

Lecture 15

Voting with Structured Preferences

Story So Far

Story So Far



No group-level
transitivity

Condorcet
paradox

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transitivity

Condorcet
paradox



No "reasonable"
voting rules

Gibbard-Satterthwaite
and Arrow's theorems

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Manipulation
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Greedy strategy

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Single-Peaked Preferences

[Black'48]

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There is an ordering " $<$ " over candidates.

$$a < b < c < d < e$$

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w.r.t. the ordering " $<$ " if for each voter v :

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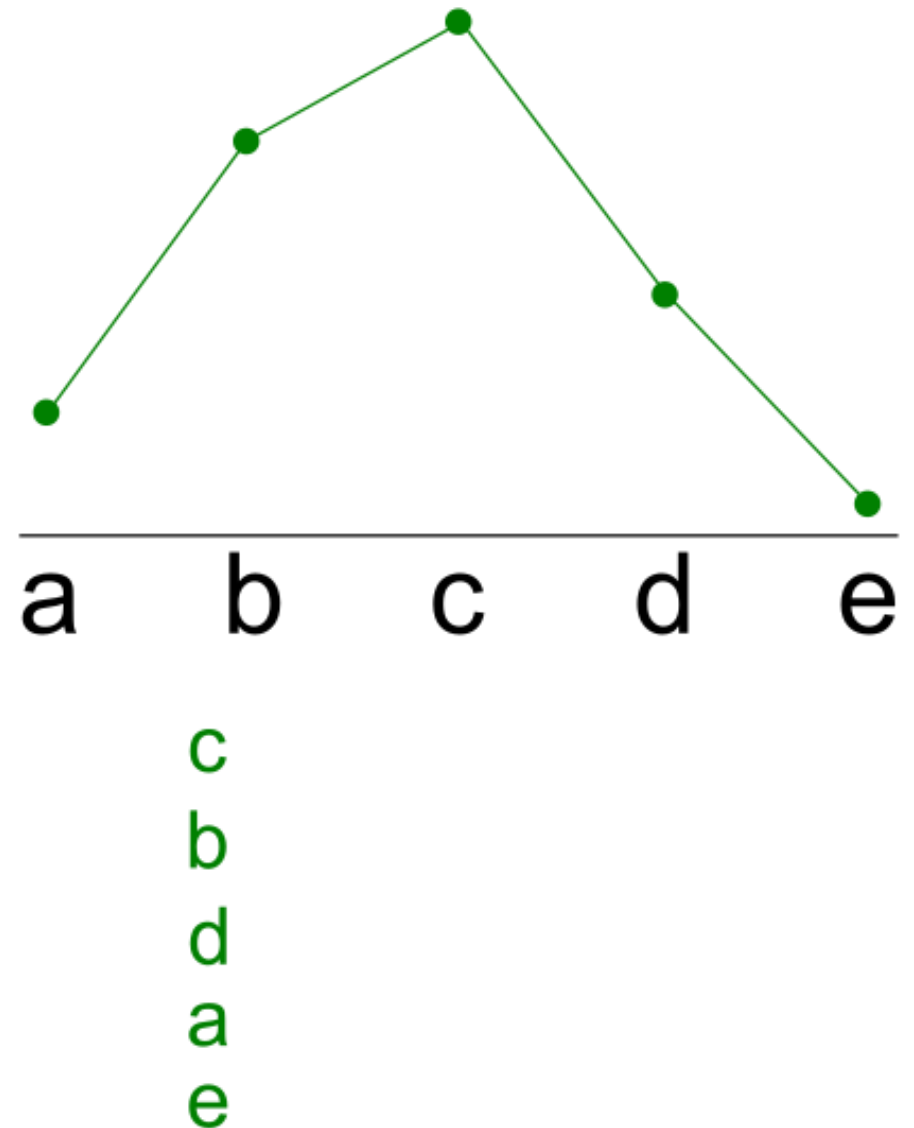
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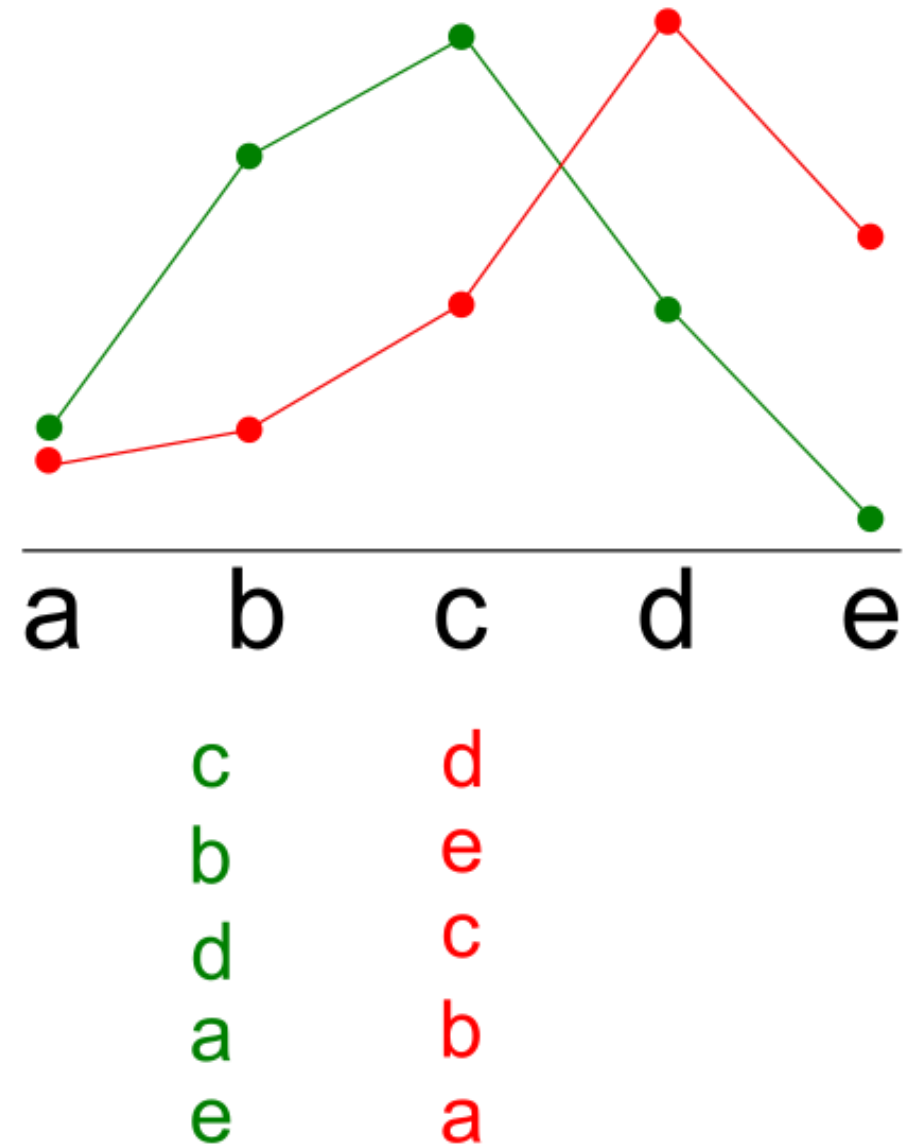
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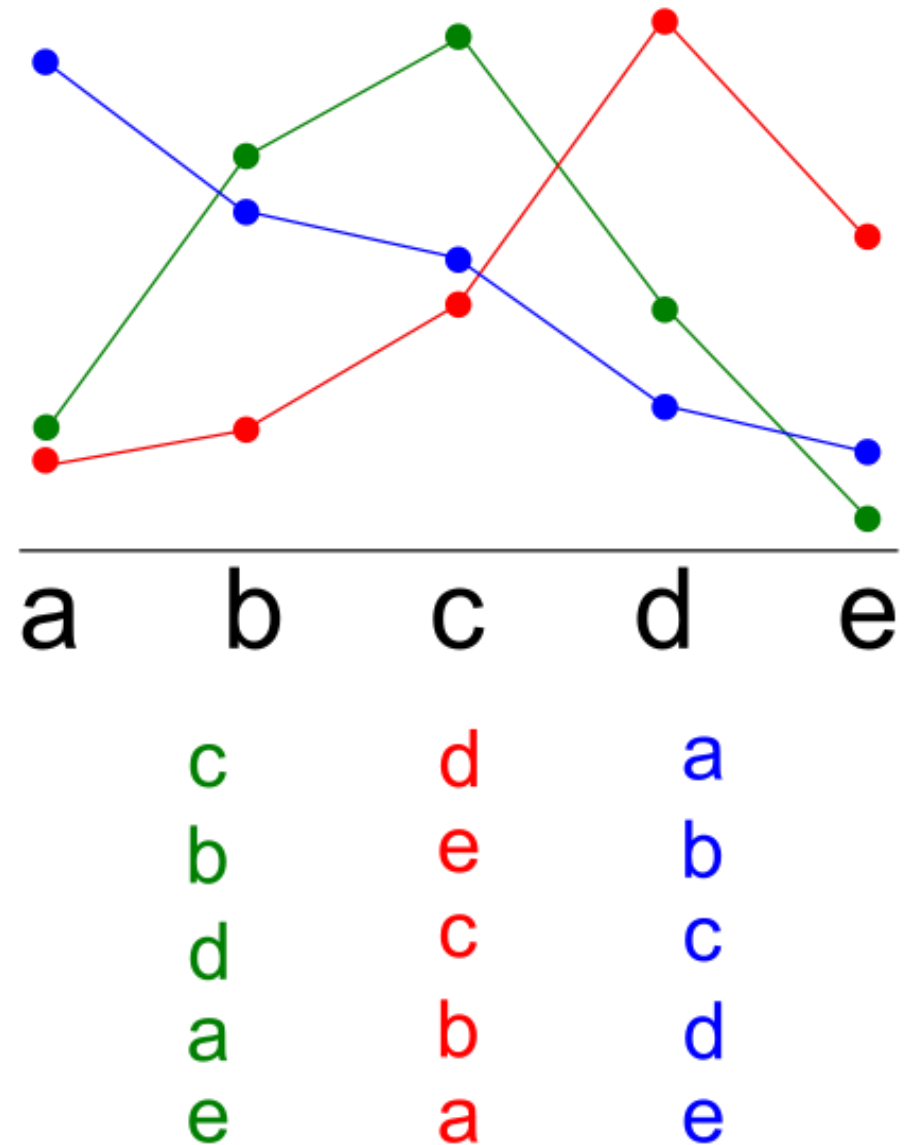
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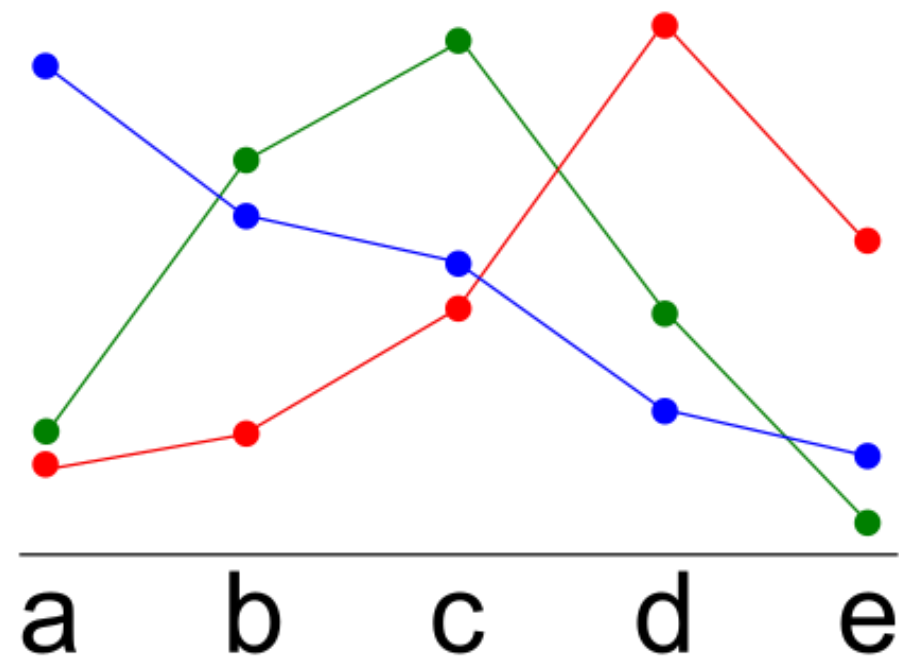
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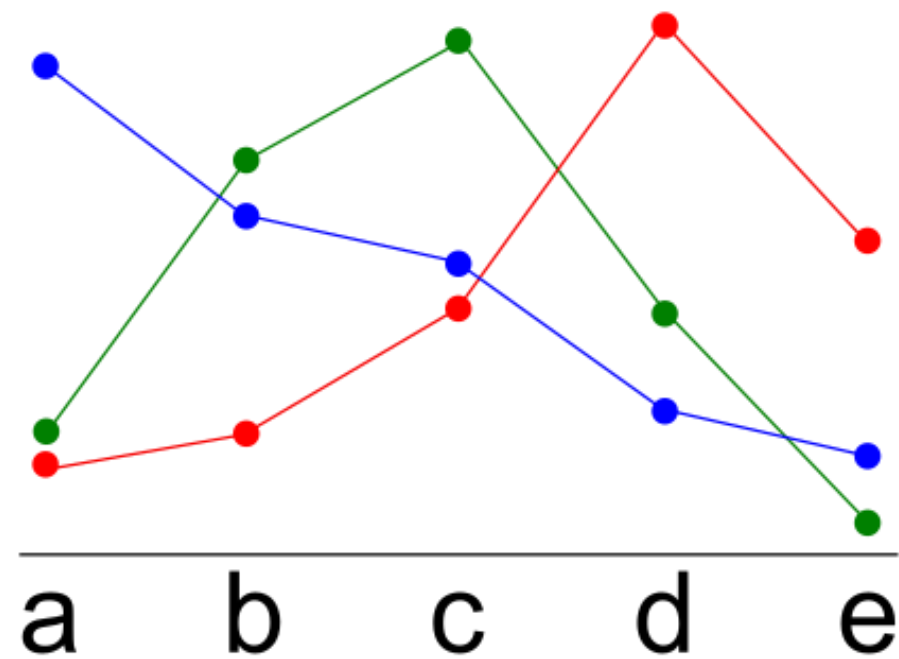
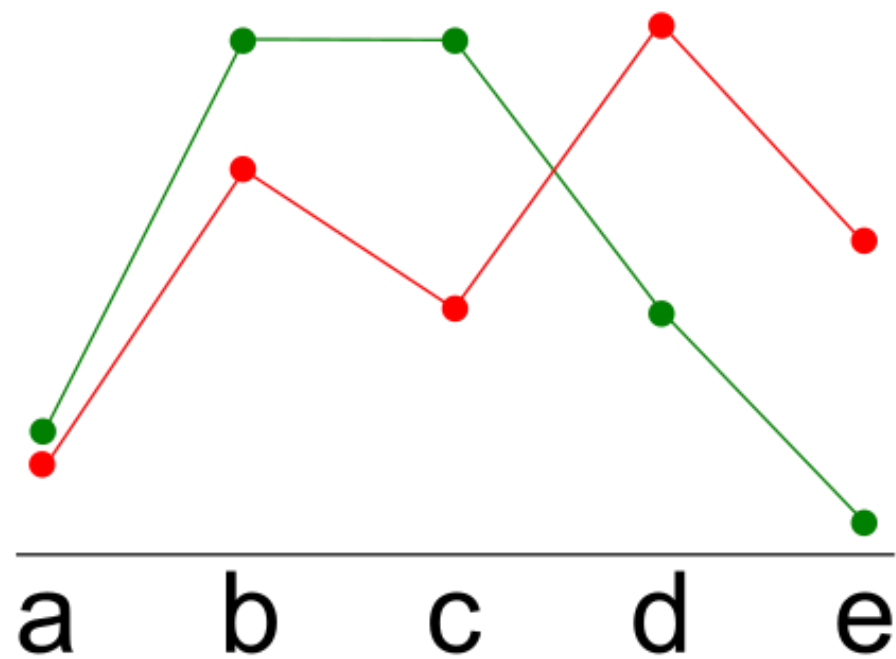
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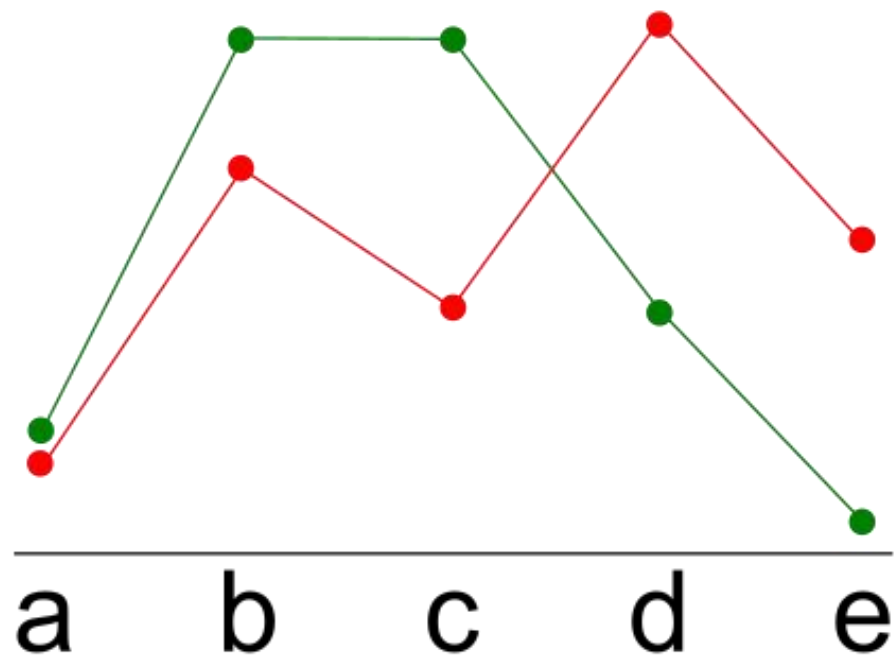
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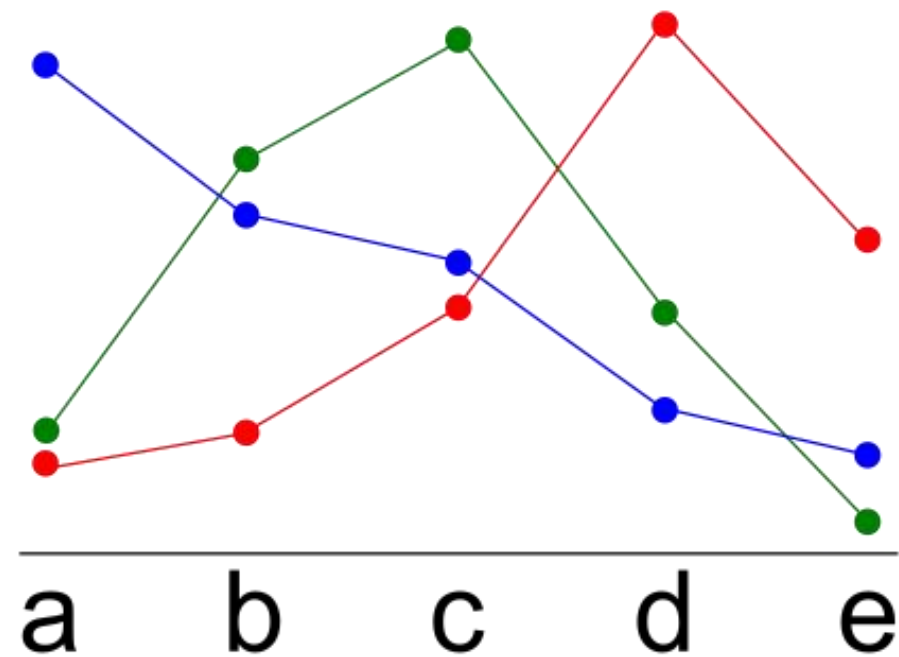


