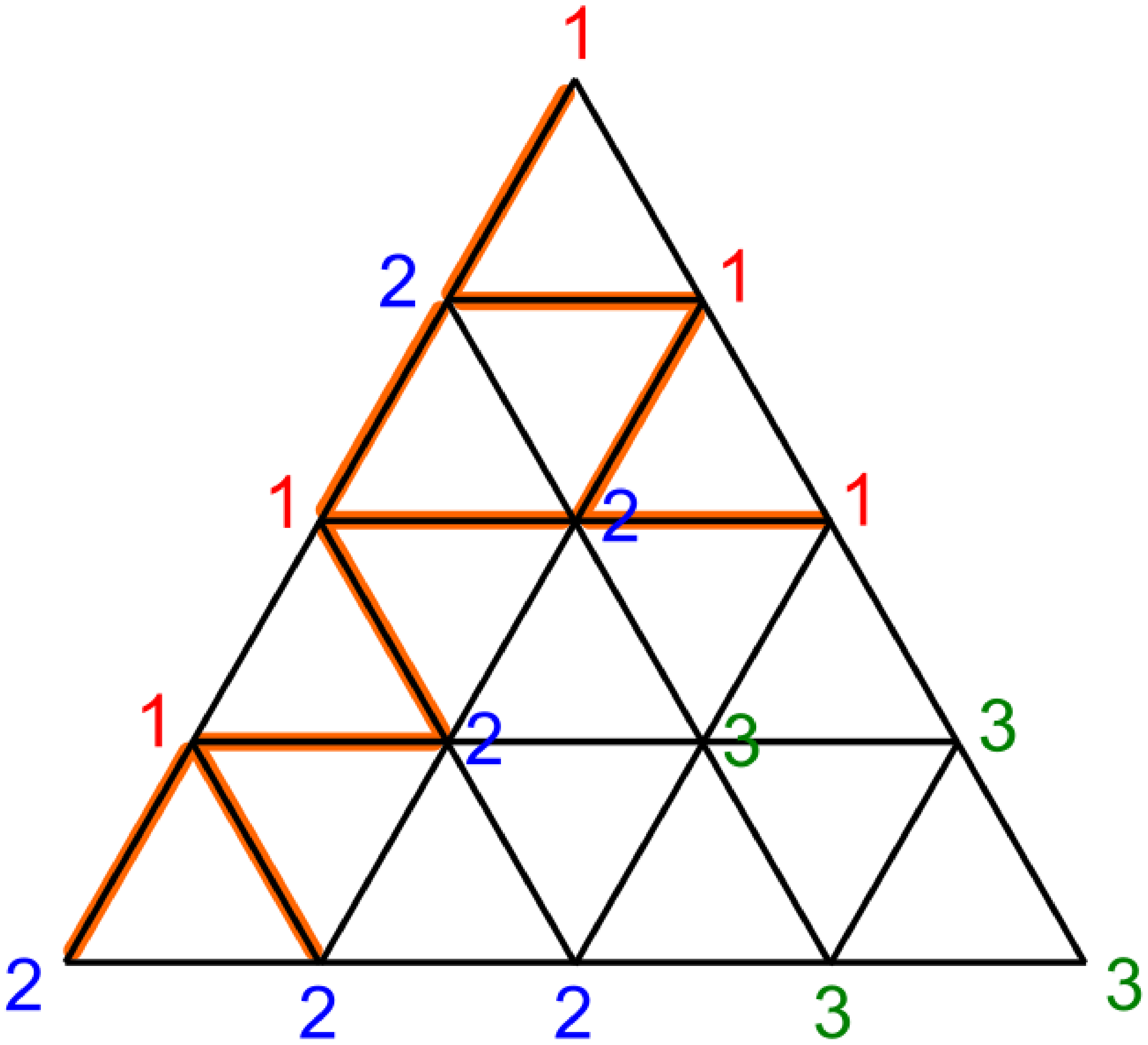

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Room with one door ⇔ Fully-labeled elementary triangle

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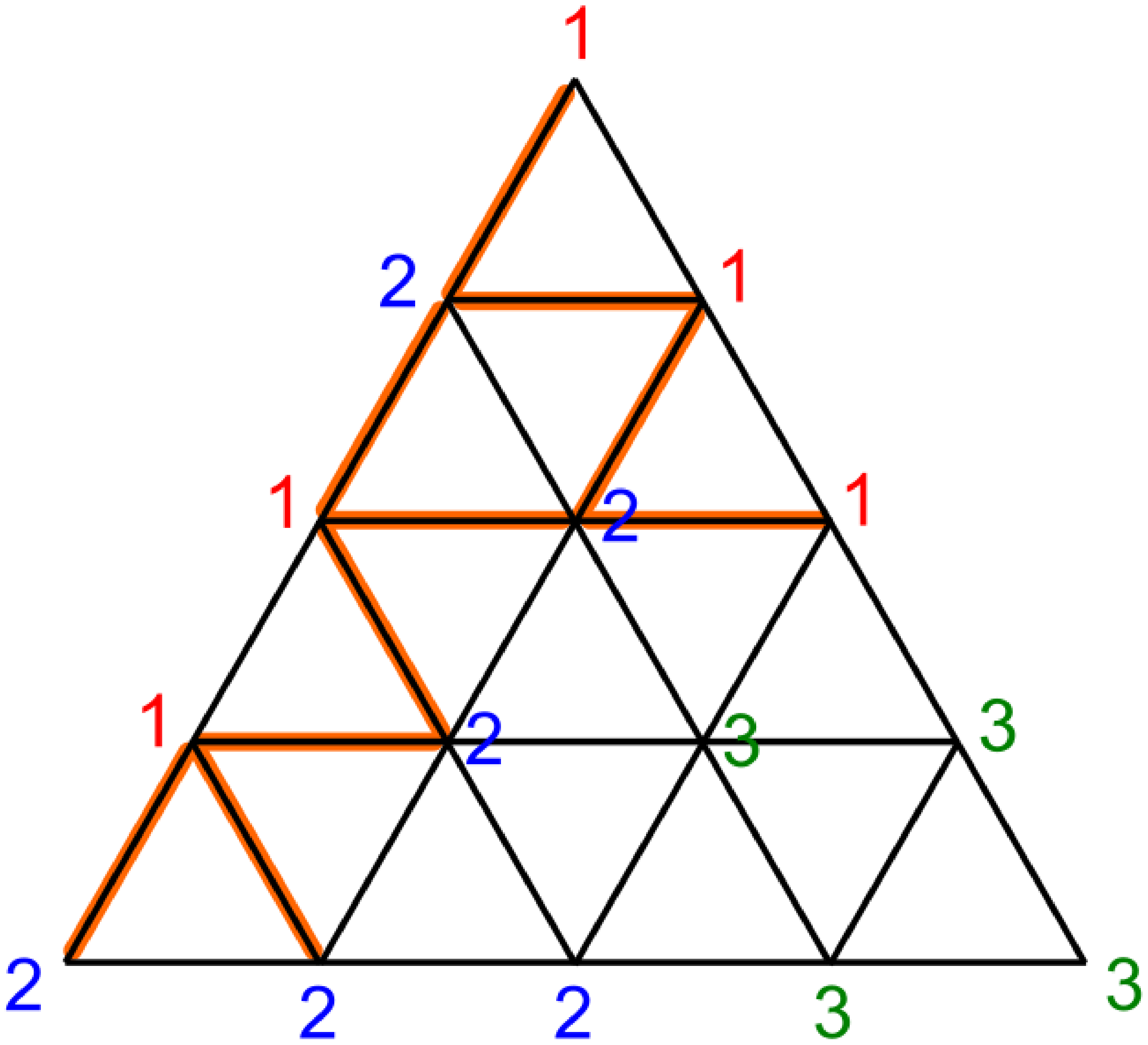

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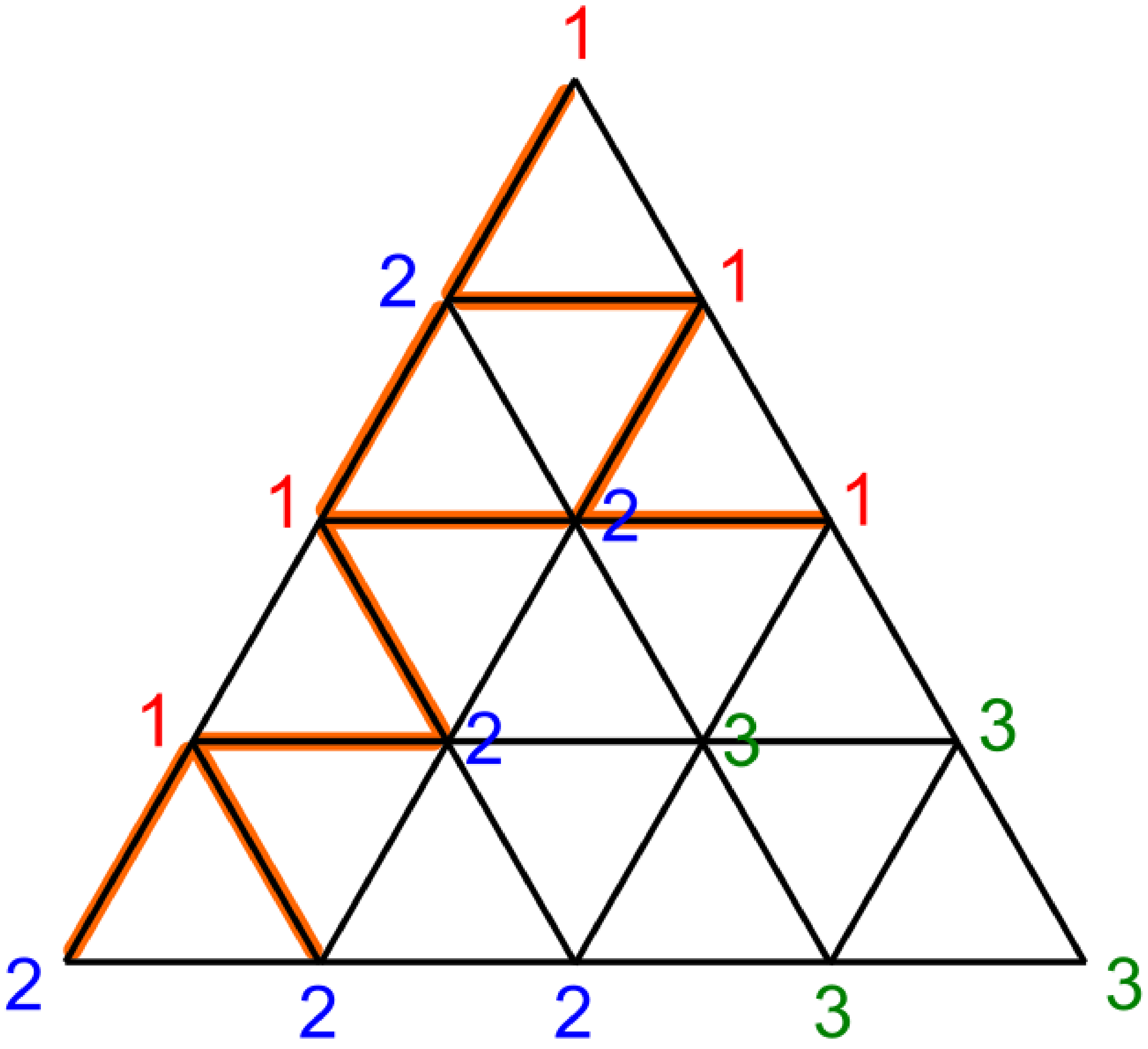

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Start walking!

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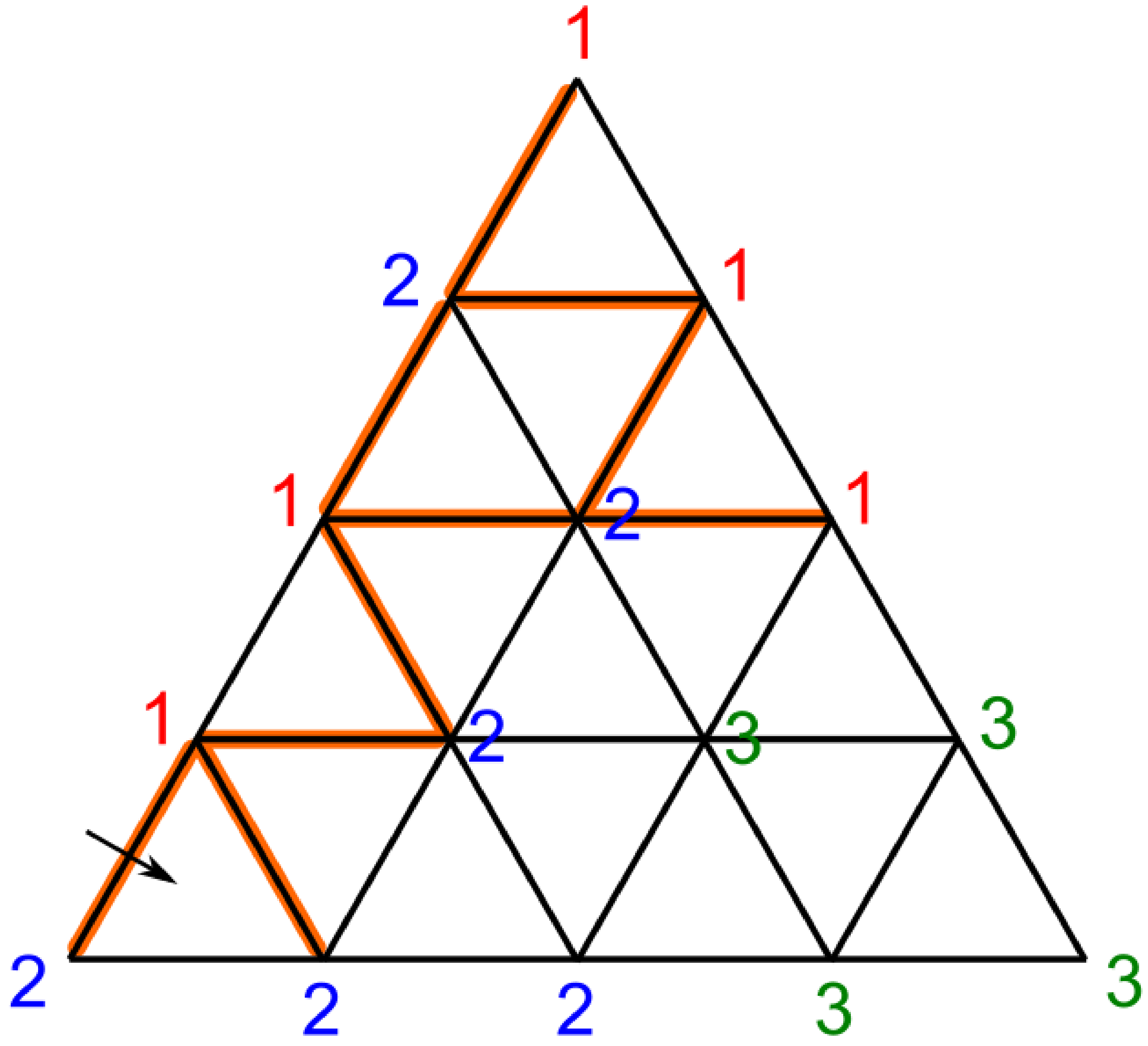

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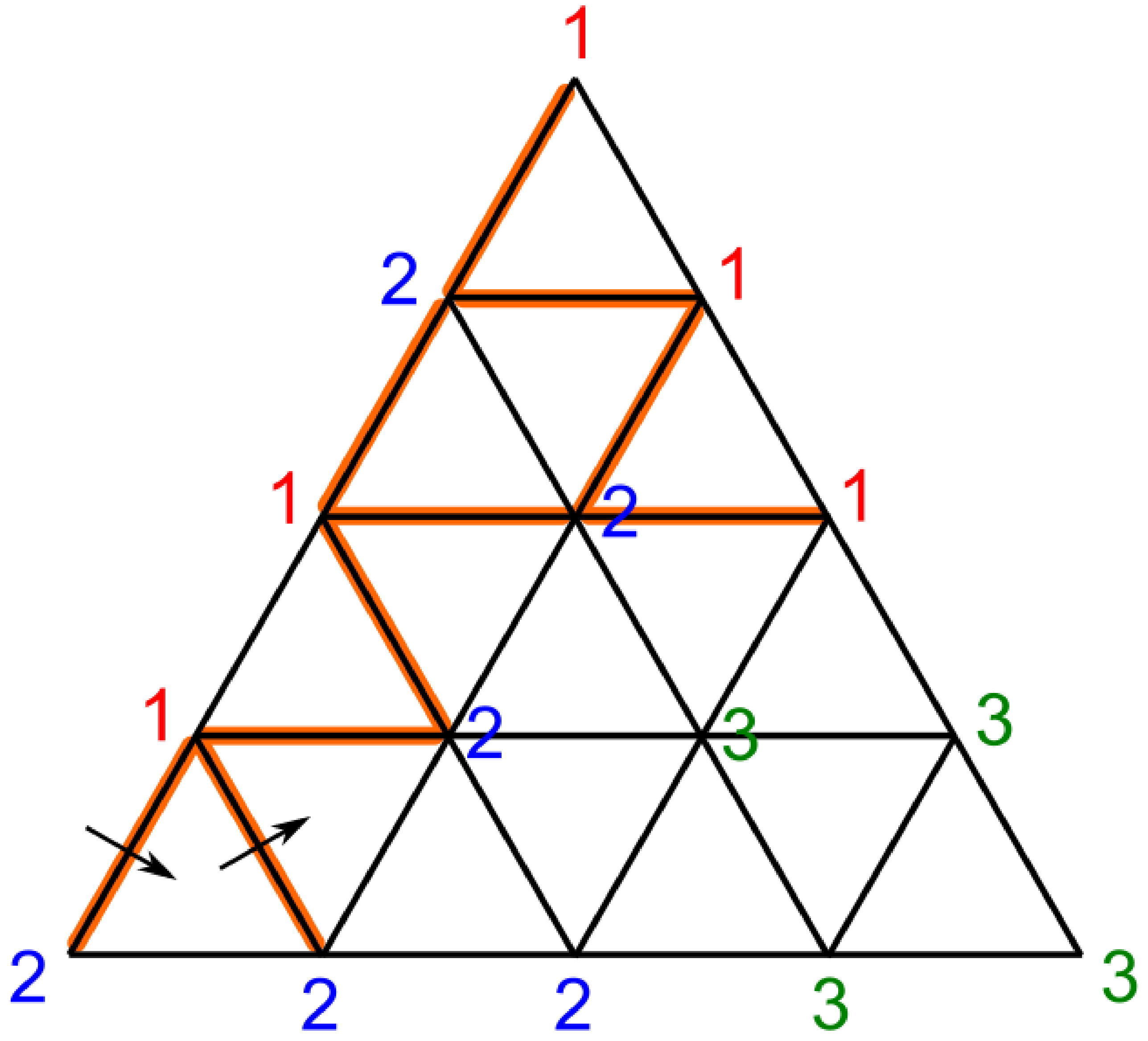

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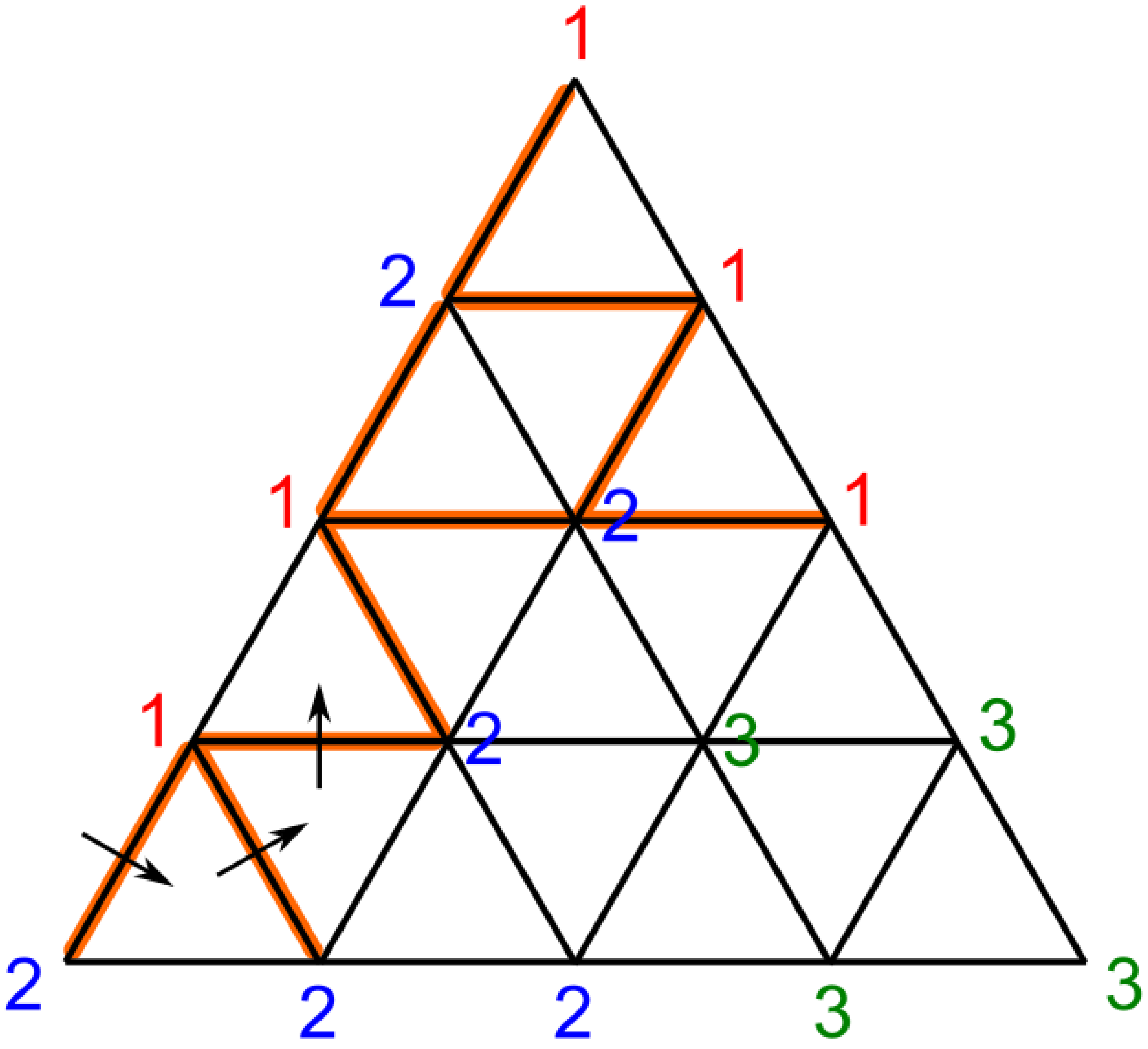

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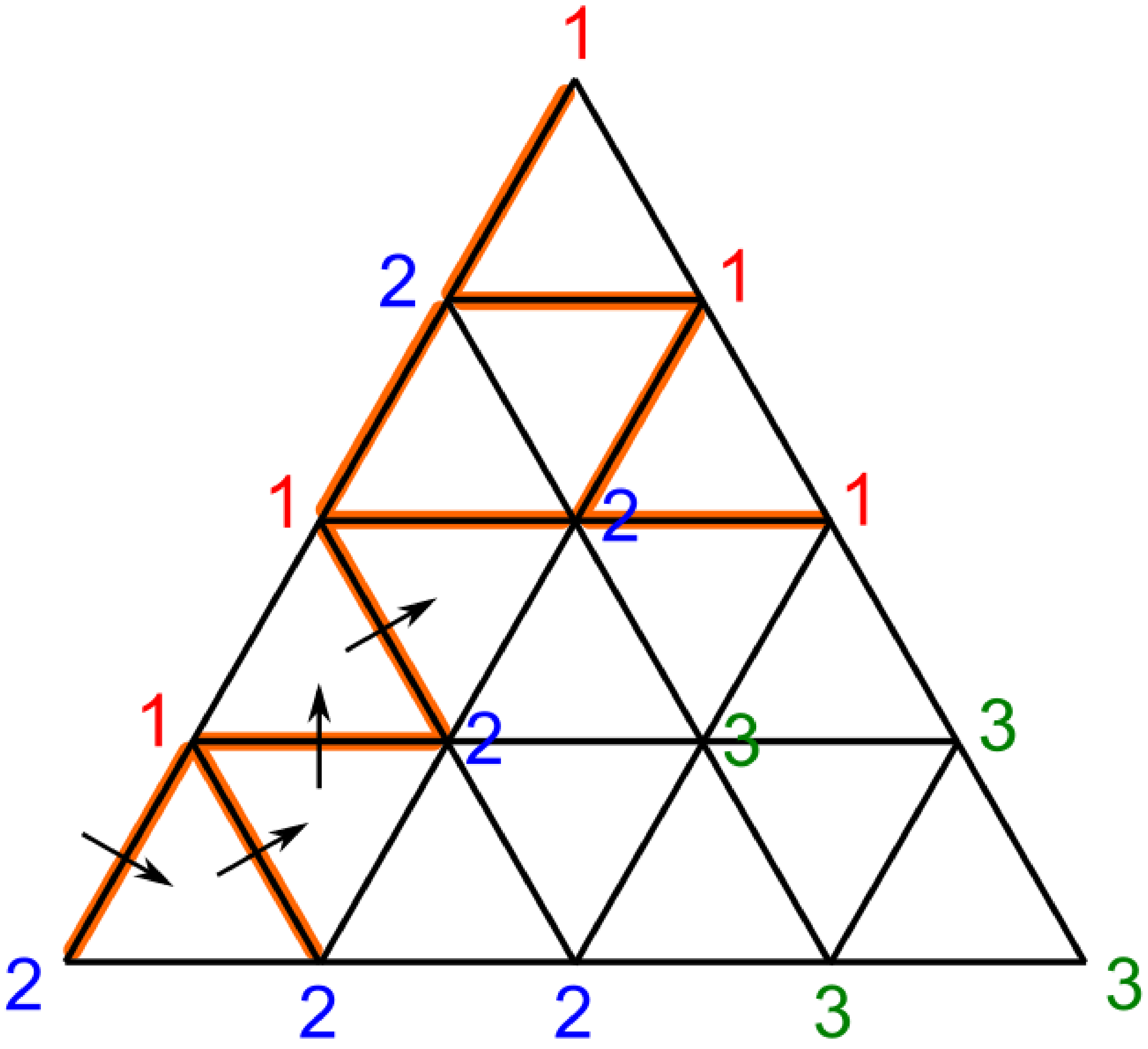

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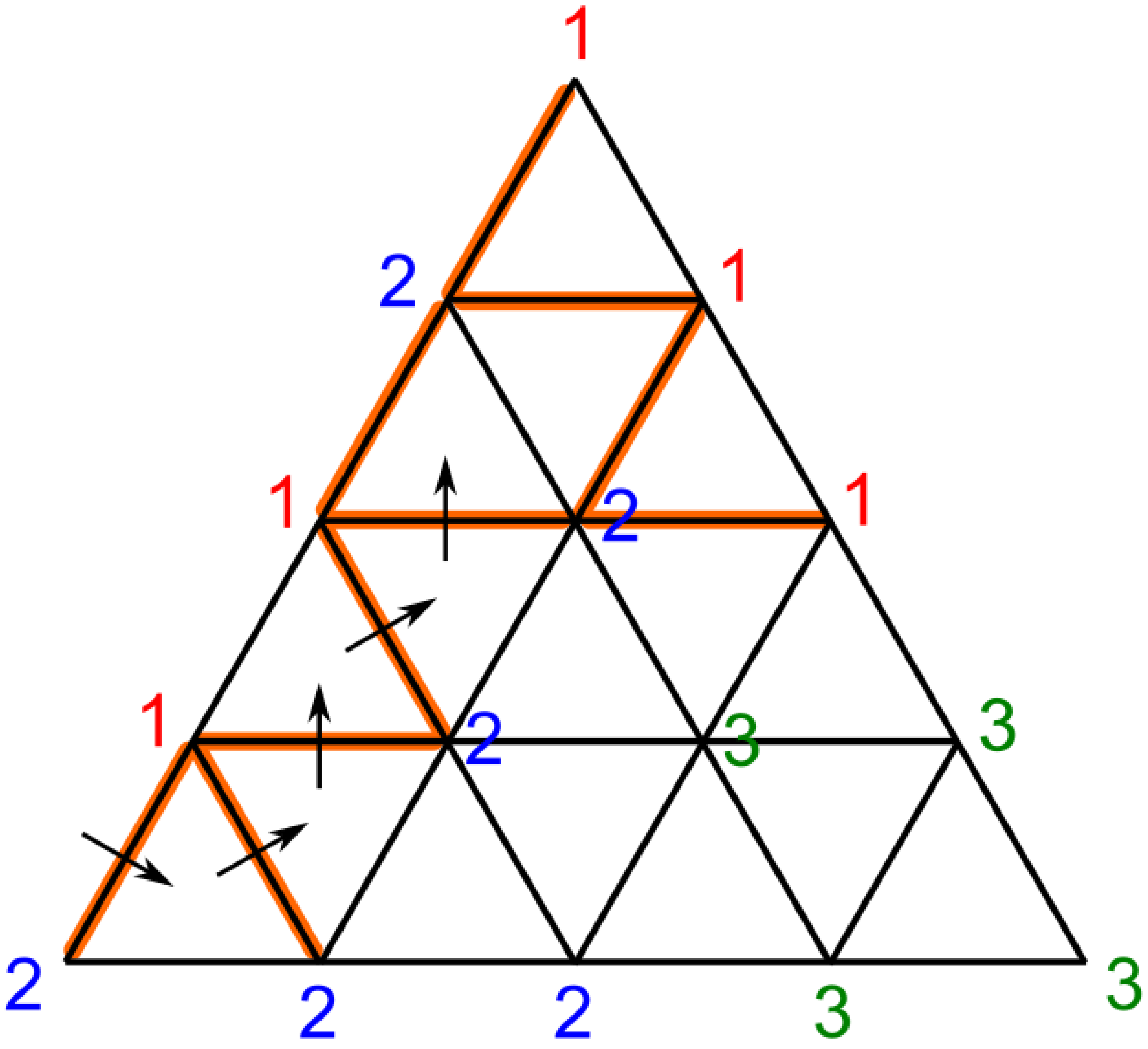

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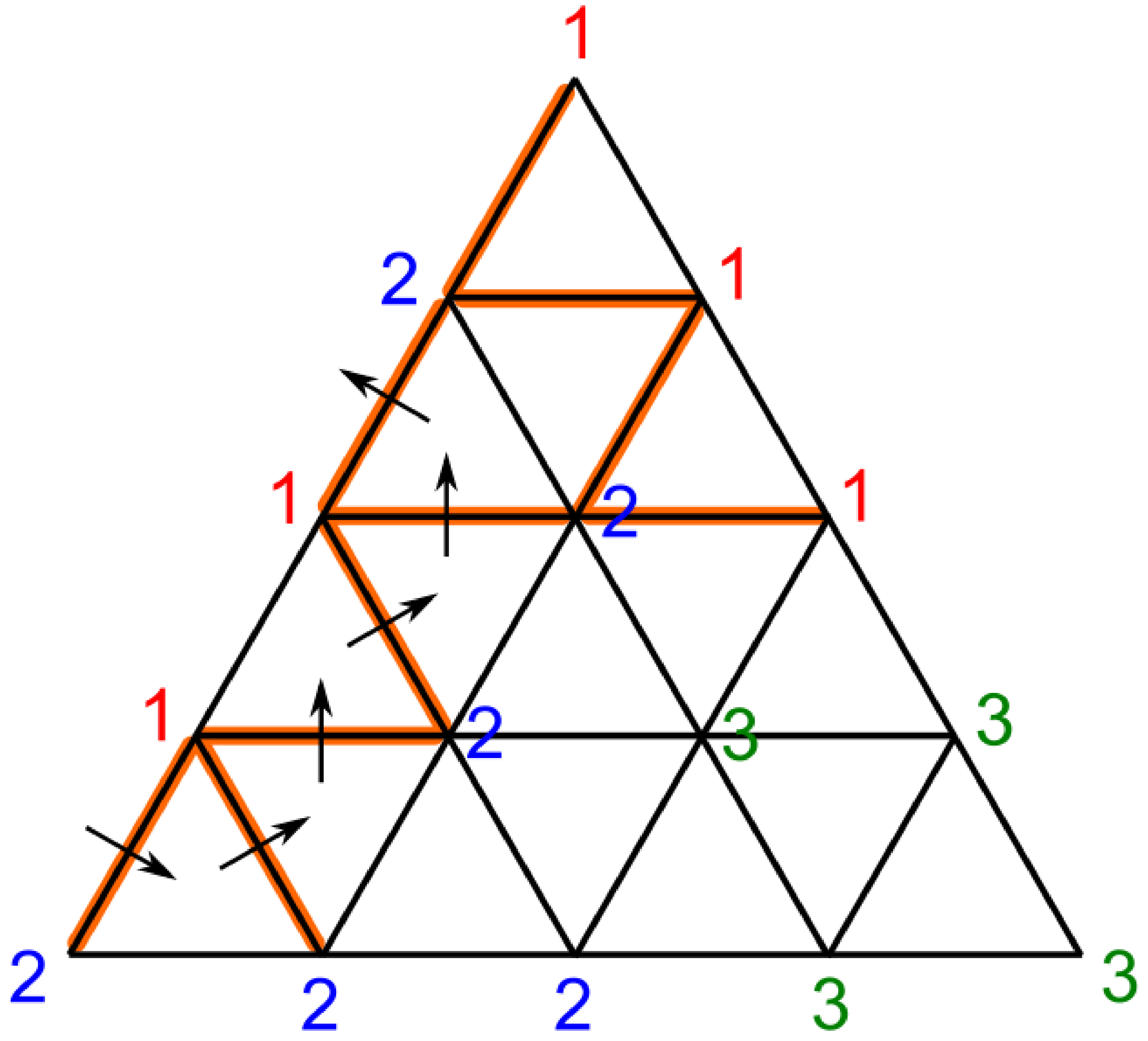

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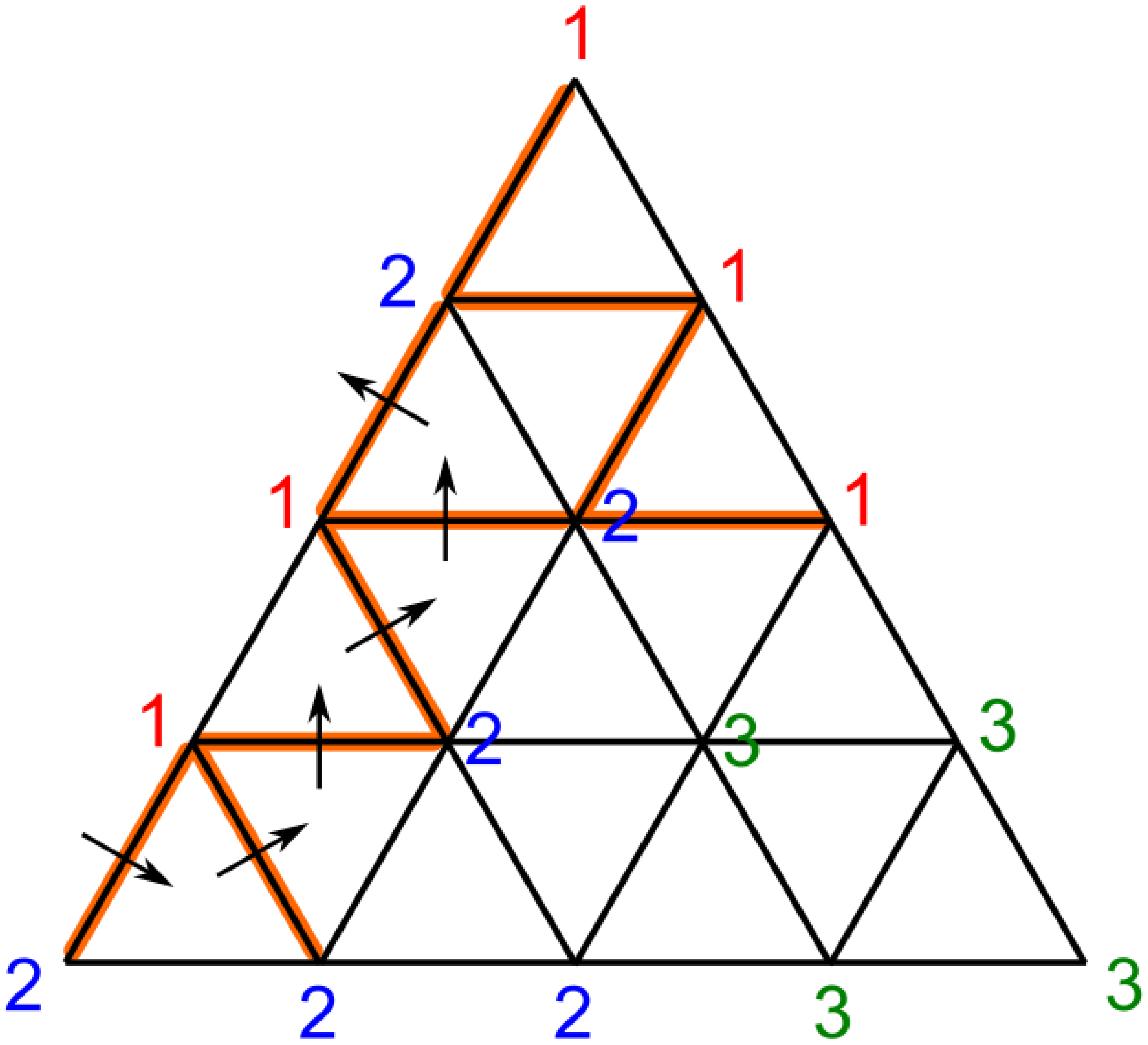

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- If thrown out, take another door.

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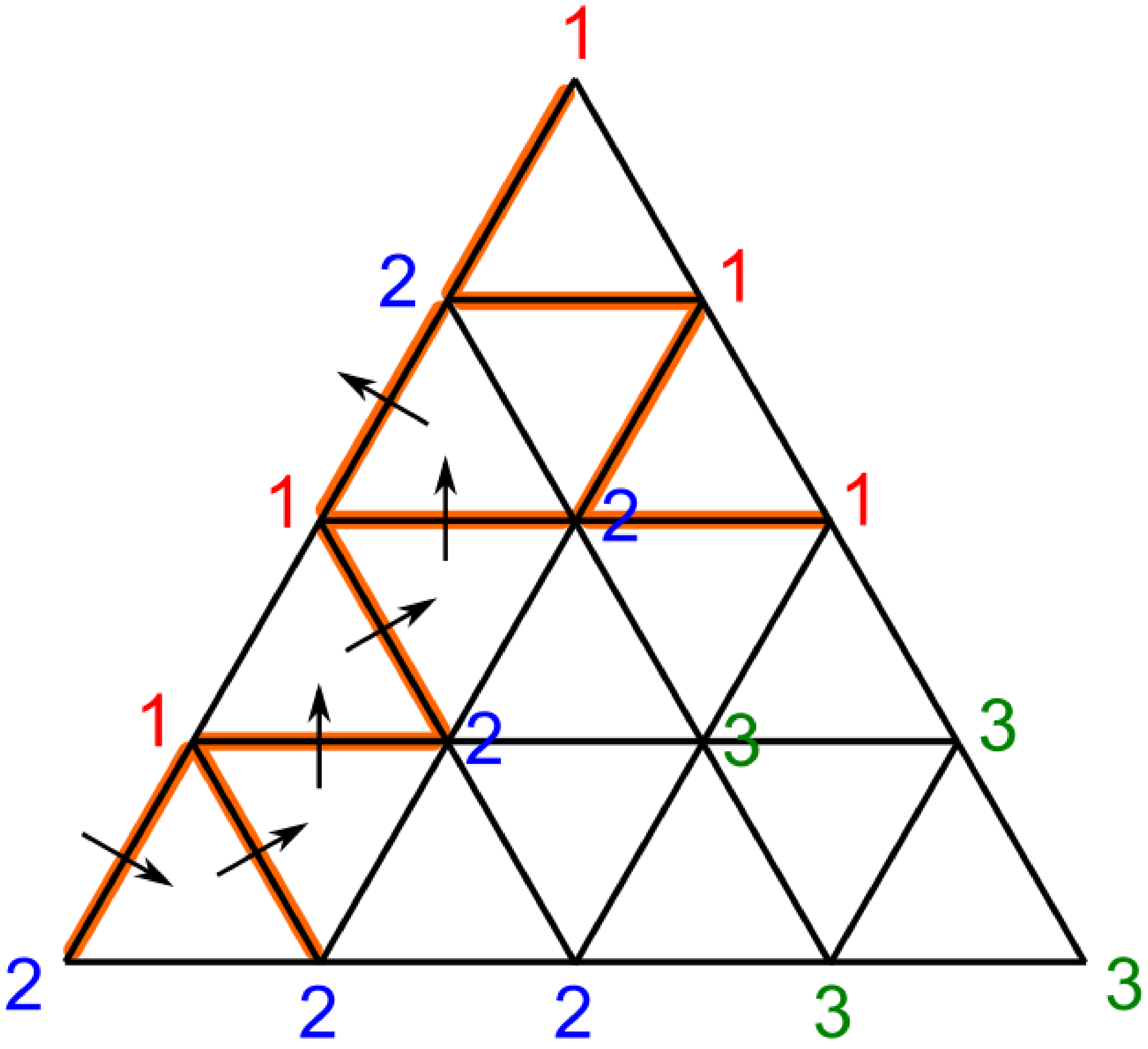

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- Some path must end up in a room with exactly one door.

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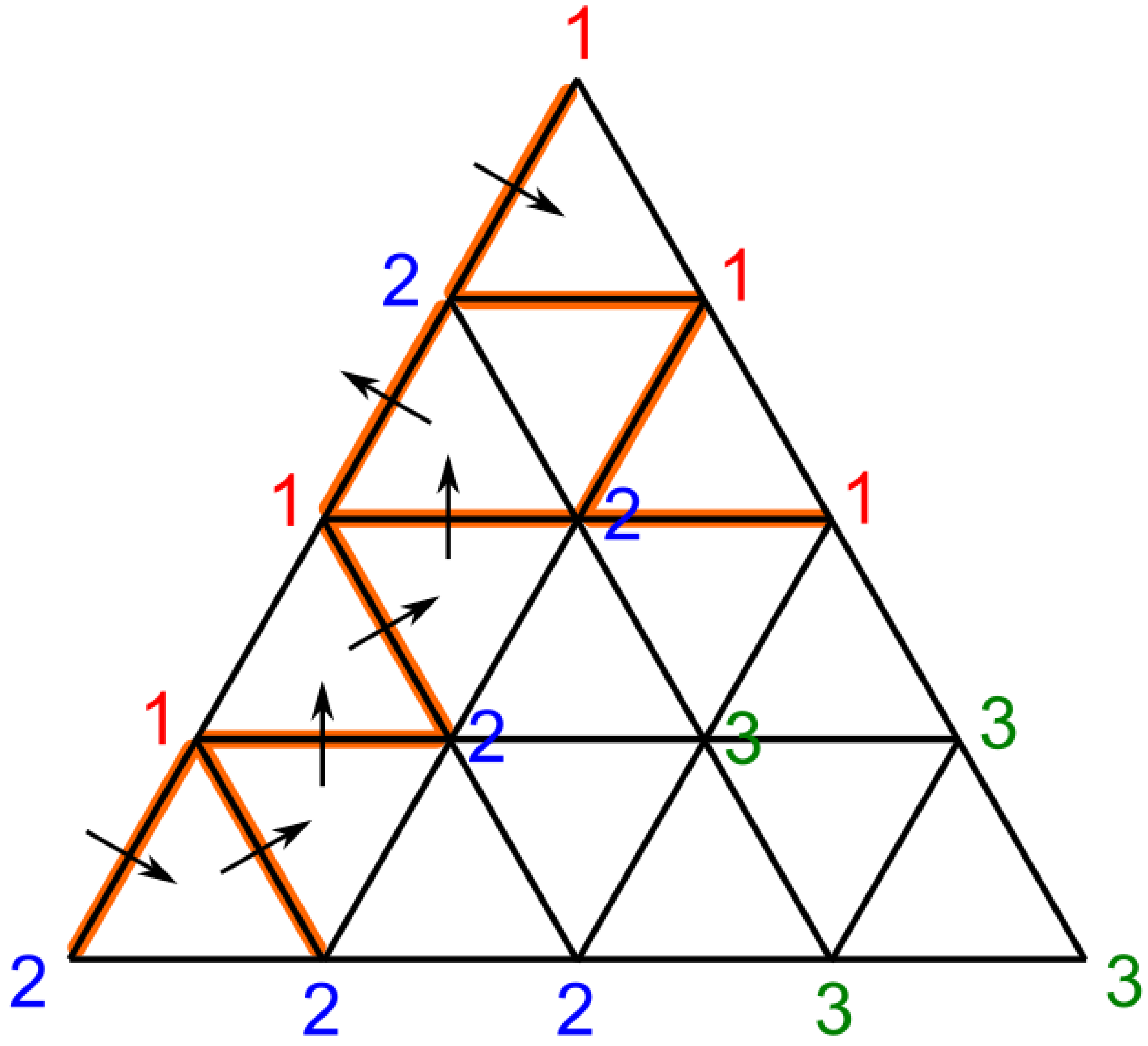

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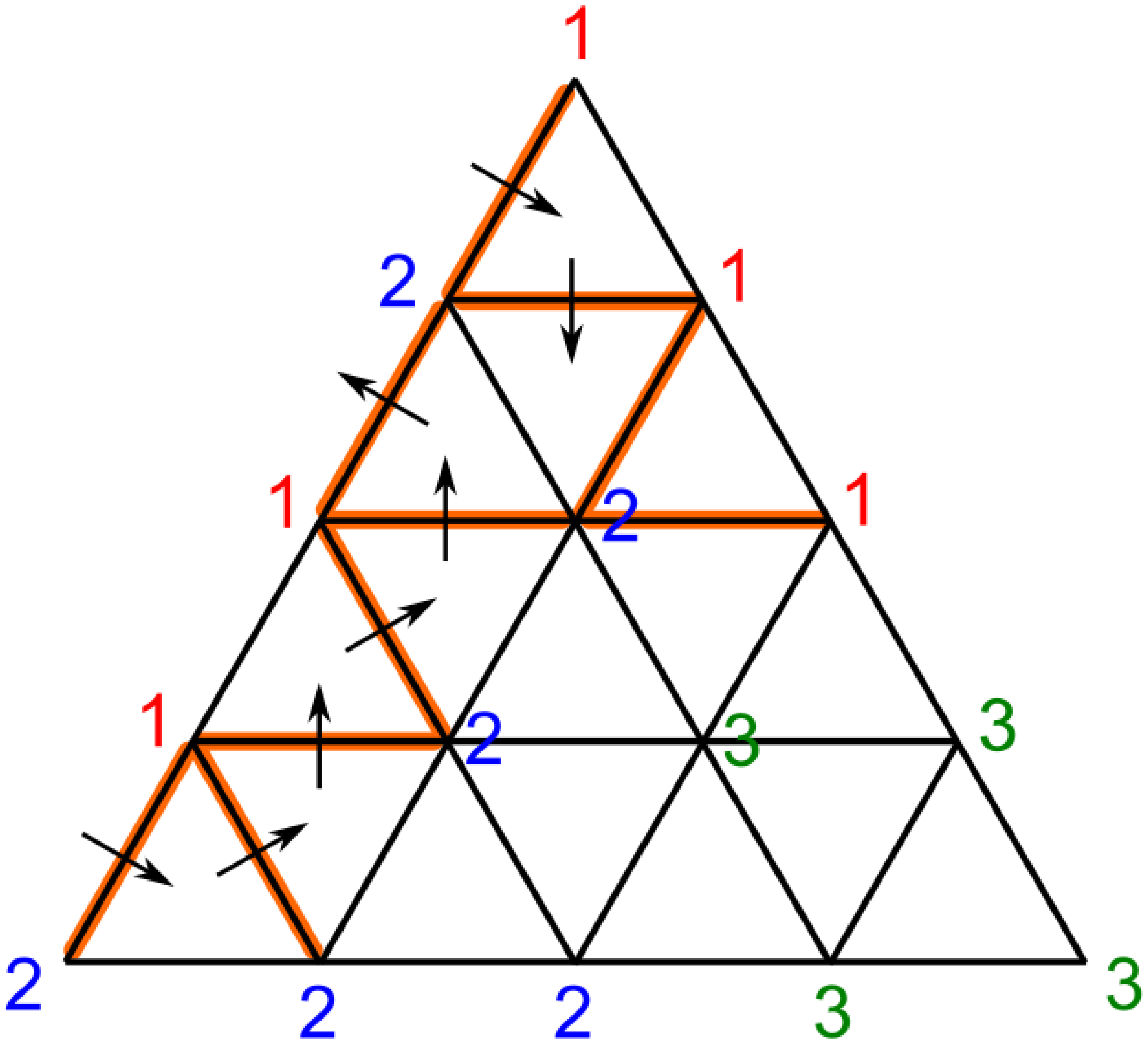

