COL866: Special Topics in Algorithms

Lecture 5

House Allocation and Kidney Exchange

Aug 23, 2022 | Rohit Vaish

Housing Market

[Shapley and Scarf, *JME* 1974]

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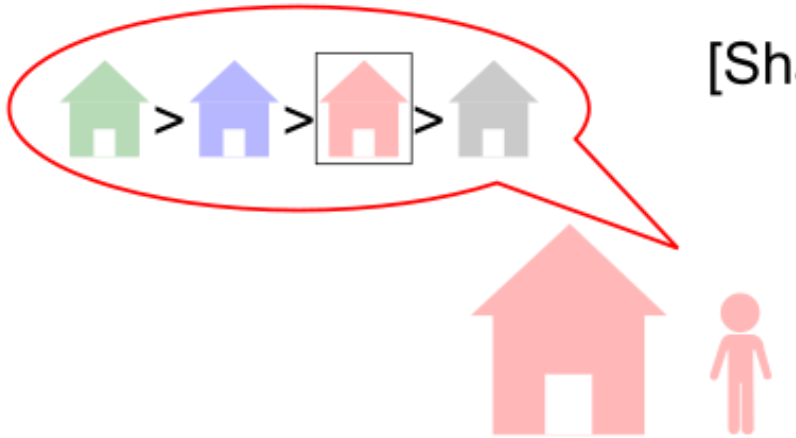

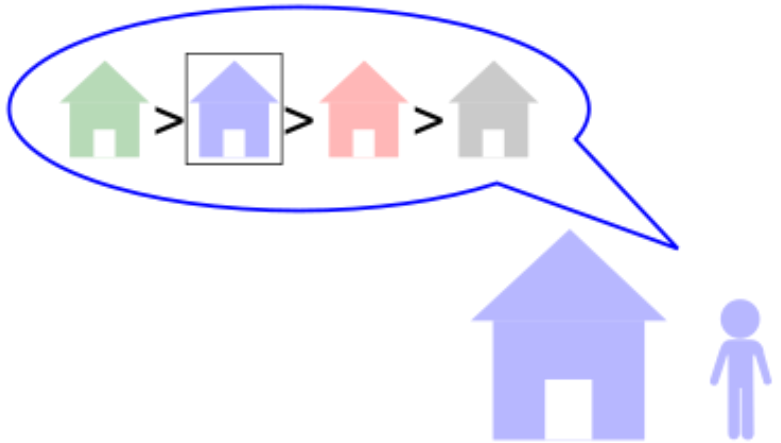

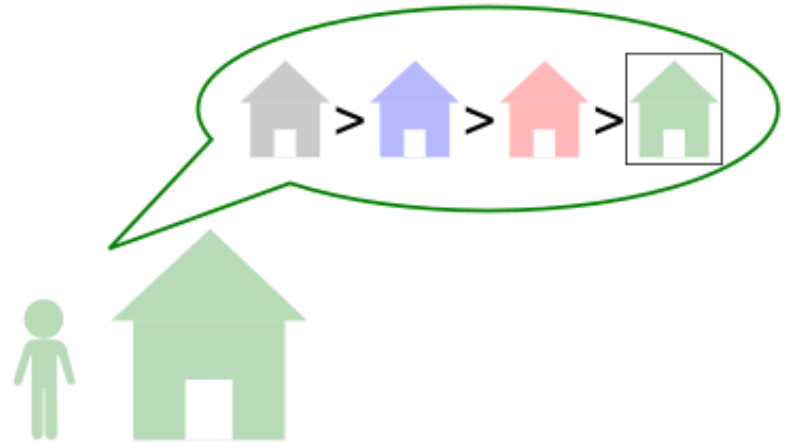

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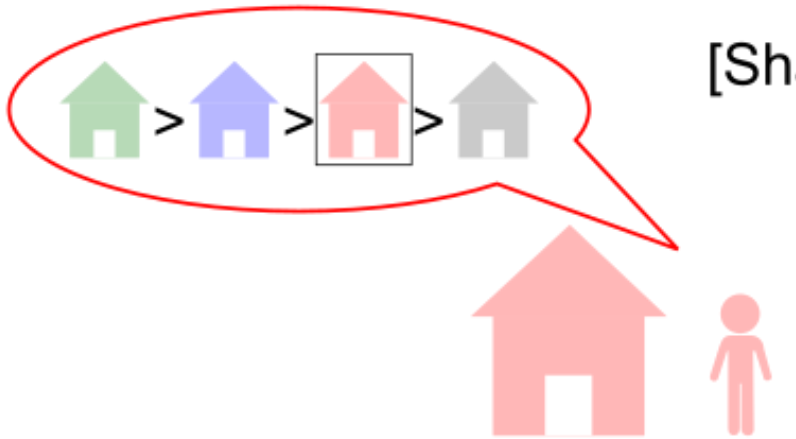

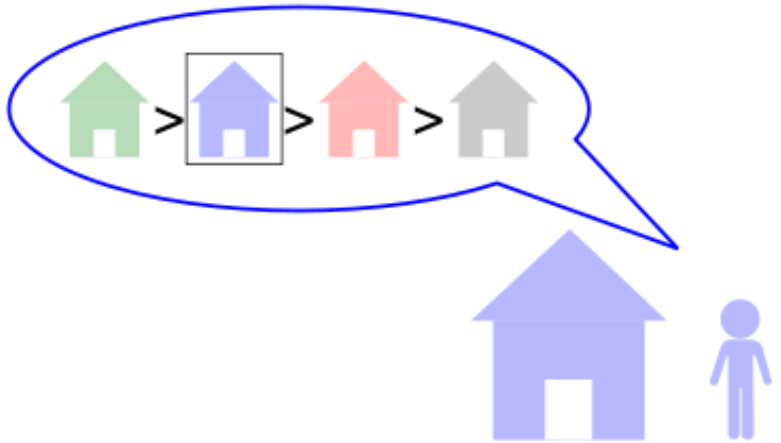

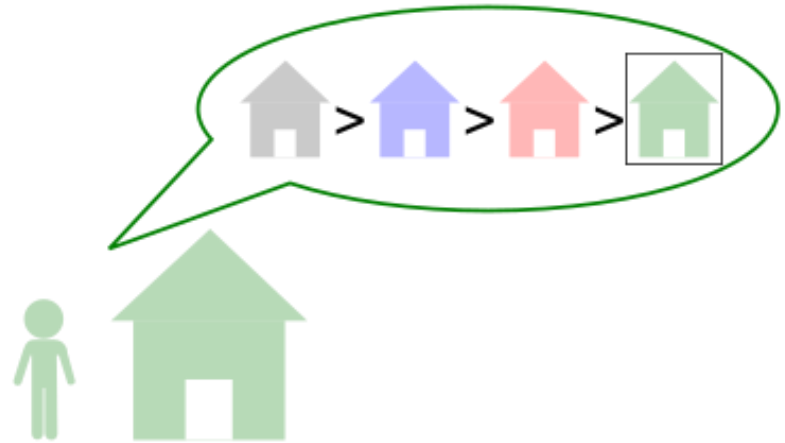

Can't use money. Only way to make agents happier is via *exchanges*.

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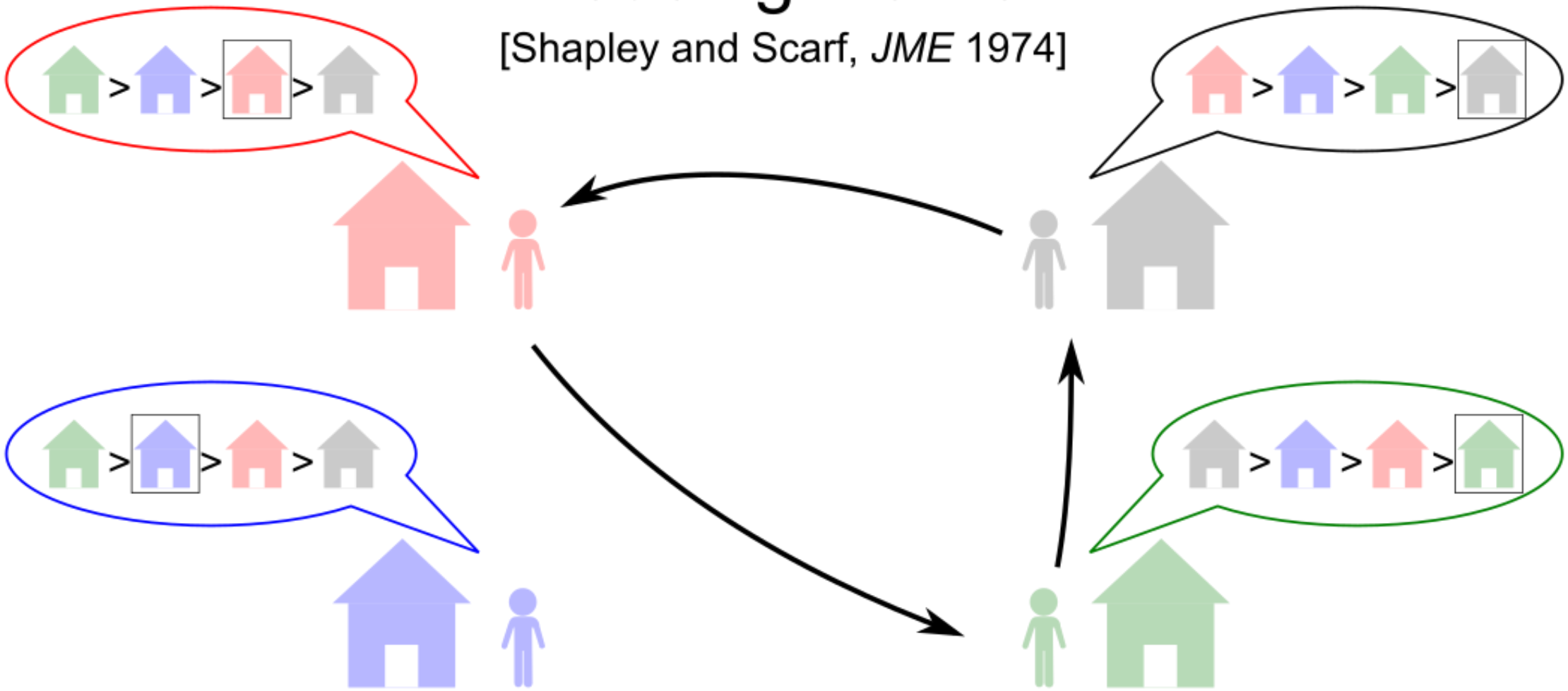

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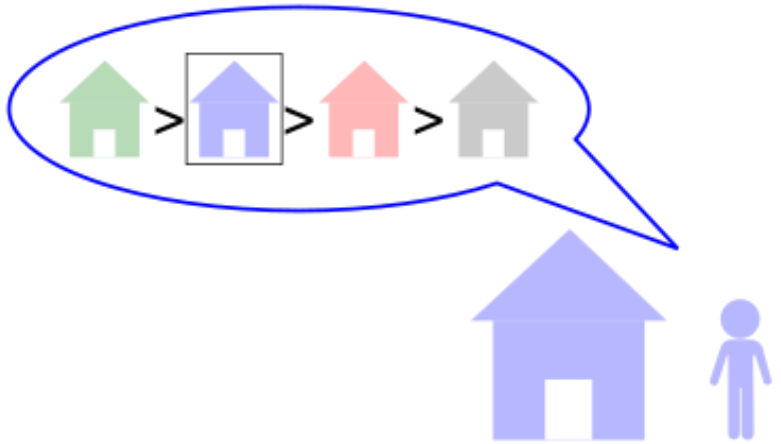

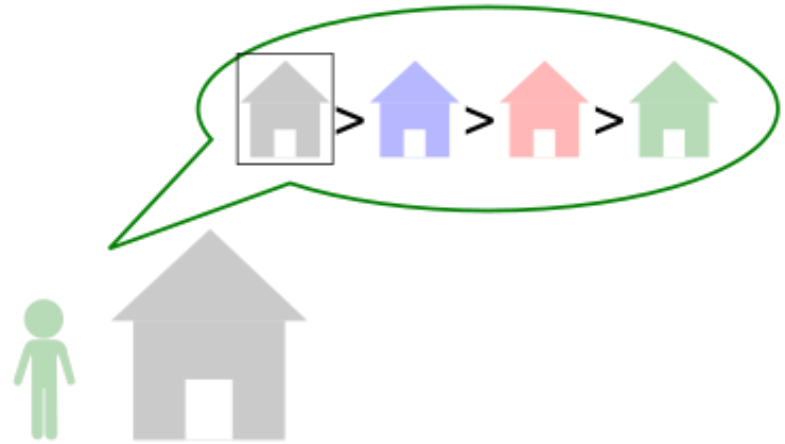

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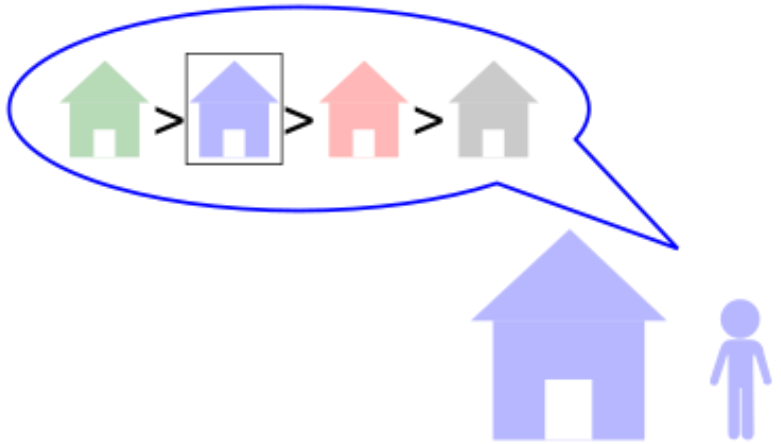

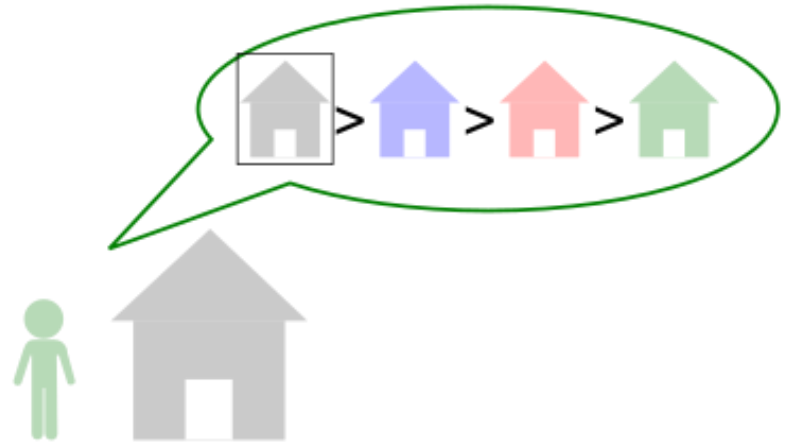

Is there a way to exchange houses to make the agents "maximally happy"?

Top-Trading Cycle Algorithm (TTCA)

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

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While there is a remaining agent:

- Each remaining agent points to the owner of its favorite house.
- There must be a cycle (include self-loops). Do the cyclic exchange.
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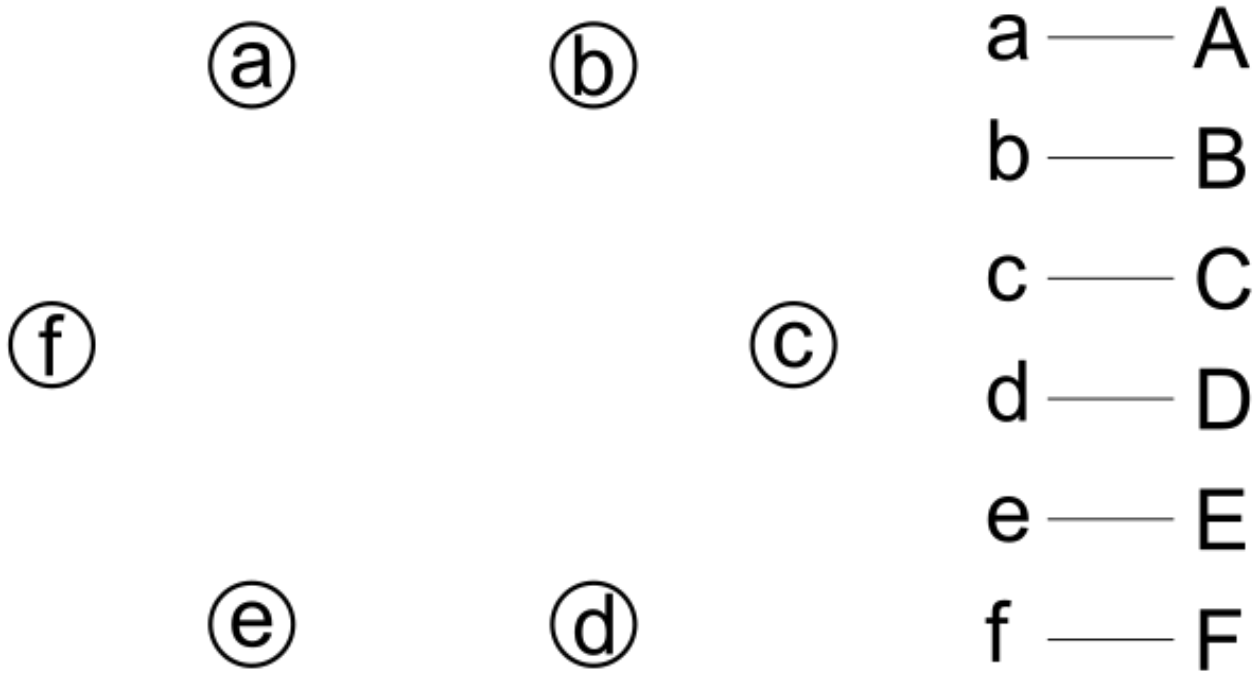

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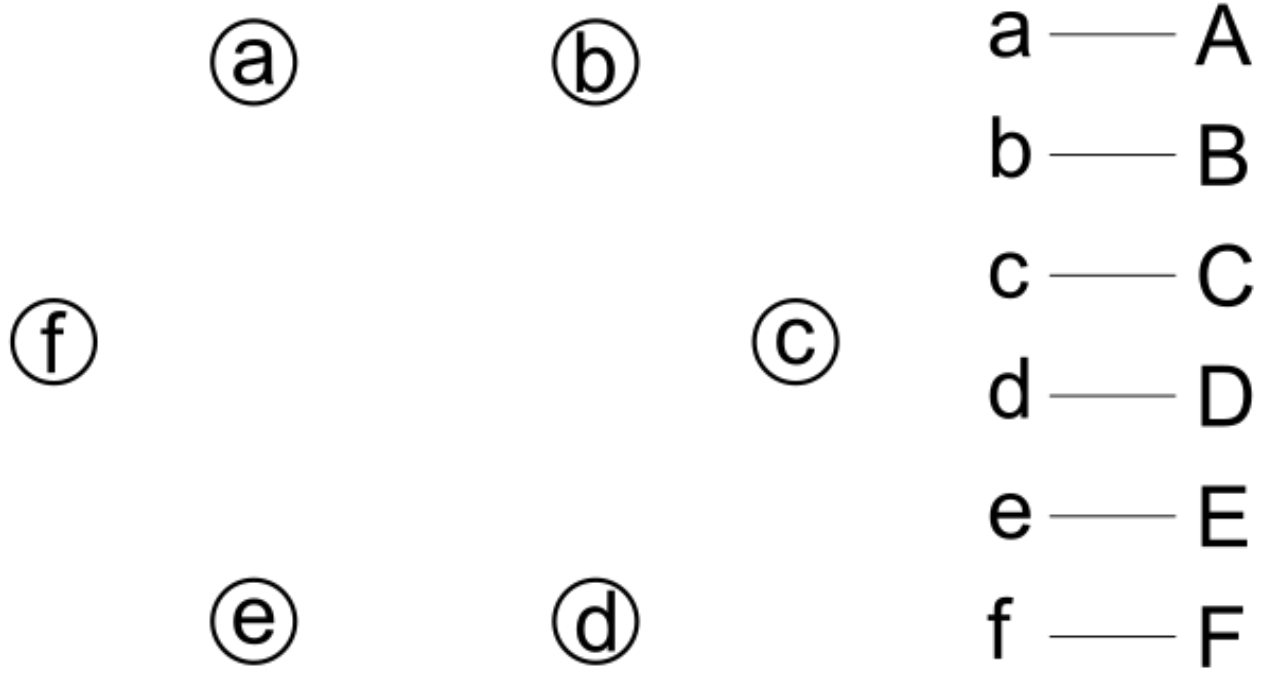

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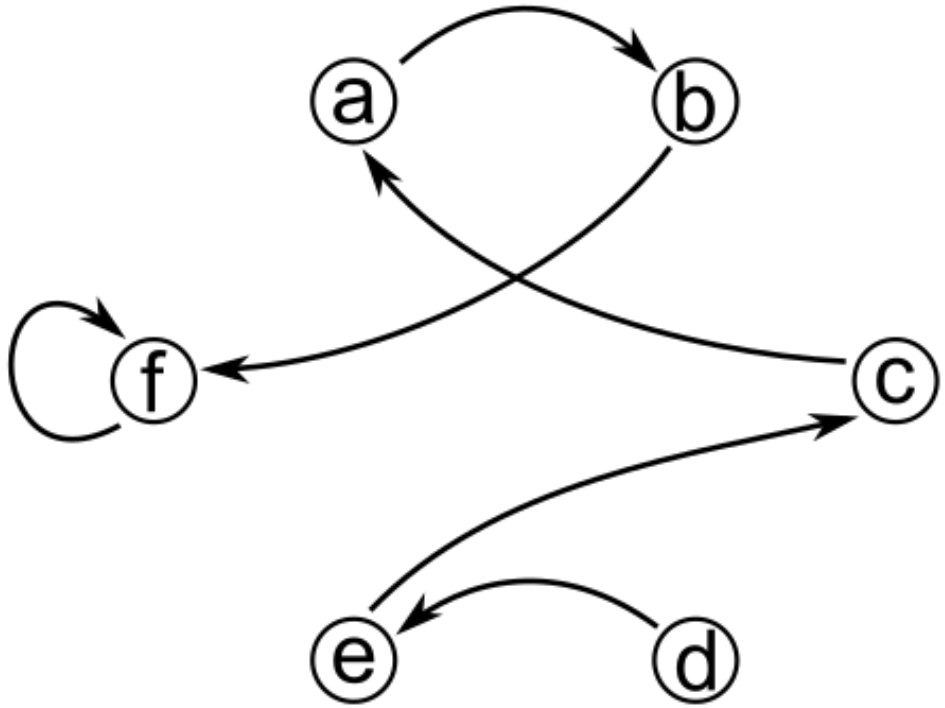

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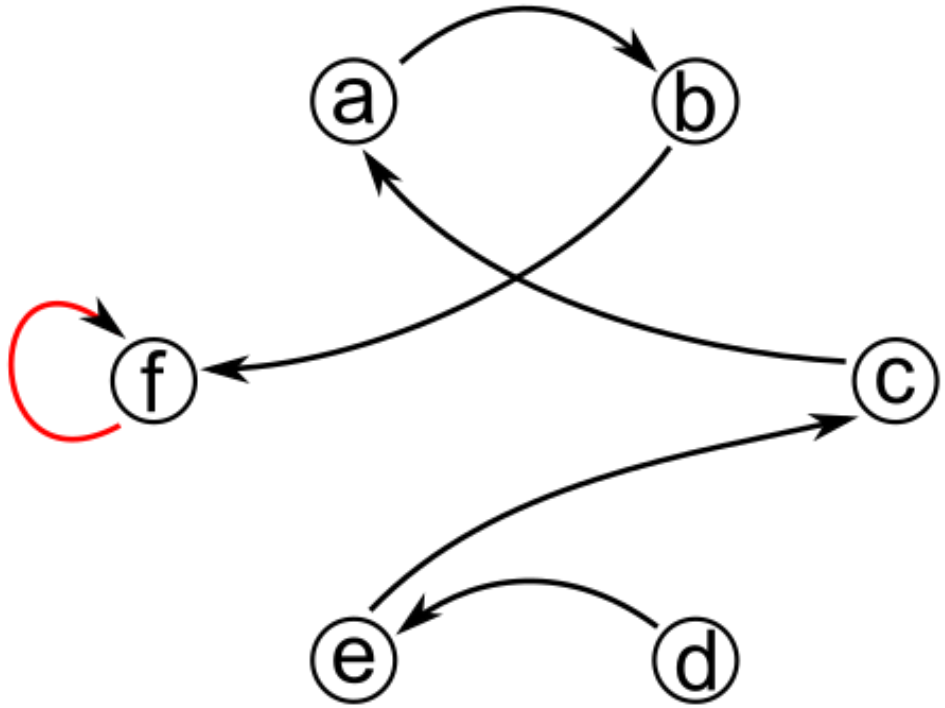

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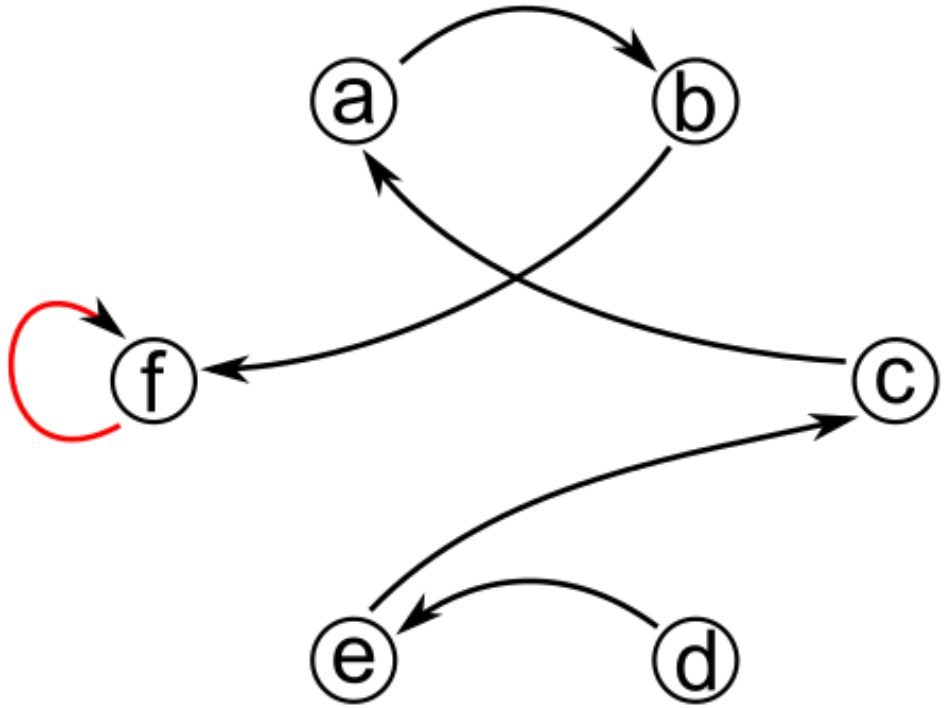

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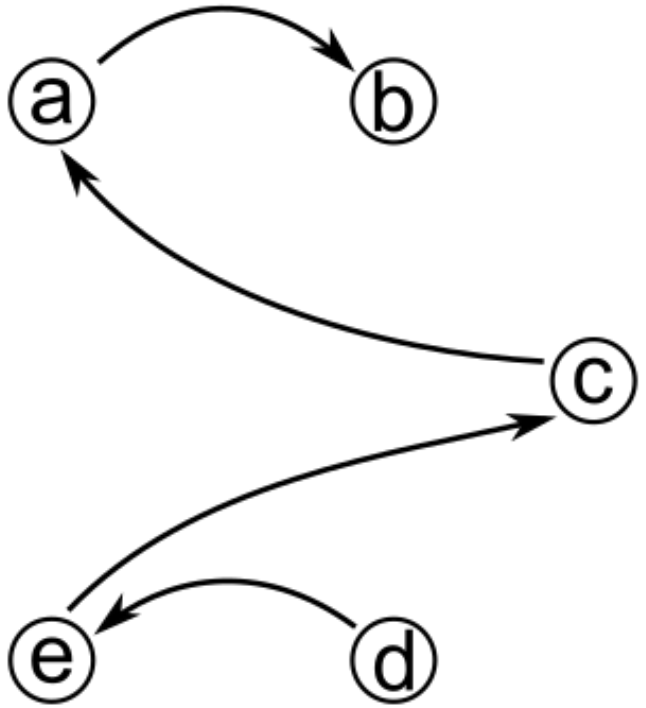

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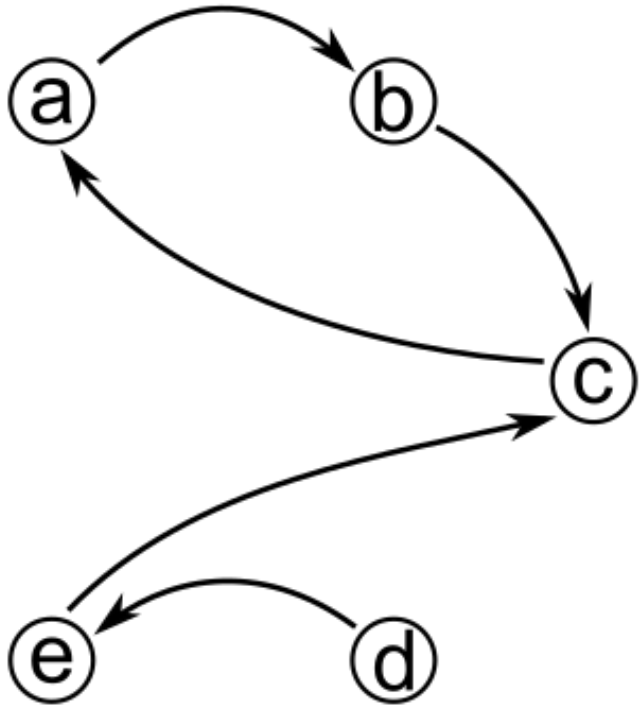

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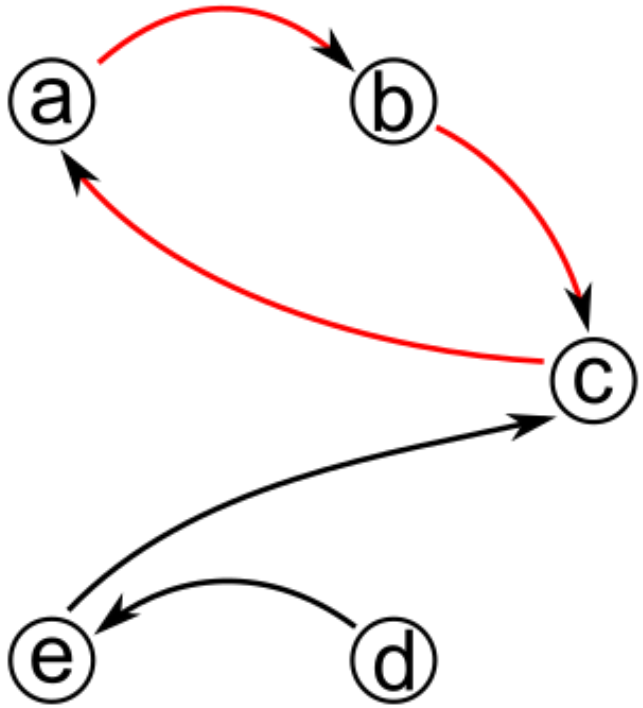

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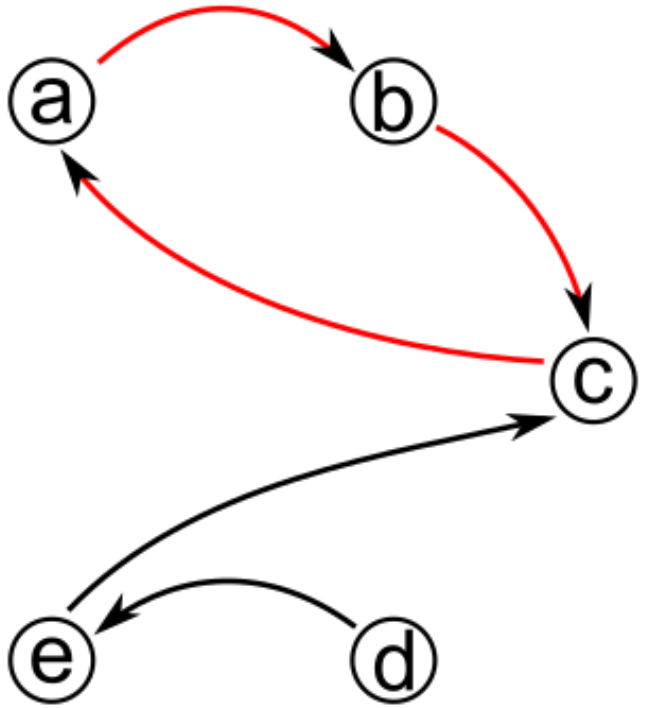

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Imagine a graph where the *vertices* are the (remaining) agents and the *edge* i → j means that agent i's favorite house is owned by agent j.