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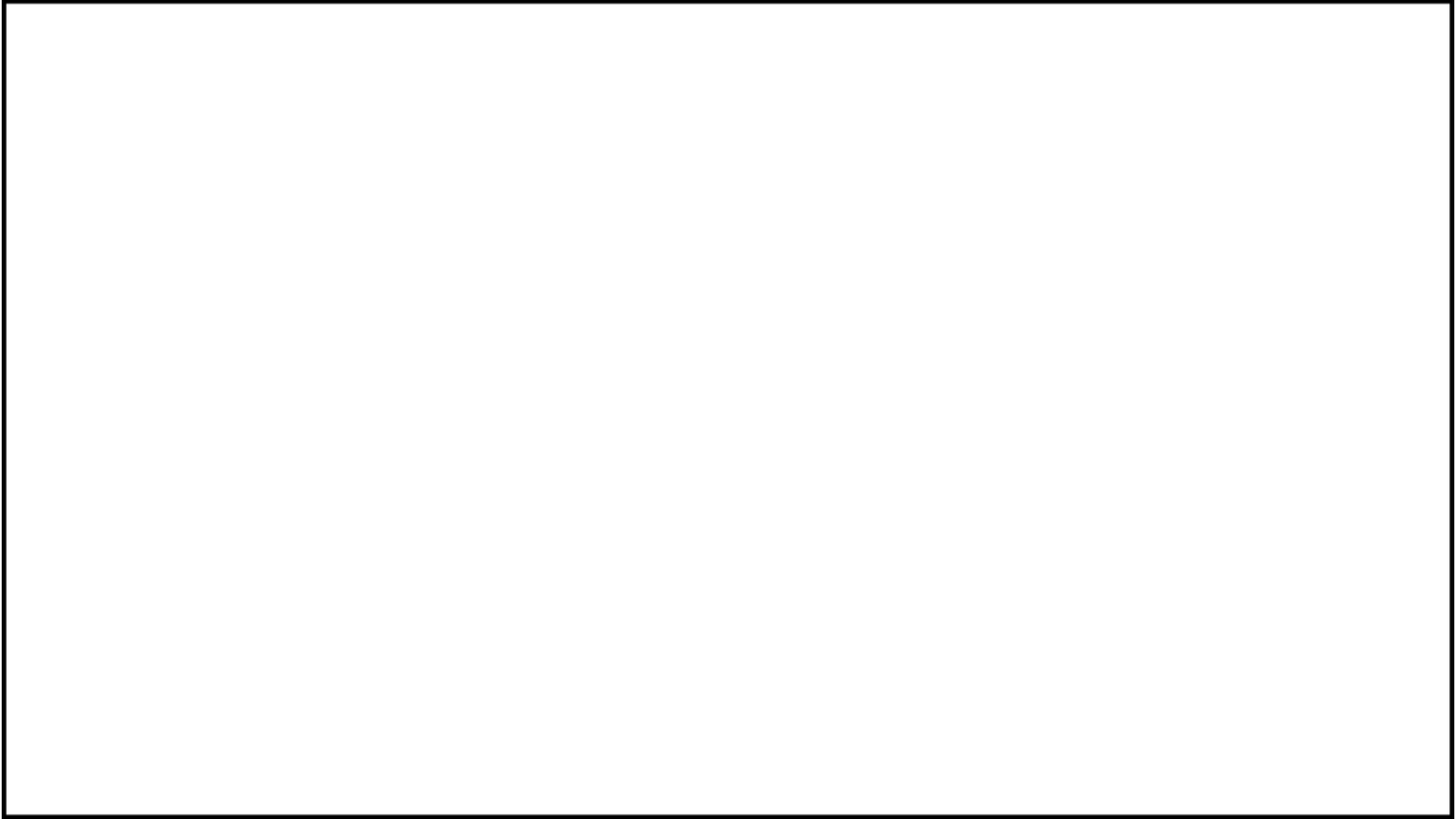
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**NOT
SINGLE-PEAKED**



SINGLE-PEAKED





17 18 19 20 21



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5% 10% 15% 20%



17 18 19 20 21



20° 21° 22° 23°



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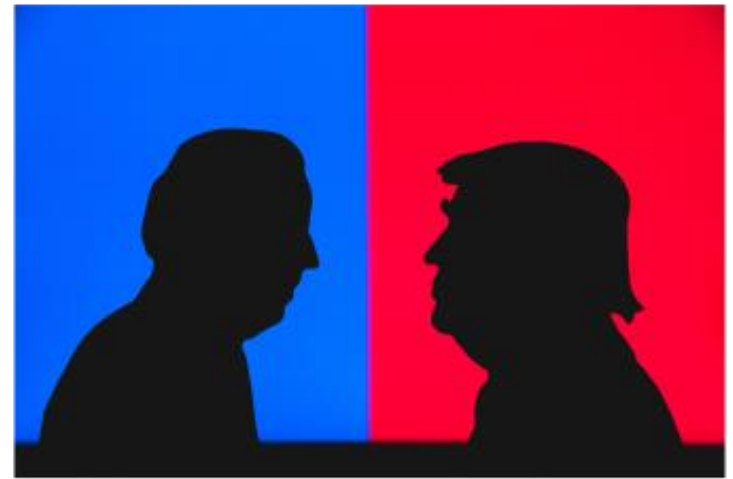
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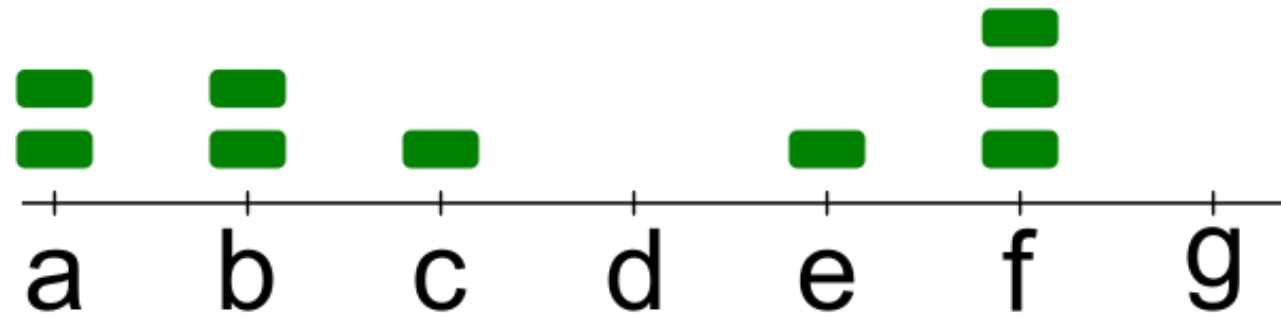


Liberal Conservative

Single-peaked profiles always admit a (weak) Condorcet winner.

