COL866: Special Topics in Algorithms

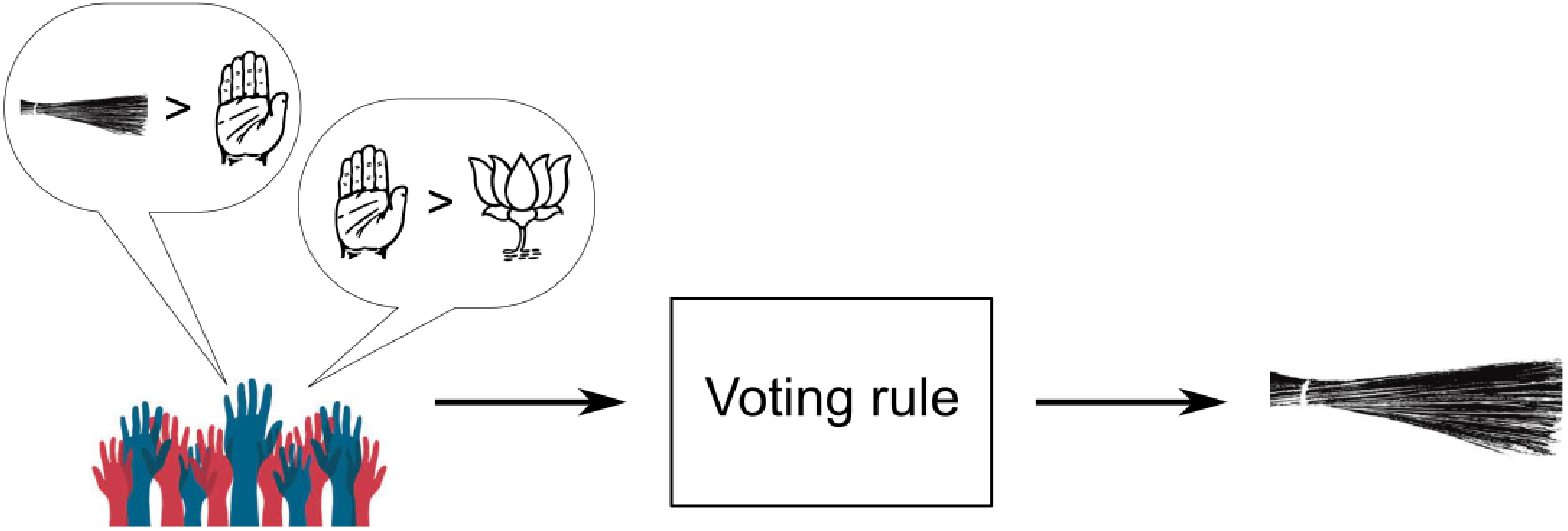
Lecture 1

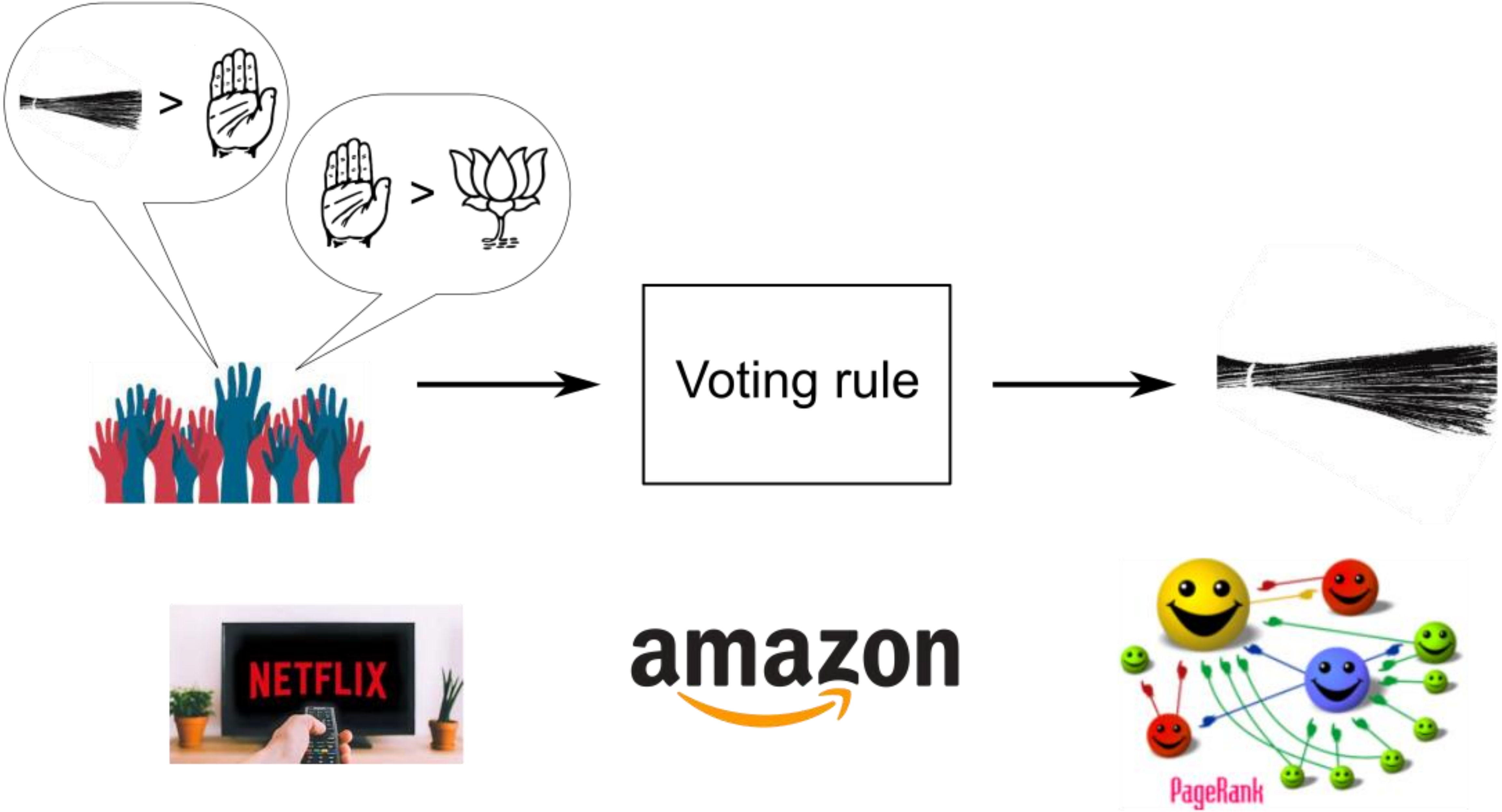
Introduction to The Course and Stable Matching Problem

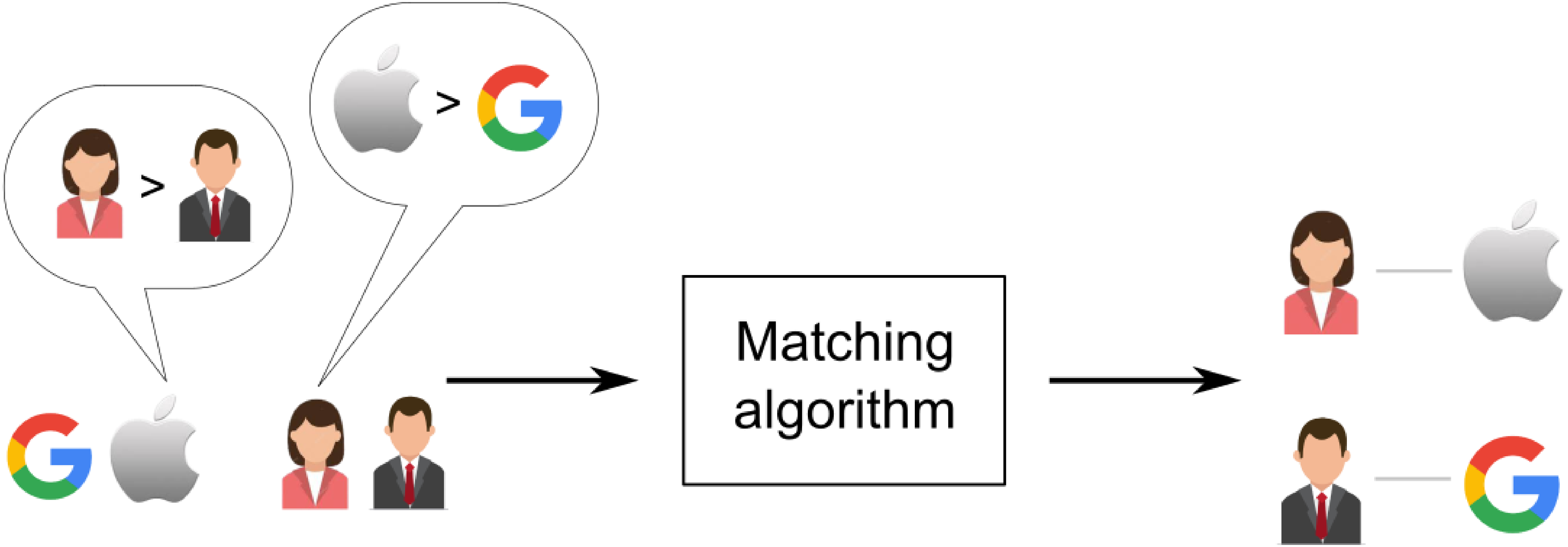
July 25, 2023 | Rohit Vaish

What this course is about

Understanding the role of **computation** in **collective decision-making** problems


Voting rule


Voting rule
NETFLIX
amazon
PageRank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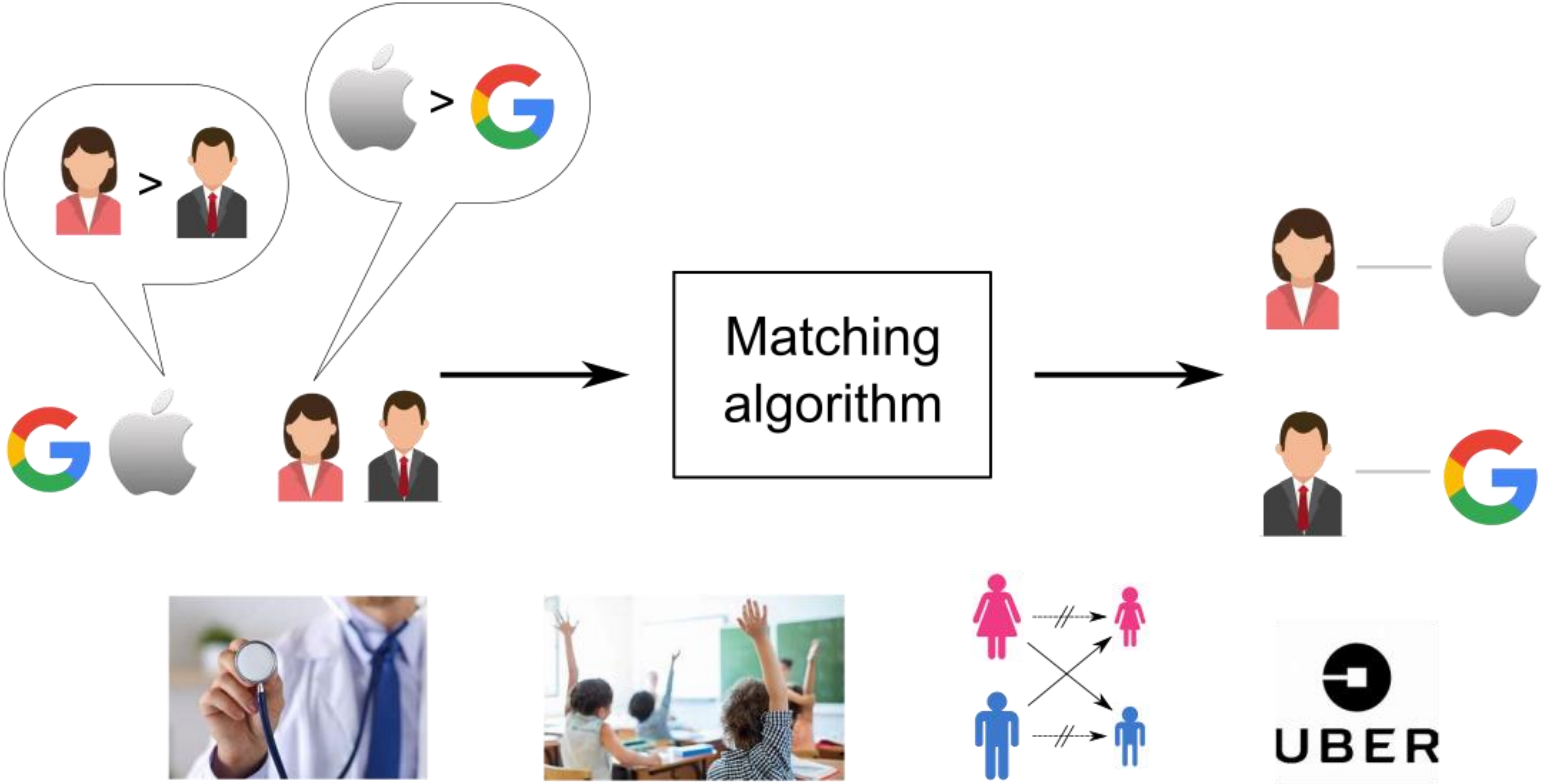
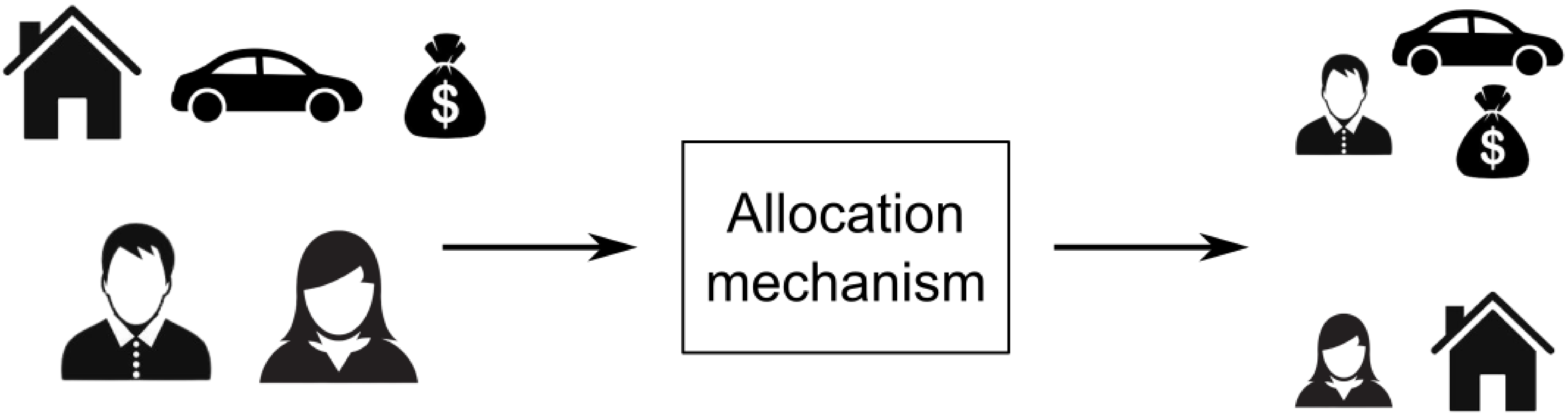
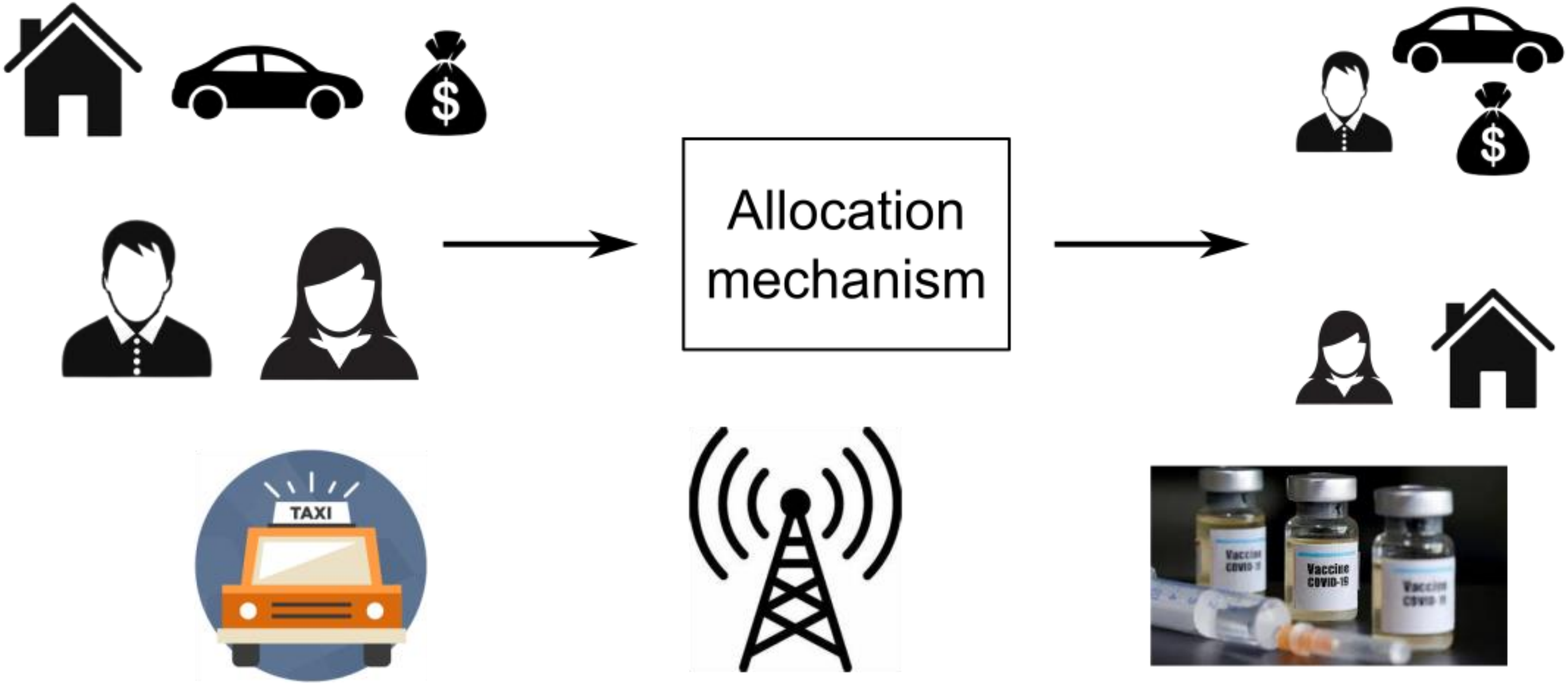
Matching algorithm


Matching
algorithm
UBER


Allocation
mechanism

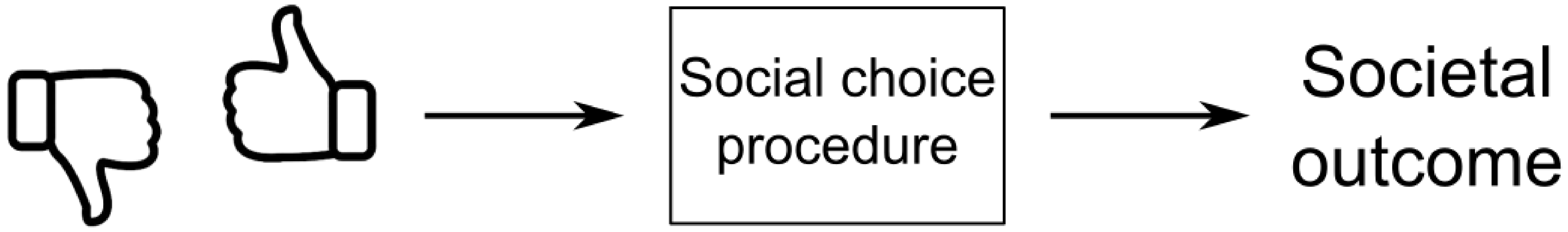

Allocation
mechanism
TAXI

Social Choice

Making a collective decision from individual preferences

Social Choice

Making a collective decision from individual preferences



Social Choice

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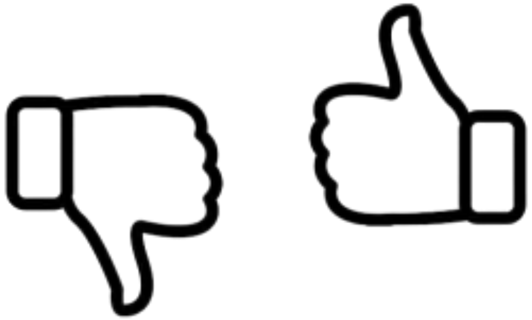
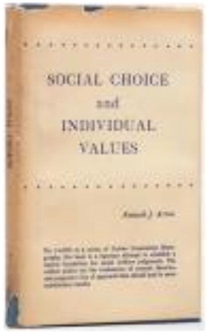
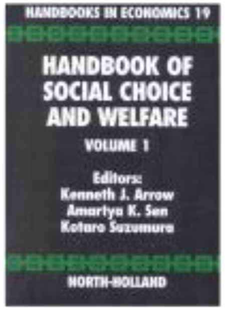
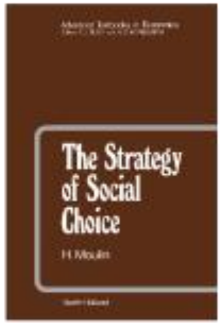


→ Social choice procedure → Societal outcome








Arrow


Sen


Maskin


Roth


Shapley

Classical Social Choice

Does there exist a social choice procedure with the desired economic properties?