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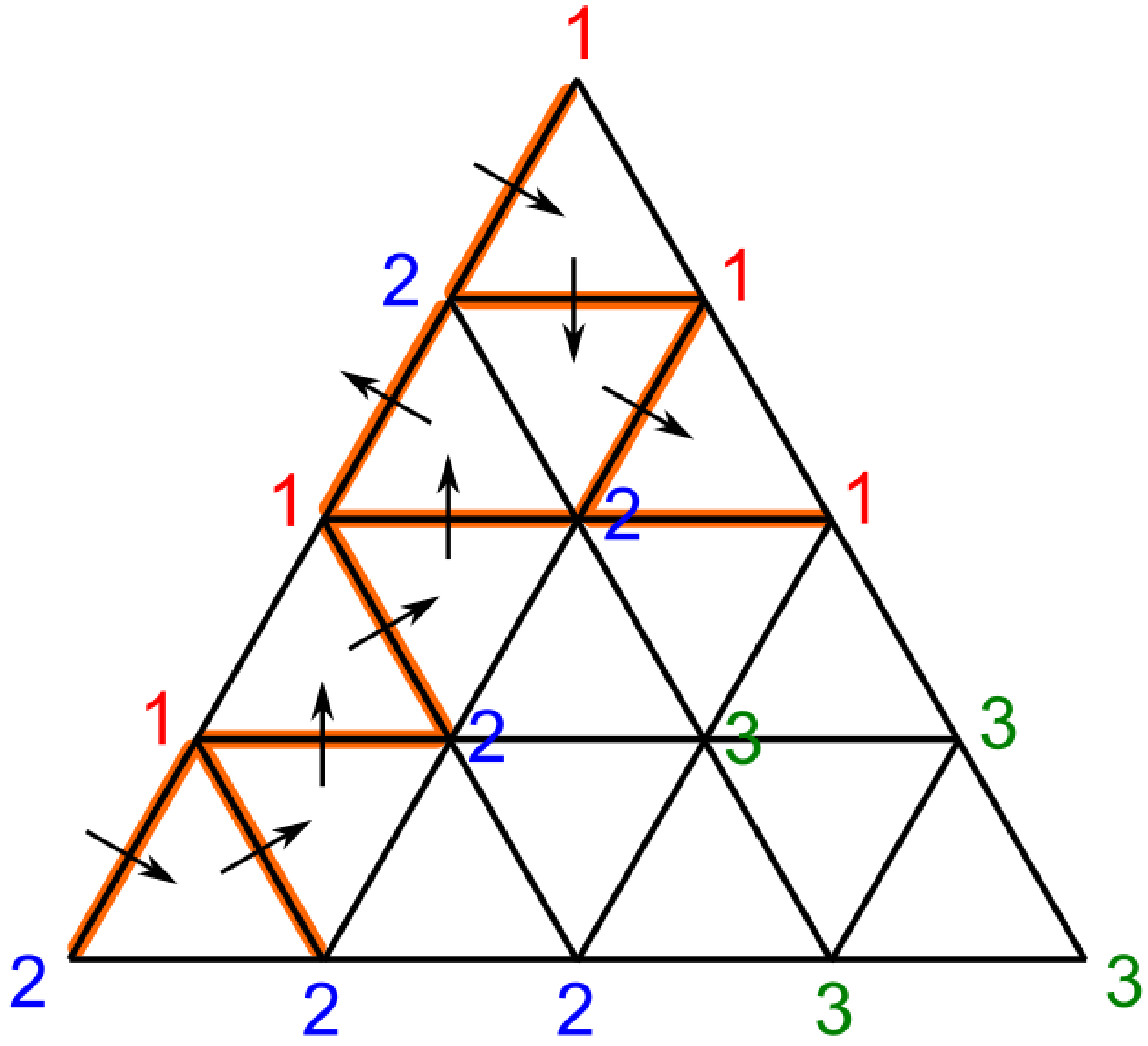

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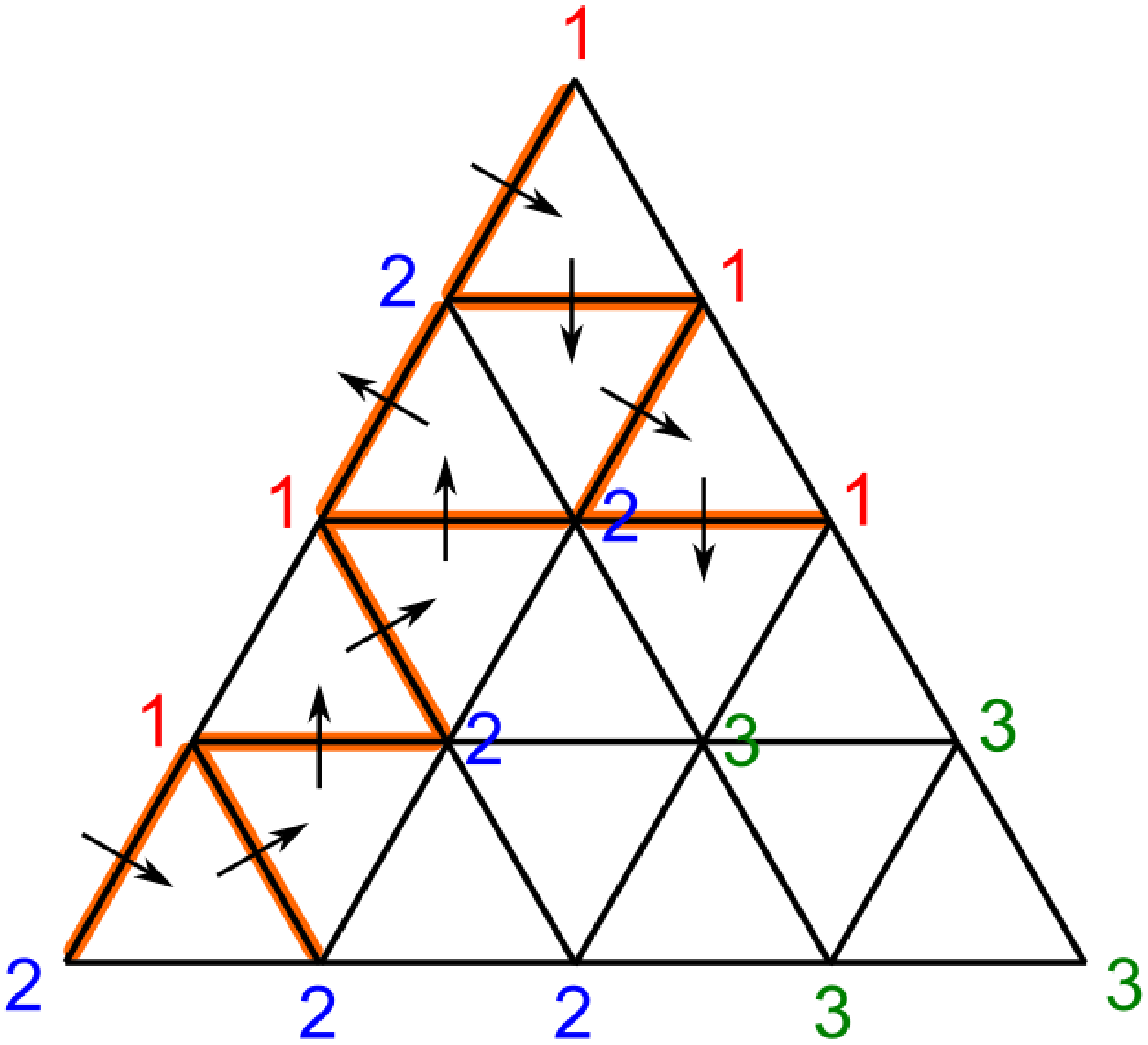

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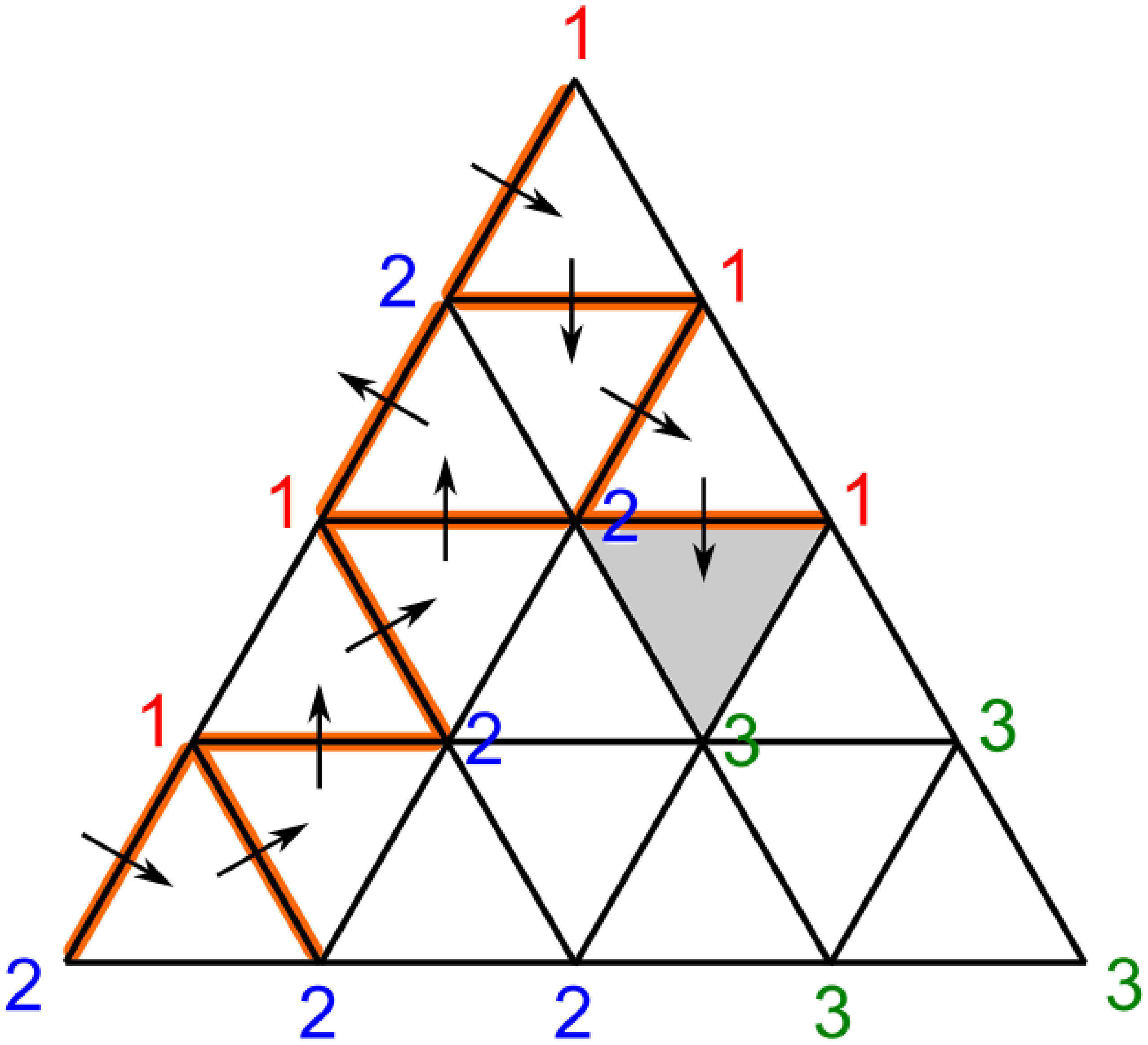

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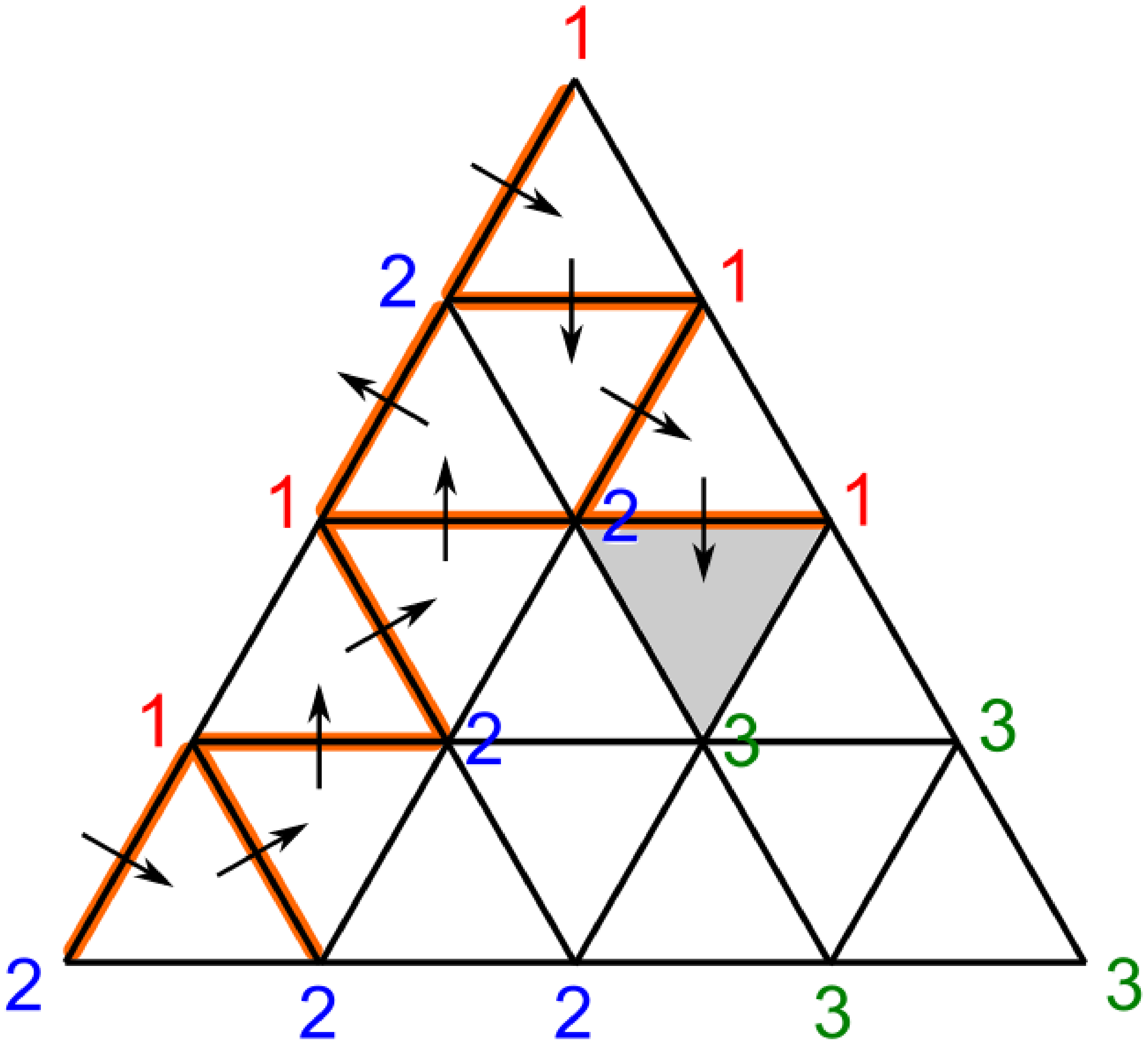

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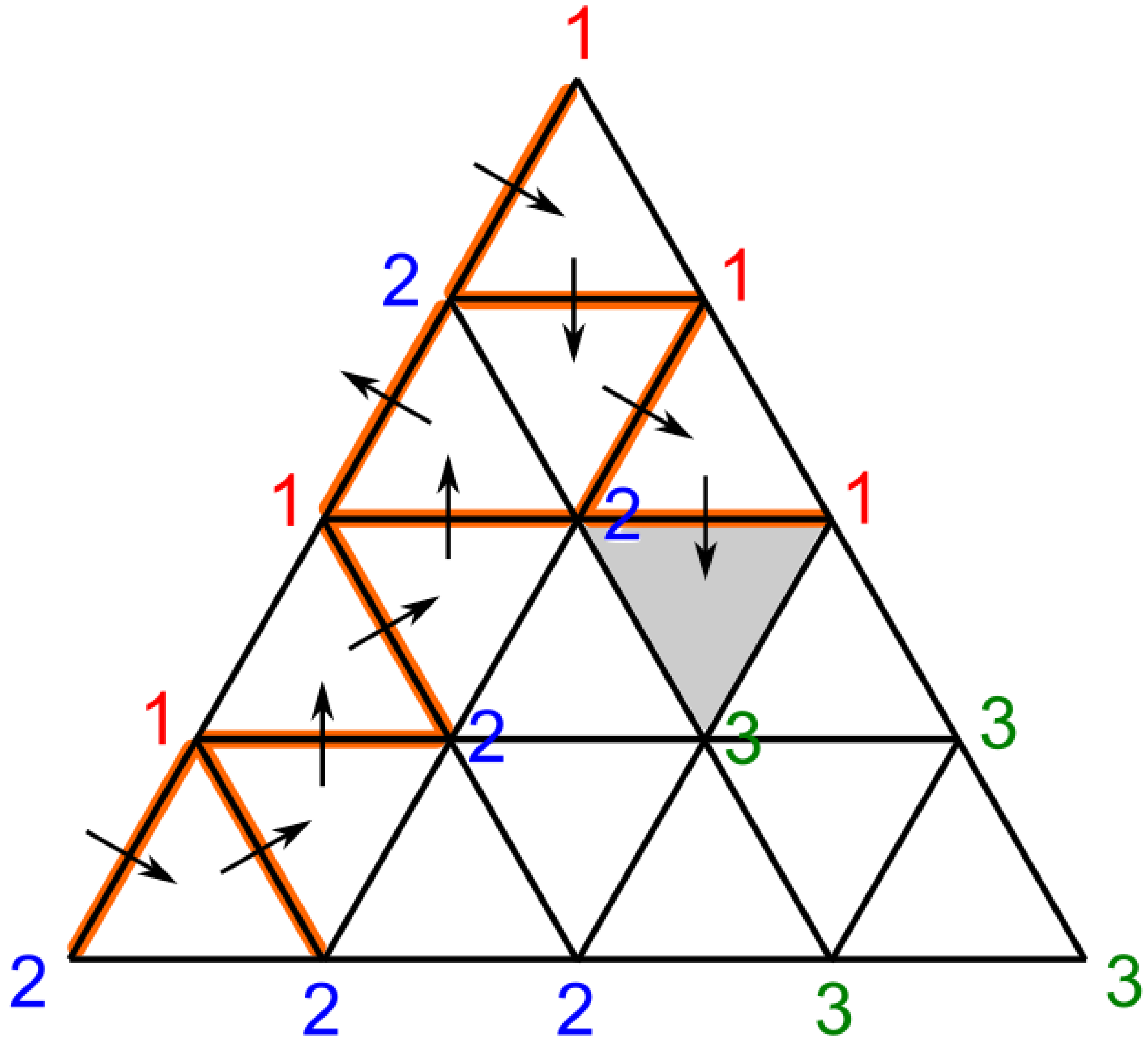

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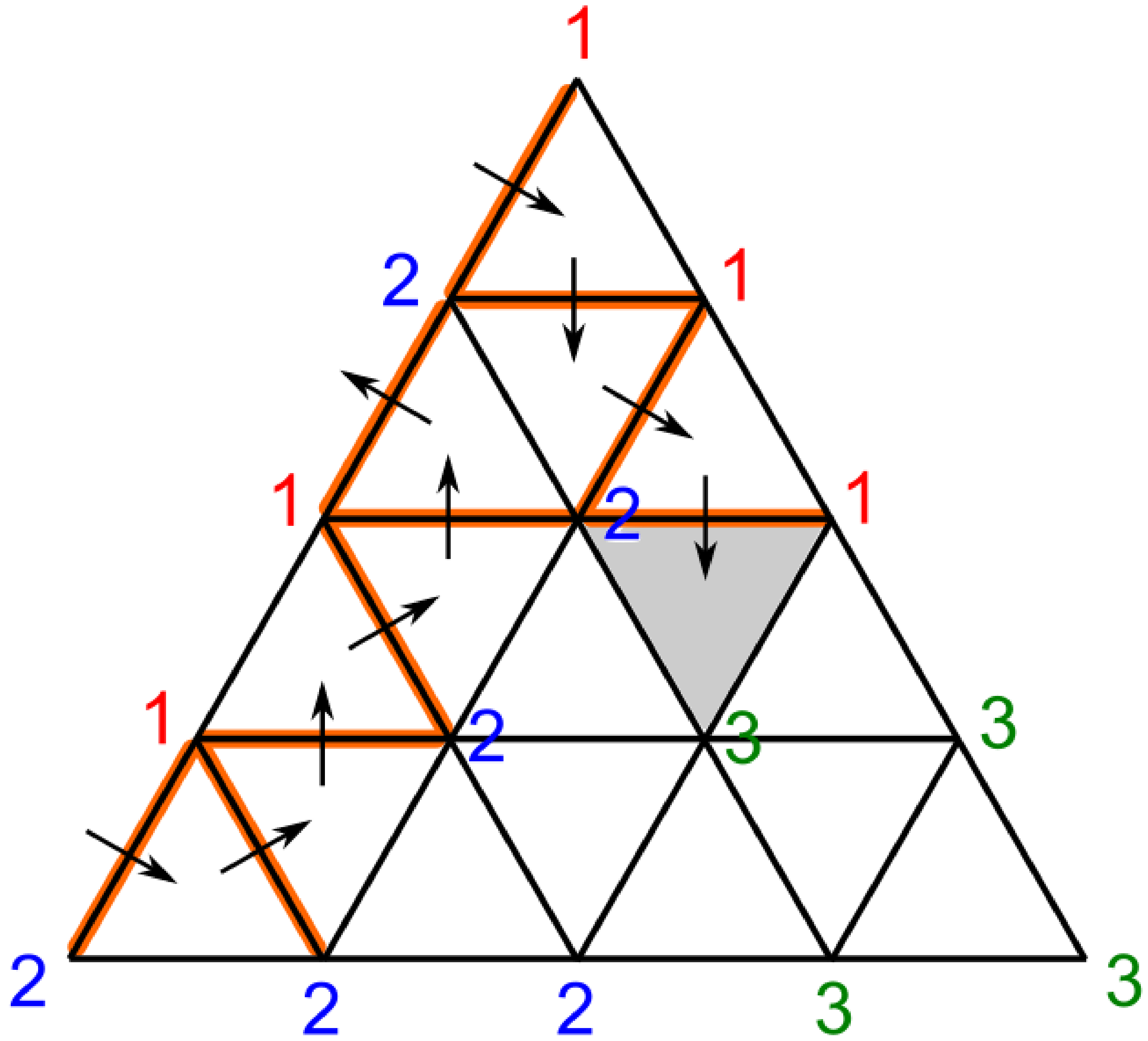

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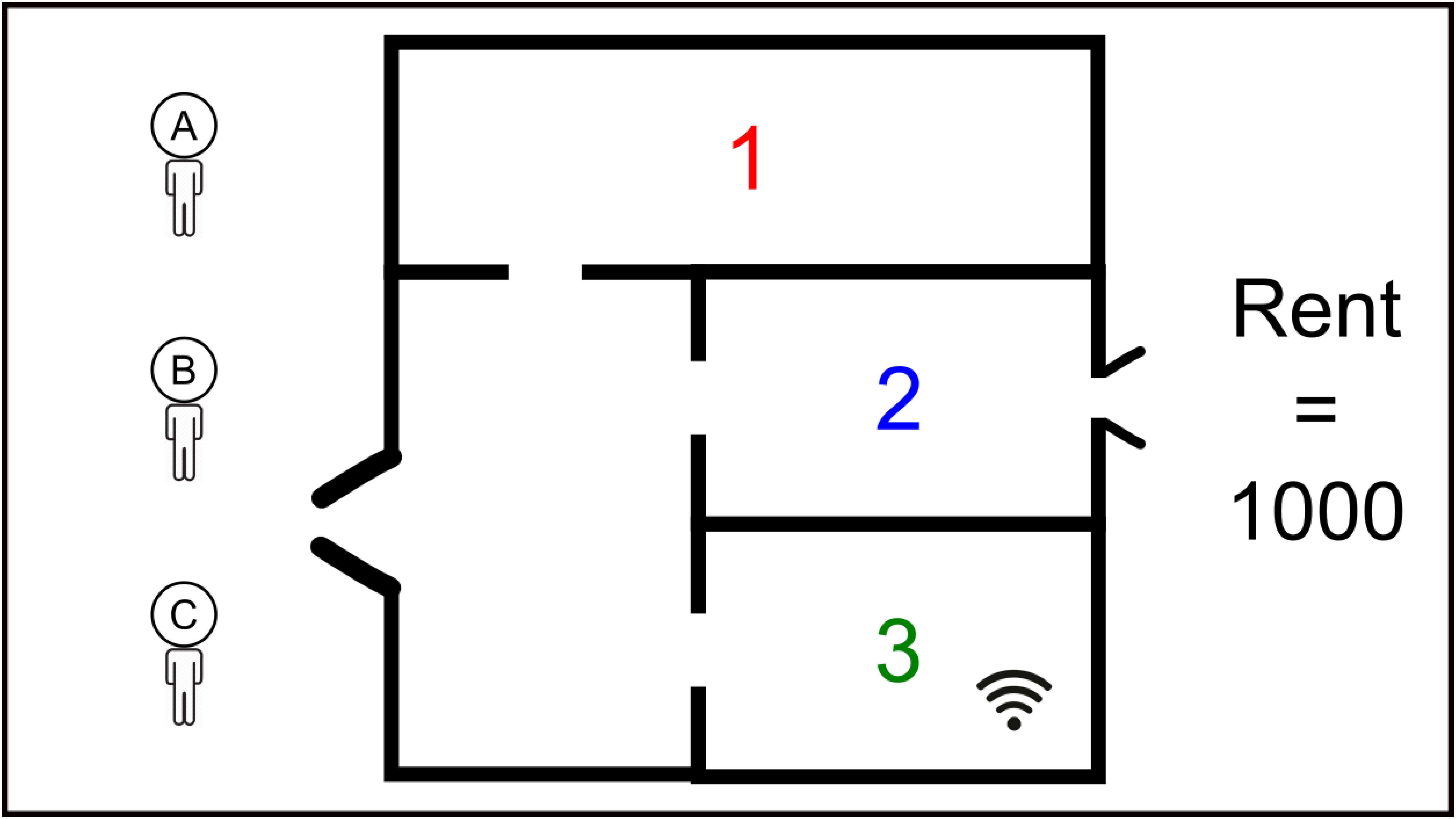
Rental Harmony via Sperner's Lemma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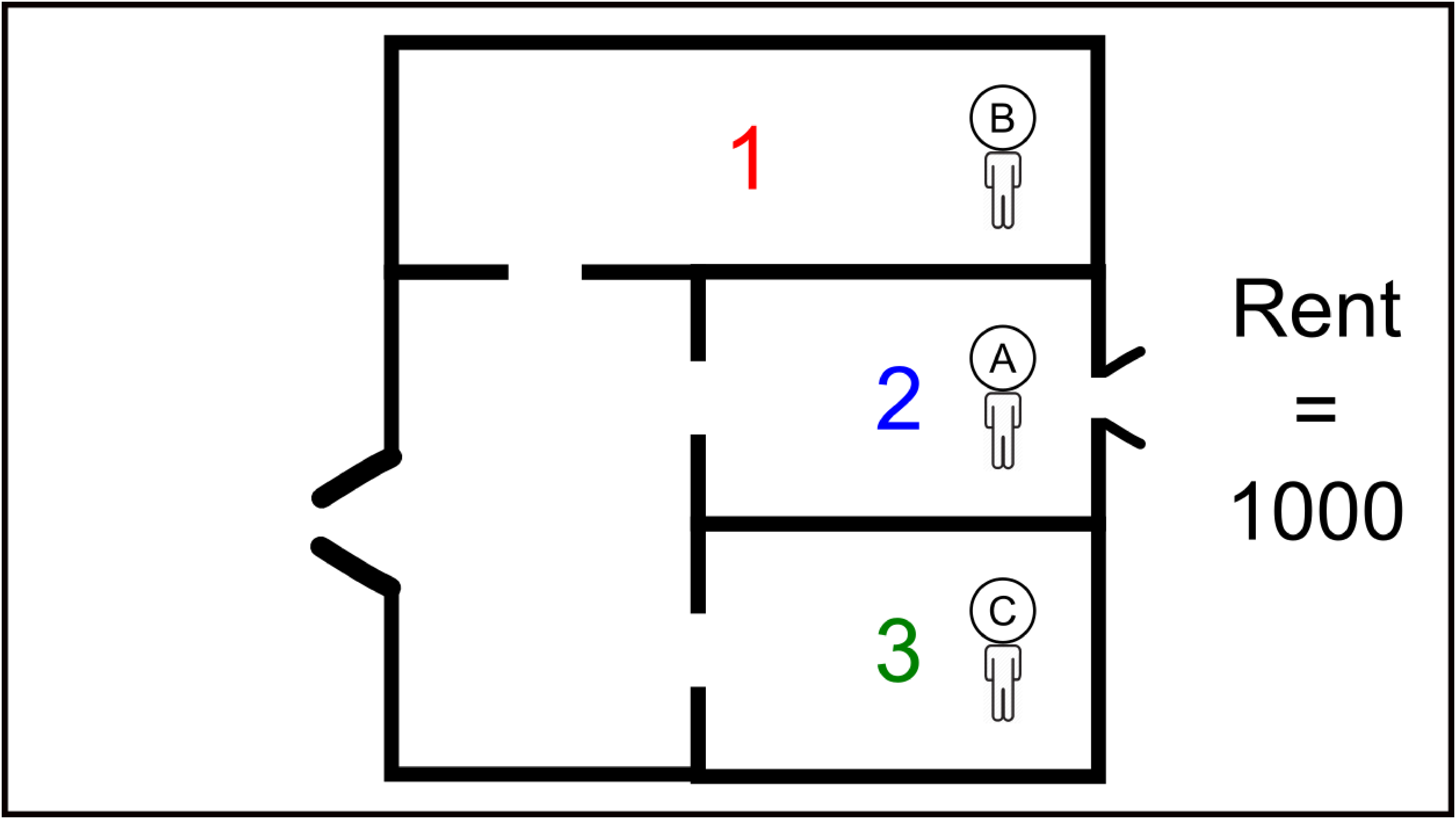

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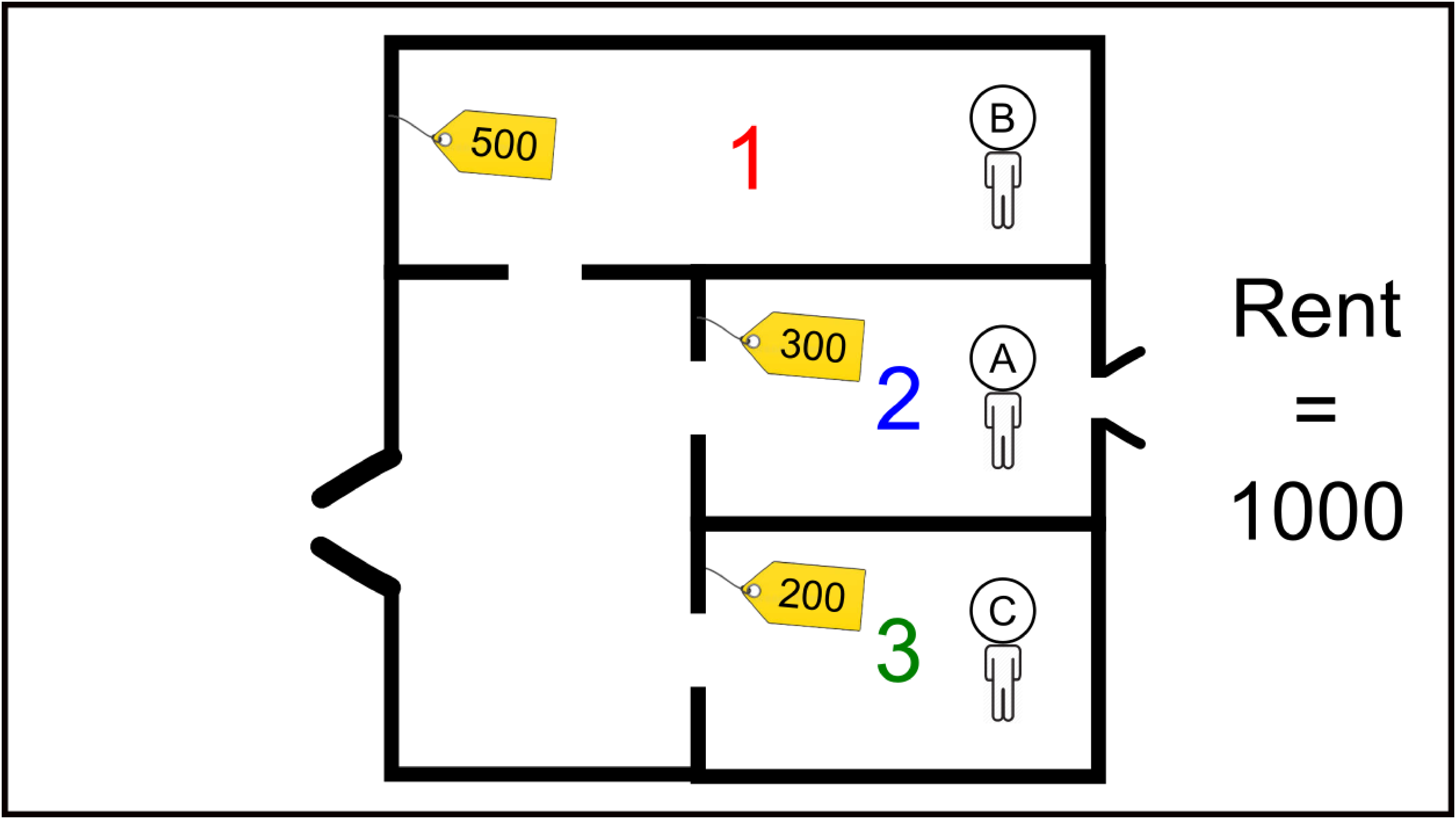
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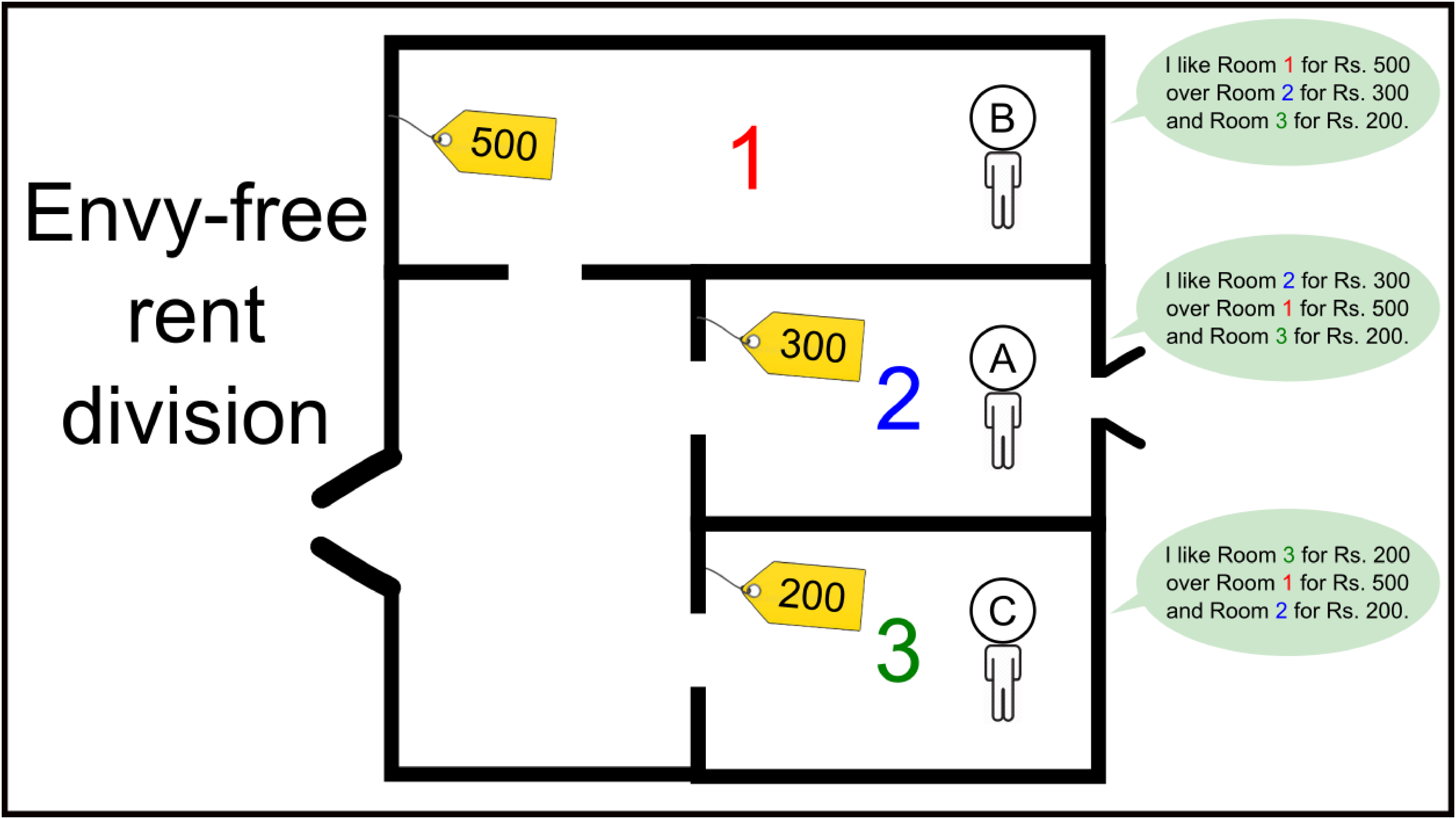
Envy-free rent division
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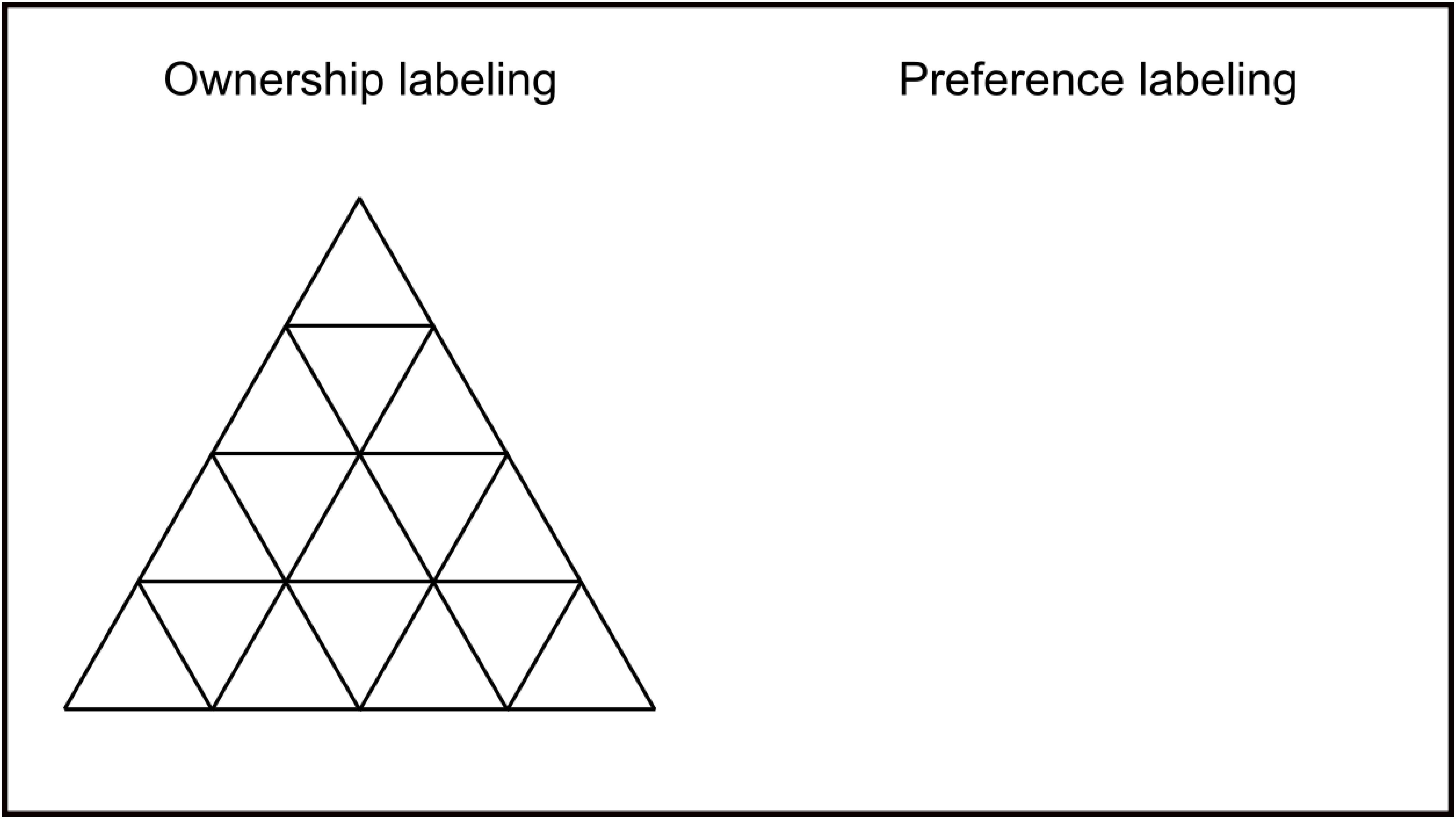
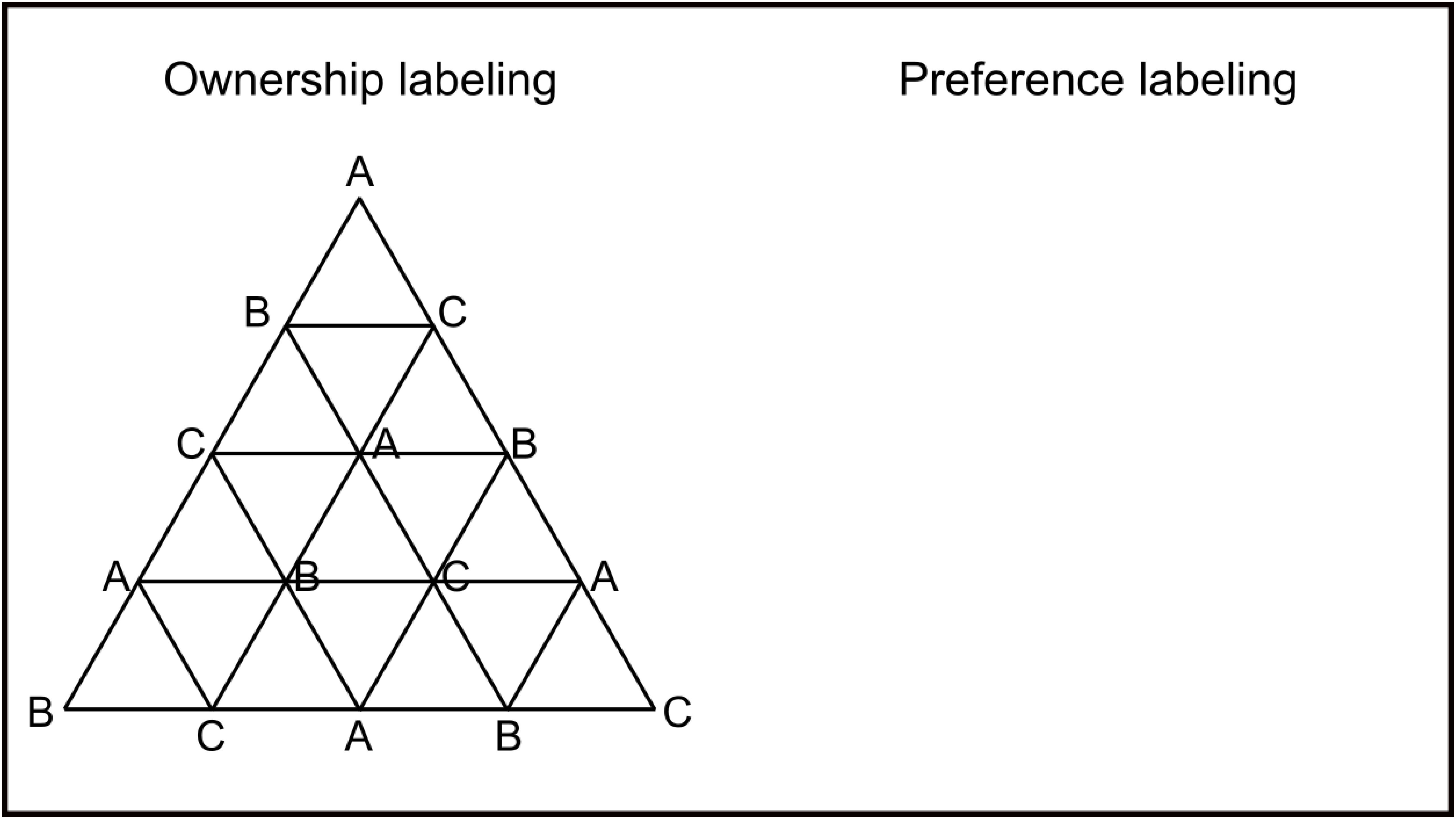
I like Room 3 for Rs. 200 over Room 1 for Rs. 500 and Room 2 for Rs. 200.