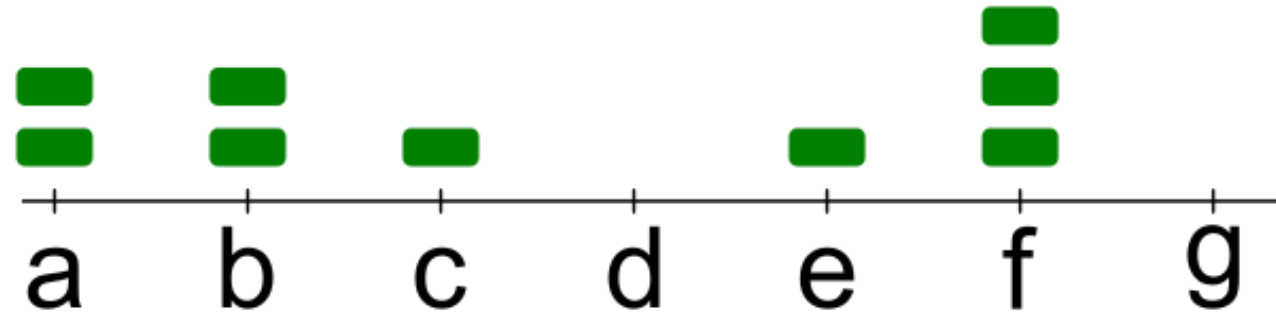
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Order the voters according to their top choices



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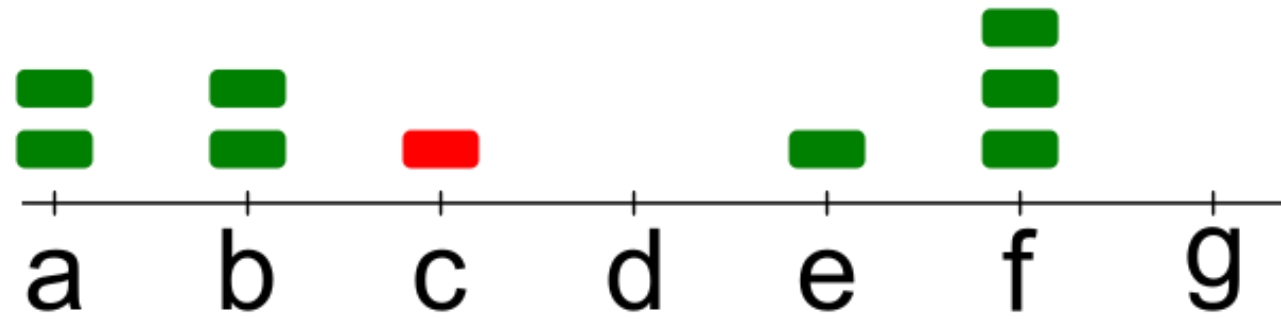
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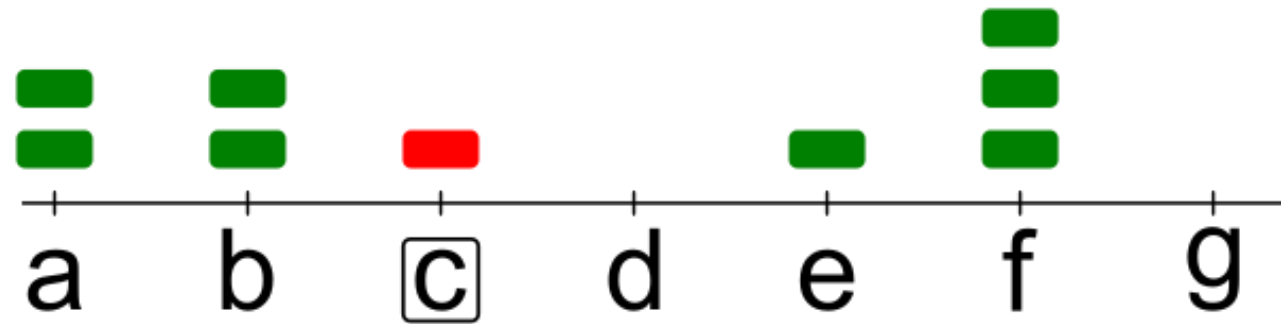
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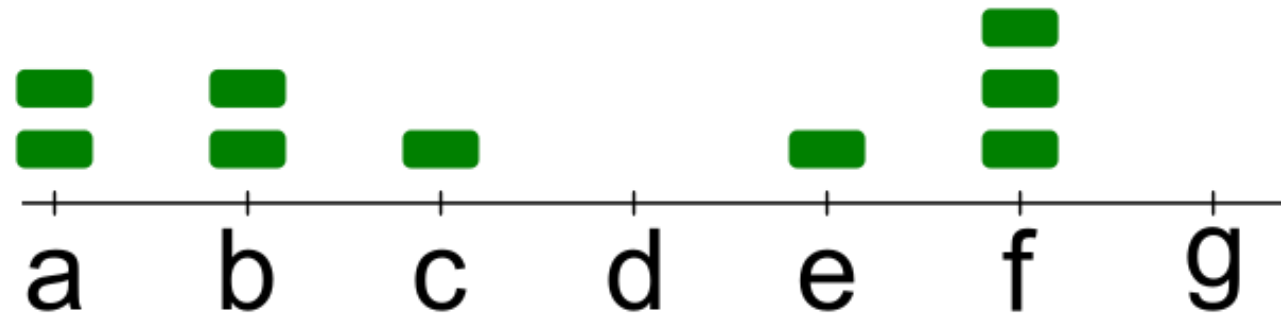


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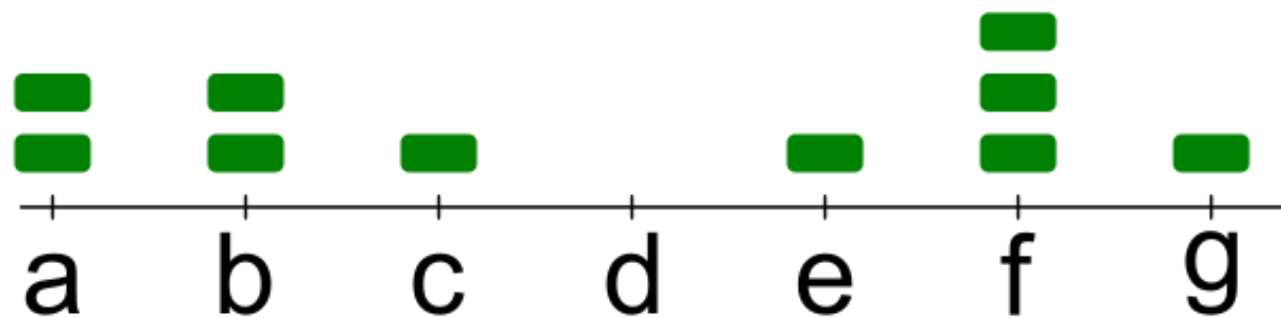


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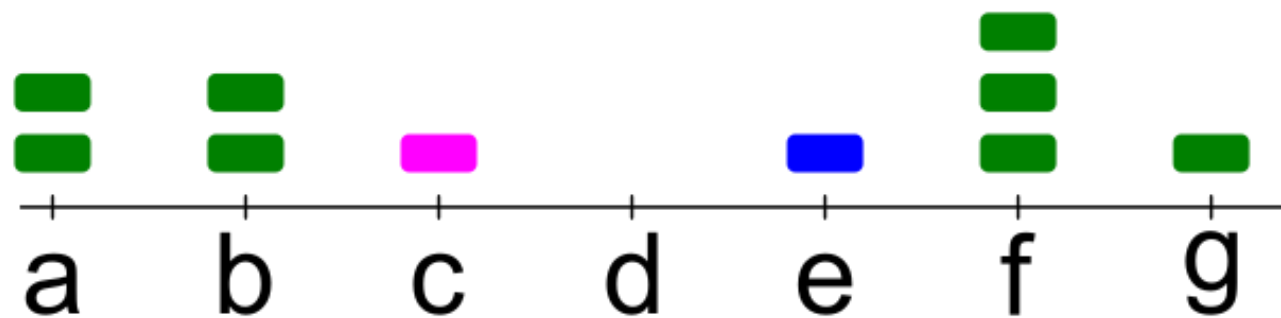
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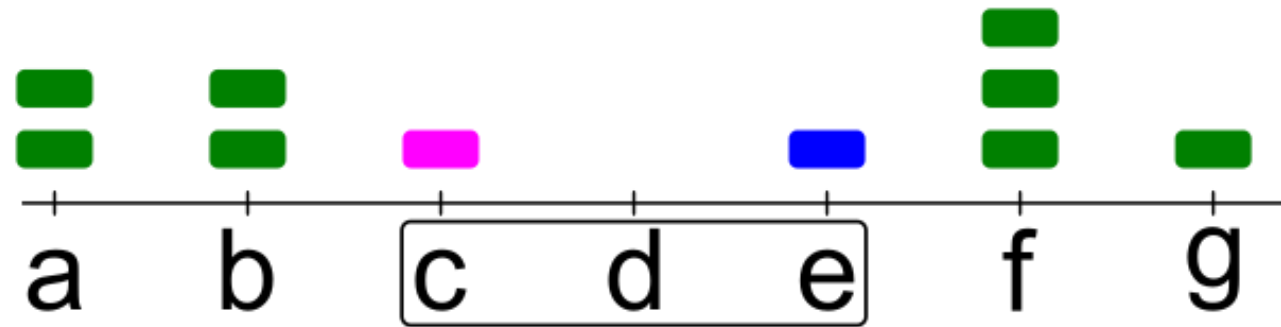
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If no. of voters is even (say, $2k$)

All candidates between
 $\text{top}(V_k)$ and $\text{top}(V_{k+1})$
are weak Condorcet winners

Single-peaked preferences admit a strategyproof voting rule.

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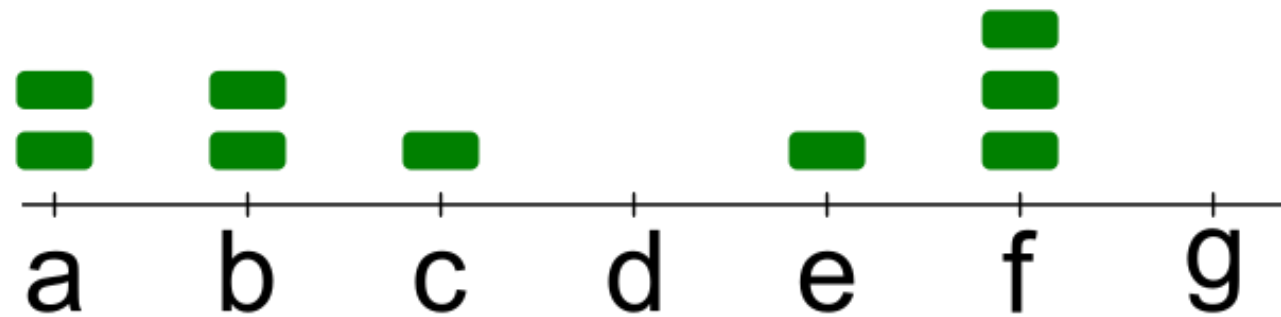
Median voter rule