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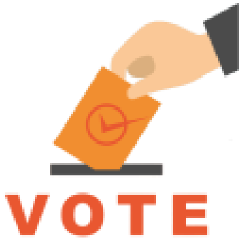

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Is there a matching procedure that is "stable"?

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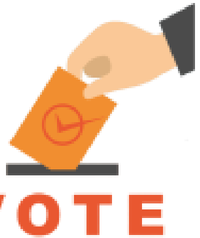

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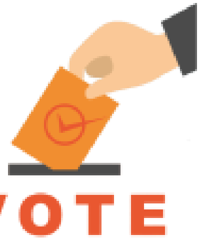

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Classical Social Choice

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with the desired economic properties?

Computational Social Choice
(This course)

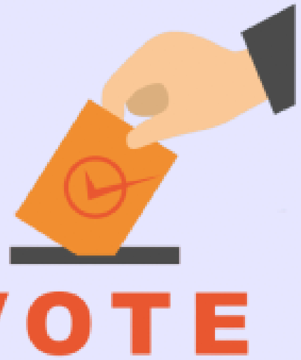
How does computation influence
the economic properties of social choice procedures?


VOTE

LEC 1-6

LEC 1-6

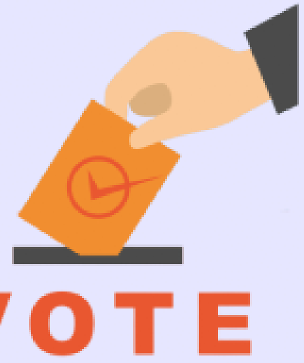
LEC 7-15



LEC 16-21

LEC 1-6

LEC 7-15

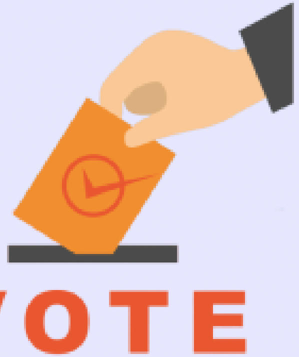


LEC 16-21

LEC 1-6

LEC 7-15

LEC 22-25



LEC 16-21

LEC 1-6

LEC 7-15

LEC 22-25

LEC 11 and 26-27: Project Presentations

LEC 16-21

LEC 1-6

LEC 7-15

LEC 22-25

LEC 11 and 26-27: Project Presentations

Stable Matching Problem

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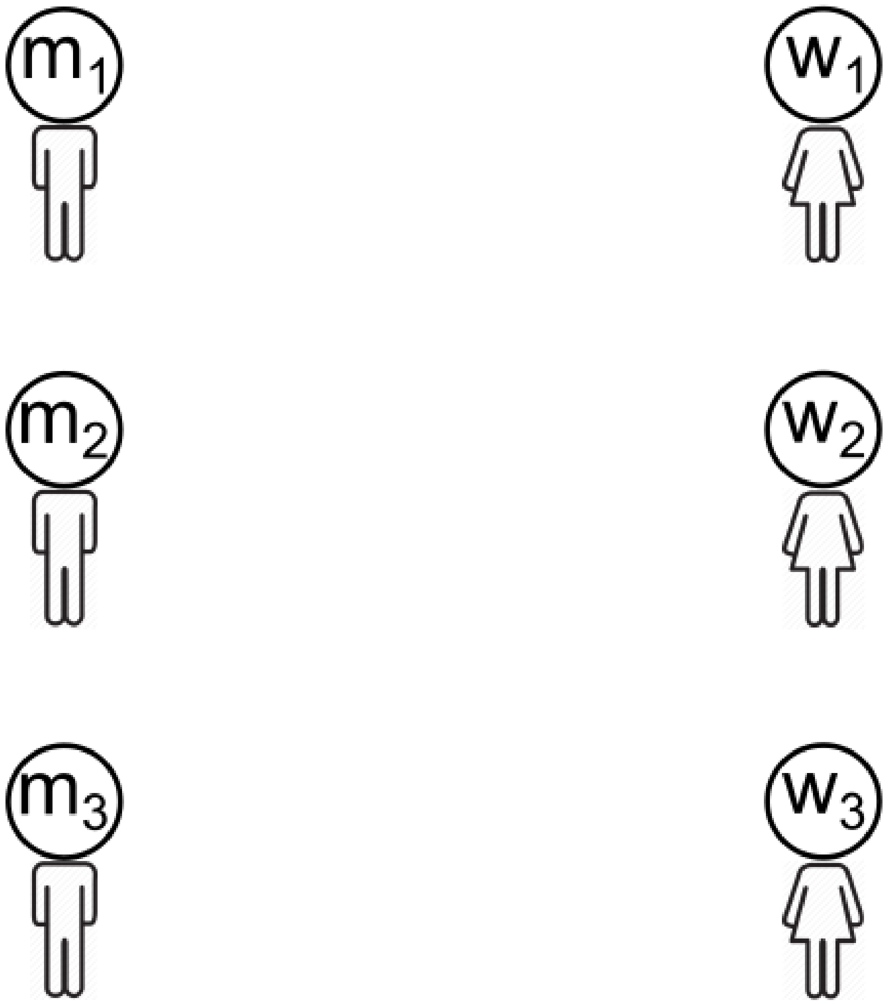
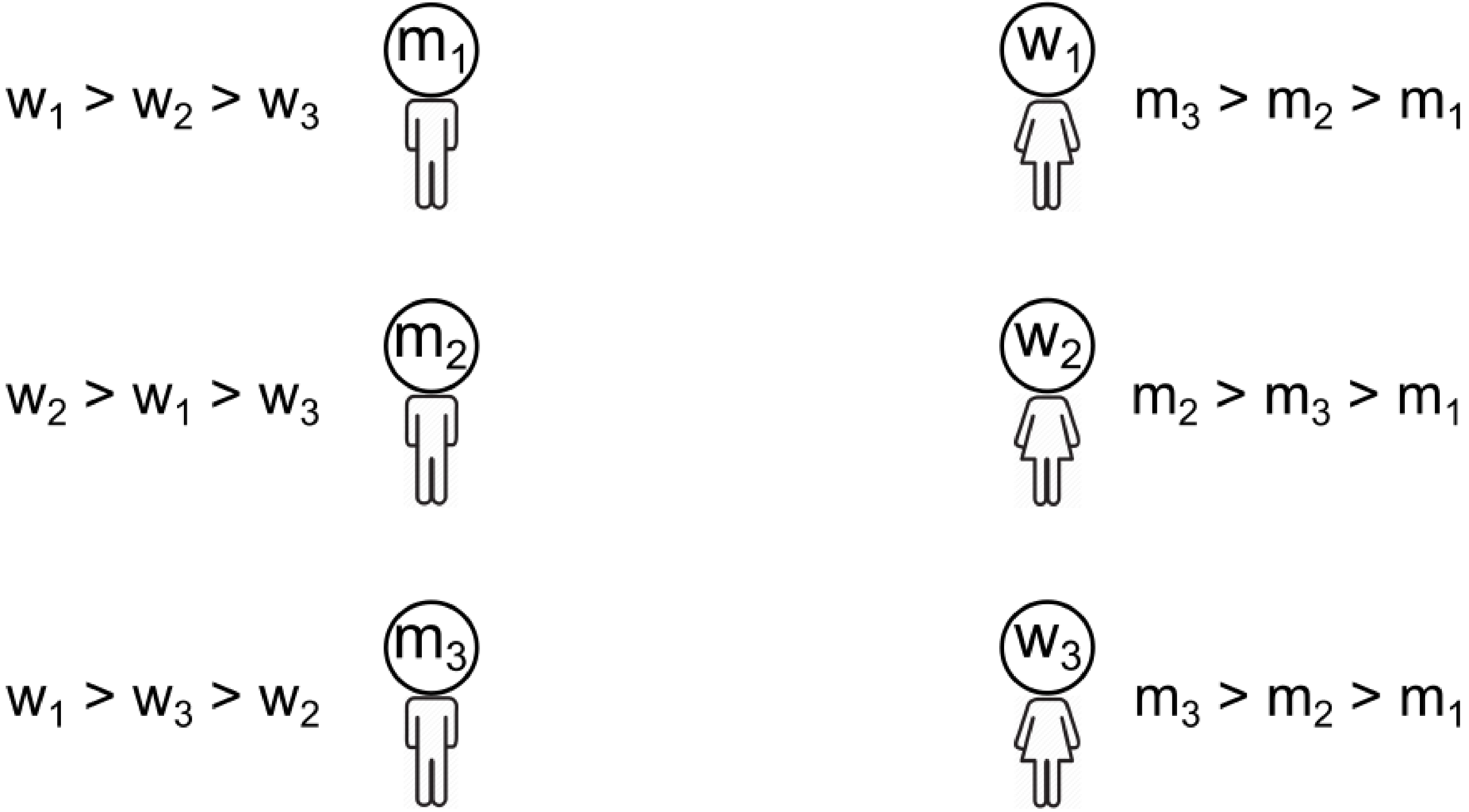
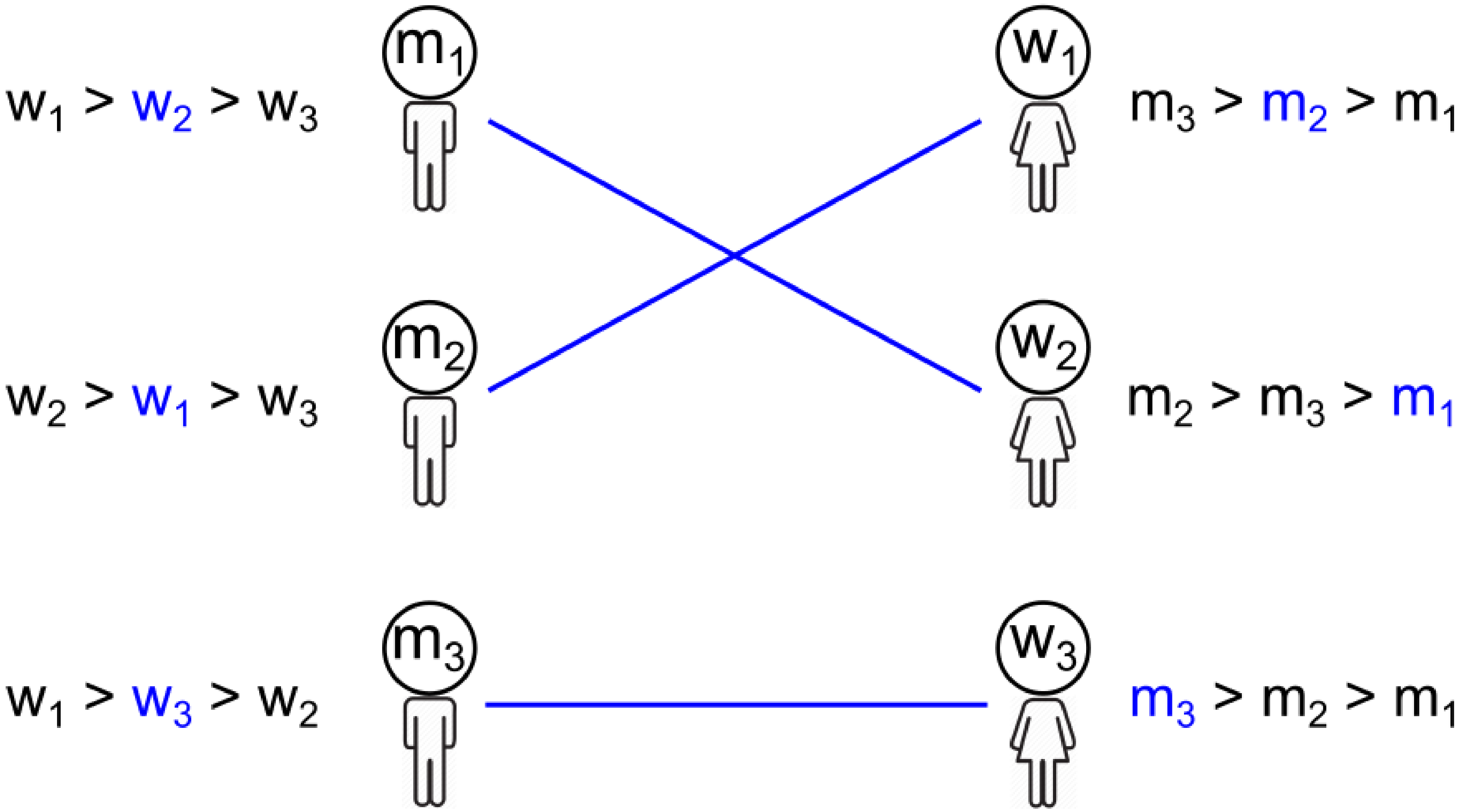