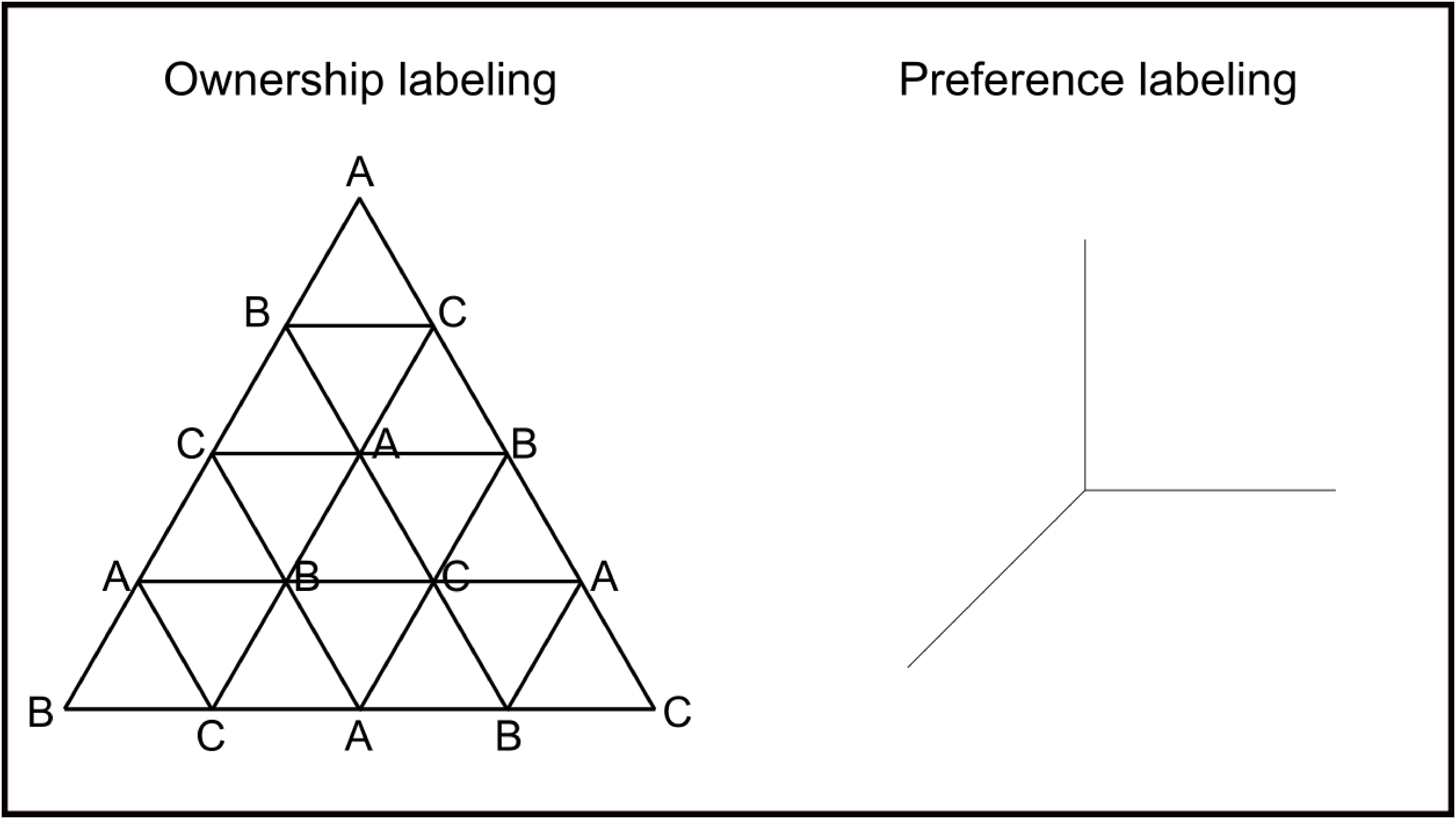
Ownership labeling

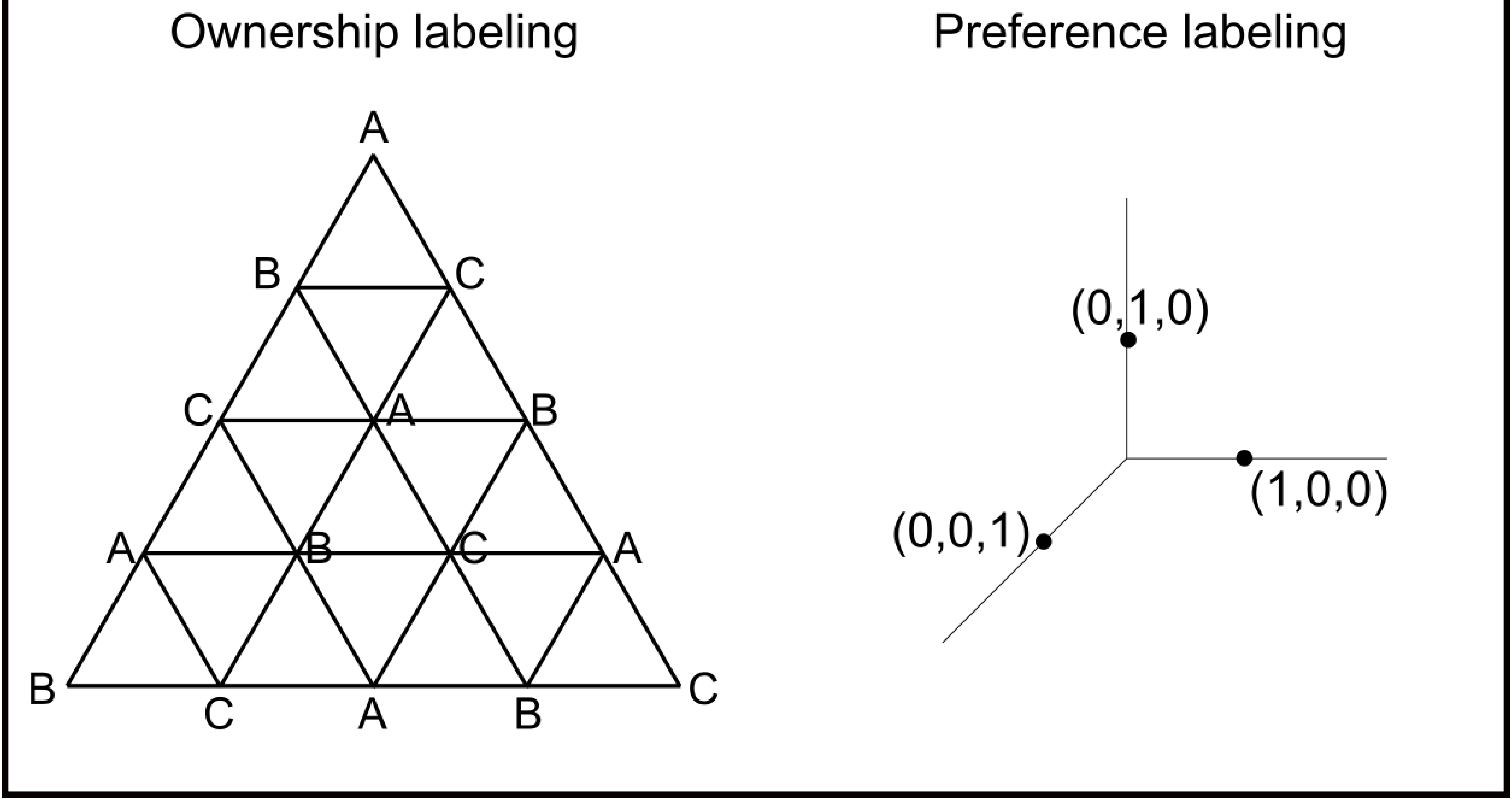
Preference labeling



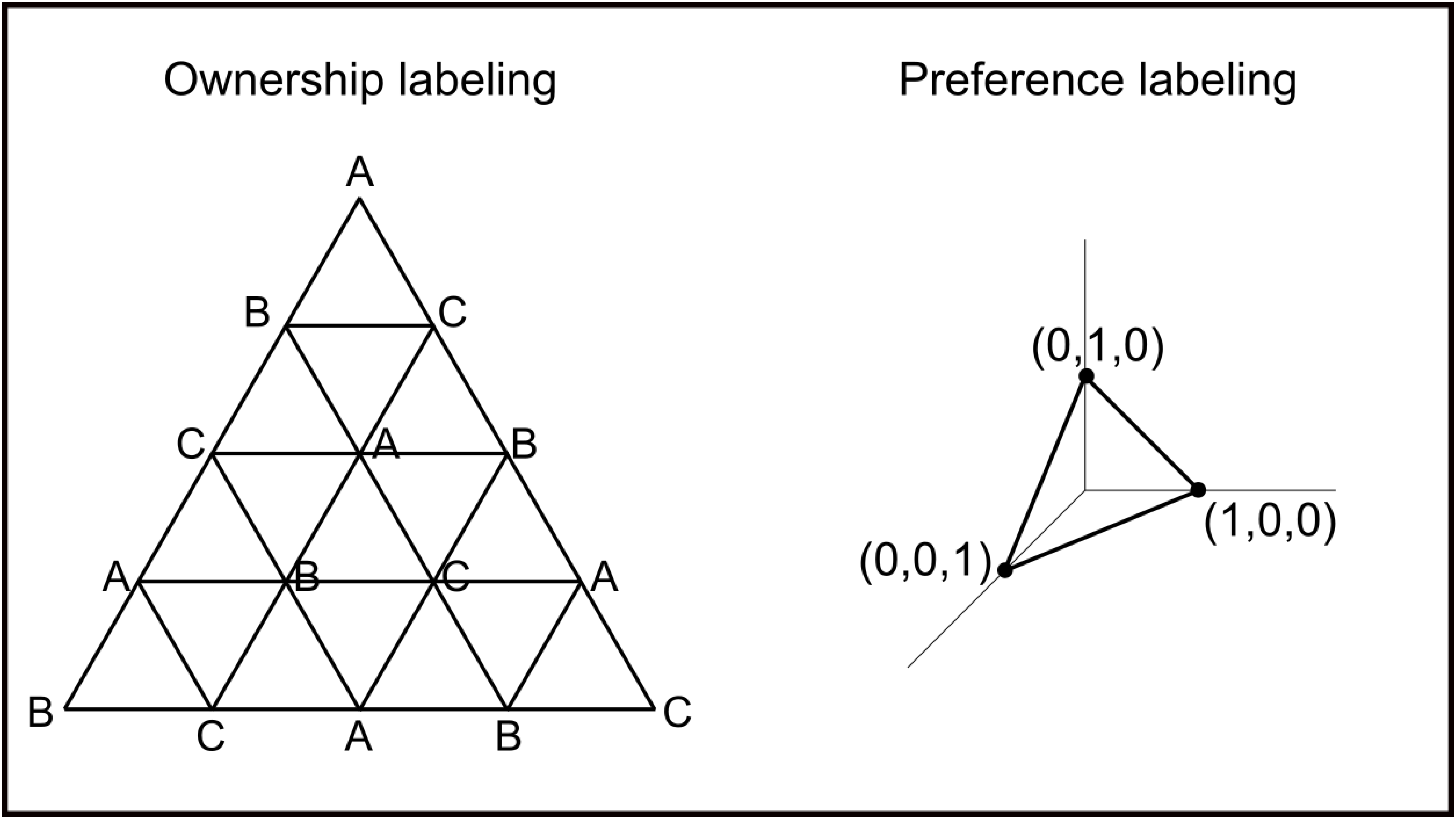
Ownership labeling
Preference labeling

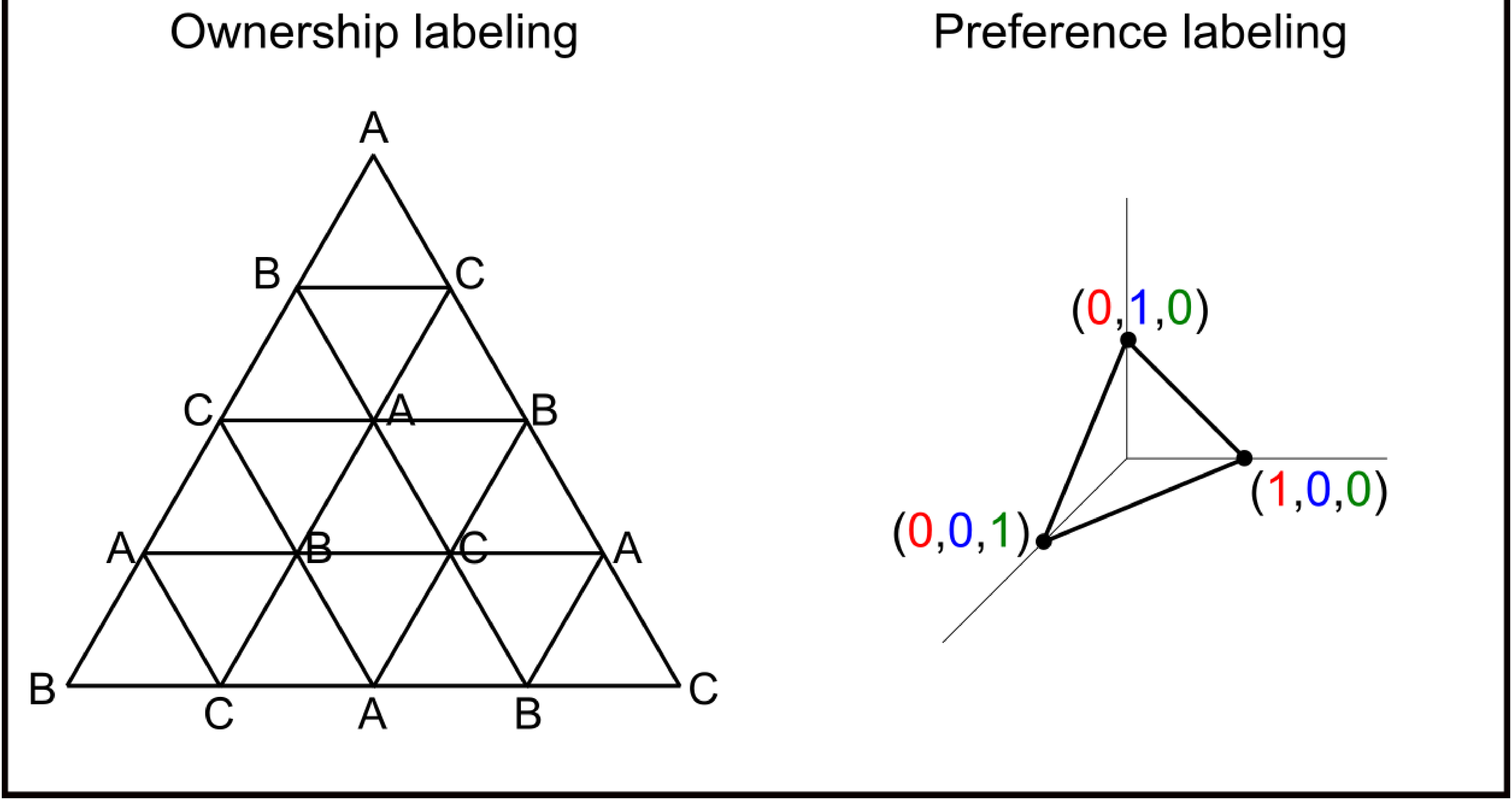

Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C

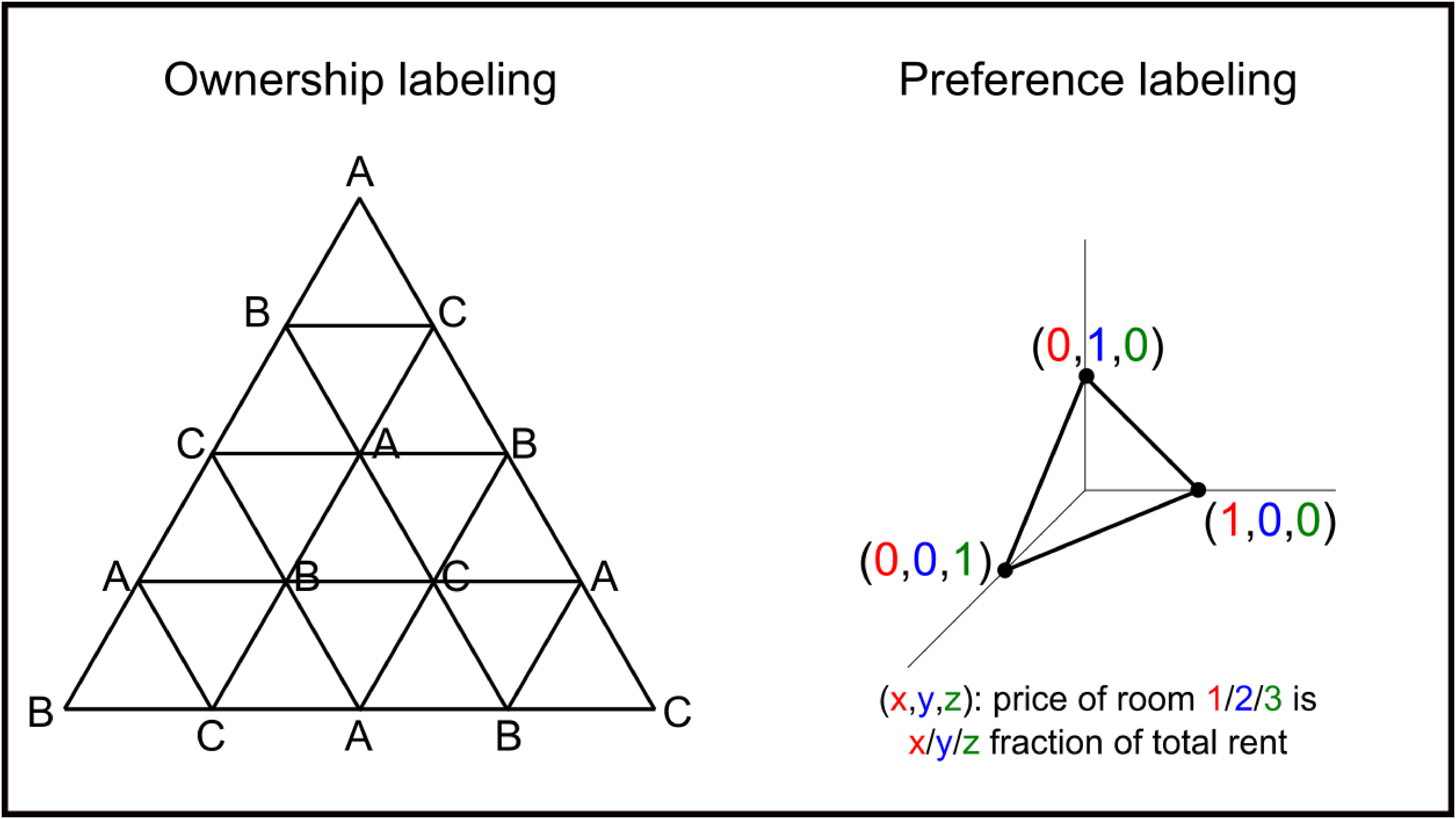

Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C

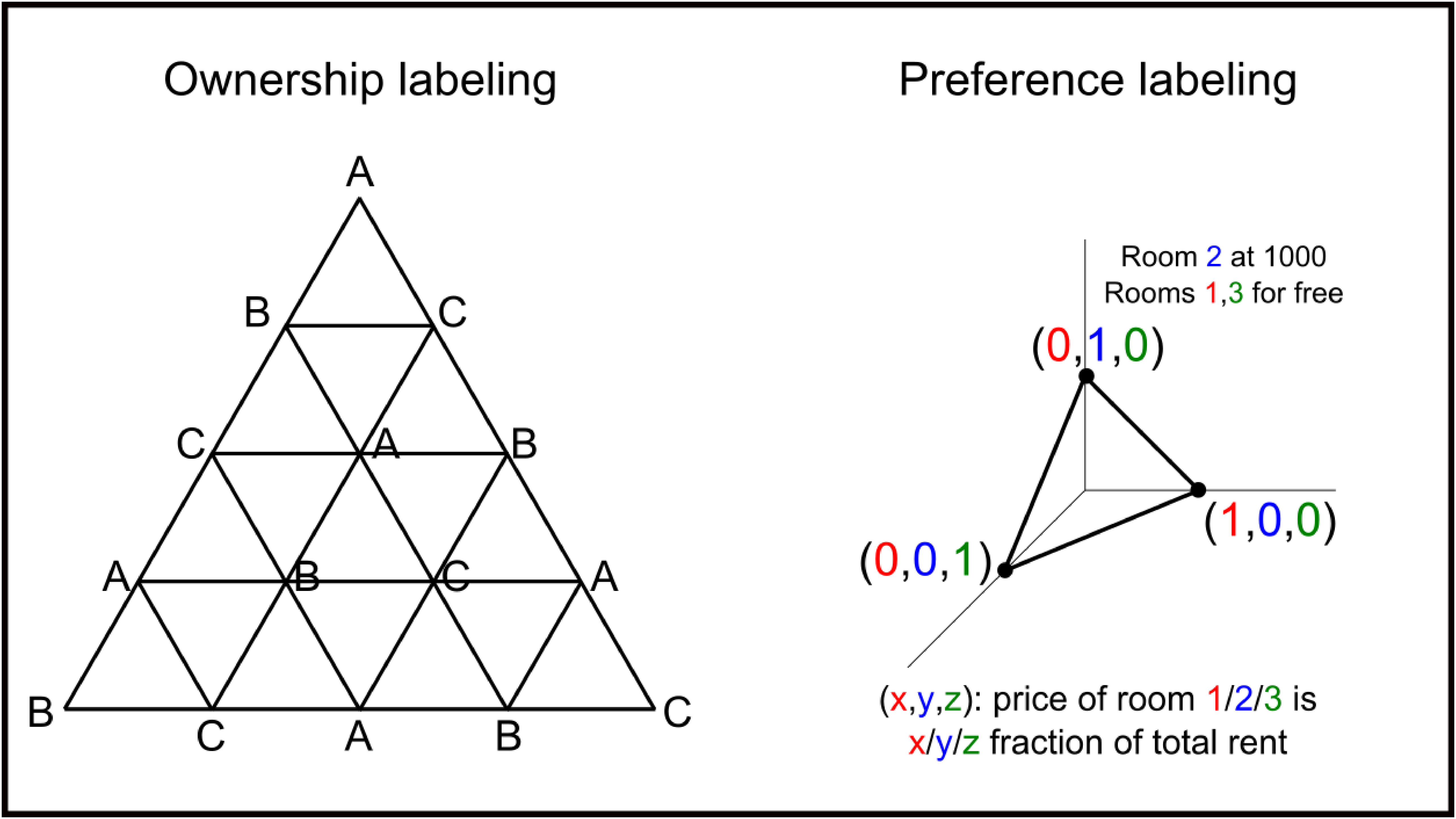


Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(0,1,0)
(1,0,0)
(0,0,1)

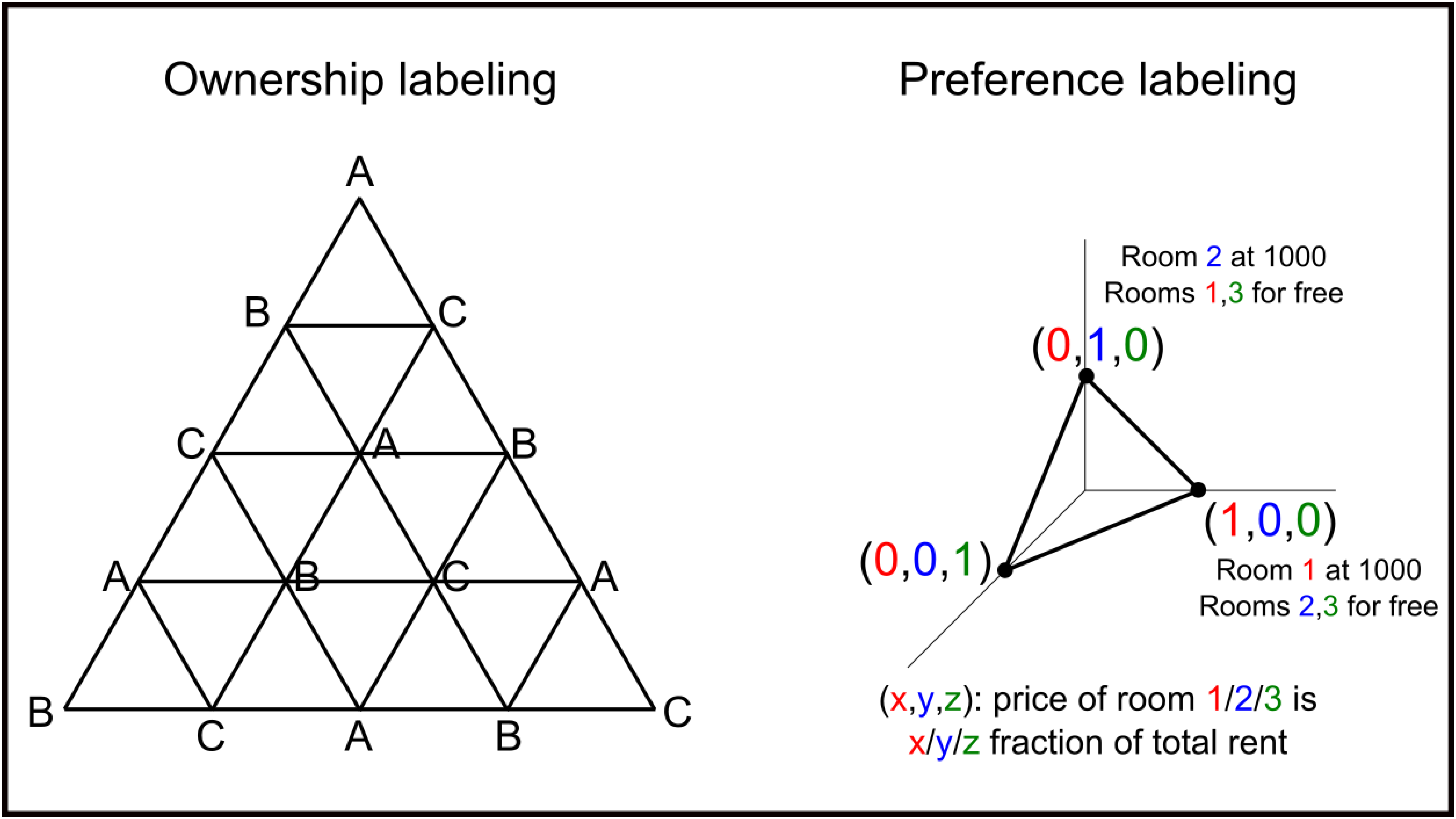

Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(0,1,0)
(1,0,0)
(0,0,1)

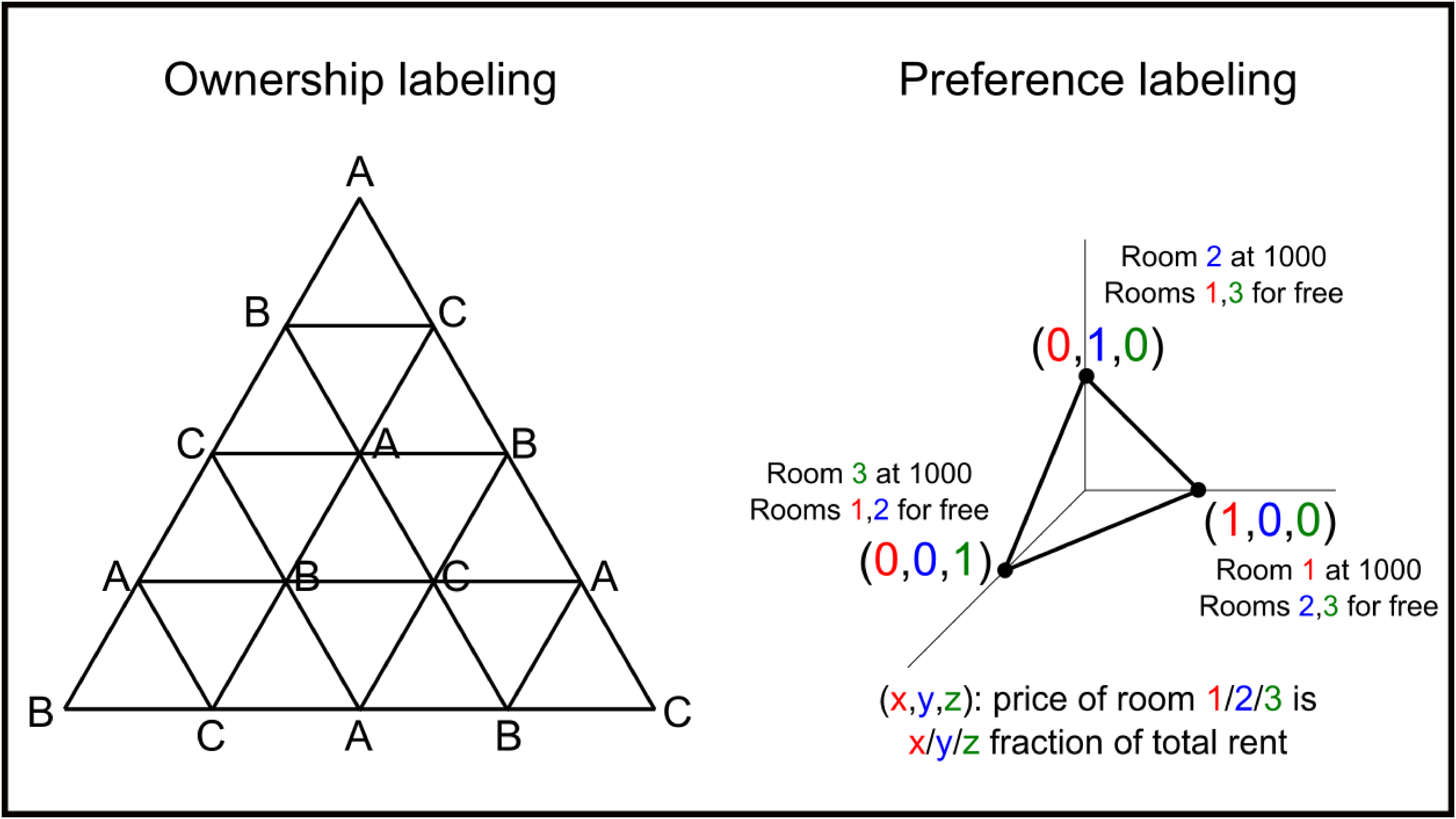

Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(0,1,0)
(1,0,0)
(0,0,1)


Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(0,1,0)
(1,0,0)
(0,0,1)
(x,y,z): price of room 1/2/3 is
x/y/z fraction of total rent

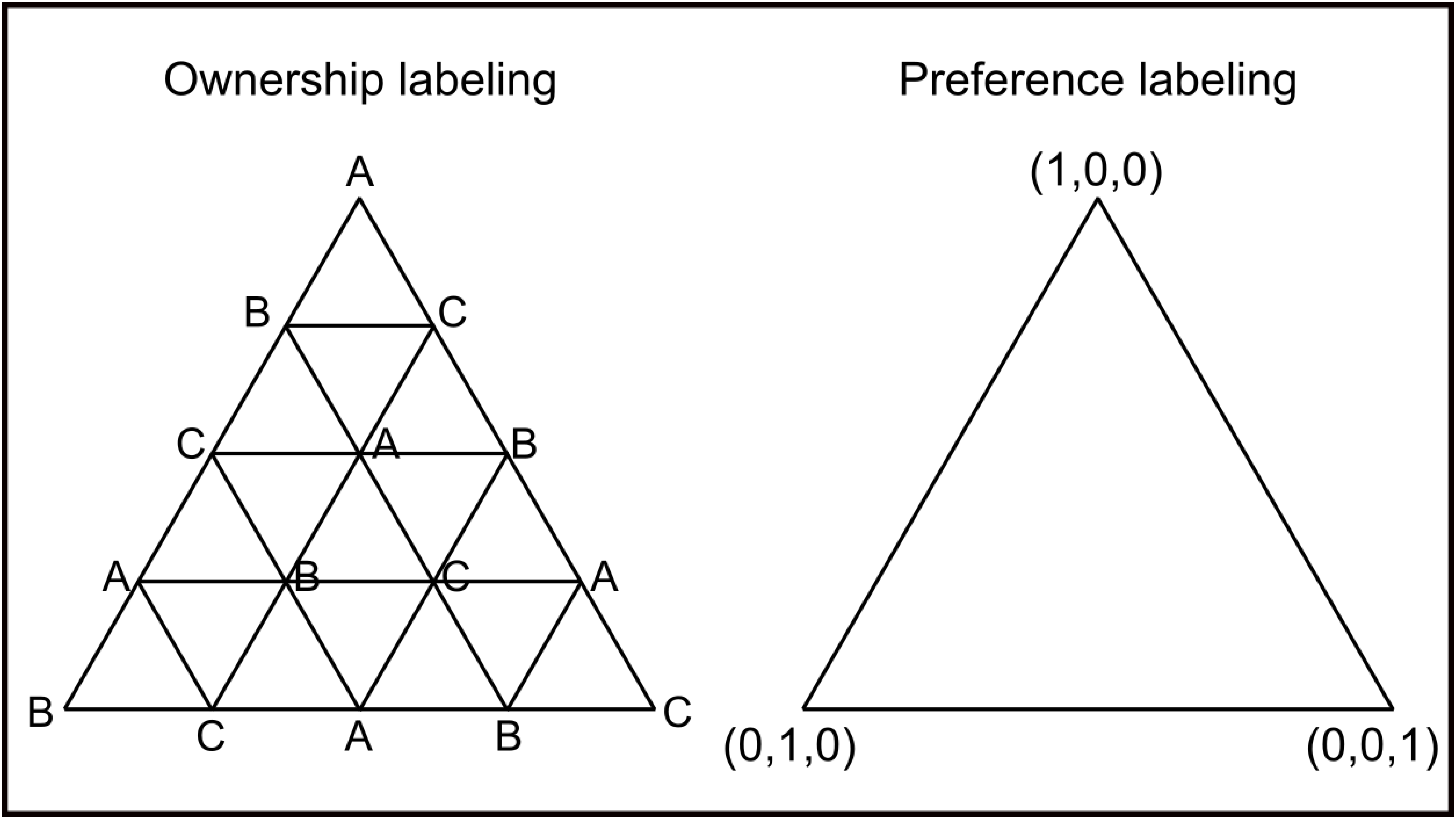


Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
Room 2 at 1000
Rooms 1,3 for free
(0,1,0)
(1,0,0)
(0,0,1)
(x,y,z): price of room 1/2/3 is
x/y/z fraction of total rent

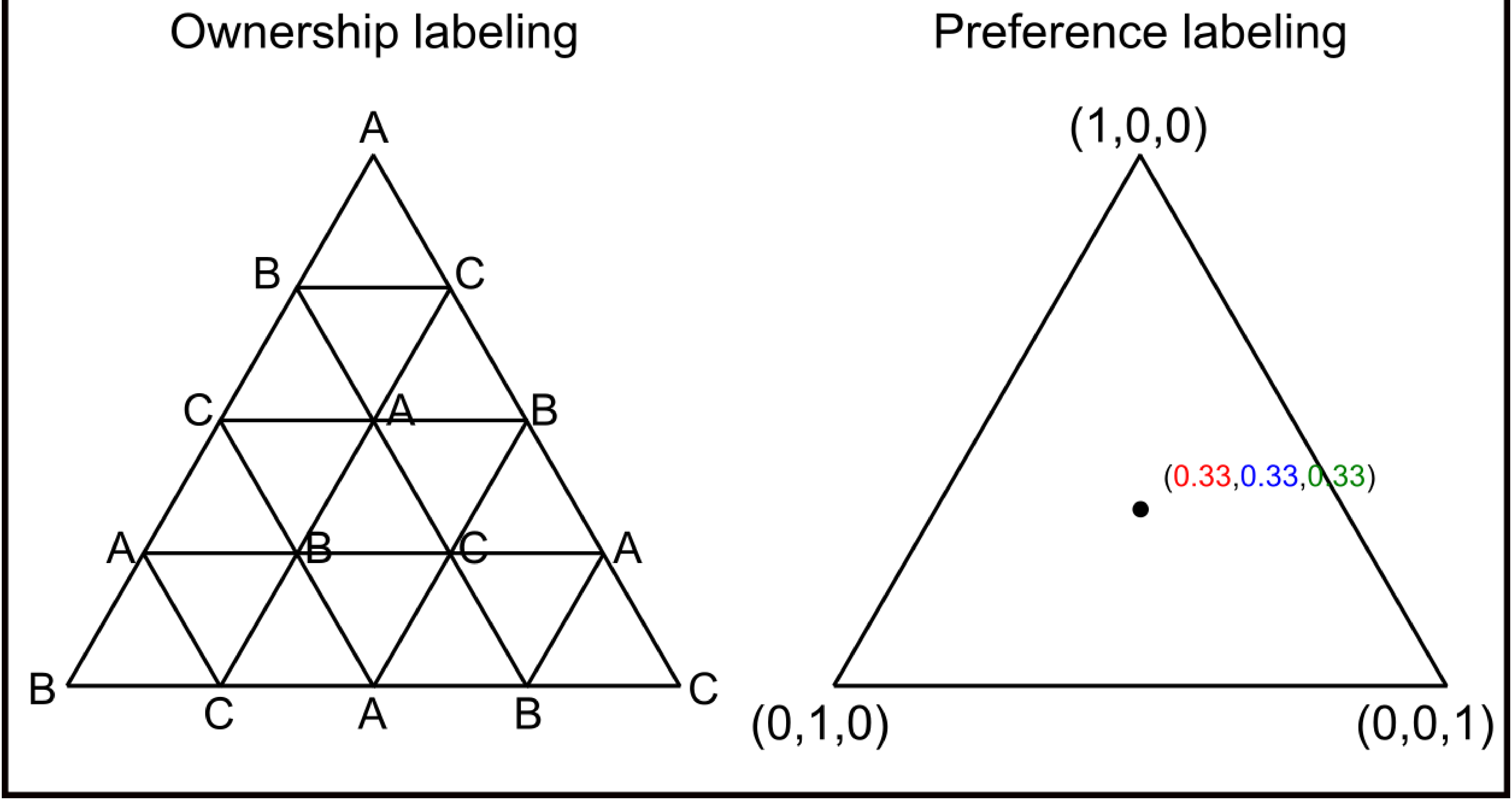

Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Room 2 at 1000
Rooms 1,3 for free
(0,1,0)
(1,0,0)
(0,0,1)
Room 1 at 1000
Rooms 2,3 for free
(x,y,z): price of room 1/2/3 is
x/y/z fraction of total rent

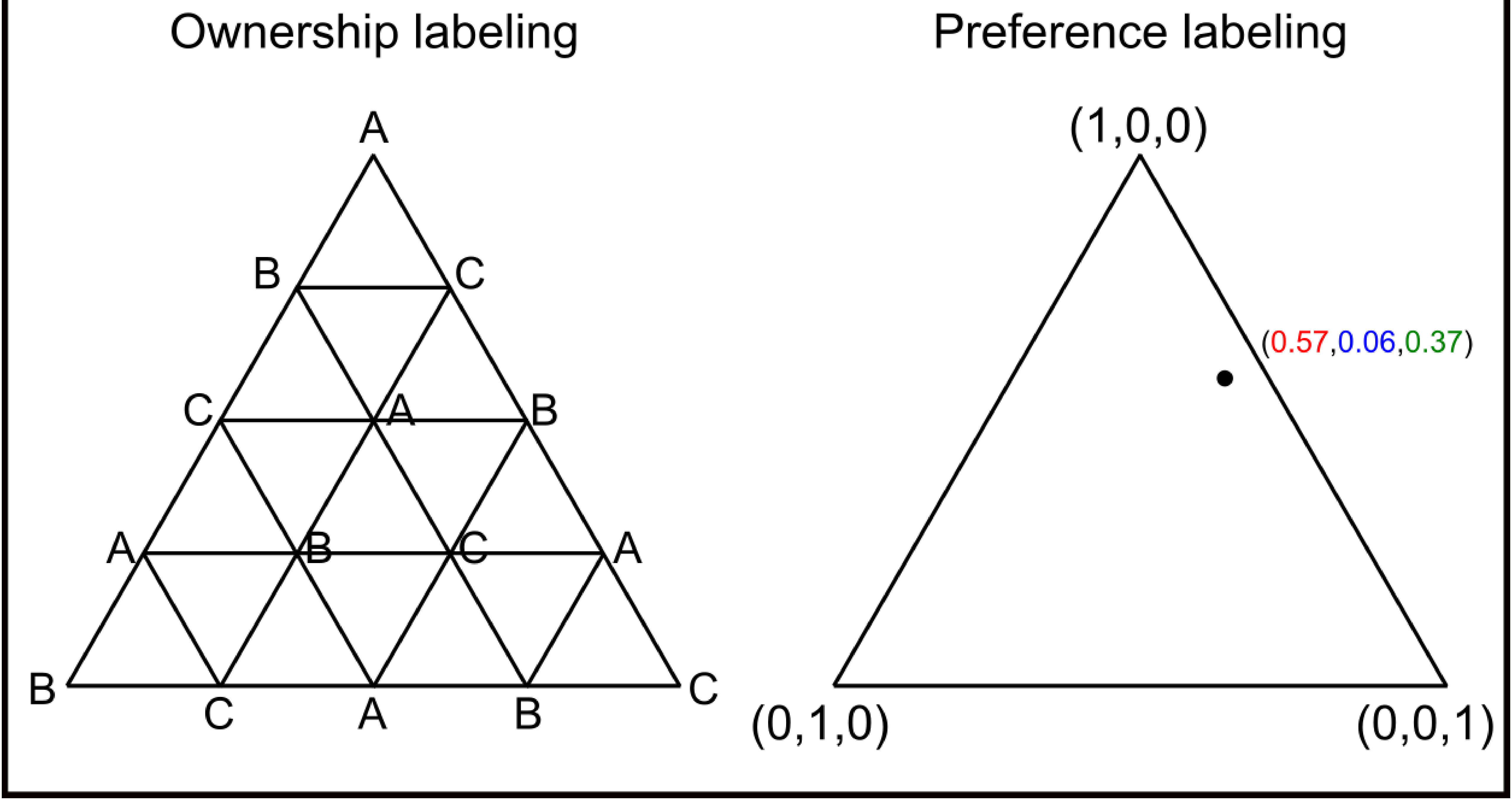


Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
Room 2 at 1000
Rooms 1,3 for free
(0,1,0)
Room 3 at 1000
Rooms 1,2 for free
(1,0,0)
(0,0,1)
Room 1 at 1000
Rooms 2,3 for free
(x,y,z): price of room 1/2/3 is
x/y/z fraction of total rent

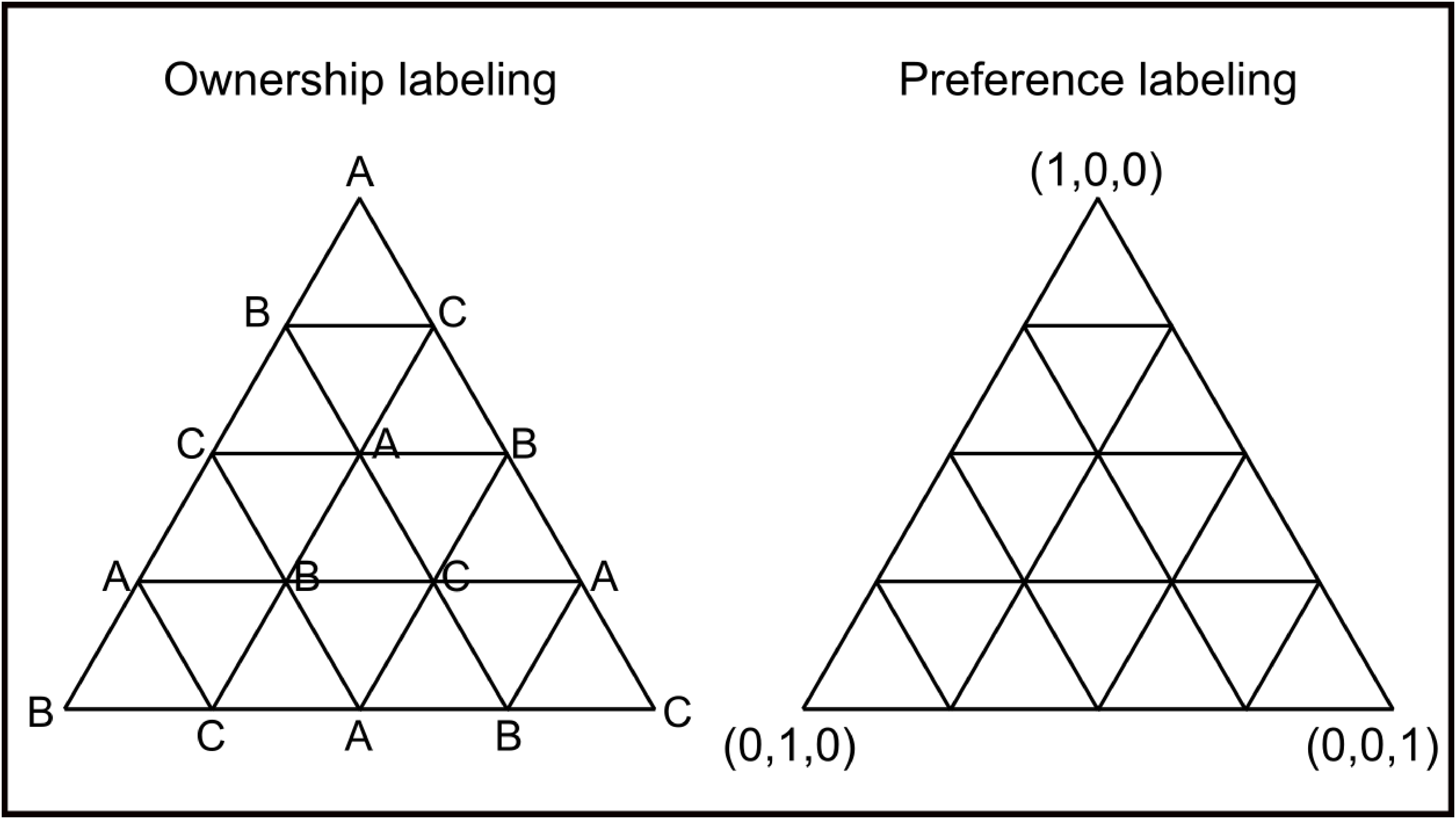


Ownership labeling
Preference labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(1,0,0)
(0,1,0)
(0,0,1)

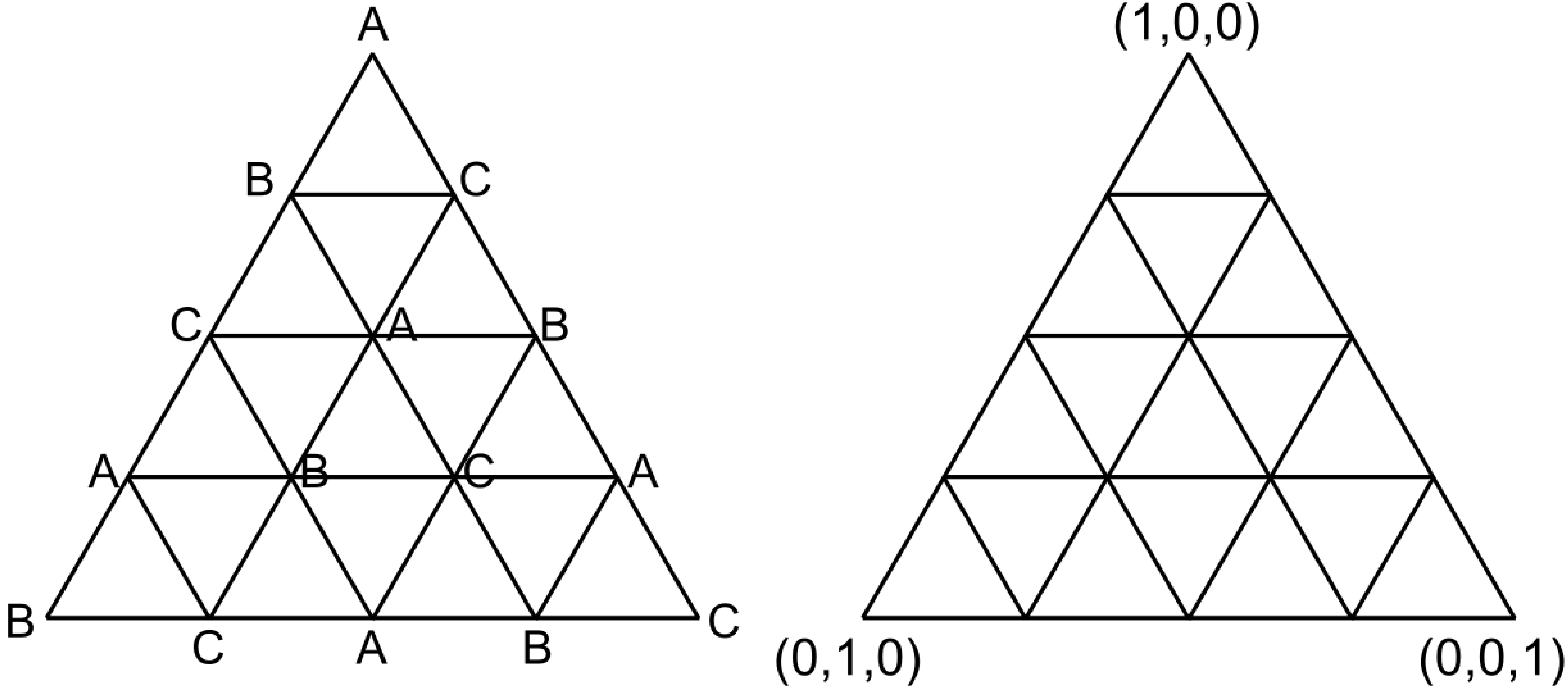

Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
(1,0,0)
(0.33,0.33,0.33)
(0,1,0)
(0,0,1)

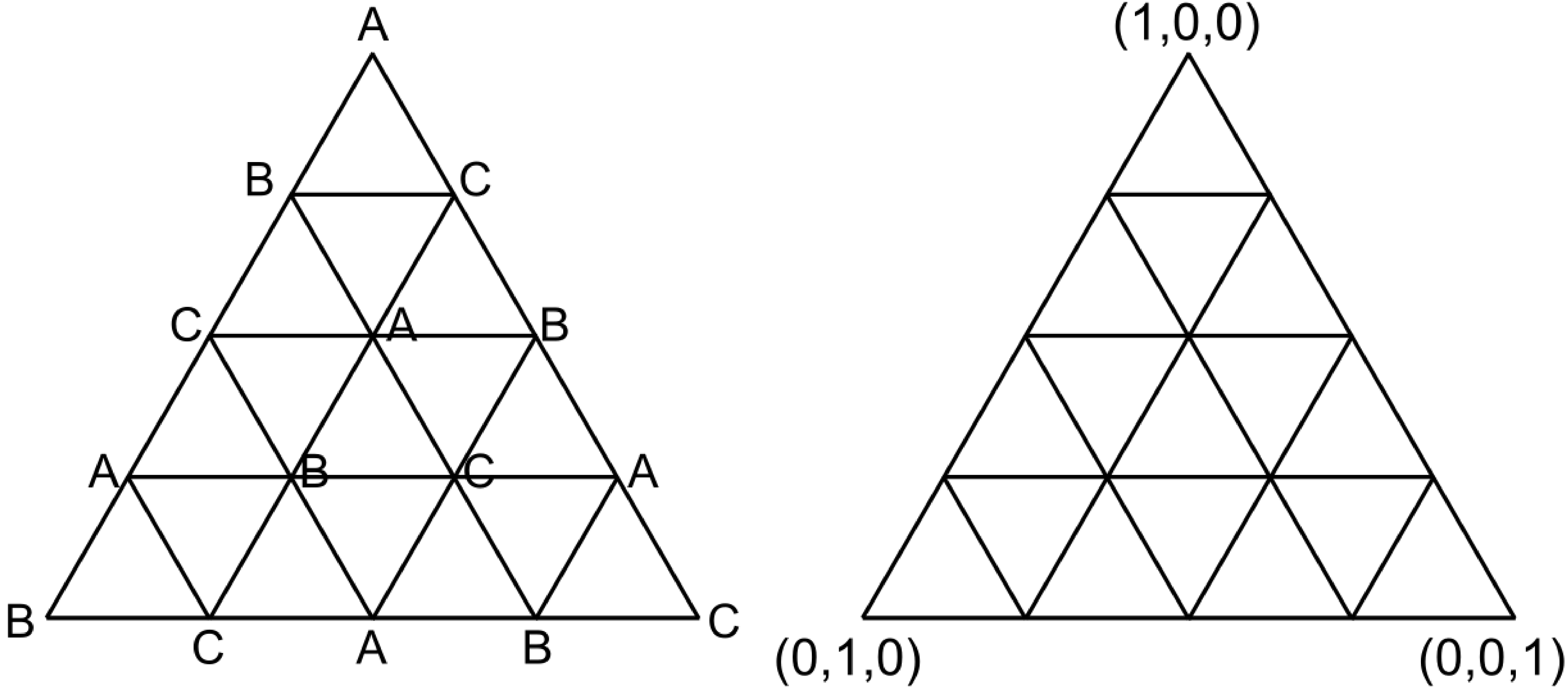


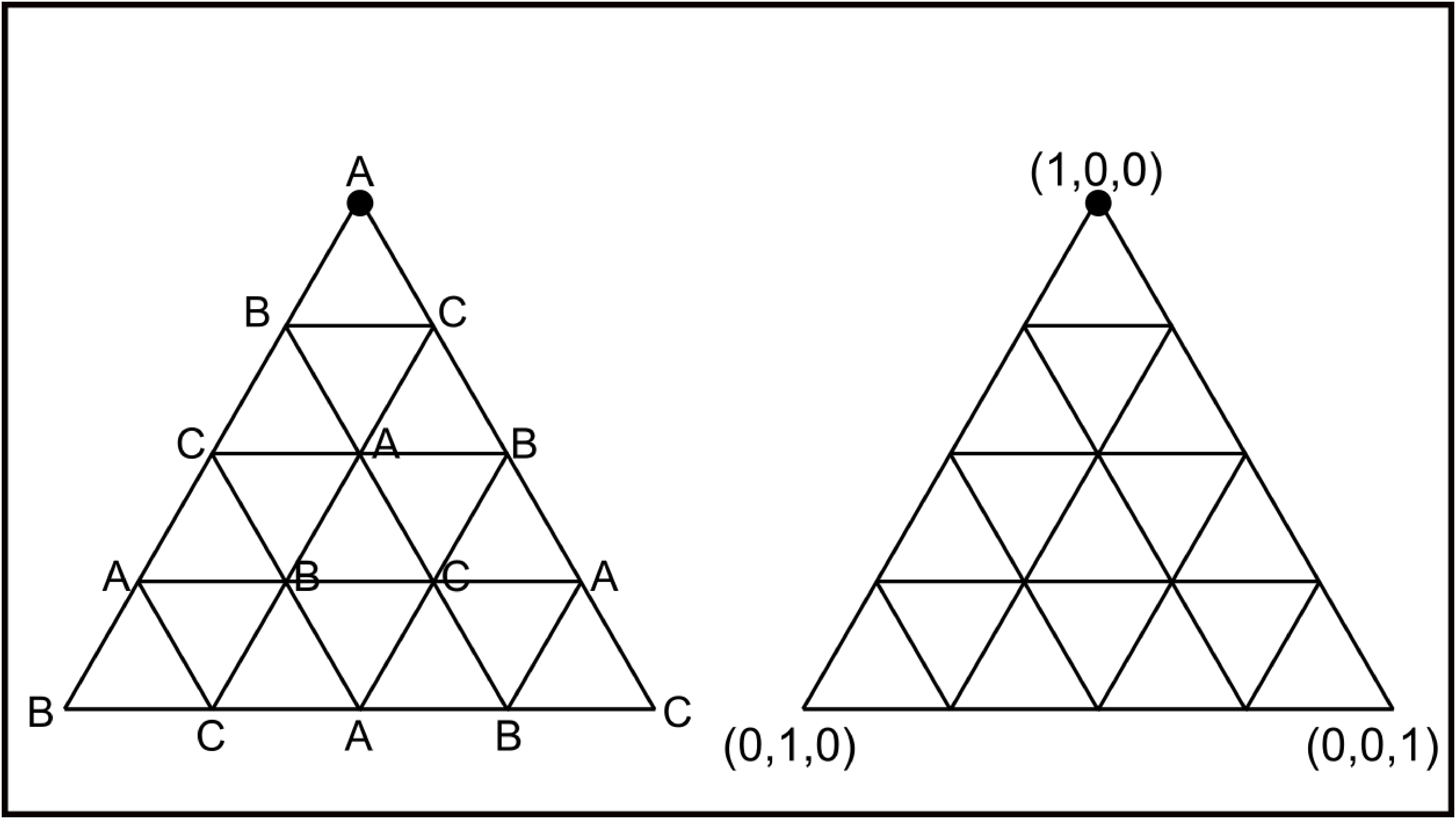
Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
(1,0,0)
(0.57,0.06,0.37)
(0,1,0)
(0,0,1)


Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
(1,0,0)
(0,1,0)
(0,0,1)

For each vertex in the ownership triangle, ask the owner its favorite room at the pricing given in the preference triangle.