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At least one agent kicked out in each round.

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Q3. Does TTCA terminate in polynomial time? Yes.

TTCA is strategyproof

[Roth, *Econ. Letters* 1982]

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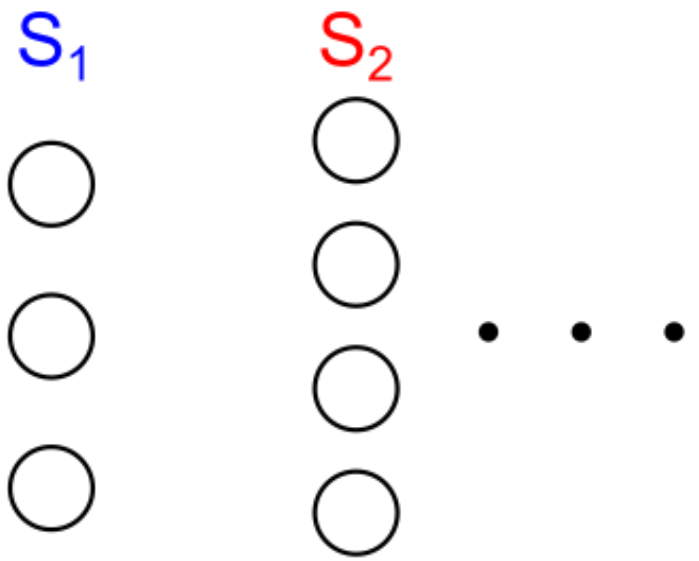

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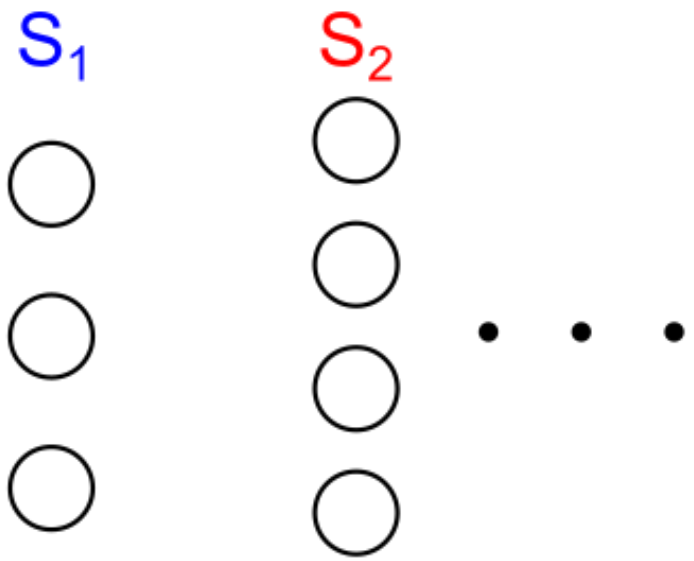

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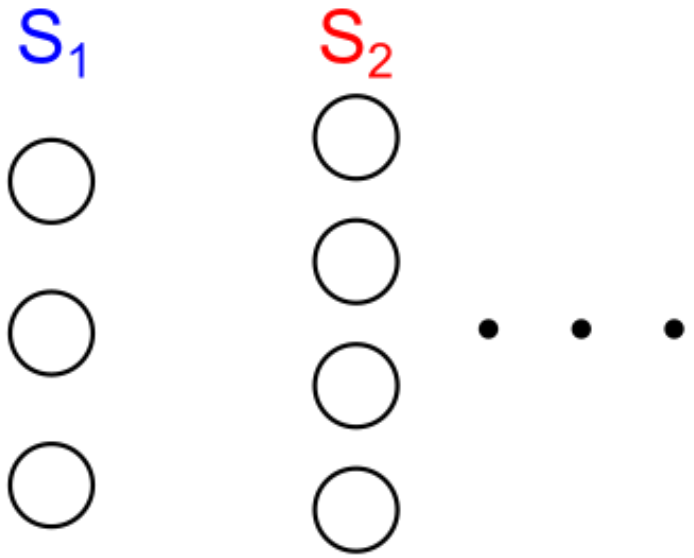

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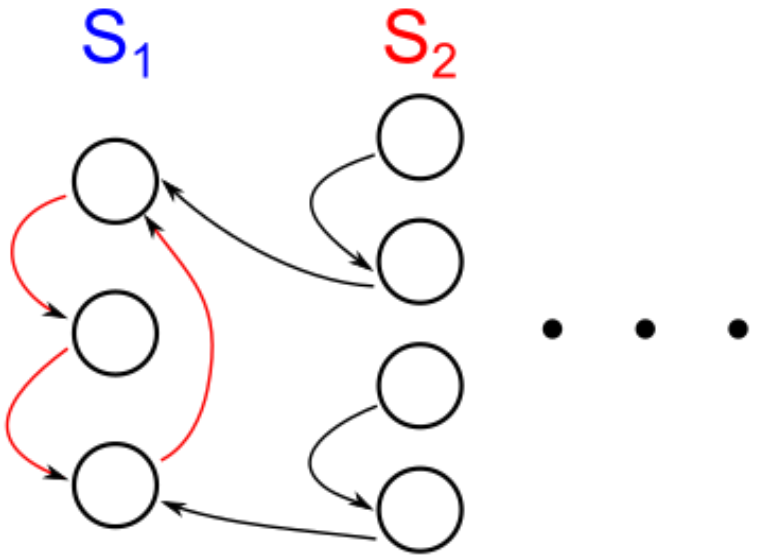

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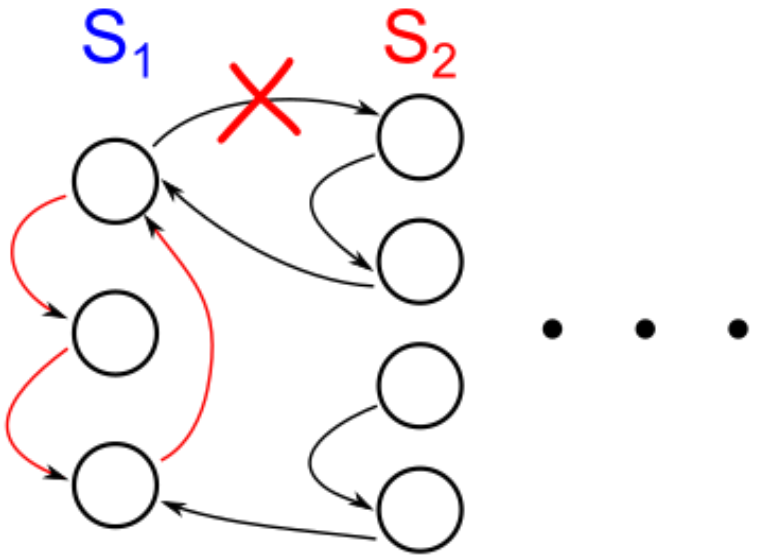

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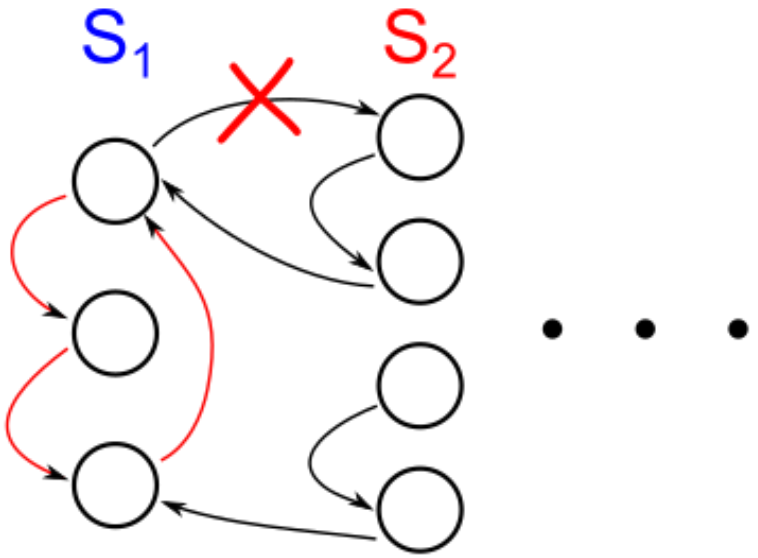

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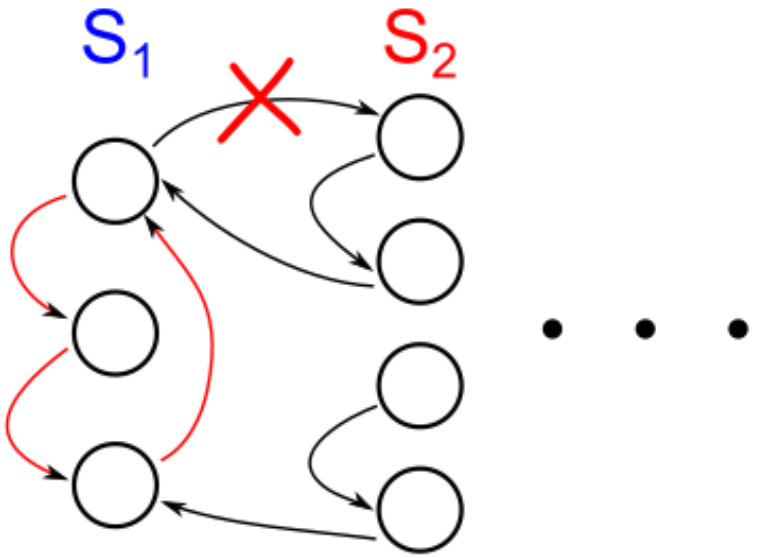

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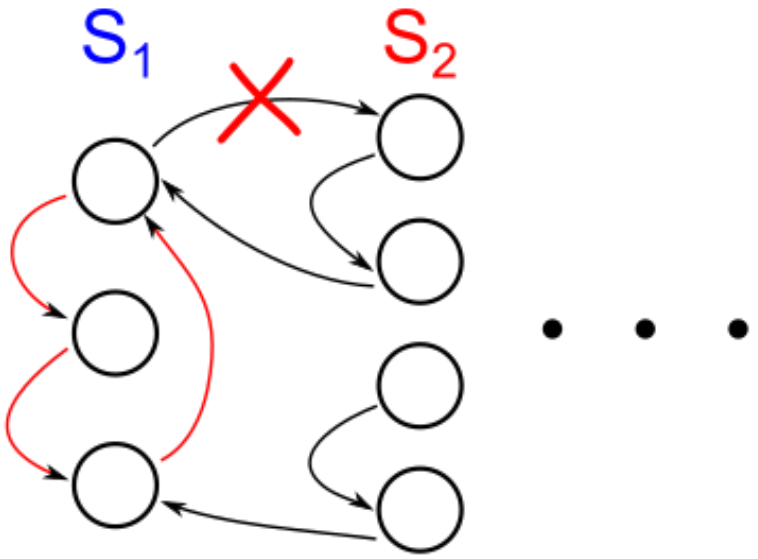

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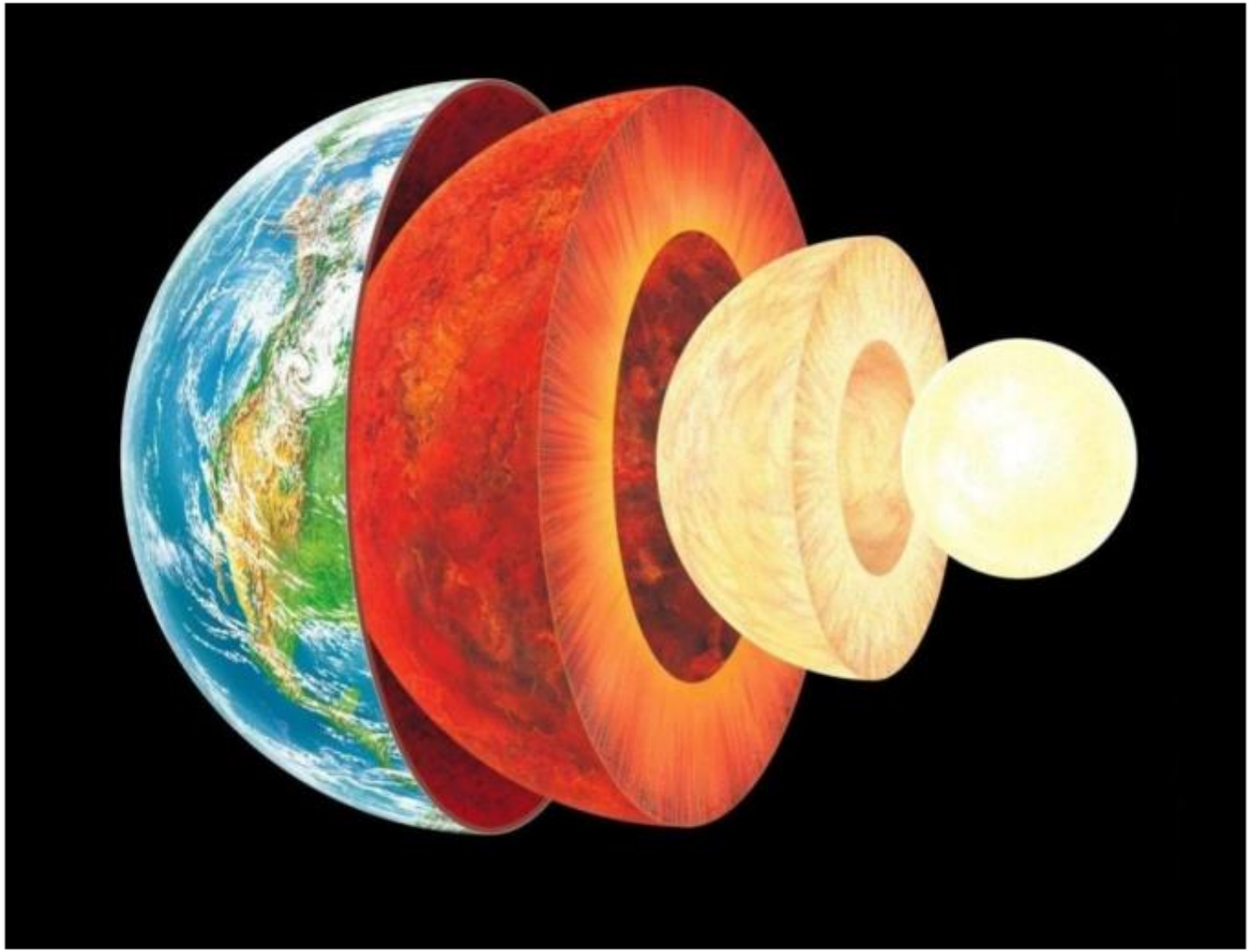
The Core



The Core

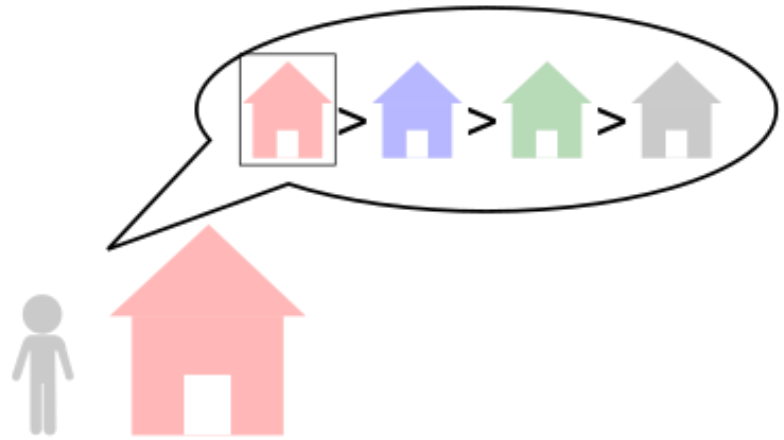
An allocation is in the core if no coalition blocks it.

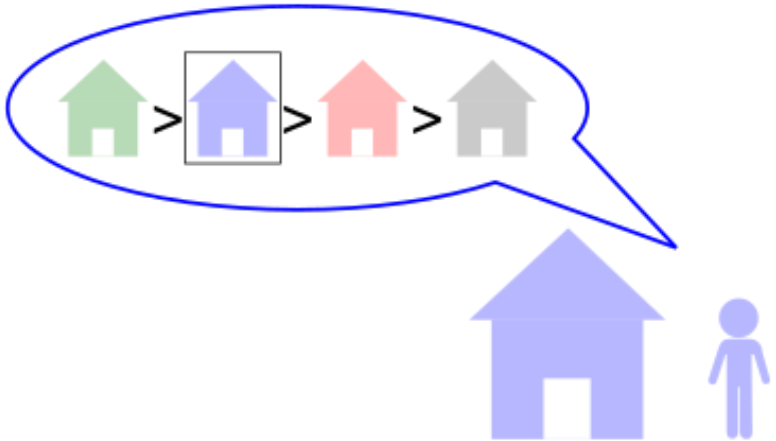
A coalition of agents blocks an allocation A if:

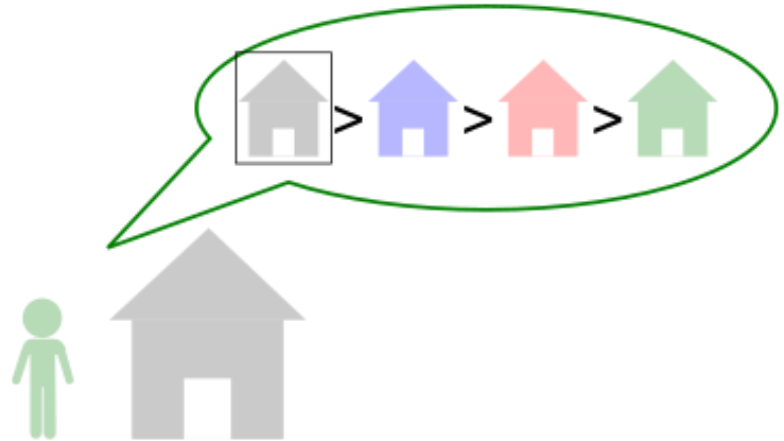
- they can redistribute their **endowed houses** among themselves such that, compared to A, none of them is worse off and at least one of them is strictly better off (i.e., redistributing endowments is a Pareto improvement over A), and

- the coalition is *minimal* (no subset can redistribute and improve)

Example of a Core Allocation







TTCA outcome is in the core.

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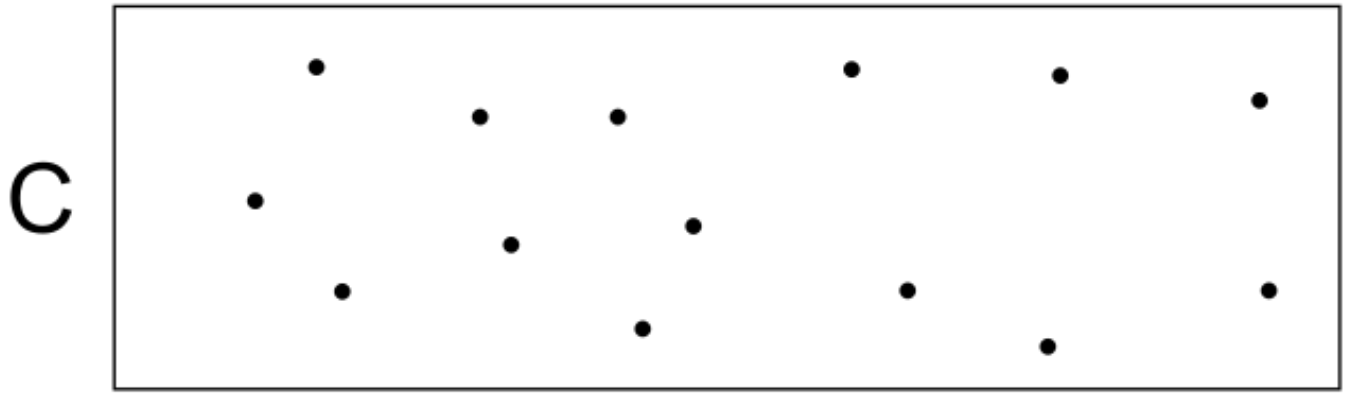
Suppose not. Then, a coalition C of agents must block the TTCA allocation T. Let R be a redistribution of endowments among agents in C that they find Pareto better than T.

TTCA outcome is in the core.

[Shapley and Scarf, *JME* 1974 (attributed to David Gale)]

Suppose not. Then, a coalition C of agents must block the TTCA allocation T. Let R be a redistribution of endowments among agents in C that they find Pareto better than T.



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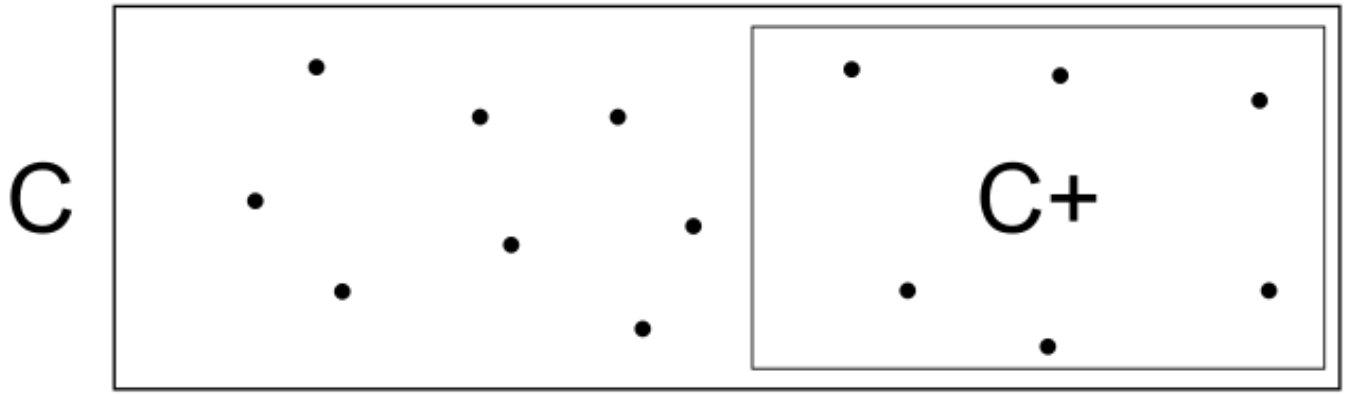
Let C+ $\subseteq$ C be the agents in C who *strictly prefer* R over T.



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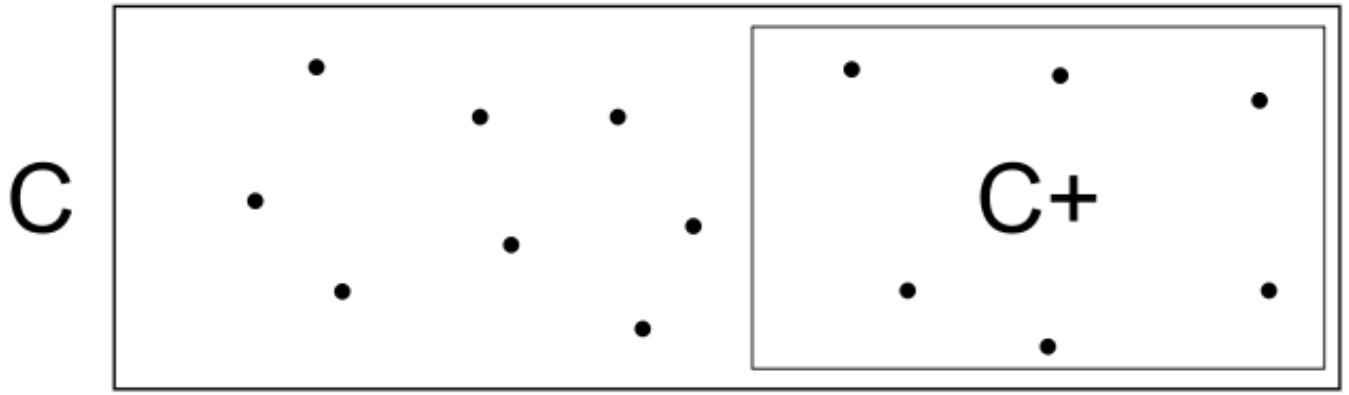

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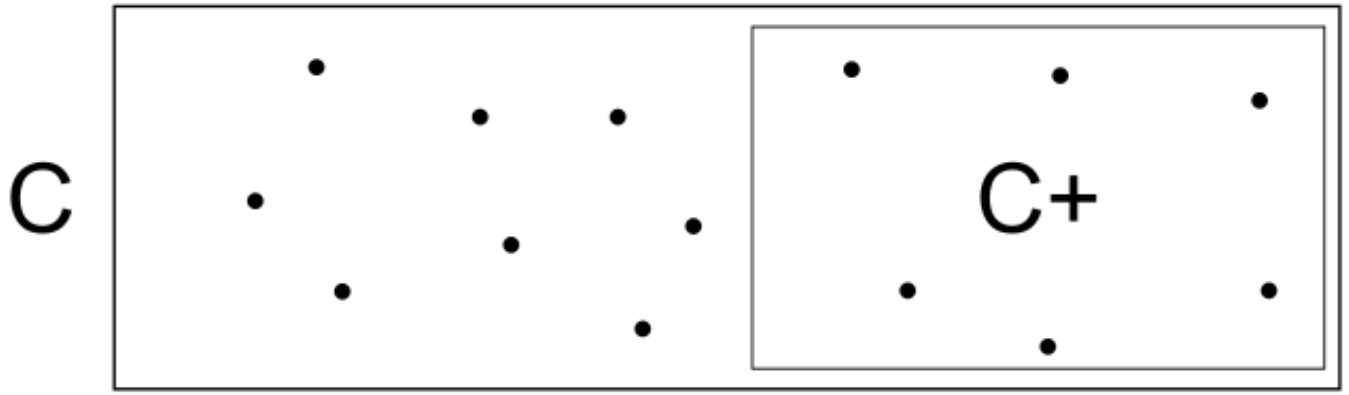
Let x ∈ C+ be the agent in C+ who is the *earliest* to be eliminated under TTCA (say, in round r).



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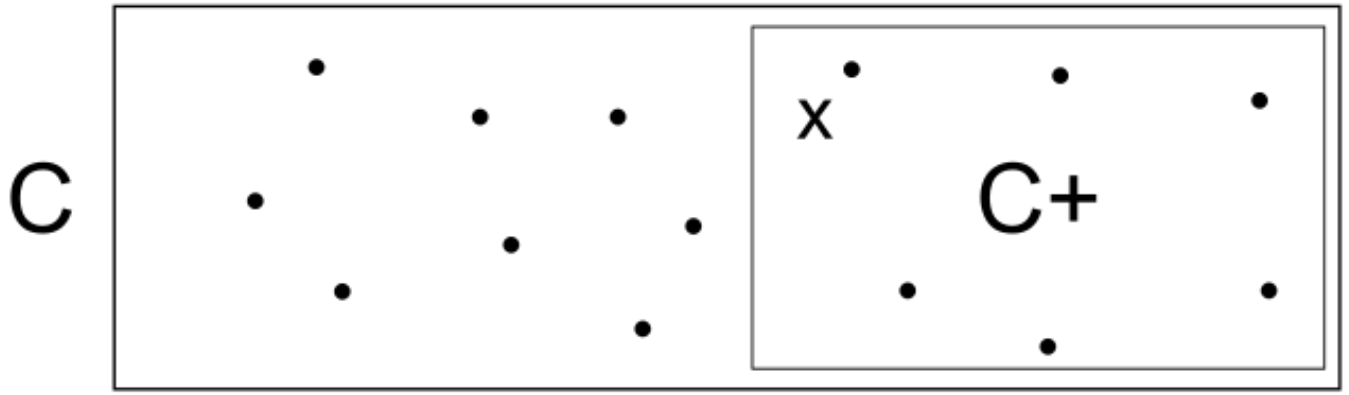

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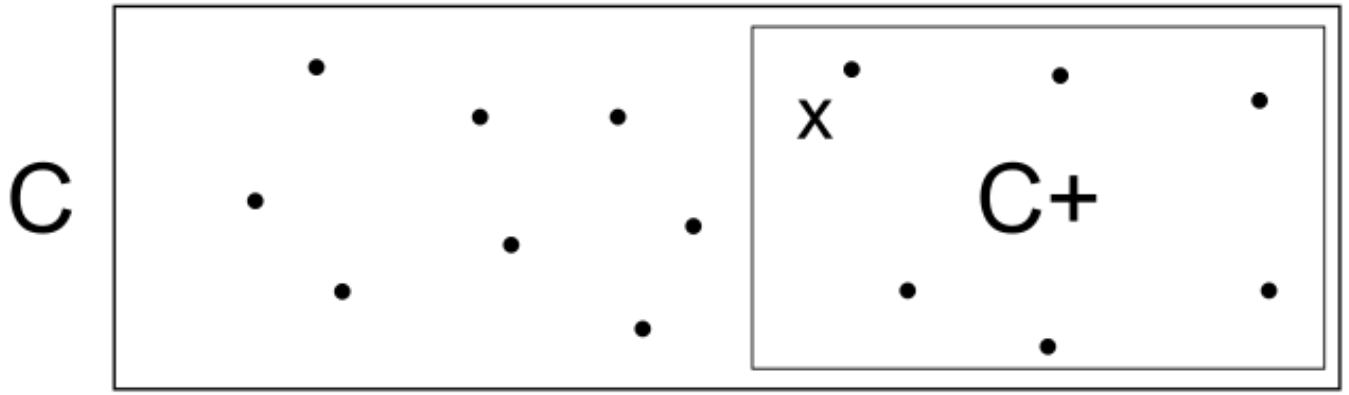
At the time x is eliminated (round r), all agents in C+ are still available under TTCA along with their endowed houses.