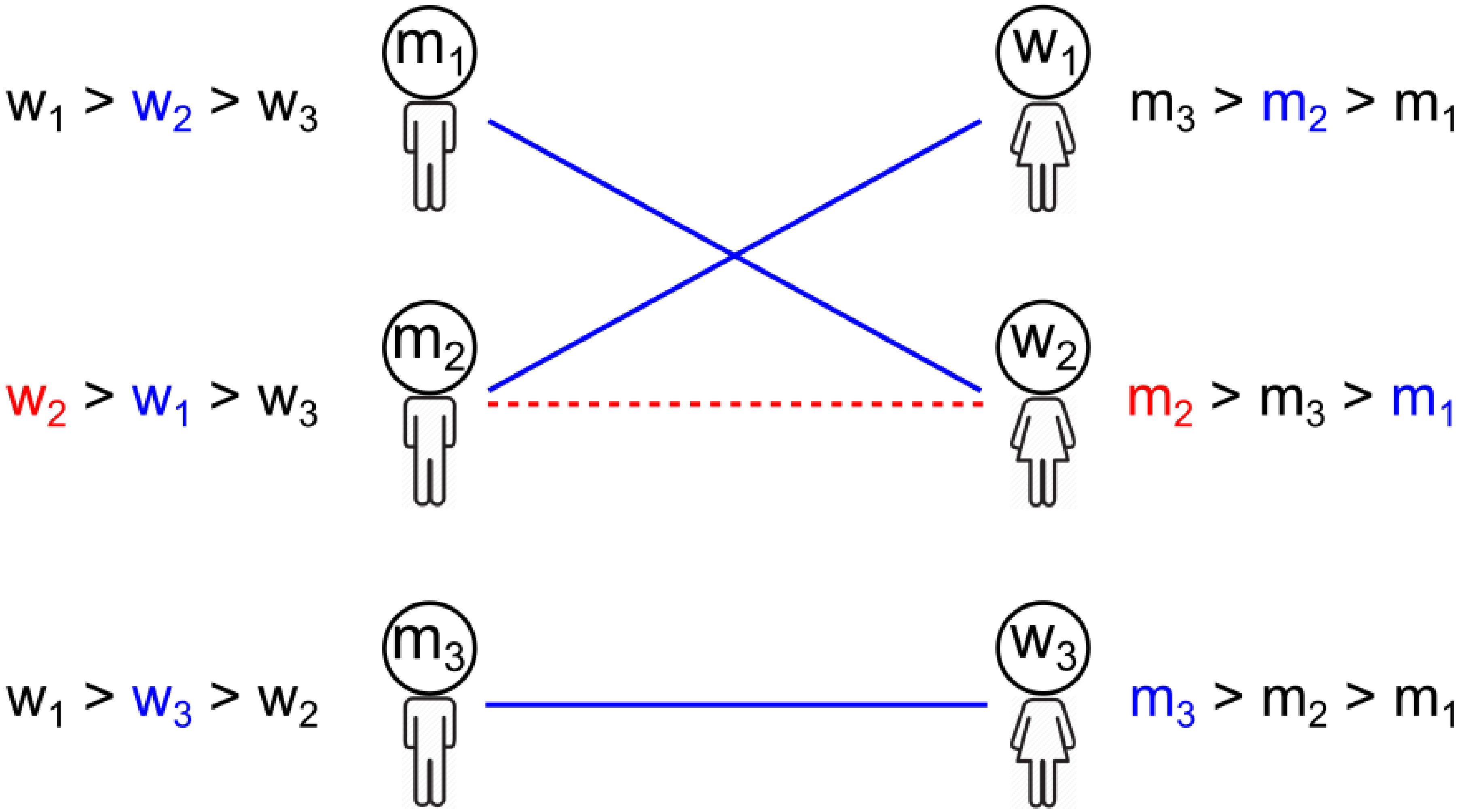
Stable Matching Problem



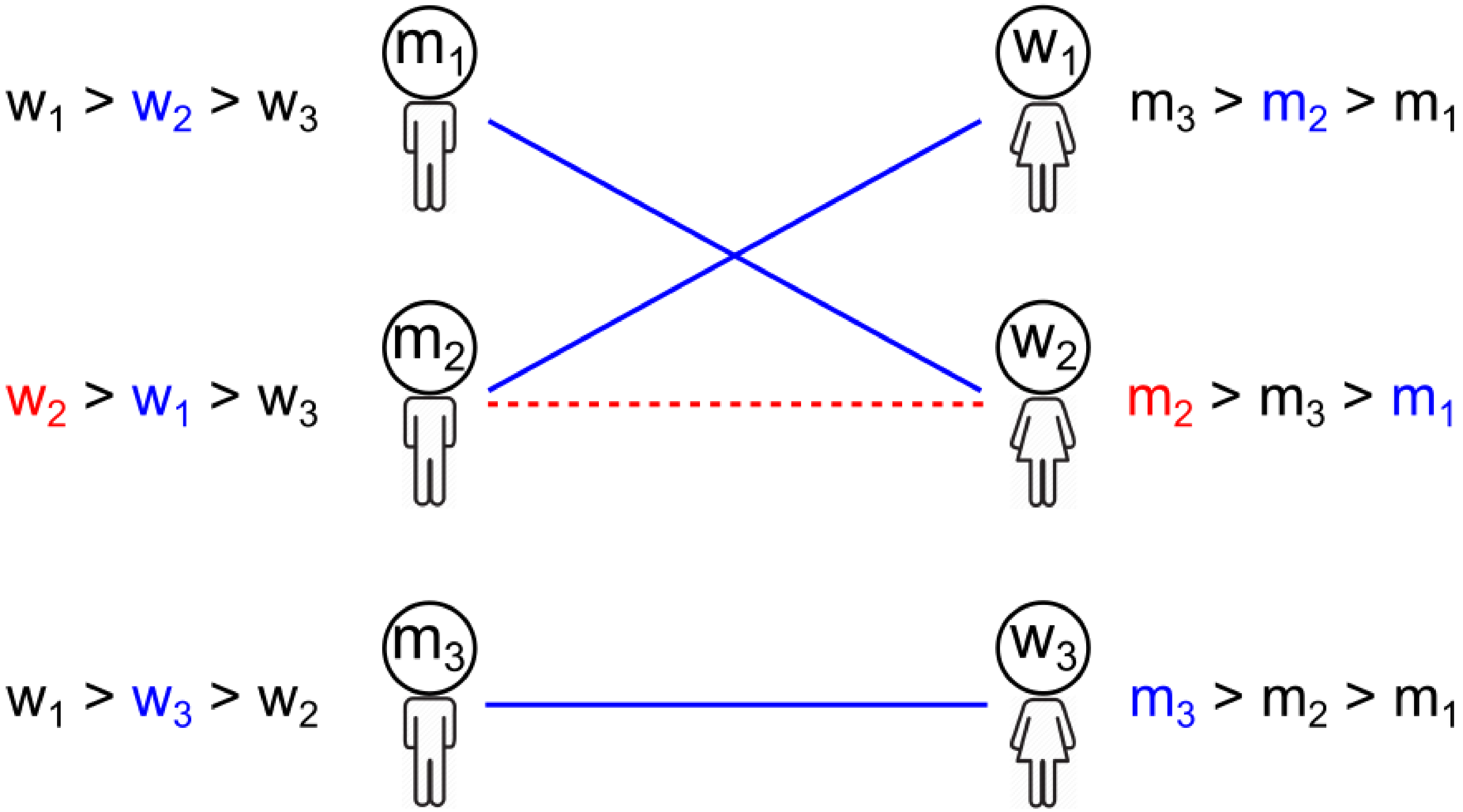
Stable Matching Problem



Stable Matching Problem

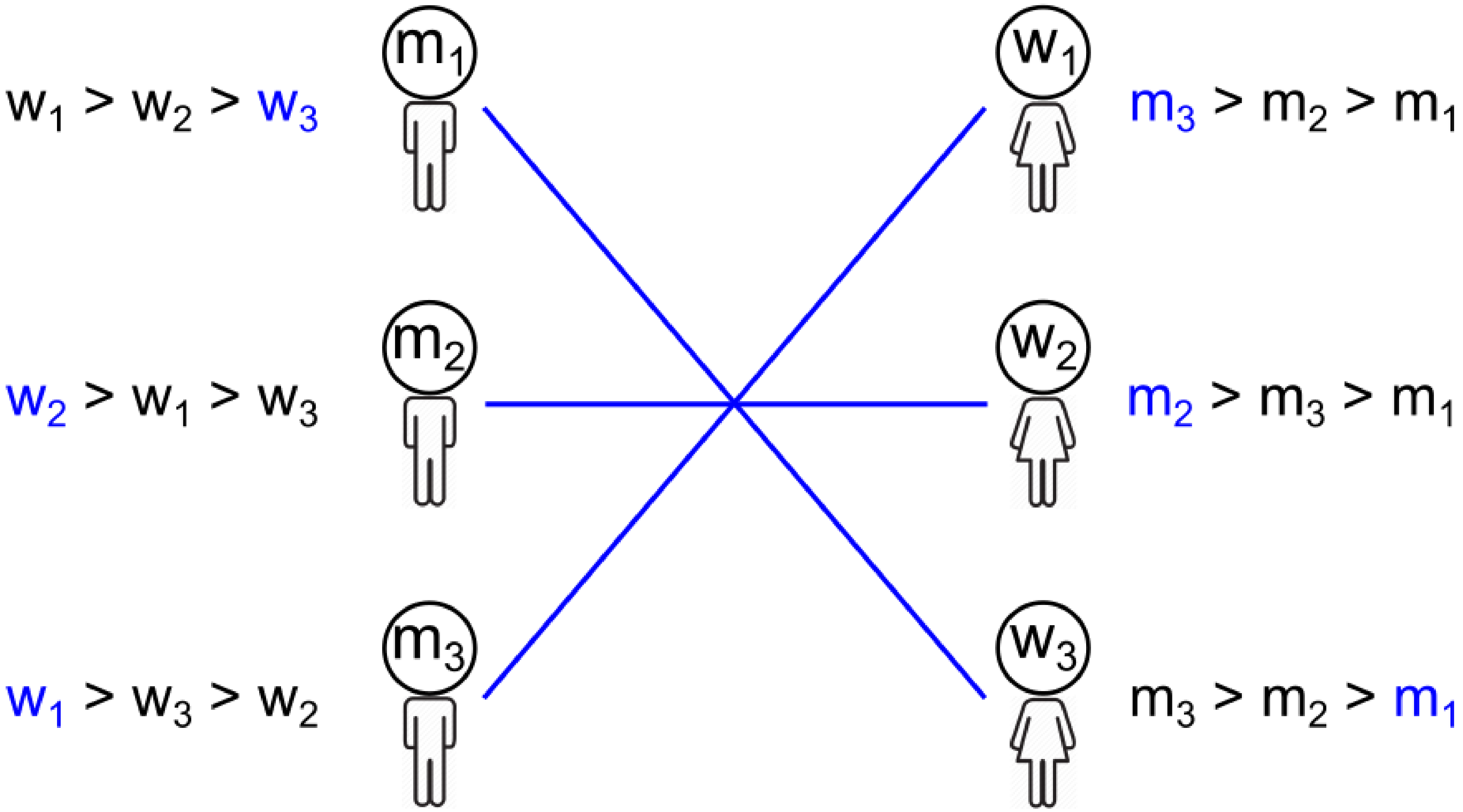


Stable Matching Problem



A matching is stable if there is no blocking pair.

Stable Matching Problem



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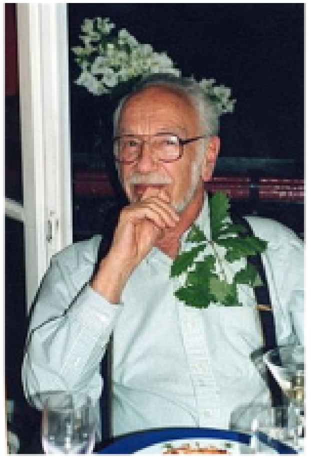

COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

Source: *The American Mathematical Monthly*, Jan., 1962, Vol. 69, No. 1 (Jan., 1962), pp. 9–15

Given any preference profile, a stable matching for that profile always exists and can be computed in polynomial time.

Deferred-Acceptance Algorithm

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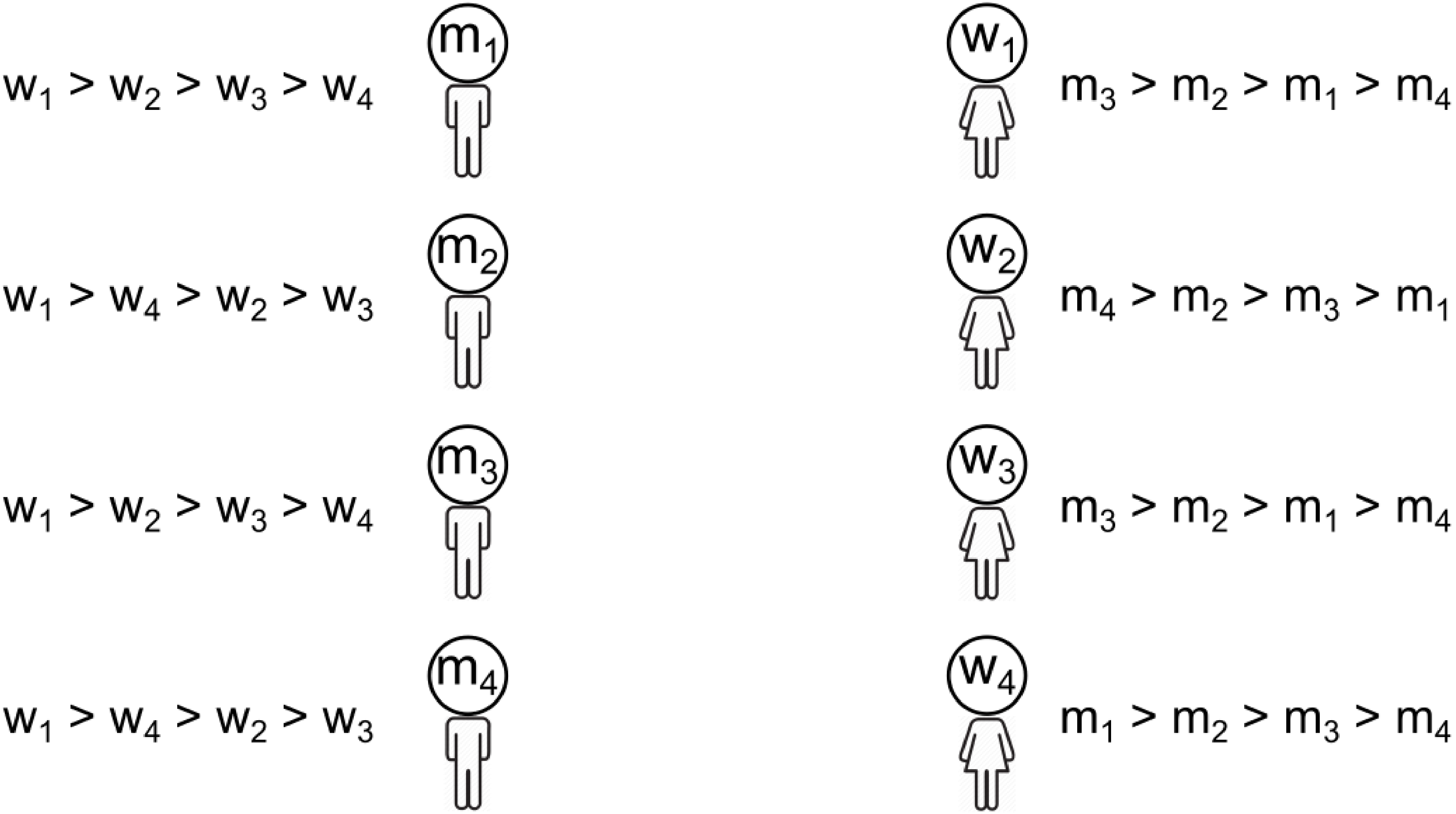
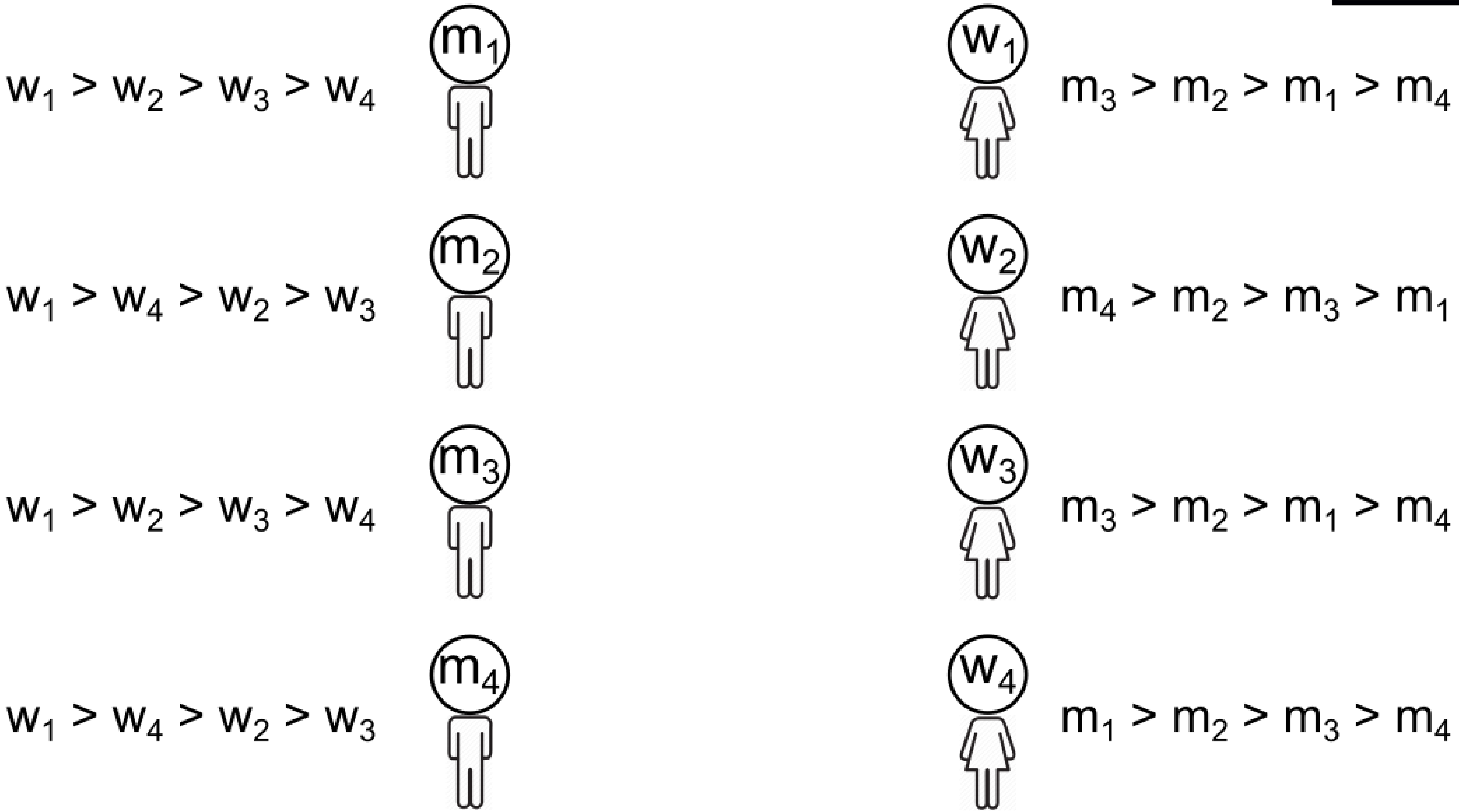


Deferred-Acceptance Algorithm

Round 1



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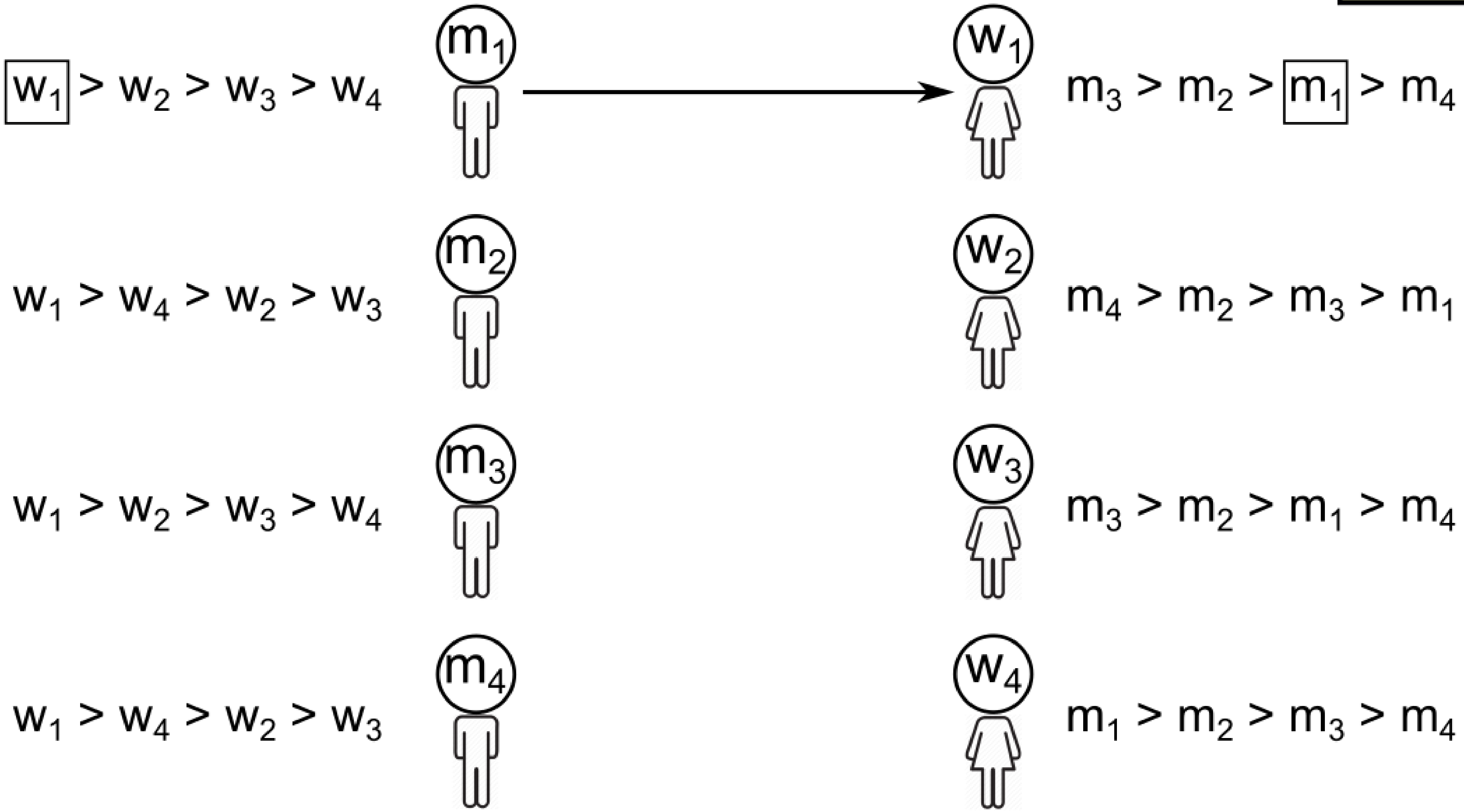