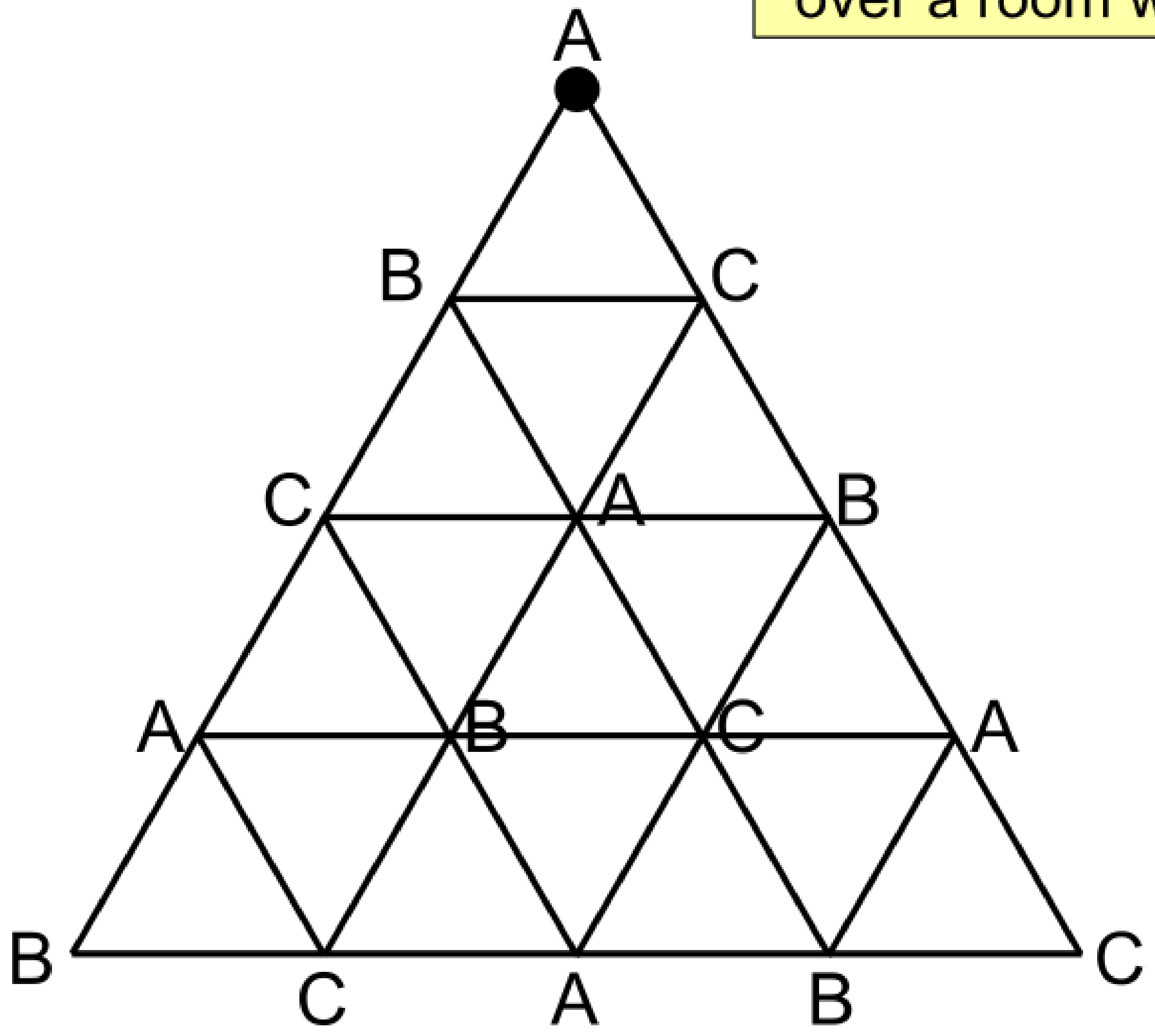


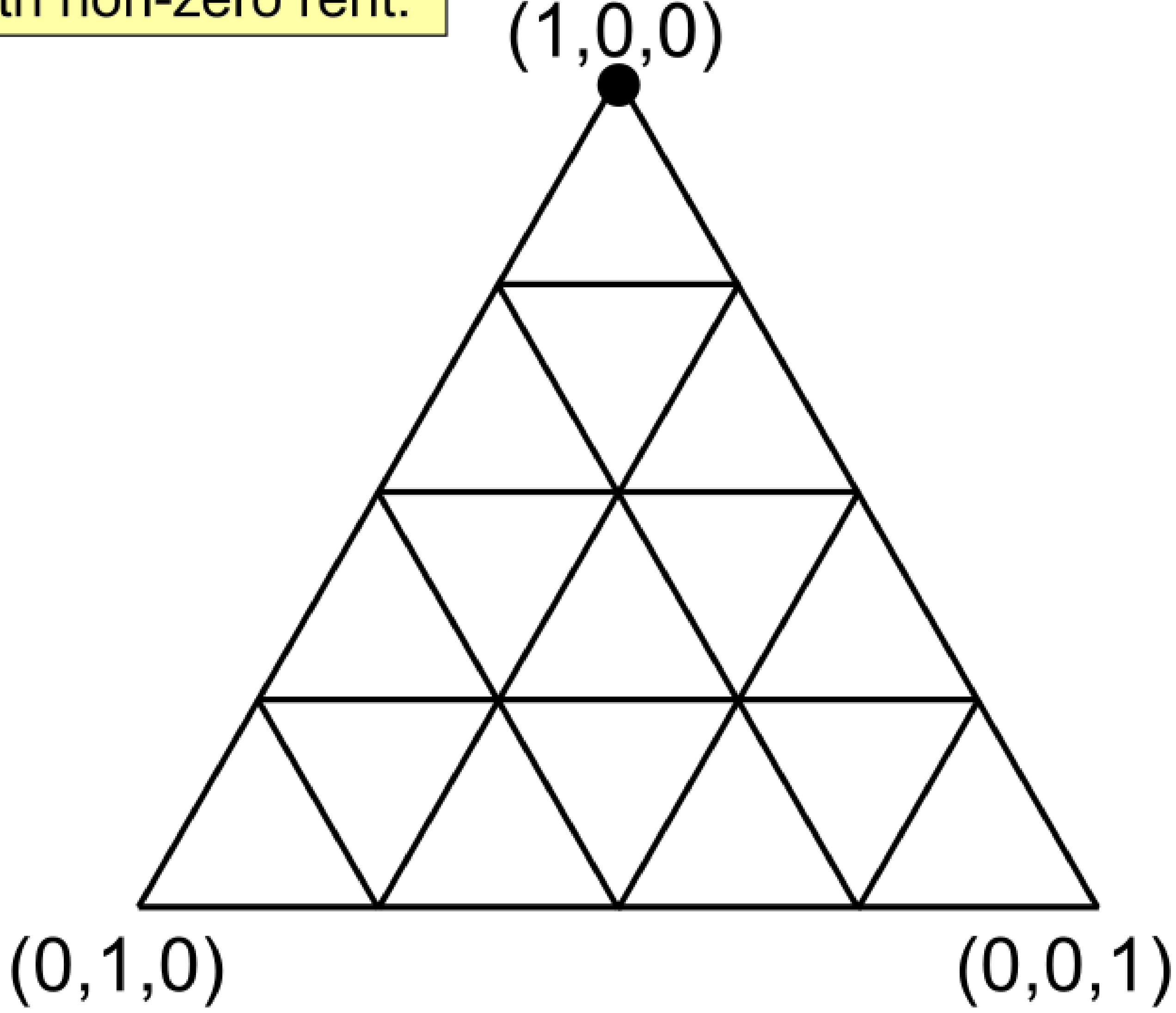

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(1,0,0)
(0,1,0)
(0,0,1)

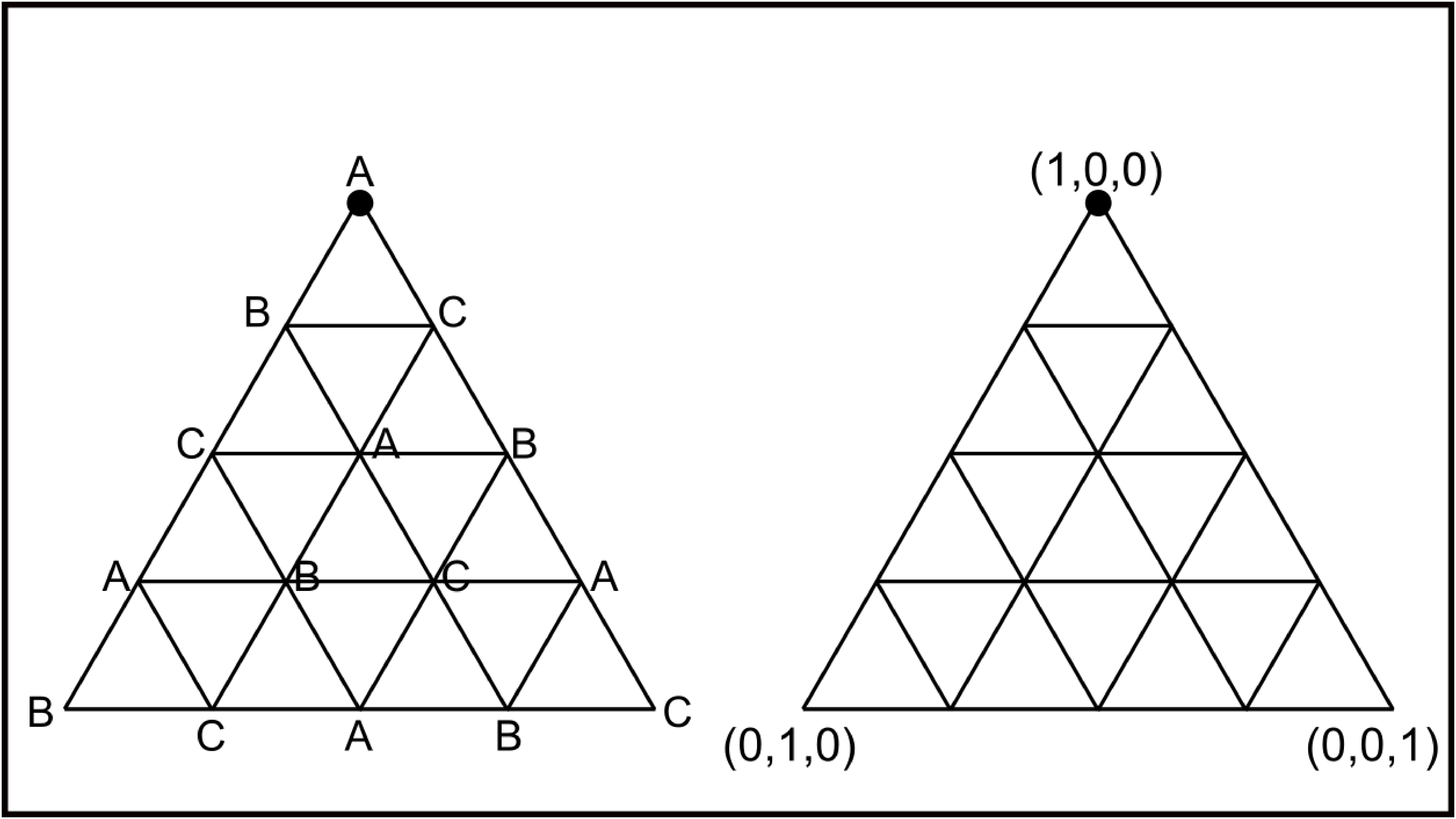

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(1,0,0)
(0,1,0)
(0,0,1)

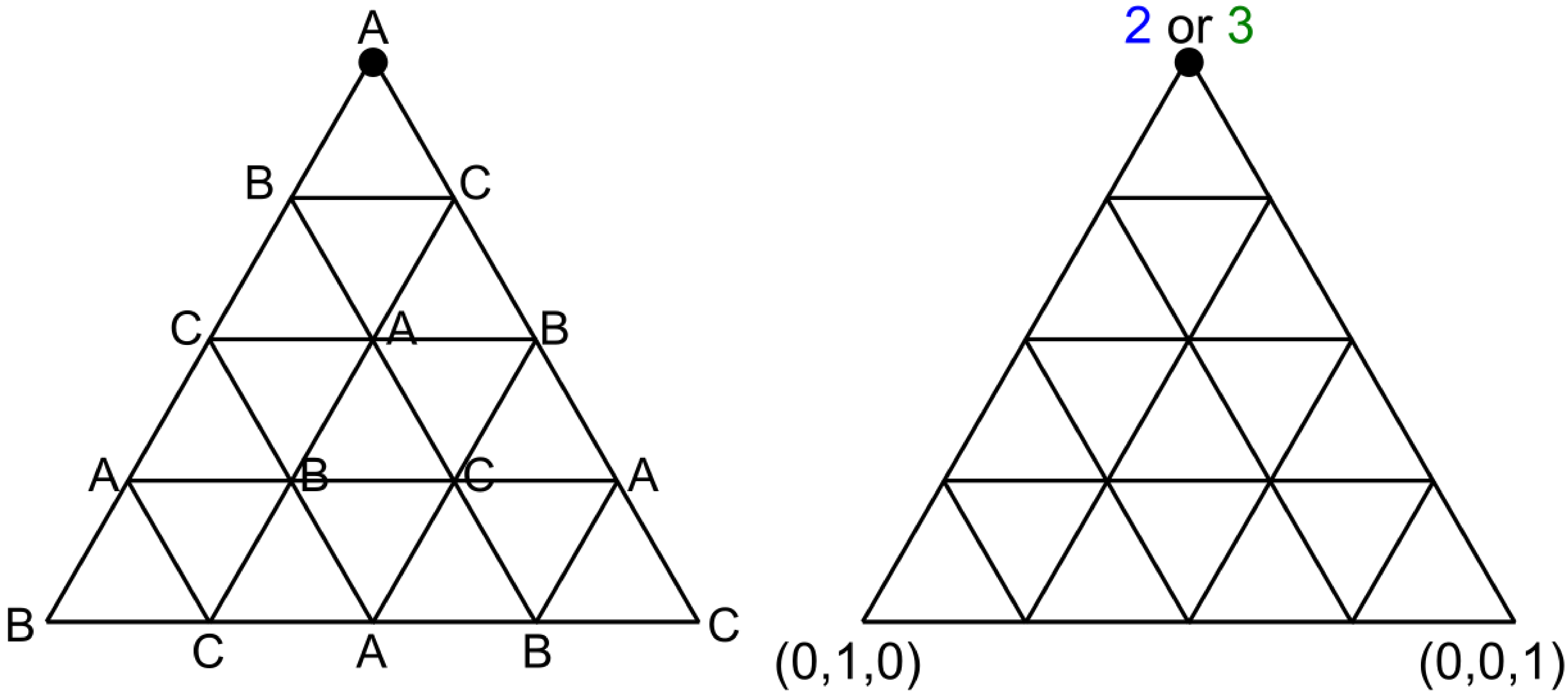
"Miserly agents" assumption

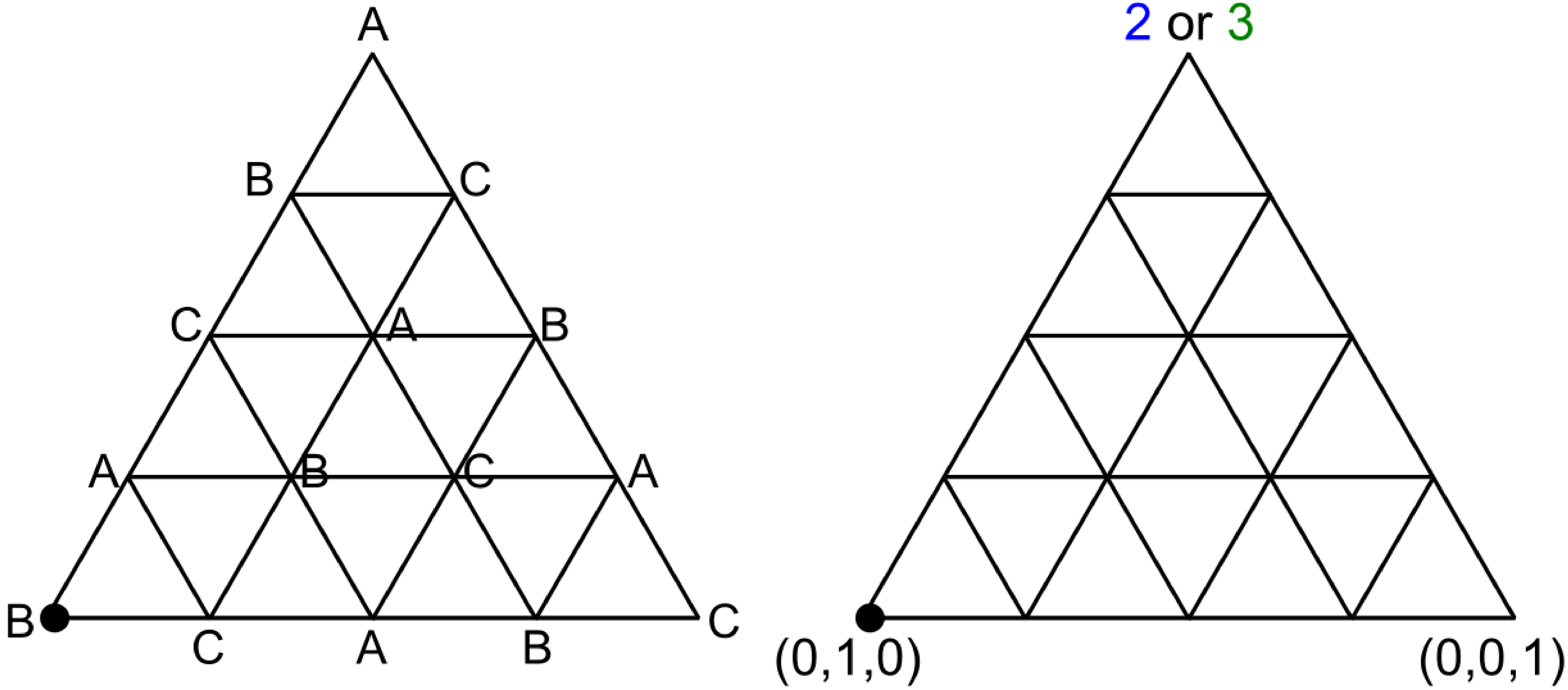
Agents always prefer a *free* room over a room with non-zero rent.

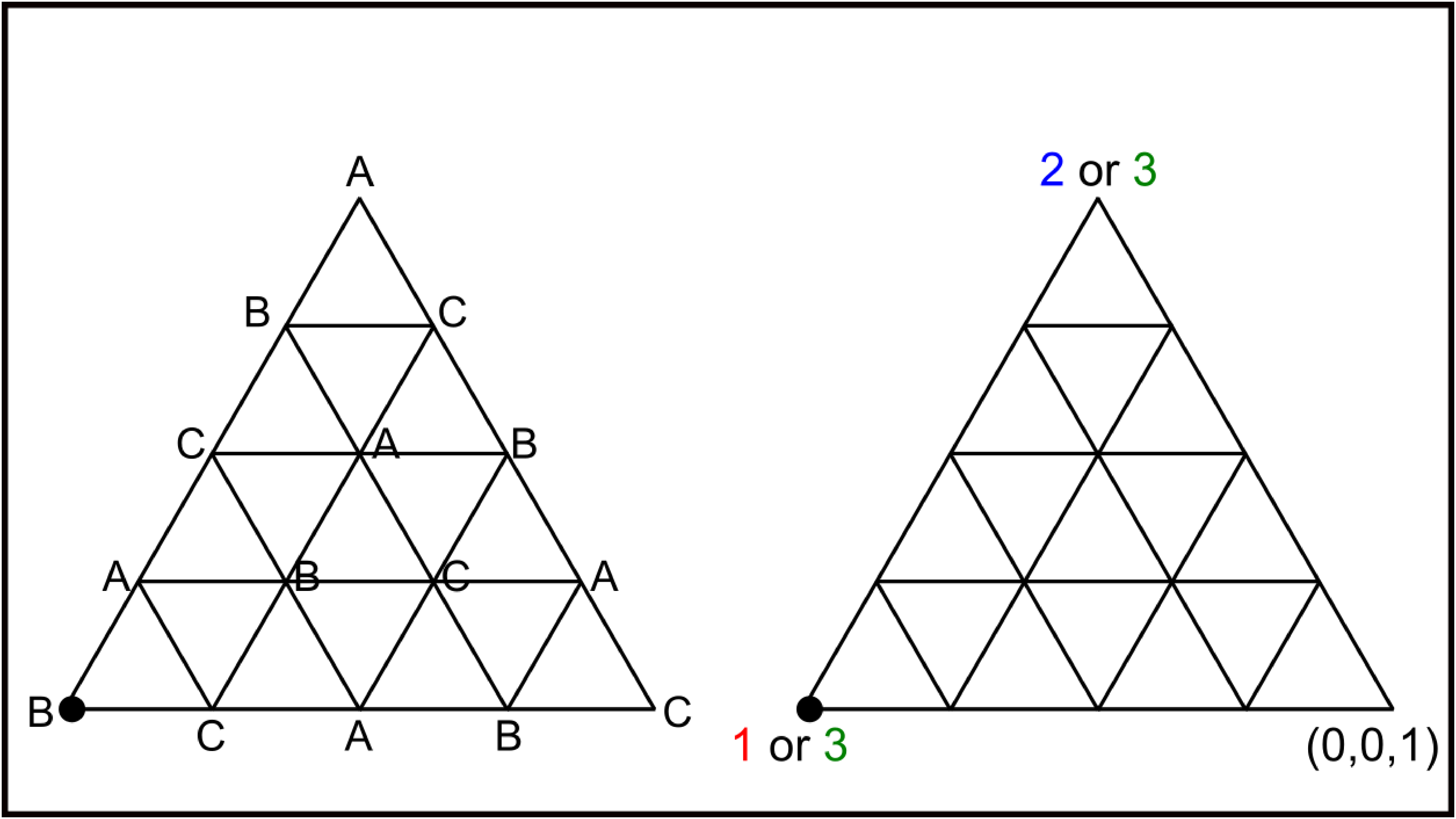




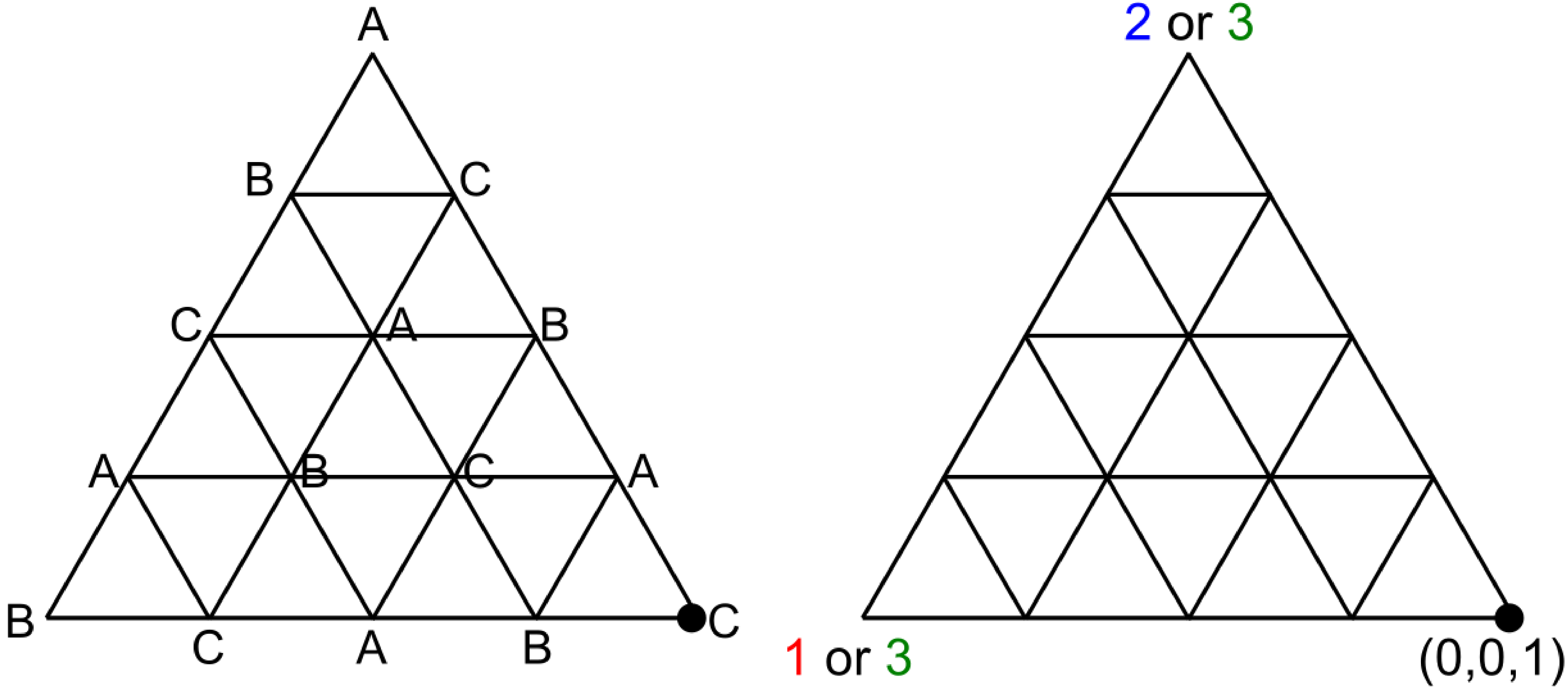

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
(1,0,0)
(0,1,0)
(0,0,1)

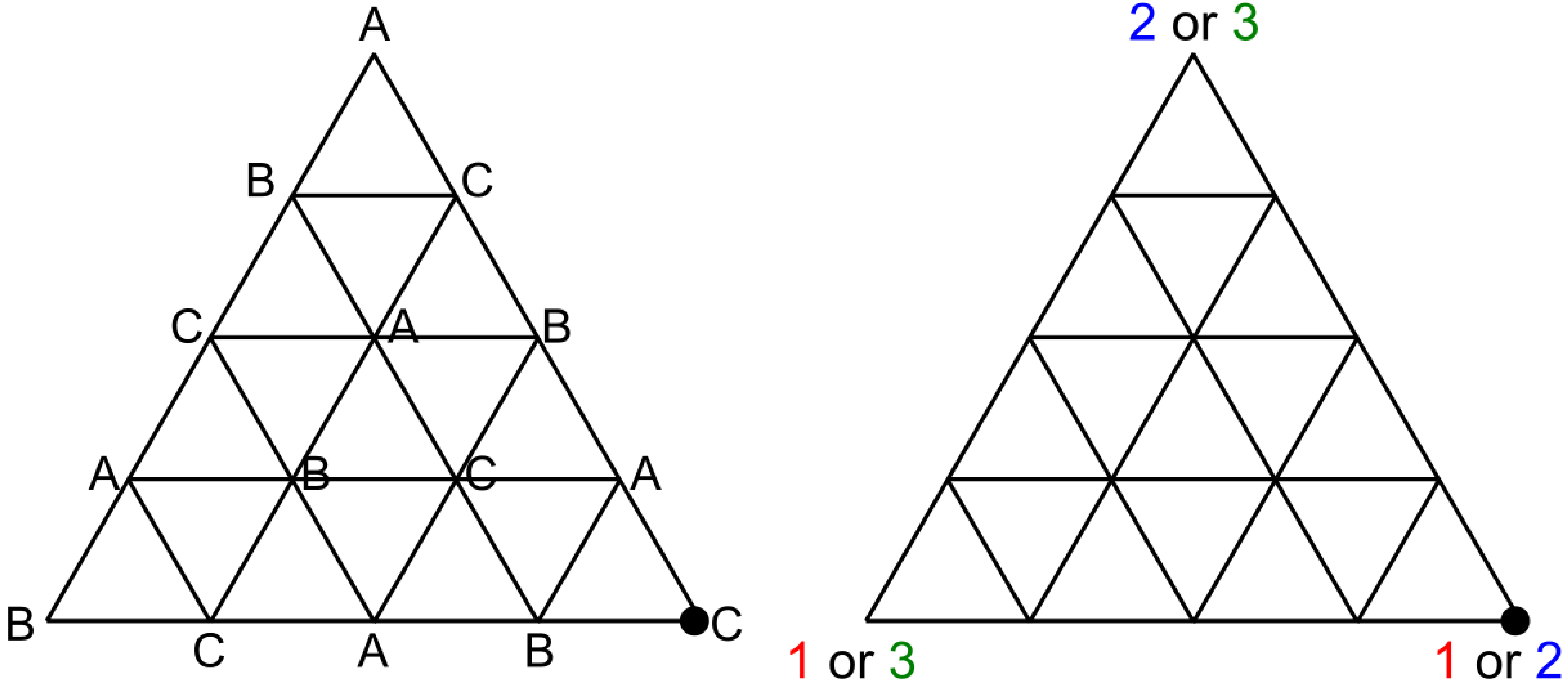

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
(0,1,0)
(0,0,1)

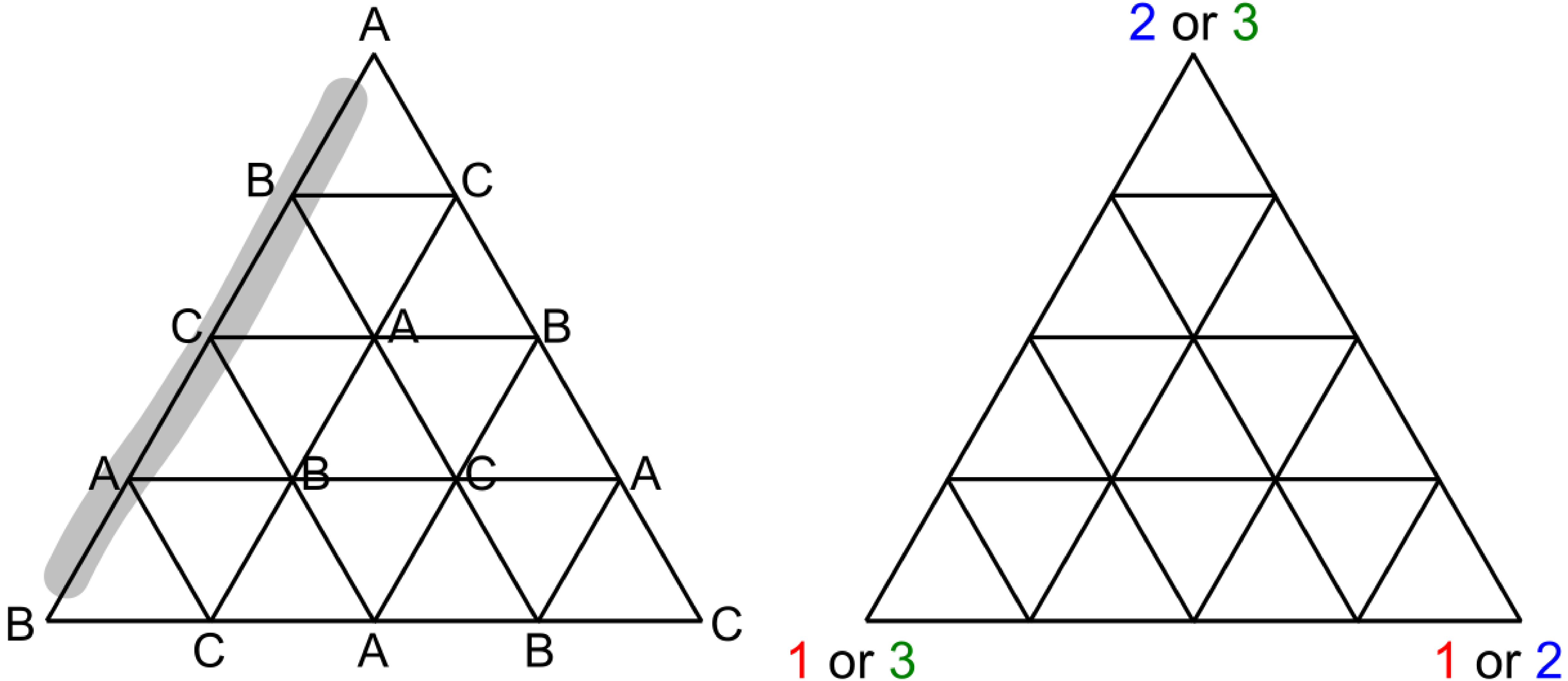

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
(0,1,0)
(0,0,1)

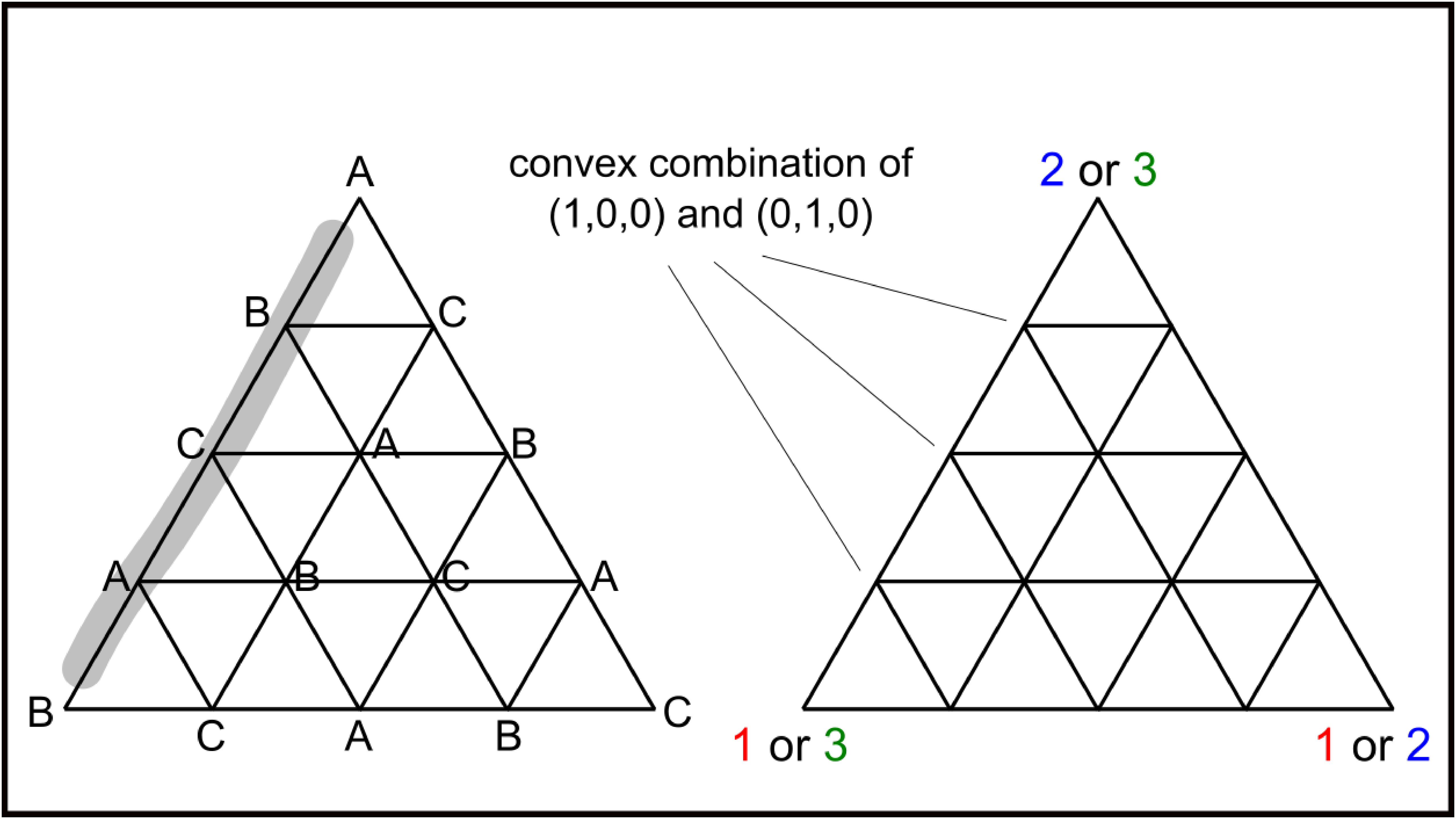


A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
1 or 3
(0,0,1)

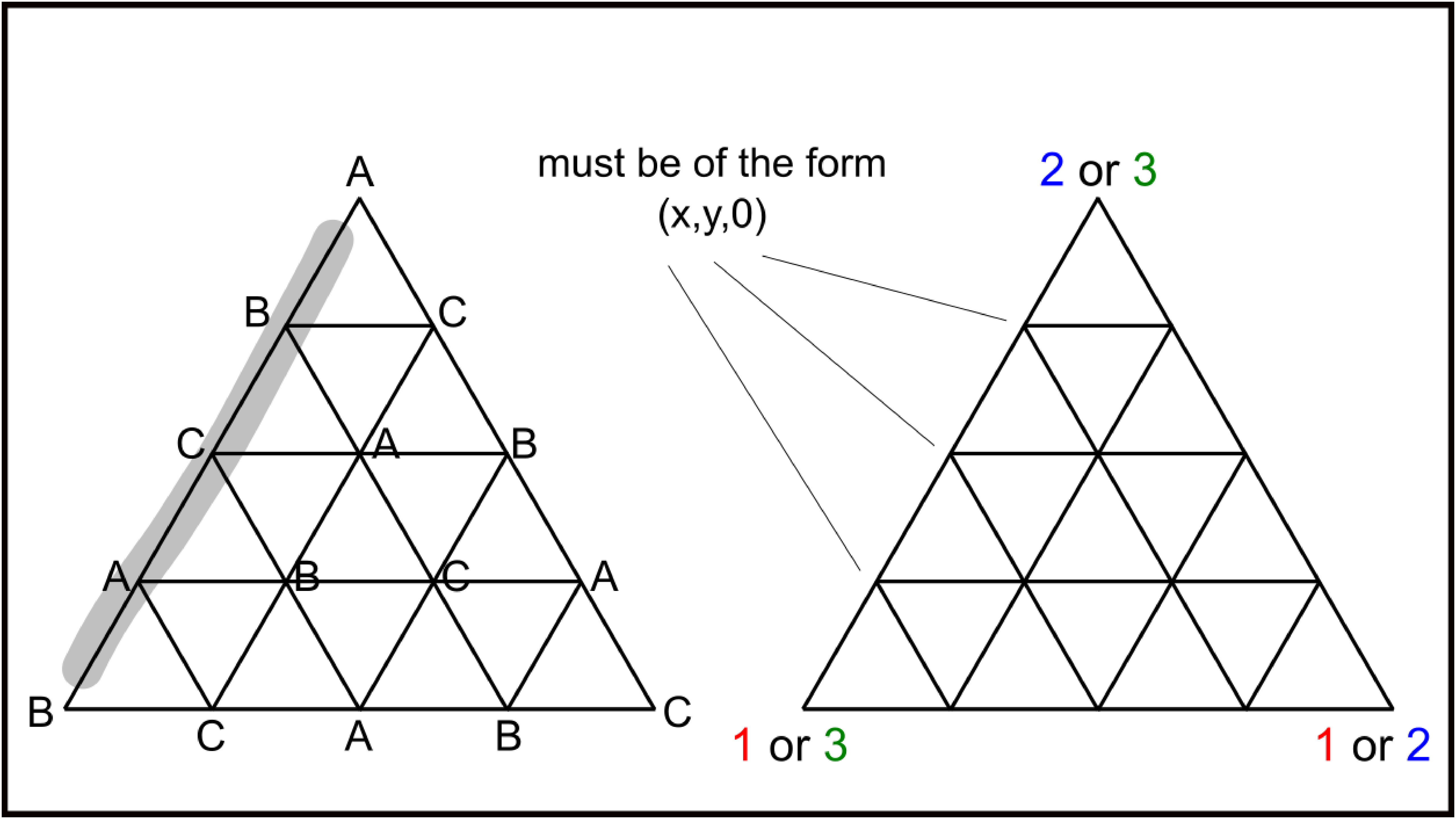

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
1 or 3
(0,0,1)

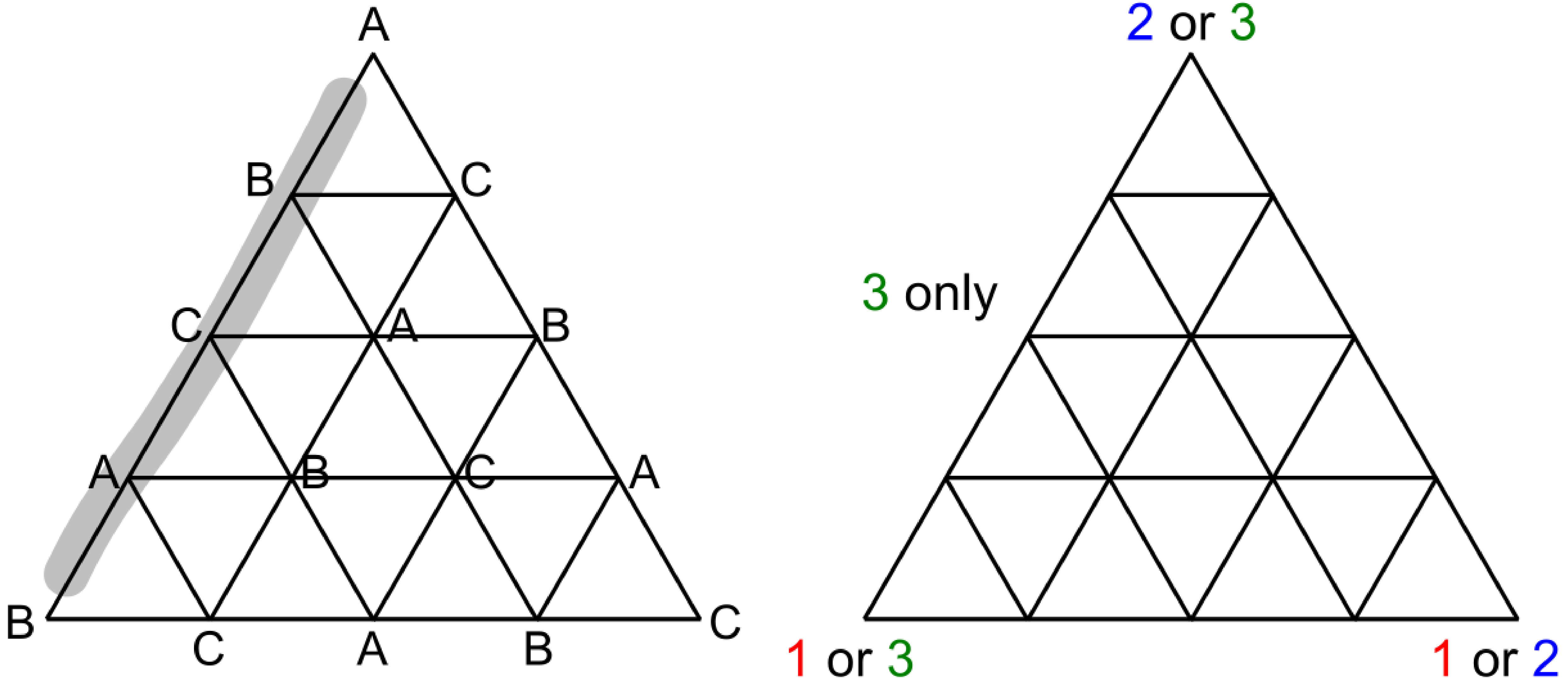

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
1 or 3
1 or 2

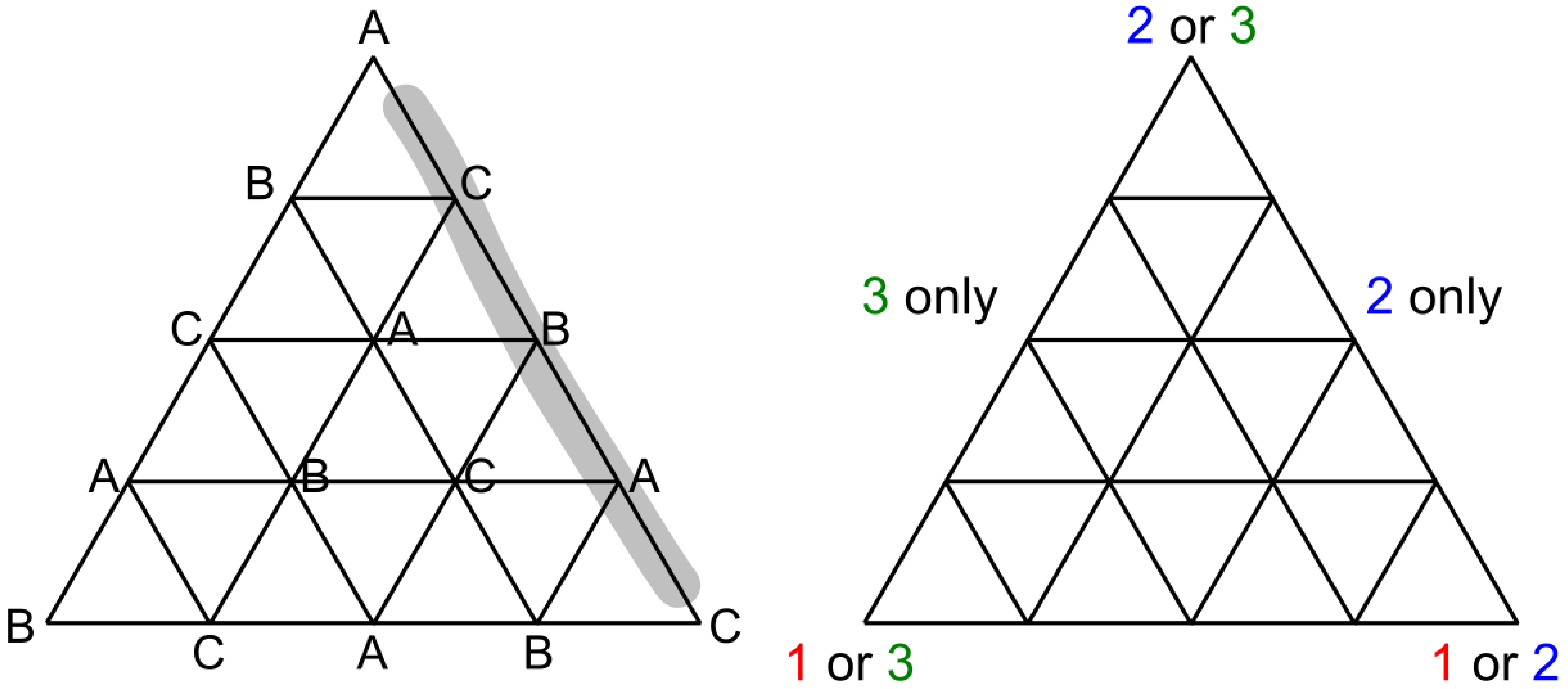

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
1 or 3
1 or 2

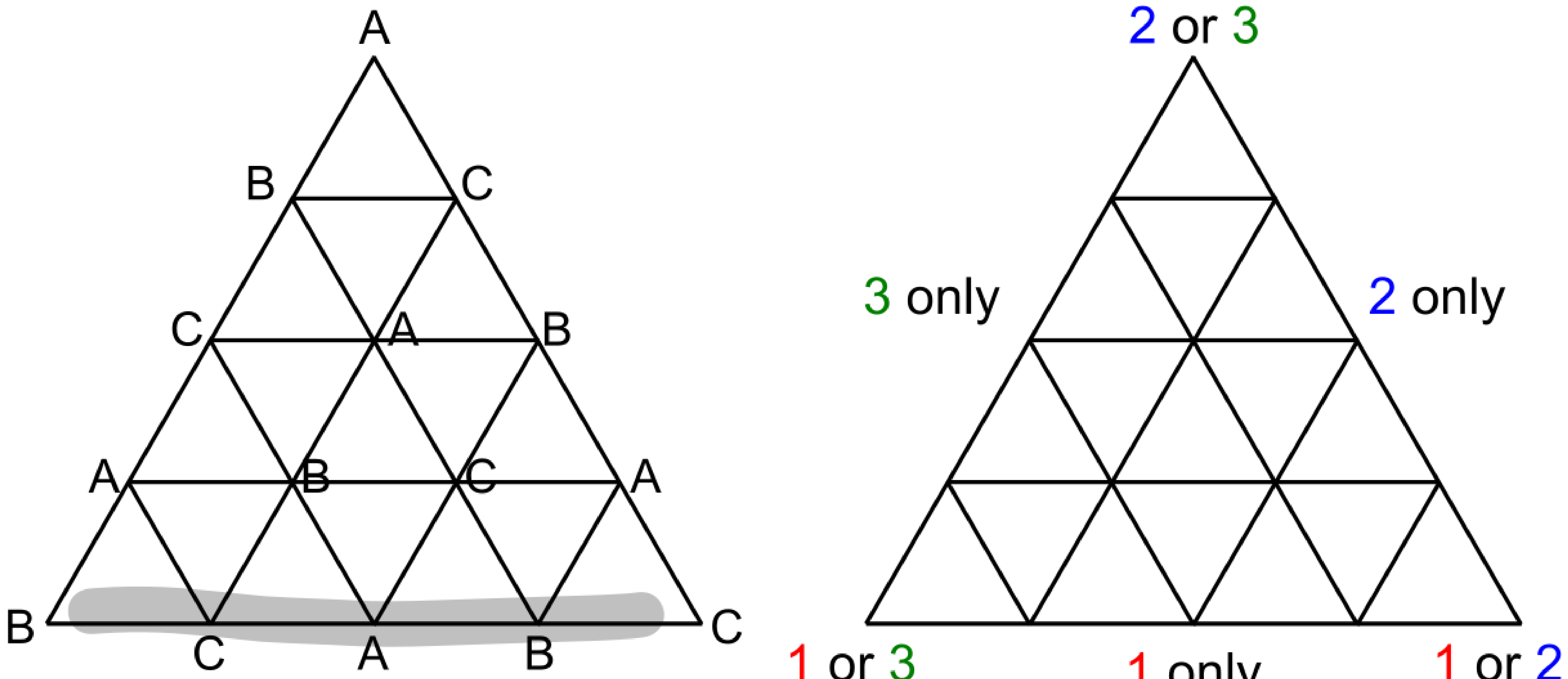


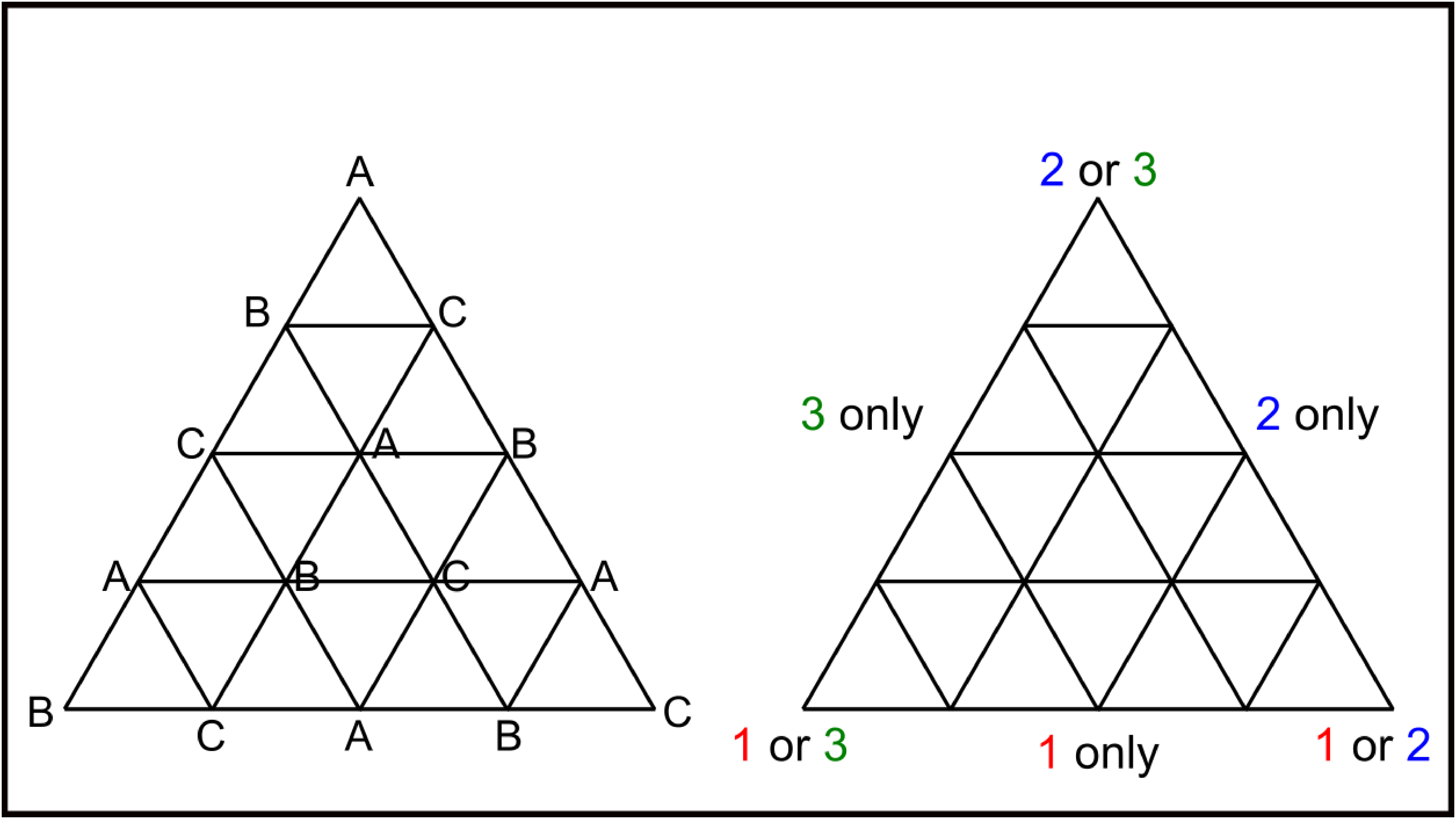
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
convex combination of
(1,0,0) and (0,1,0)
2 or 3
1 or 3
1 or 2