1. Each voter reports its favorite candidate (or "peak").
2. The* median of the reported peaks is the winner.

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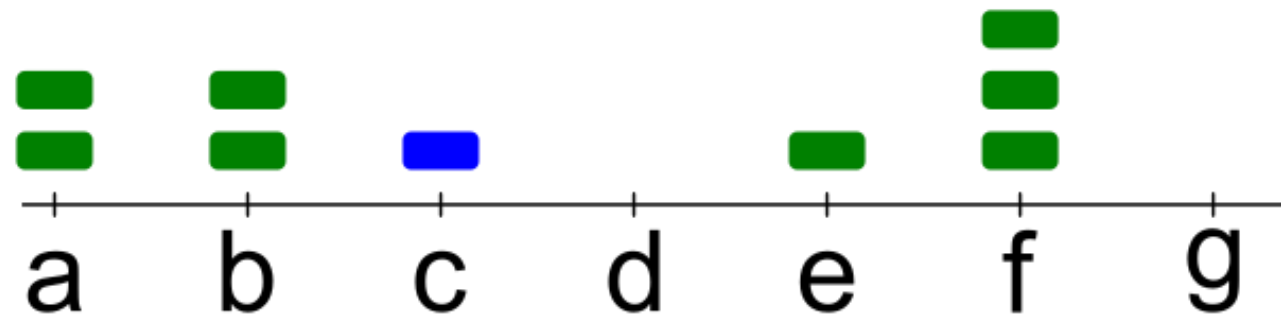
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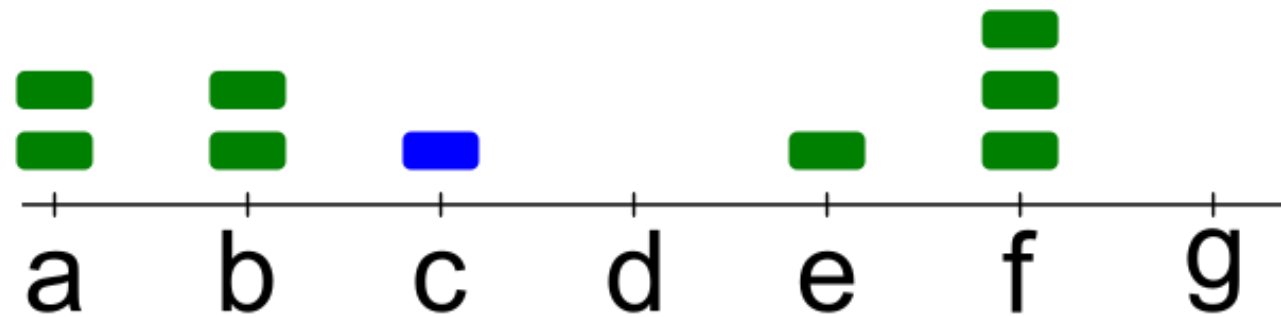
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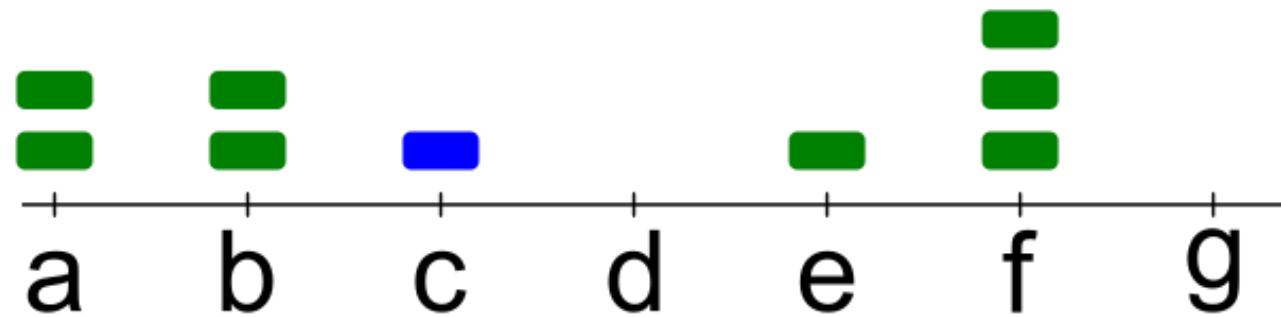
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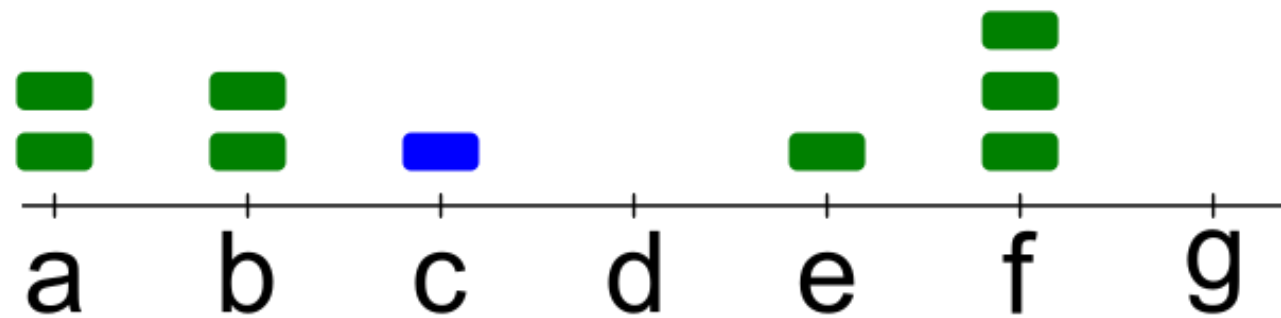


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For any **misreport** on the *same* side, the **outcome** doesn't change.

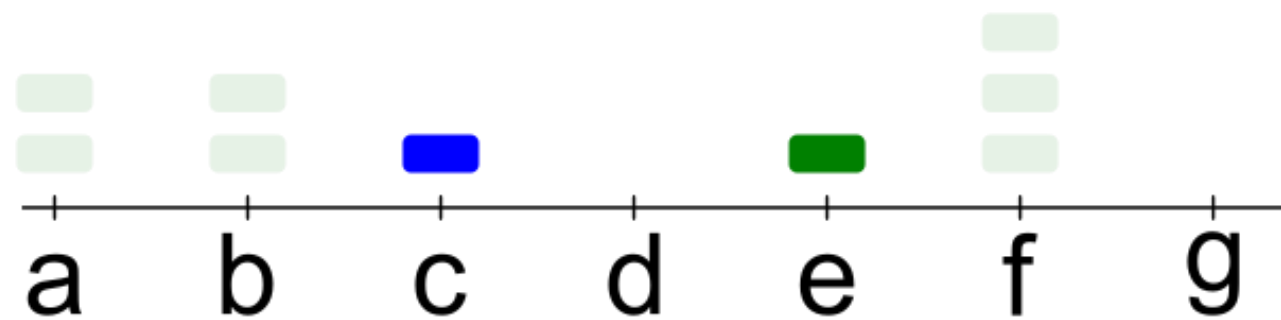


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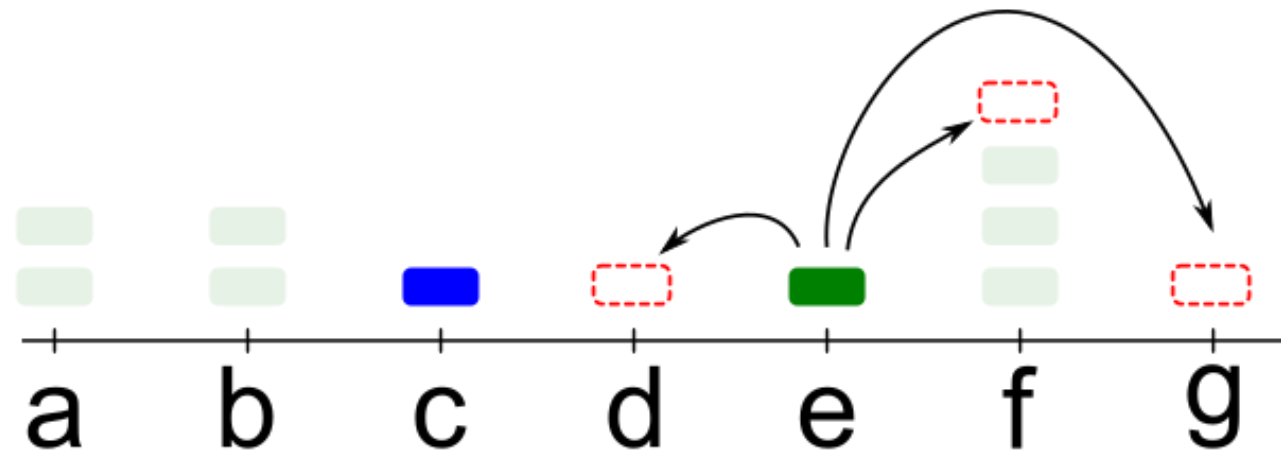


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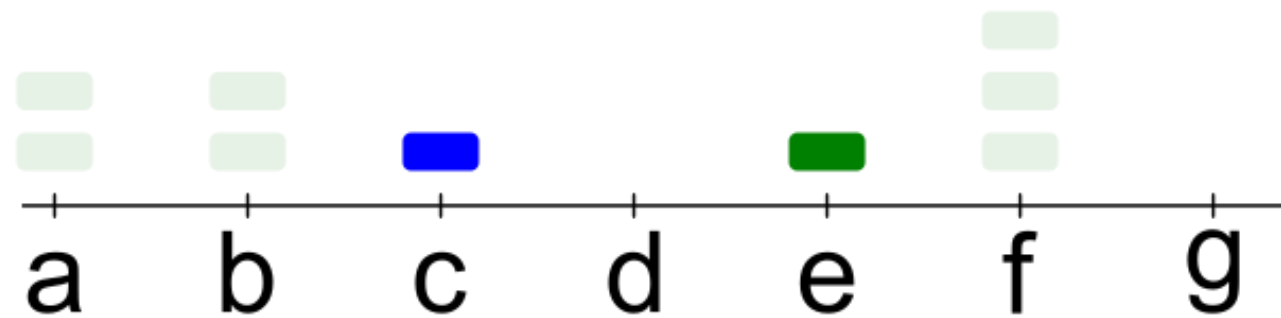


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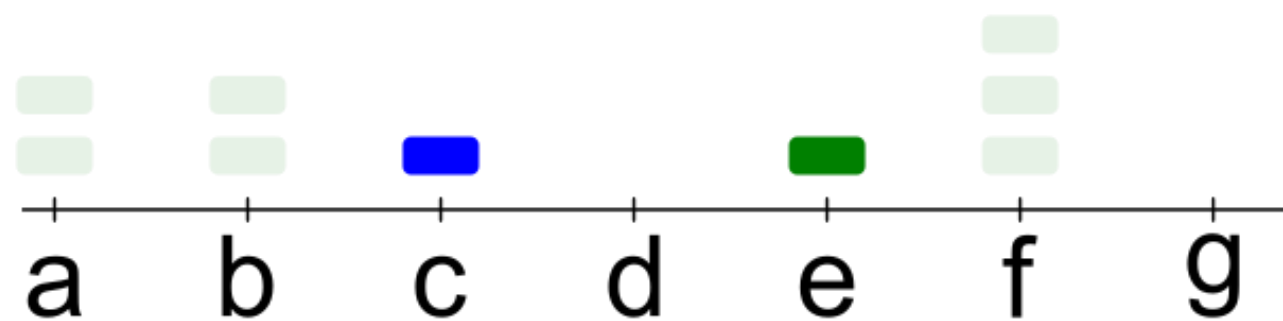
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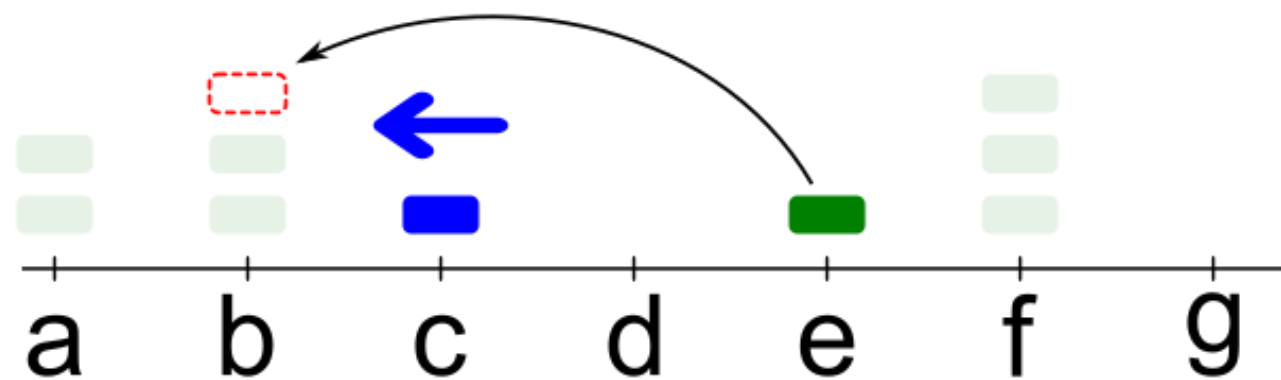
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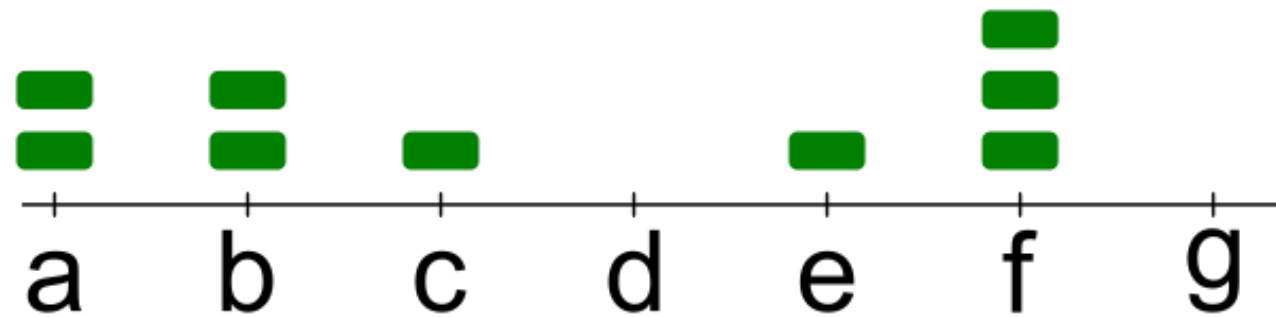
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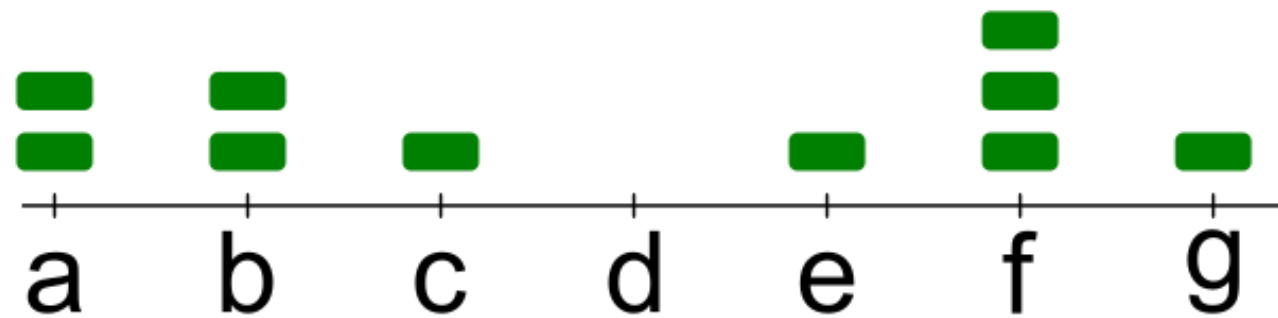
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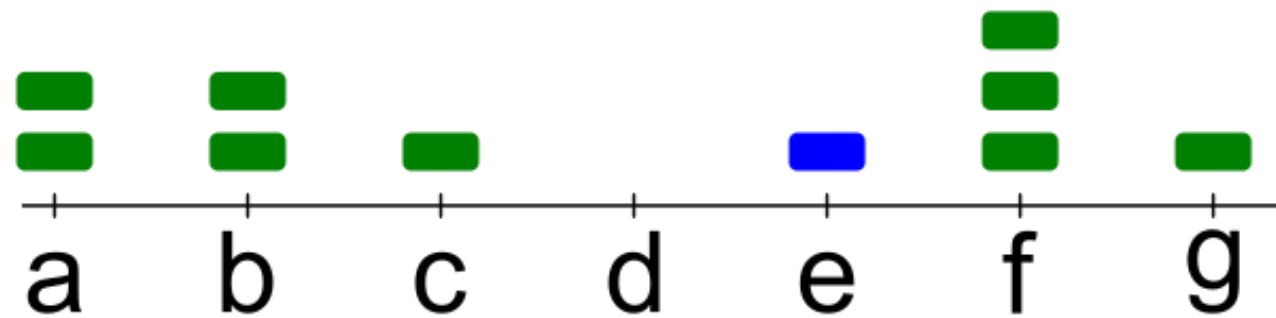