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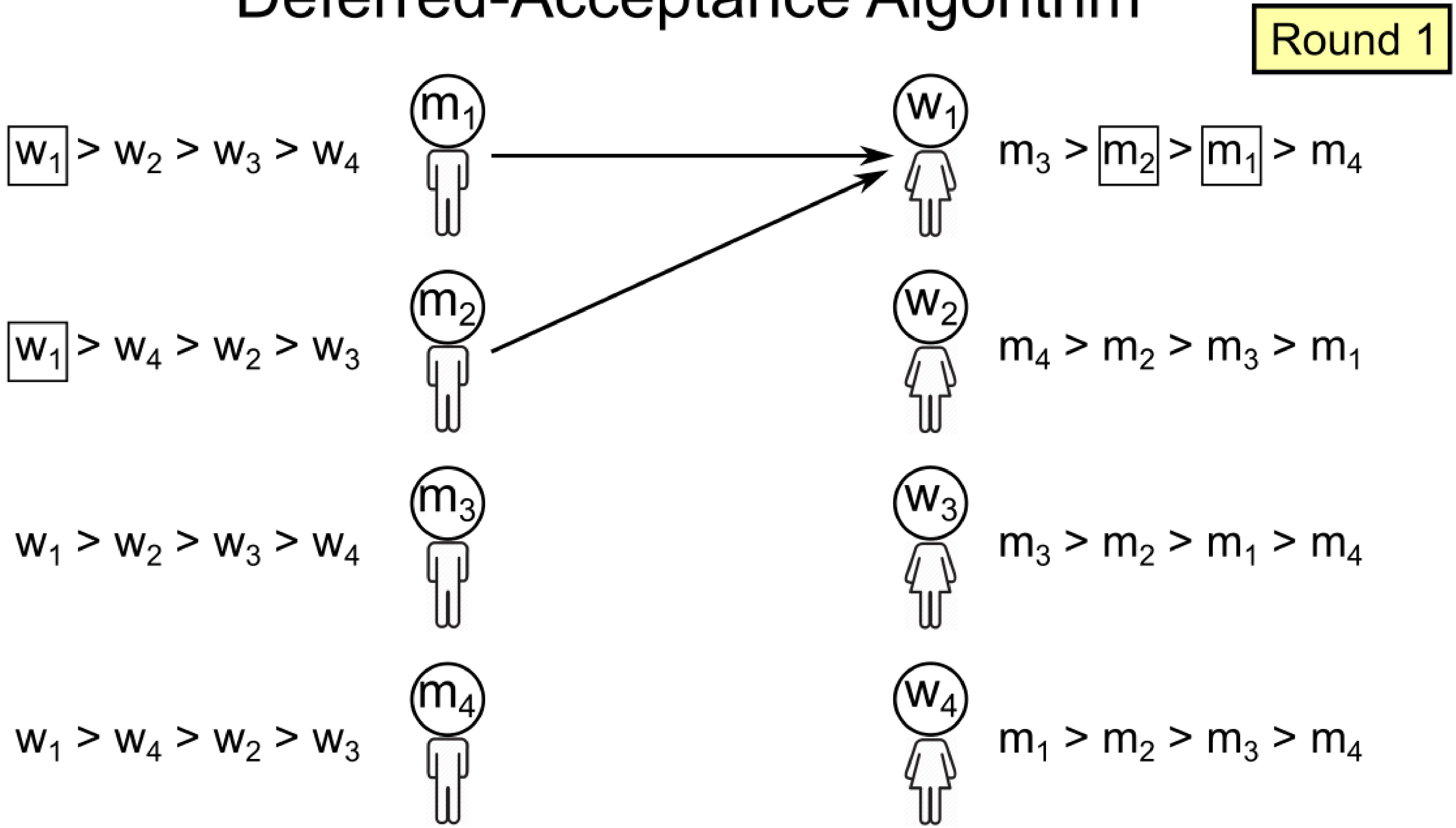

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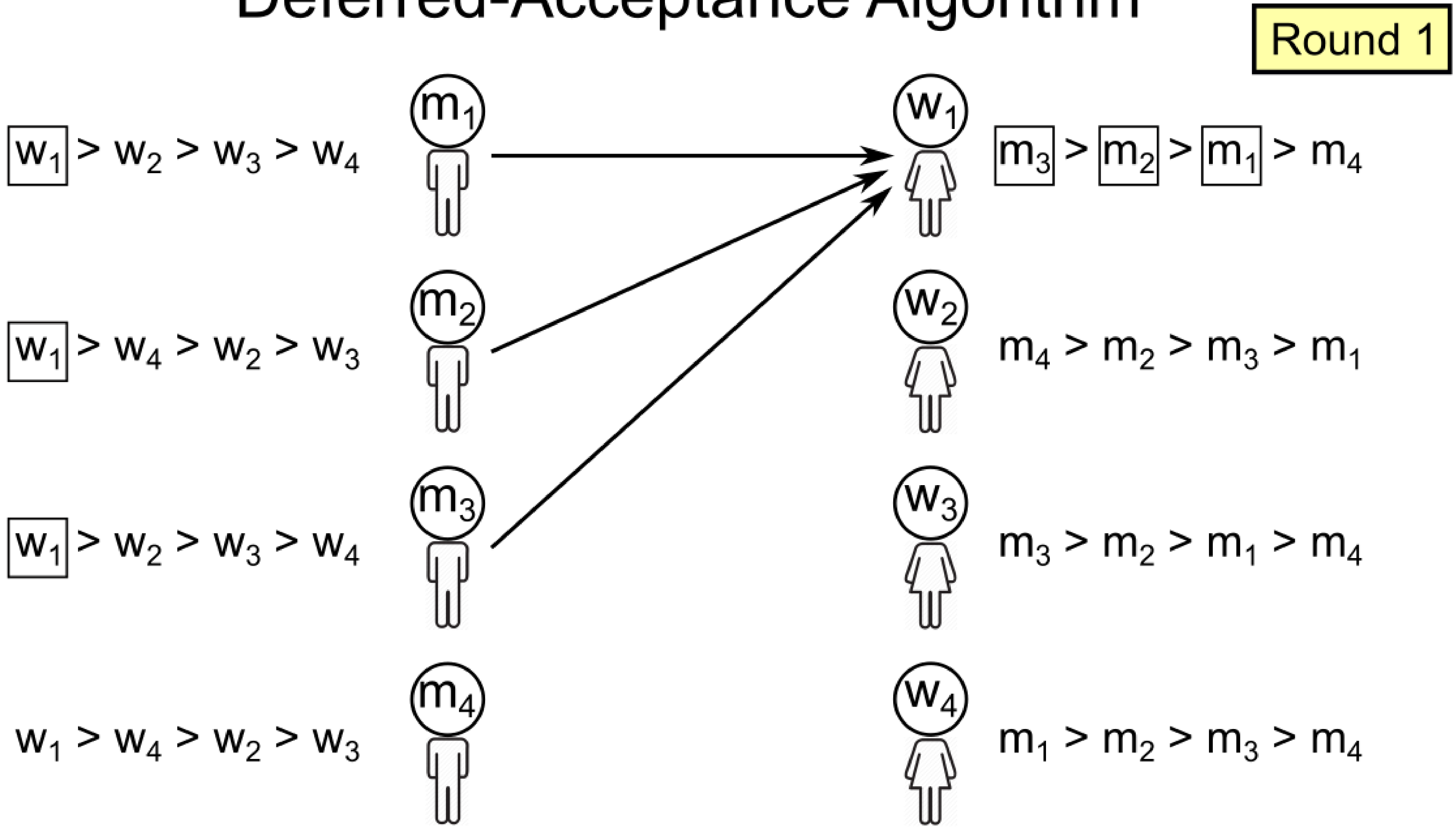

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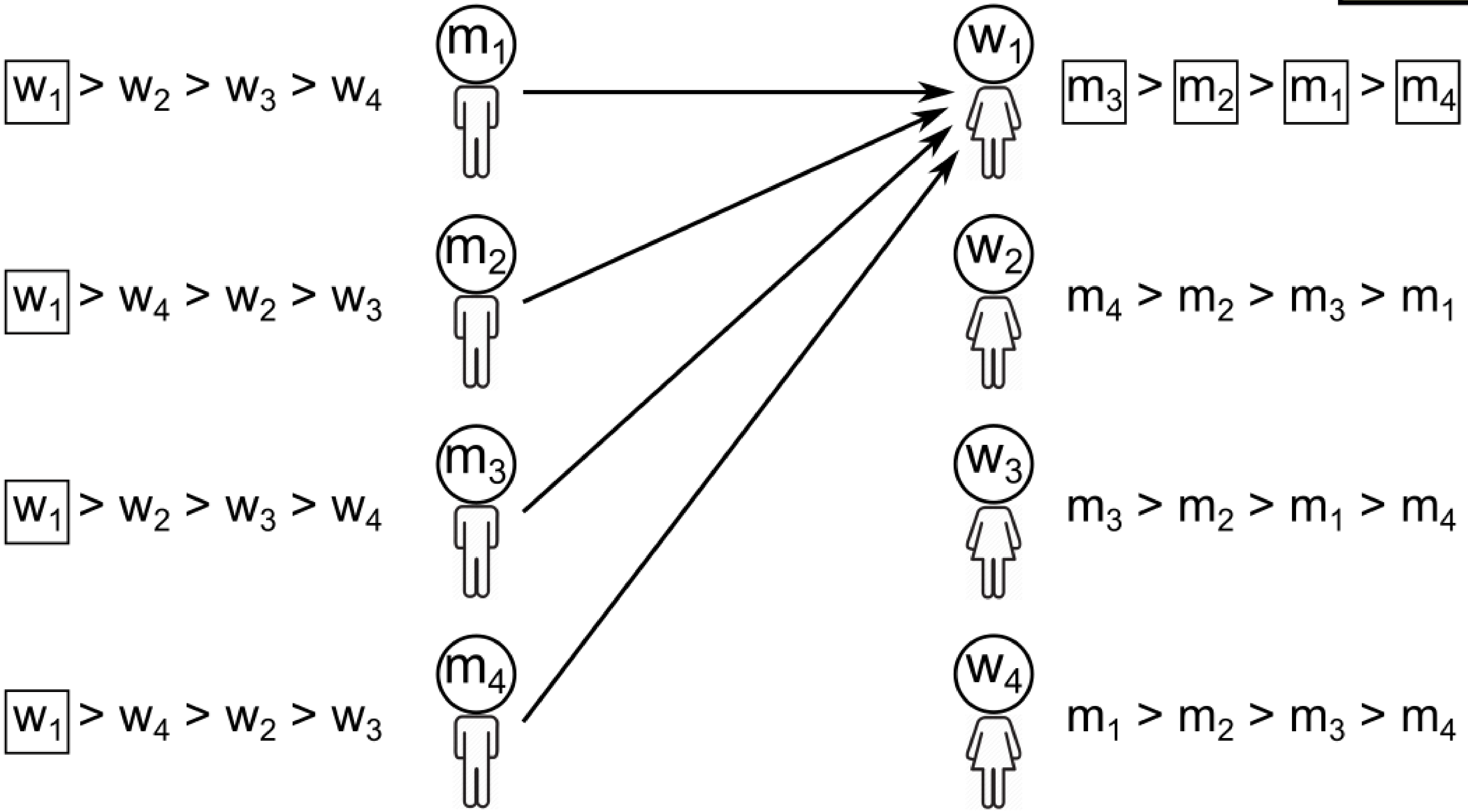

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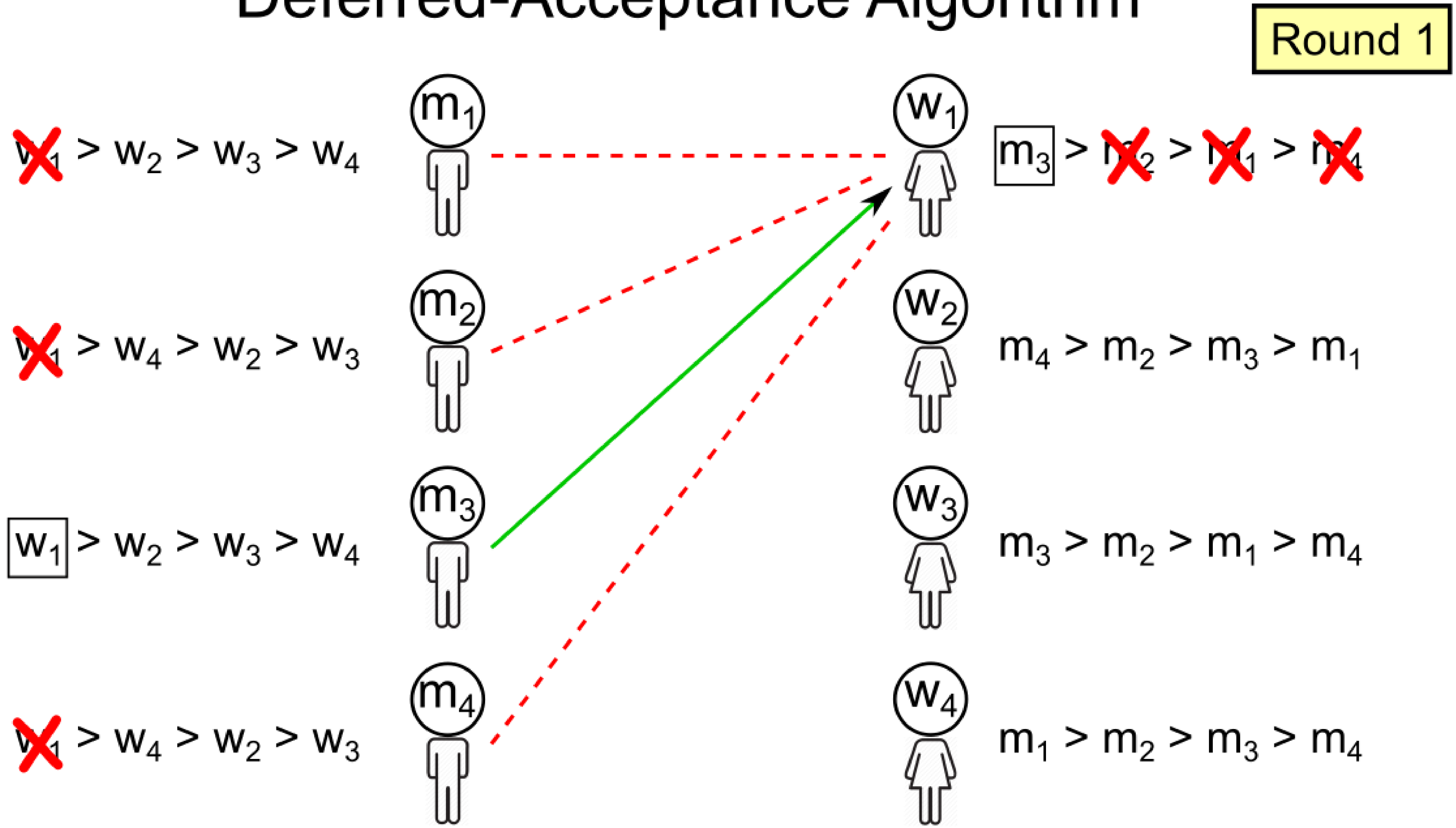

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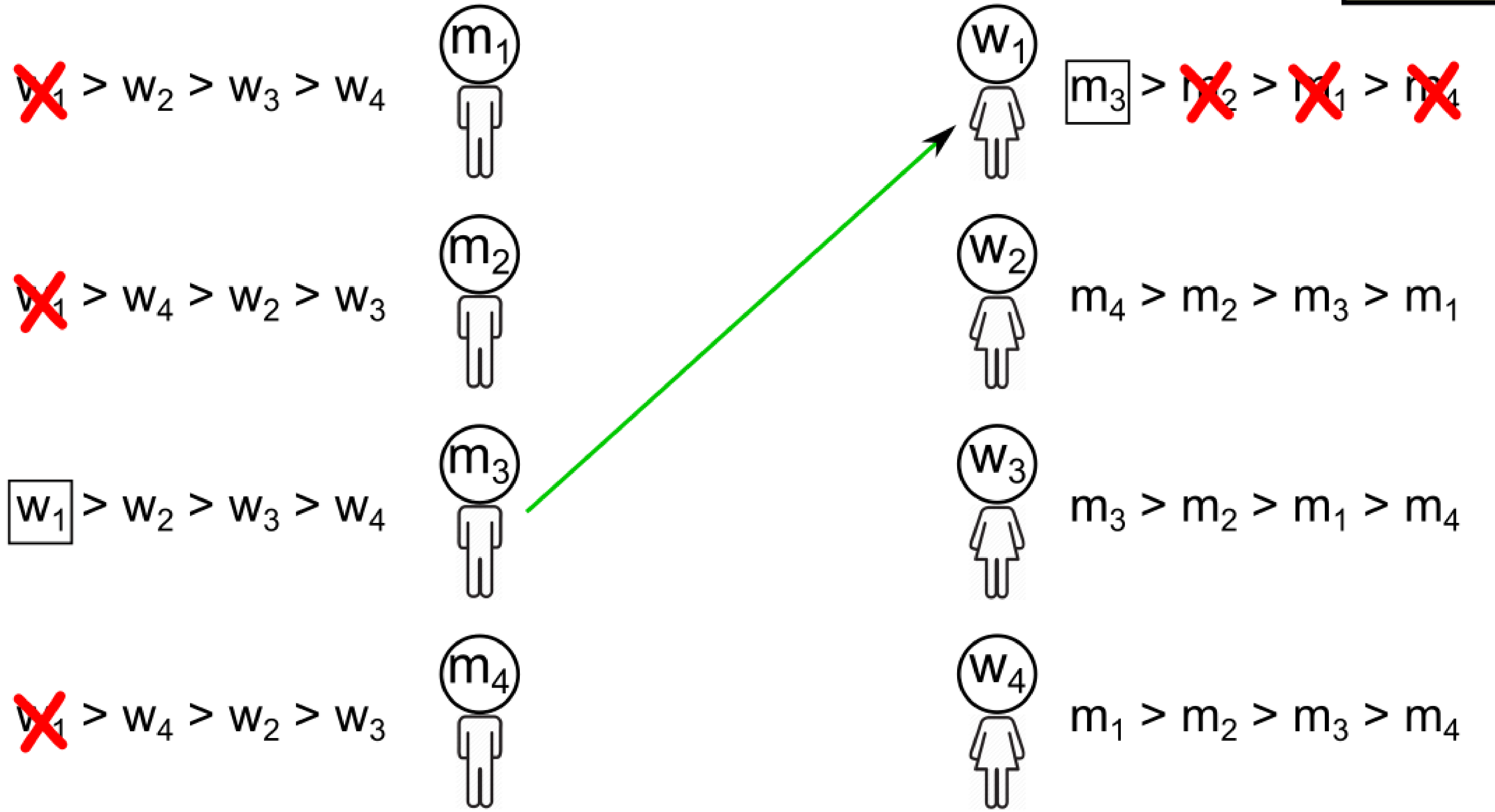