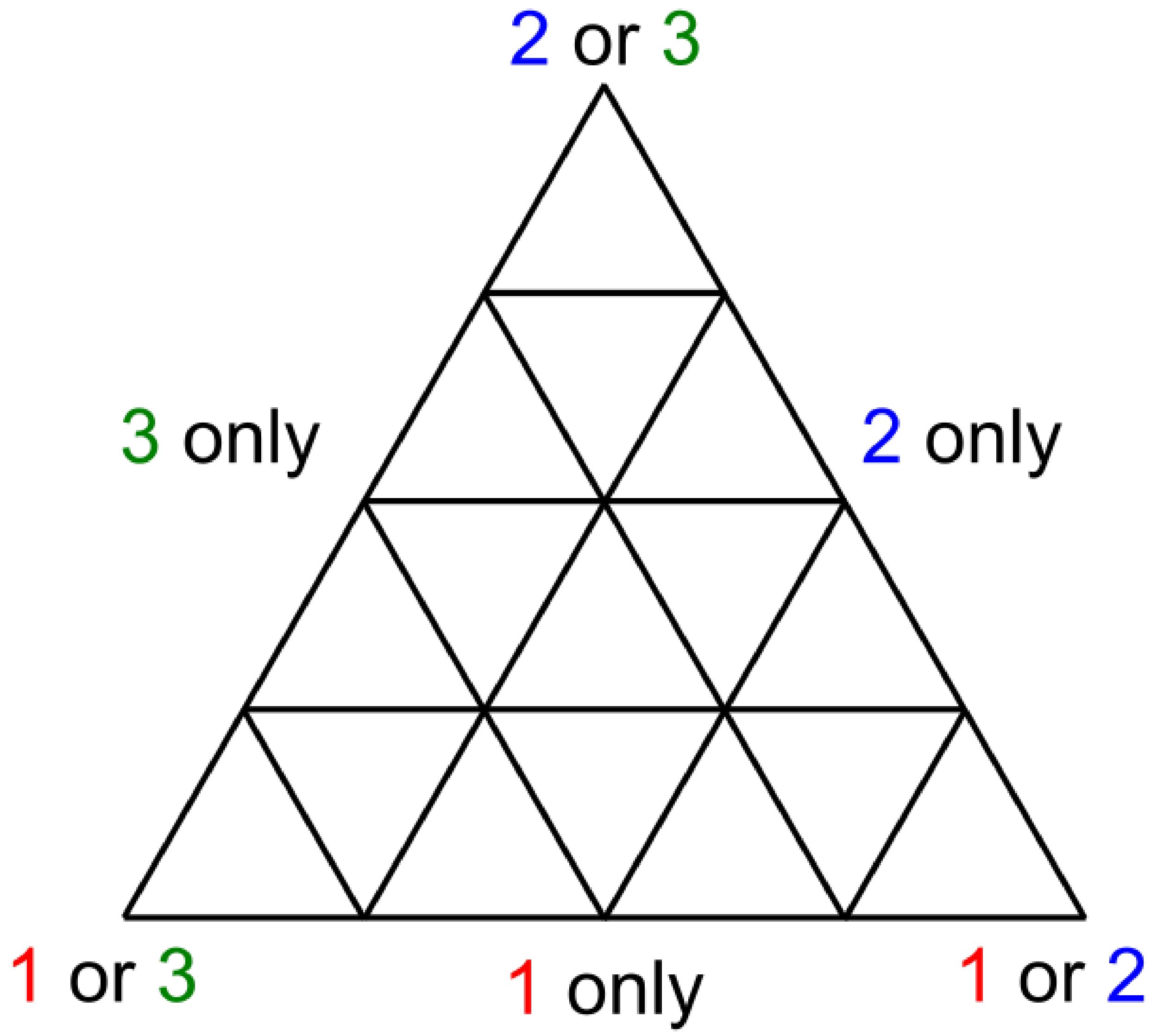

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
must be of the form
(x,y,0)
2 or 3
1 or 3
1 or 2


A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
3 only
1 or 3
1 or 2


A
B C
C A B
A B C A
B C A B C
2 or 3
3 only
2 only
1 or 3
1 or 2

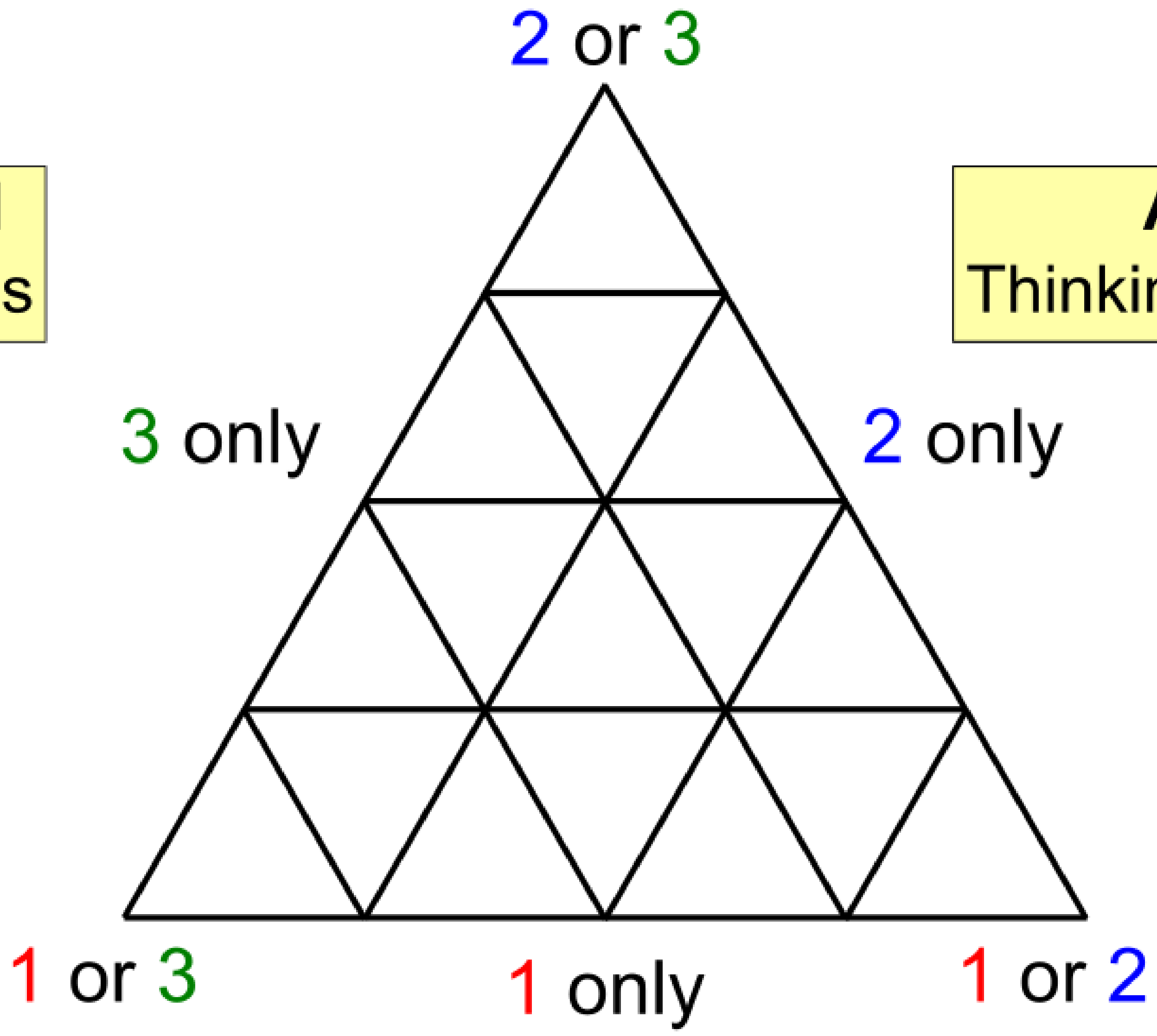

A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2


A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2

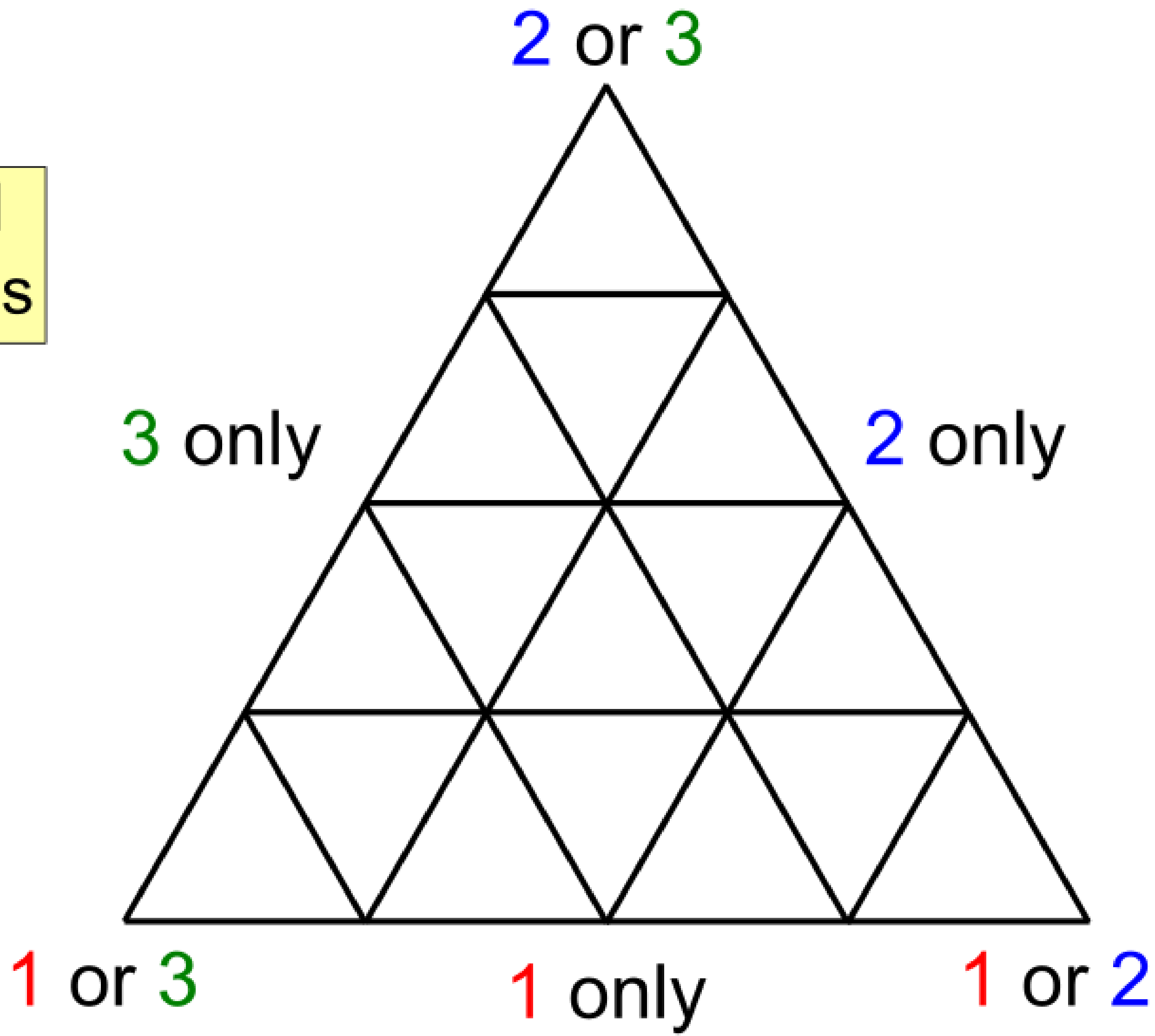

2 or 3
3 only
2 only
1 or 3
1 only
1 or 2

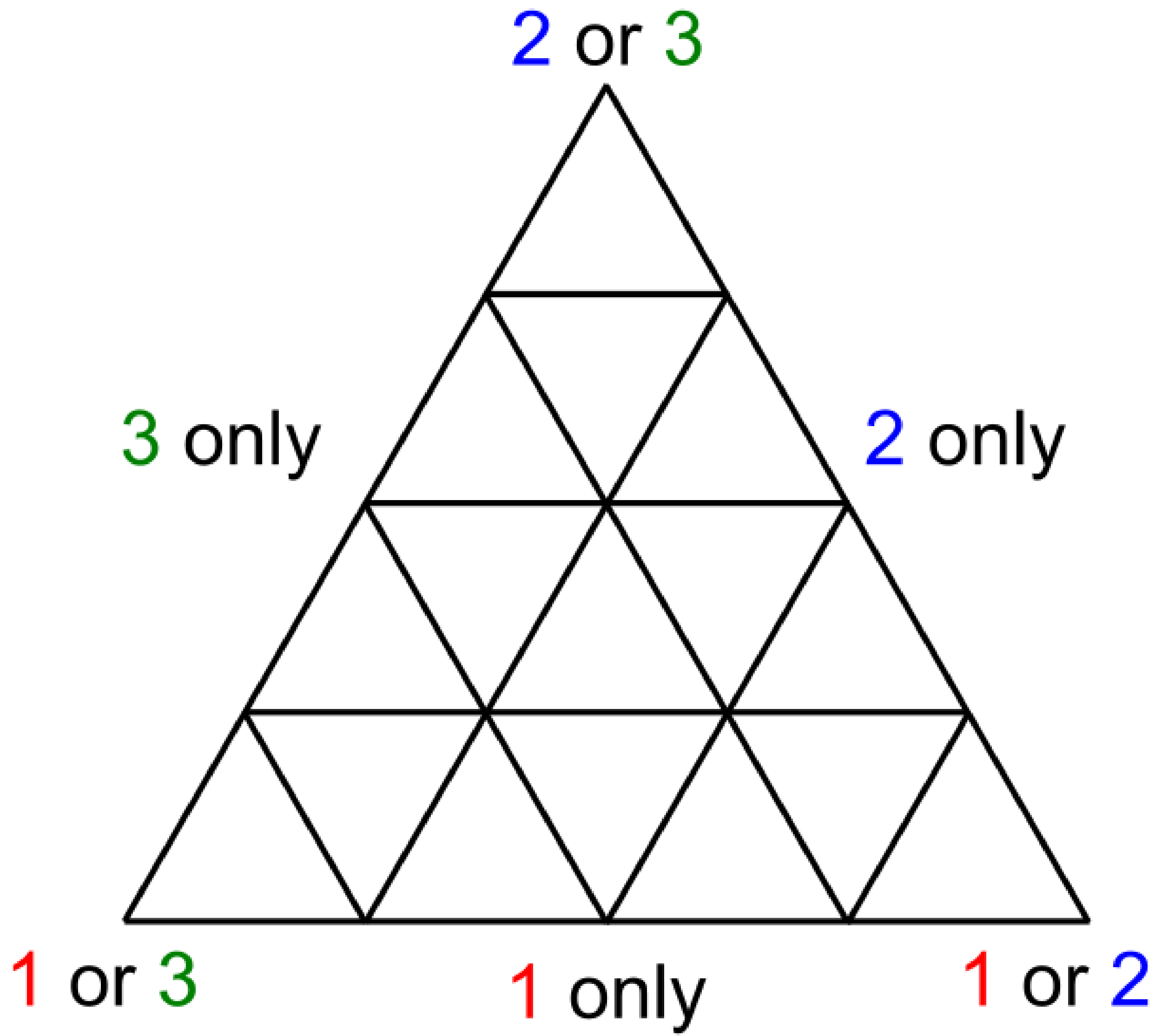
Approach 1
Case analysis

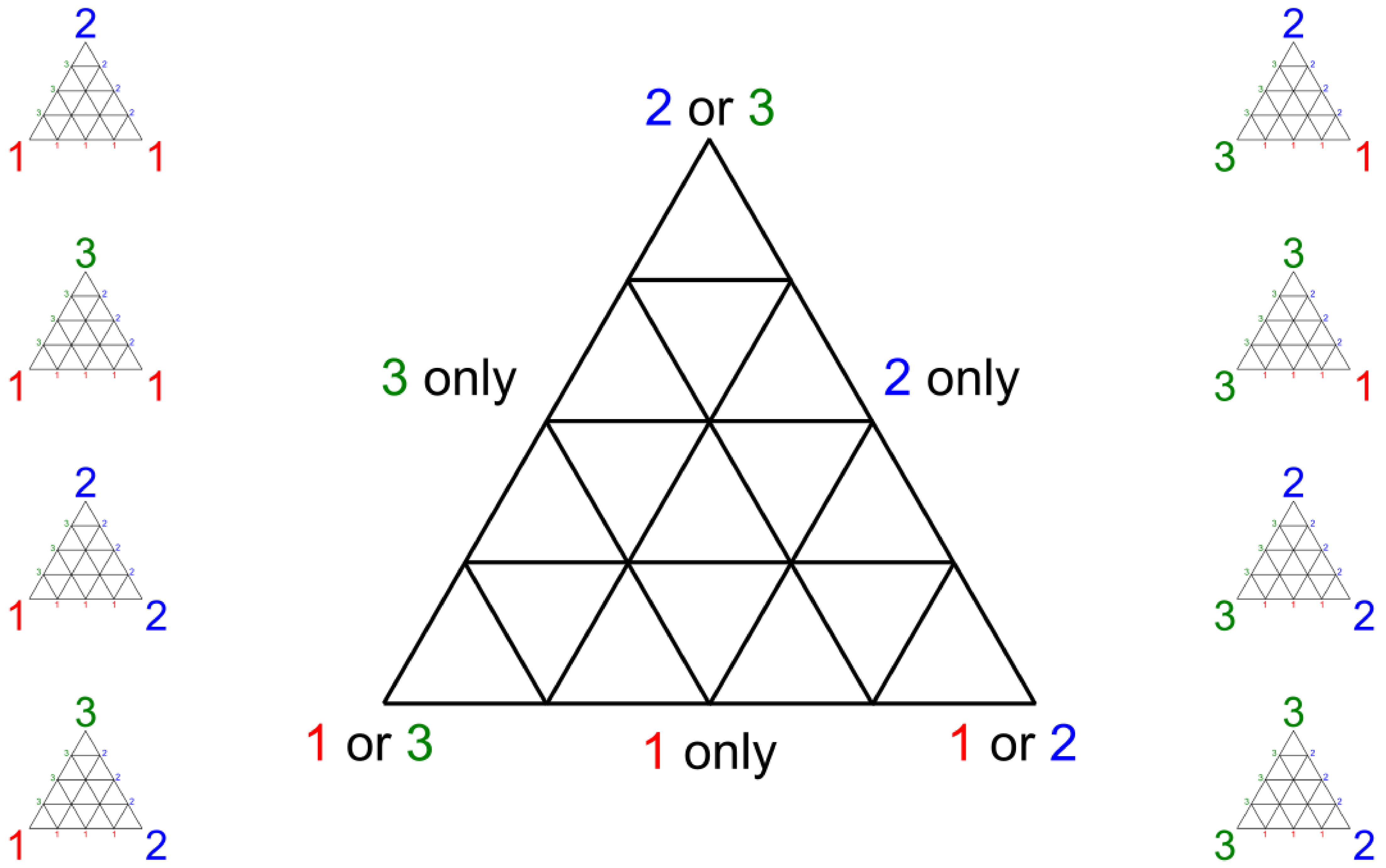
Approach 2
Thinking outside the box

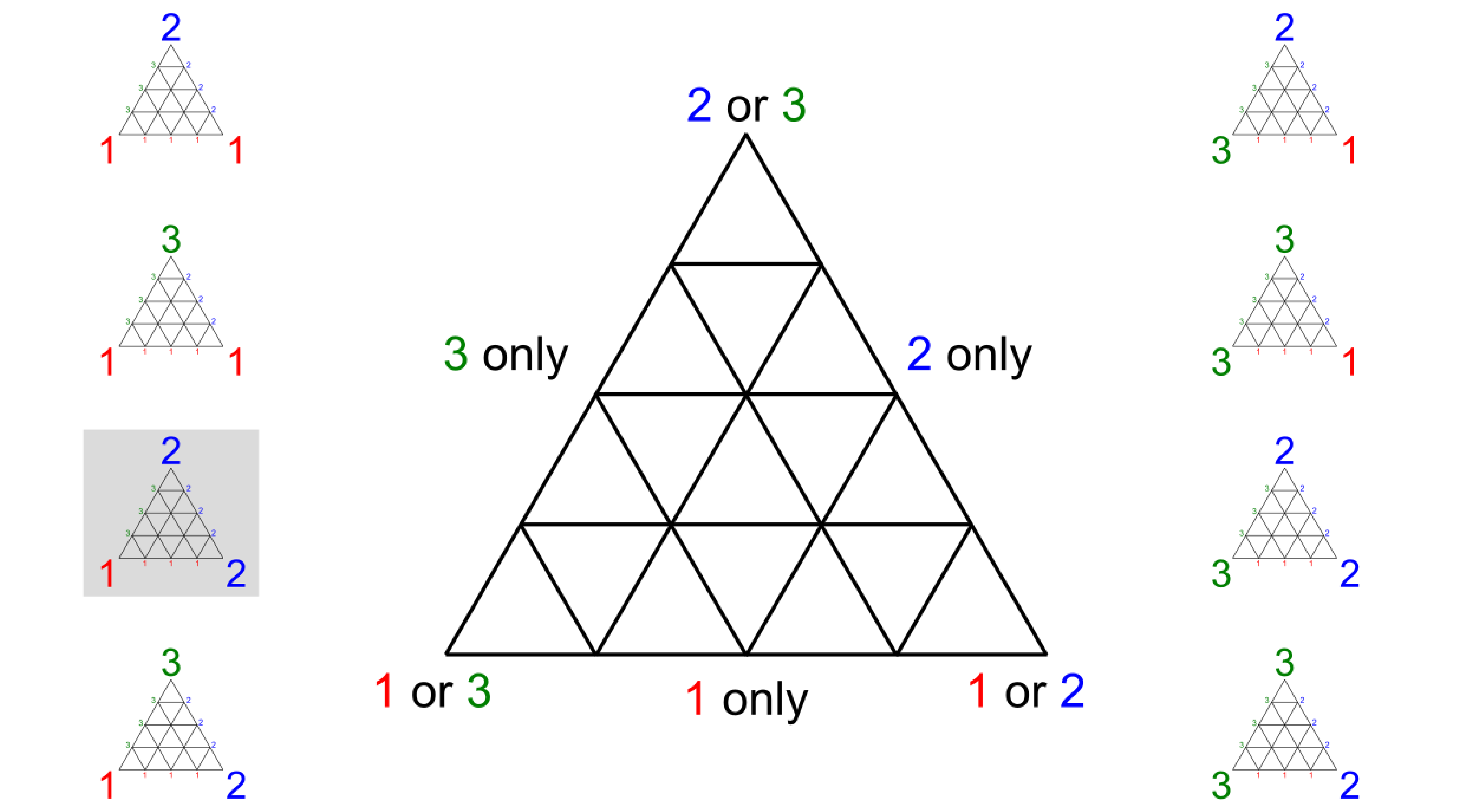


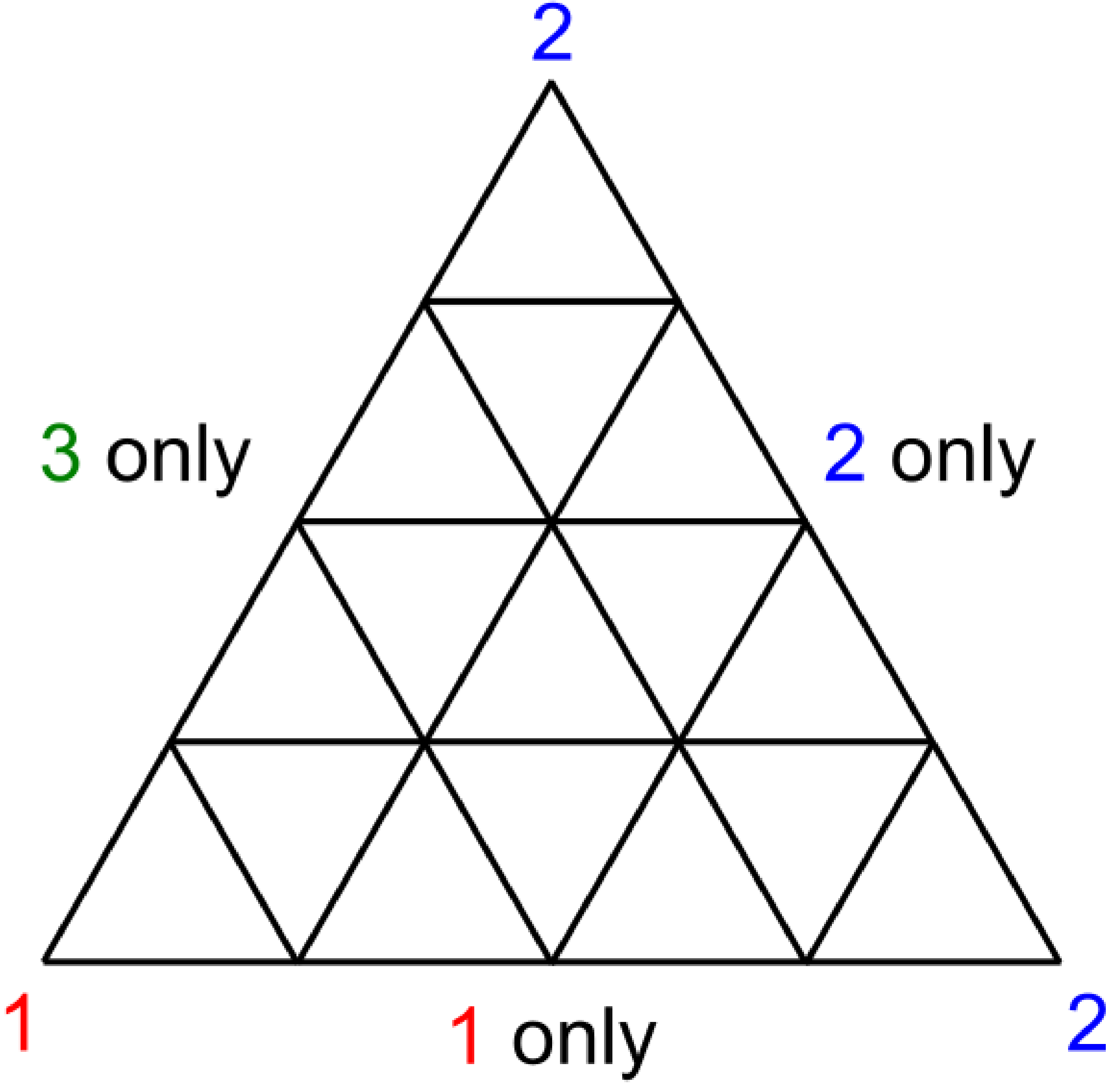
Approach 1
Case analysis

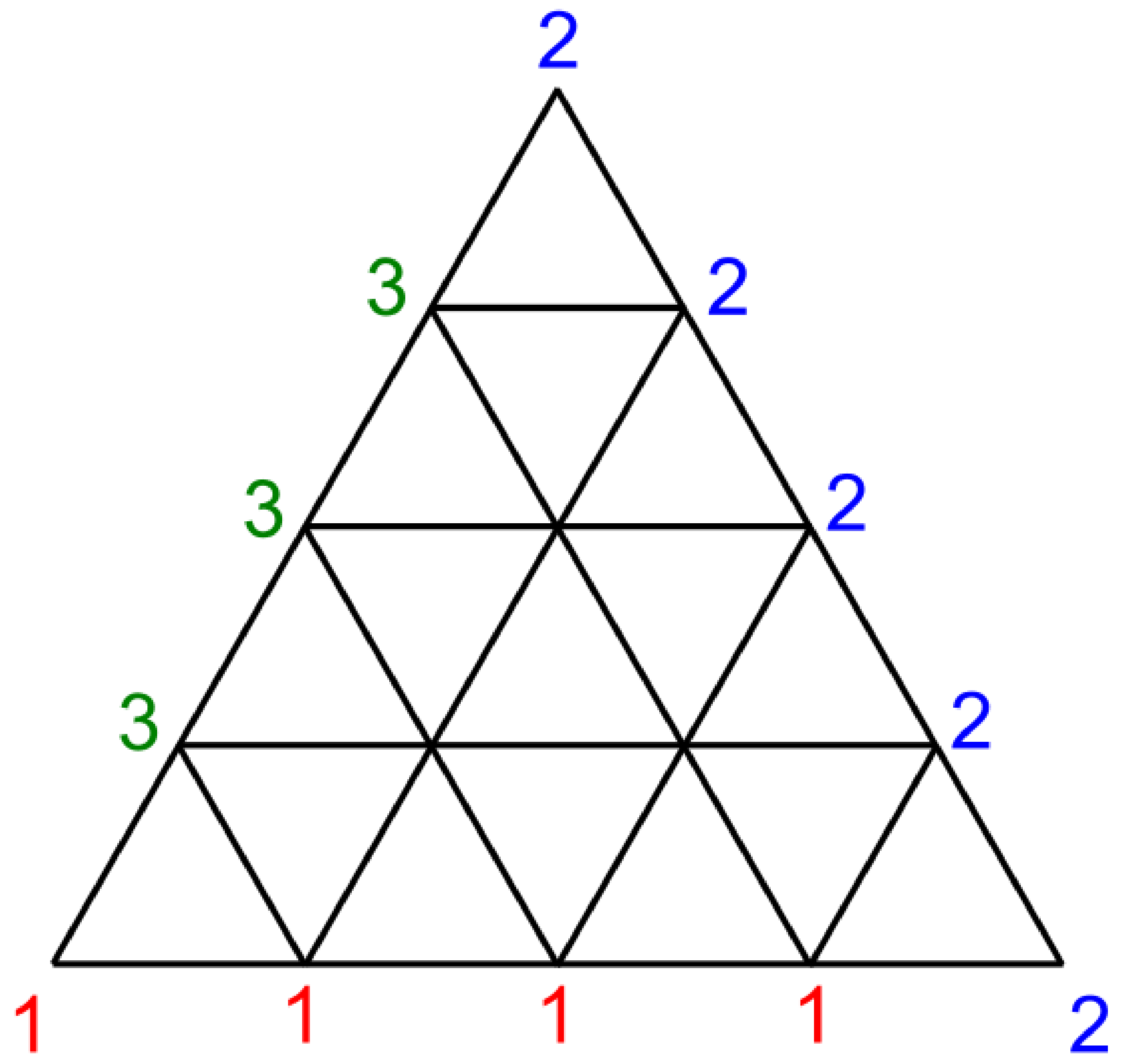



2 or 3
3 only
2 only
1 or 3
1 only
1 or 2

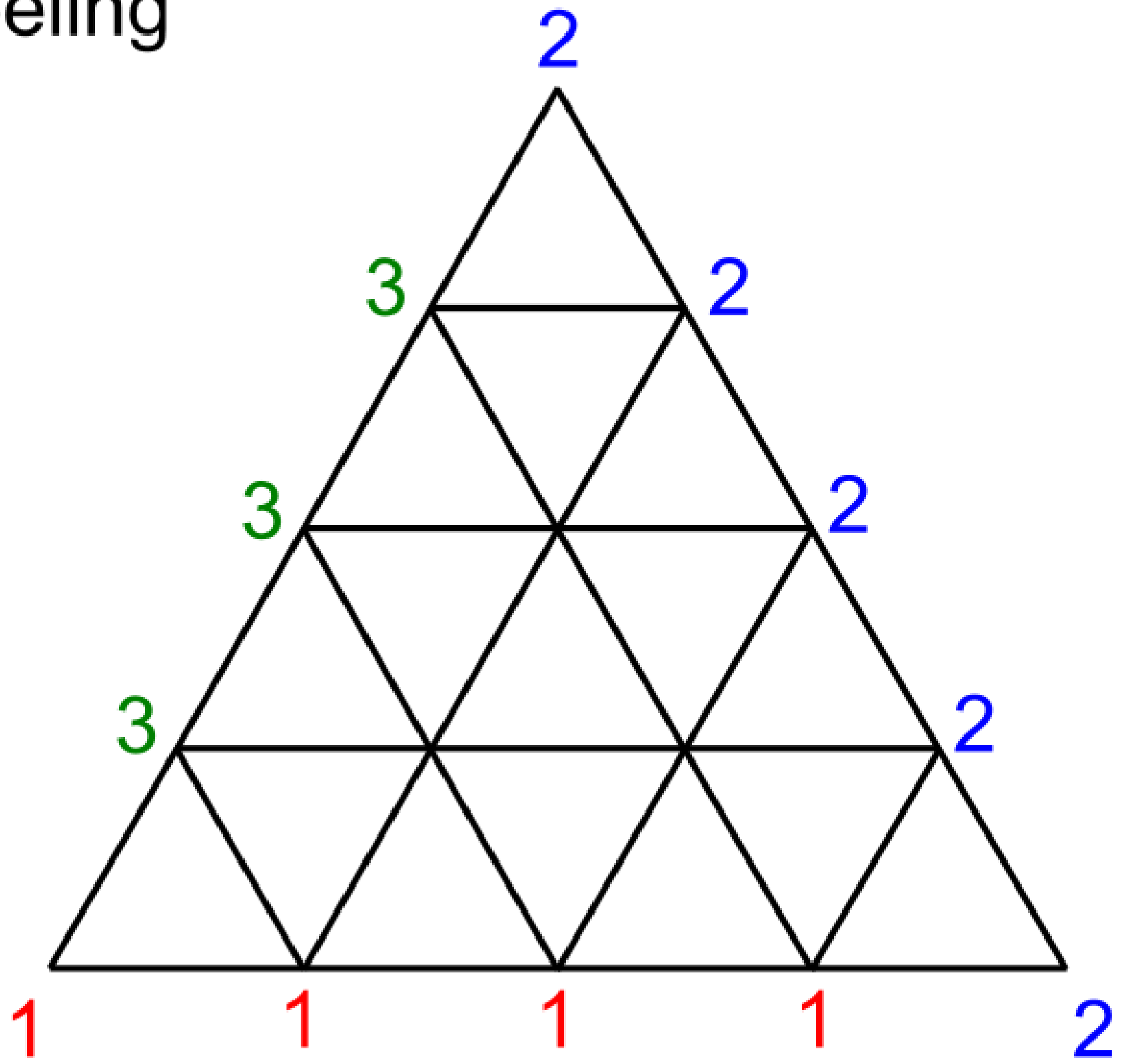

2 or 3
3 only
2 only
1 or 3
1 only
1 or 2


2 or 3
3 only
2 only
1 or 3
1 only
1 or 2


2
3 only
2 only
1
1 only
2

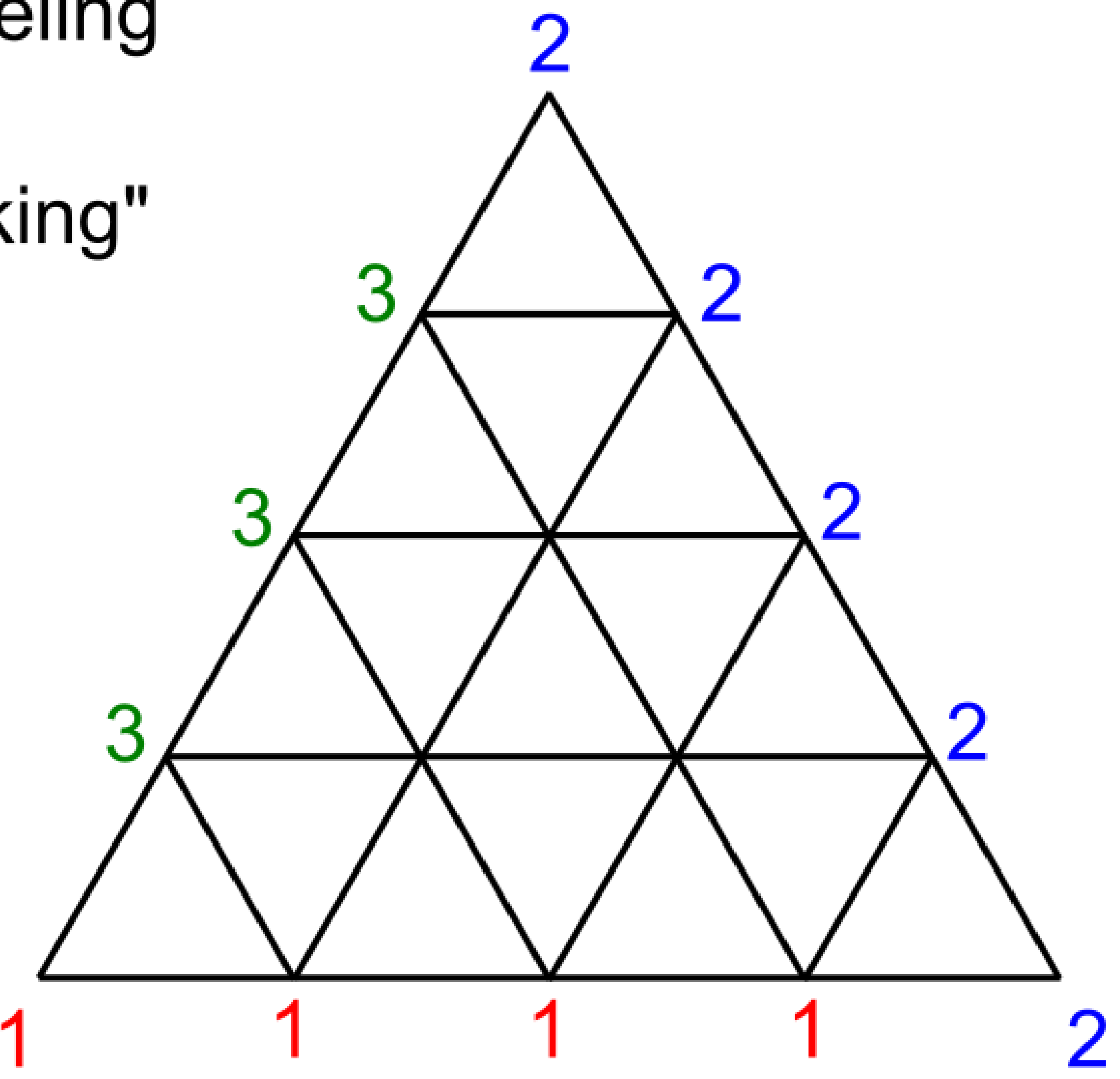

2
3
2
3
2
3
2
1
1
1
1
2

Not a Sperner labeling



Not a Sperner labeling

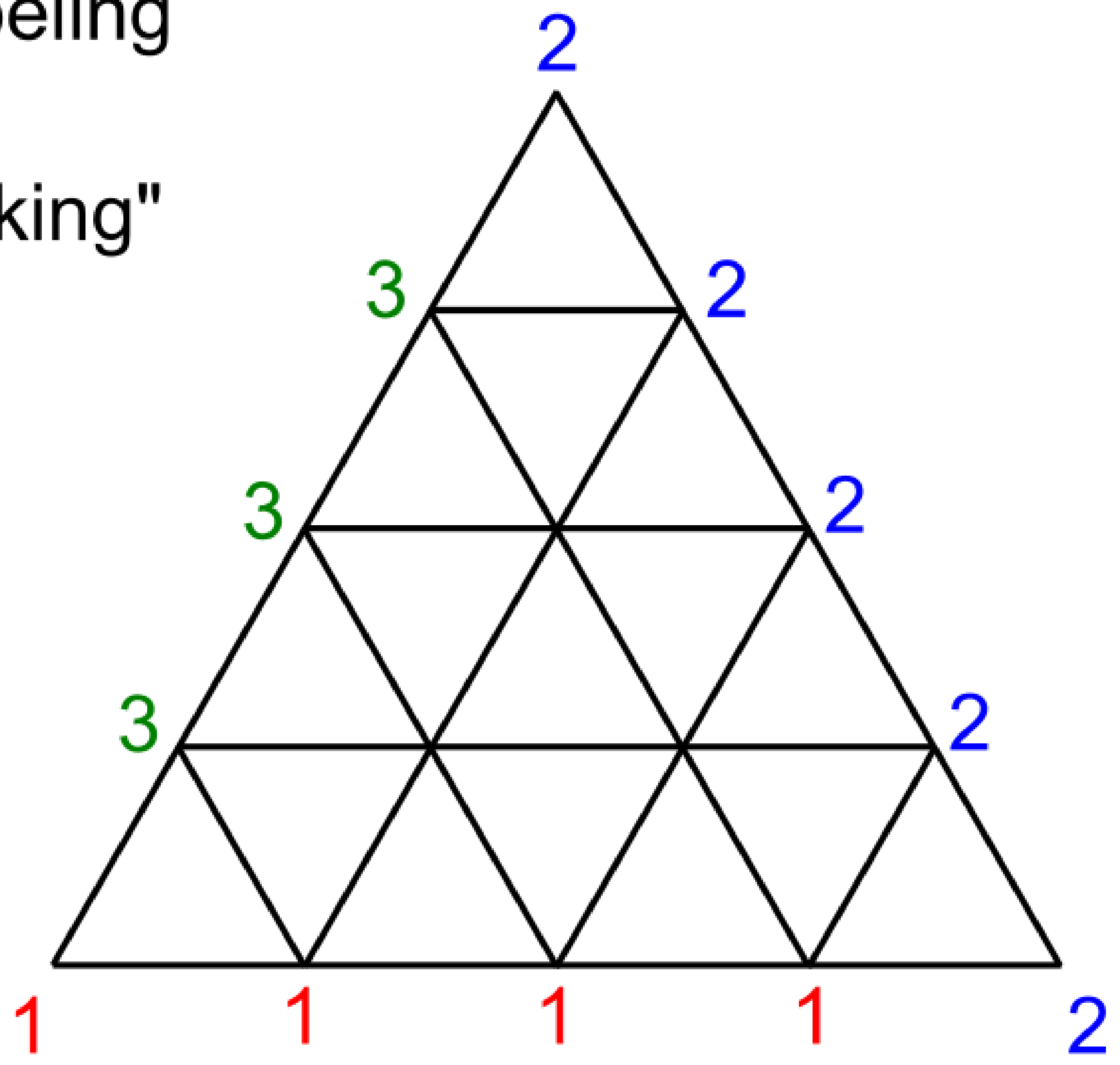
But "proof by walking" still applies!



Not a Sperner labeling

But "proof by walking" still applies!