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TTCA outcome is in the core.

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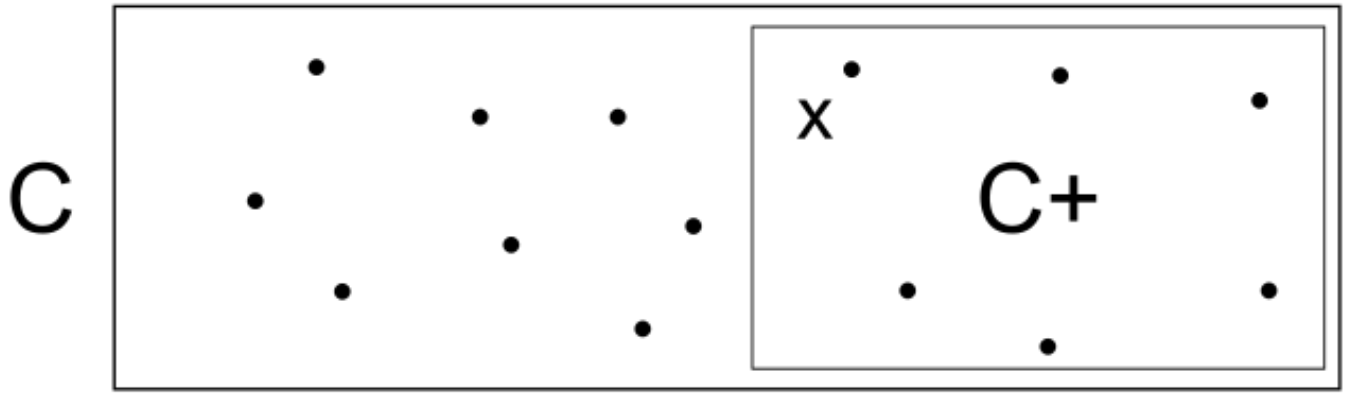
The house x gets under T is at least as good (according to x's preference) as the endowed house of any agent in C+.



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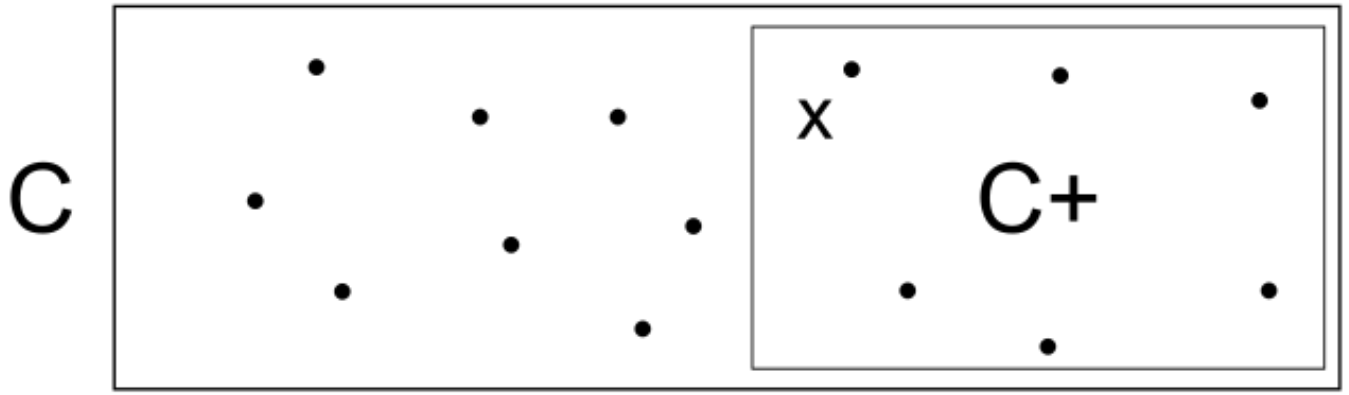

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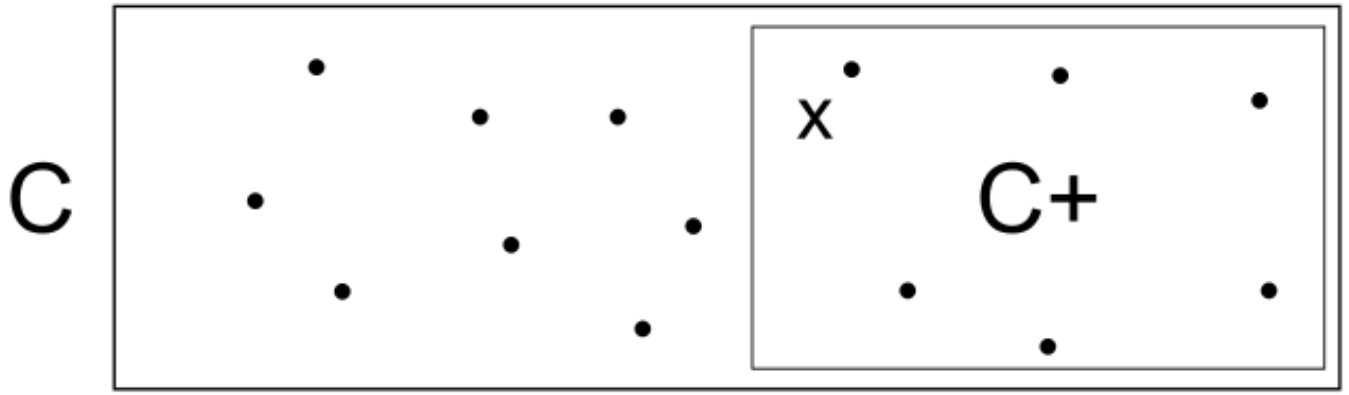

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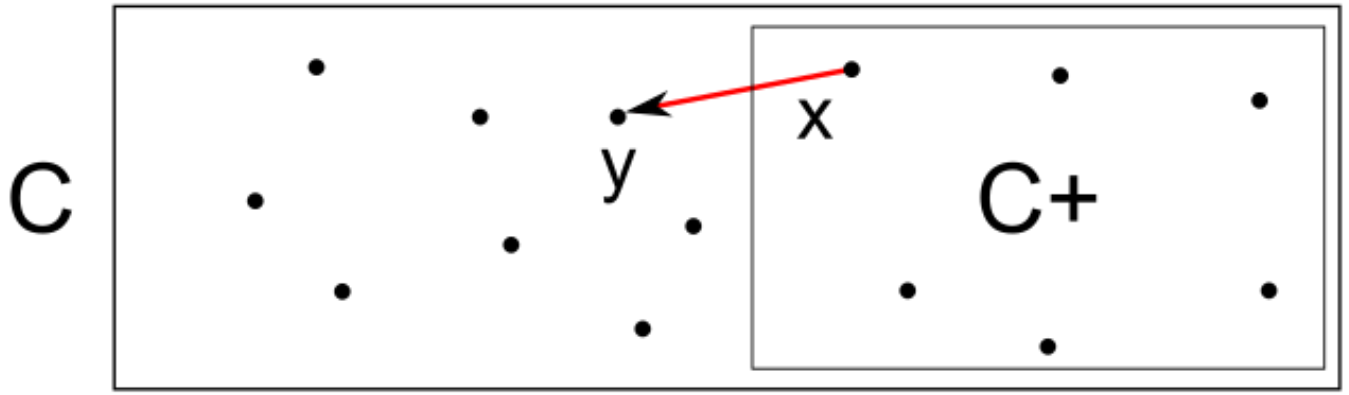

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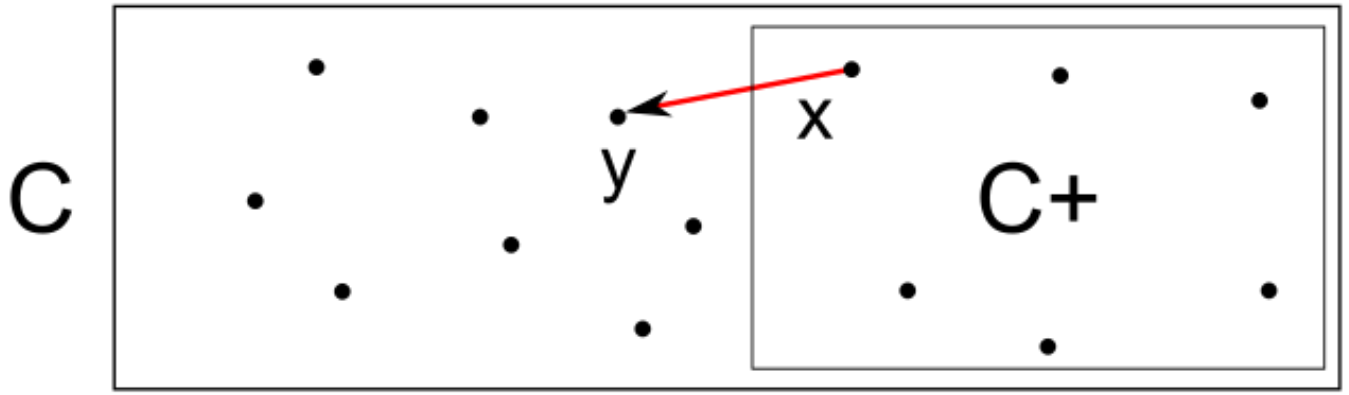
Then, under TTCA, agent y must have been eliminated in round r-1 or earlier (i.e., strictly before x).



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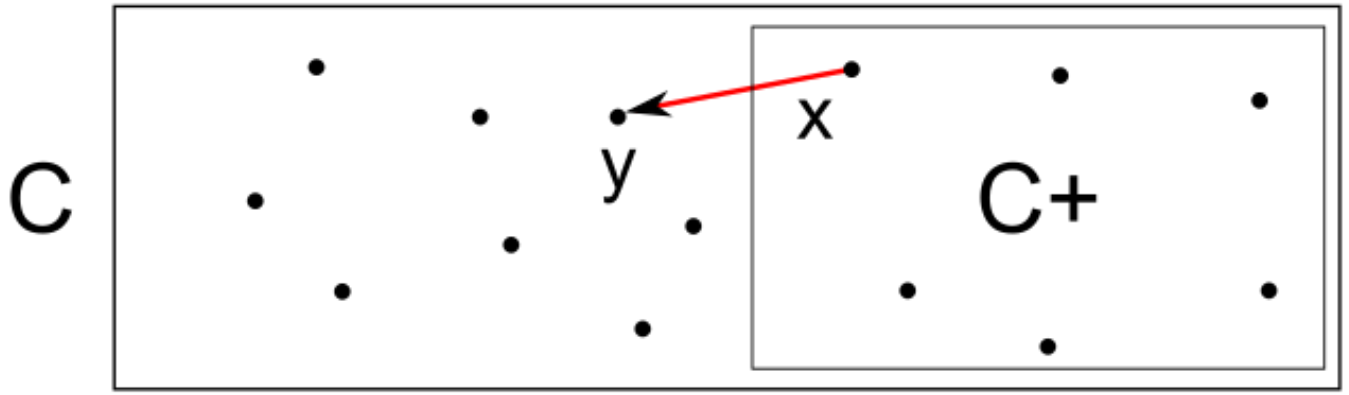
Agent y cannot get its own endowed house under the TTCA outcome T (due to minimality).



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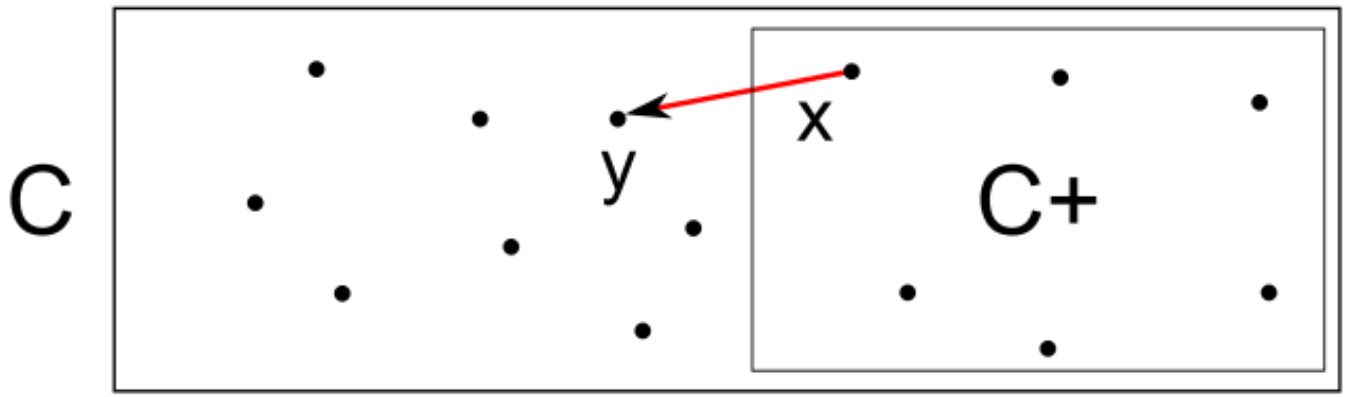
So, under T (as well as R), y gets the endowed house of agent z $\in$ C.



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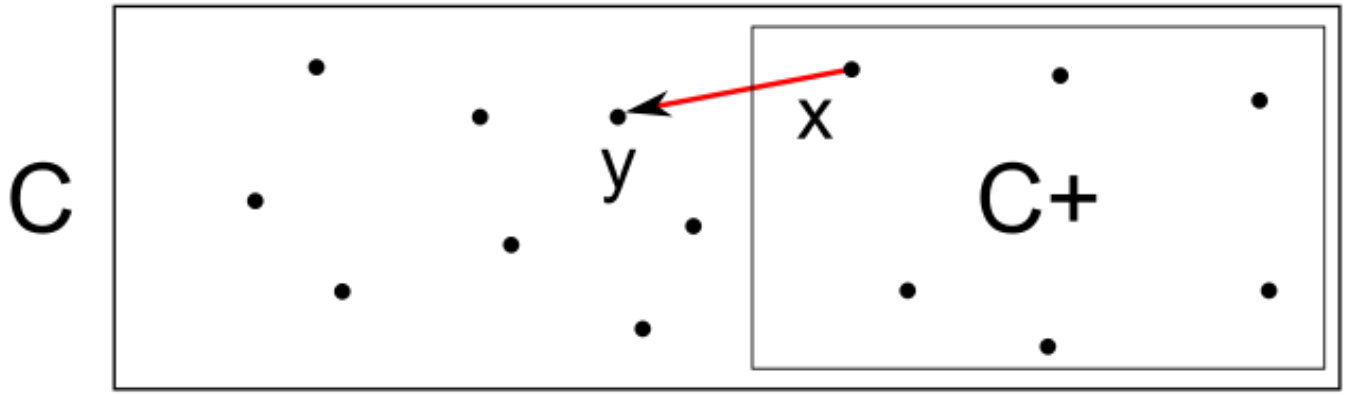
Note that z ∈ C\C+ by semantics of TTCA.



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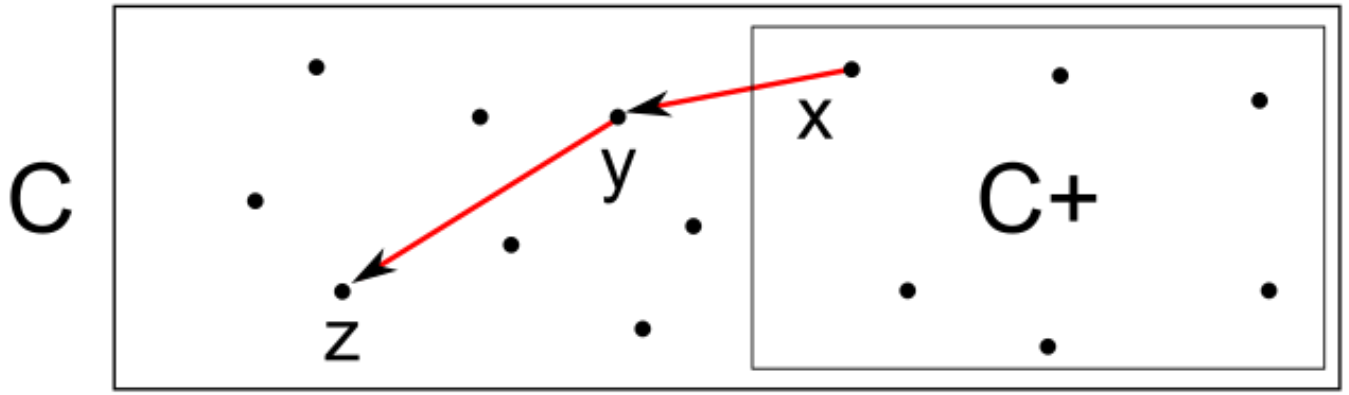

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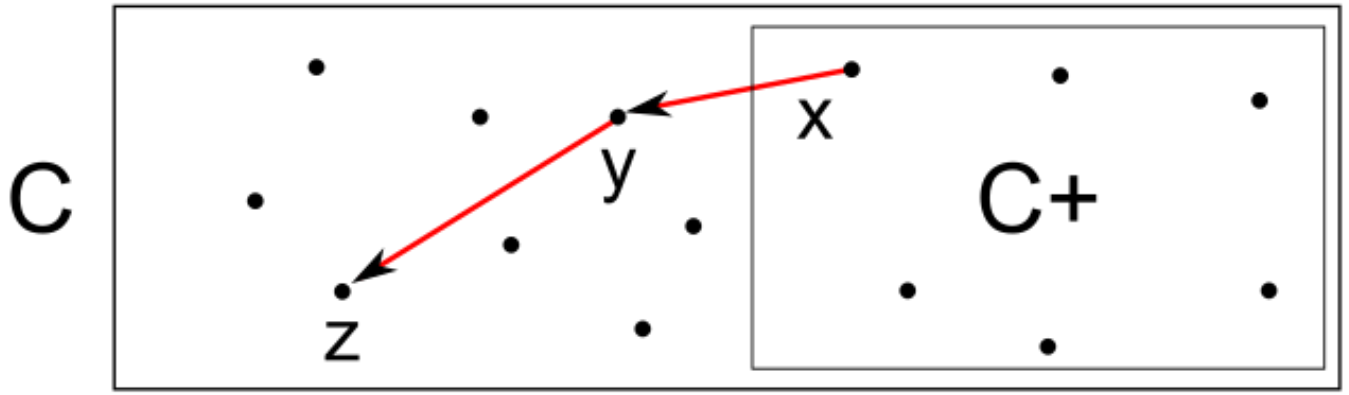

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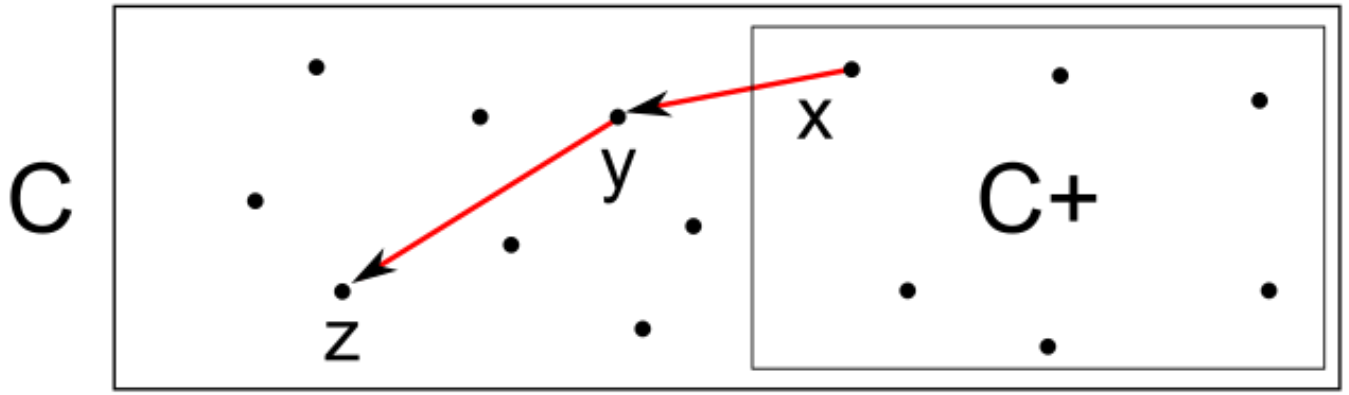
That means agent z gets the endowed house of w $\in$ C\C+ such that w≠y.



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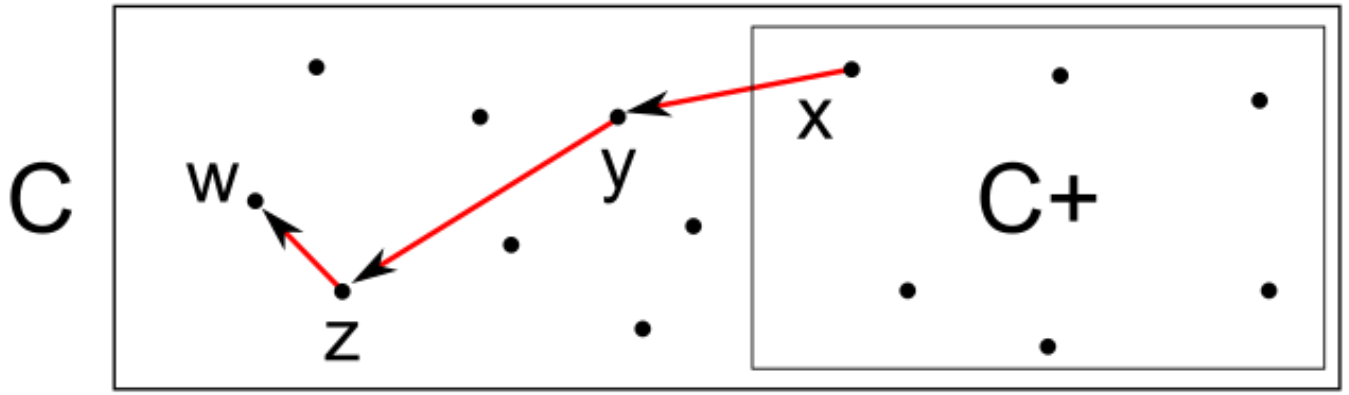

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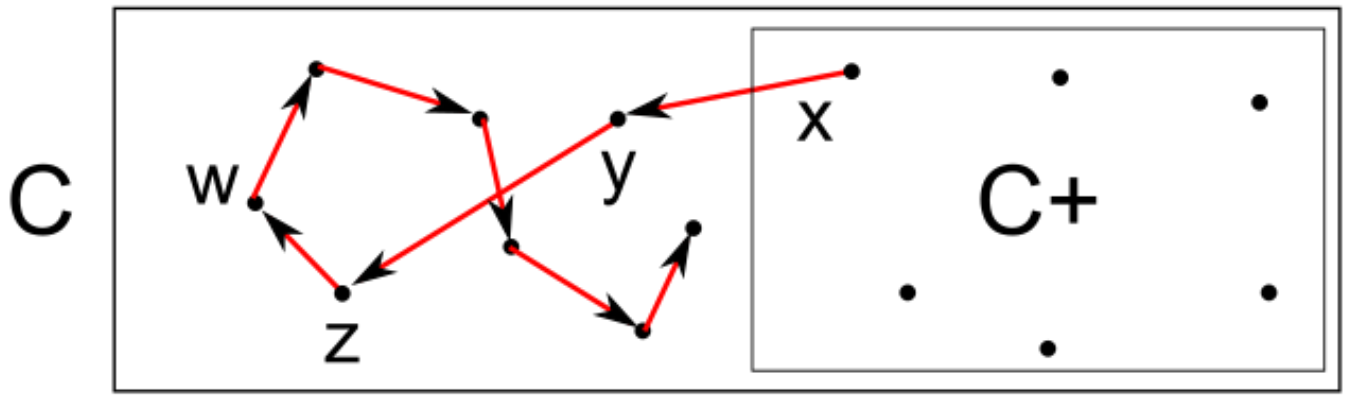
Continuing in this manner, we get an "unending" chain within C\C+.



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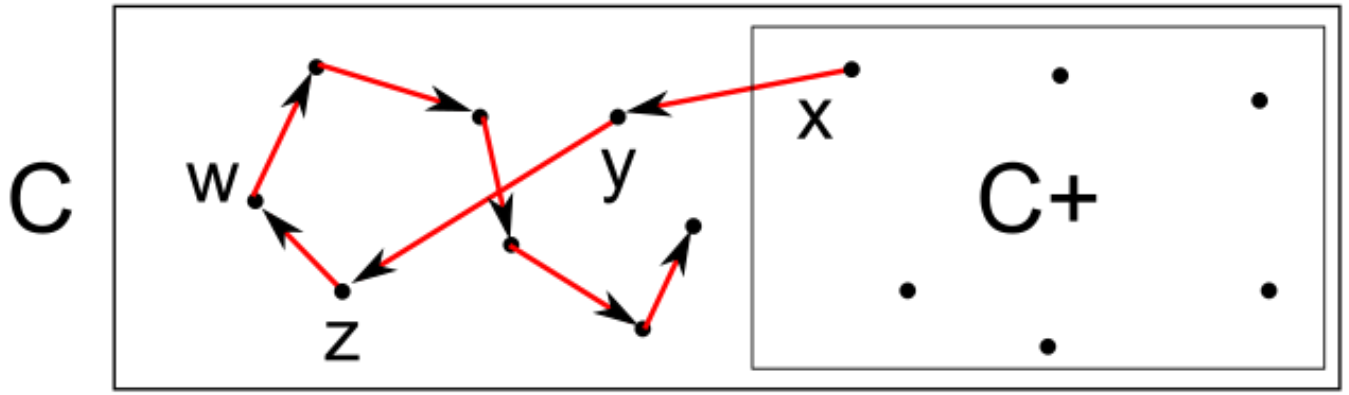
Contradicts the fact that there is a finite number of agents.



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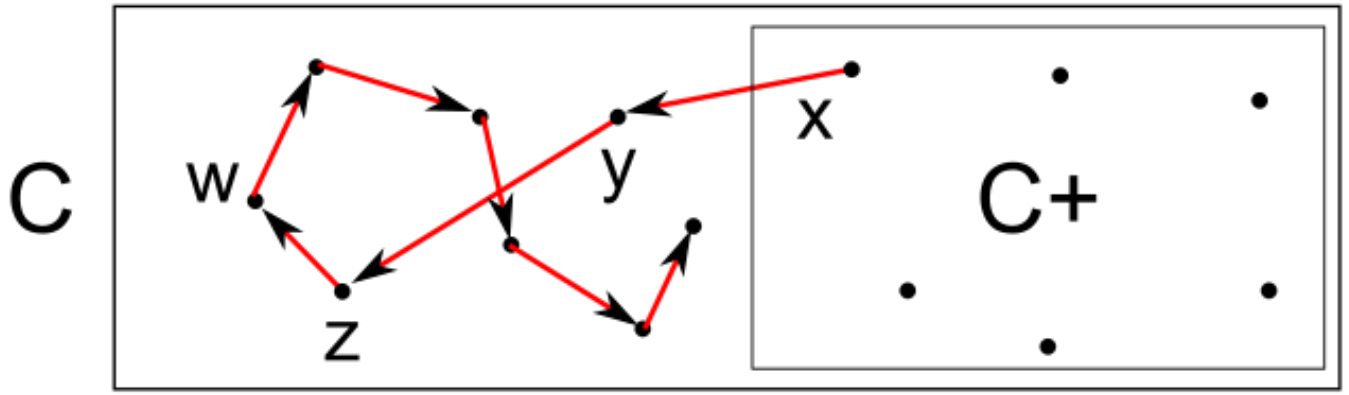

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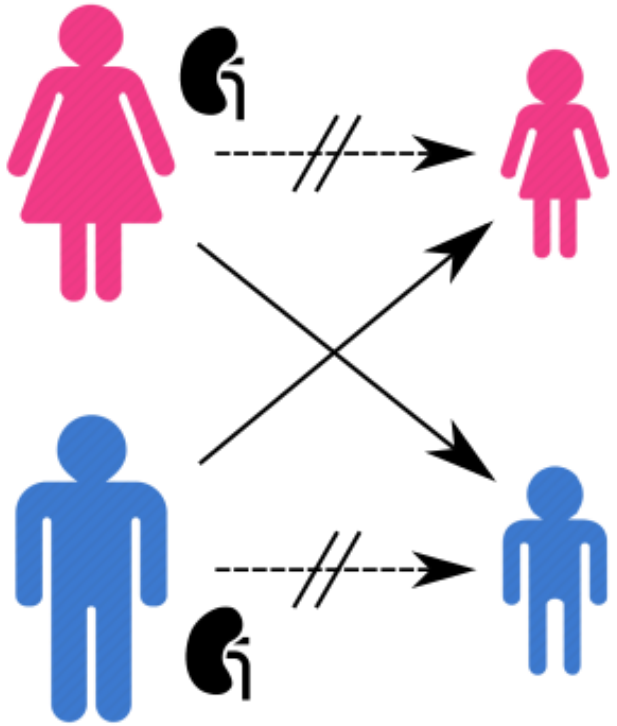
In fact, TTCA recovers the unique core outcome.