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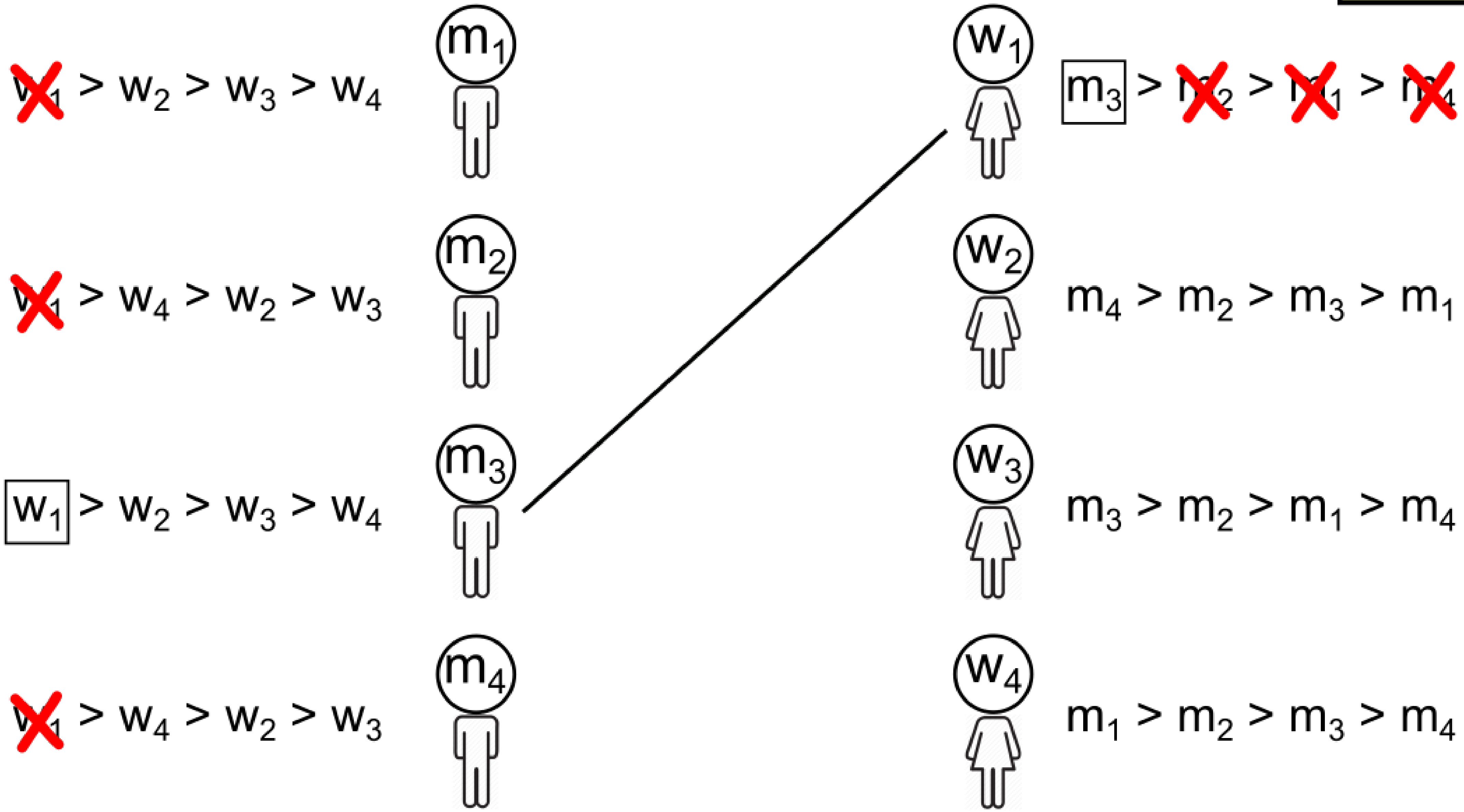

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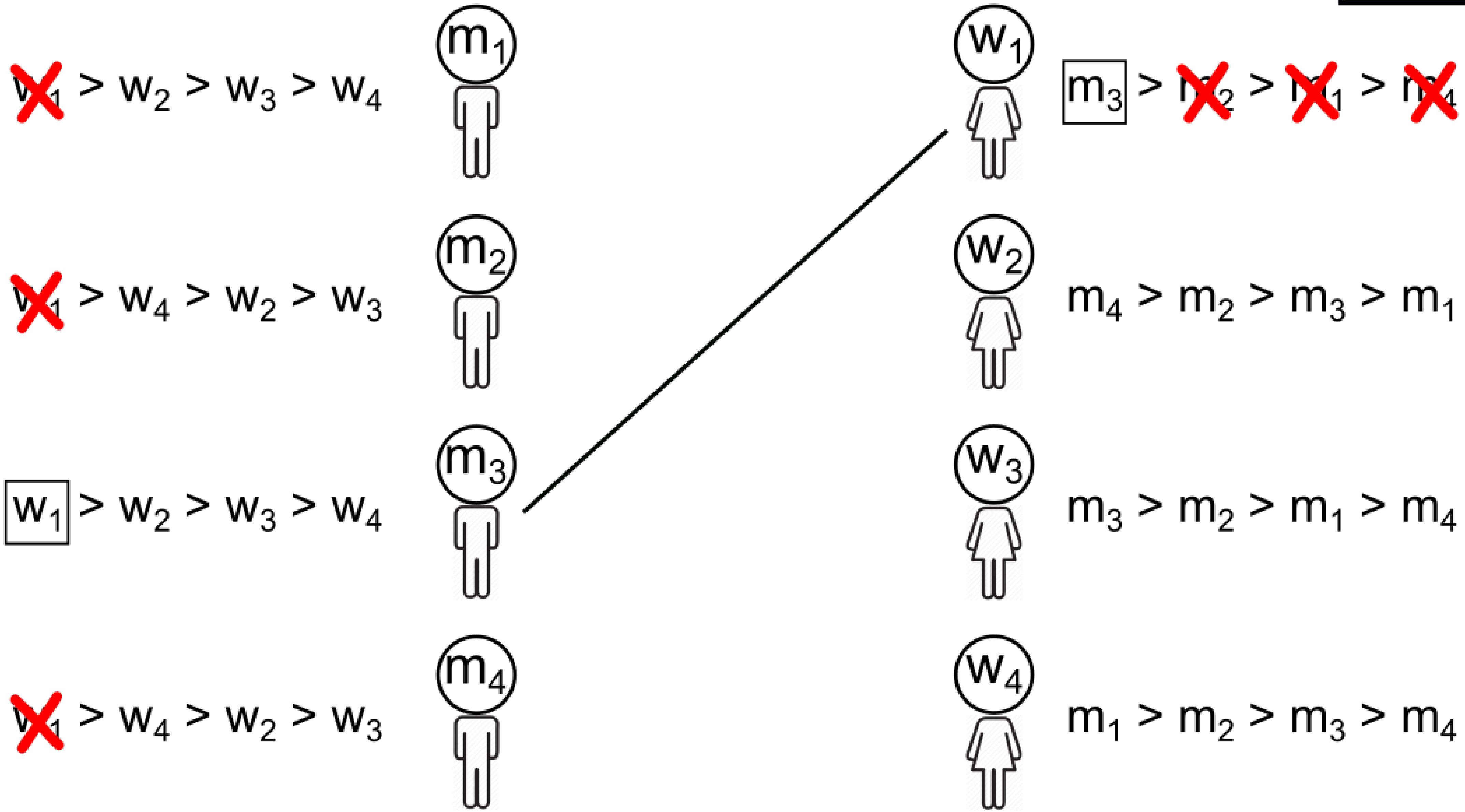
Round 2



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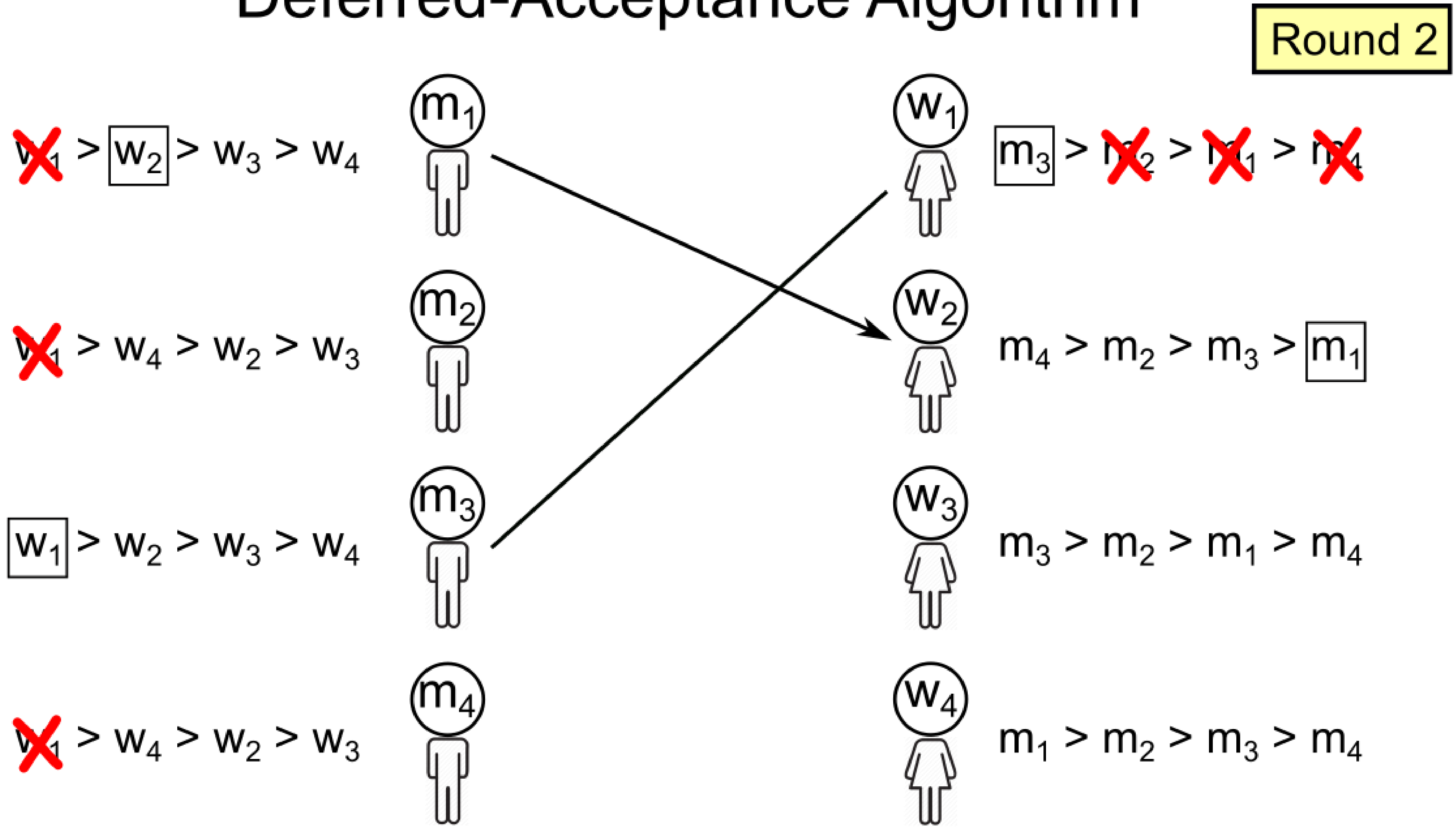

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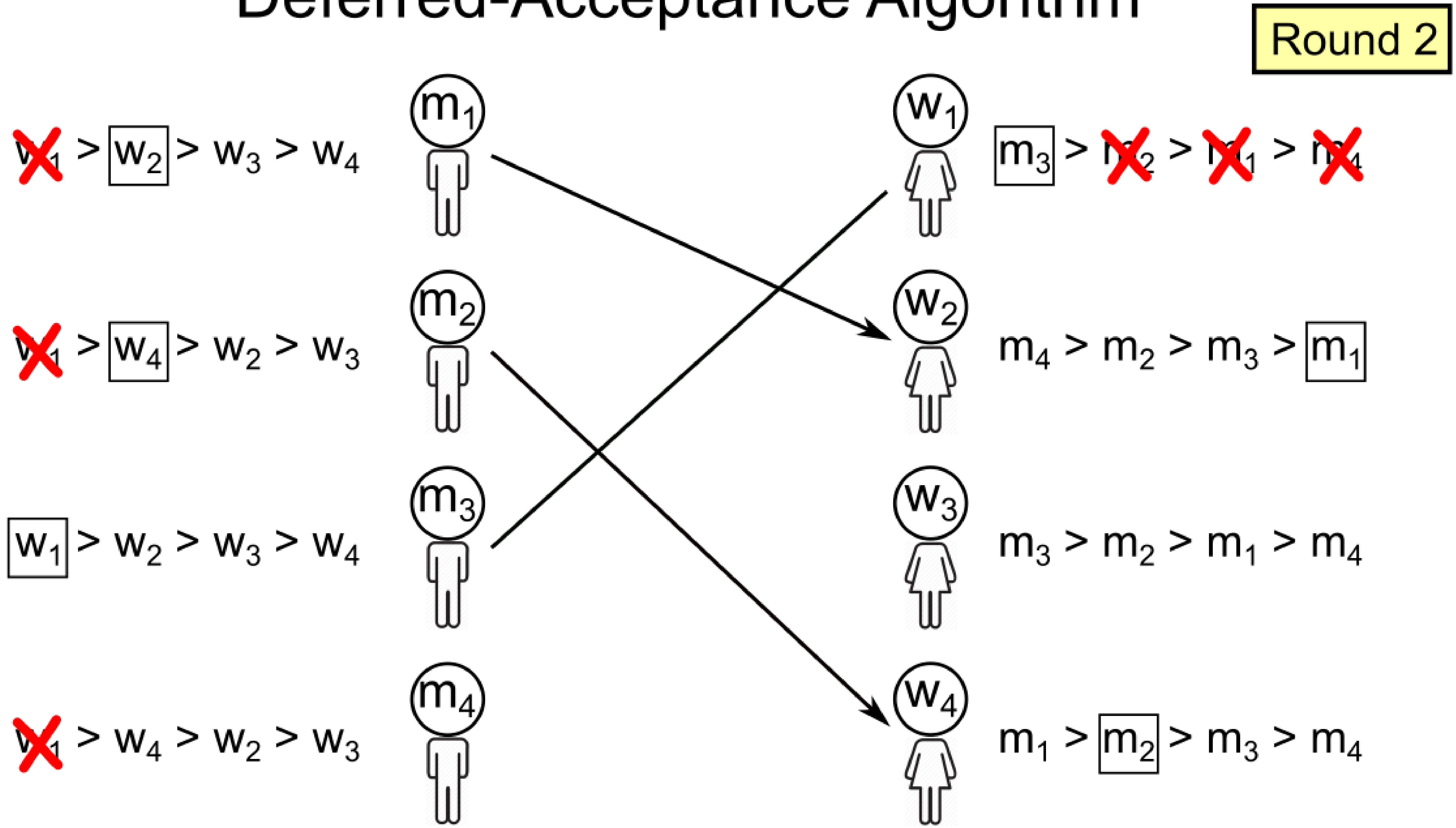

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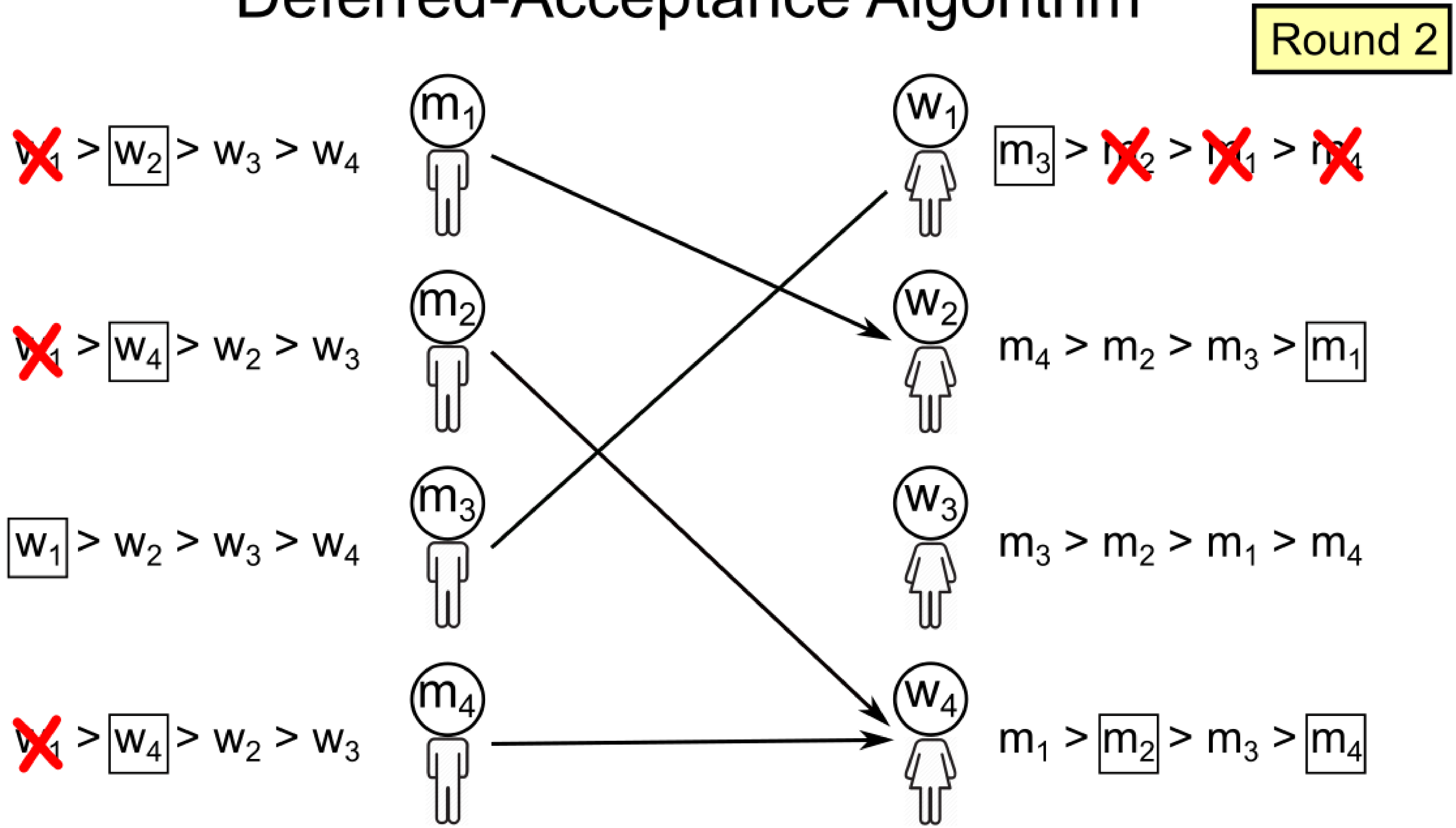