Single-peaked preferences admit a strategyproof voting rule.

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Suppose the no. of voters is even.

Deterministically pick a fixed median (either left or right).



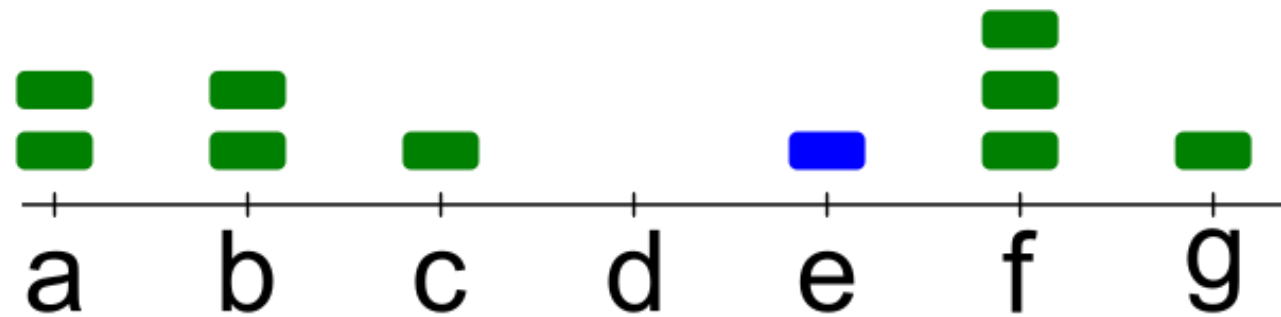
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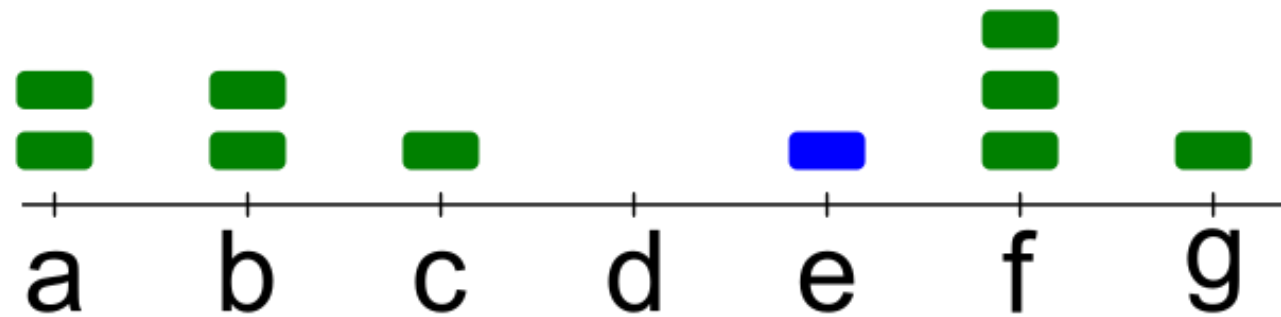
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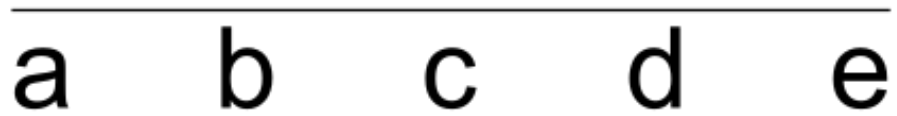


w.r.t. a **given** axis

w.r.t. **some** axis


Recognizing Single-Peaked Prefs w.r.t. a given axis

Recognizing Single-Peaked Prefs w.r.t. a given axis



Recognizing Single-Peaked Prefs w.r.t. a given axis

a b c d e

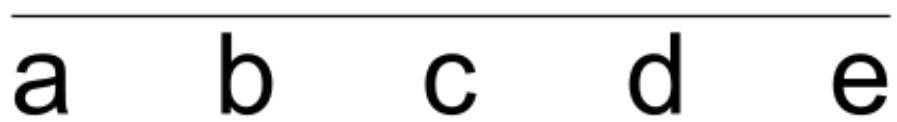


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Recognizing Single-Peaked Prefs w.r.t. a given axis

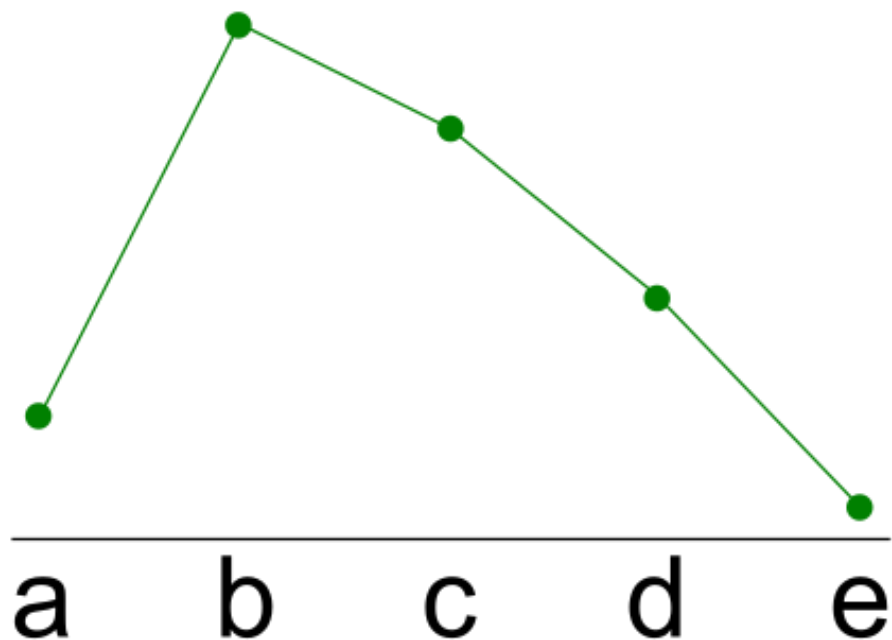
a b c d e



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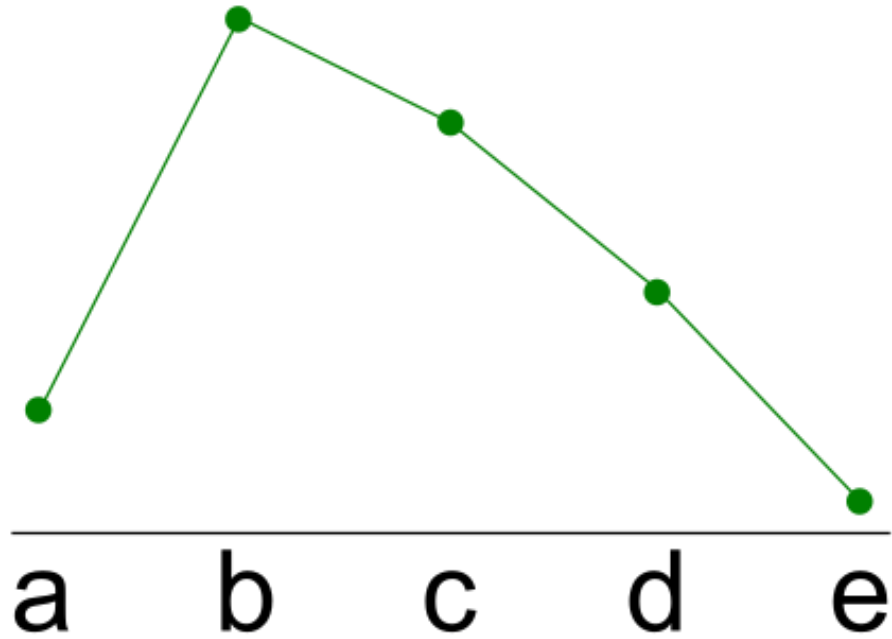