[Discuss on Teams channel or on Piazza]

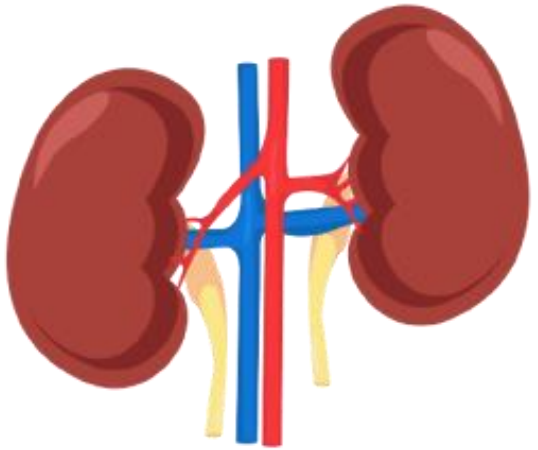
Application of TTCA



Kidney Exchange

Kidney Exchange

Many people suffer from kidney failure
(waiting list in US: ~90,000; in India: ~200,000).



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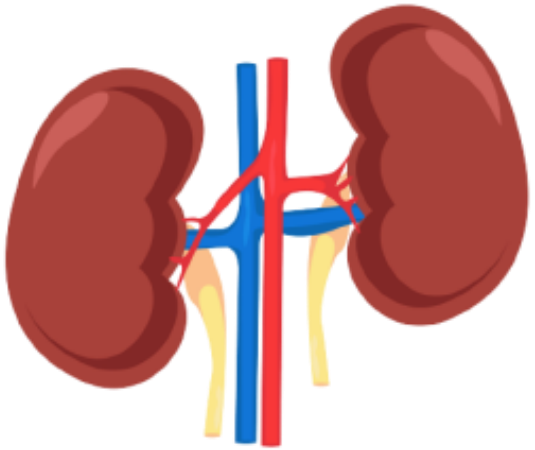

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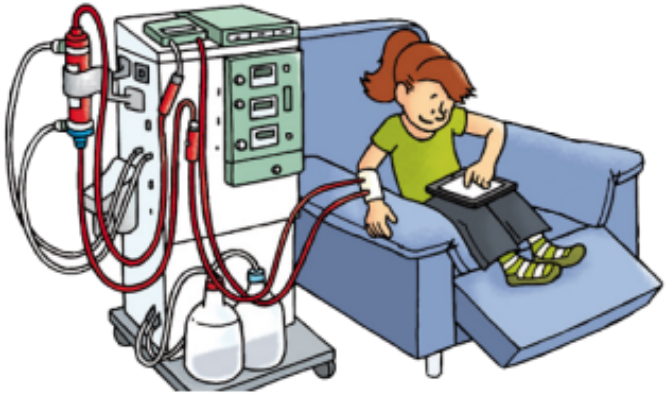
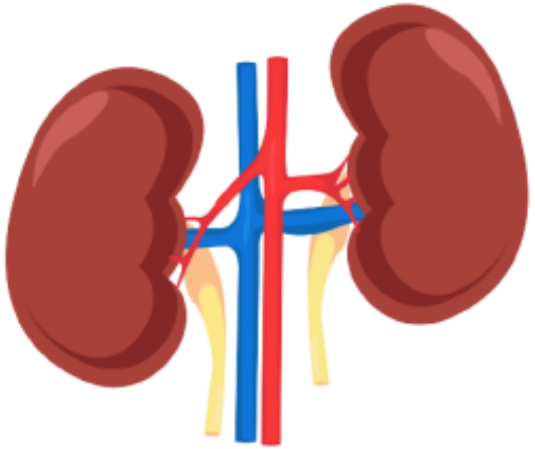


Dialysis

Kidney Exchange



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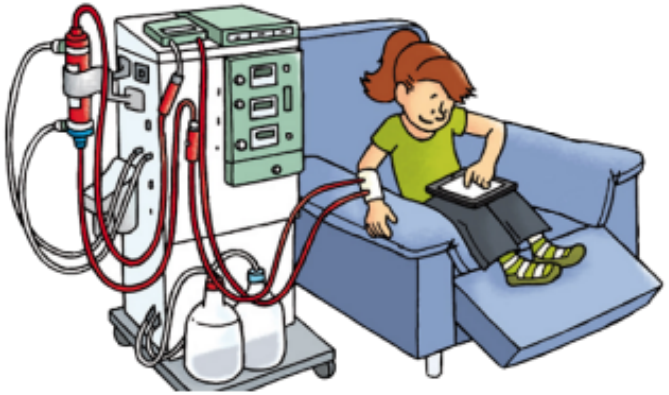

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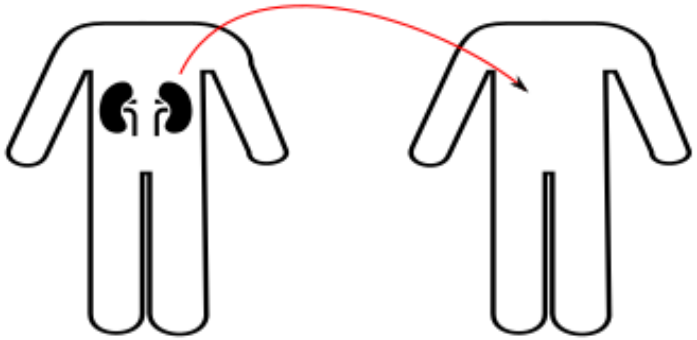
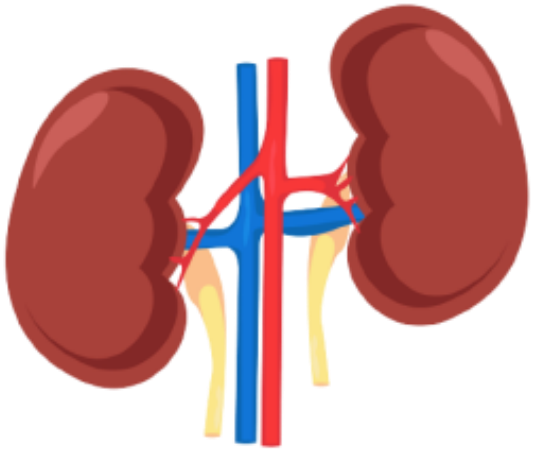


Transplant

Kidney Exchange



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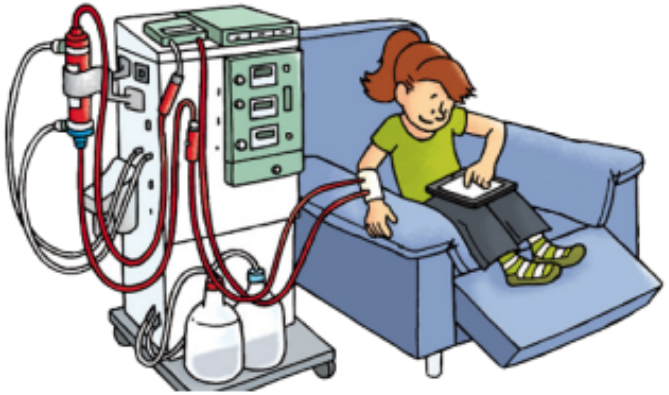
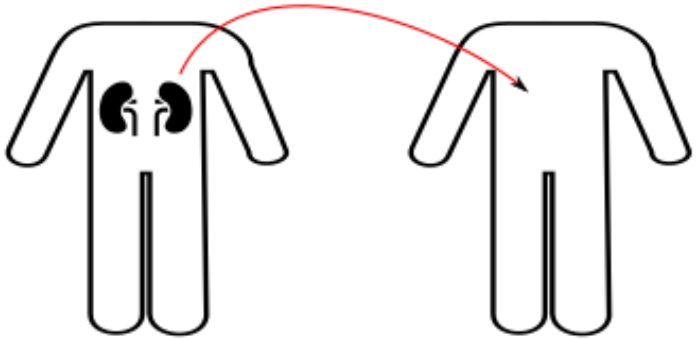




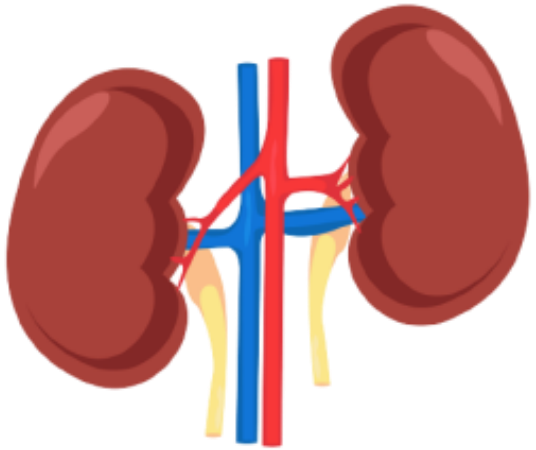
Dialysis

Poor quality of life
High cost

Transplant

Better life quality
Lower cost

Kidney Exchange



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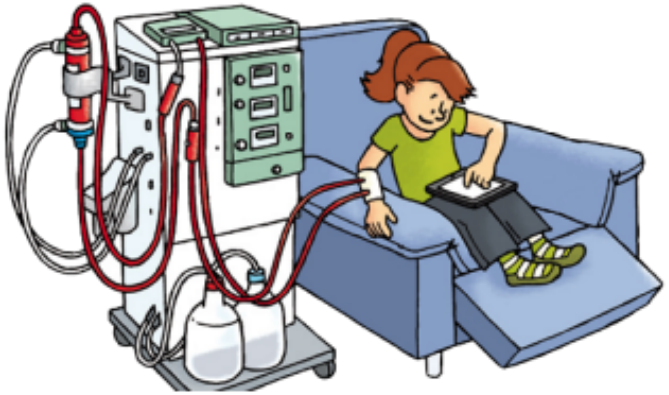

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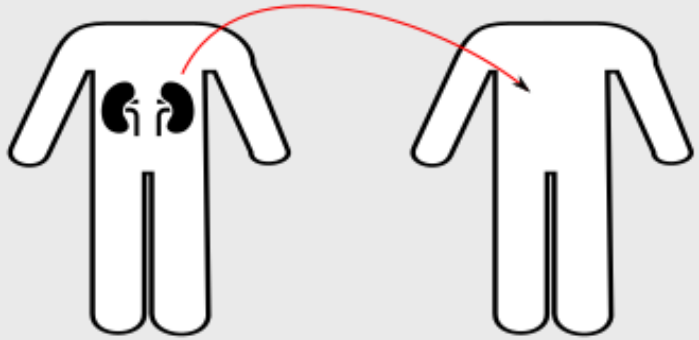

Transplant

Better life quality
Lower cost

- From a deceased donor
- From a living donor

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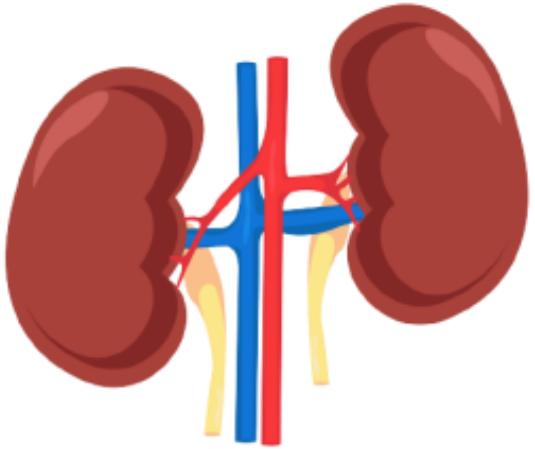

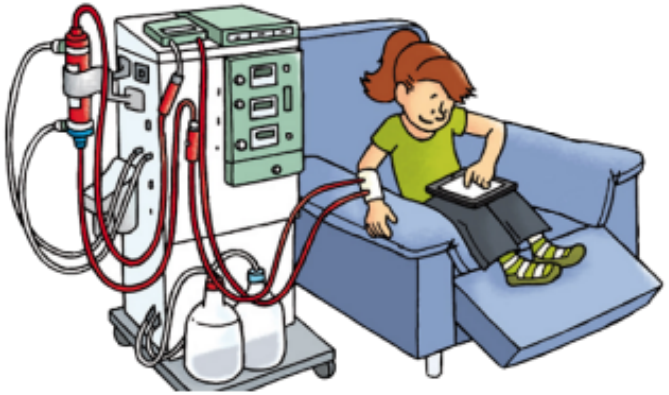

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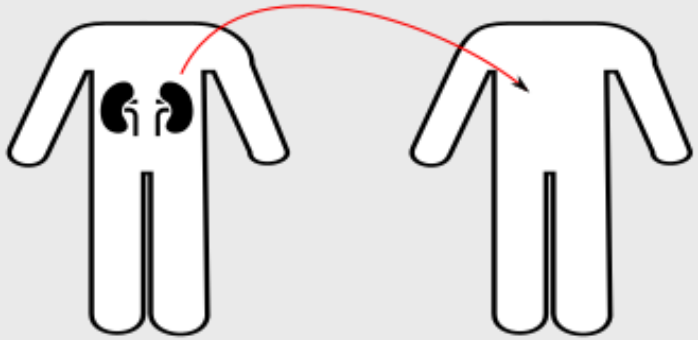

Transplant

Better life quality
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- From a deceased donor

- From a living donor

E.g., first-degree relatives, spouse etc.

Kidney Exchange

Simply having a willing donor is not enough.
The organ must be medically compatible.

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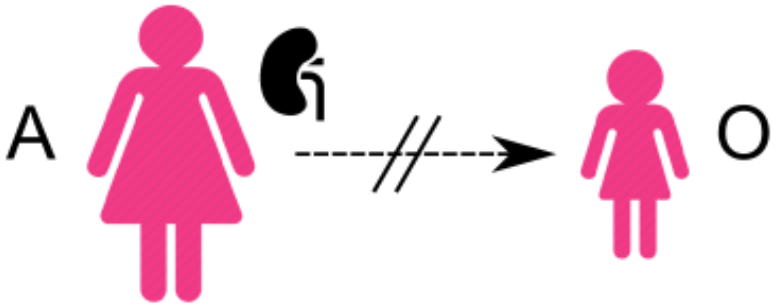
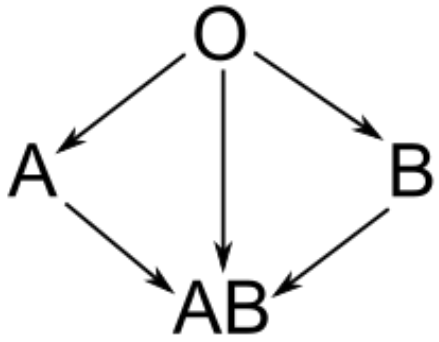


Blood-type incompatibility



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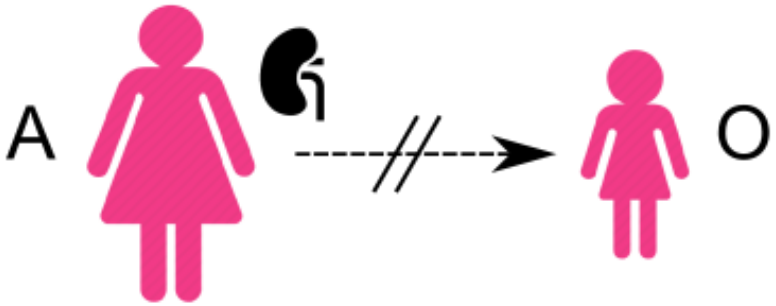

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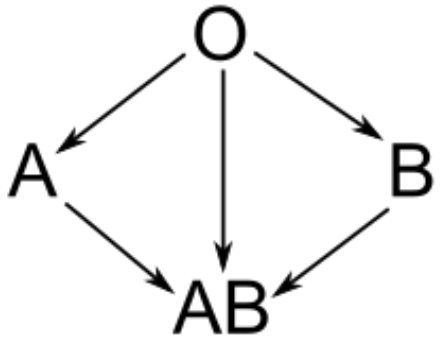
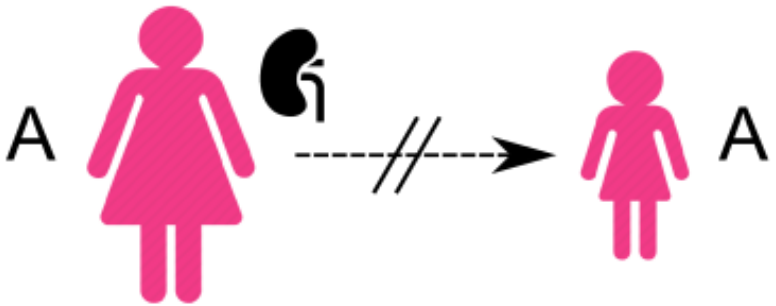


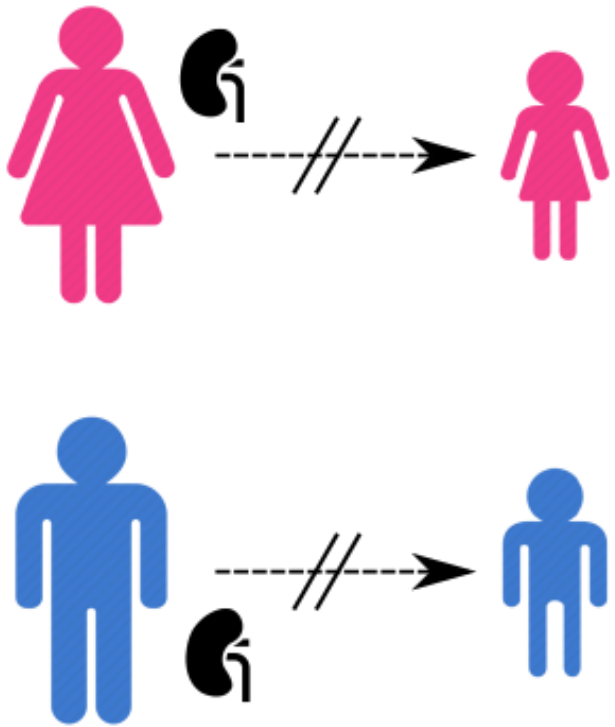


Tissue-type incompatibility

Sensitized patients with antibodies that will attack a foreign tissue (often due to prior exposure during blood transfusion, pregnancy, previous transplants).

Kidney Exchange

[Rapaport, *Transpl. Proc.* 1986; Ross et al. *NEJM* 1997]



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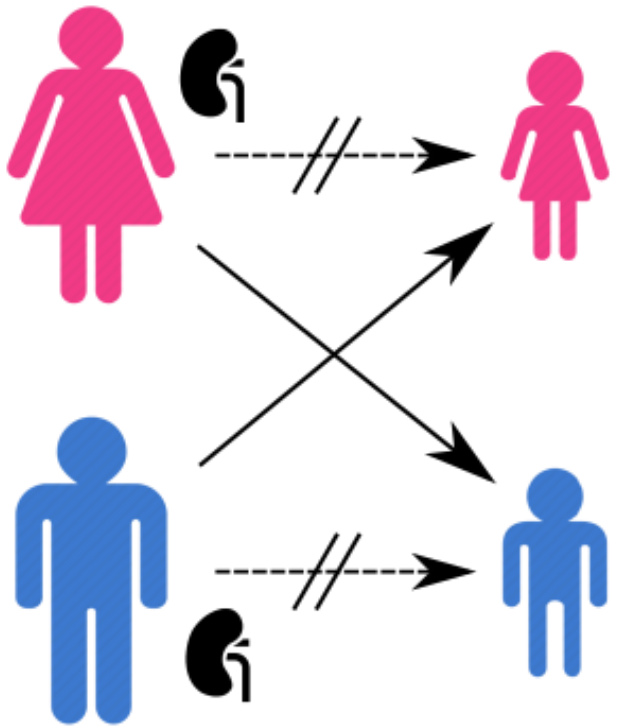

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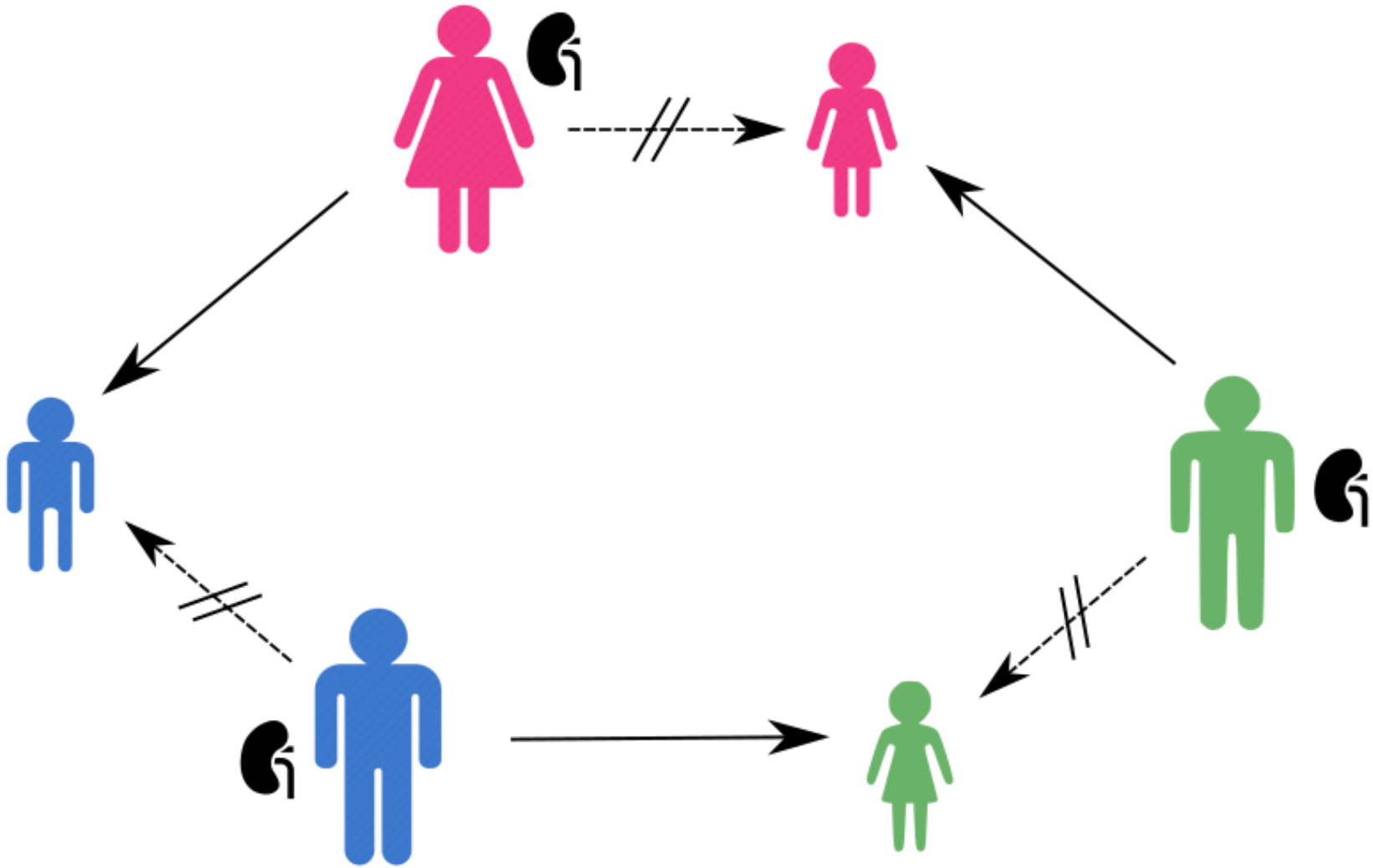

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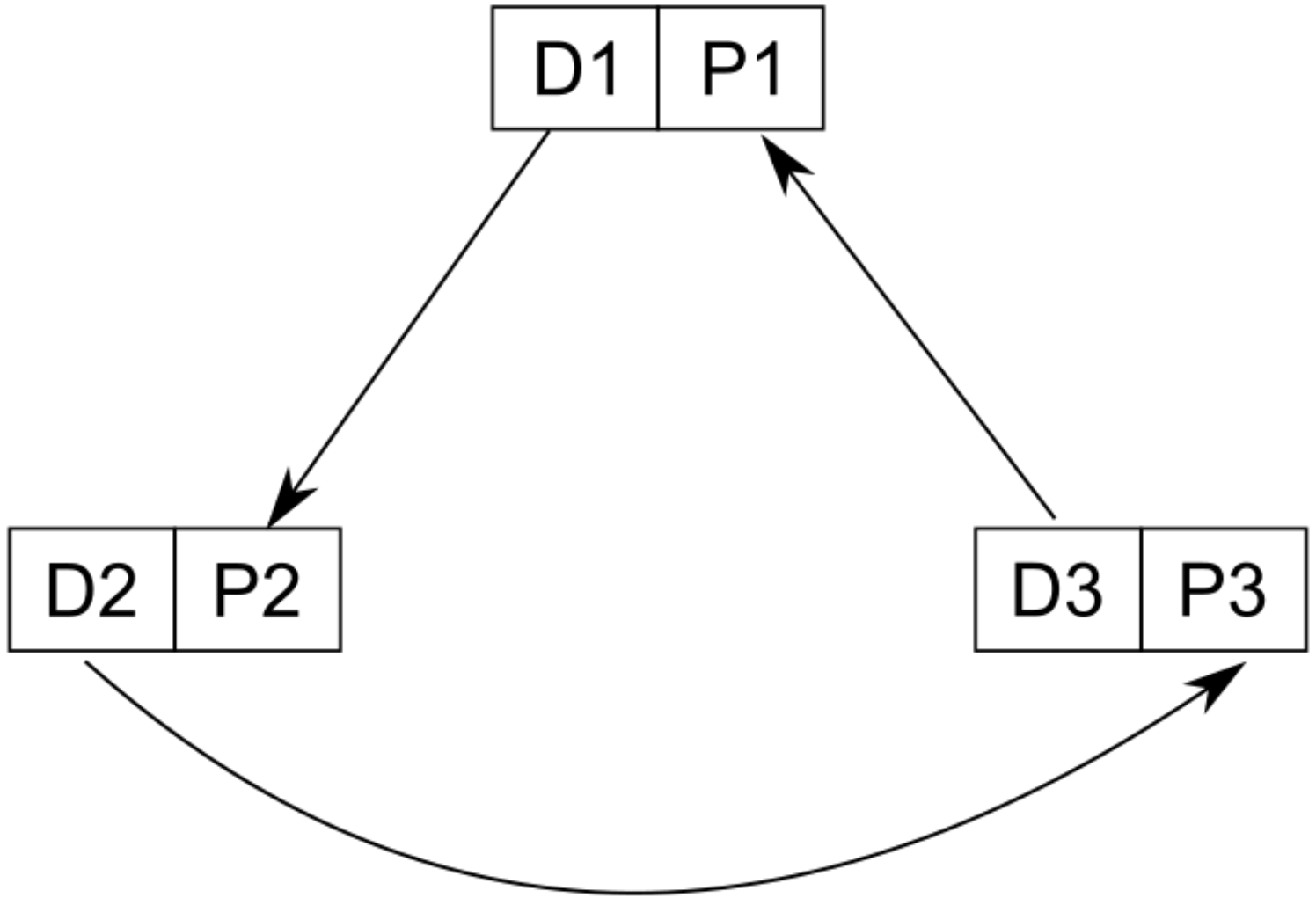

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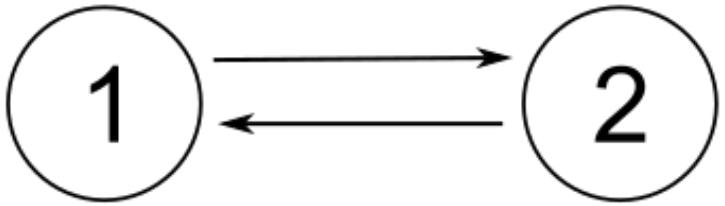

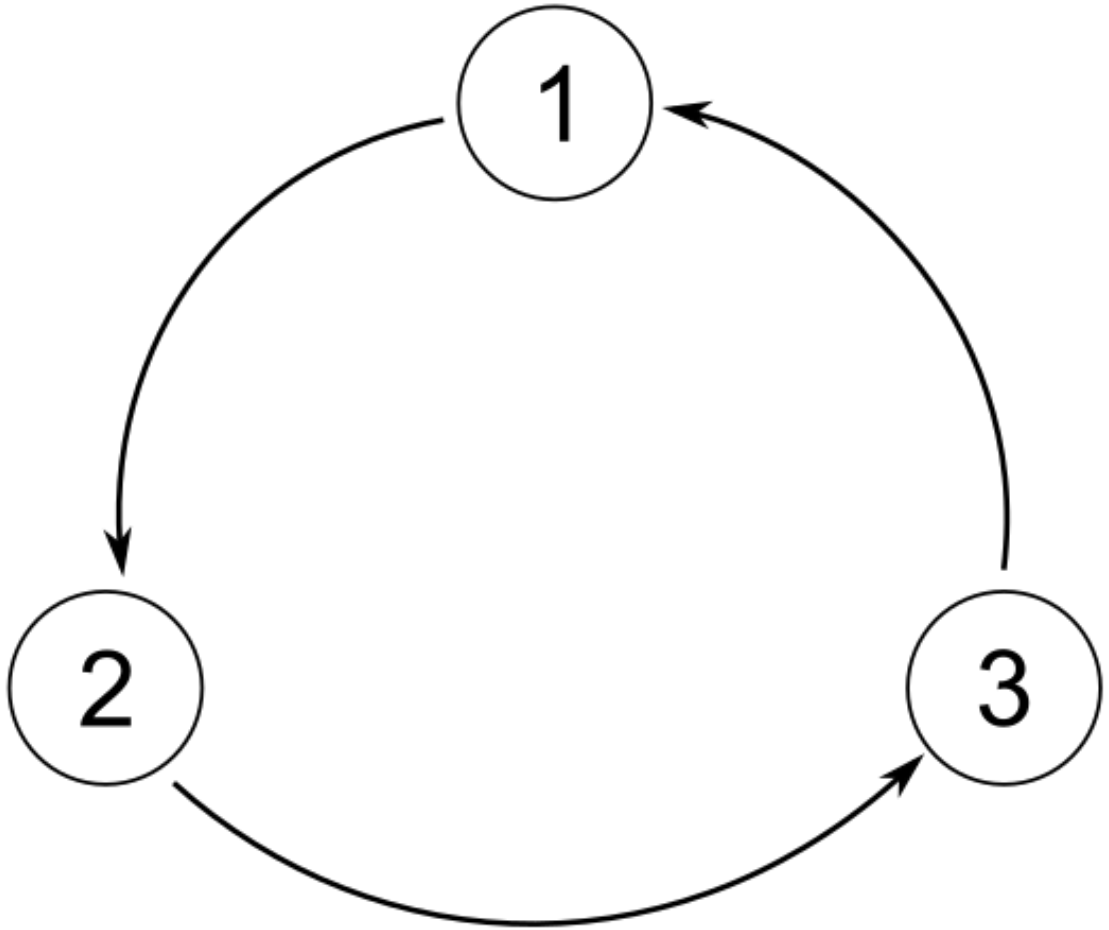

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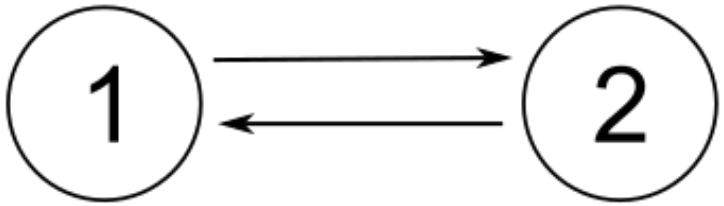

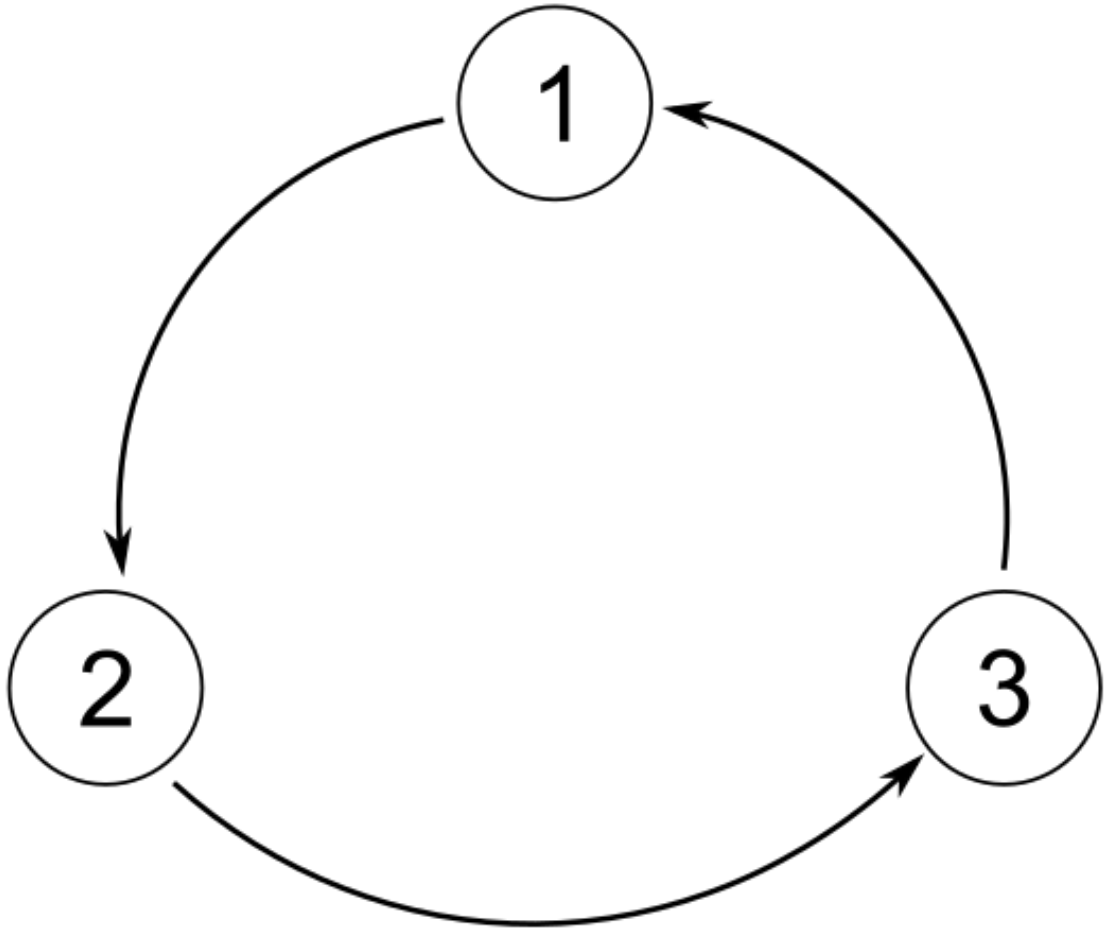

Intuitively, having more and more of such willing-but-incompatible pairs will create more opportunities for an exchange.