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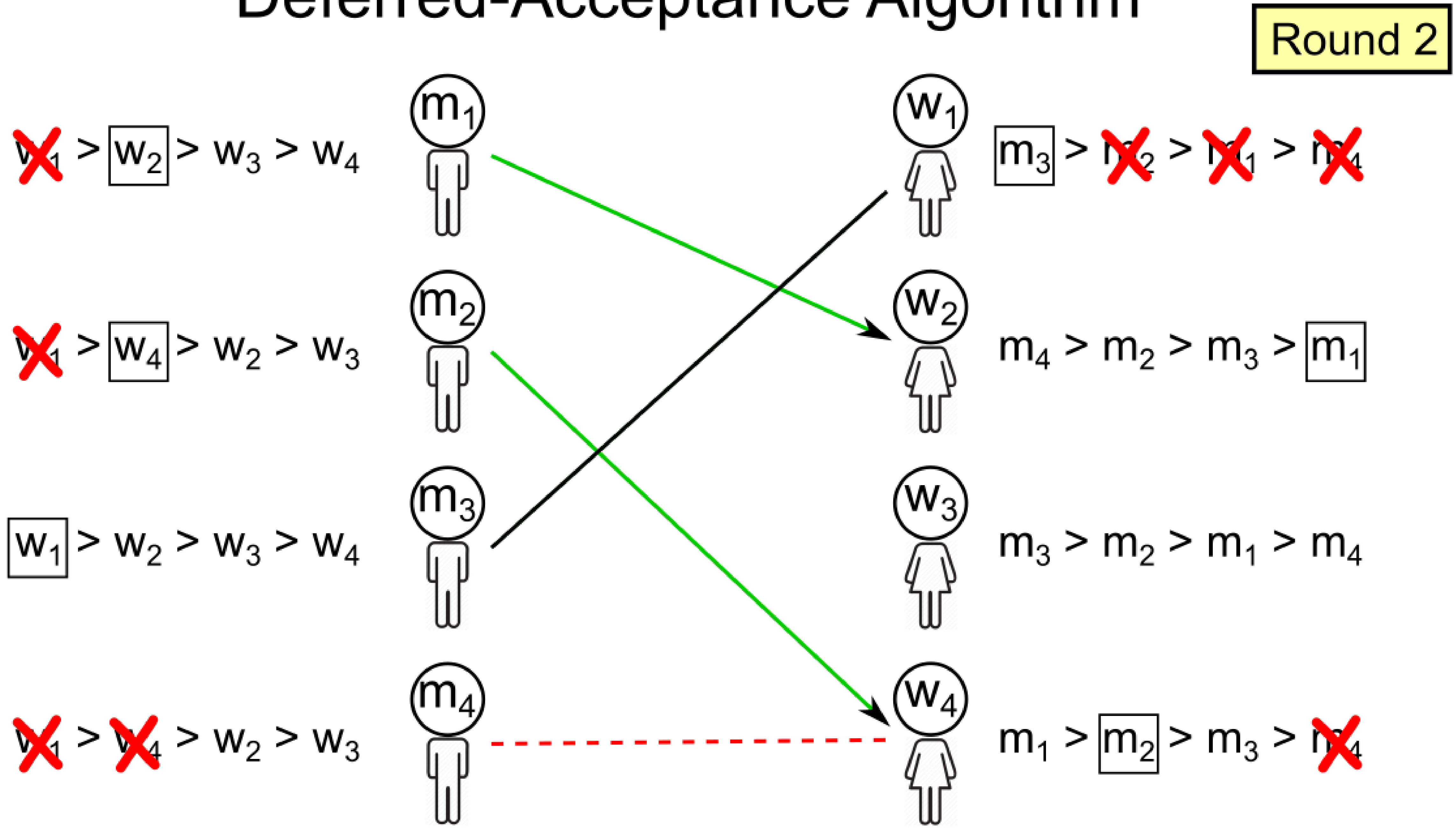

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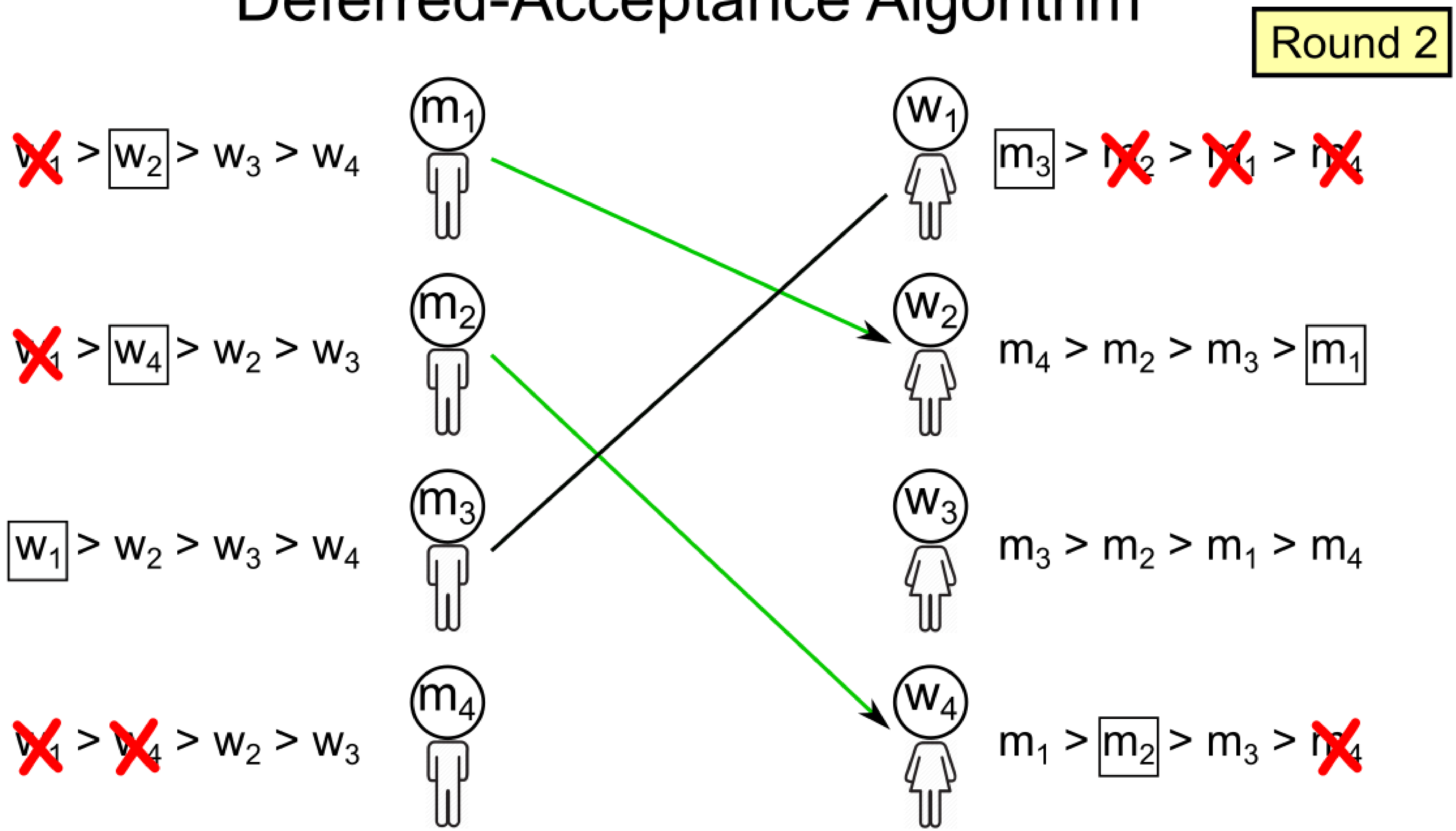

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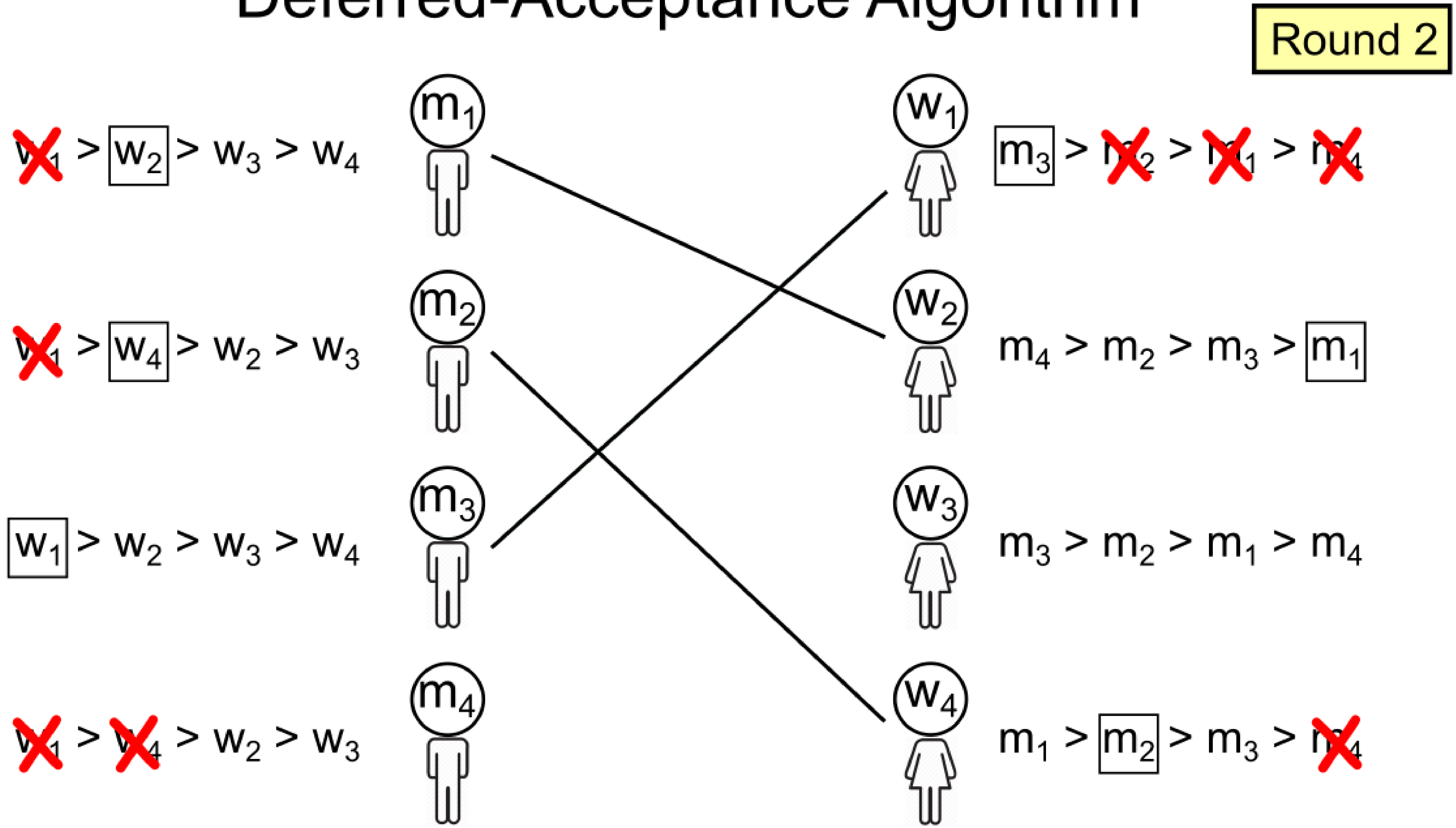
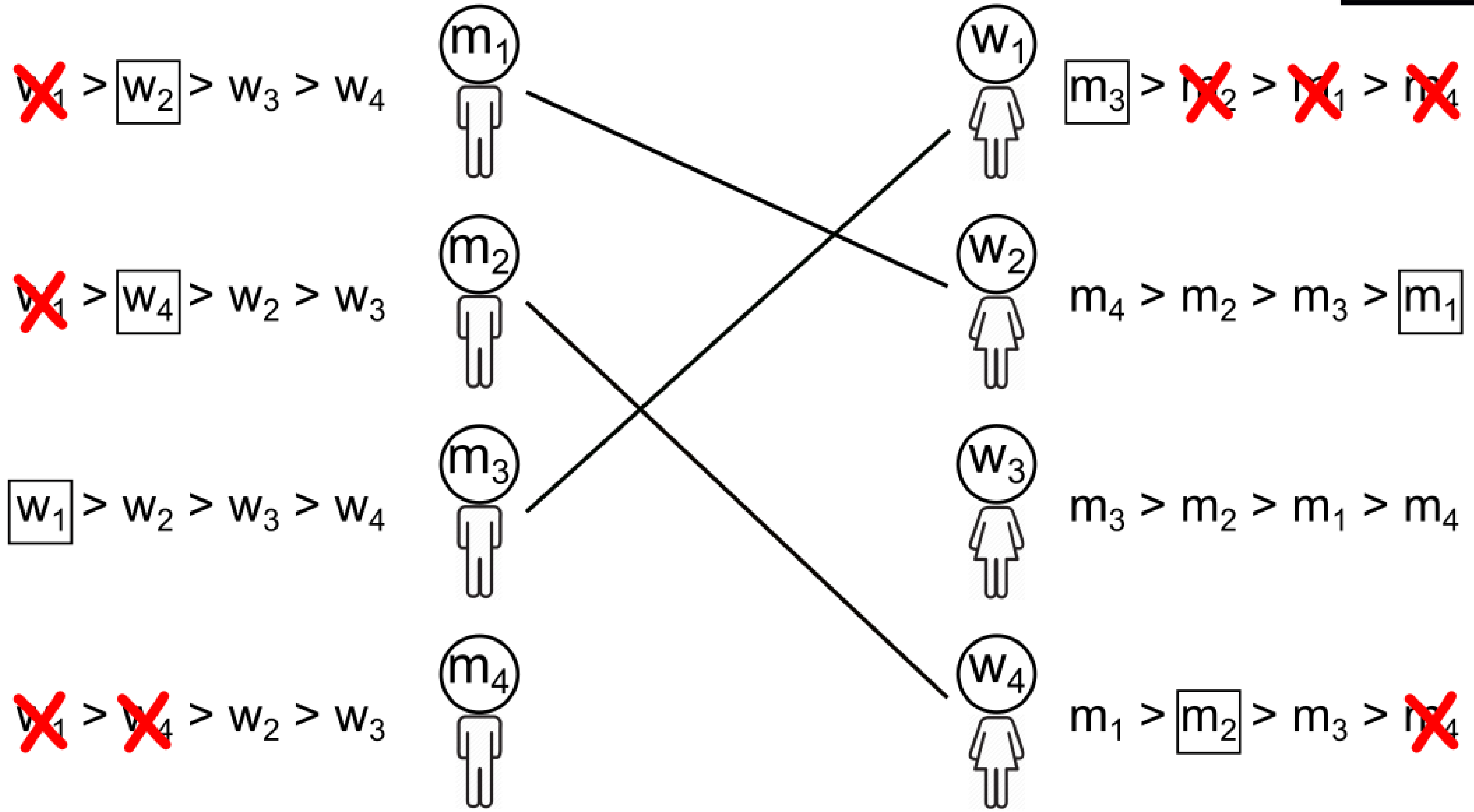


Deferred-Acceptance Algorithm

Round 3



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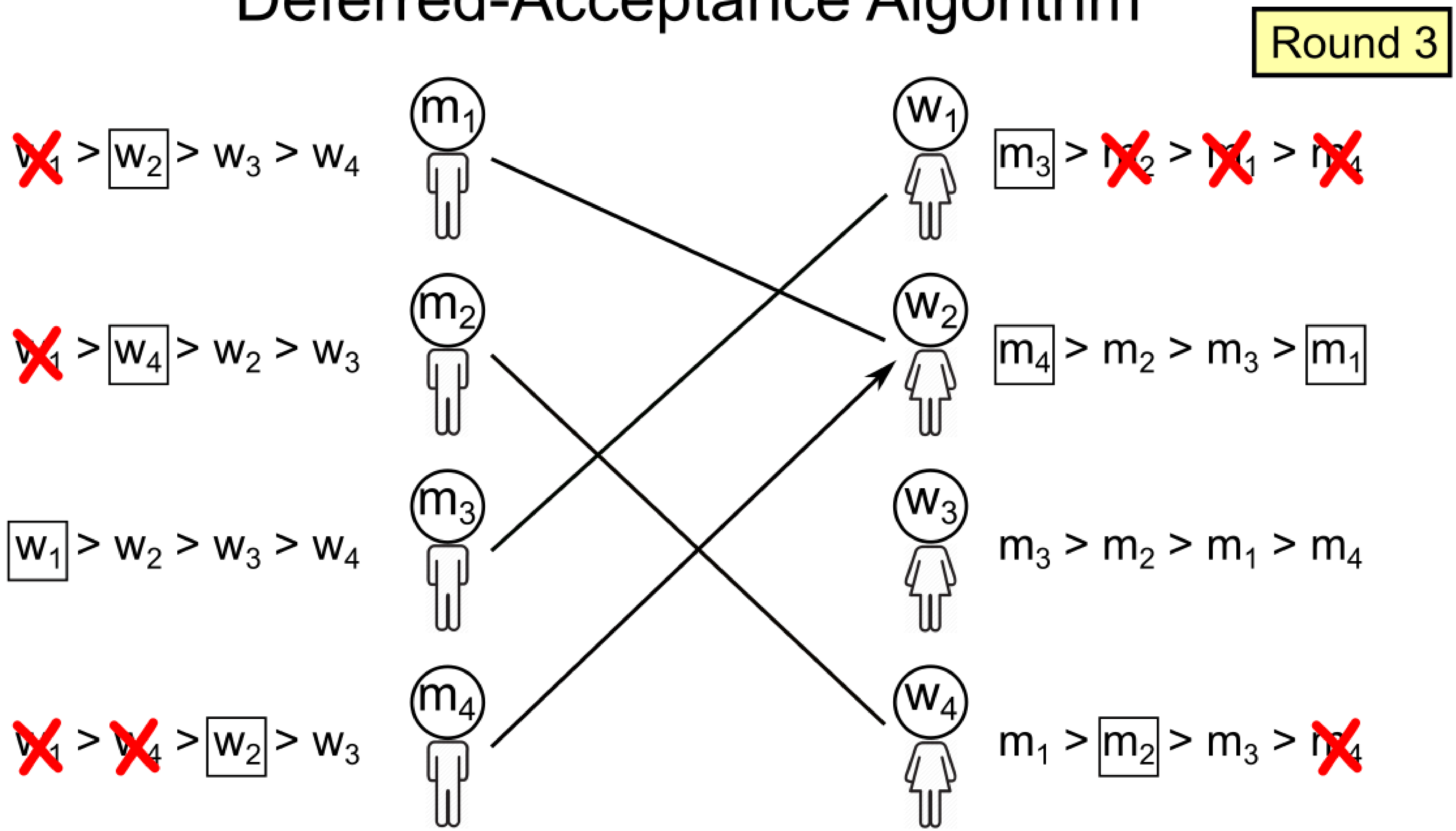