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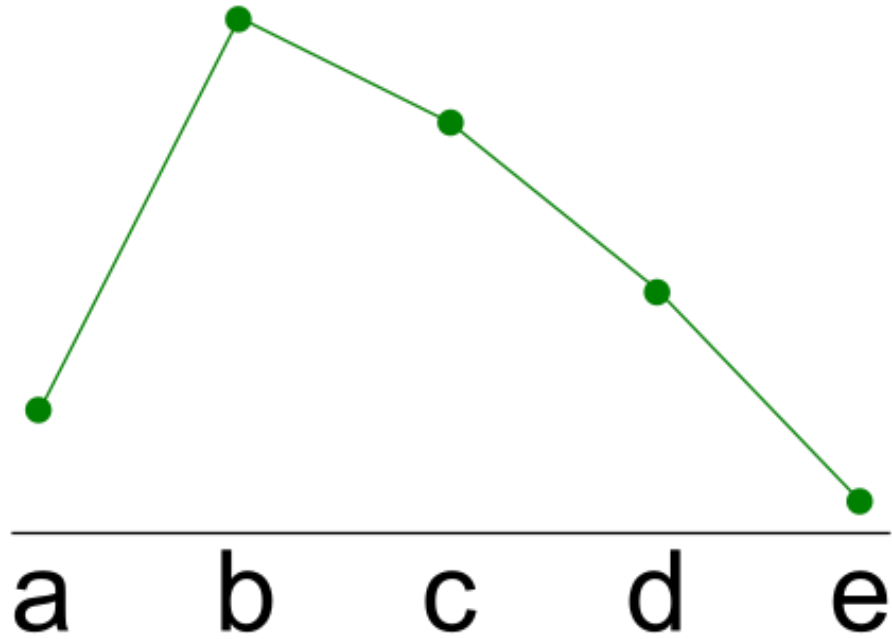
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b
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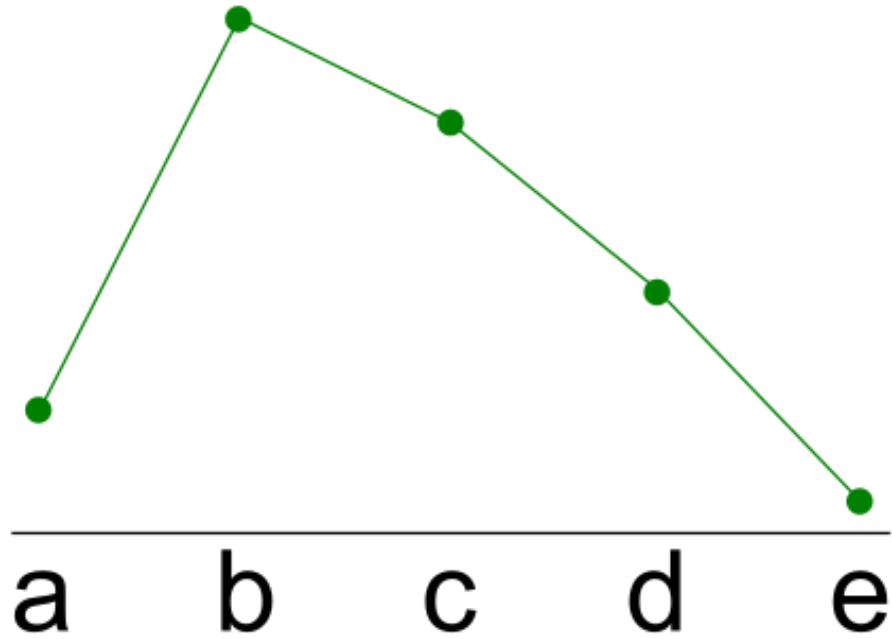
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b
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d
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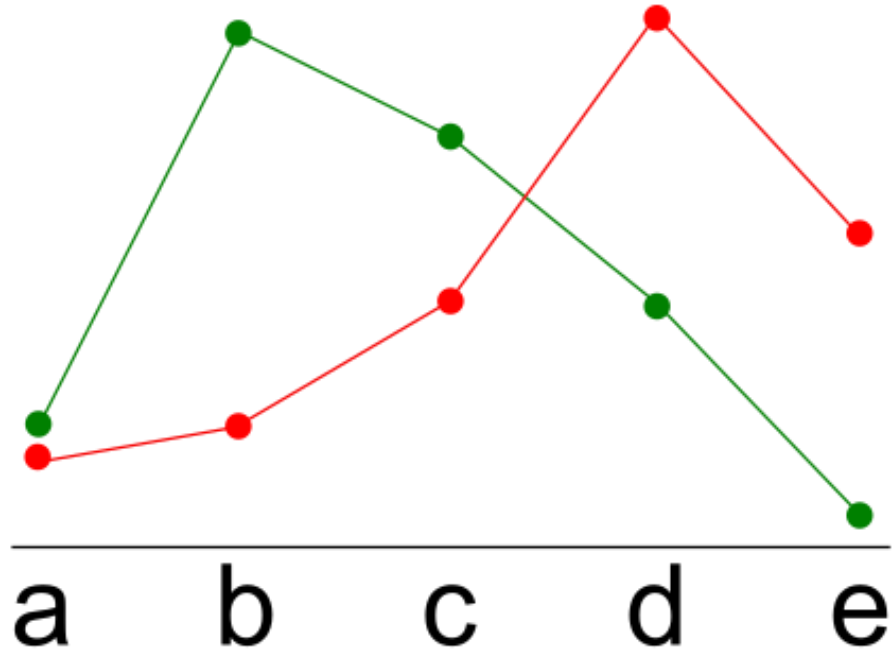
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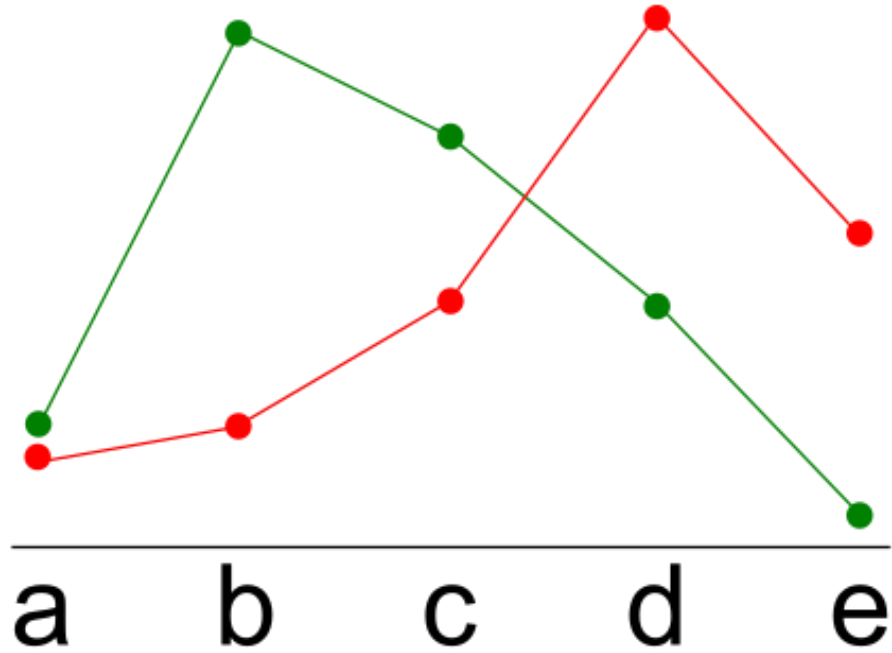


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Recognizing Single-Peaked Prefs w.r.t. a given axis



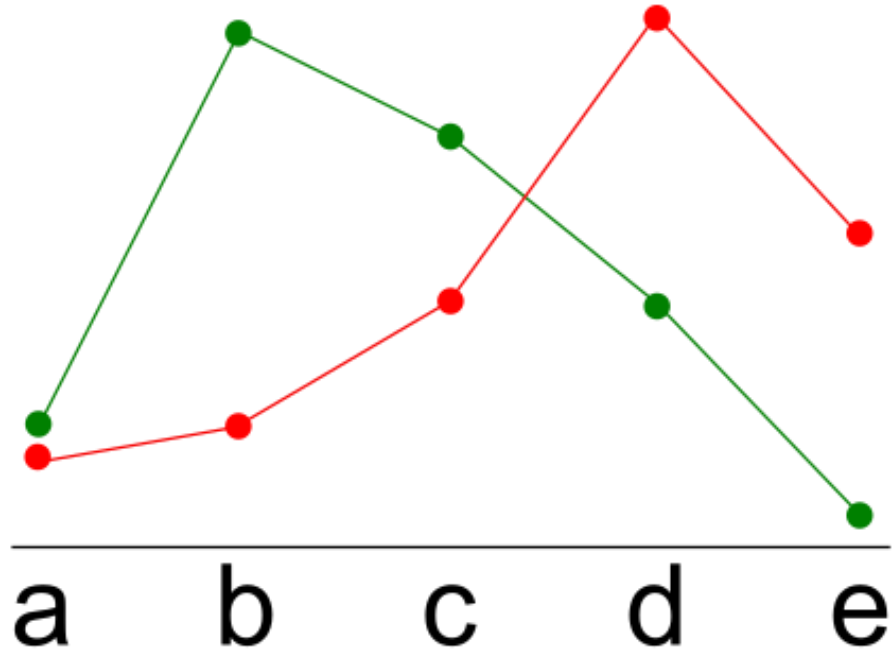
b
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d
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Recognizing Single-Peaked Prefs w.r.t. a given axis



b
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d
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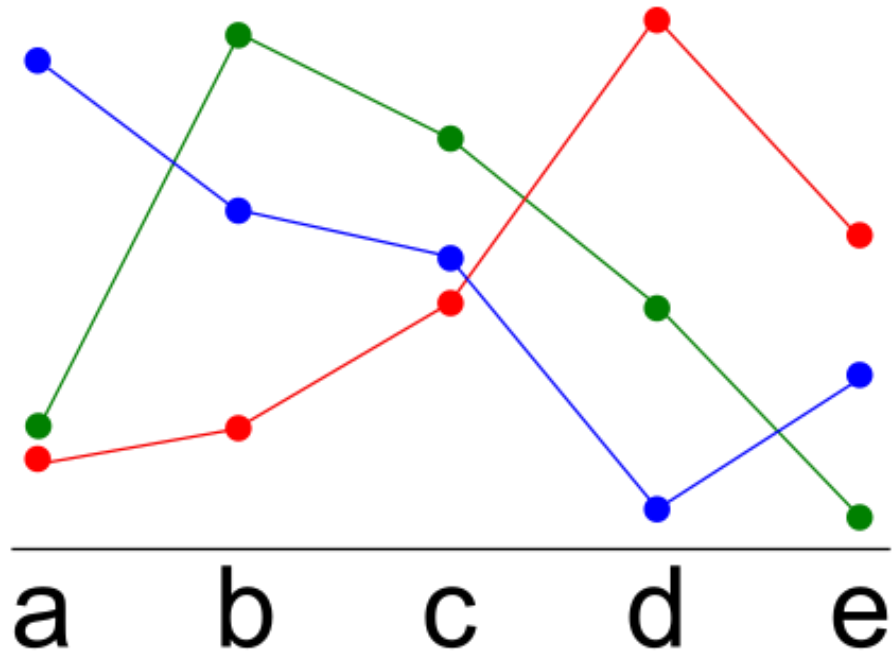


d
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b
c
e
d

Recognizing Single-Peaked Prefs w.r.t. a given axis



b
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d
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e



d
e
c
b
a



a
b
c
e
d

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

For this, let us discuss an equivalent definition of single-peaked preferences.

Contiguous Segments Property

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A preference profile satisfies contiguous segments property w.r.t. $<$ if, for each vote and for every k , the set of top- k candidates in that vote forms a contiguous segment w.r.t. $<$.

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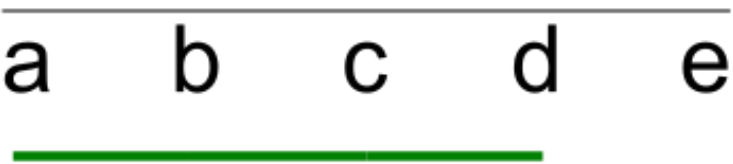
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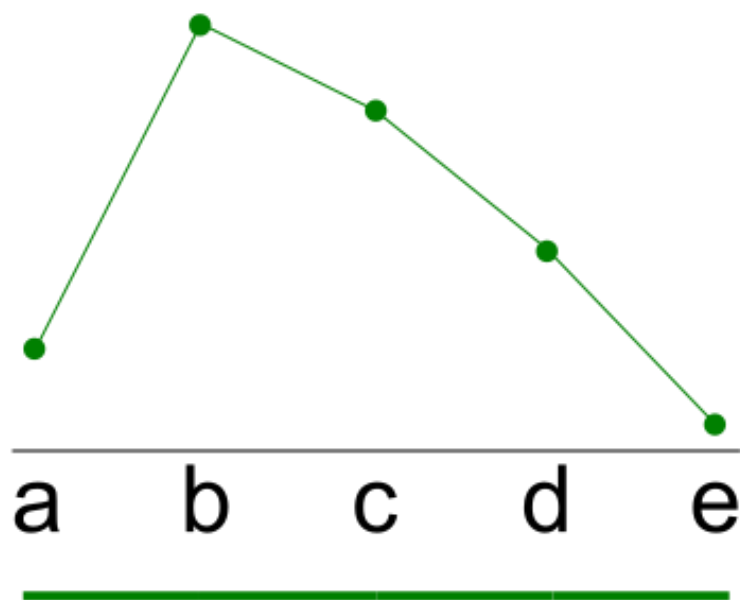
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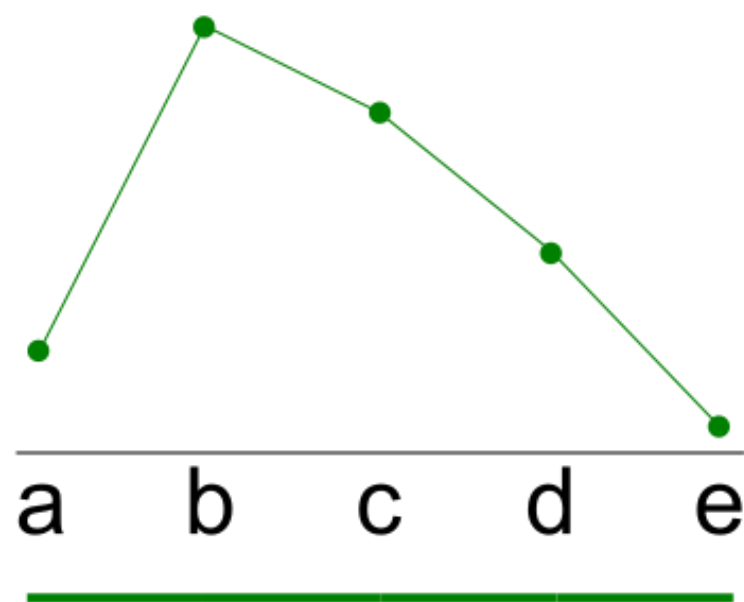
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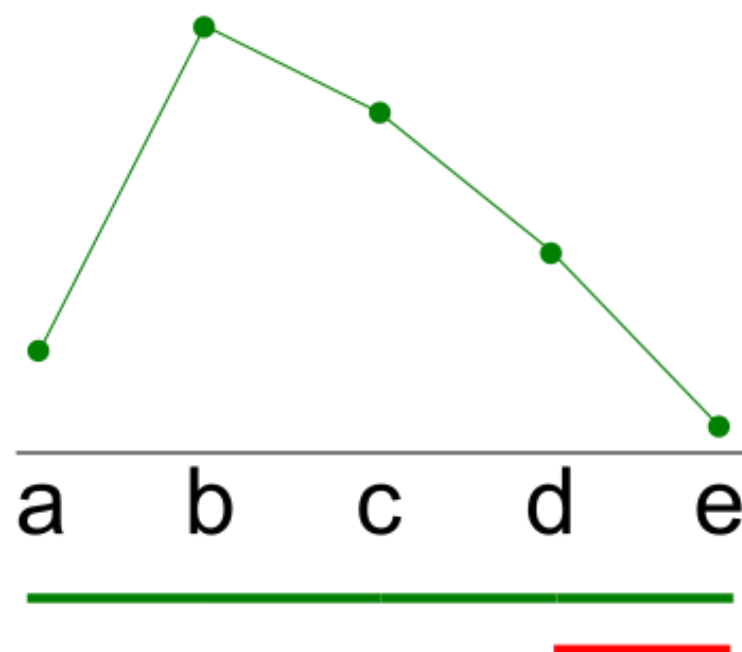