History [edit]

Year	Event
1986	KPD first suggested by Rapaport[14]
1991	First KPD program started in South Korea by Dr. Park[8]
1999	First European KPD transplants performed in Switzerland[8]
2000	First KPD transplants performed in USA at the Rhode Island Hospital[8]
2001	Hopkins completes KPD transplants and begins first KPD program in USA[15]
2004	Dutch established first national KPD program[8]
2007	NEAD chain started by APD utilizing first bridge donor[16]
2007	Charlie Norwood Living Organ Donation Act clarifies legality of KPD in USA [17]
2008	National Kidney Registry organizes its first transplants on Valentine's Day[18]
2009	Hopkins leads first 16 patient multicenter Domino Chain [19][20]
2010	UNOS organizes its first KPD transplants[8]
2012	National Kidney Registry completes largest chain involving 60 participants[6]
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Early Proposals for Kidney Exchange

Proposal 1

Using TTCA

Proposal 2

?

Proposal 1: Kidney Exchange using TTCA

[Roth, Sönmez, and Ünver, *QJE* 2004]

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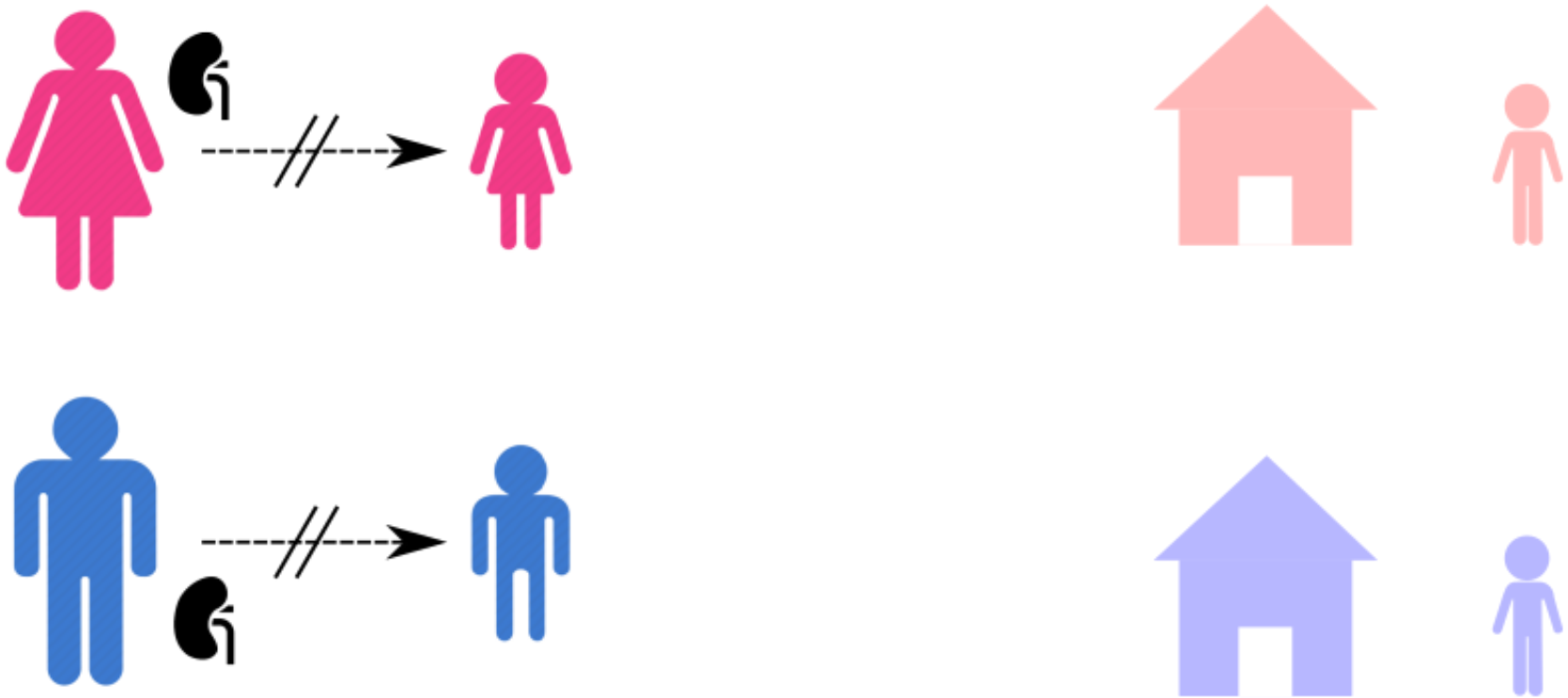

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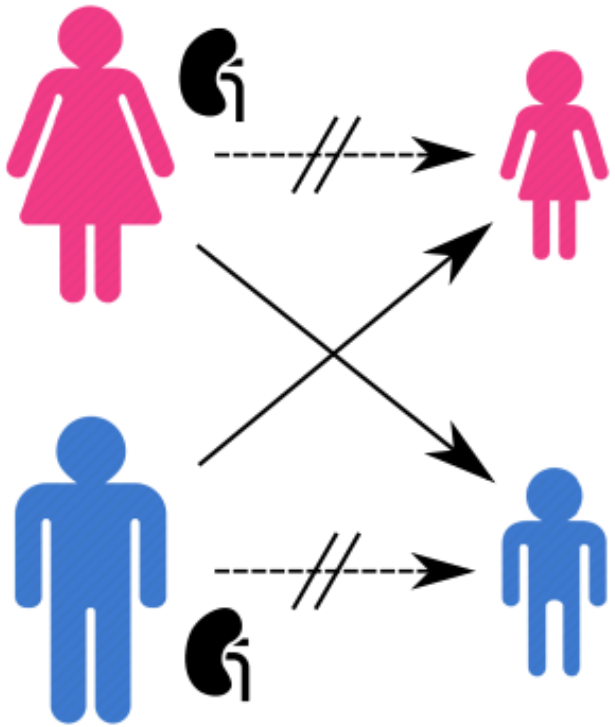

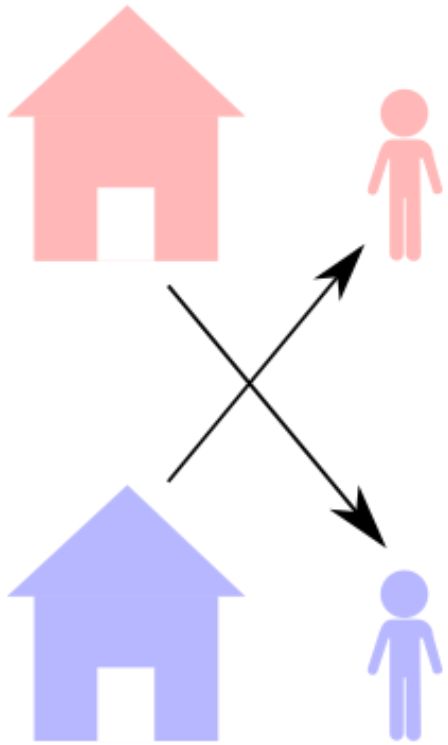

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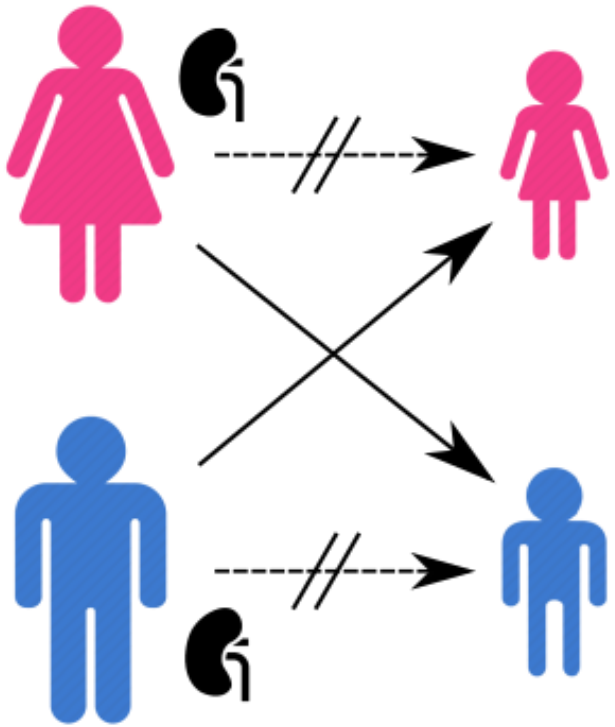

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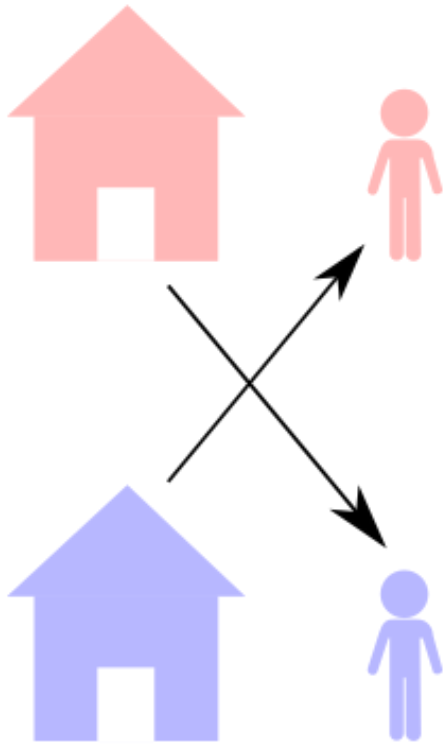

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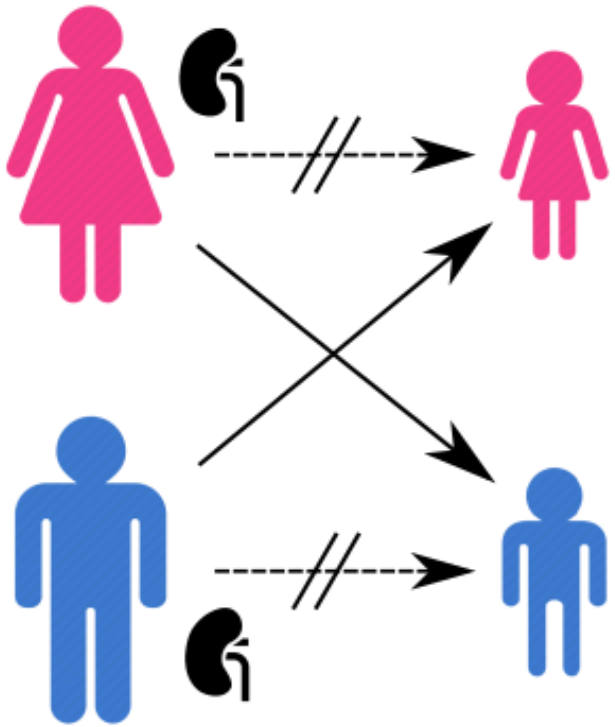

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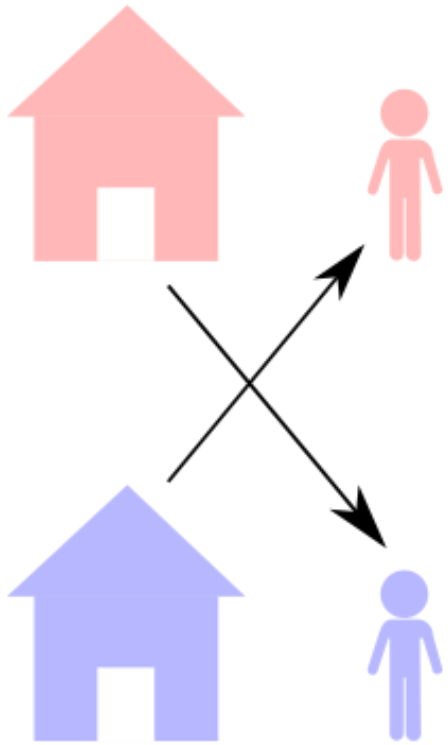

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Advantages of Kidney Exchange using TTCA

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No group of patients can strictly improve
by reallocating donors among themselves.

Every patient is (weakly) better off than before.

Polynomial time.

Limitations of Kidney Exchange using TTCA

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In practice, there may be:

- patients without a donor ("agents without a house")
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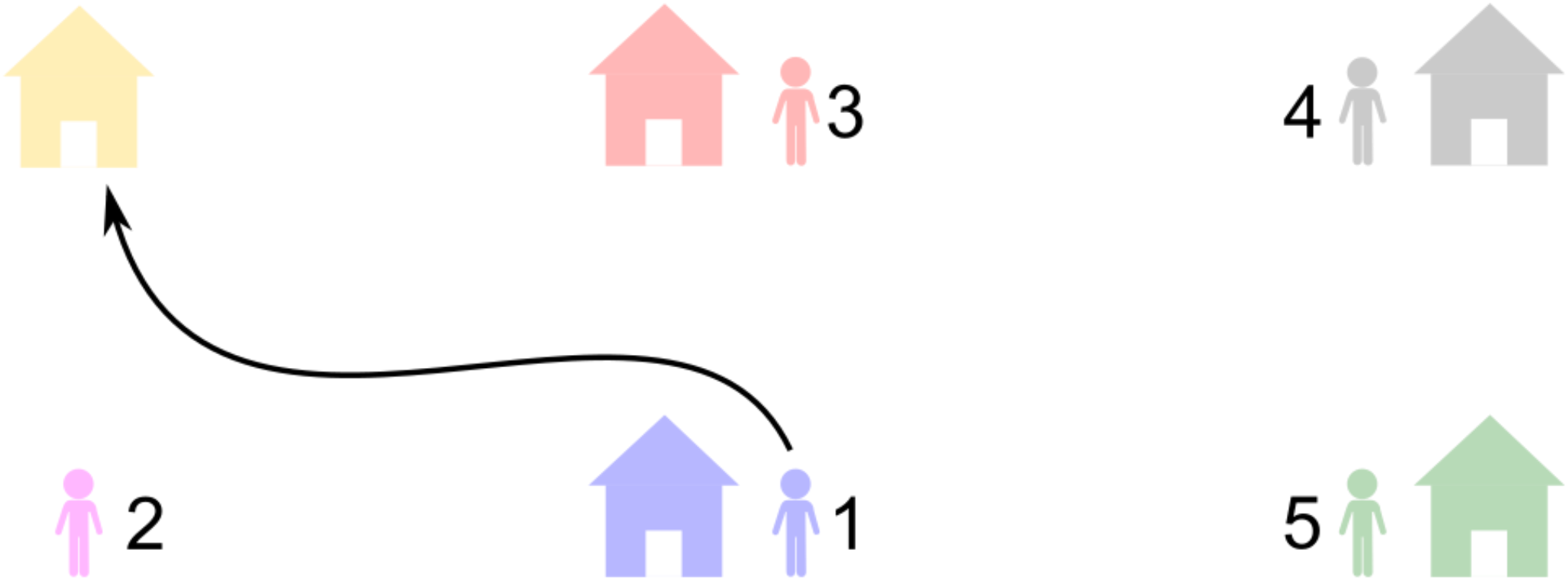

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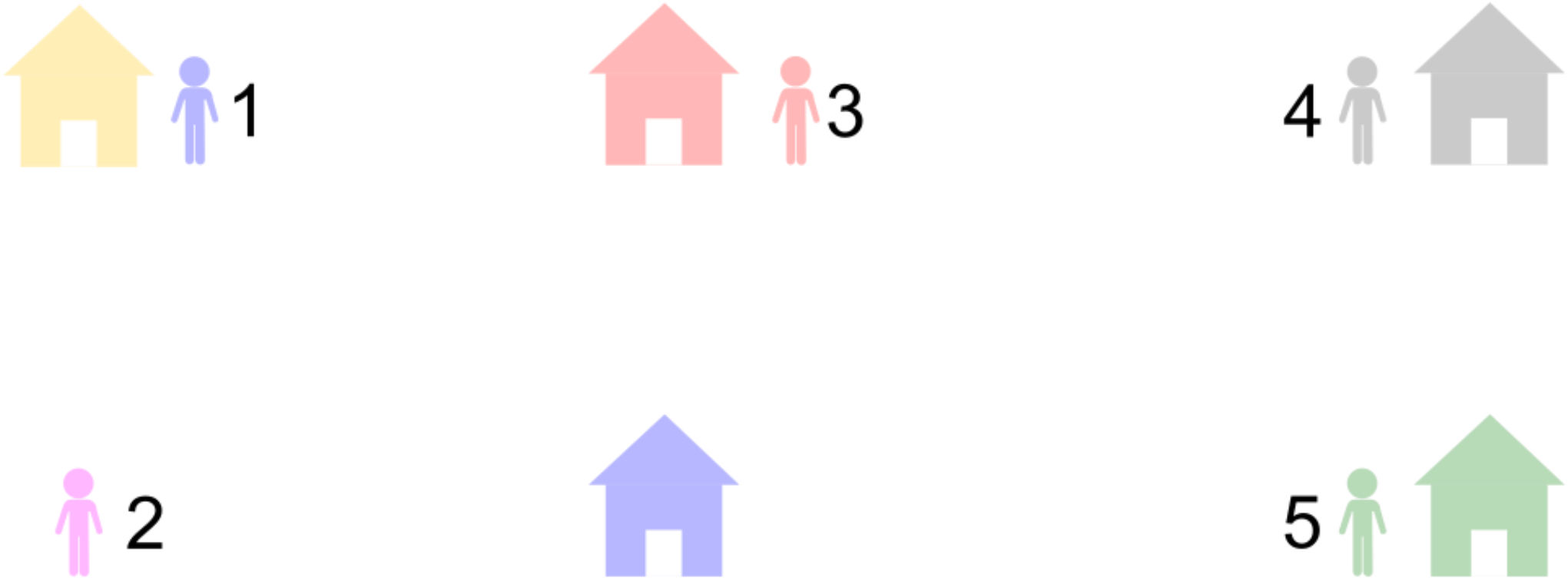

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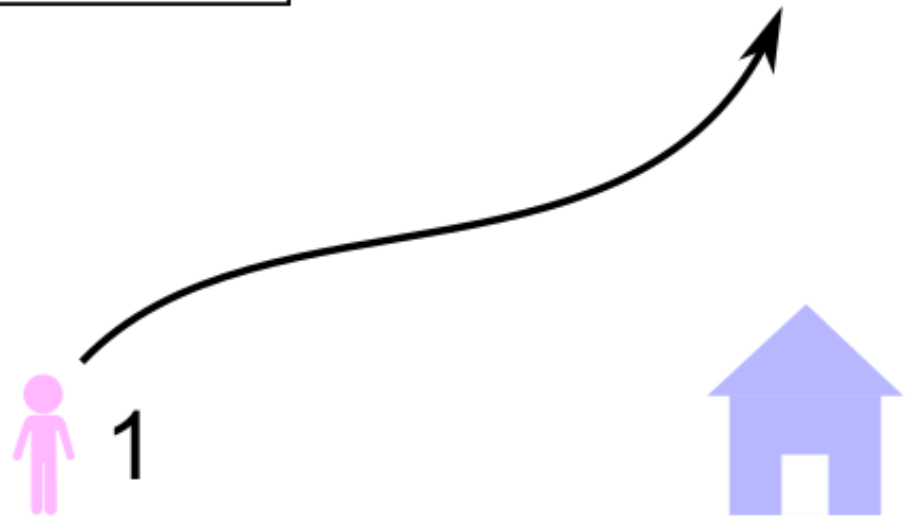

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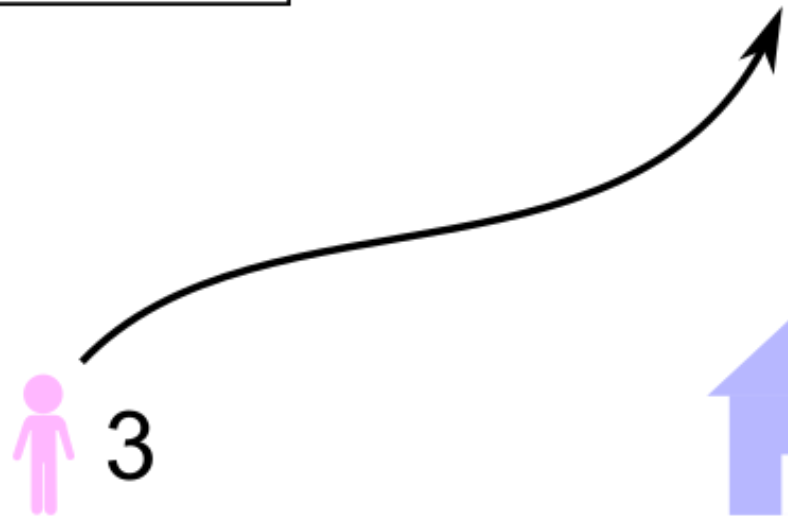

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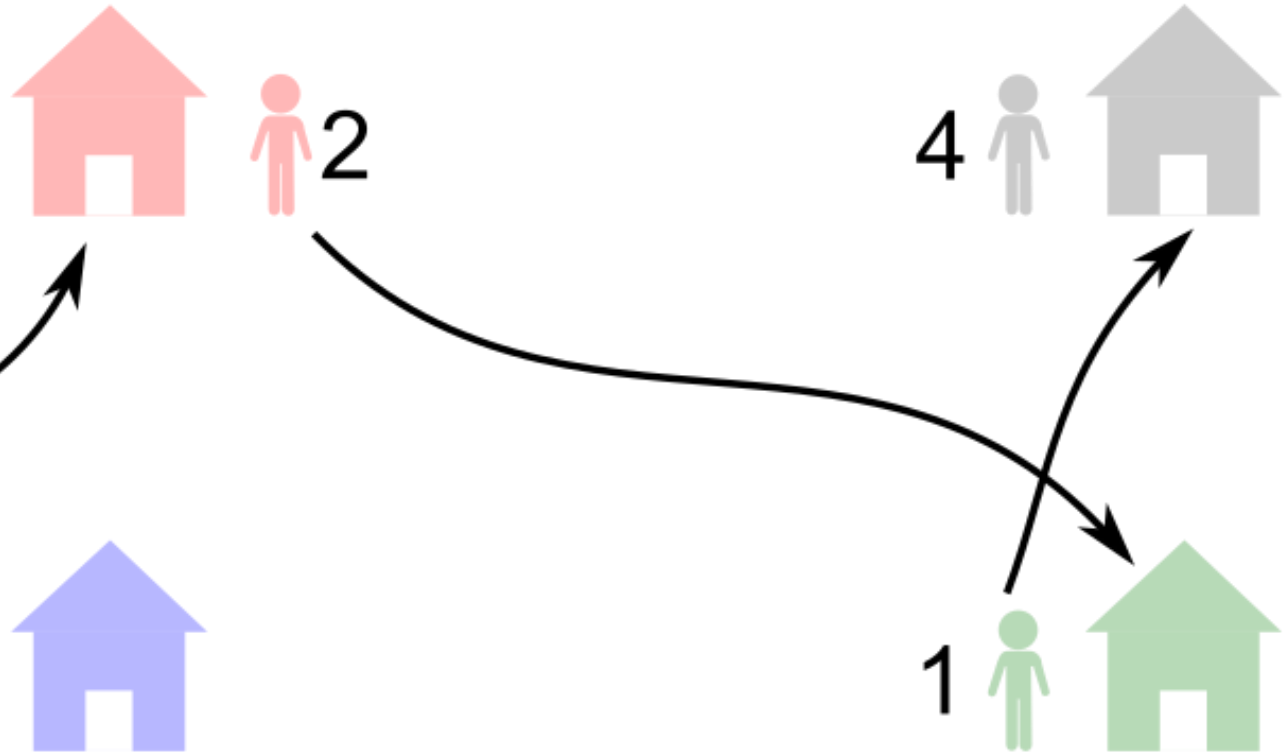

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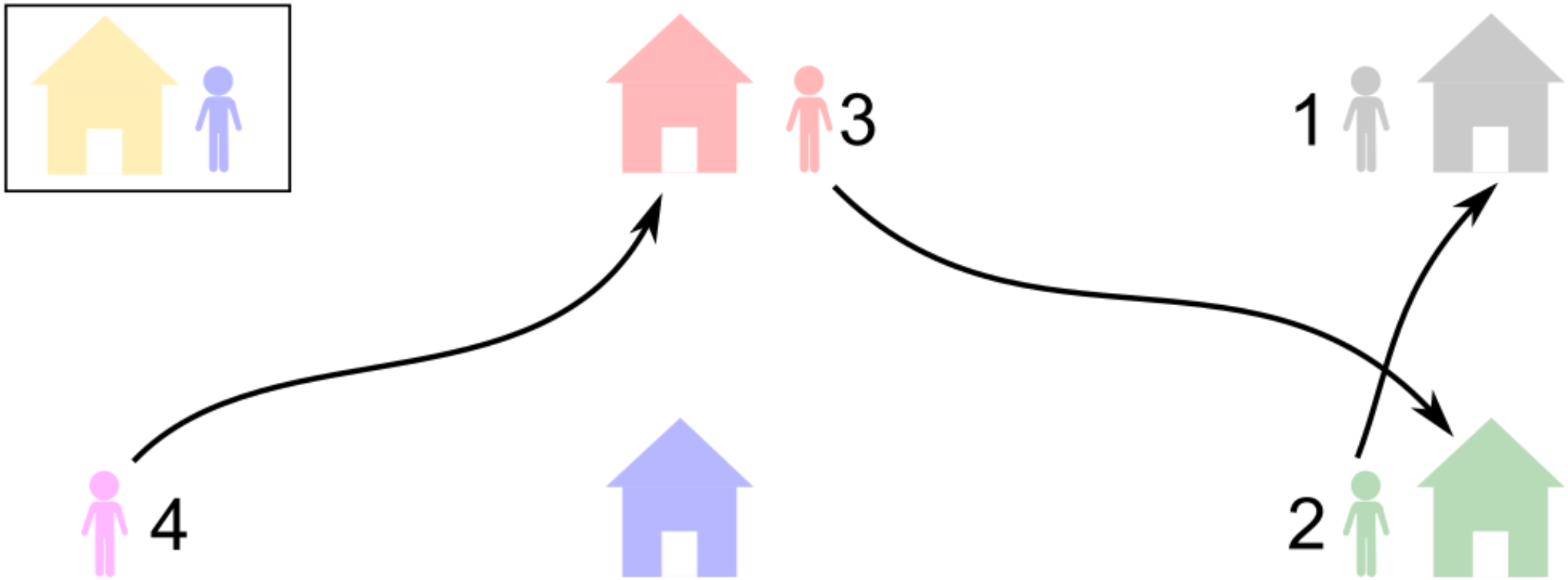