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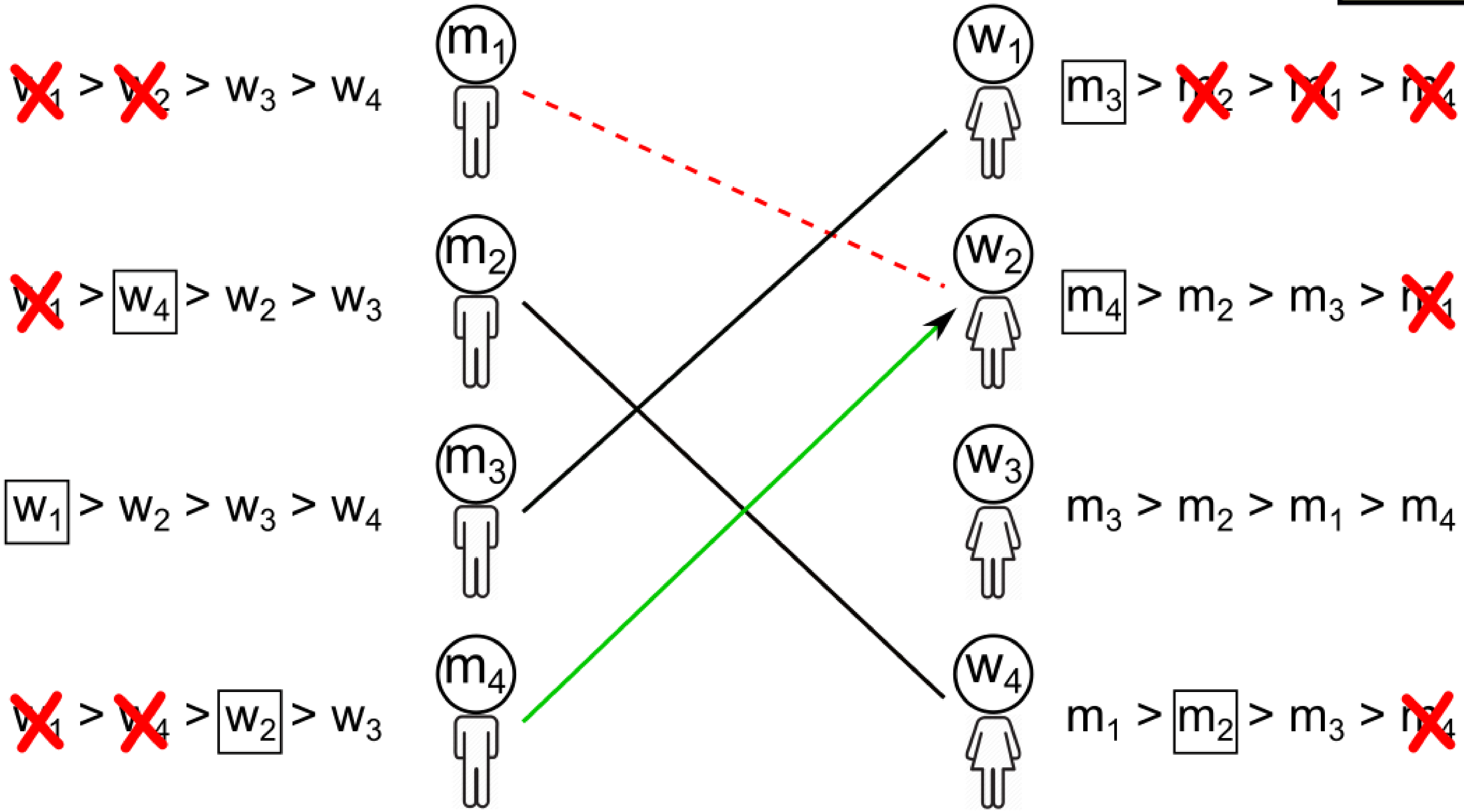

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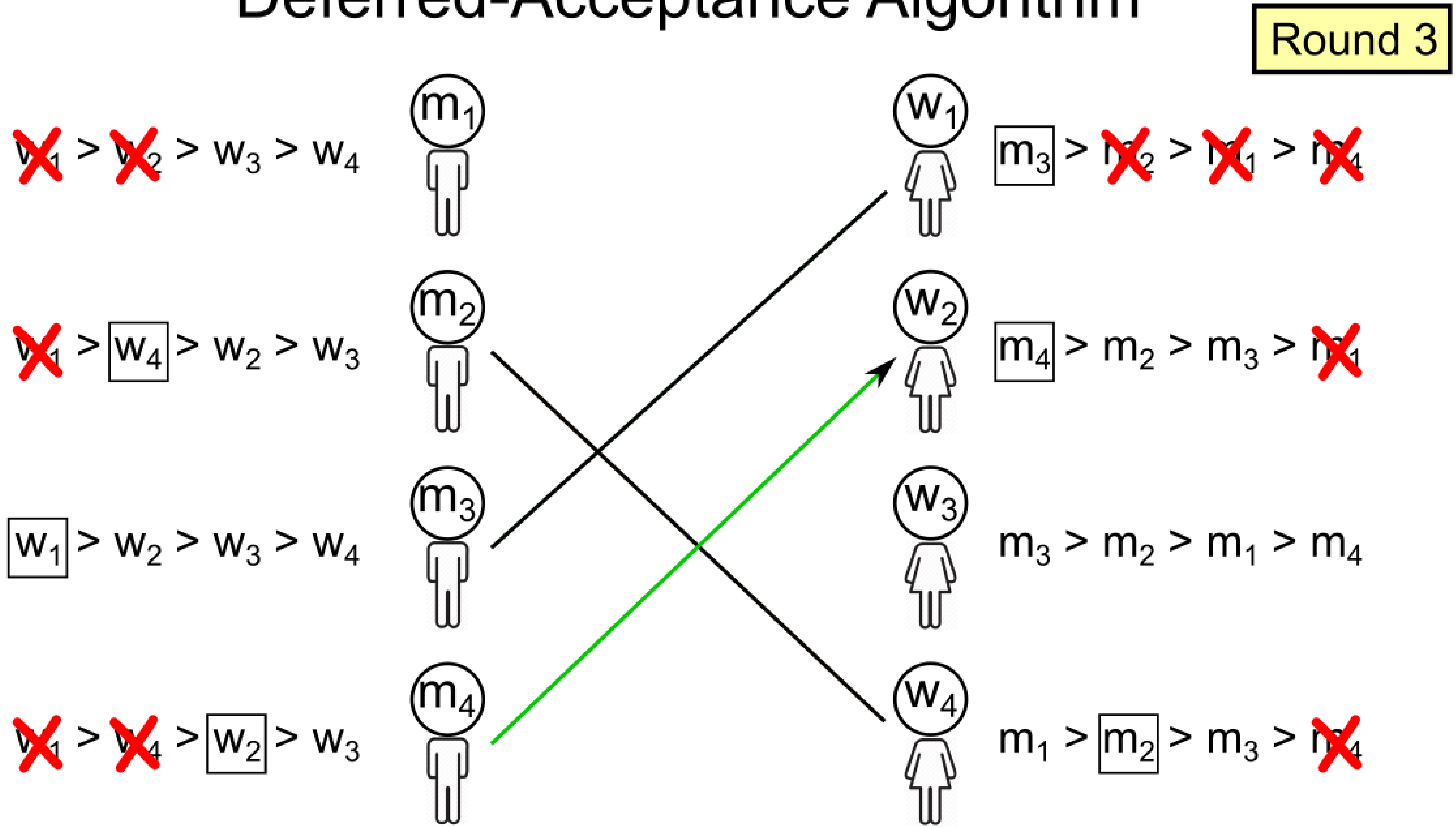

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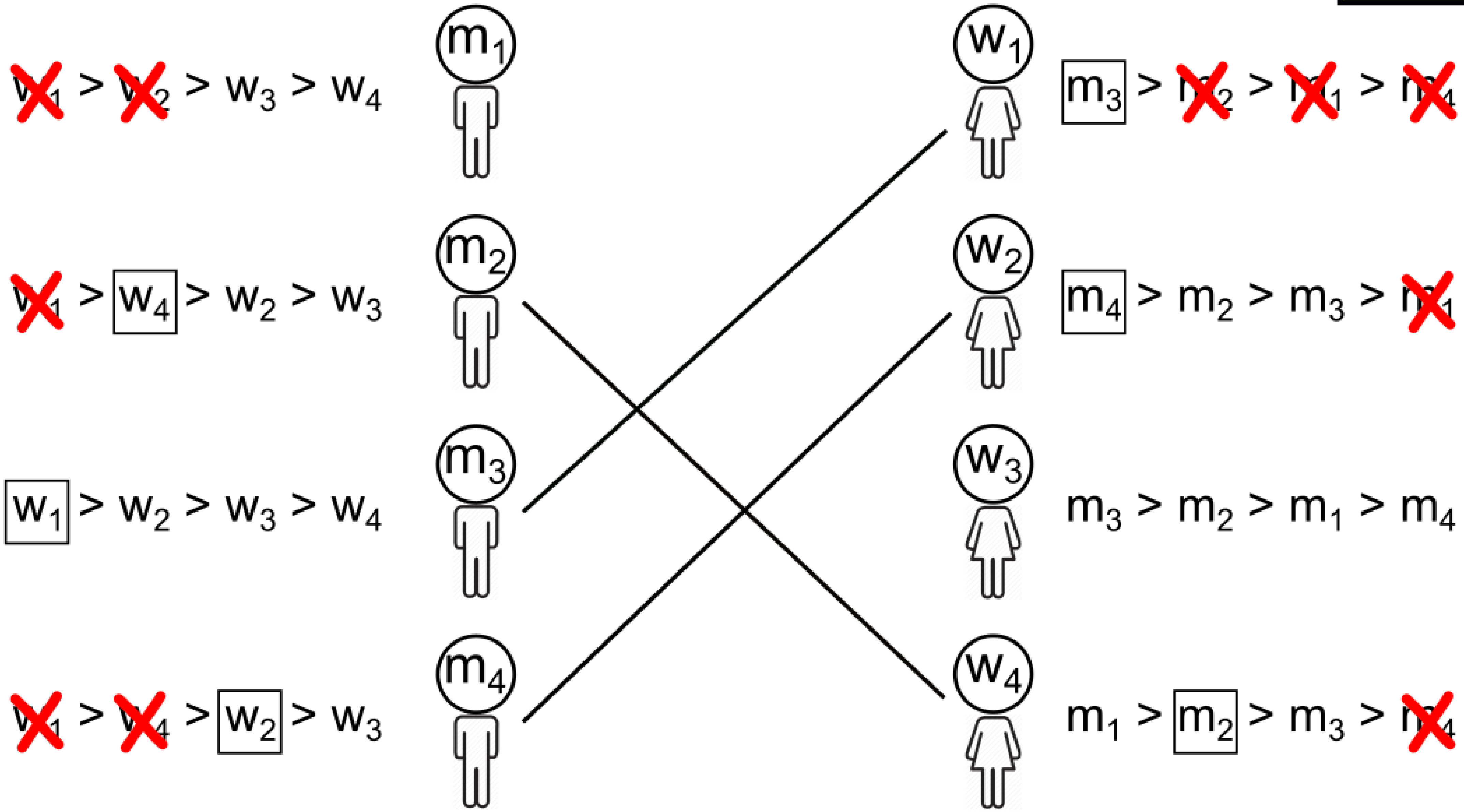

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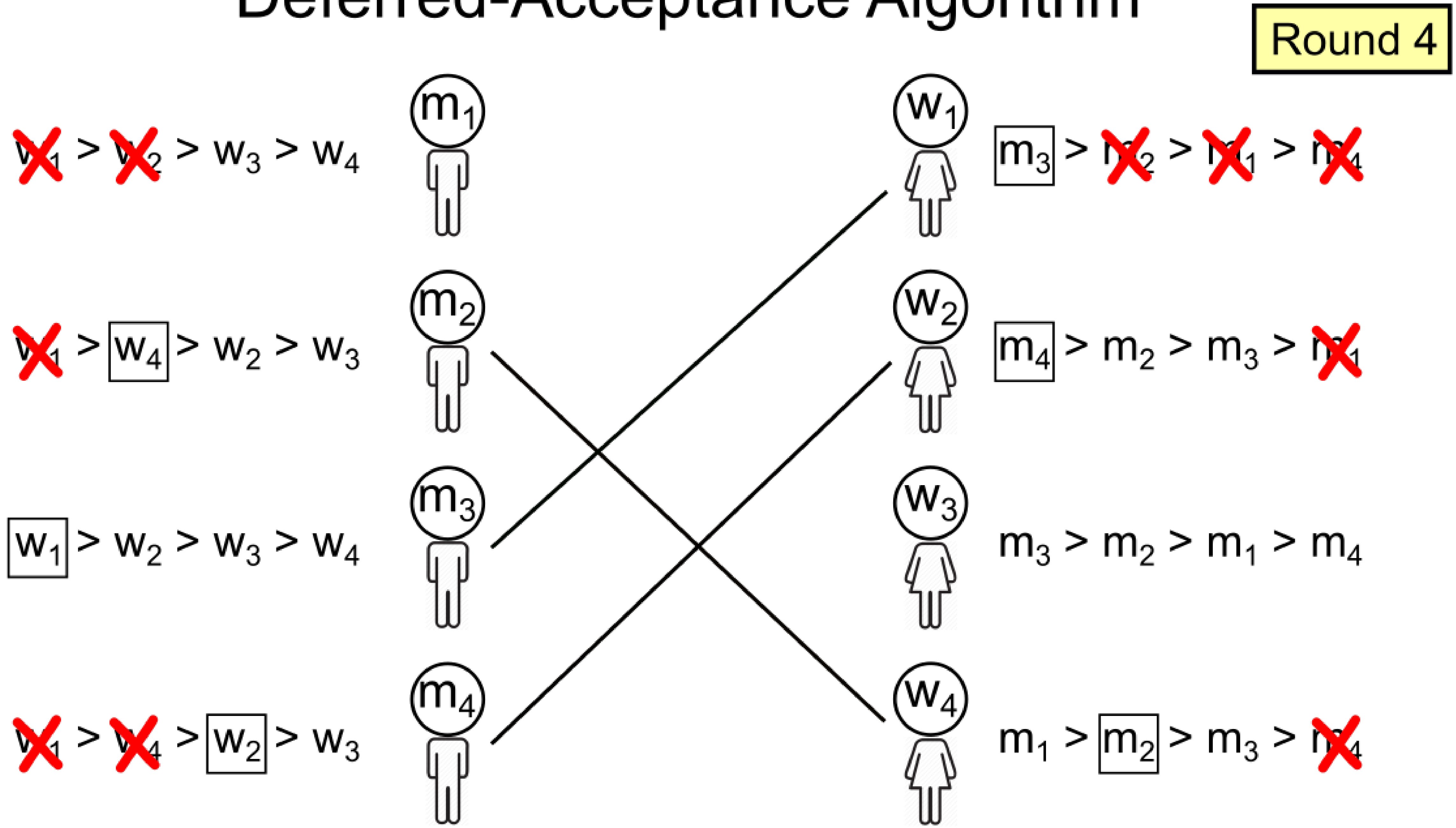

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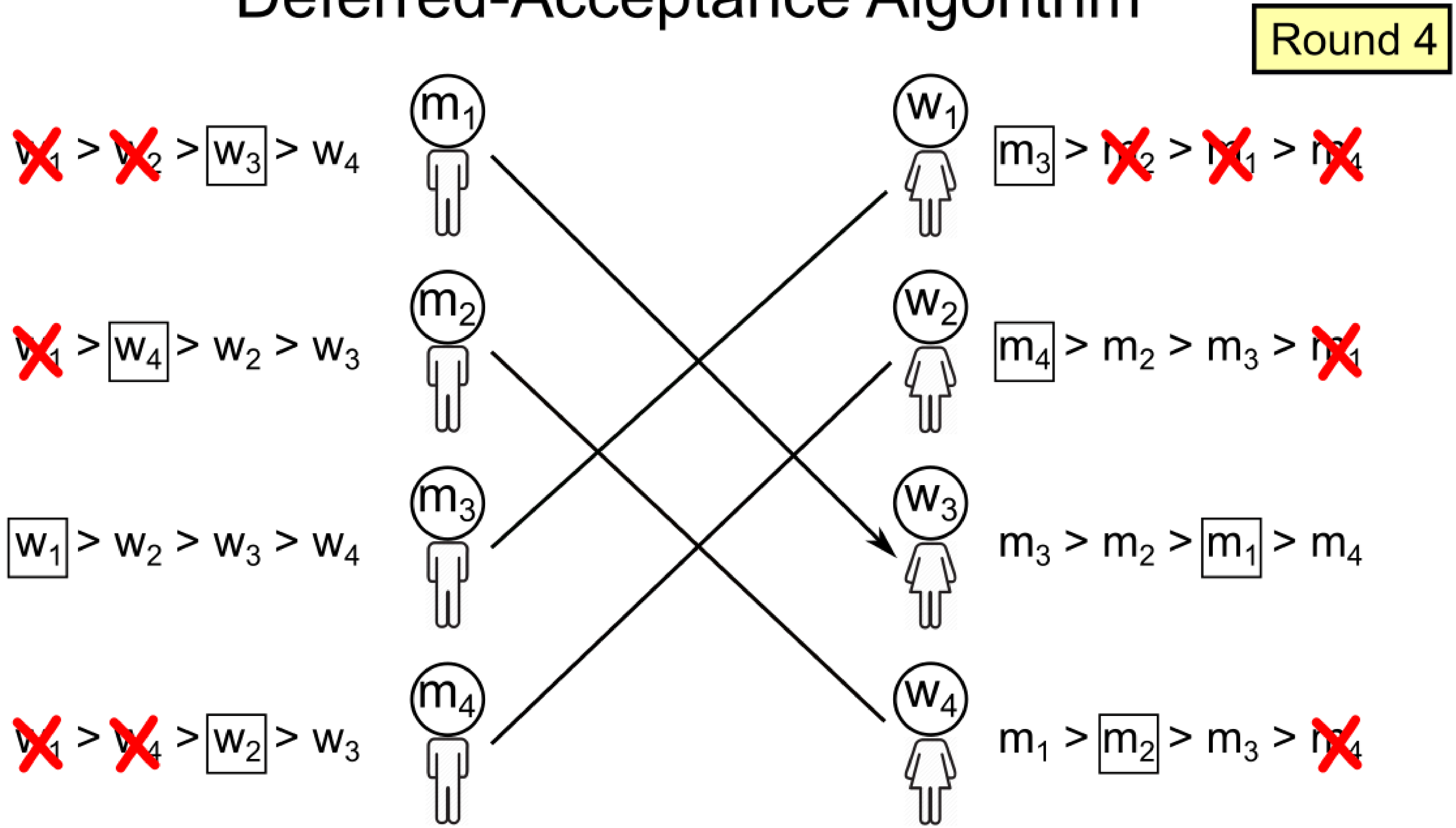