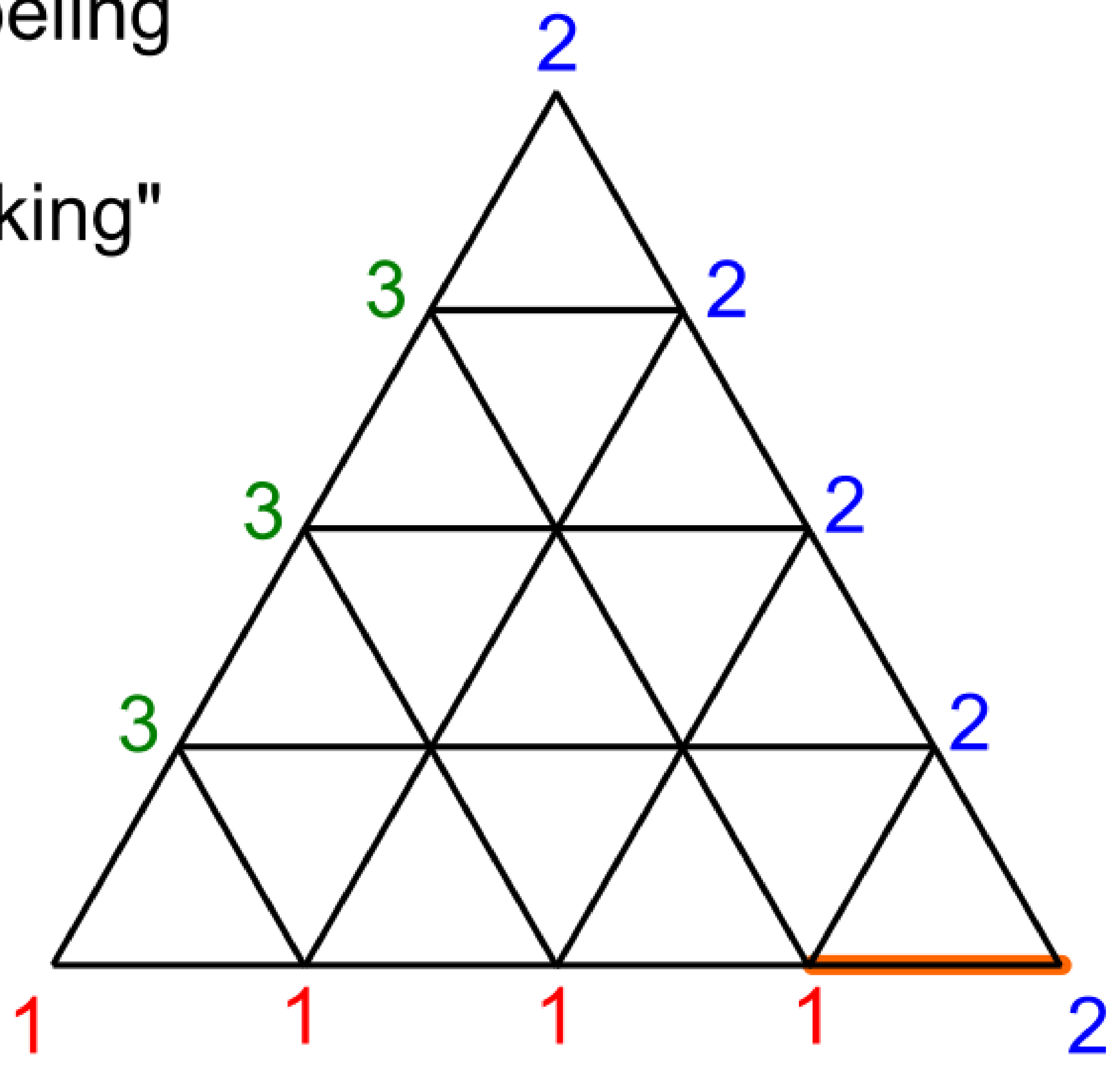
Why?



Not a Sperner labeling

But "proof by walking" still applies!

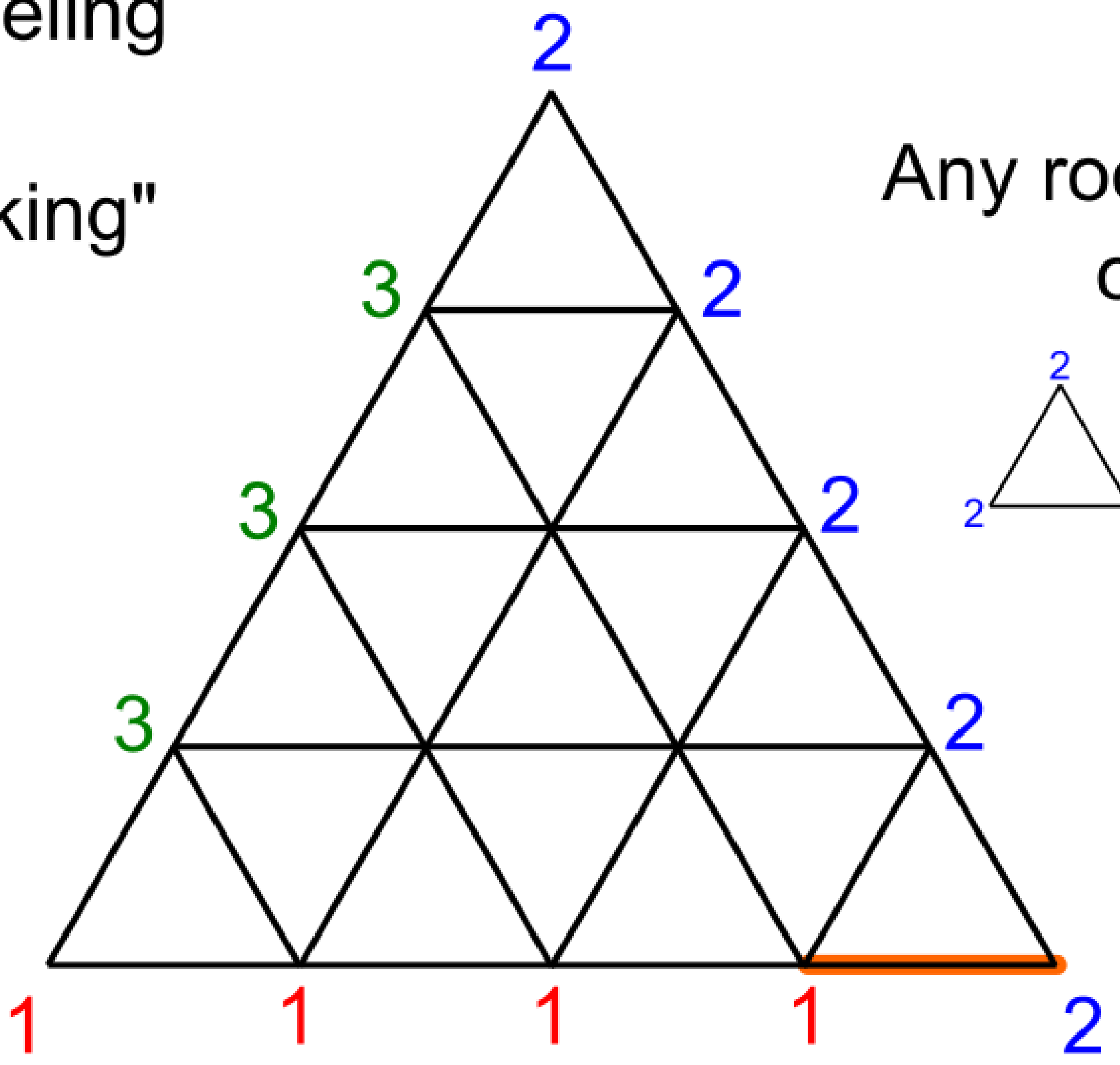
Why?



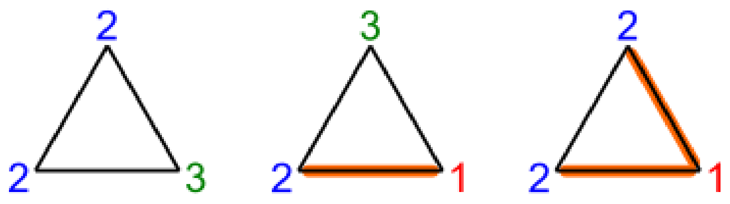
Not a Sperner labeling

But "proof by walking" still applies!

Why?



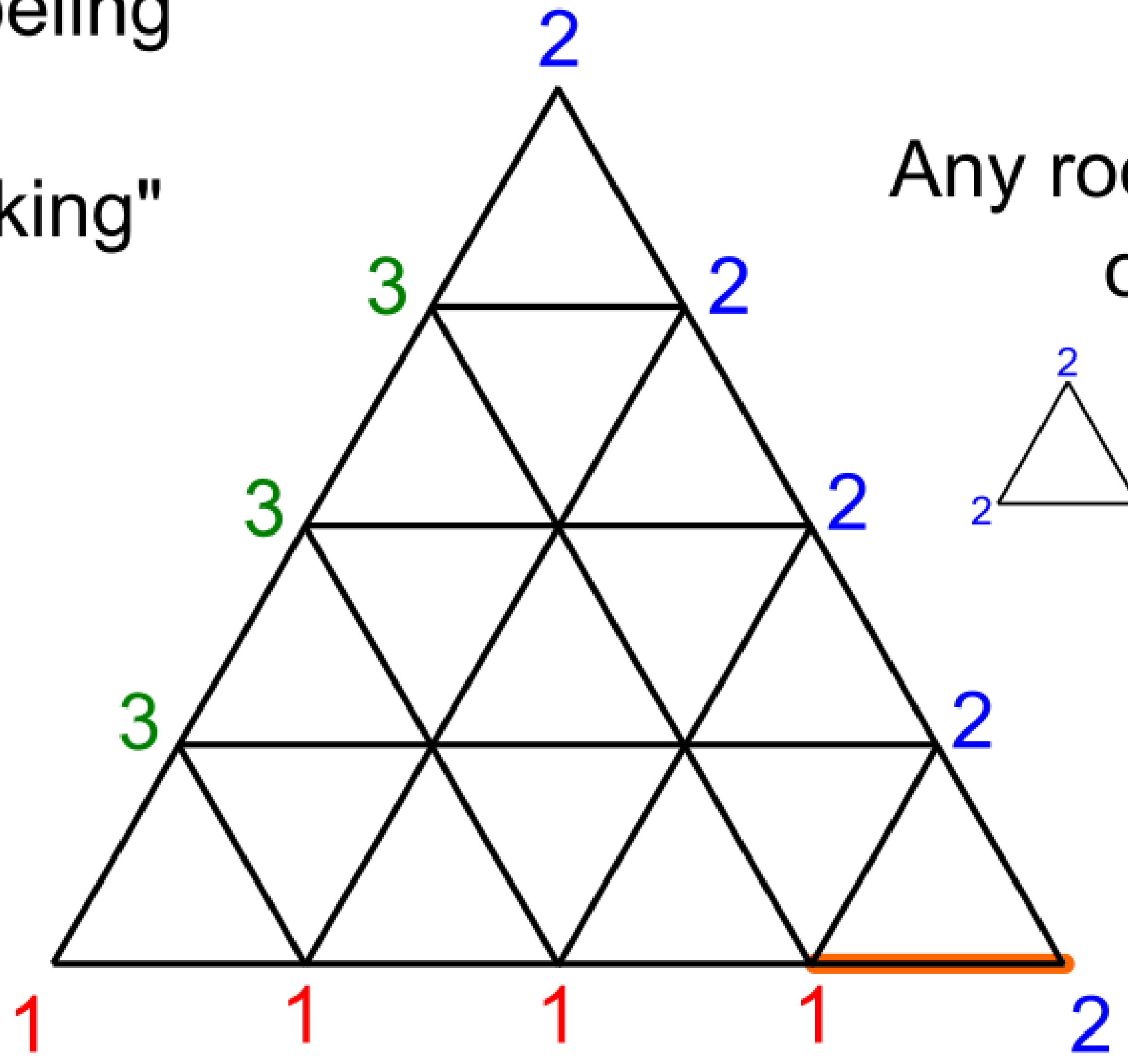
Any room has zero, one, or two doors



Not a Sperner labeling

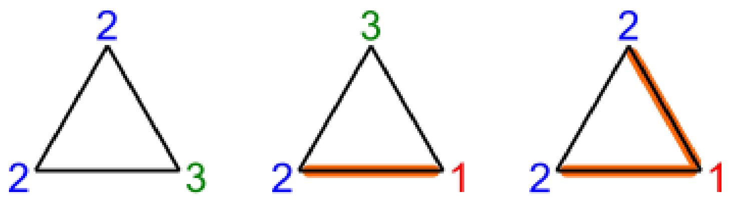
But "proof by walking" still applies!

Why?



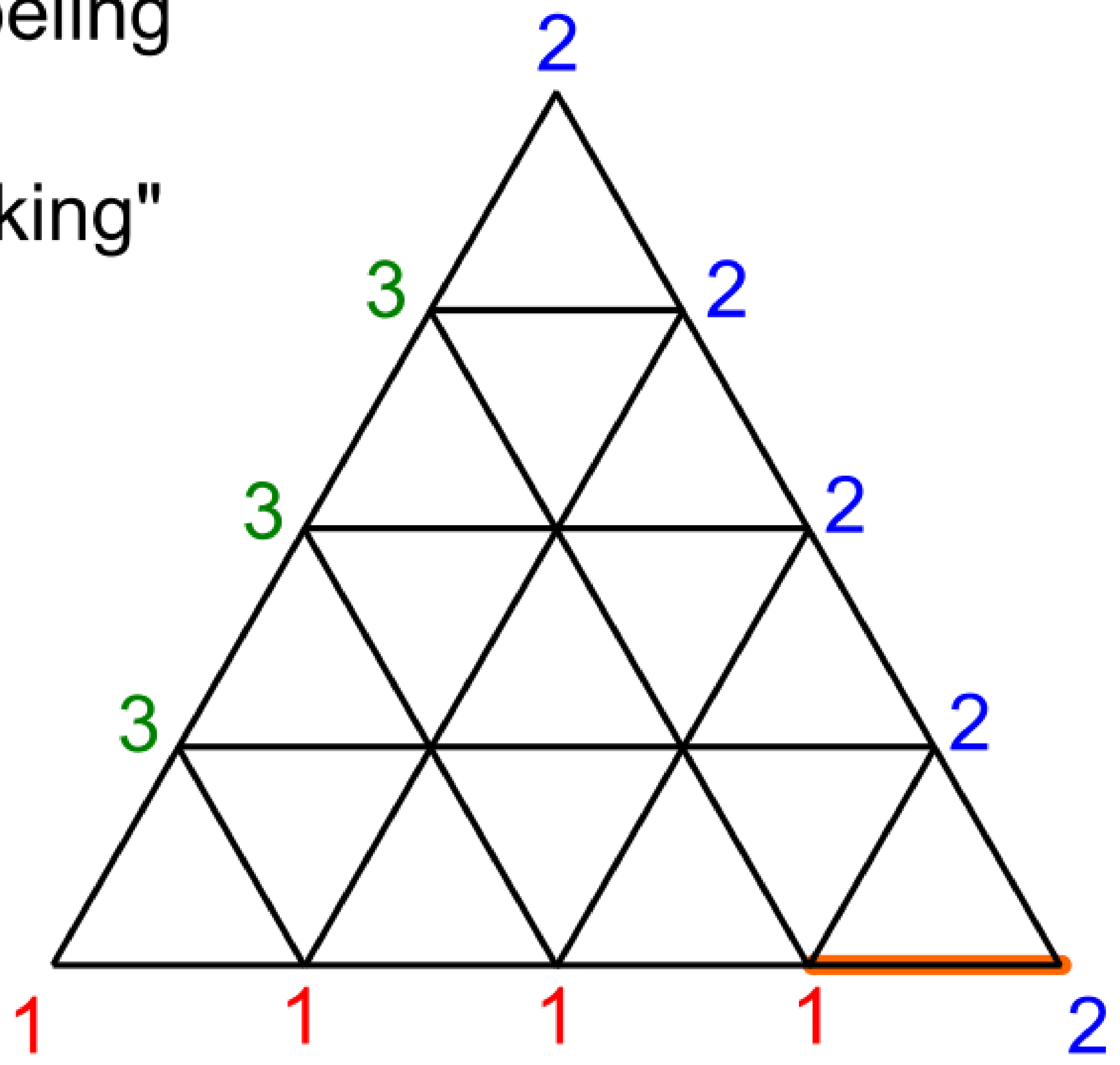
Property of triangles!

Any room has zero, one, or two doors



Not a Sperner labeling

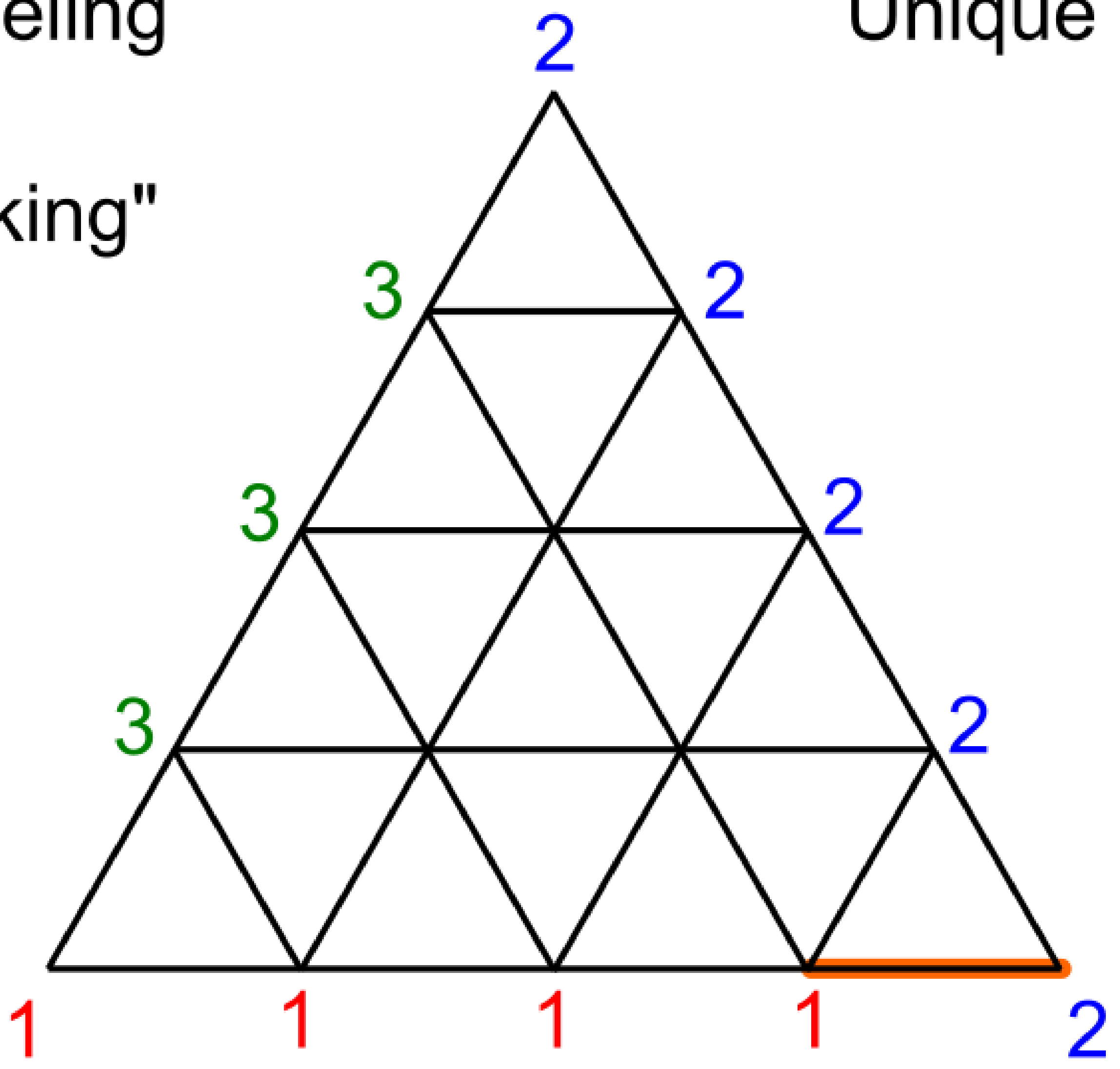
But "proof by walking"
still applies!



Not a Sperner labeling

But "proof by walking" still applies!

Unique door on boundary

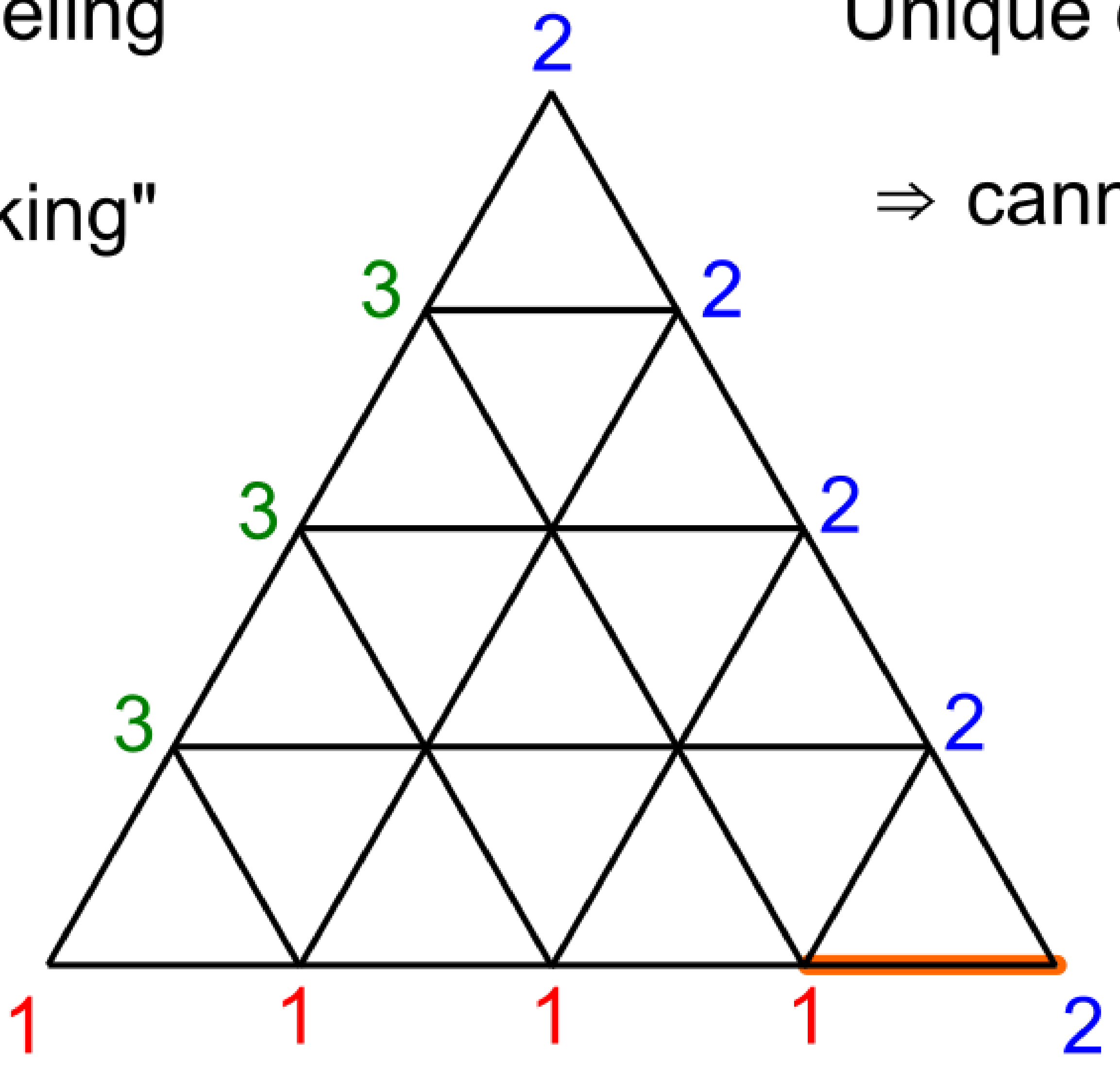


Not a Sperner labeling

But "proof by walking" still applies!

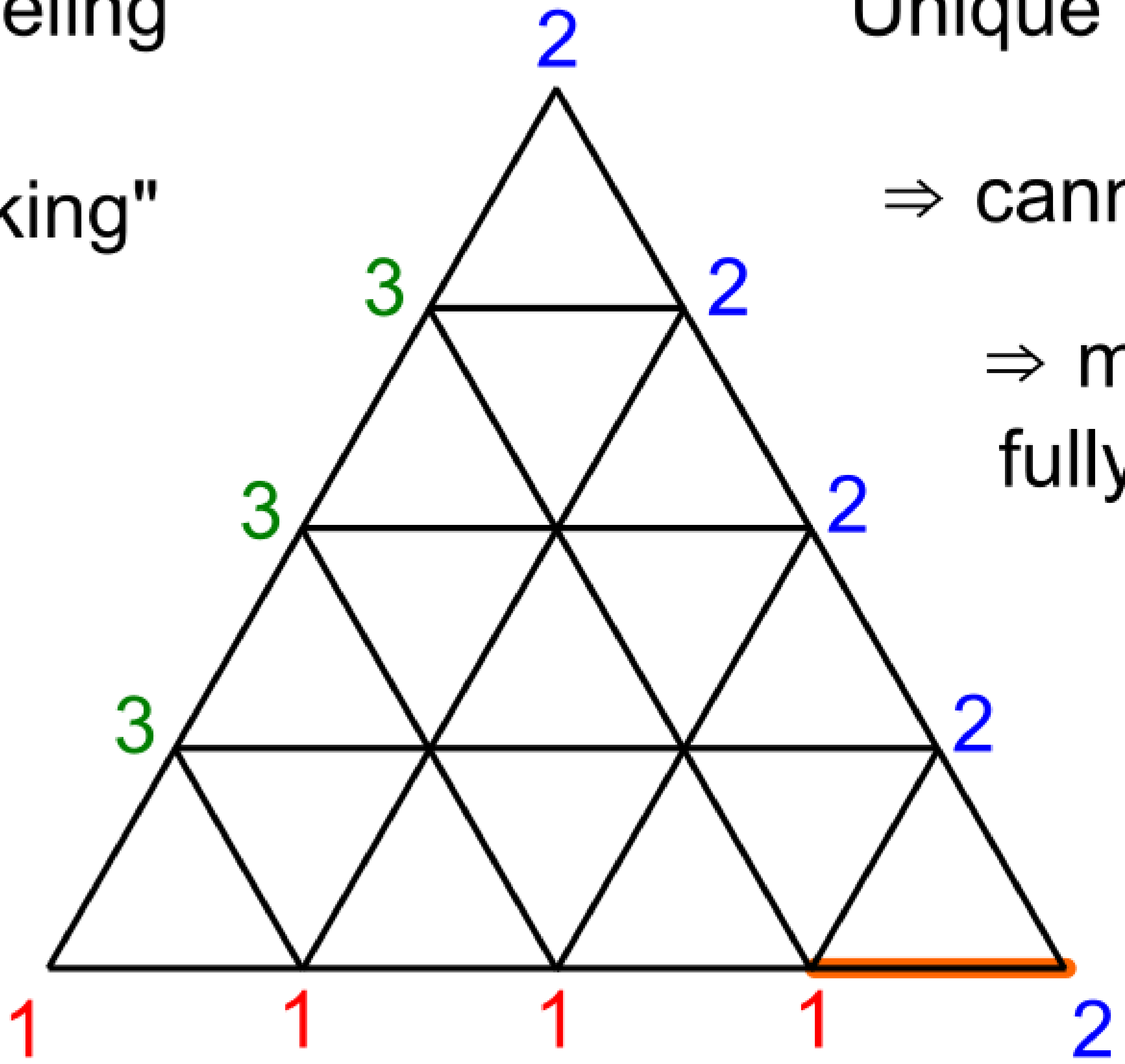
Unique door on boundary

⇒ cannot be thrown out



Not a Sperner labeling

But "proof by walking" still applies!



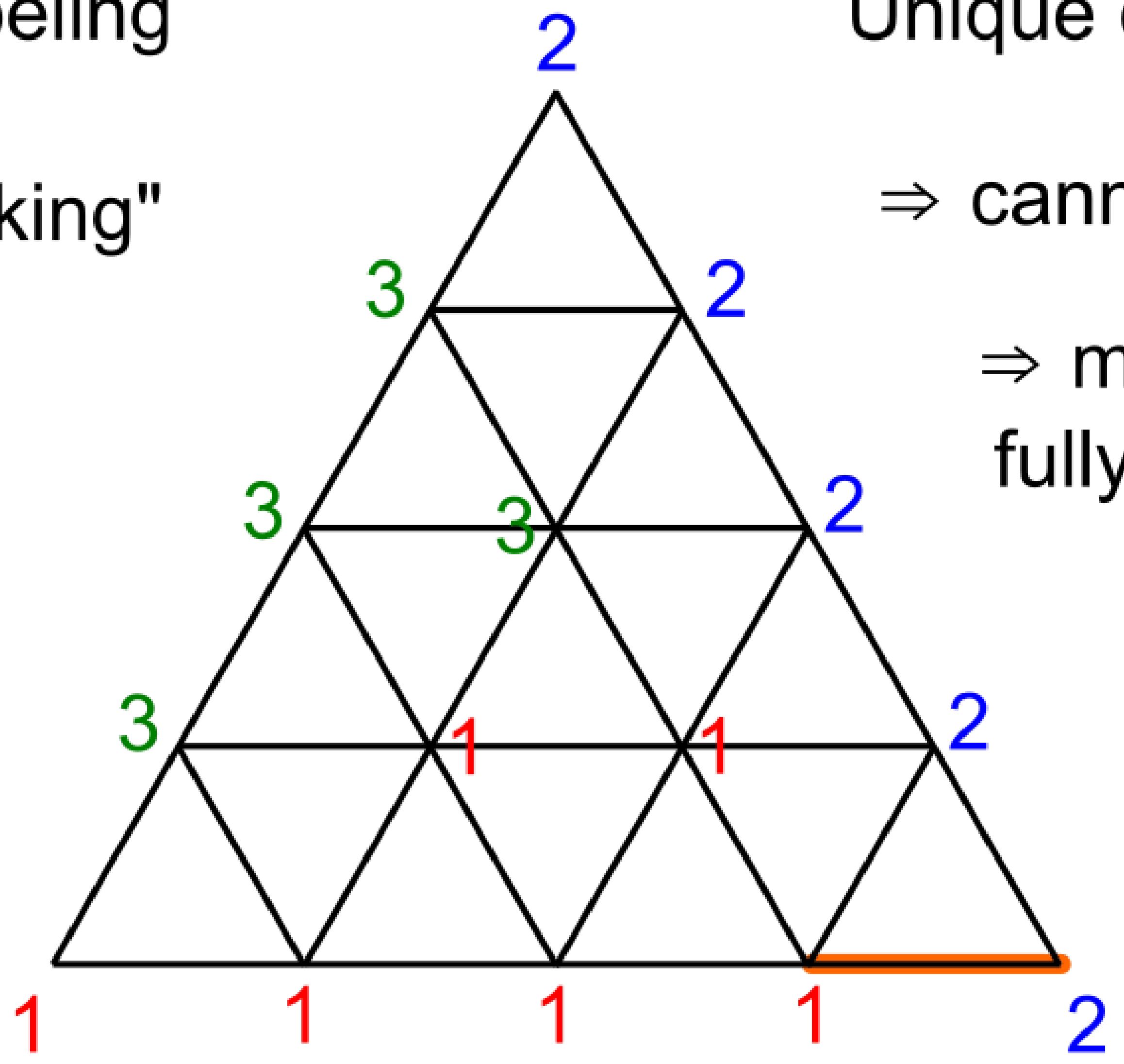
Unique door on boundary

⇒ cannot be thrown out

⇒ must end up in a fully labeled room!

Not a Sperner labeling

But "proof by walking" still applies!



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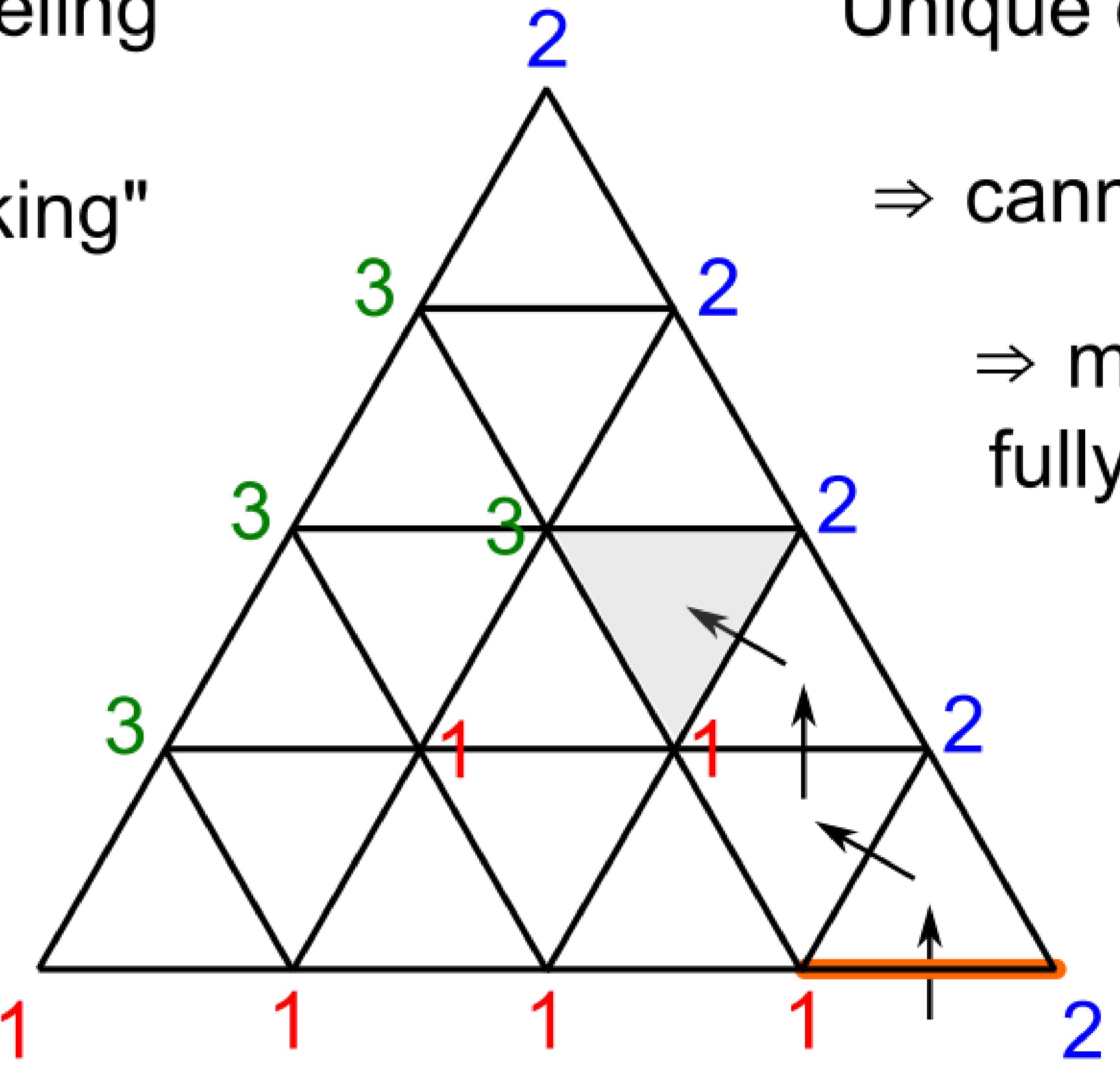

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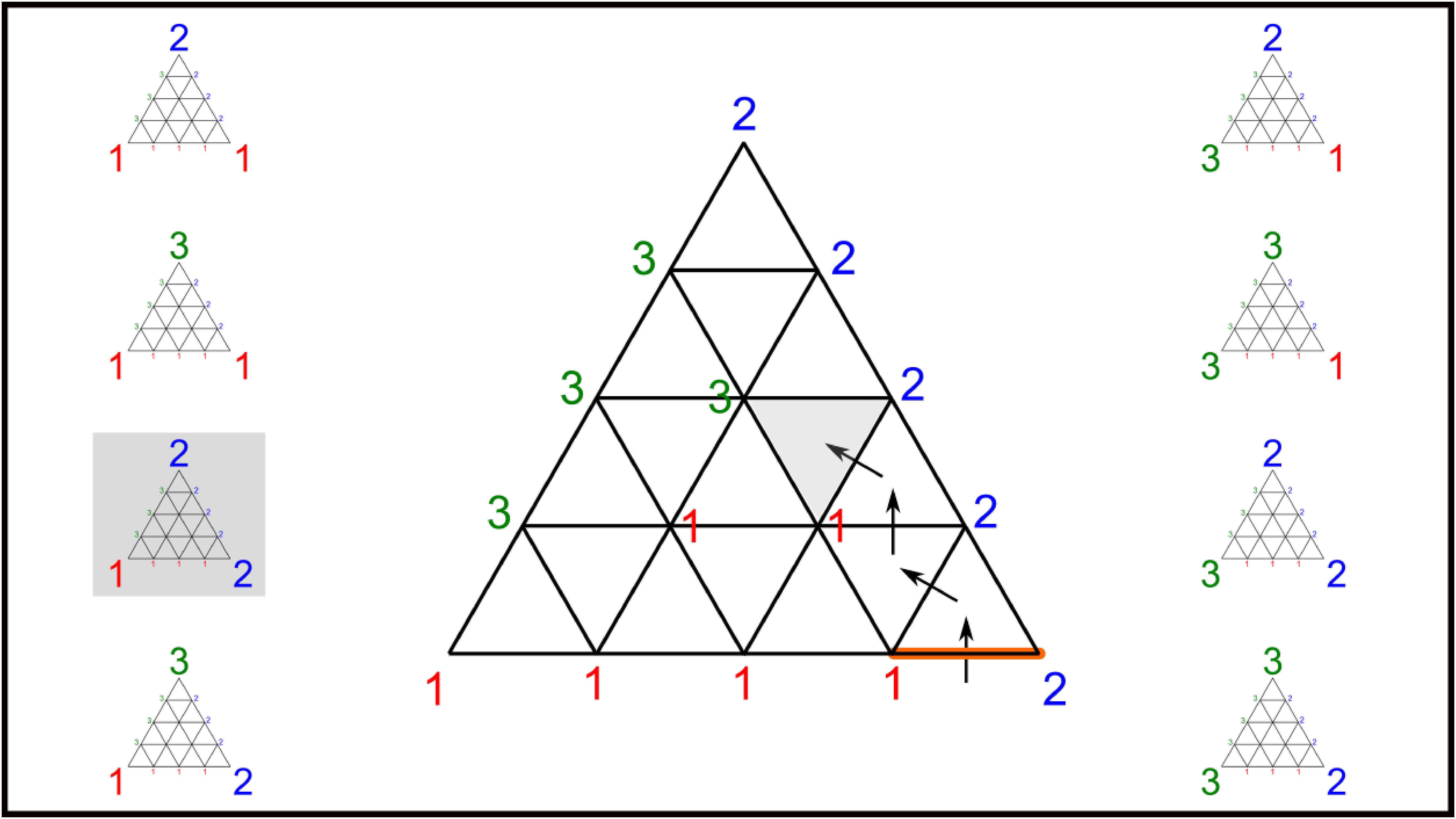
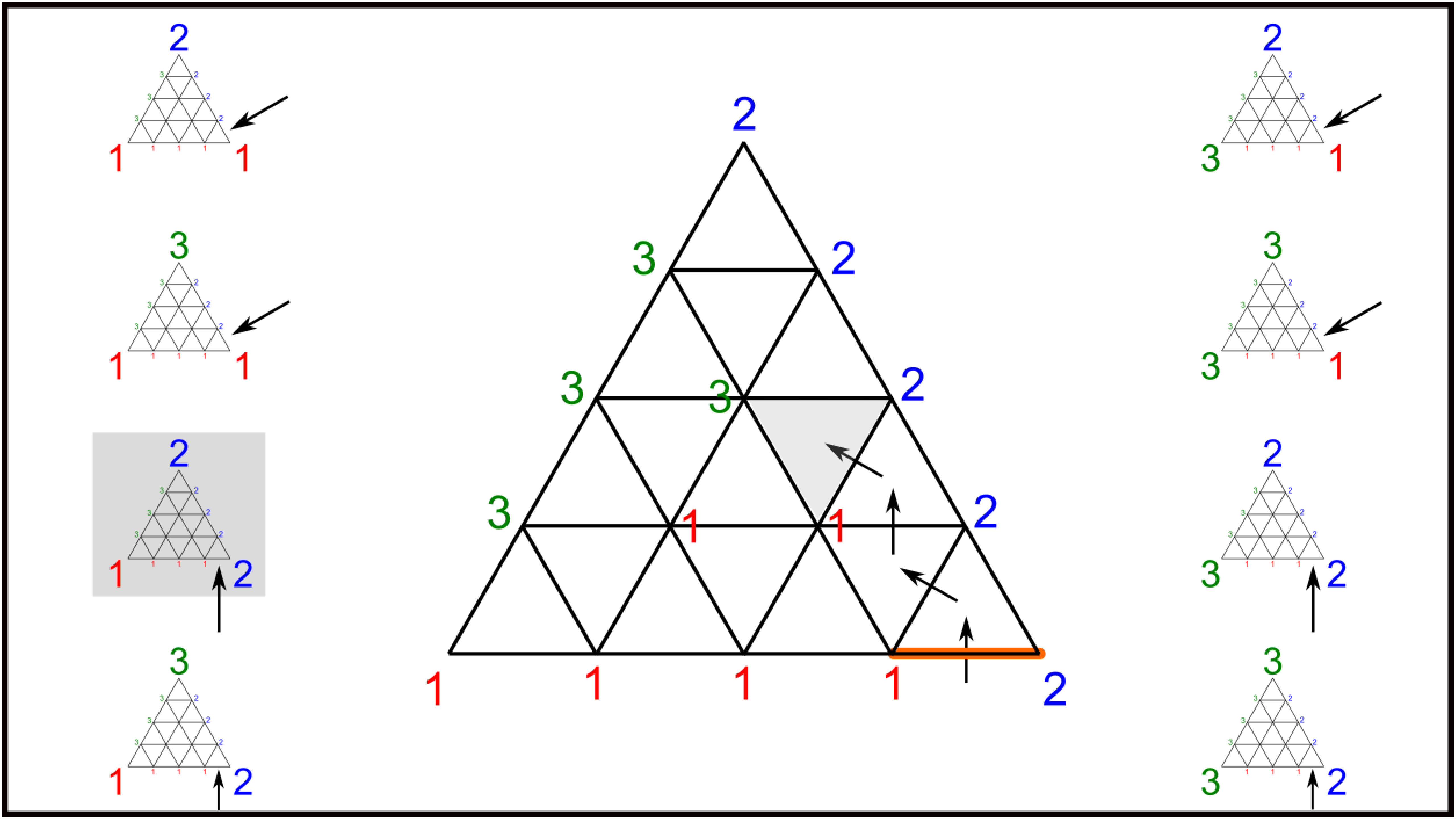
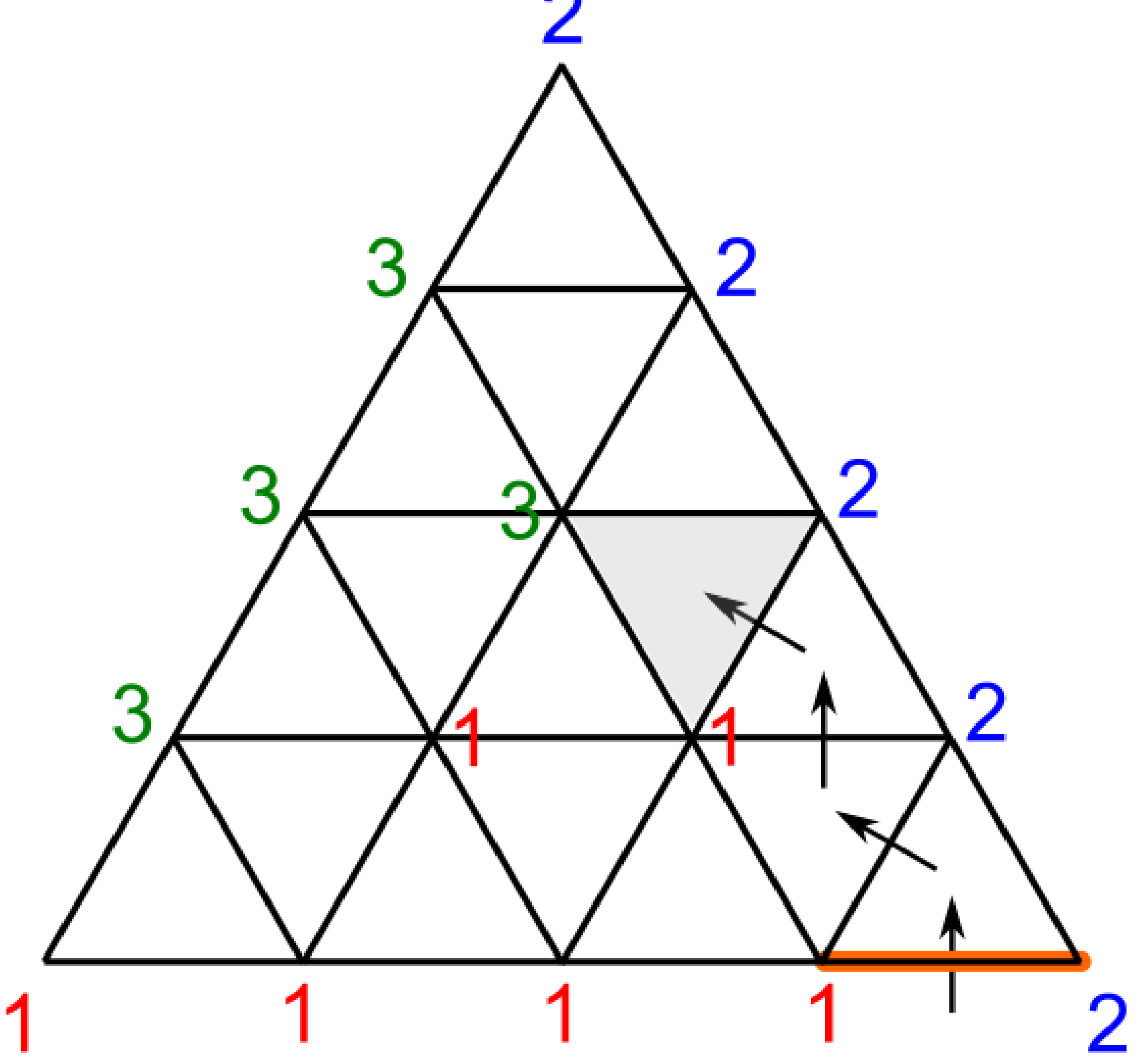
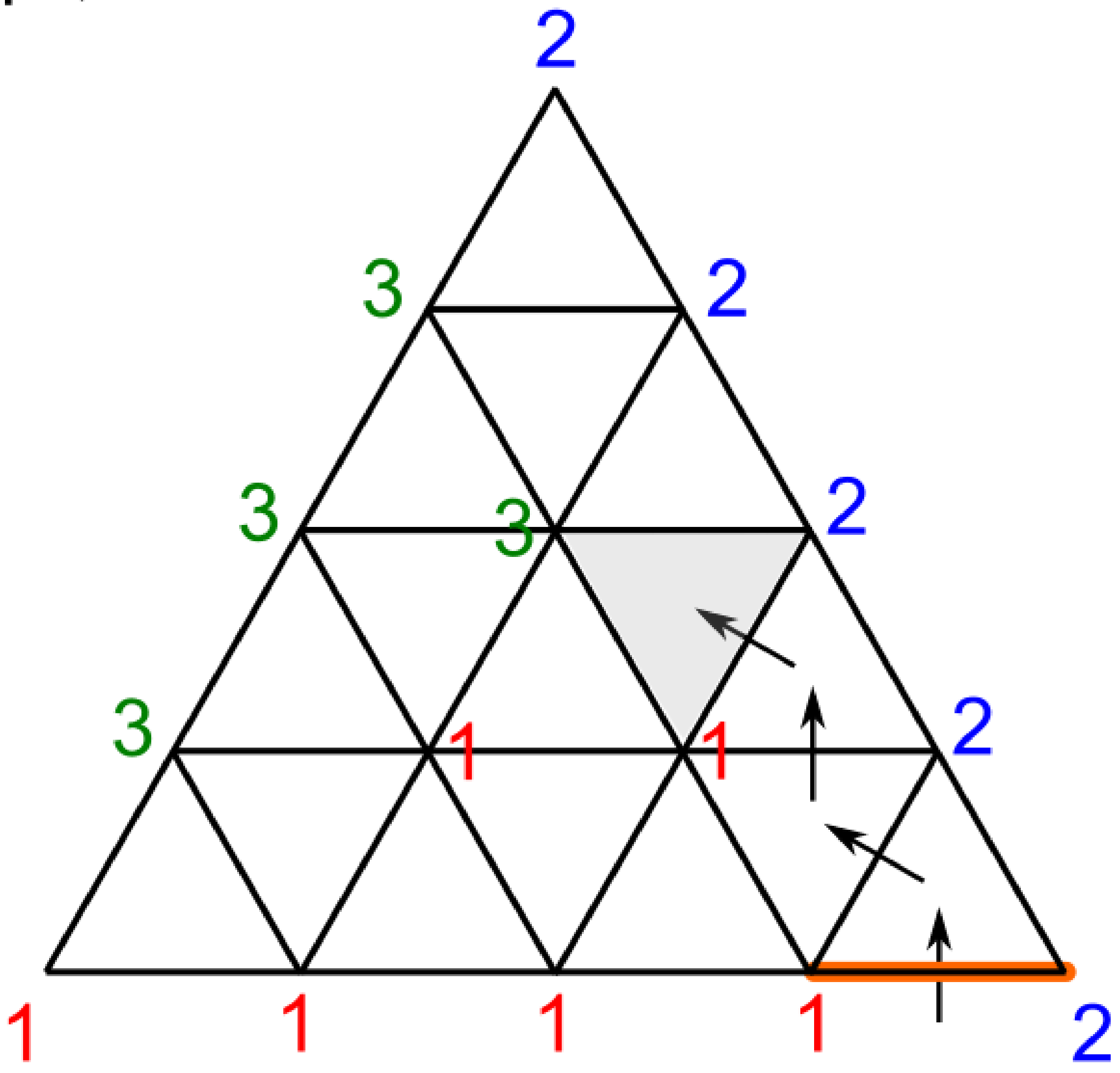