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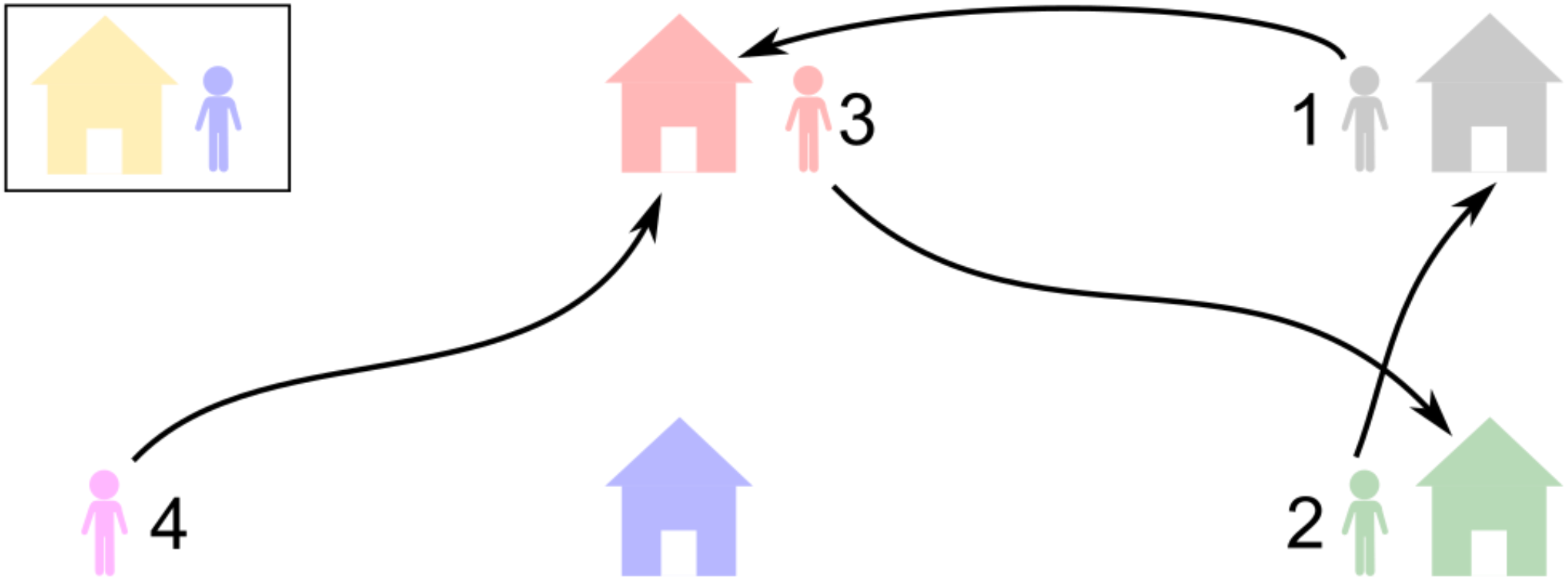

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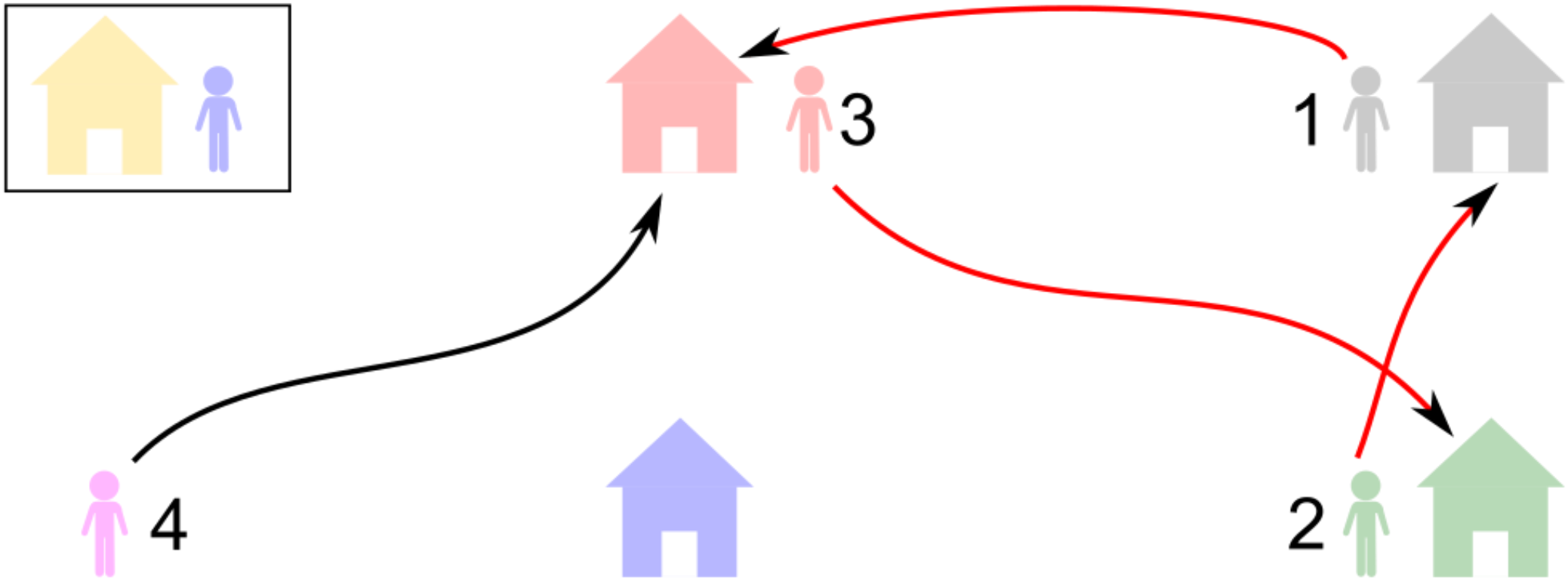

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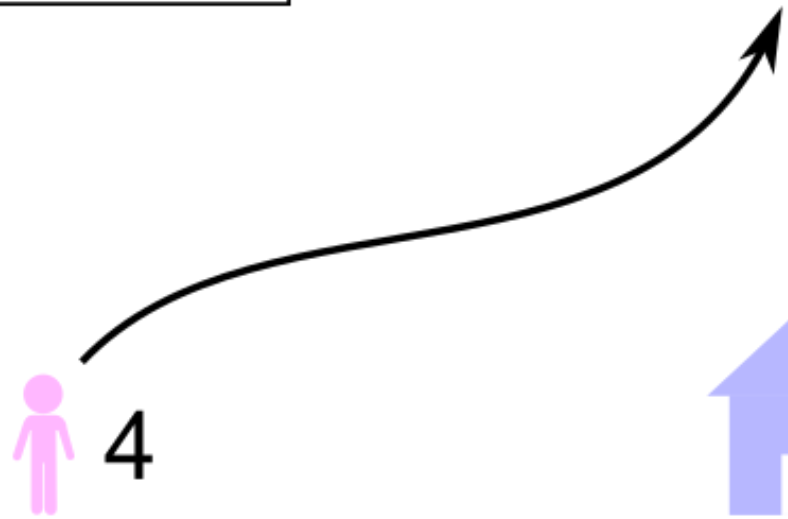

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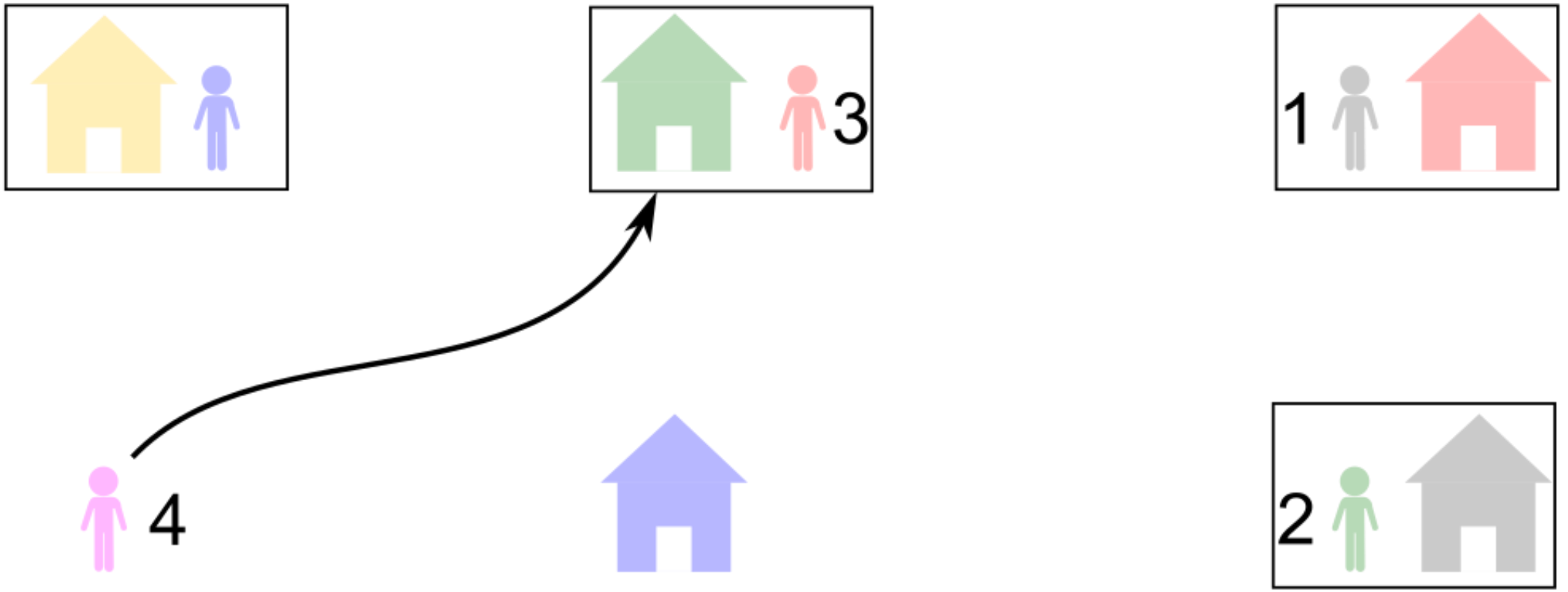

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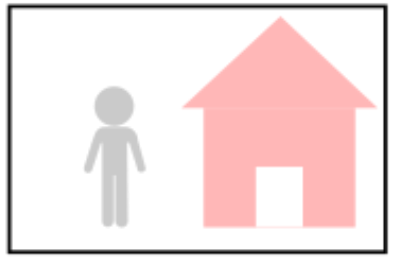

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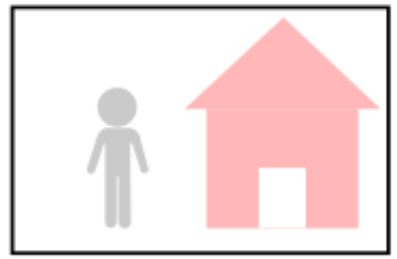

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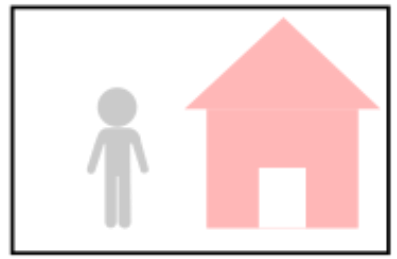

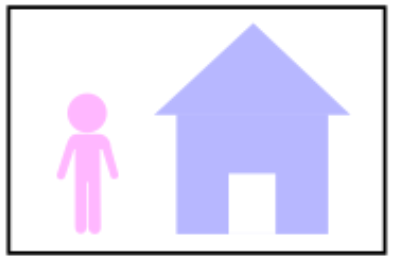

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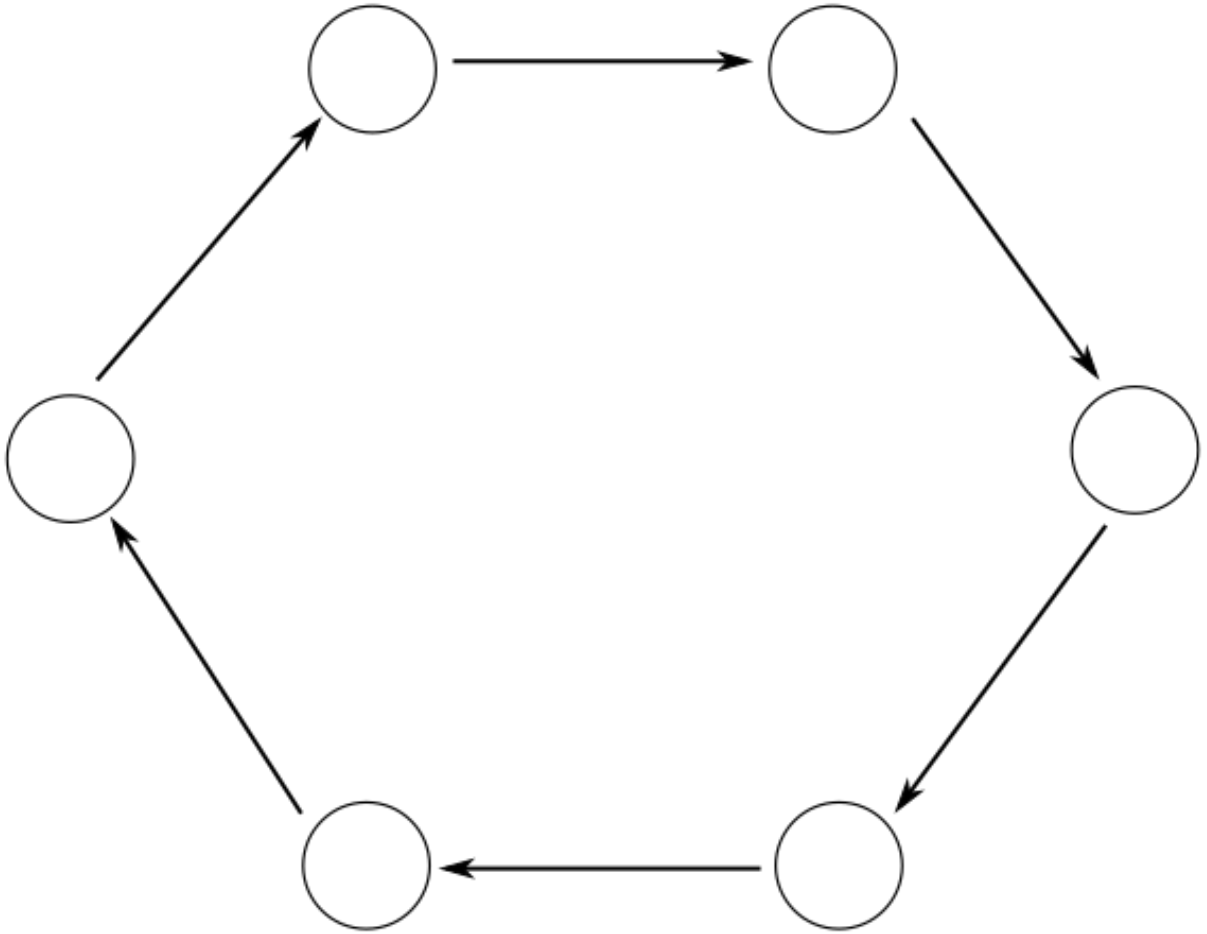

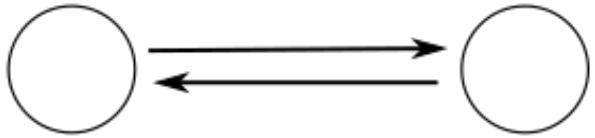

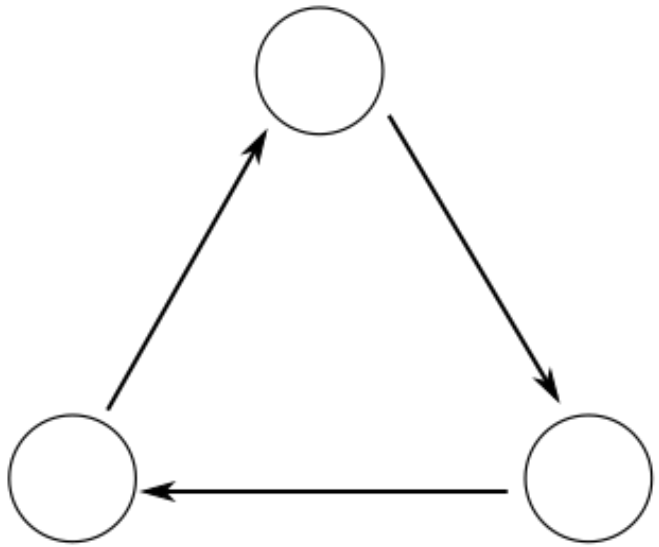

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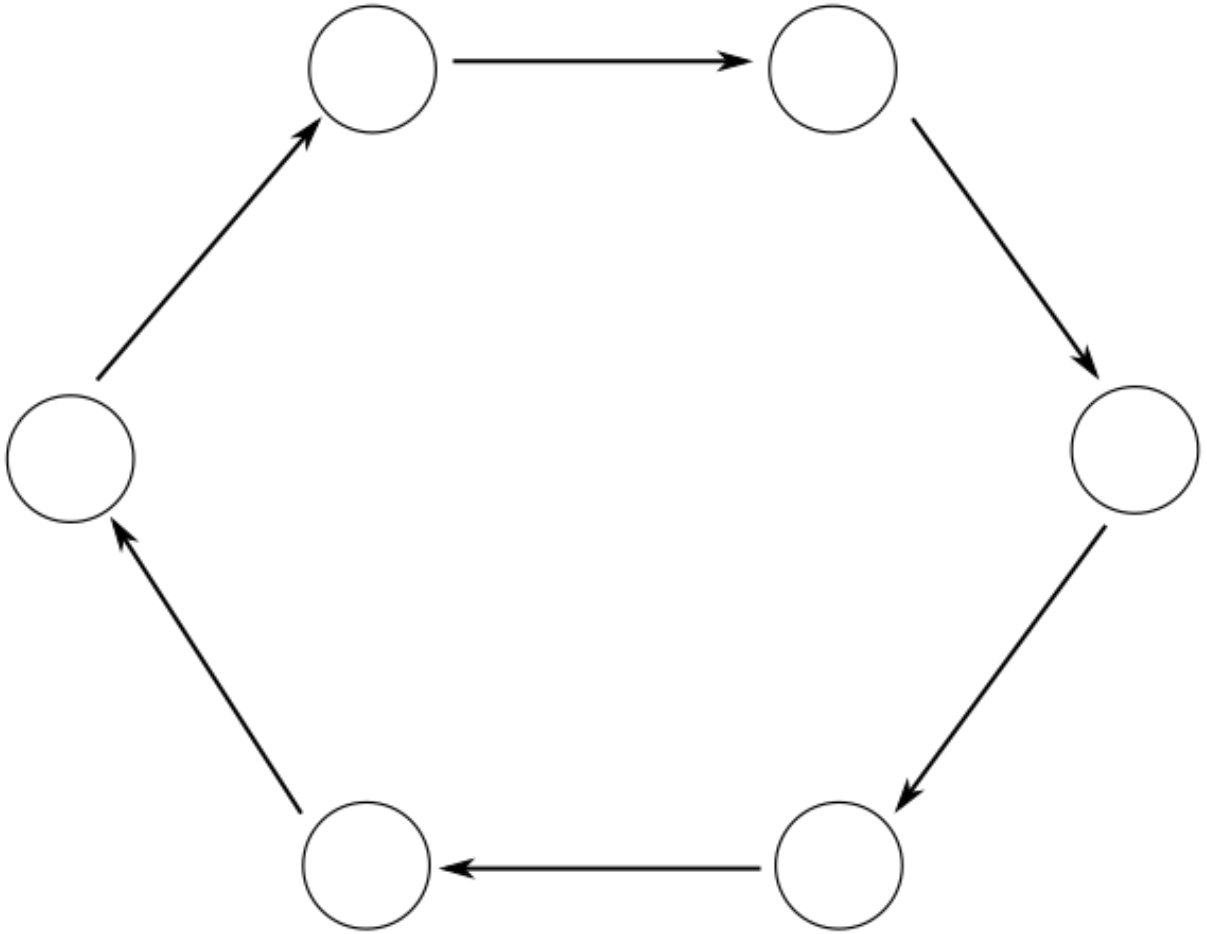

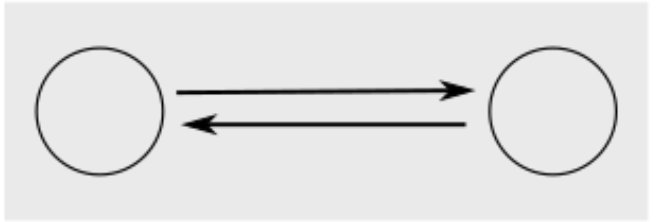

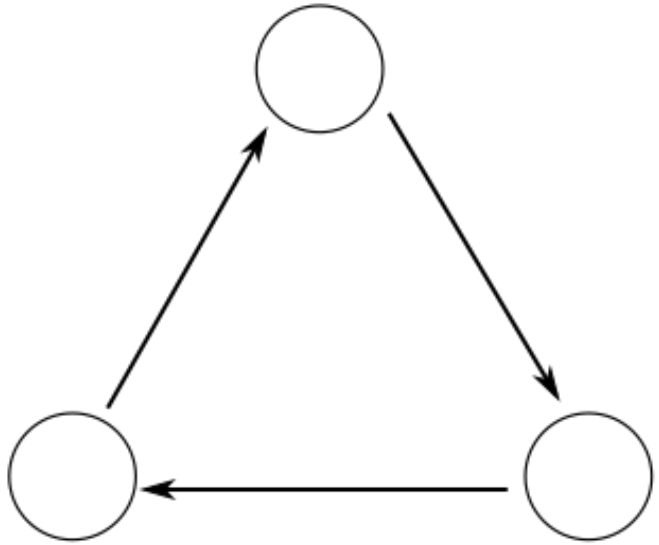

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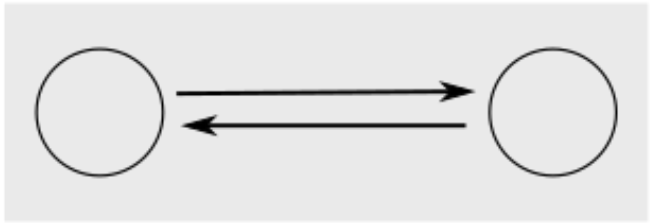

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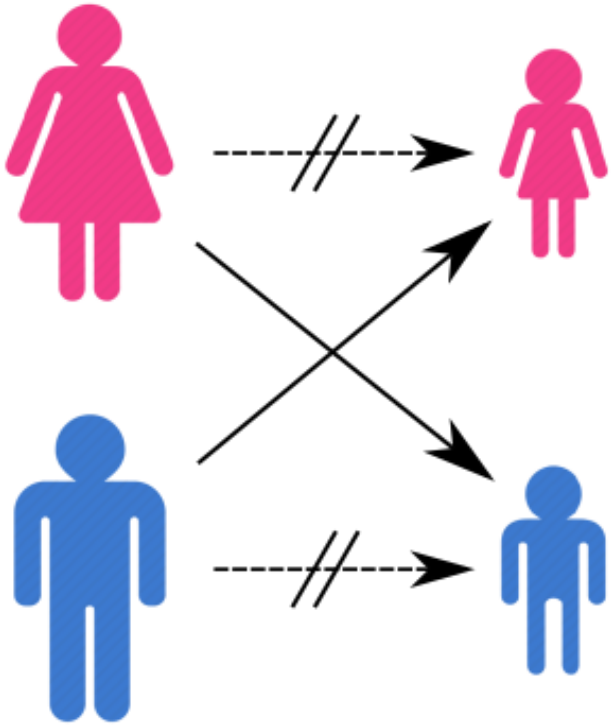

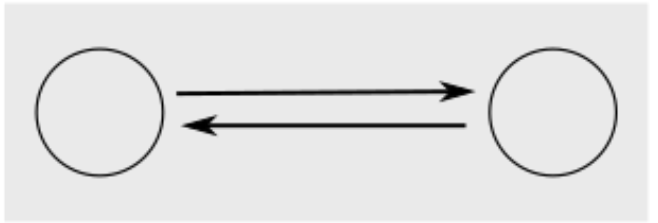

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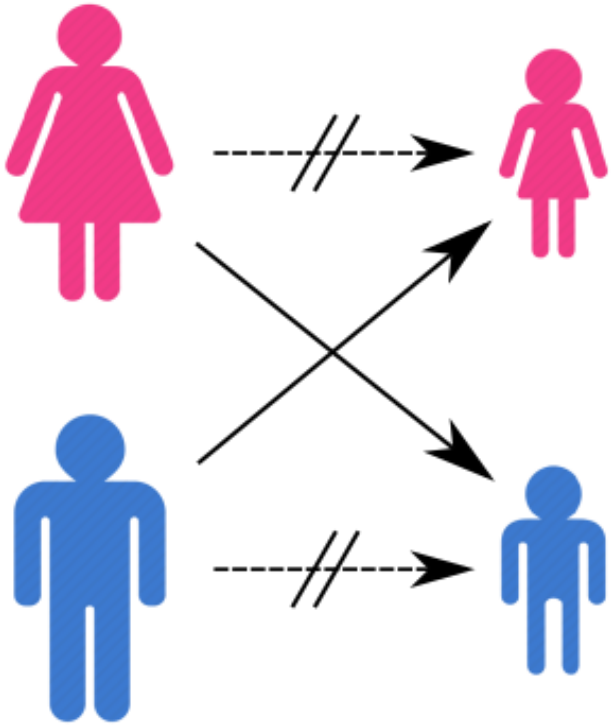

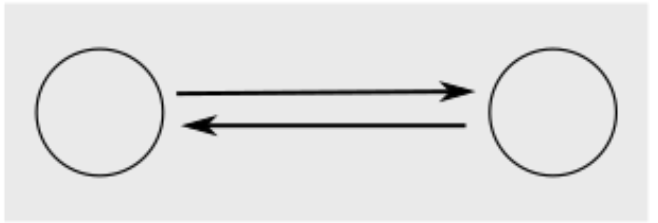

Four surgeries

Limitations of Kidney Exchange using TTCA

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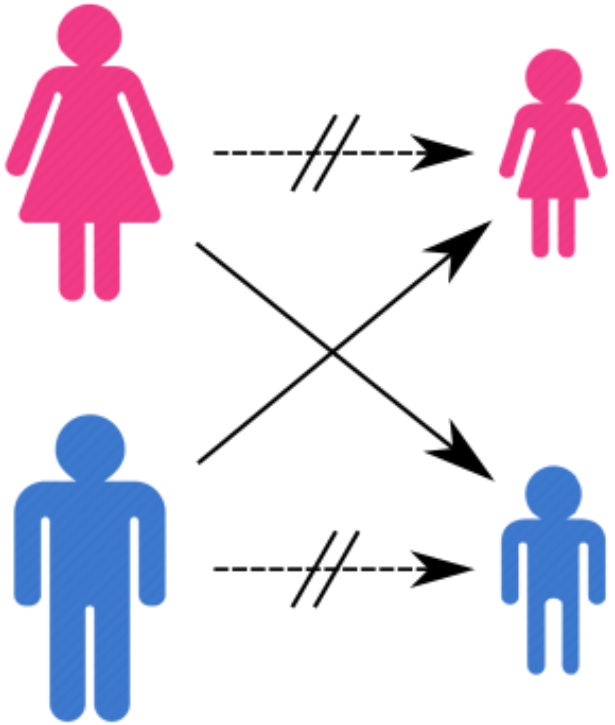

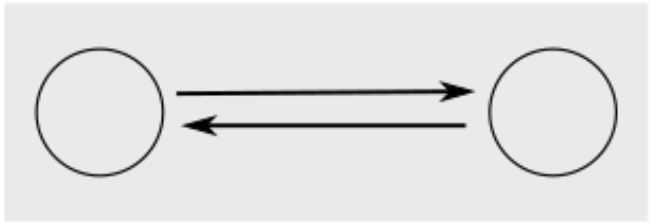

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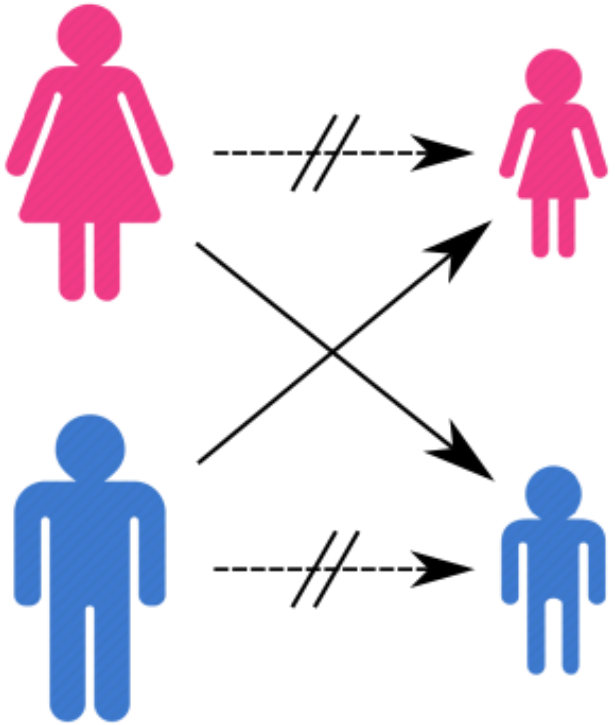