Deferred-Acceptance Algorithm

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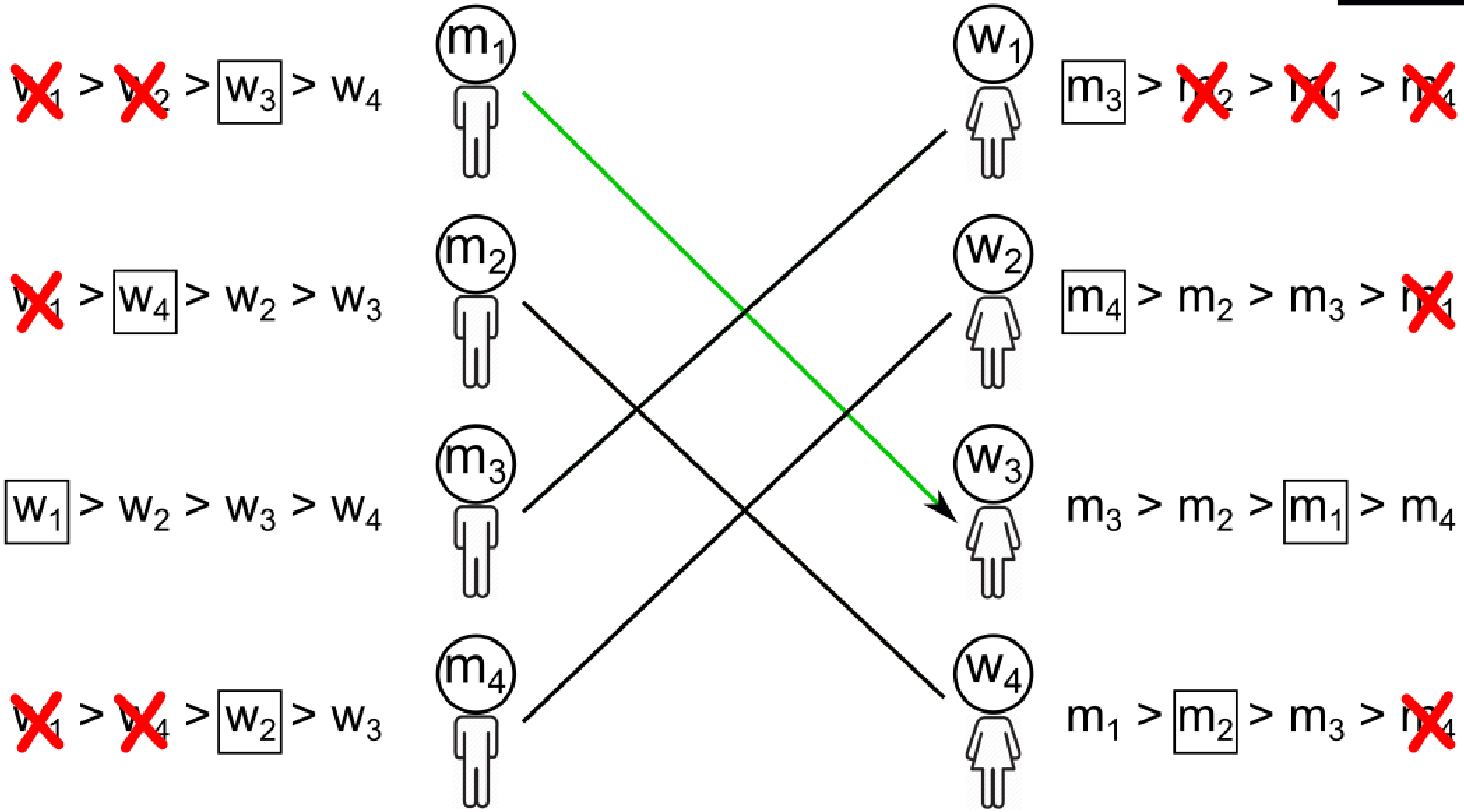

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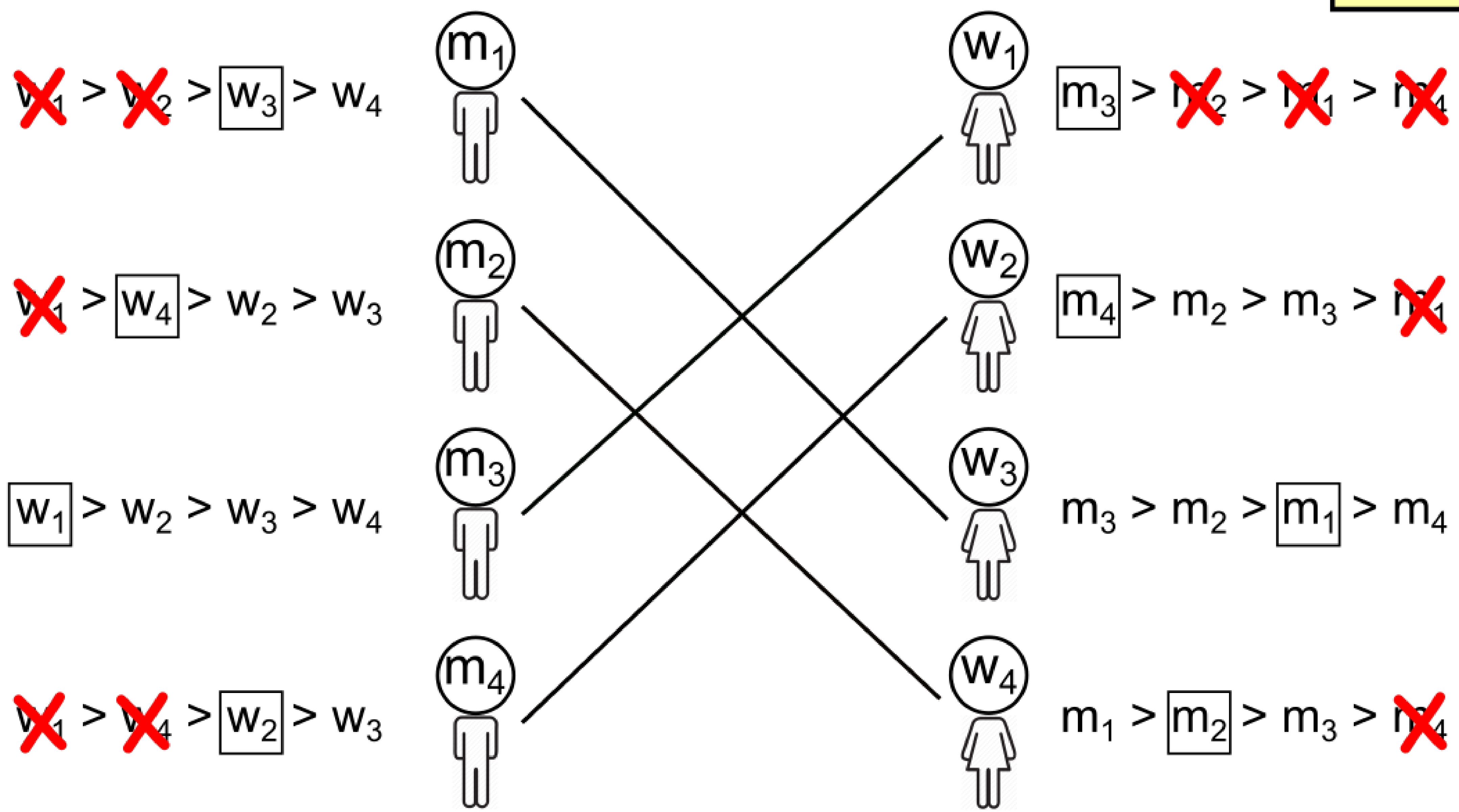

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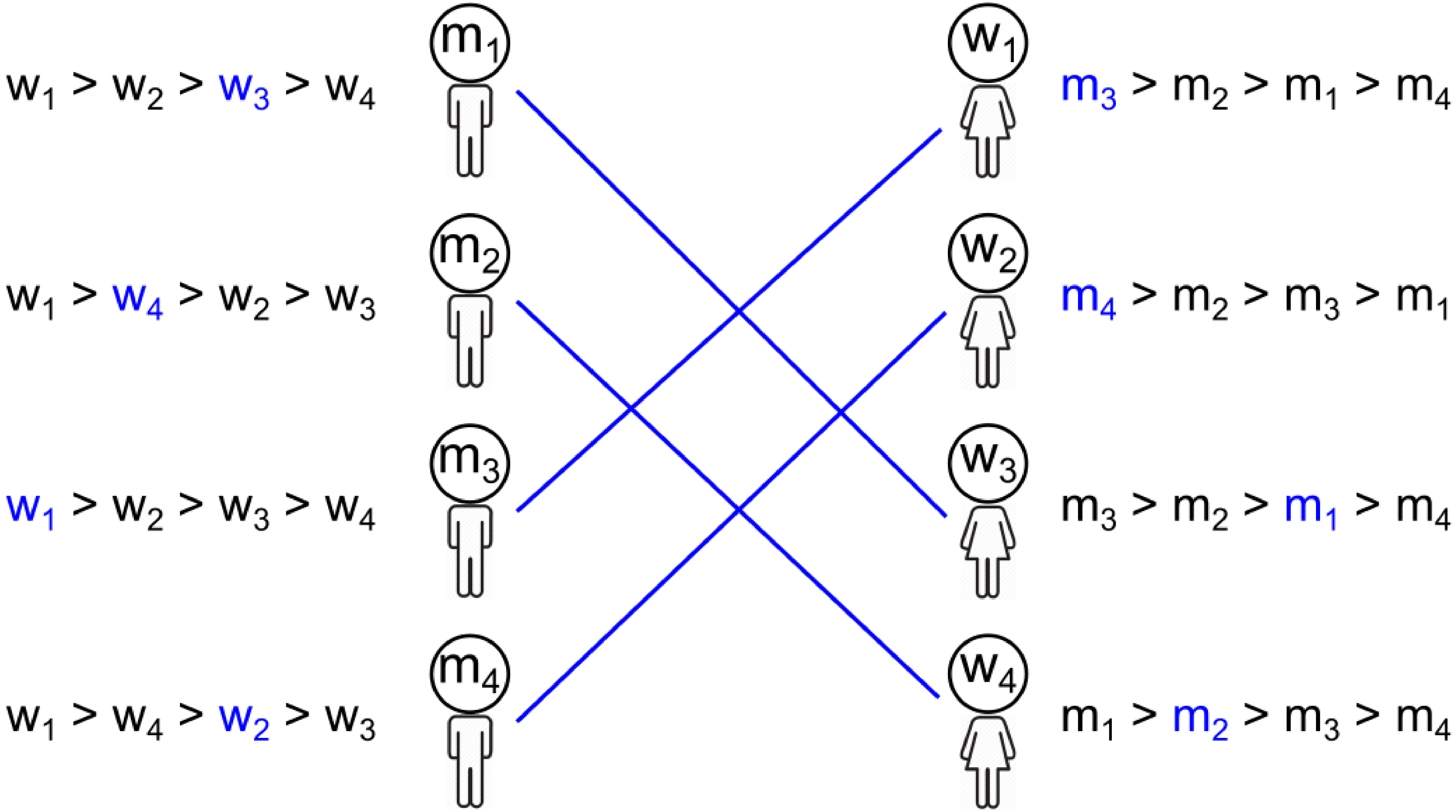

1

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1

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Deferred-acceptance algorithm terminates in polynomial time.

2

Does the deferred-acceptance algorithm always find a perfect matching?

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Once tentatively matched, a woman never becomes unmatched.

3

Does the deferred-acceptance algorithm always find a stable matching?

3

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So, m must have proposed to (and been rejected by) w.

Then, w must have received a better-than-m proposal in some round.

Women only "trade up" during the DA algorithm.

Applications


HOSPITAL


UNHC
UNHCR
UNHCR

Trends in Computational Social Choice

CHAPTER 18

Applications of Matching Models under Preferences

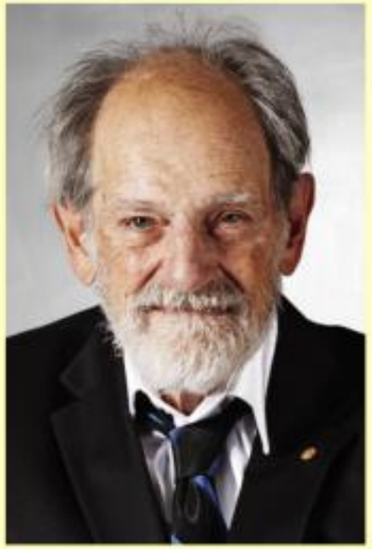
Péter Biró

18.1 Introduction

Matching problems under preferences have been studied widely in mathematics, computer science and economics, starting with the seminal paper by Gale and Shapley (1962). A comprehensive survey on this topic was published also in Chapter 14 of the Handbook of Computational Social Choice (Klaus et al., 2016), and for the interested reader we recommend consulting the following four comprehensive books on the computational (Gusfield and Irving, 1989; Manlove, 2013) and game-theoretical, market design aspects (Roth and Sotomayor, 1990; Roth, 2015) of this topic. In this chapter our goal is to give a general overview of the related applications.

Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012

Alvin E. Roth



Lloyd S. Shapley

"for the theory of stable allocations and the practice of market design."

Logistics

Logistics

- *Course website*: https://rohitvaish.in/Teaching/2023-Fall/

Logistics

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- *Announcements + Discussions*: Microsoft Teams

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Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)

Logistics

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Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)

Preferably in LaTeX
(5-6 problems, ~*one week*)

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Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)

~3 ques/lecture
lightly subjective

Preferably in LaTeX
(5-6 problems, ~*one week*)

Logistics

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Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)
Preferably in LaTeX (5-6 problems, ~*one week*)	~3 ques/lecture *lightly subjective*	Understand a paper *thoroughly* + Contribute *new* insights (suggestive list on course website)

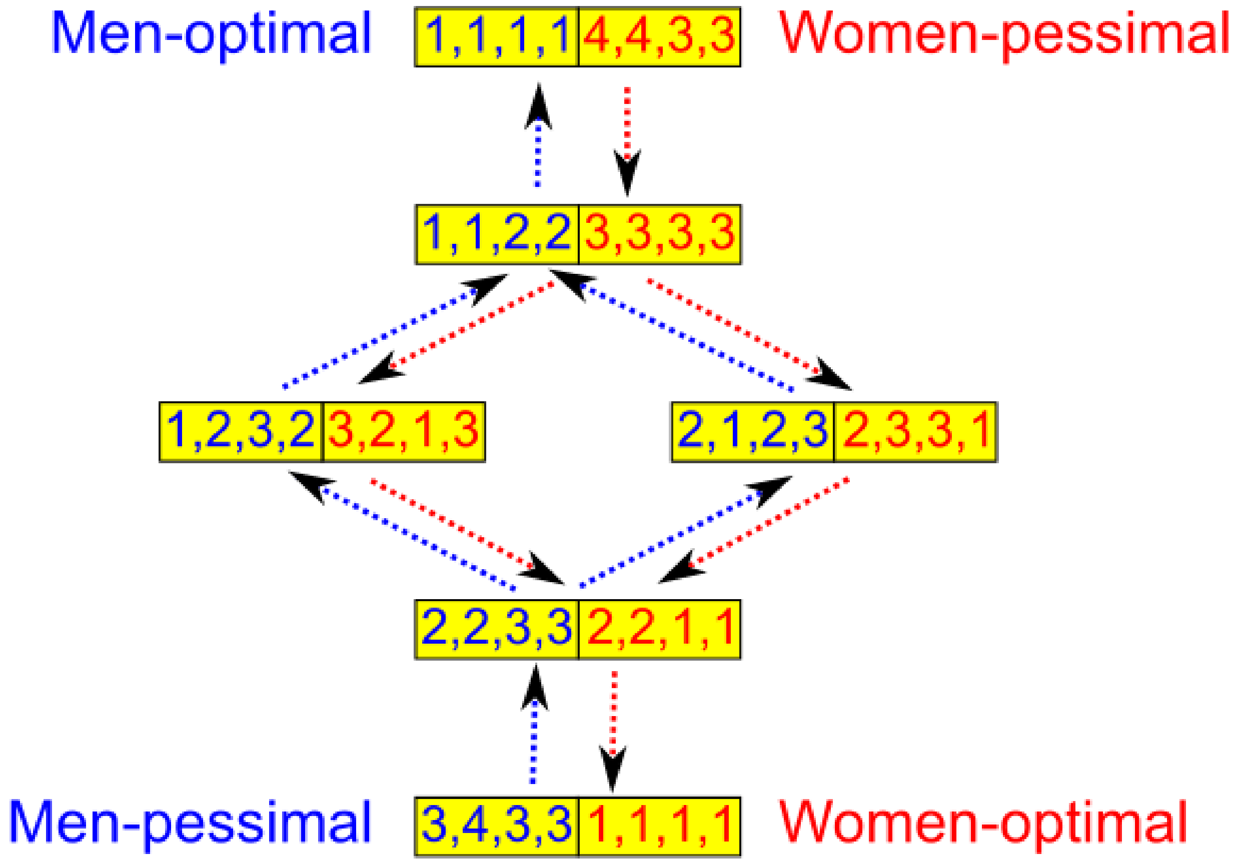
Logistics

- *Course website*: https://rohitvaish.in/Teaching/2023-Fall/
- *Announcements* + *Discussions*: Microsoft Teams
- *Audit policy*: No audits
- *Evaluation policy*: No exams

Assignments (4 x 10% = 40%)	Quizzes (30%)	Project: Present+Report (30%)
Preferably in LaTeX (5-6 problems, ~*one week*)	~3 ques/lecture *lightly subjective*	Understand a paper *thoroughly* + Contribute *new* insights (suggestive list on course website)

Next Time

Structure of Stable Matchings



References

- Stability and the Deferred Acceptance Algorithm

David Gale and Lloyd Shapley
“College Admissions and the Stability of Marriage”
American Mathematical Monthly, 69(1), 1962 pg 9-15