b
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e

d
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c
b
a

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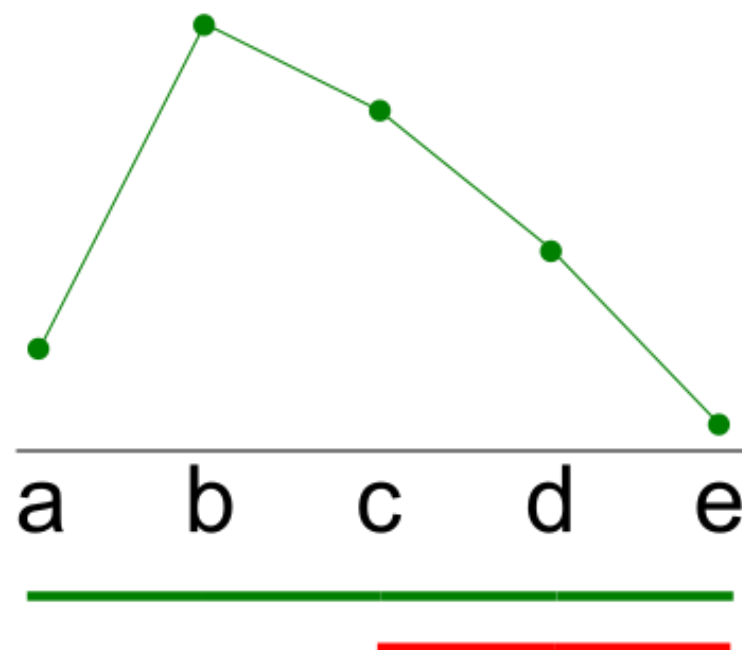


b
c
d
a
e

d
e
c
b
a

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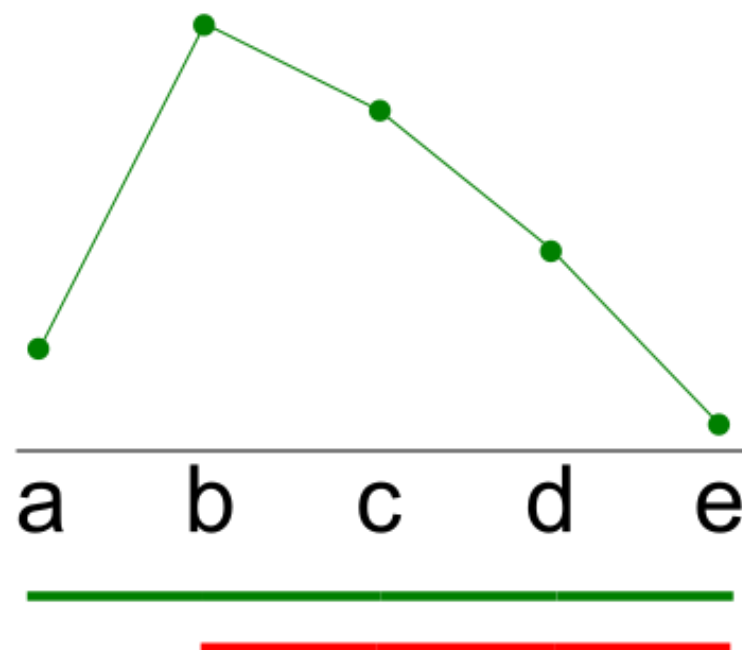


b
c
d
a
e

d
e
c
b
a

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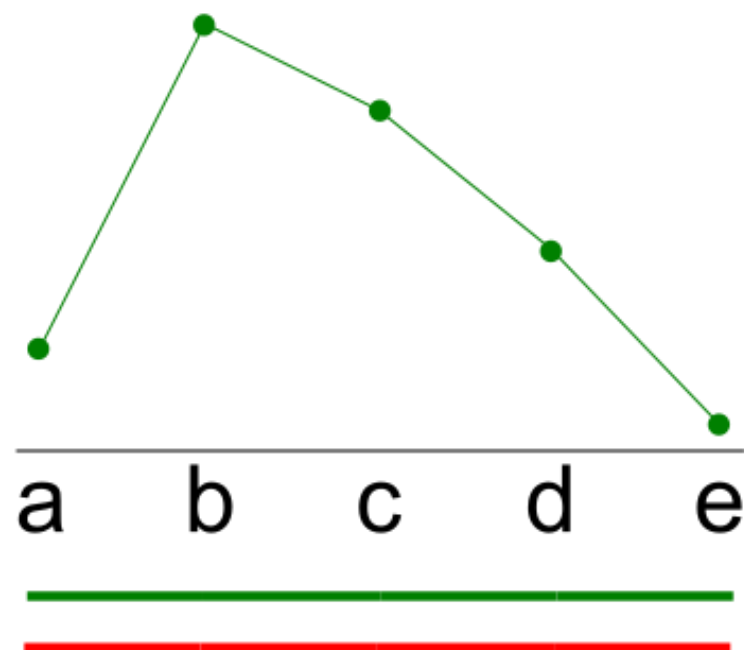


b
c
d
a
e

d
e
c
b
a

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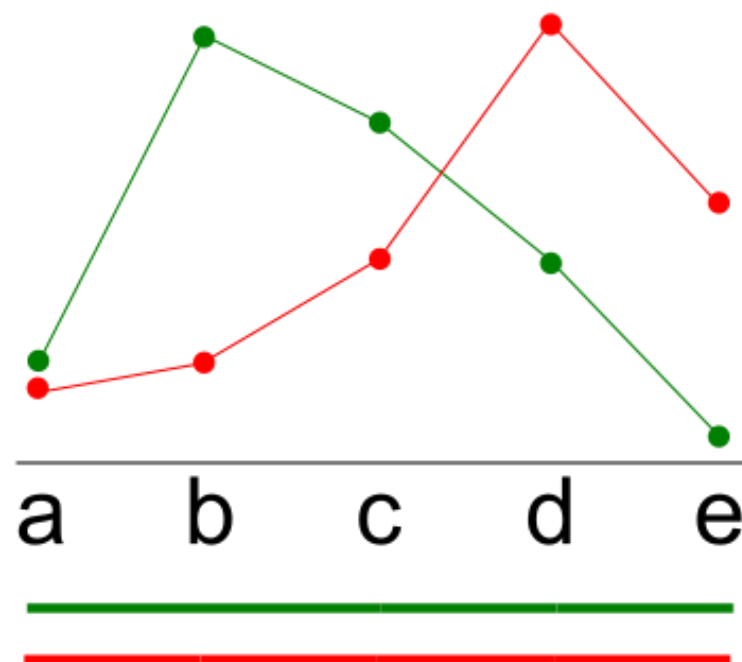


b
c
d
a
e

d
e
c
b
a

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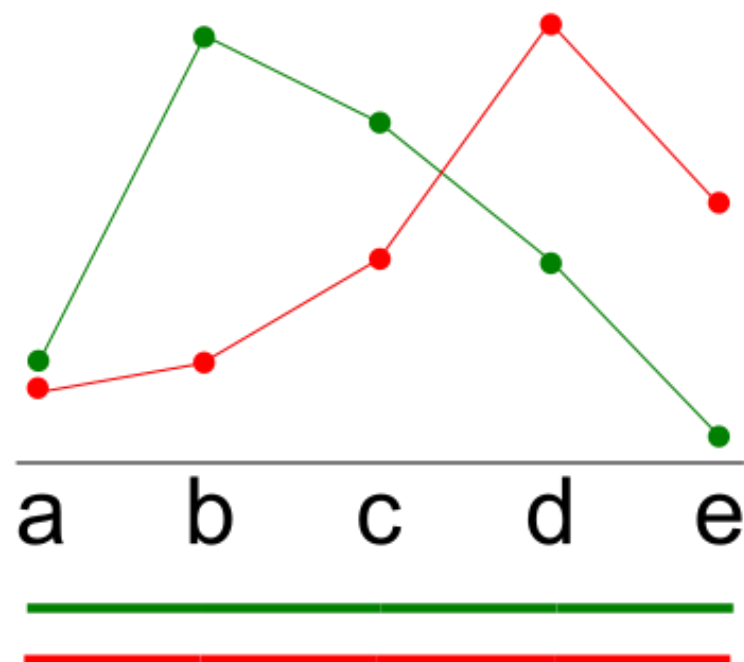