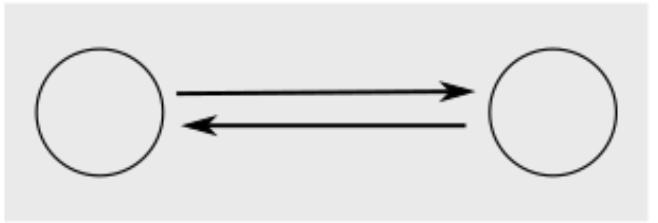

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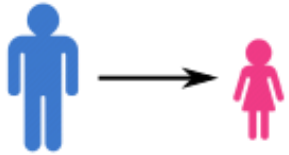
Cannot risk doing

 today

 tomorrow

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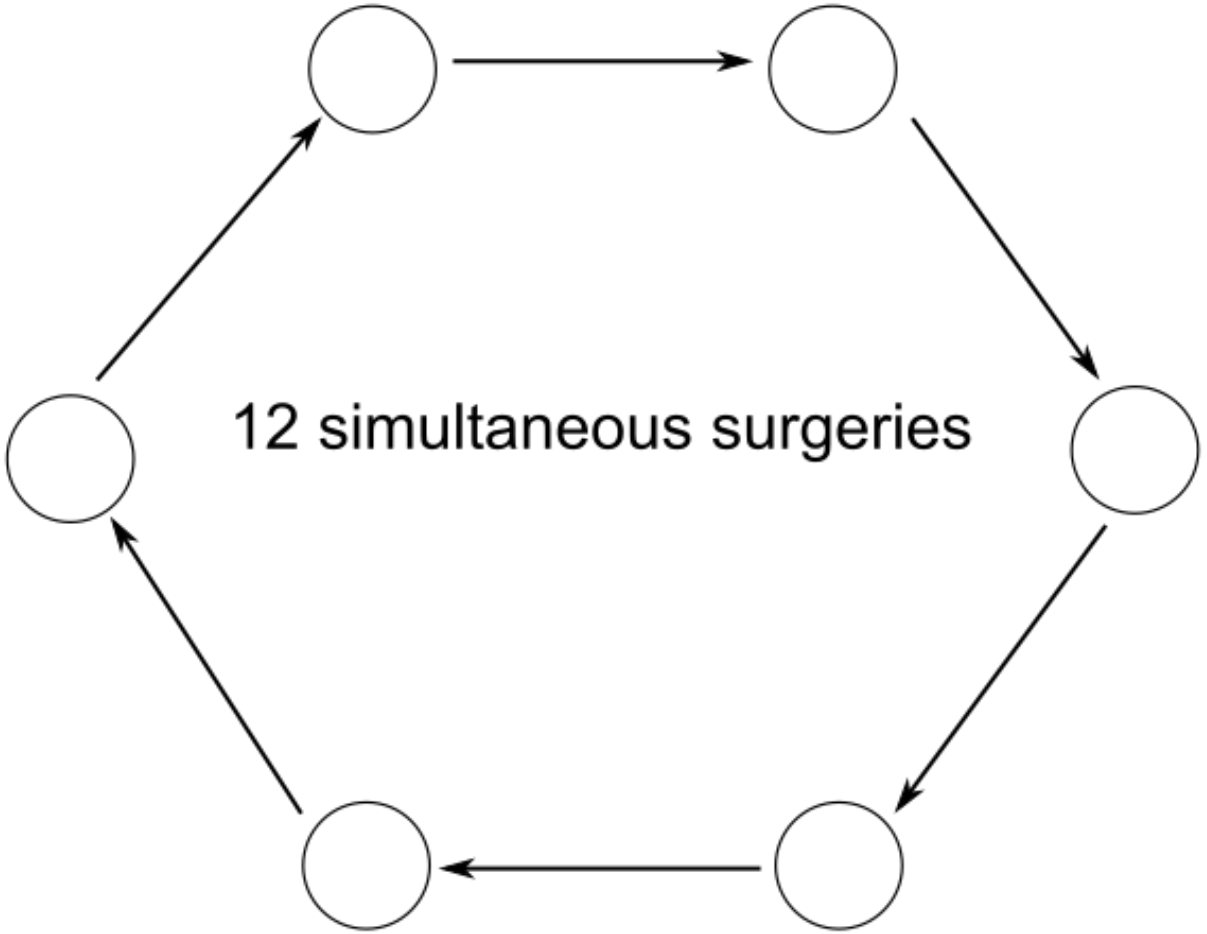

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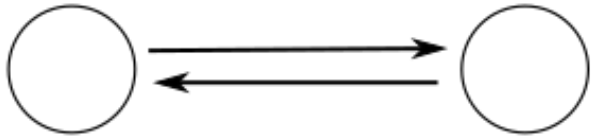

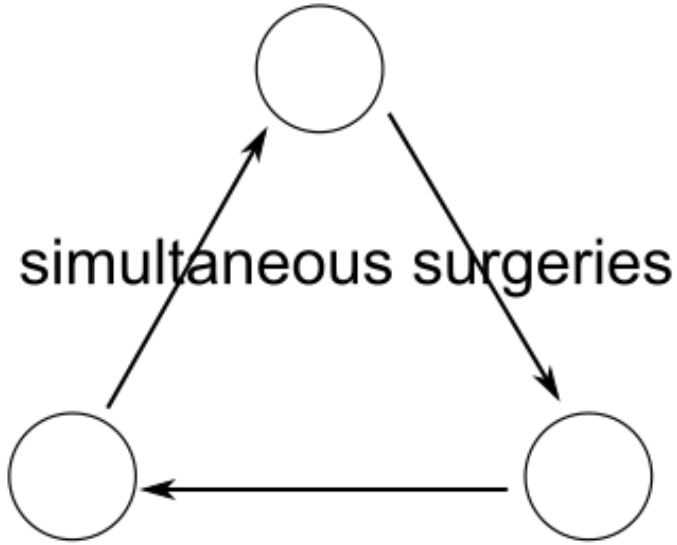

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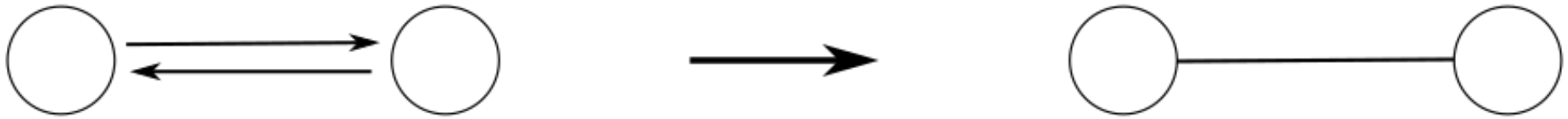

Early Proposals for Kidney Exchange

Proposal 1

Using TTCA

Proposal 2

?

Early Proposals for Kidney Exchange

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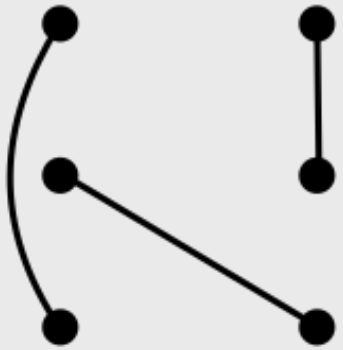
Proposal 2

Using Matchings



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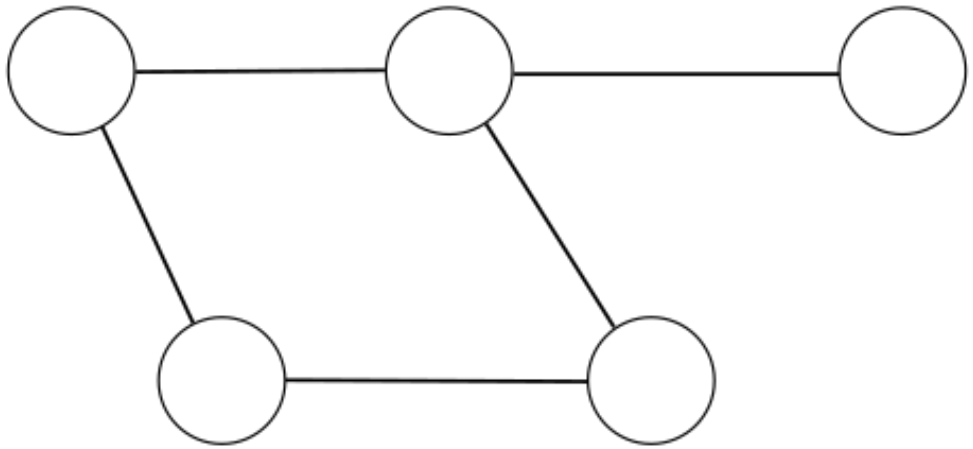

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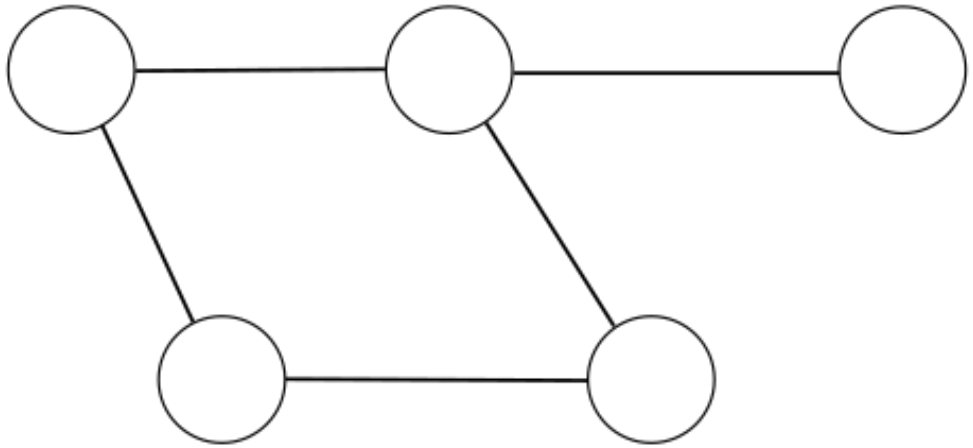

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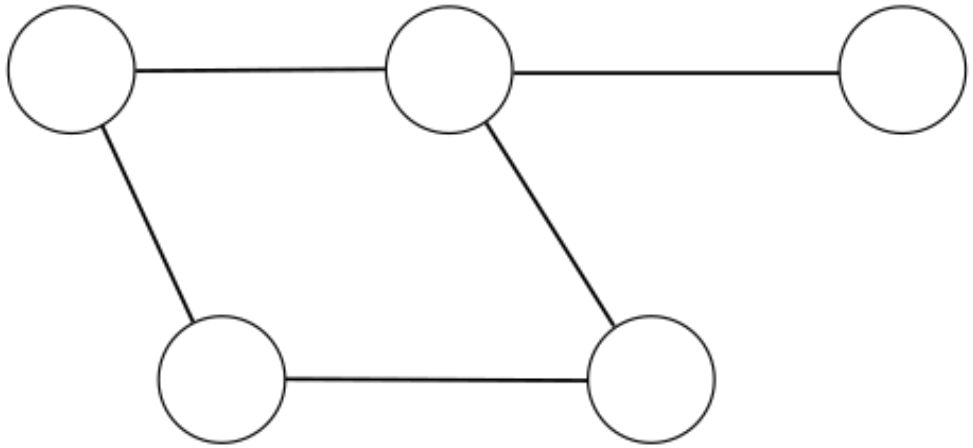

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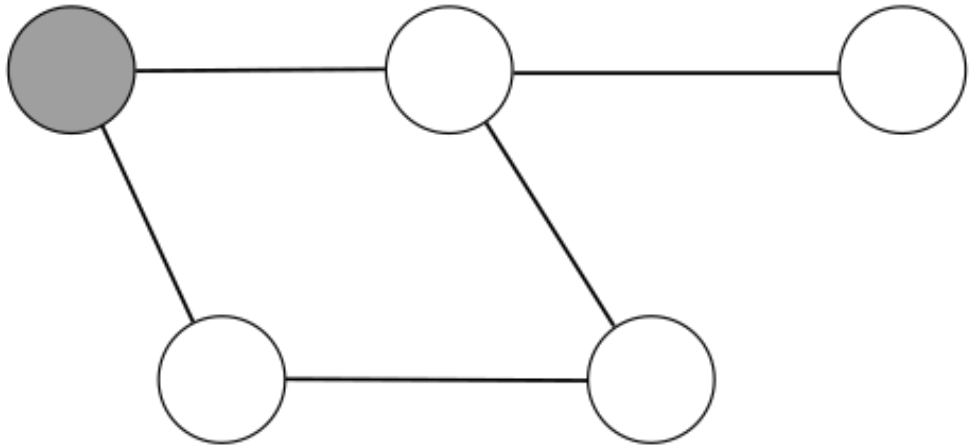

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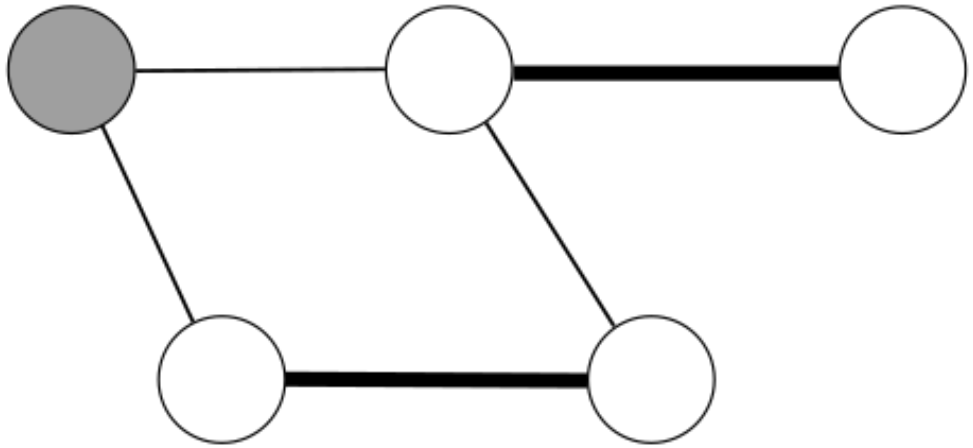

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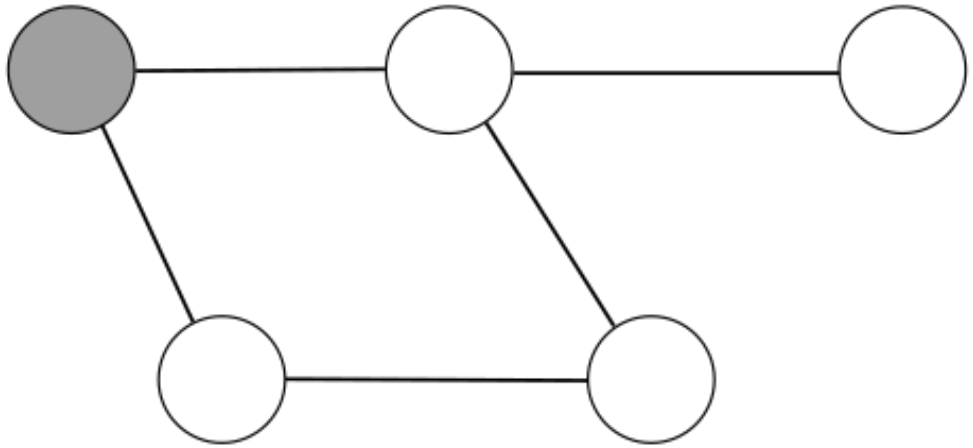

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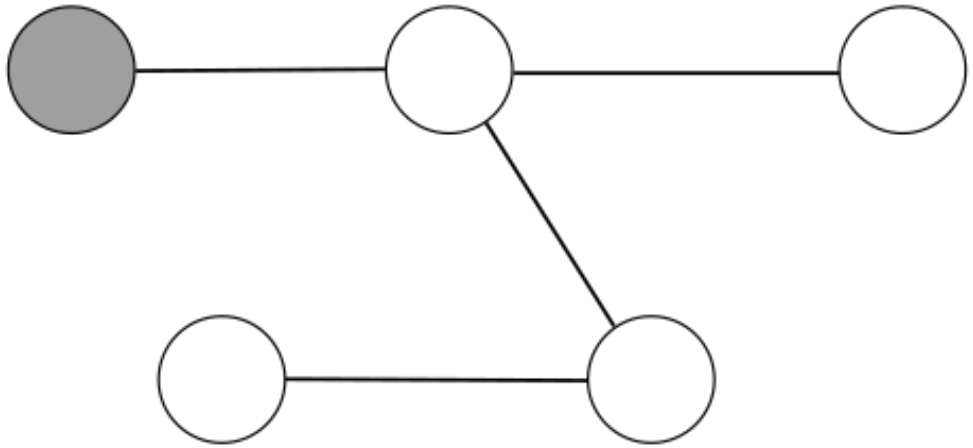

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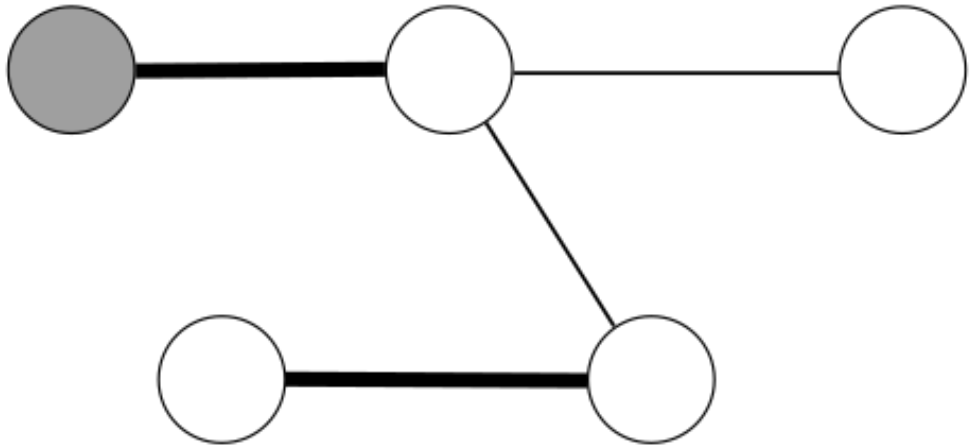

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Polynomial time
(uses Edmonds-Gallai decomposition from Graph Theory)

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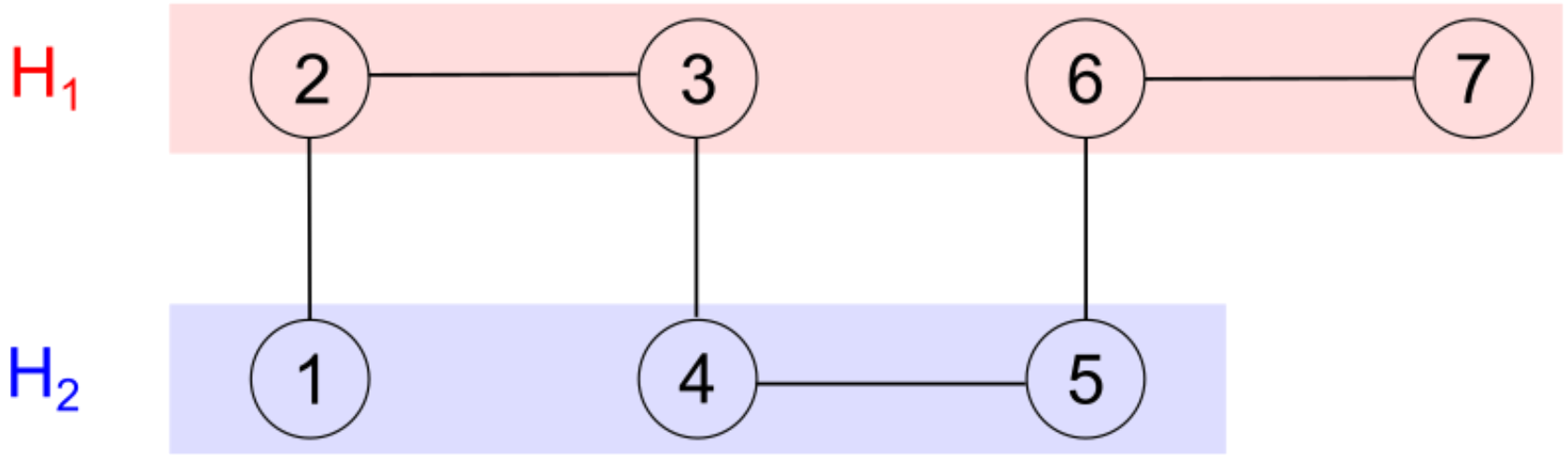

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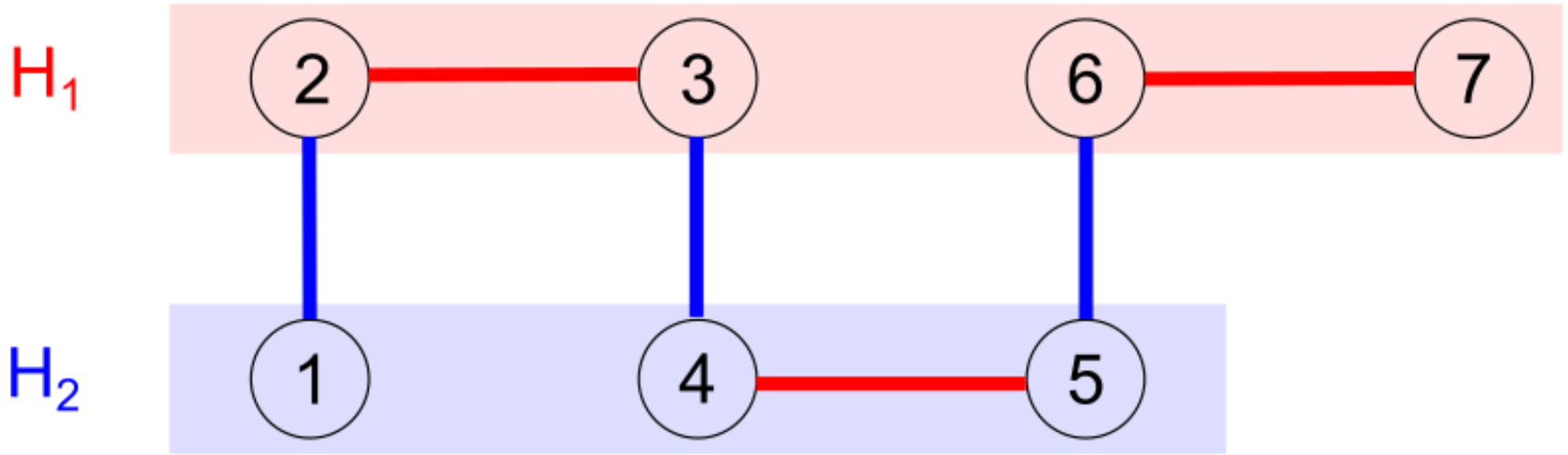

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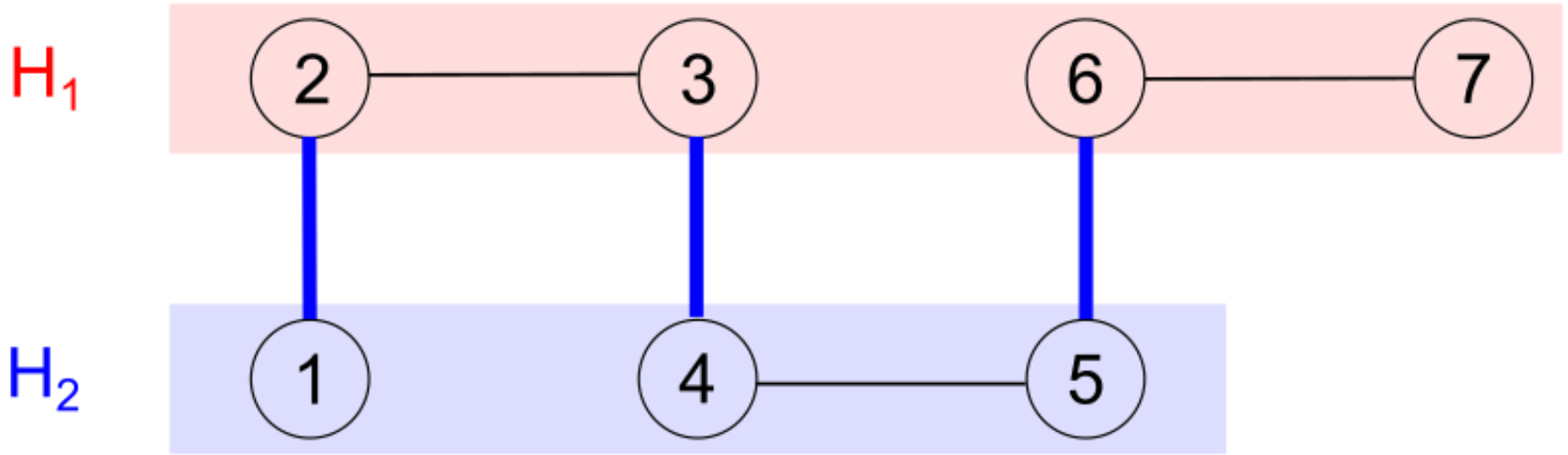

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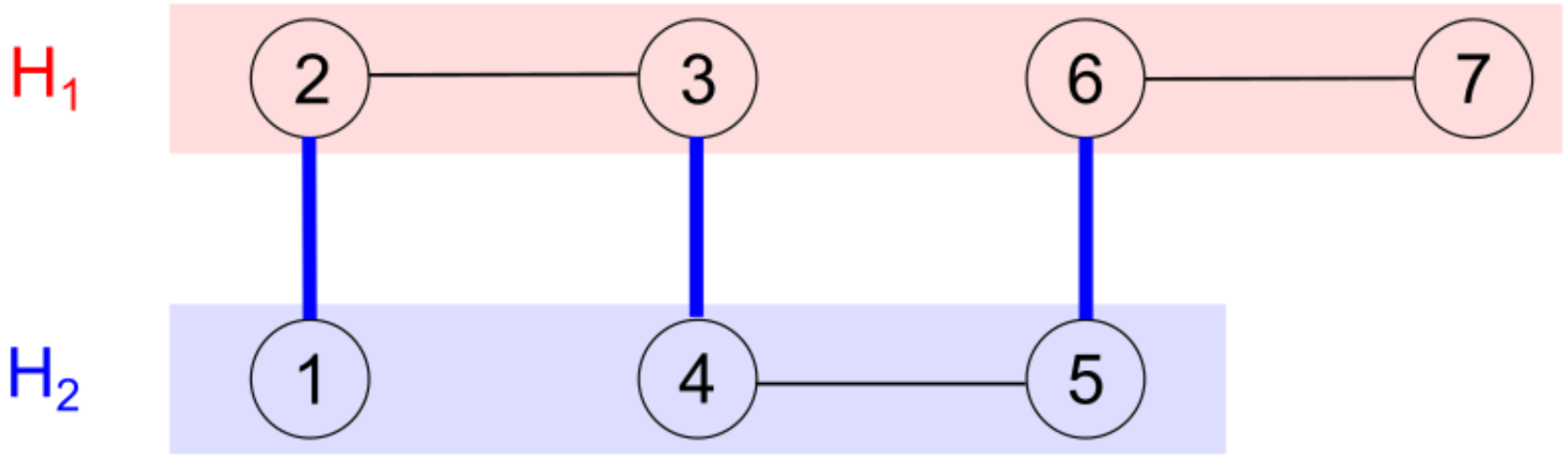

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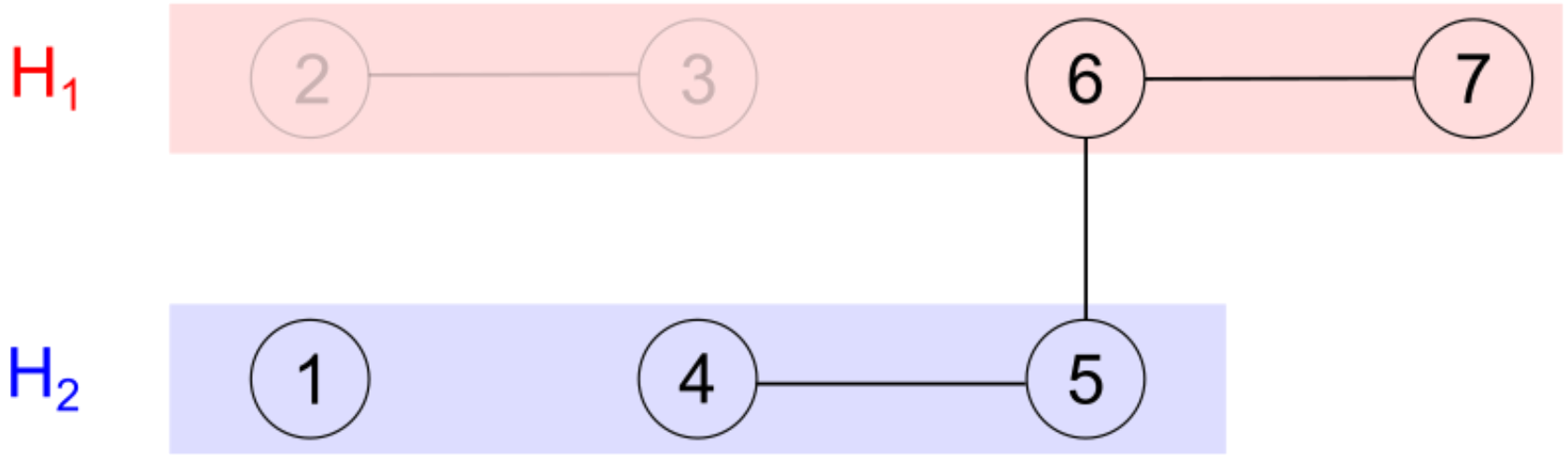

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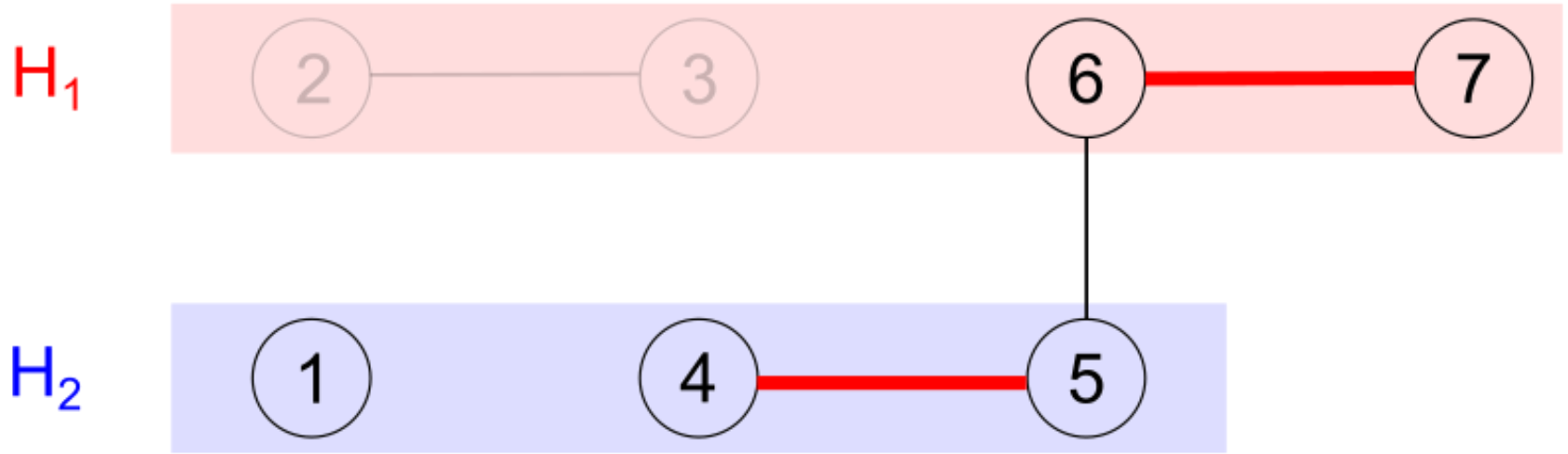