2
3
2
3
3
2
3
1
1
2
1
1
1
1
2



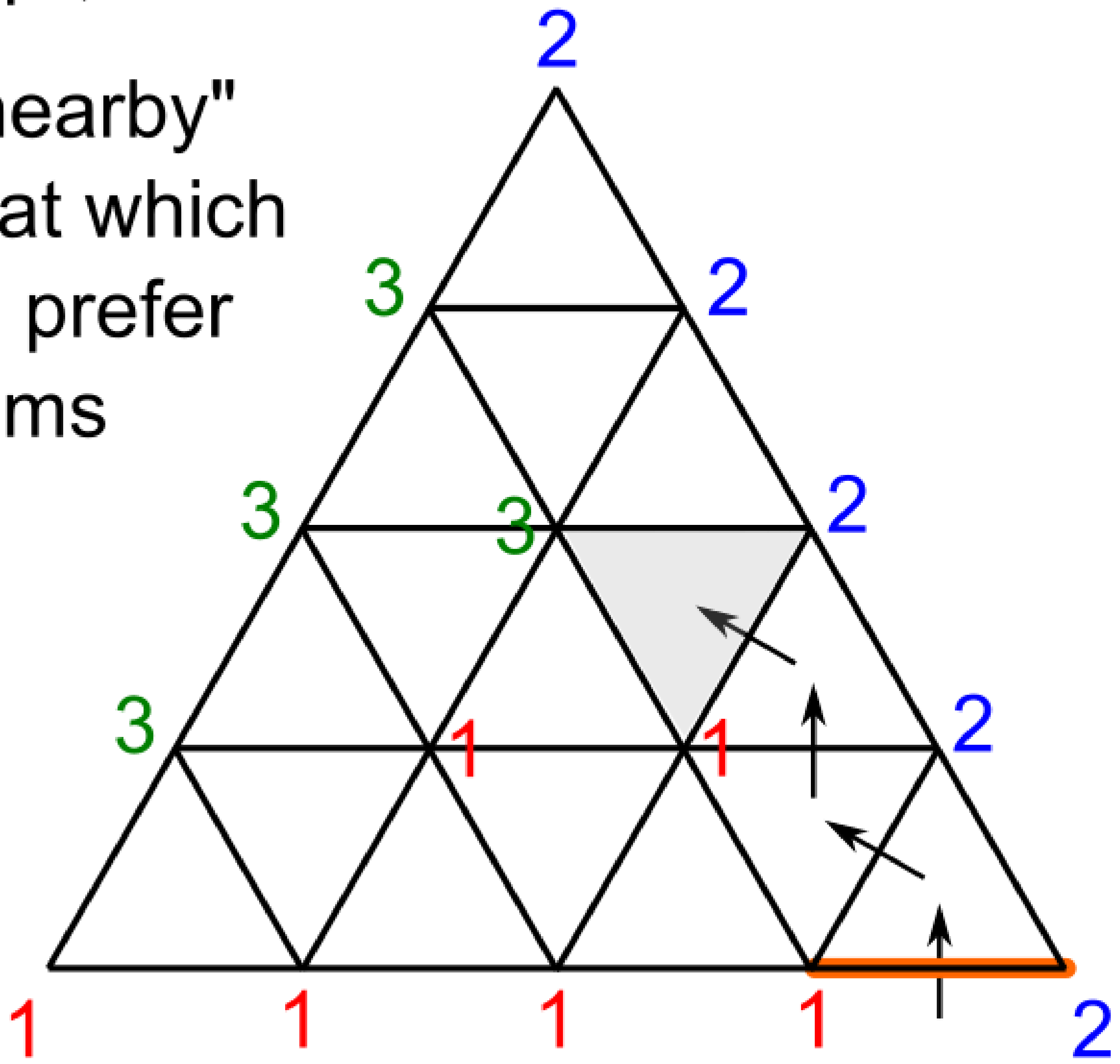
2
3
2
3
3
2
3
1
1
2
1
1
1
1
2

Fully labeled room ⇒



Fully labeled room ⇒

a set of three "nearby" pricing schemes at which different agents prefer different rooms



Fully labeled room ⇒

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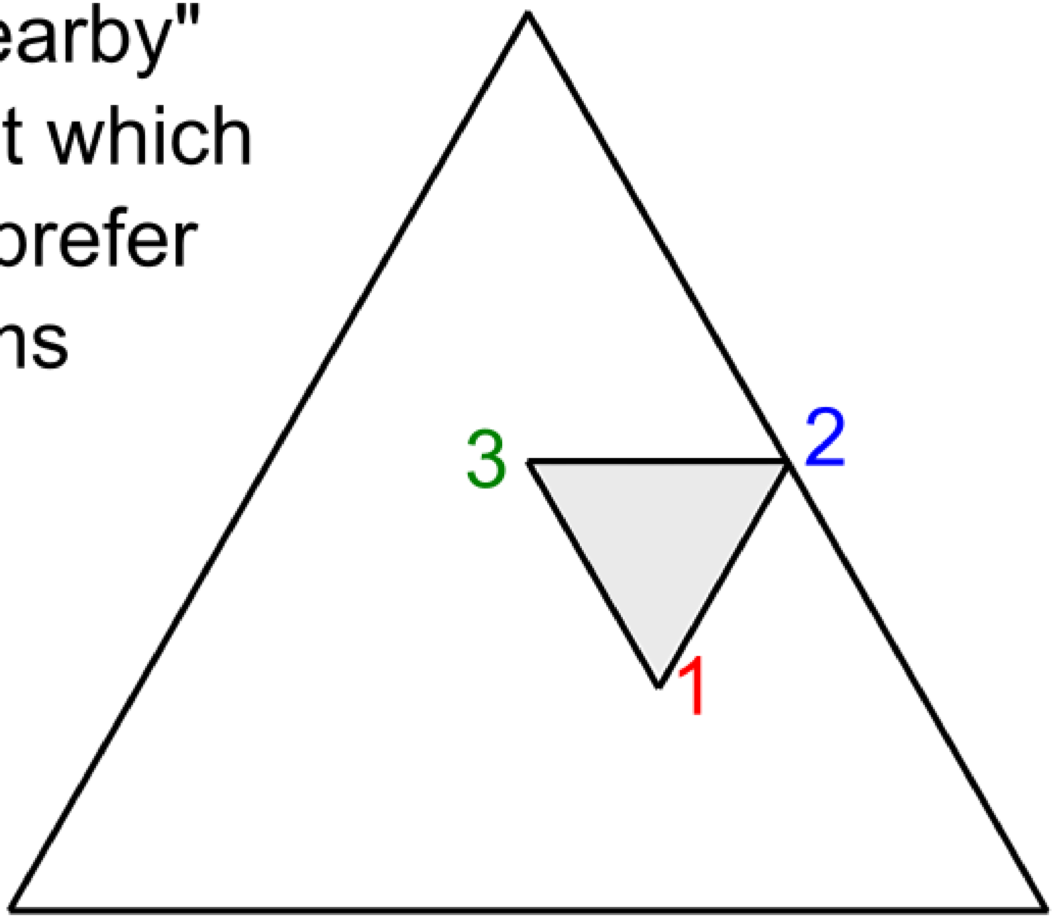
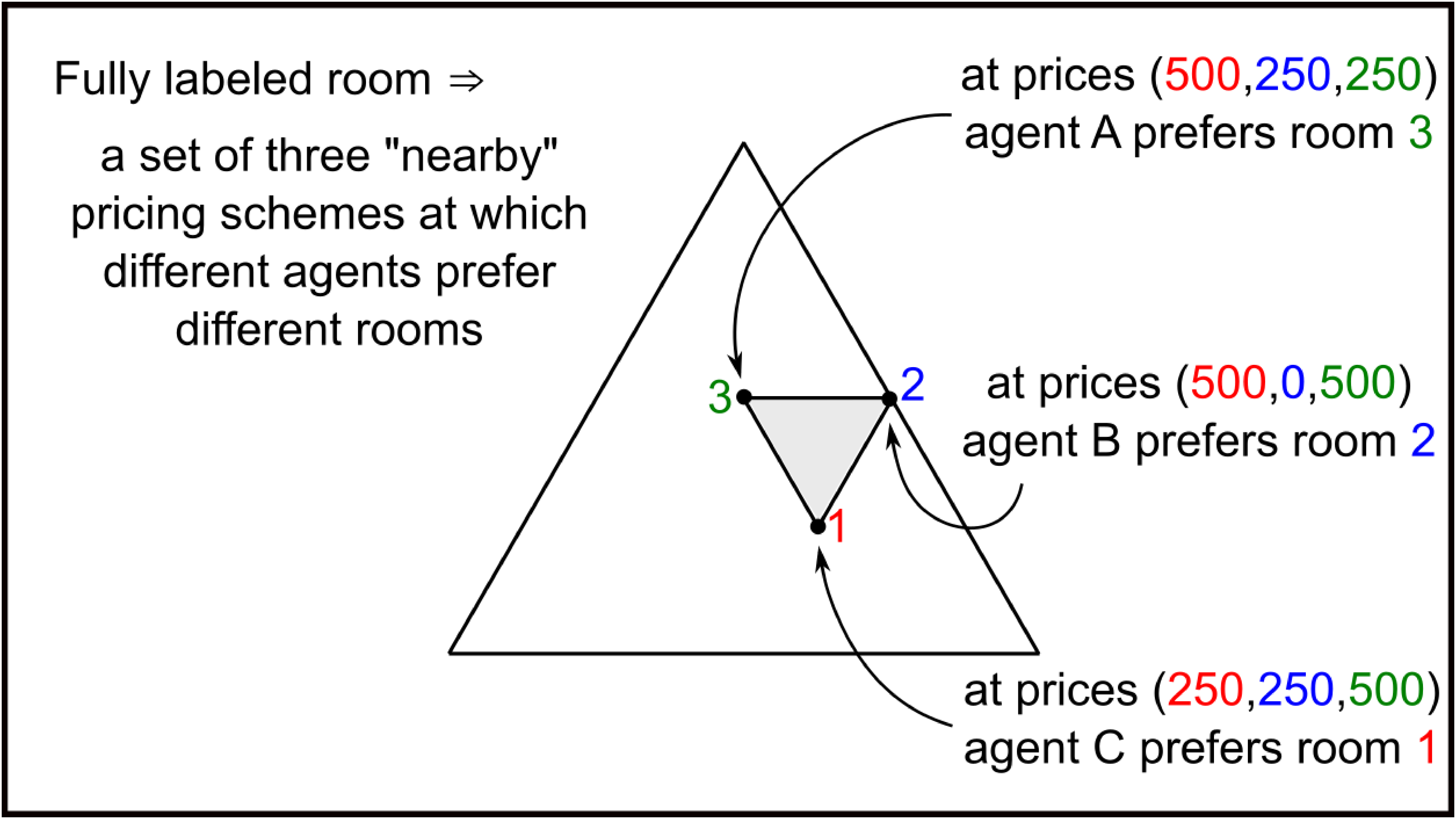
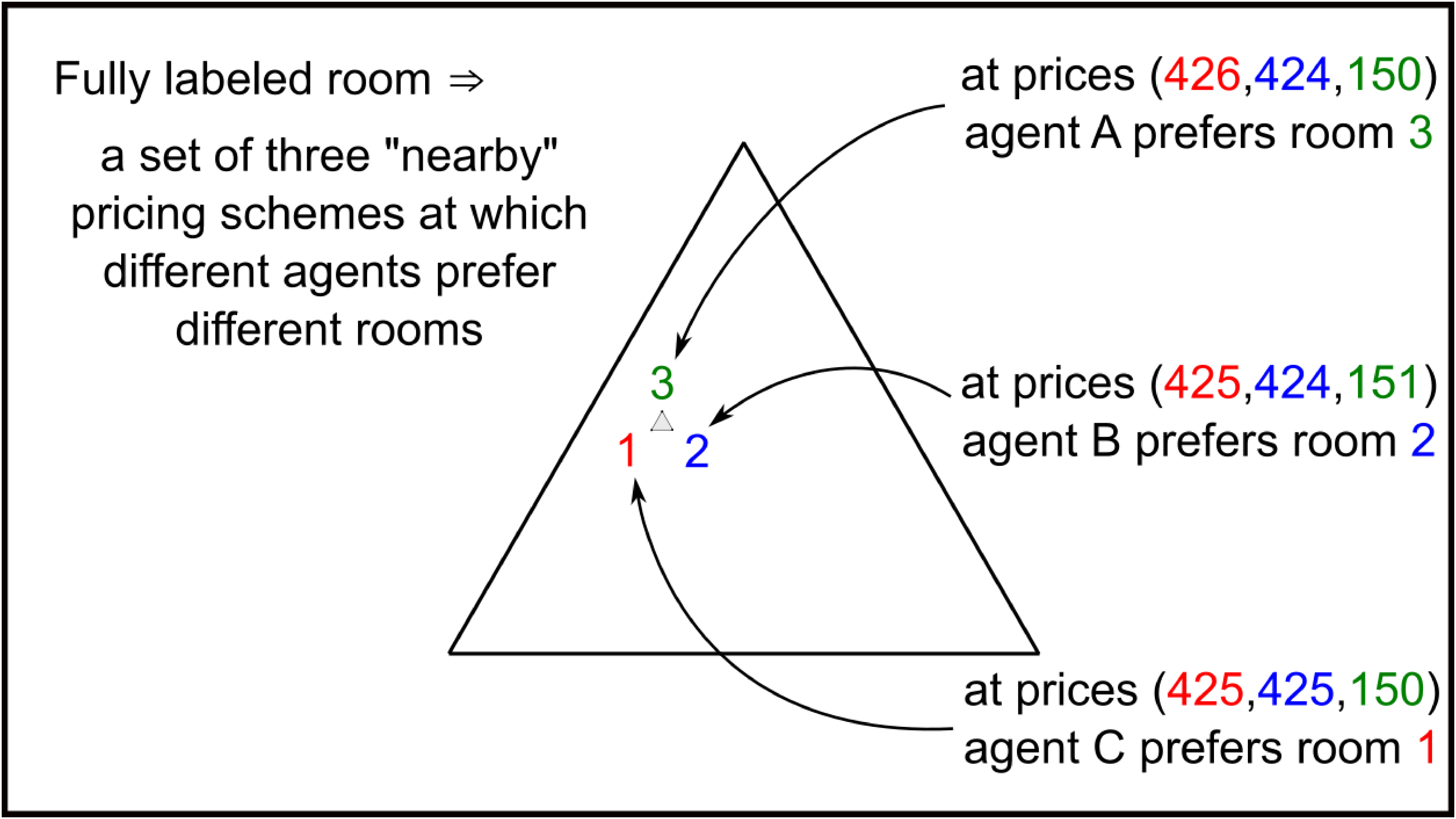
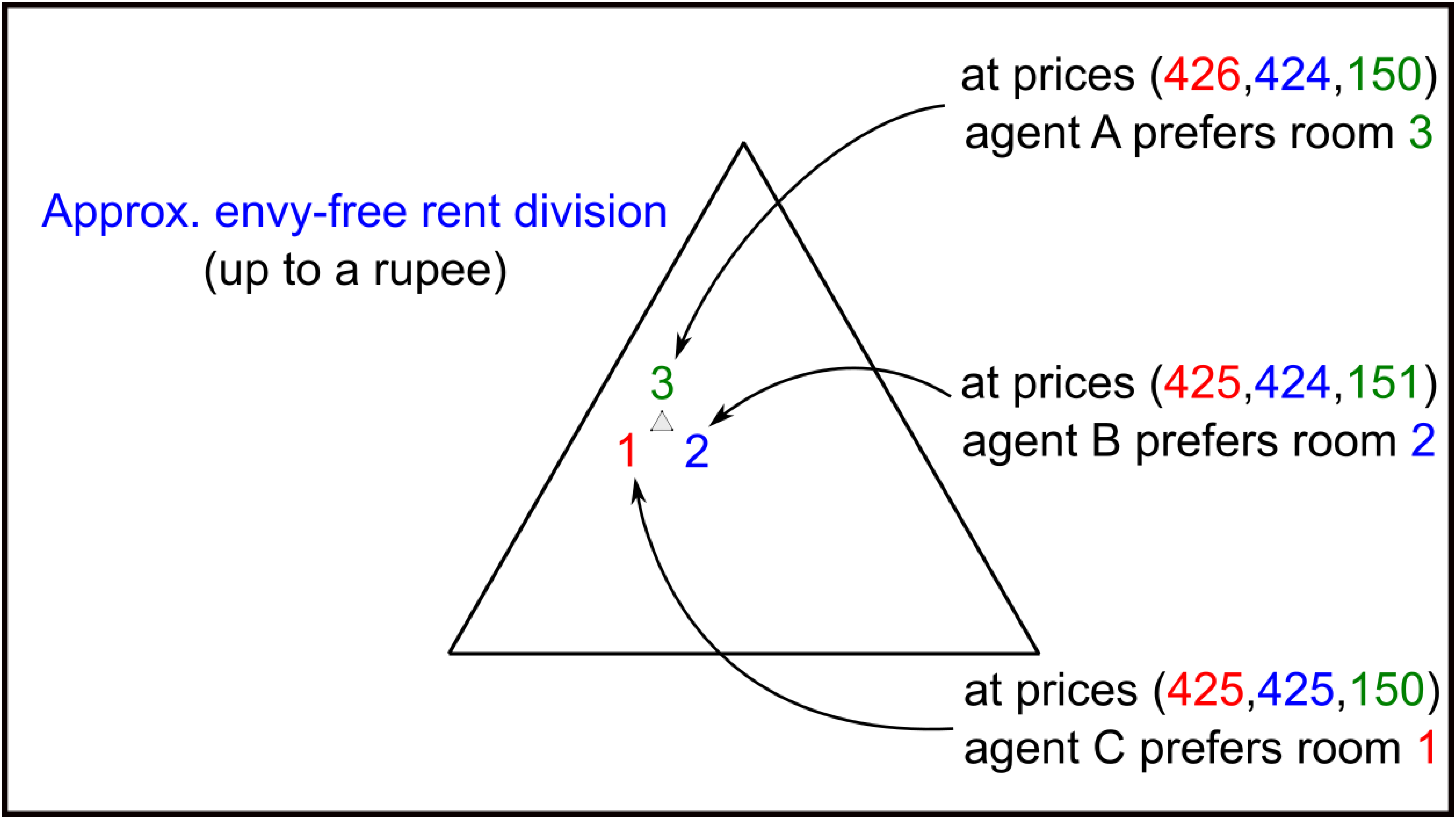
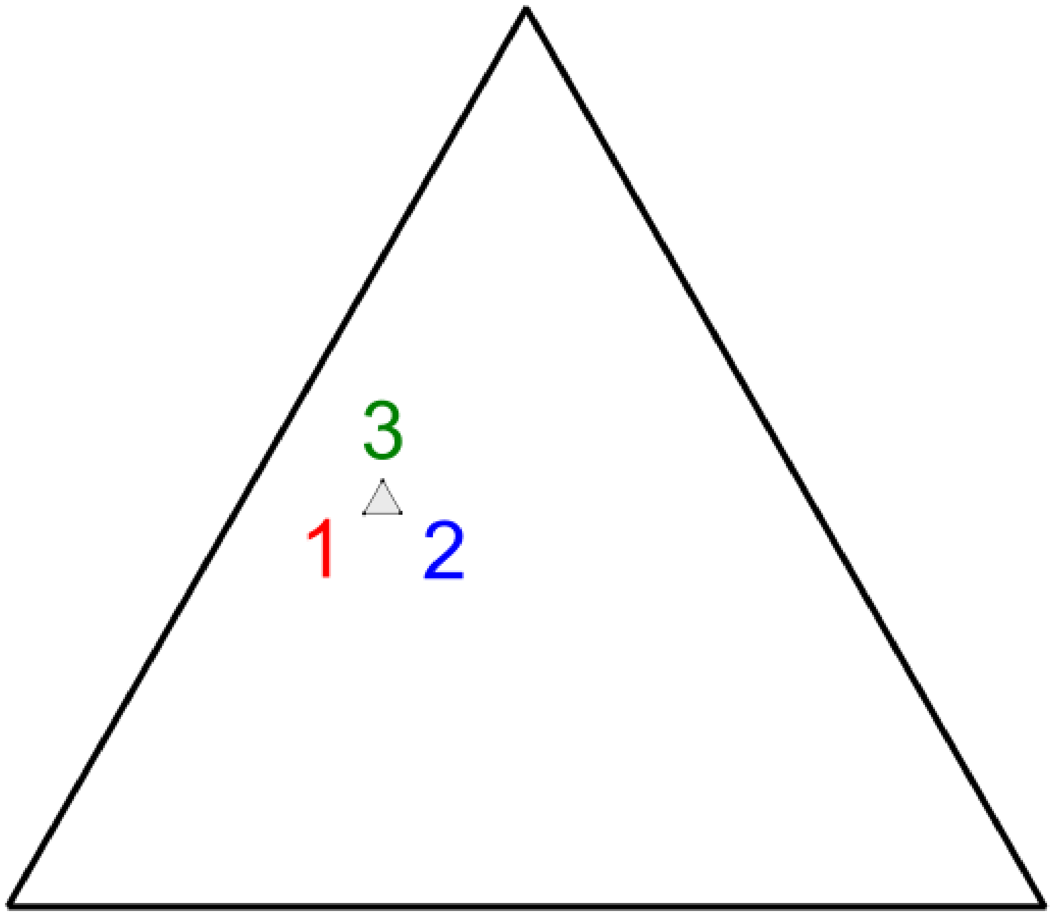



Fully labeled room ⇒
a set of three "nearby" pricing schemes at which different agents prefer different rooms
at prices (500,250,250)
agent A prefers room 3
at prices (500,0,500)
agent B prefers room 2
at prices (250,250,500)
agent C prefers room 1
3
2
1

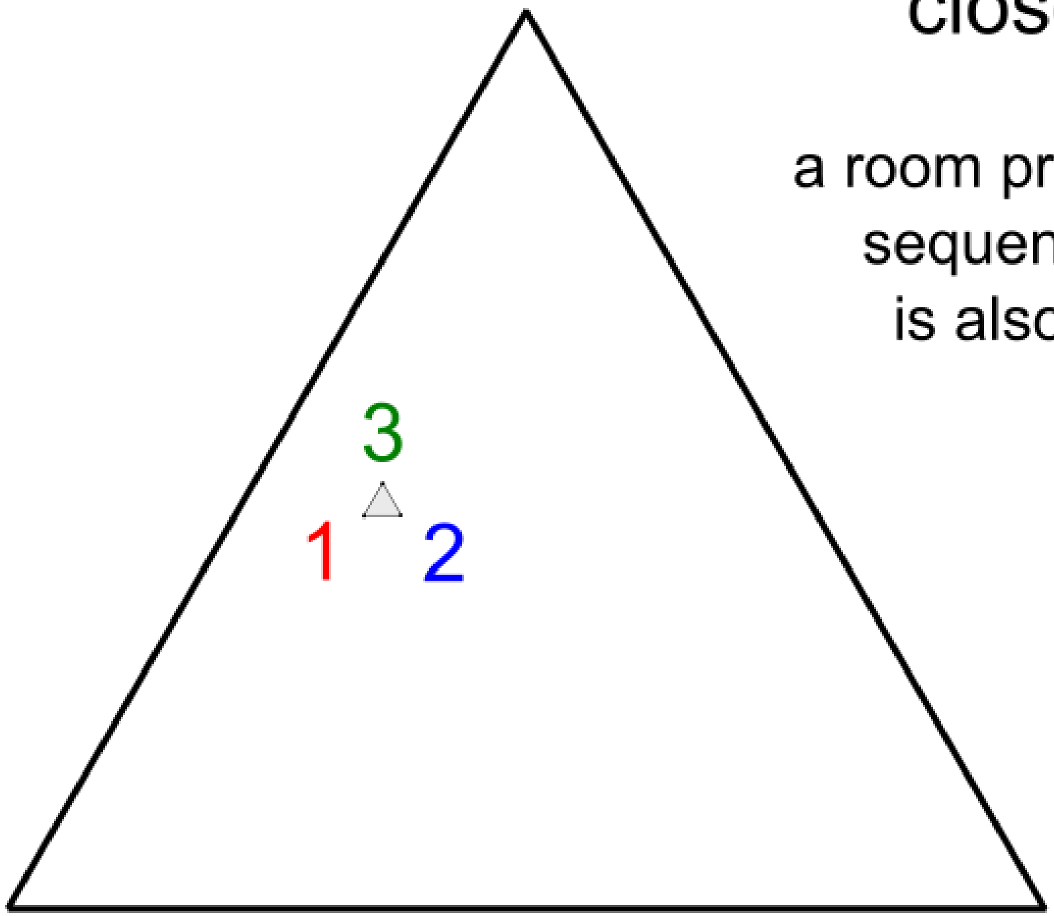

Fully labeled room ⇒
a set of three "nearby" pricing schemes at which different agents prefer different rooms
at prices (426,424,150)
agent A prefers room 3
at prices (425,424,151)
agent B prefers room 2
at prices (425,425,150)
agent C prefers room 1
3
1
2


Approx. envy-free rent division
(up to a rupee)
at prices (426,424,150)
agent A prefers room 3
at prices (425,424,151)
agent B prefers room 2
at prices (425,425,150)
agent C prefers room 1
3
1
2



[Su, *Amer. Math. Mon.* 1999]

For *closed* preferences, an *exact* envy-free rent division exists.



"closed preferences"

a room preferred for a convergent sequence of pricing schemes is also preferred in the limit

[Su, *Amer. Math. Mon.* 1999]

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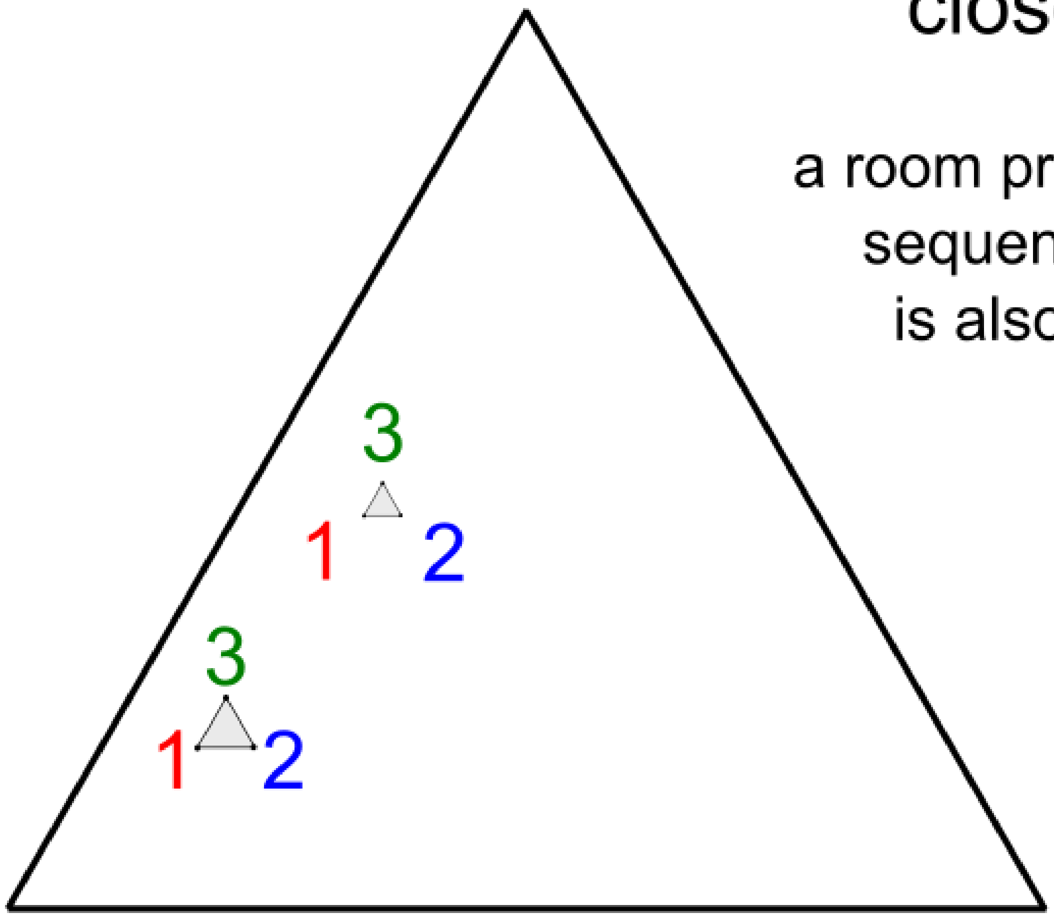

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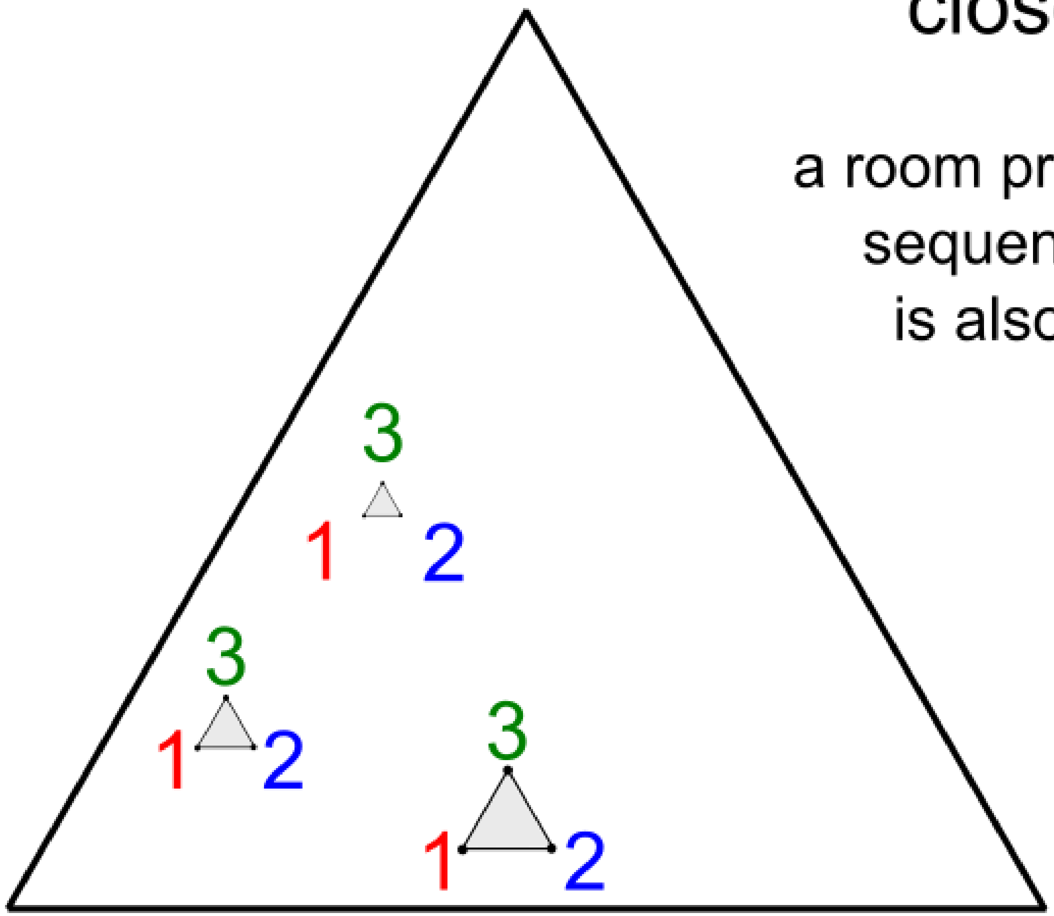

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"closed preferences"

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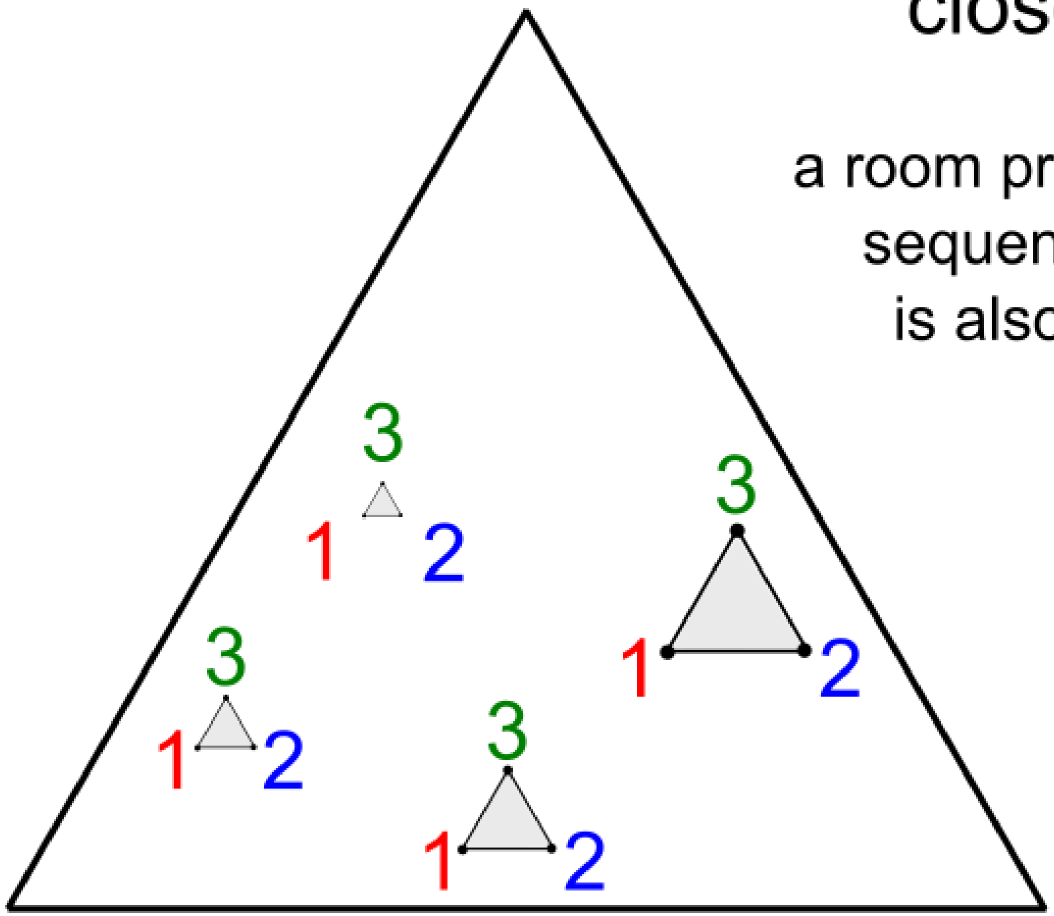
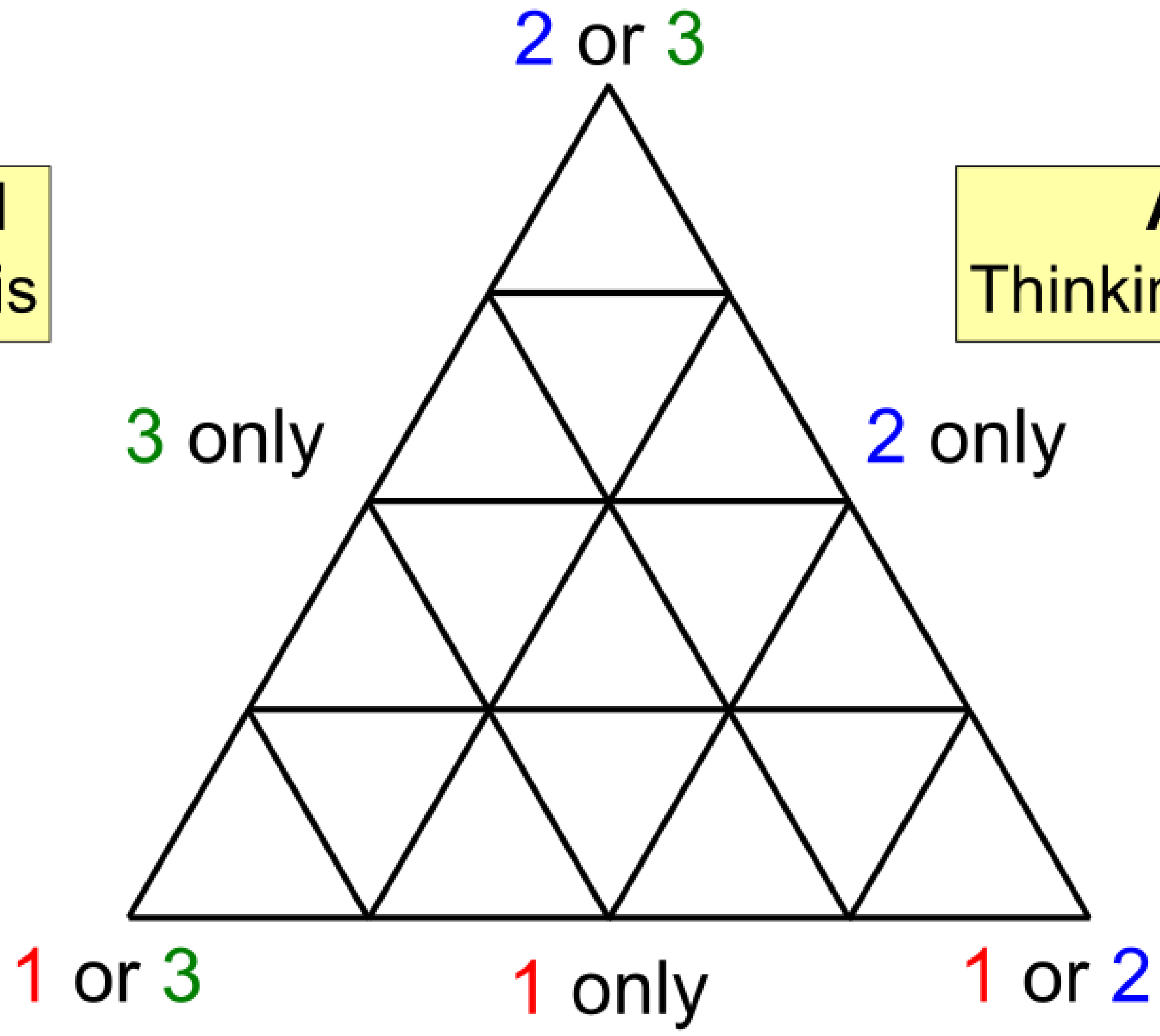


[Su, *Amer. Math. Mon.* 1999]

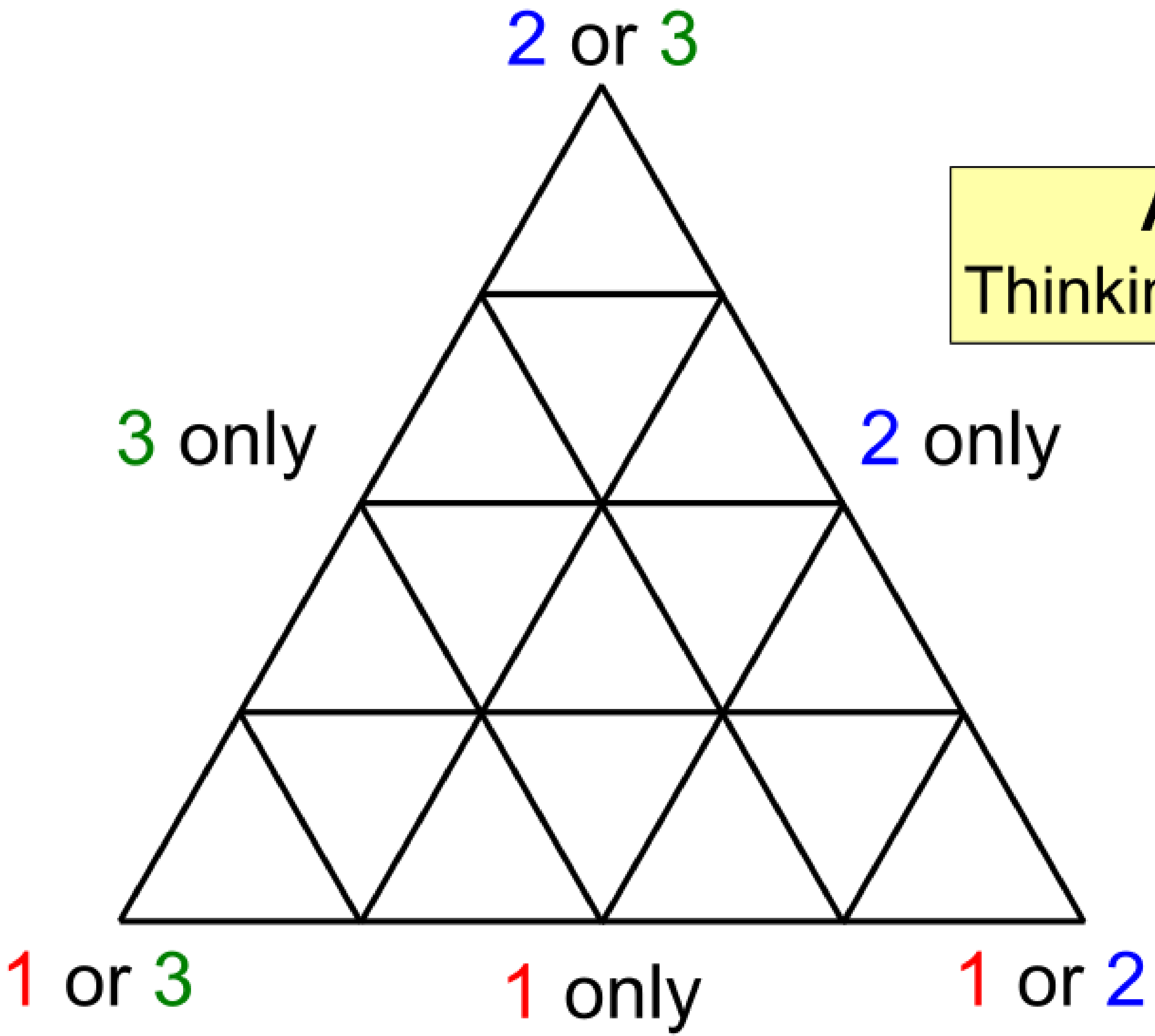
For *closed* preferences, an *exact* envy-free rent division exists.