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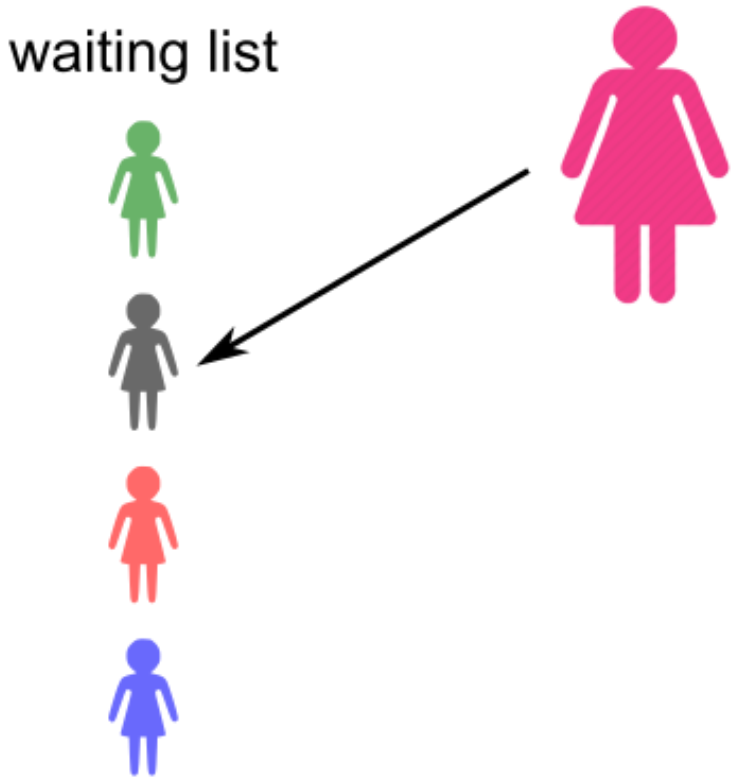

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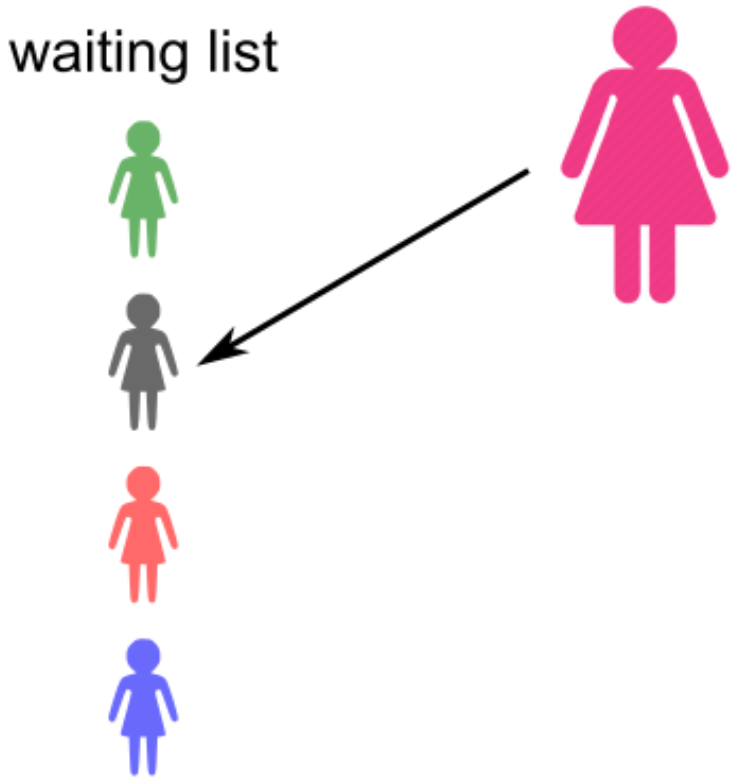

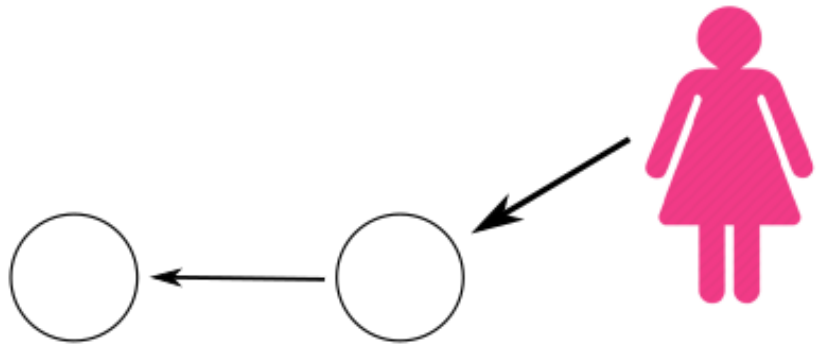

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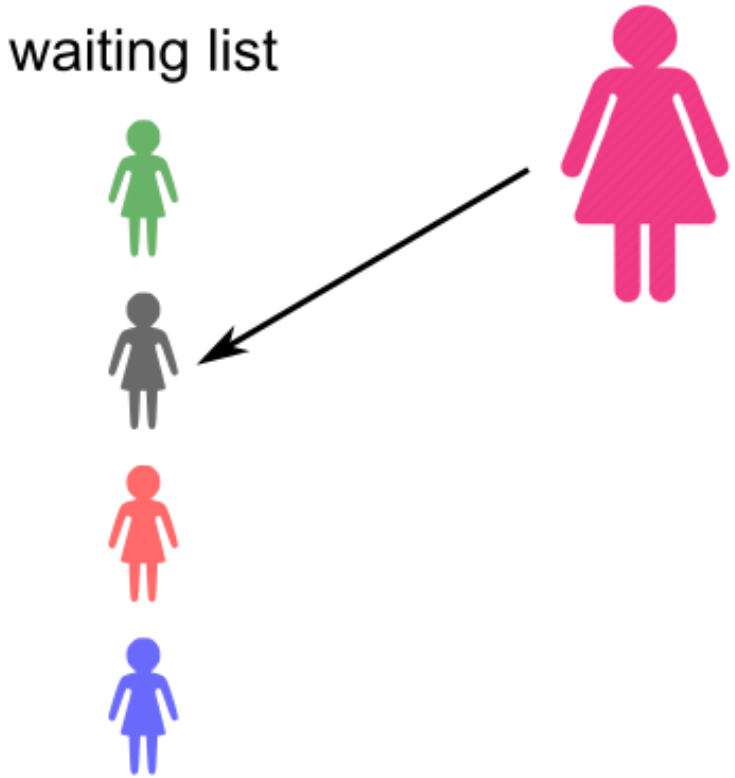

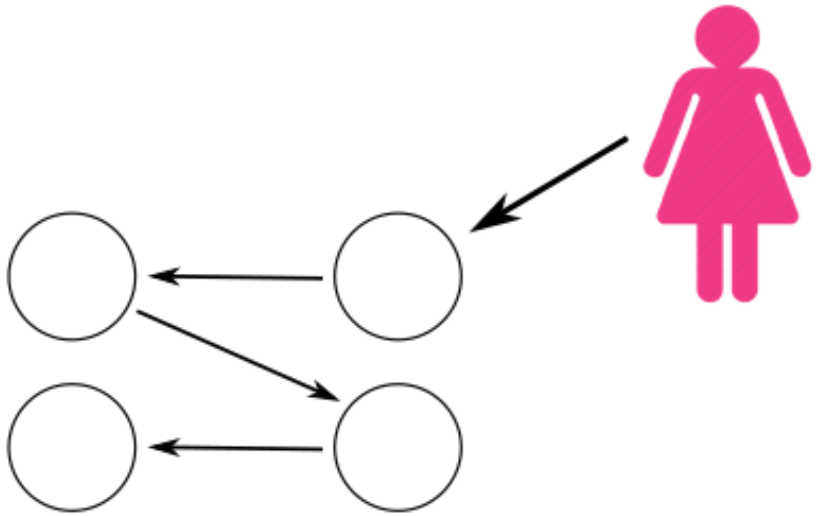

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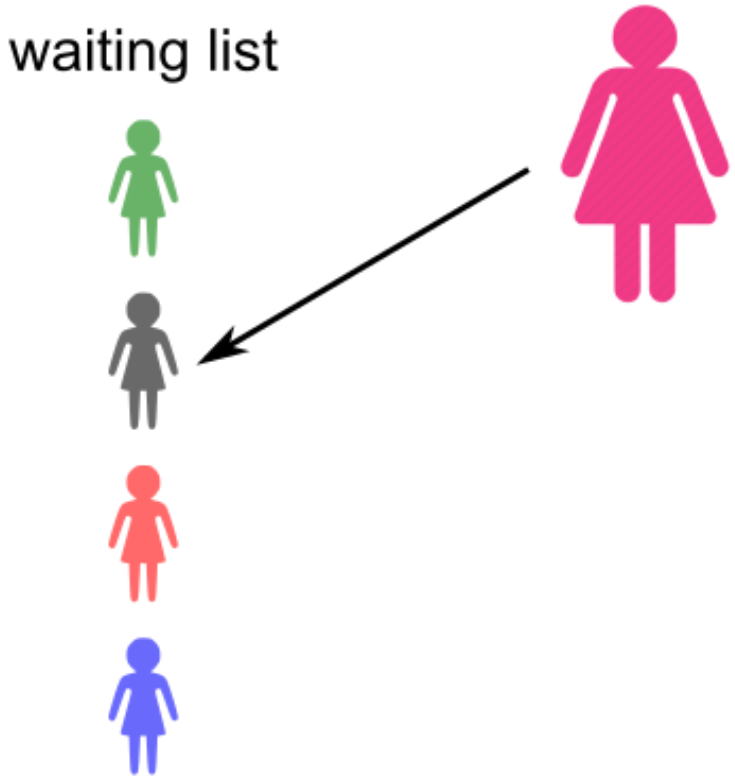

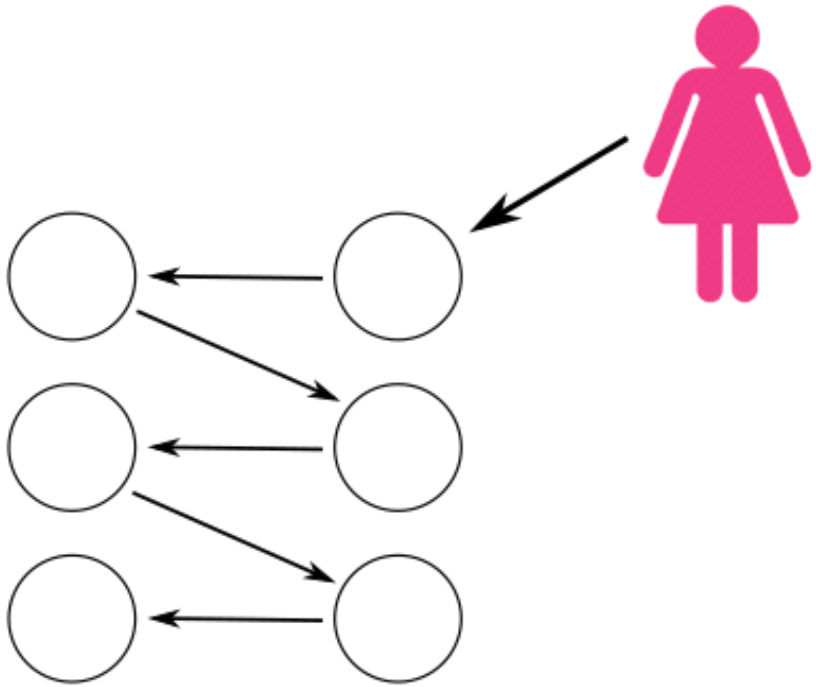

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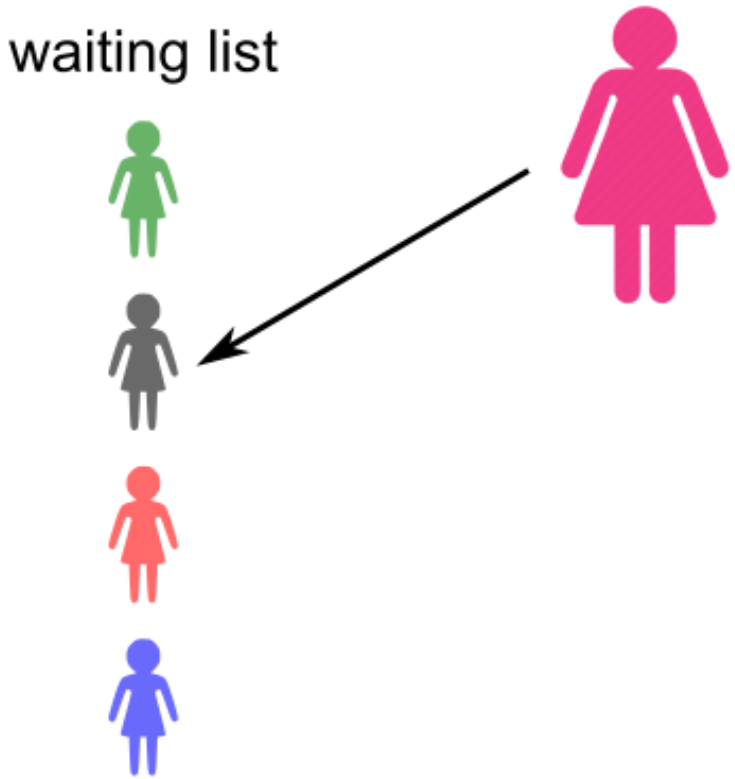

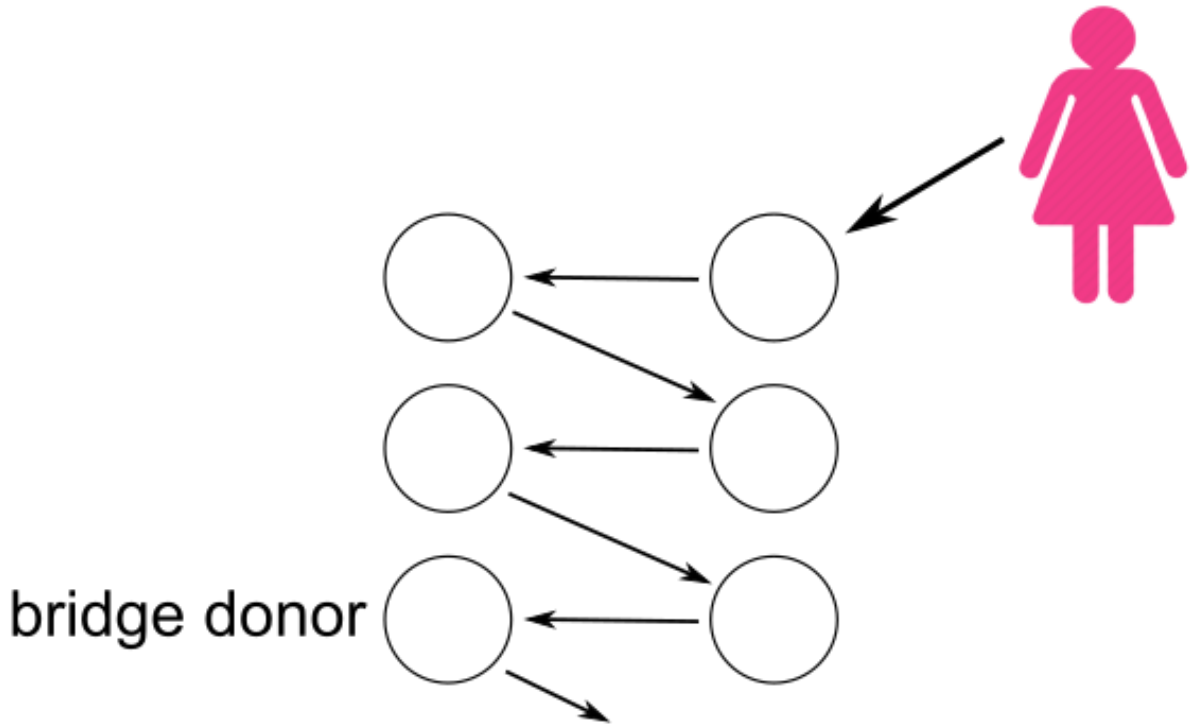

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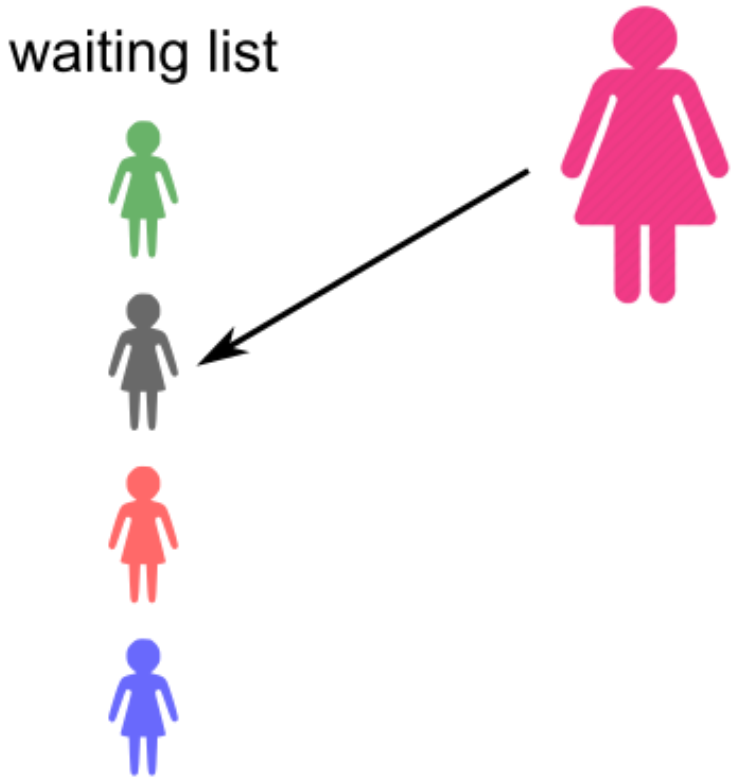
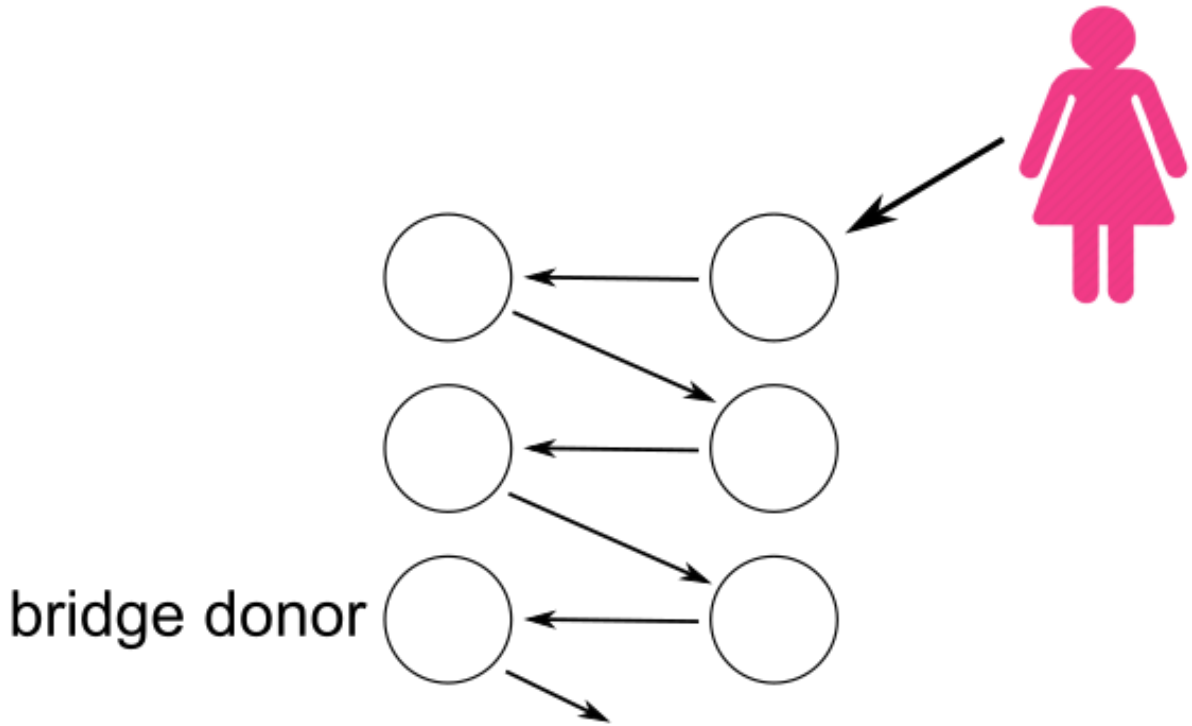




Transplants in an altruistic donor chain can be done non-simultaneously.

THE KIDNEY CHAIN

How a single organ donation changed 20 lives and created the longest-running transplant chain

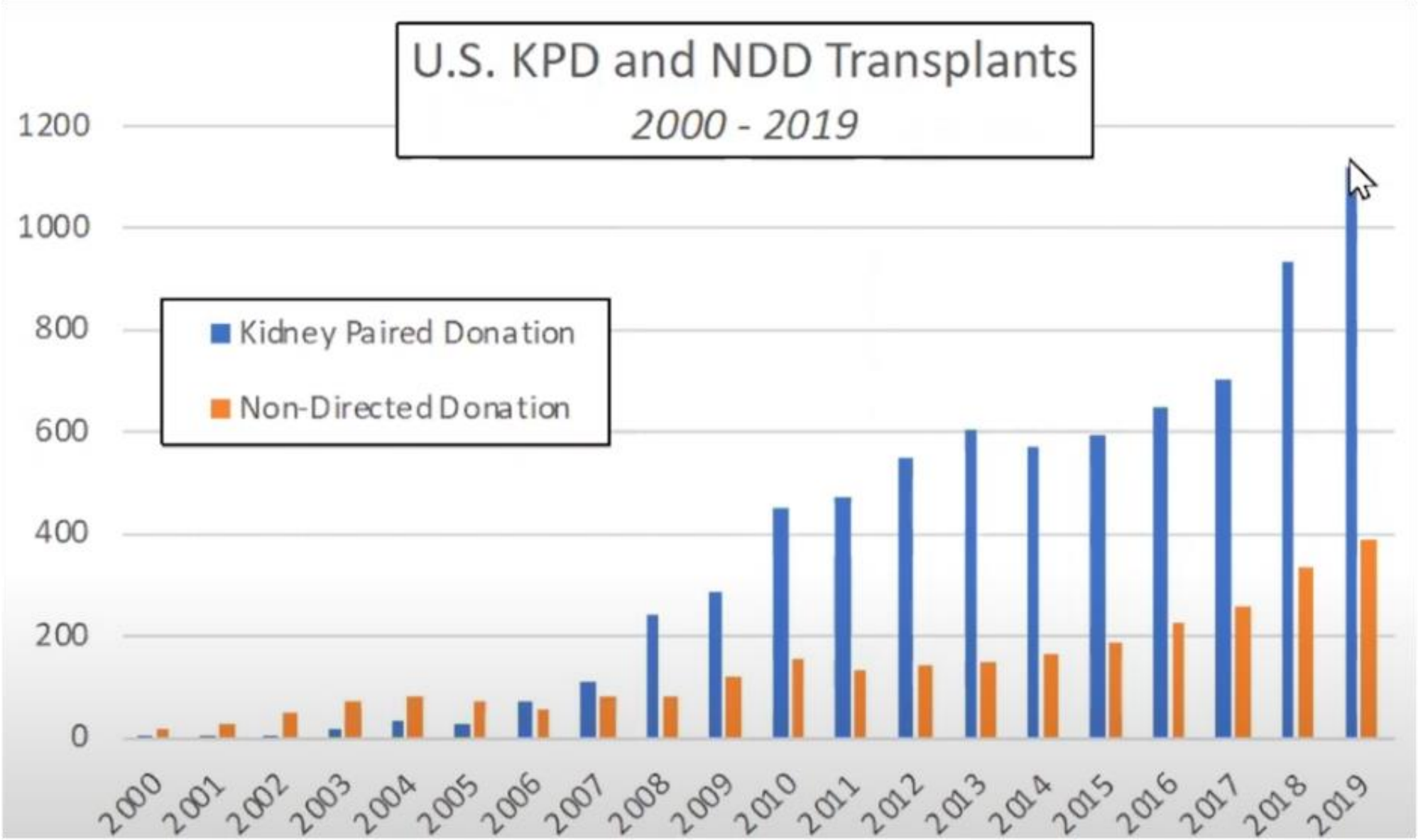


Figure source: Talk by Alvin Roth on "*Kidney Exchange: An Operations Perspective*" hosted by INFORMS Health Applications Society; June 2021

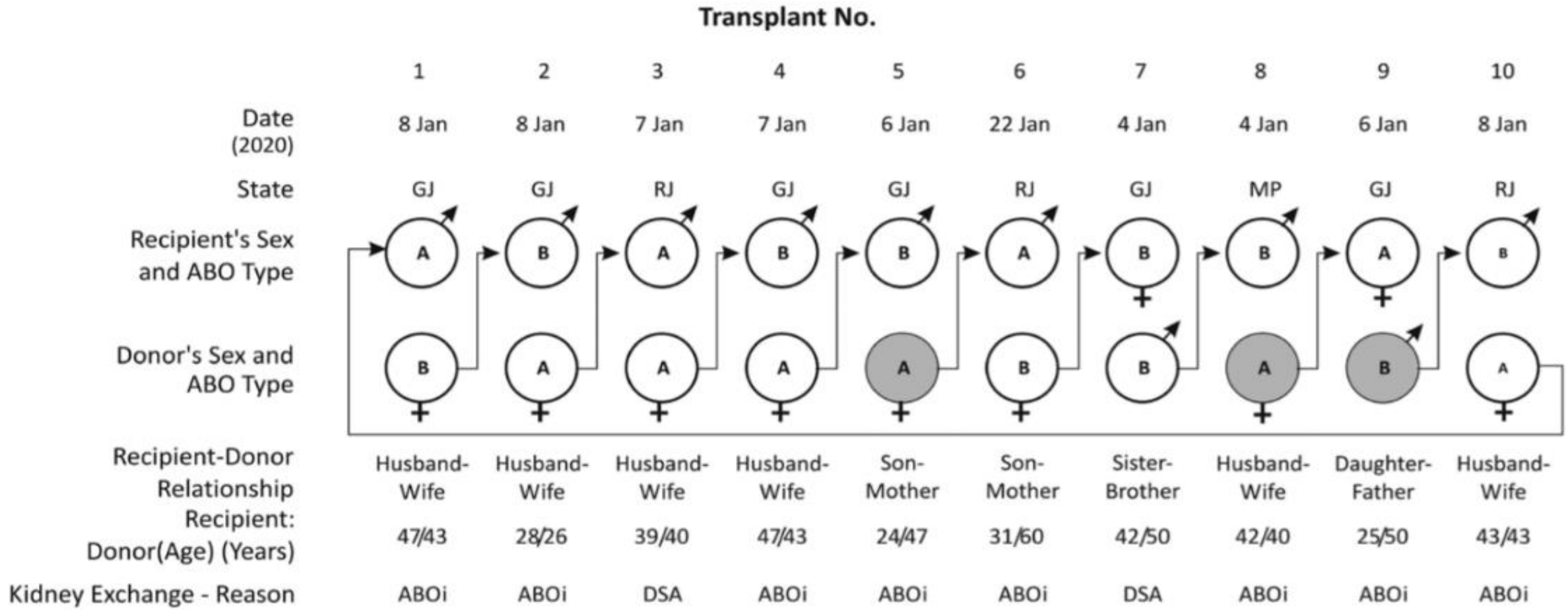
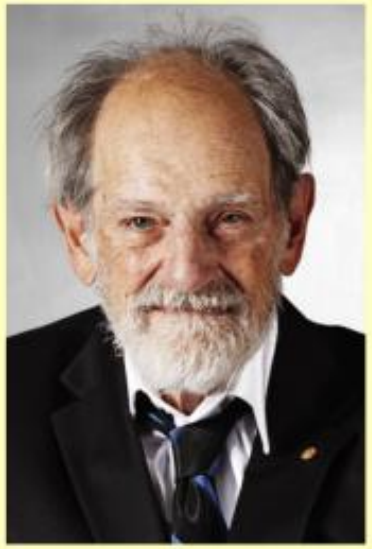


FIGURE 1. A nonsimultaneous 10-way kidney exchange cycle. GJ, Gujarat; MP, Madhya Pradesh; RJ, Rajasthan.

Kute et al. *"Paired Kidney Exchange in India: Future Potential and Challenges Based on the Experience at a Single Center"*, Transplantation, May 2021, 105(5), pp 929-932

Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012

Alvin E. Roth



Lloyd S. Shapley

"*for the theory of stable allocations and the practice of market design*."

Challenges in Kidney Exchange

Economic: Incentives of hospitals vs maximizing overall welfare.

Computational: Finding a maximum matching using 3-cycles is NP-hard.

Informational: Patients and donors arrive sequentially.

Logistical: Long cycles require many simultaneous surgeries.

Legal: Many countries do not permit altruistic donation.

Ethical: Is it OK to buy and sell kidneys?

Reminders

Assignment 1: Due by Friday (Aug 26)

Project groups: Due by Wednesday (Aug 31)

Next Time

Cake cutting

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- Housing markets and TTCA

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- Truthfulness under TTCA

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  "*Incentive Compatibility in a Market with Indivisible Goods*"
  Economics Letters, 9(2), 1982, pg 127-132
  https://www.sciencedirect.com/science/article/pii/0165176582900039

References

- The presentation is heavily inspired from Tim Roughgarden's lectures in the course on Algorithmic Game Theory (Fall 2013); see Lec 9 and 10. https://timroughgarden.org/f13/f13.html

- Data about waiting list for organs in the US: https://optn.transplant.hrsa.gov/data/

- Data about waiting list for kidneys in India (April 2019): https://www.narayanahealth.org/blog/kidney-transplants-in-india/

- Wikipedia article on "Kidney Paired Donation": https://en.wikipedia.org/wiki/Kidney_paired_donation

References

- Proposal 1: Kidney exchange using top-trading cycles and chains

  Alvin E. Roth, Tayfun Sönmez, and Utku Ünver
  "*Kidney Exchange*"
  The Quarterly Journal of Economics, 119(2), 2004 pg 457-488
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- Proposal 2: Kidney exchange using matchings

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  Journal of Economic Theory, 125(2), 2005 pg 151-188
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- The "*You request my house, I get your turn*" extension of TTCA was originally proposed in the context of dormitory assignment:

  Atila Abdulkadiroğlu and Tayfun Sönmez
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  Journal of Economic Theory, 88(2), 1999 pg 233-260
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- Altruistic donor chains:

  Rees et al.
  "*A Non-Simultaneous, Extended, Altruistic Donor Chain*"
  The New England Journal of Medicine, 360(11), 2009 pg 1096-1101
  https://www.nejm.org/doi/full/10.1056/NEJMoa0803645

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  https://www.donordiaries.com/

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  https://www.youtube.com/watch?v=exB1O3pTf7E
  https://www.youtube.com/watch?v=4NbJTcfN6UA
  https://www.donordiaries.com/episodes

  http://marketdesigner.blogspot.com/search/label/kidney%20exchange