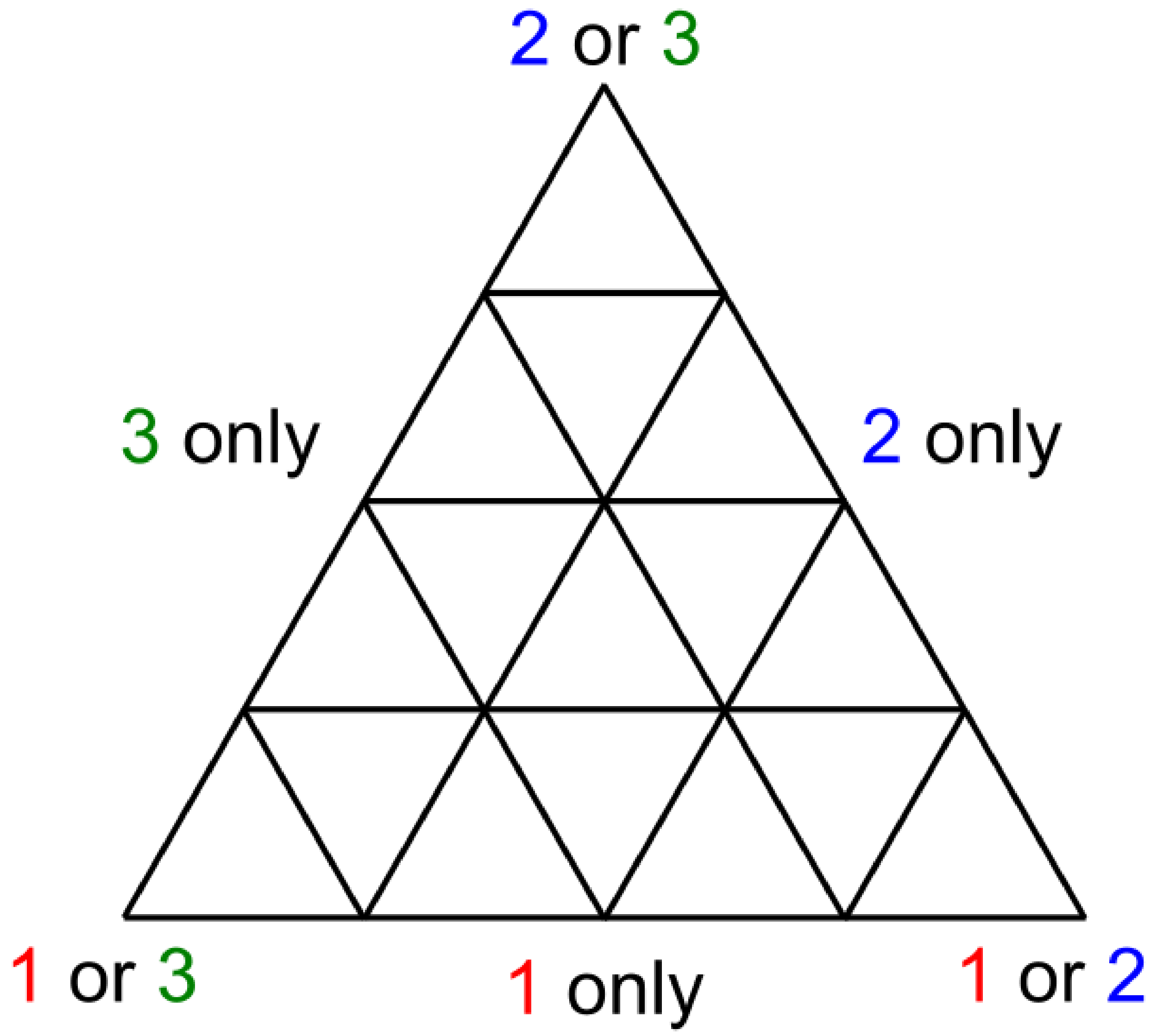
Approach 1
Case analysis

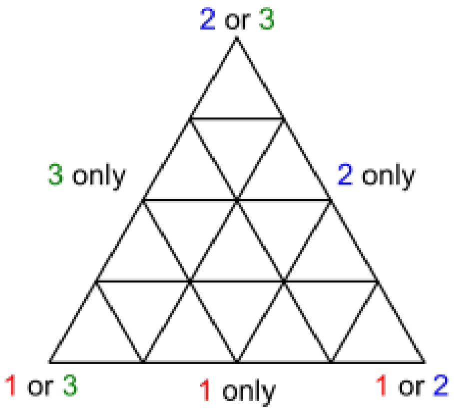
Approach 2
Thinking outside the box

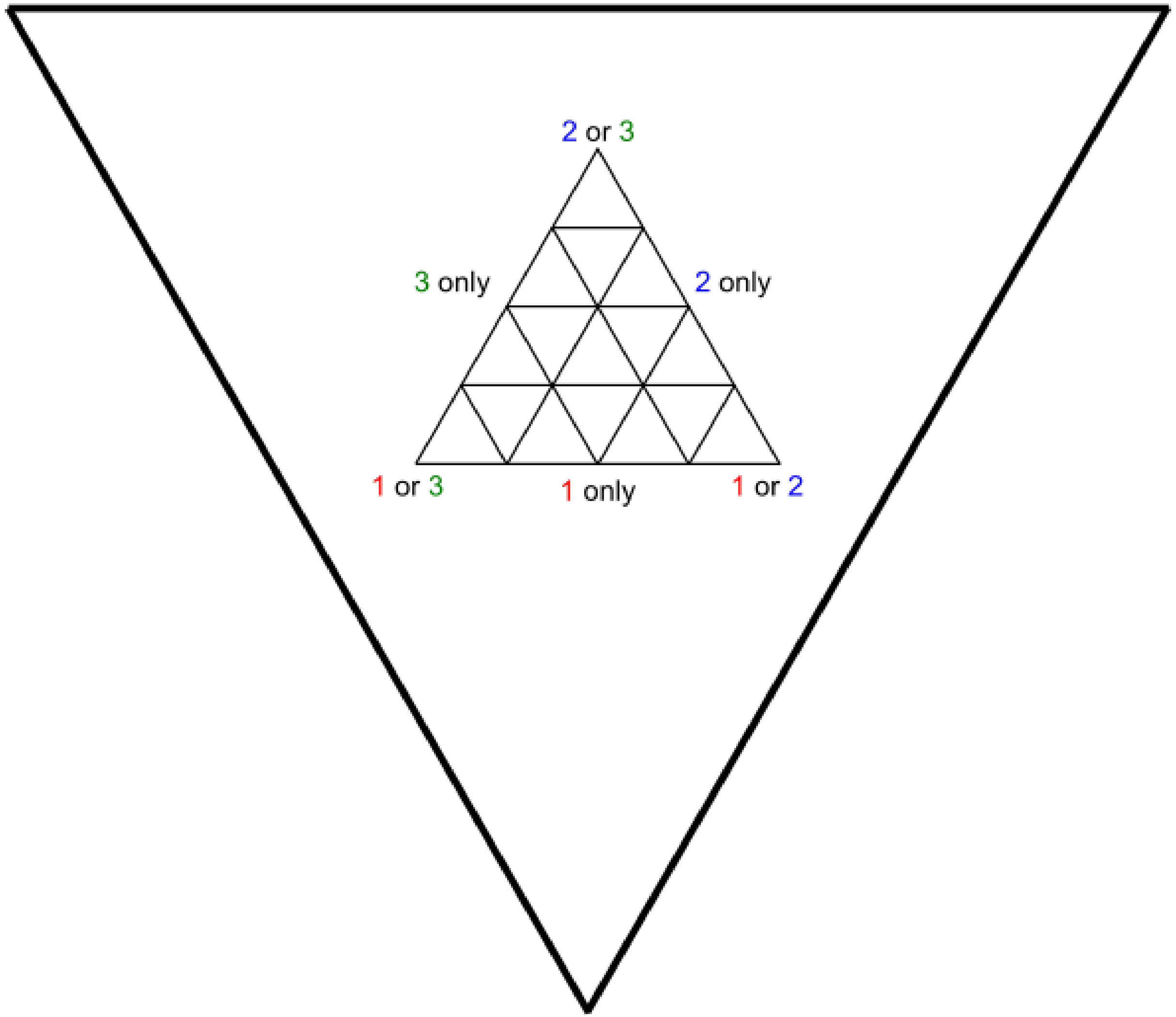


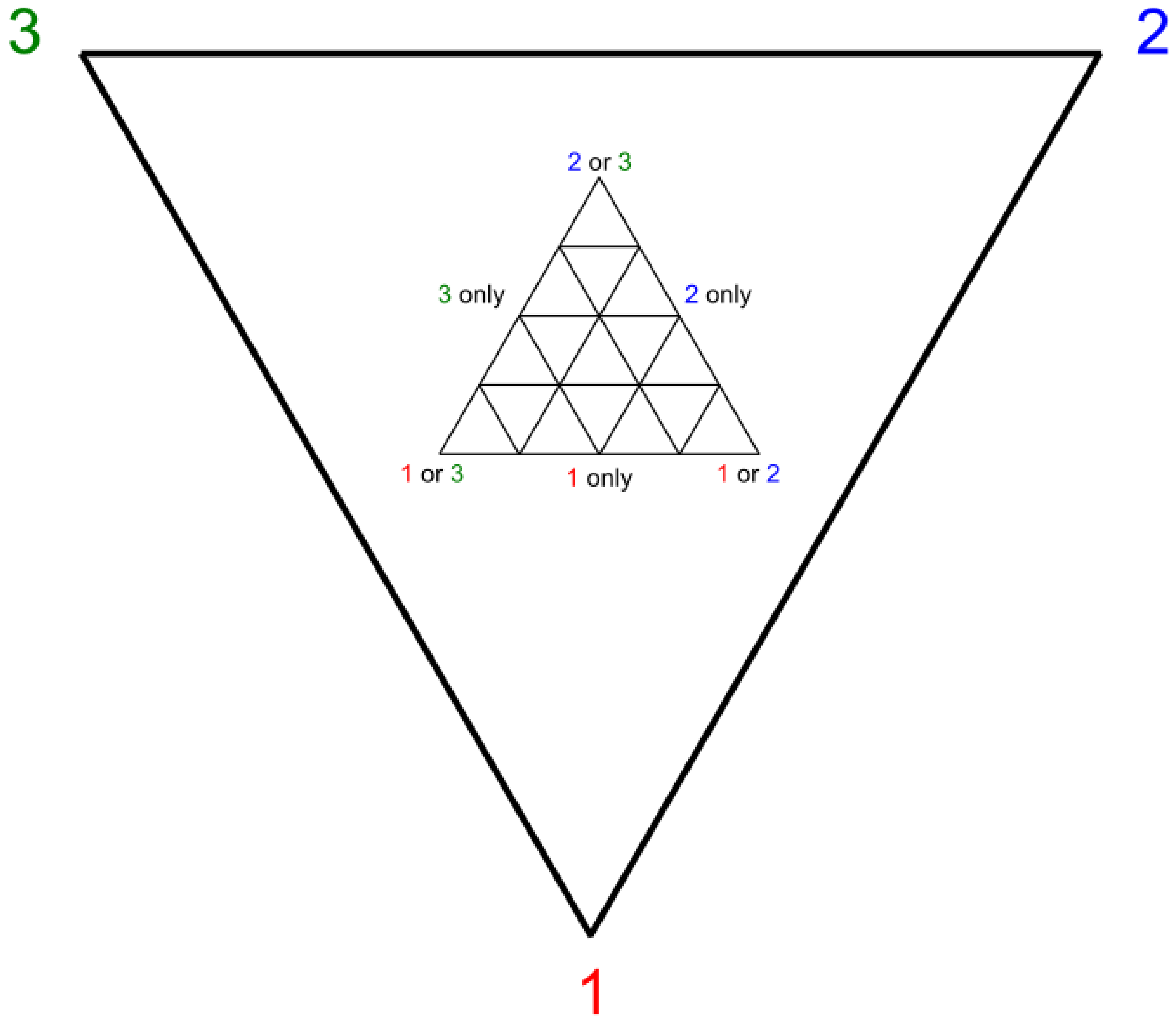
Approach 2
Thinking outside the box




2 or 3
3 only
2 only
1 or 3
1 only
1 or 2


2 or 3
3 only
2 only
1 or 3
1 only
1 or 2


2 or 3
3 only
2 only
1 or 3
1 only
1 or 2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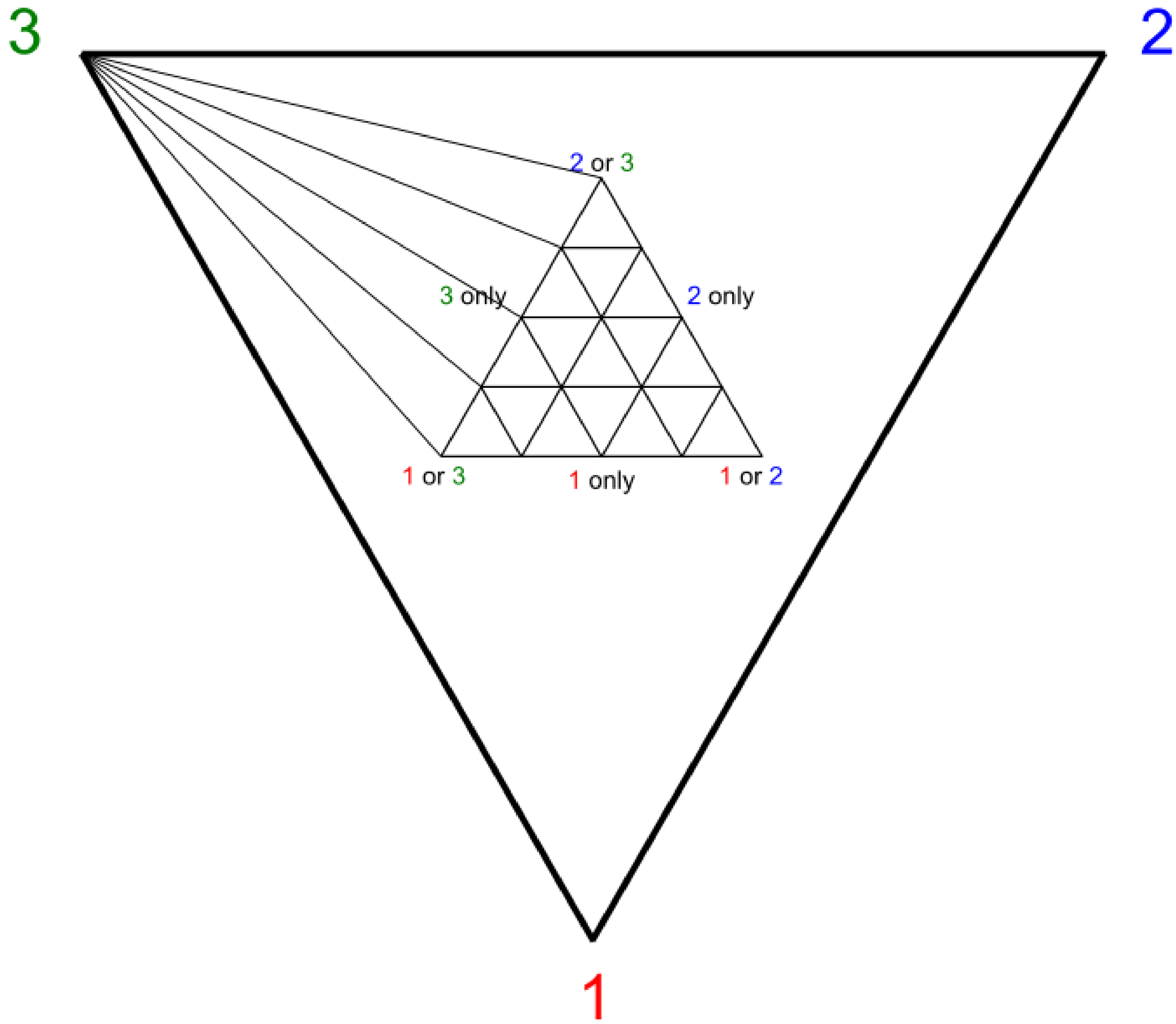
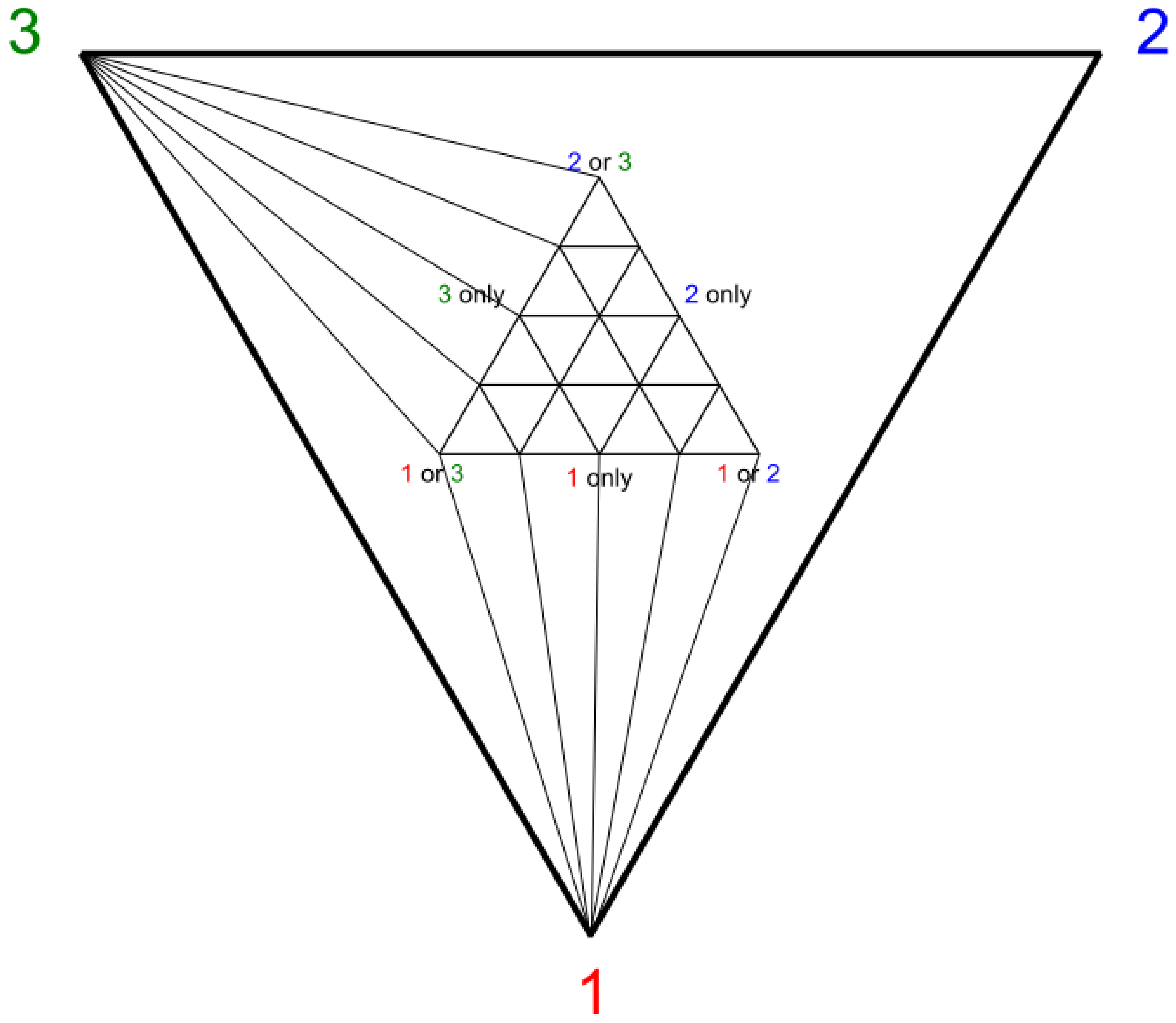
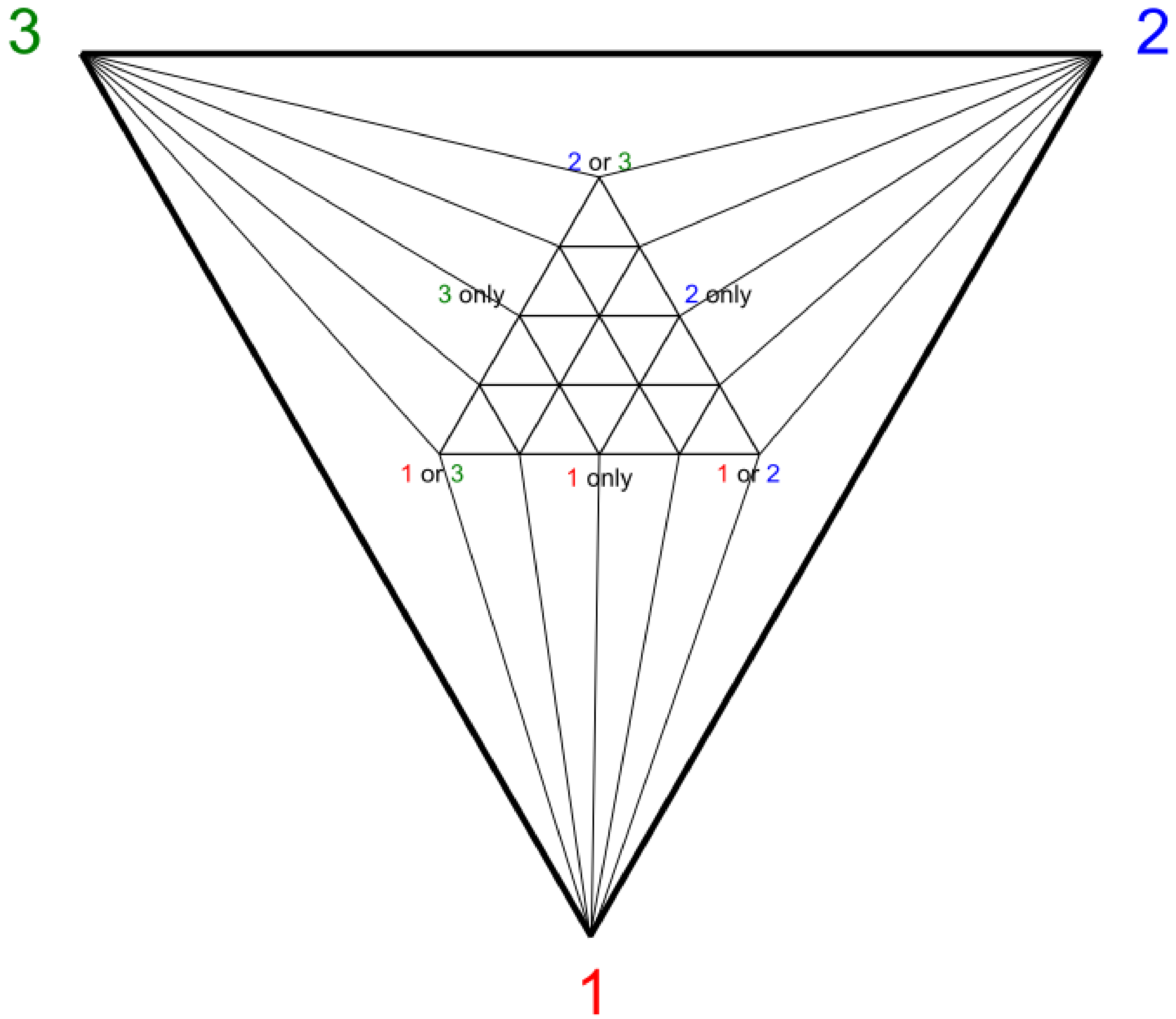
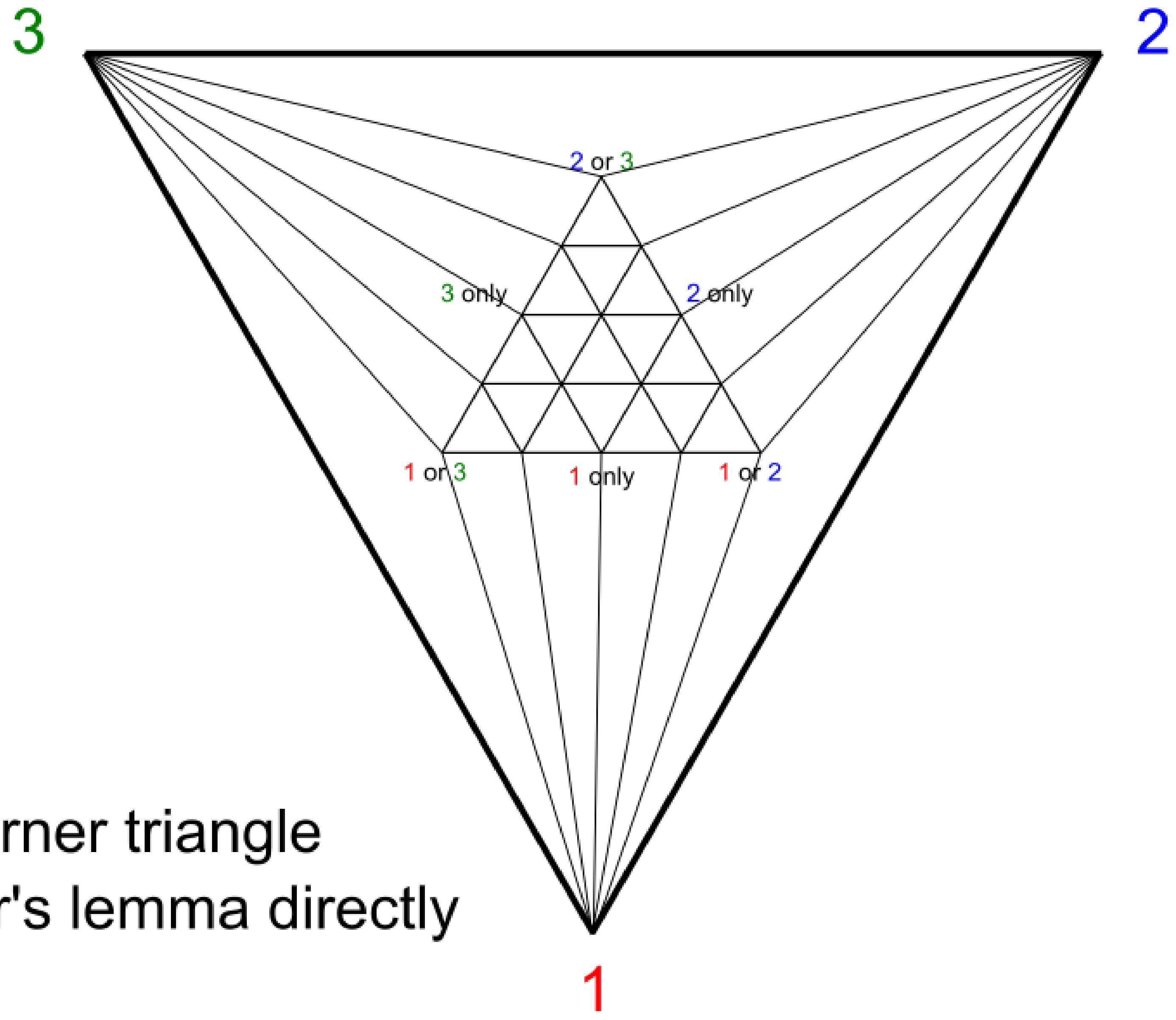
2
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
1


3
2
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
1


3
2
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
1

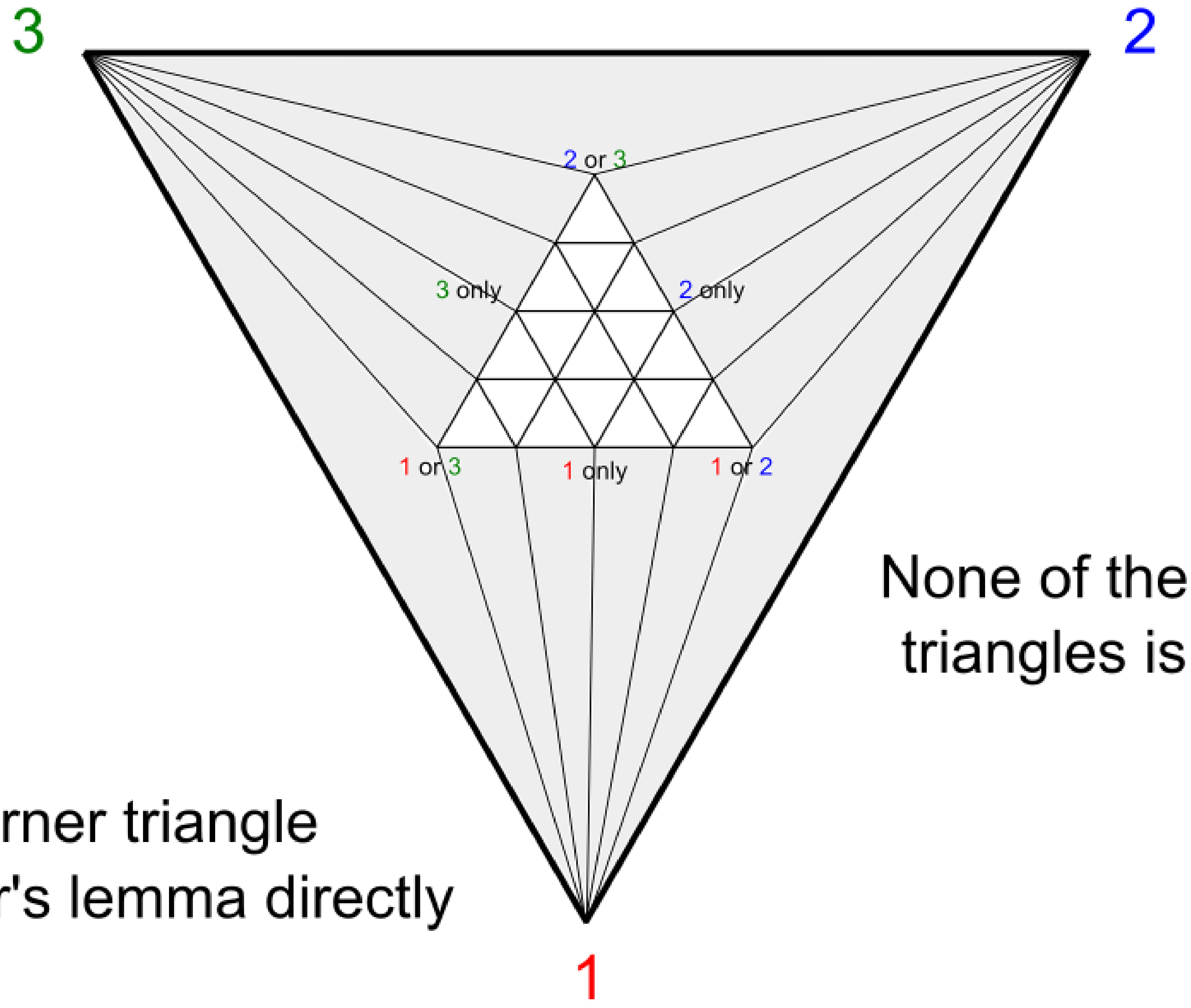


3
2
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
1



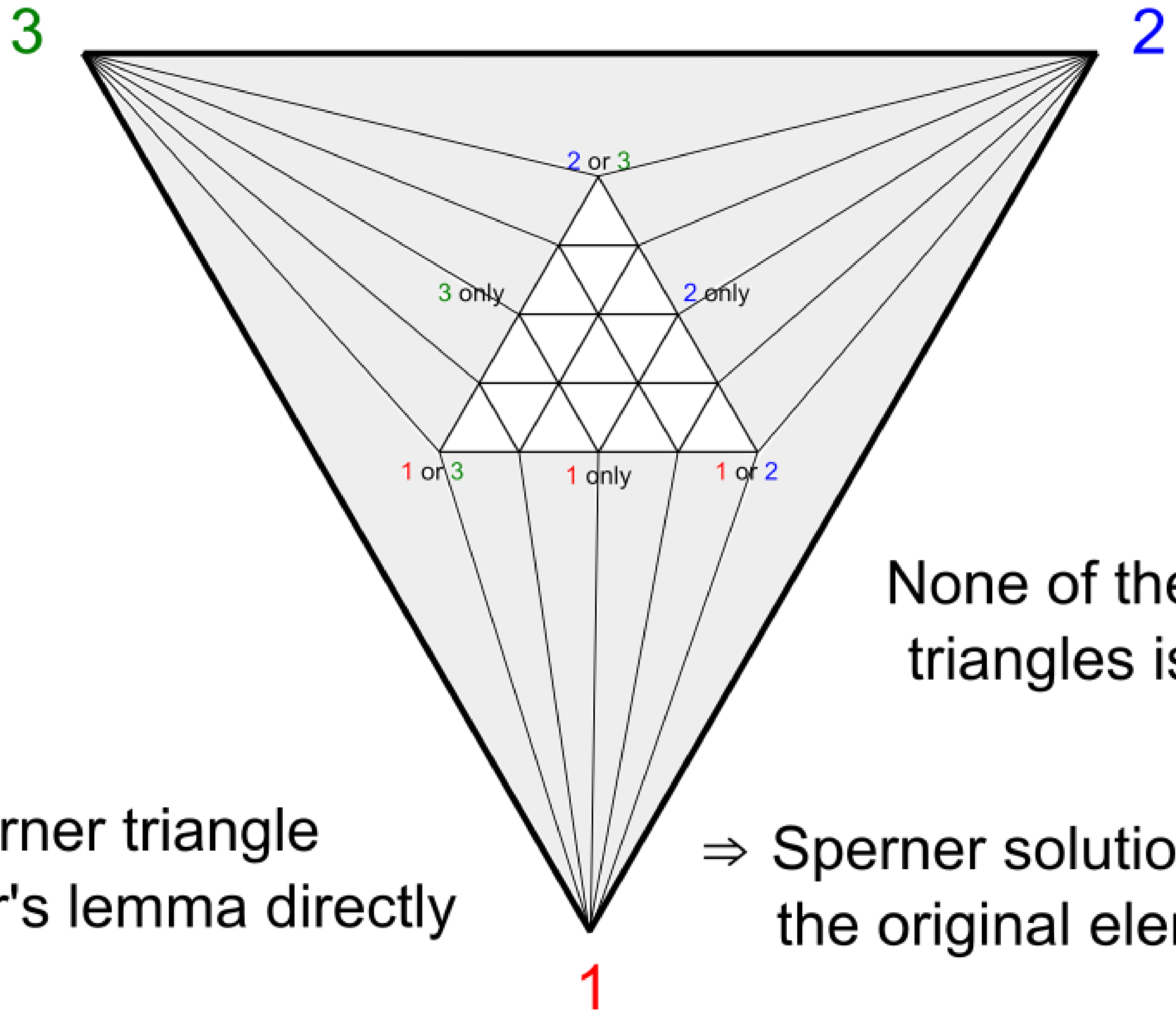
A valid Sperner triangle

⇒ apply Sperner's lemma directly



None of the newly added triangles is fully labeled

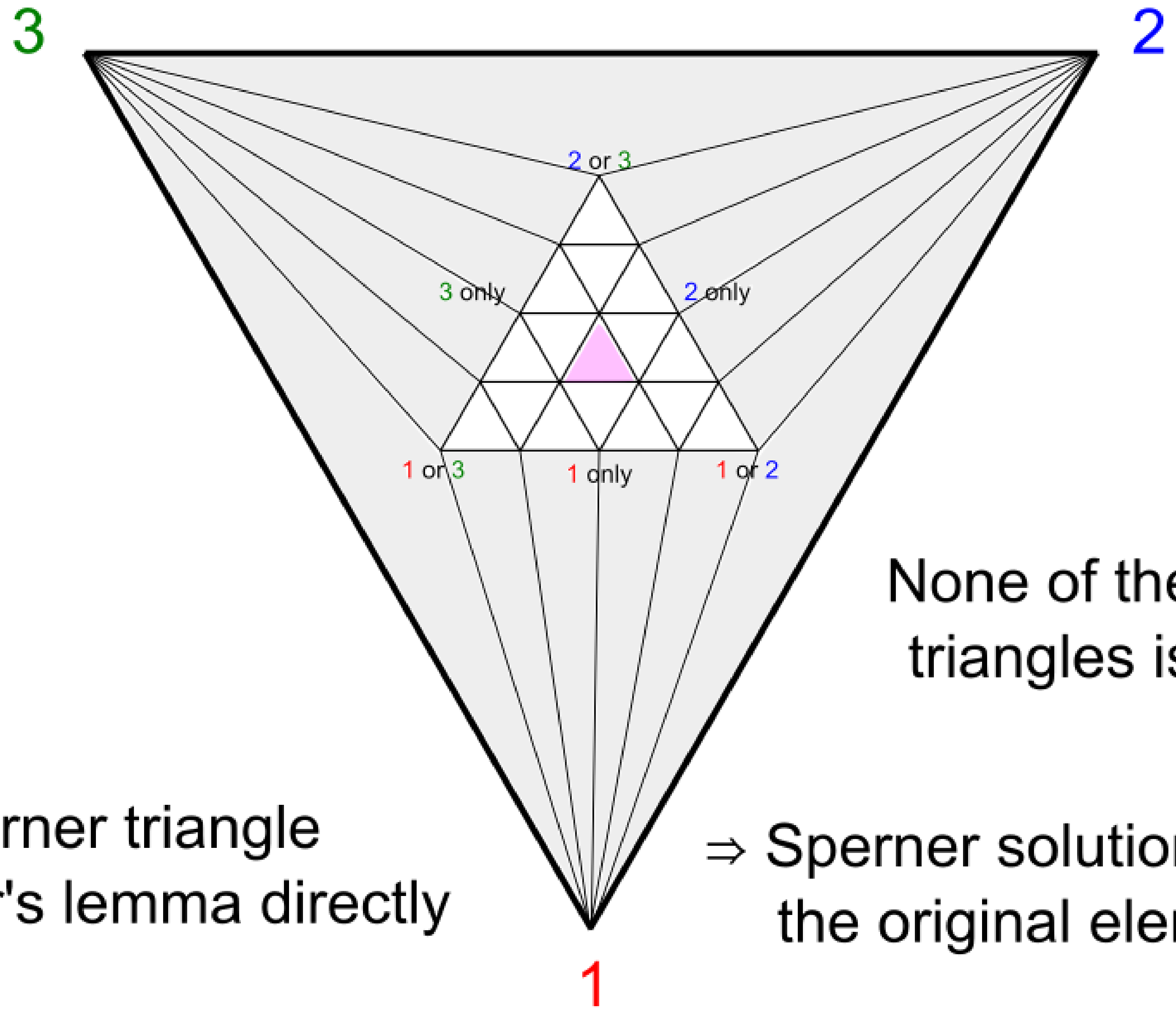
A valid Sperner triangle
⇒ apply Sperner's lemma directly



None of the newly added triangles is fully labeled

A valid Sperner triangle
⇒ apply Sperner's lemma directly

⇒ Sperner solution must be one of the original elementary triangles



None of the newly added triangles is fully labeled

A valid Sperner triangle
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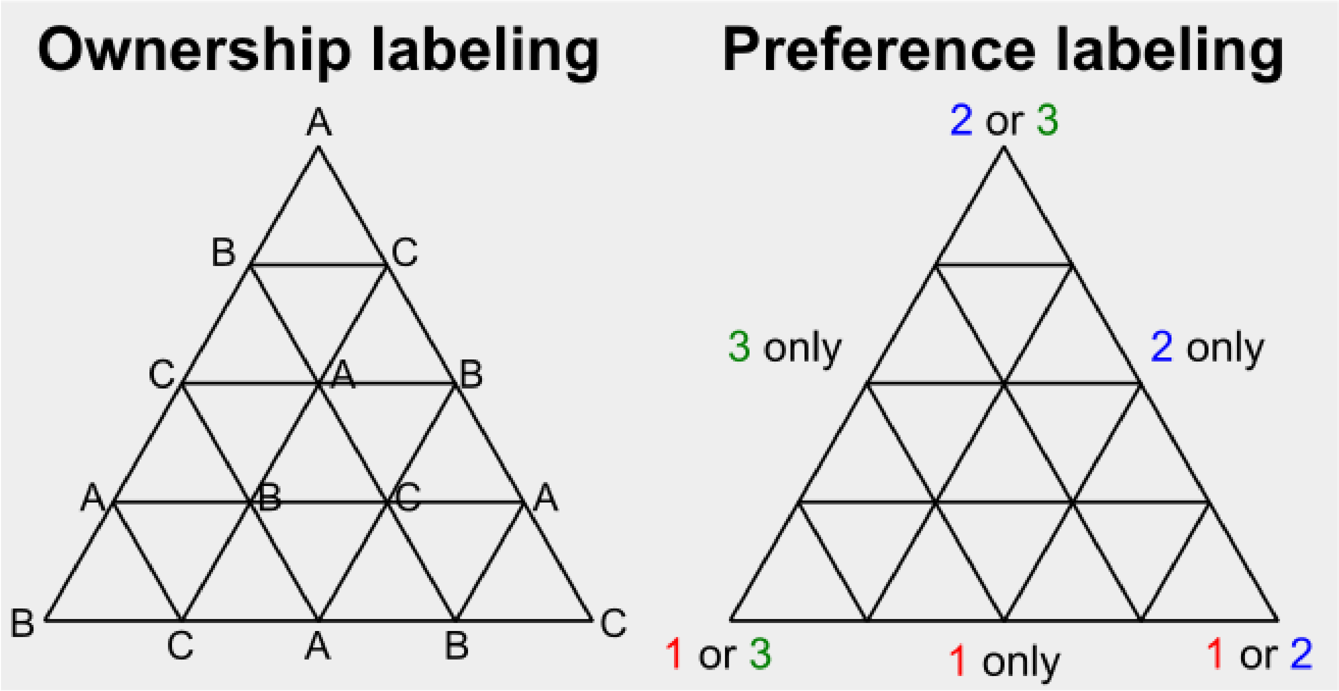
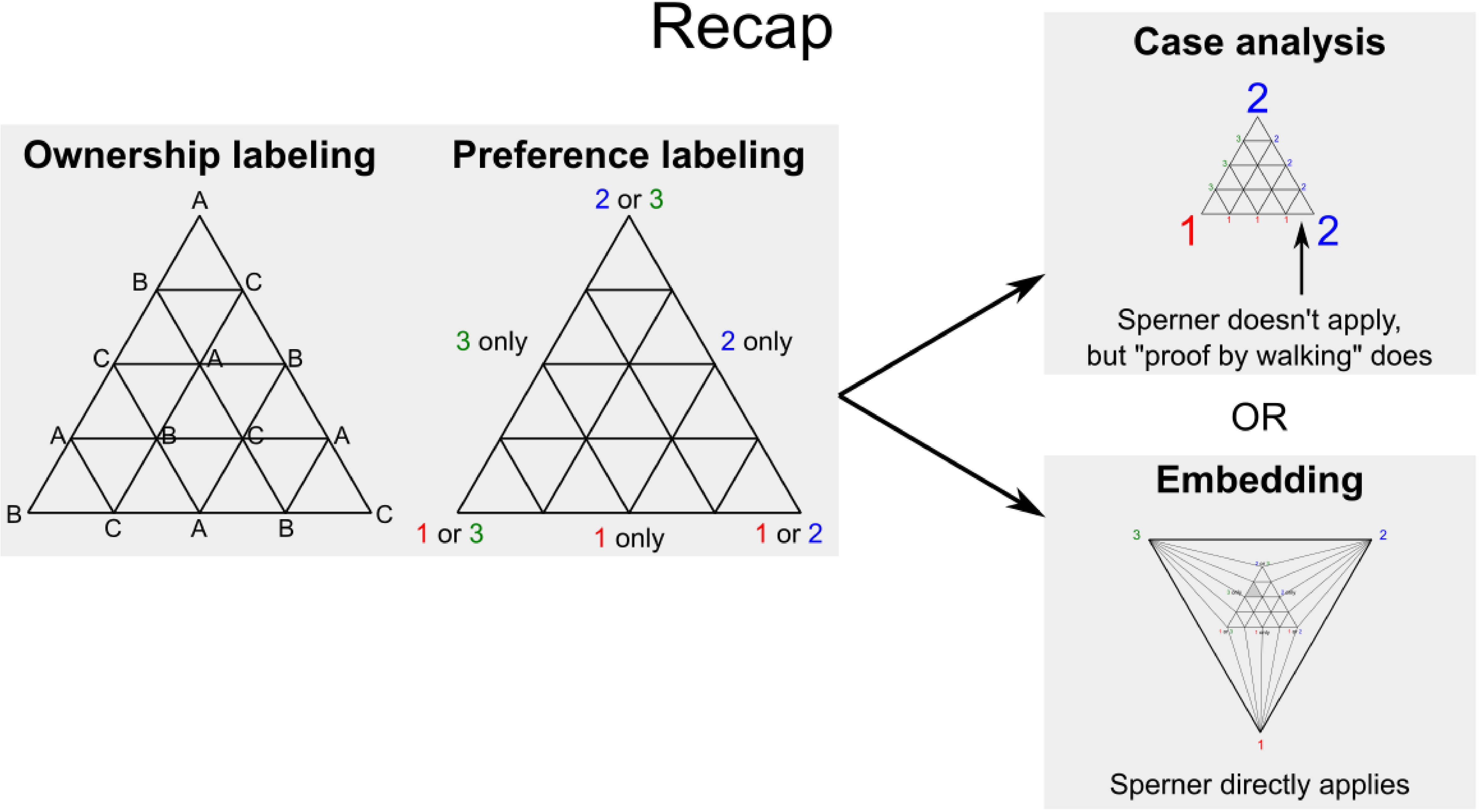
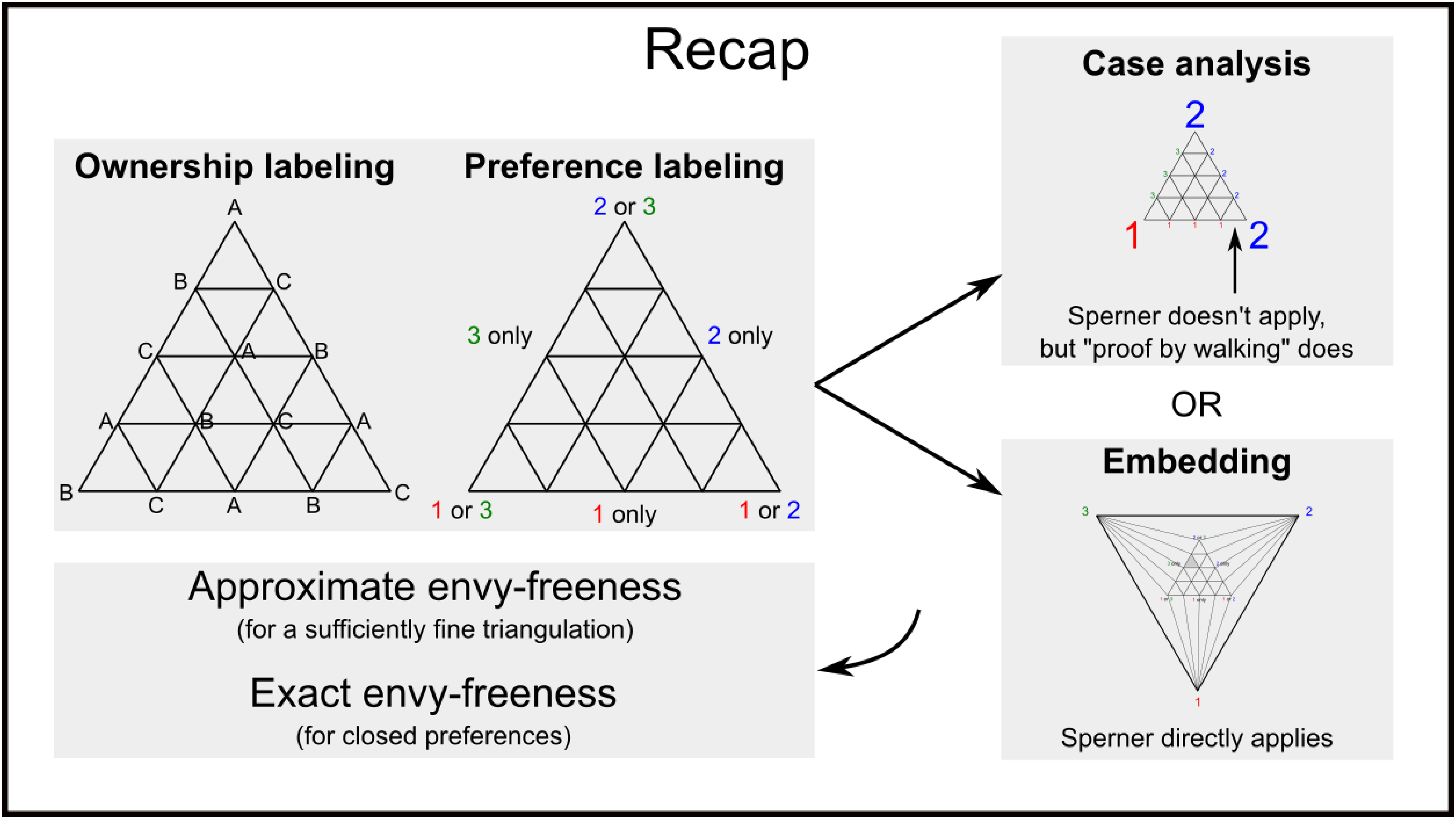
Recap

Recap


Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2

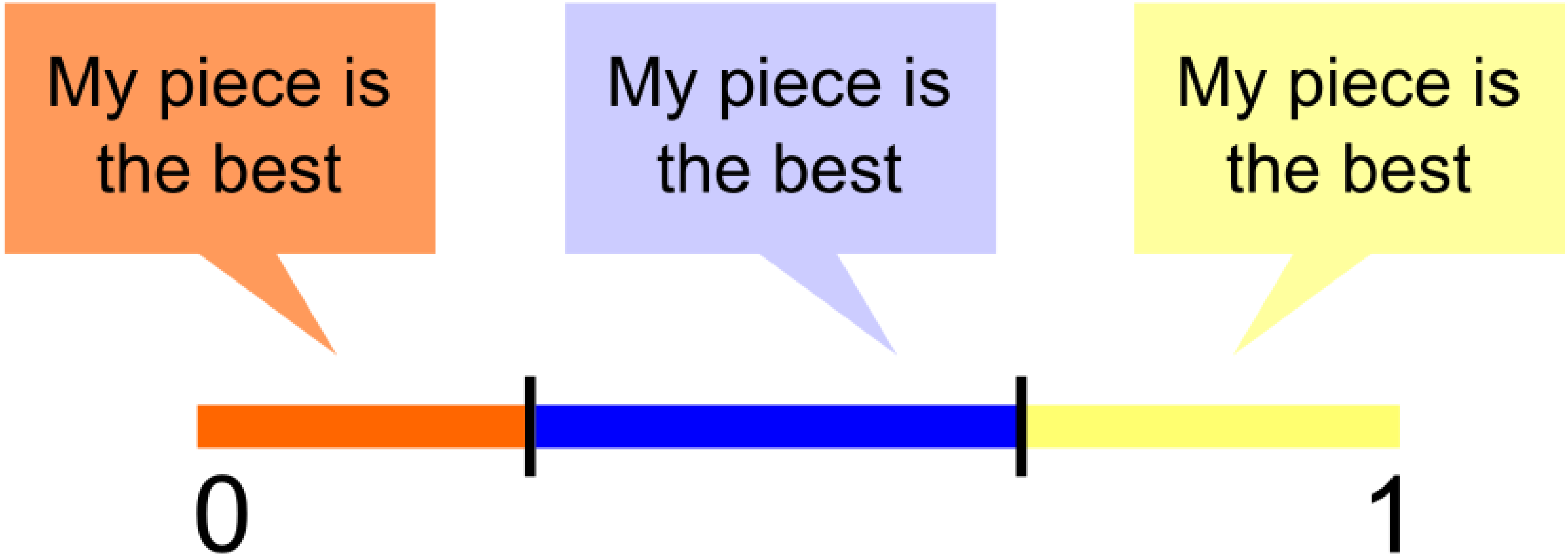
Recap


Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
Case analysis
2
1
2
Sperner doesn't apply,
but "proof by walking" does
OR
Embedding
3
2
1
Sperner directly applies


Recap
Ownership labeling
A
B
C
C
A
B
A
B
C
A
B
C
A
B
C
Preference labeling
2 or 3
3 only
2 only
1 or 3
1 only
1 or 2
Case analysis
2
1
2
Sperner doesn't apply,
but "proof by walking" does
OR
Embedding
3
2
1
Sperner directly applies
Approximate envy-freeness
(for a sufficiently fine triangulation)
Exact envy-freeness
(for closed preferences)

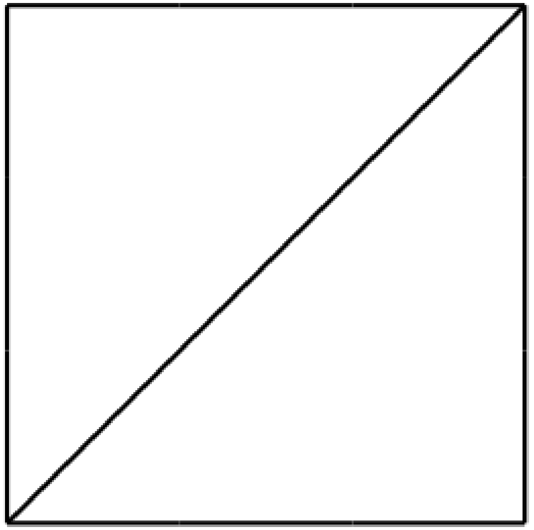
Other Applications of Sperner's Lemma

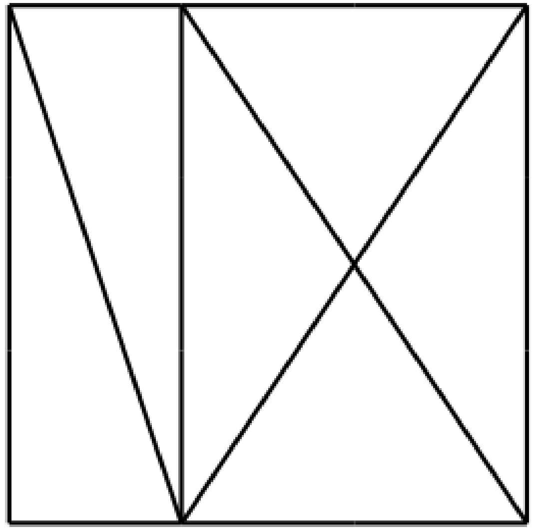
Envy-free cake cutting with connected pieces



Monsky's theorem

A square cannot be dissected into an odd number of triangles with equal area.





Brouwer's fixed point theorem

Every continuous function from a compact convex set to itself has a fixed point.

Brouwer's fixed point theorem

$f(x) = x$

Every continuous function from a compact convex set to itself has a fixed point.

Brouwer's fixed point theorem

$f(x) = x$

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Brouwer's fixed point theorem

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Brouwer's fixed point theorem

$f(x) = x$

Every continuous function from a compact convex set to itself has a fixed point.

- Used to prove the existence of Nash equilibrium in game theory

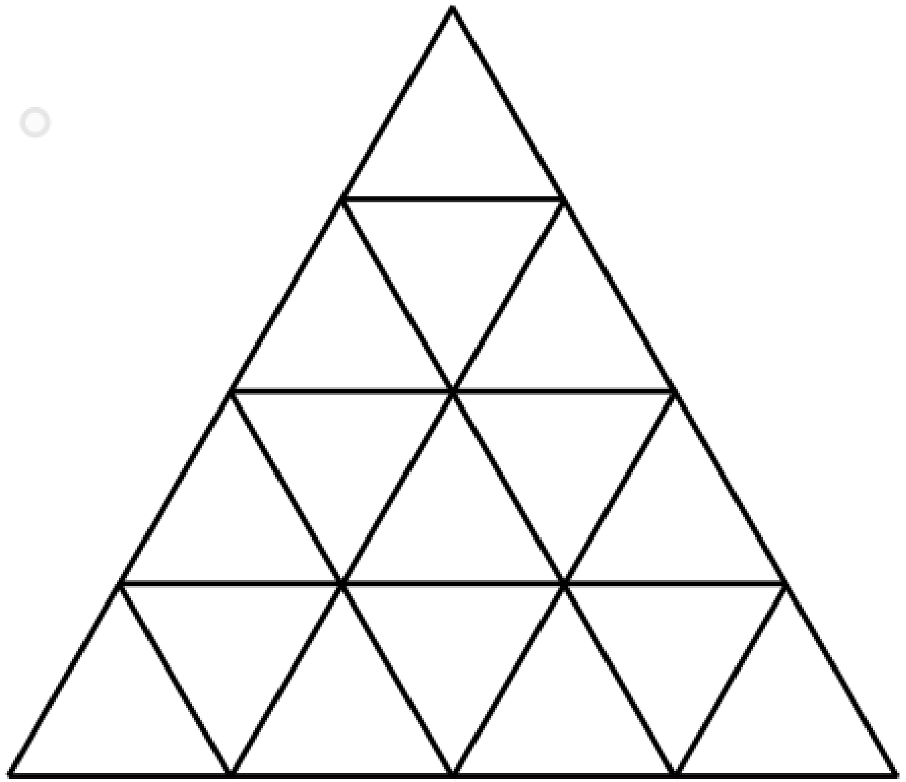
Next Time

Fairness via Randomness

Quiz

Quiz

Demonstrate a Sperner coloring that maximizes the number of fully-labeled elementary triangles.



References

- Fair cake cutting and rent division via Sperner's Lemma

  Francis Su
  "Rental Harmony: Sperner's Lemma in Fair Division"
  American Mathematical Monthly, 106, 1999 pg 930-942

- Fun videos on the topic:

  Mathologer: https://www.youtube.com/watch?v=7s-YM-kcKME
  MoMath: https://www.youtube.com/watch?v=CBVg8x4LWZE
  PBS Infinite Series: https://www.youtube.com/watch?v=48oBEvpdYSE