b
c
d
a
e

d
e
c
b
a

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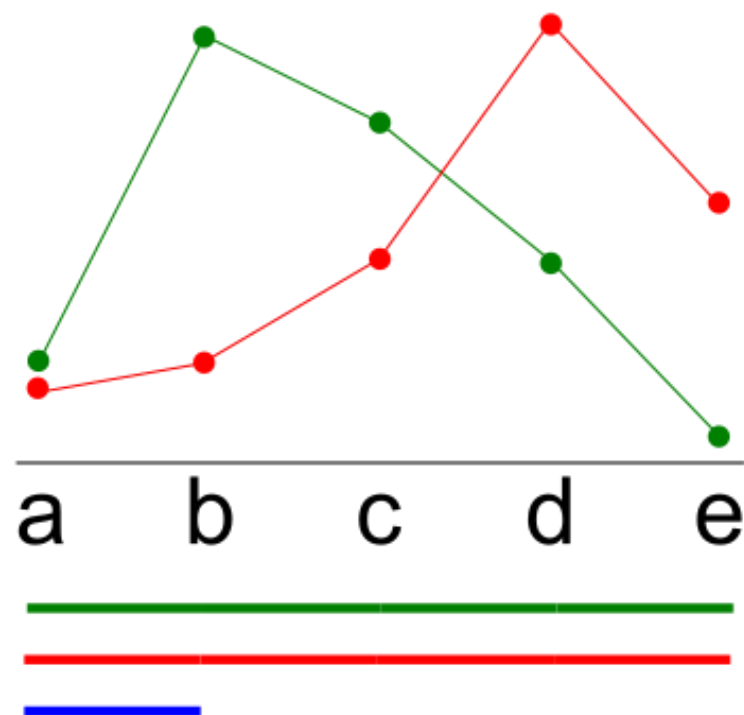
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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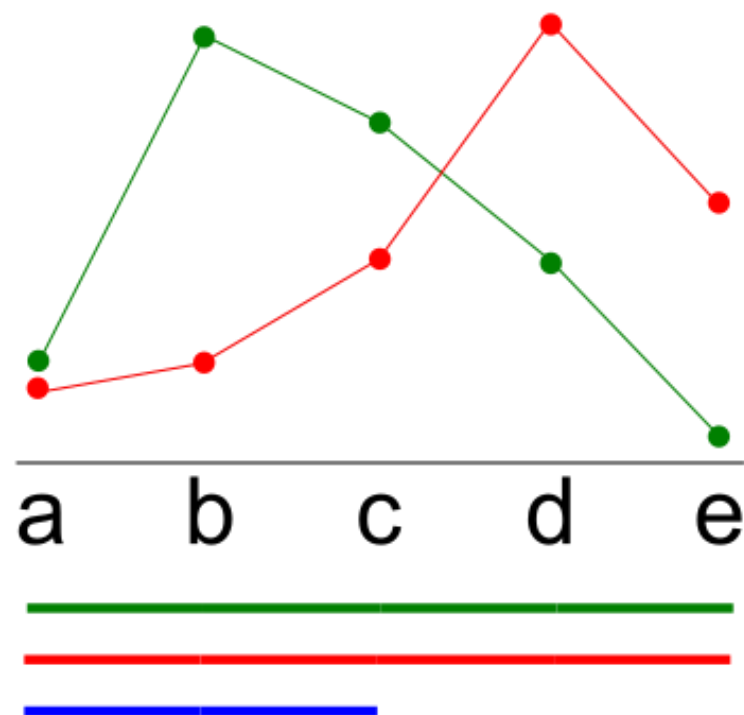
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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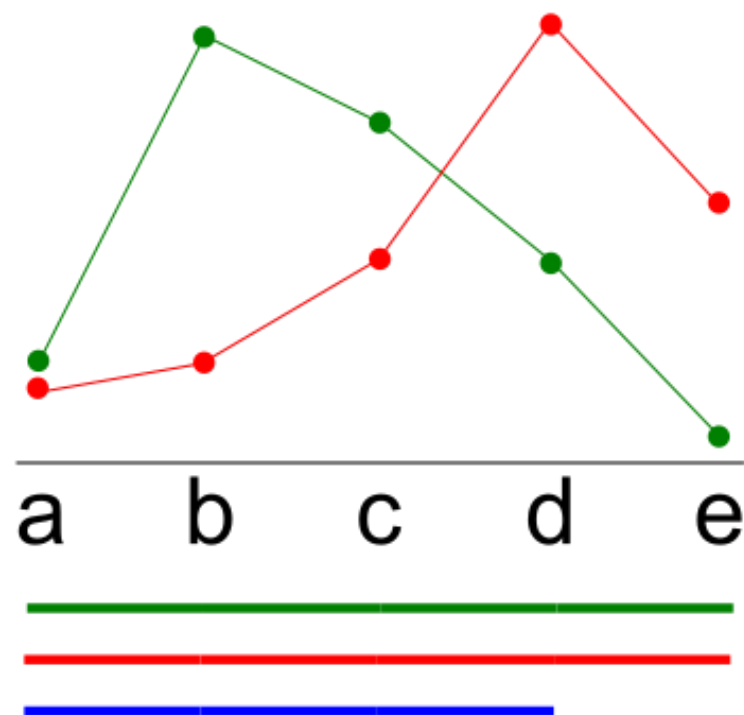
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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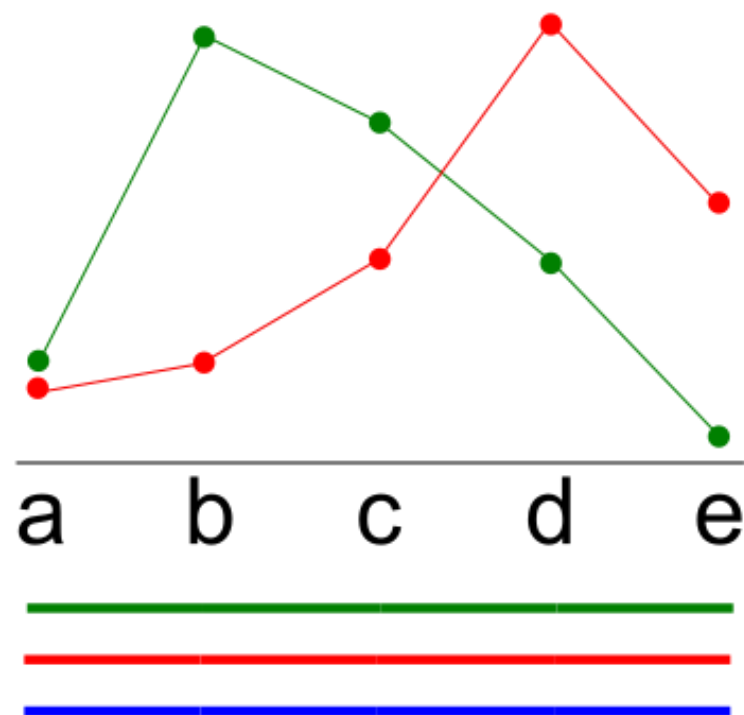
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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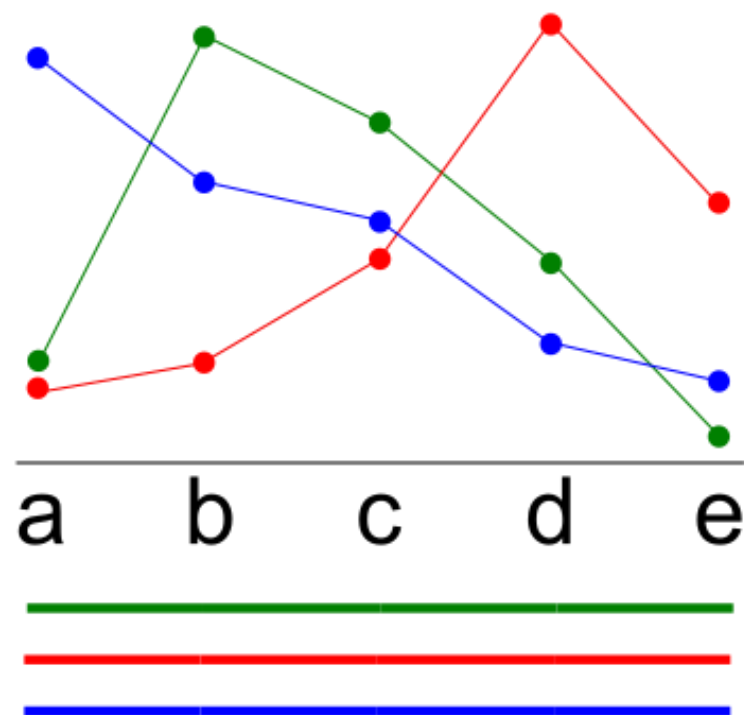
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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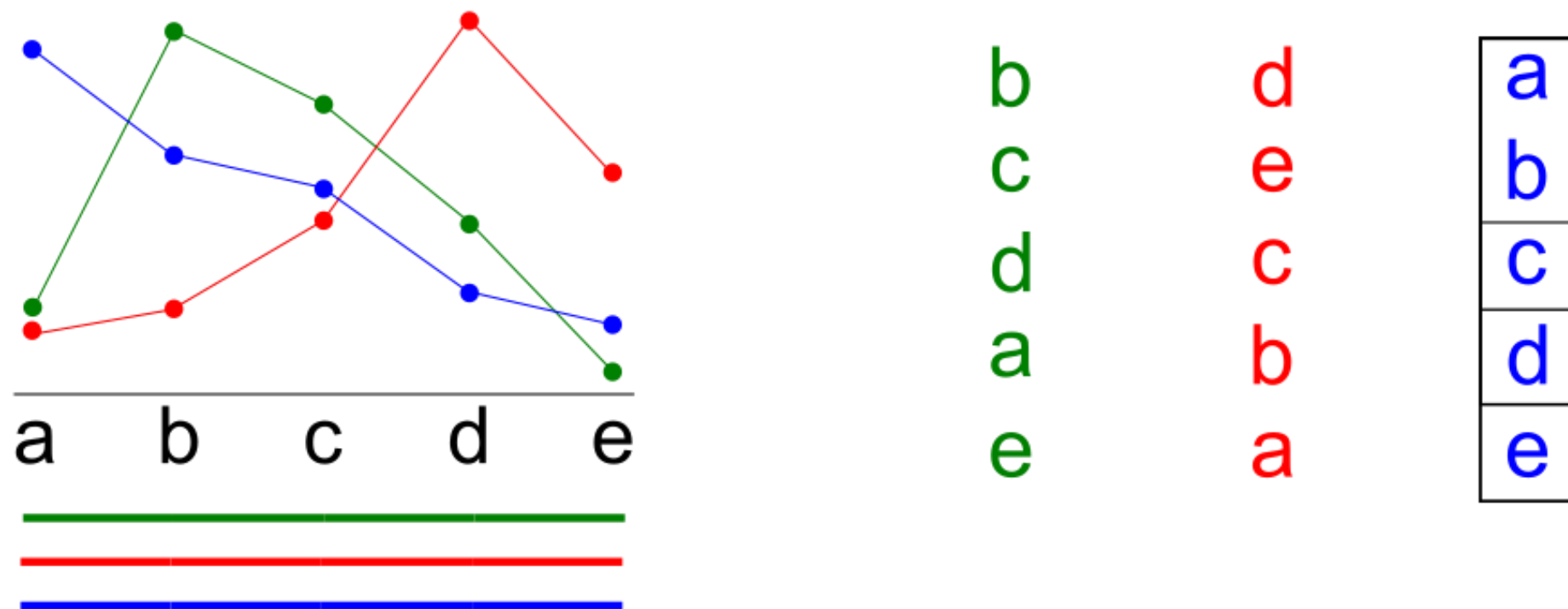
b
c
d
a
e

d
e
c
b
a

a
b
c
d
e

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Contiguous segments (w.r.t. $<$) \Leftrightarrow Single-peaked (w.r.t. $<$)

Contiguous Segments \Rightarrow Single-Peaked

Contiguous Segments \Rightarrow Single-Peaked

Suppose the contiguous segments property holds w.r.t. the axis \leftarrow .

Contiguous Segments \Rightarrow Single-Peaked

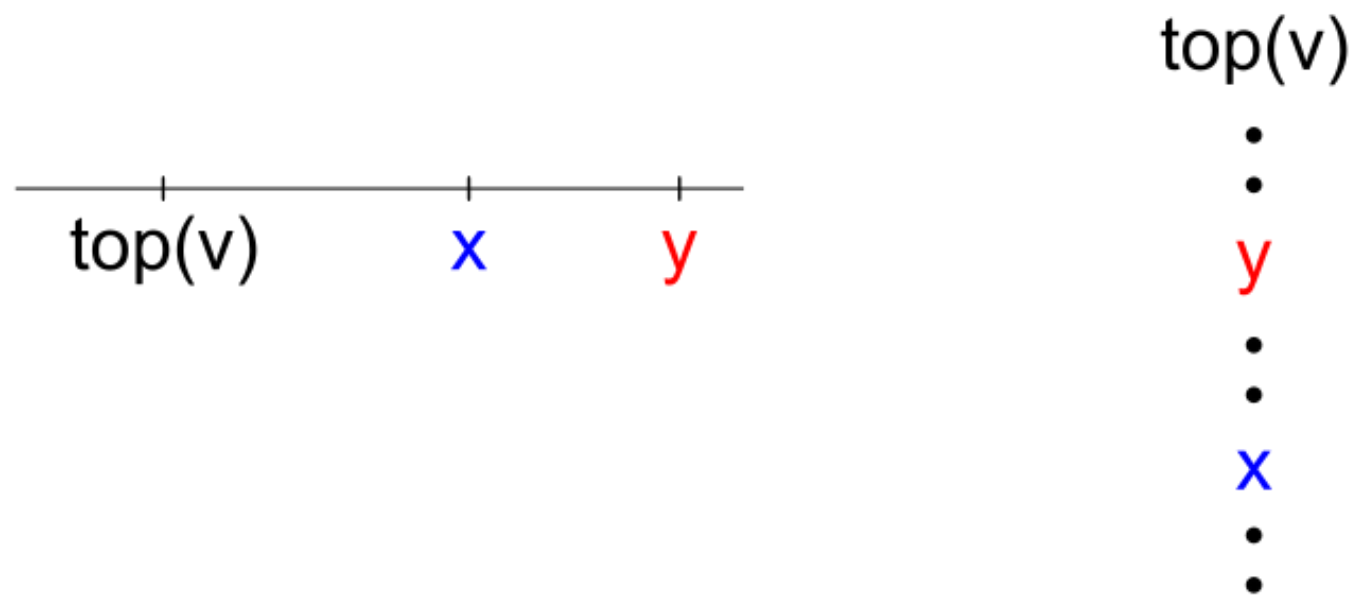
Suppose the contiguous segments property holds w.r.t. the axis $<$.

Suppose, for contradiction, that for some pair of candidates x, y and some voter v , $\text{top}(v) < x < y$ but v prefers y over x .

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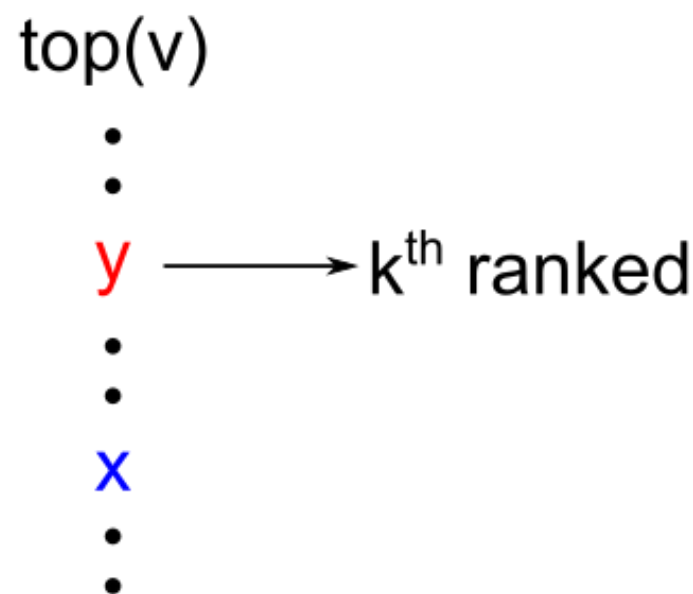
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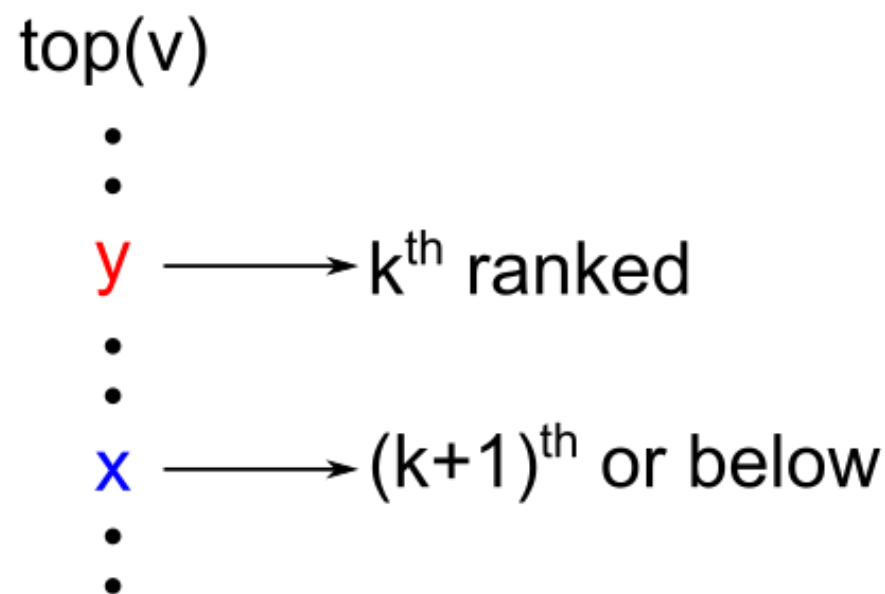
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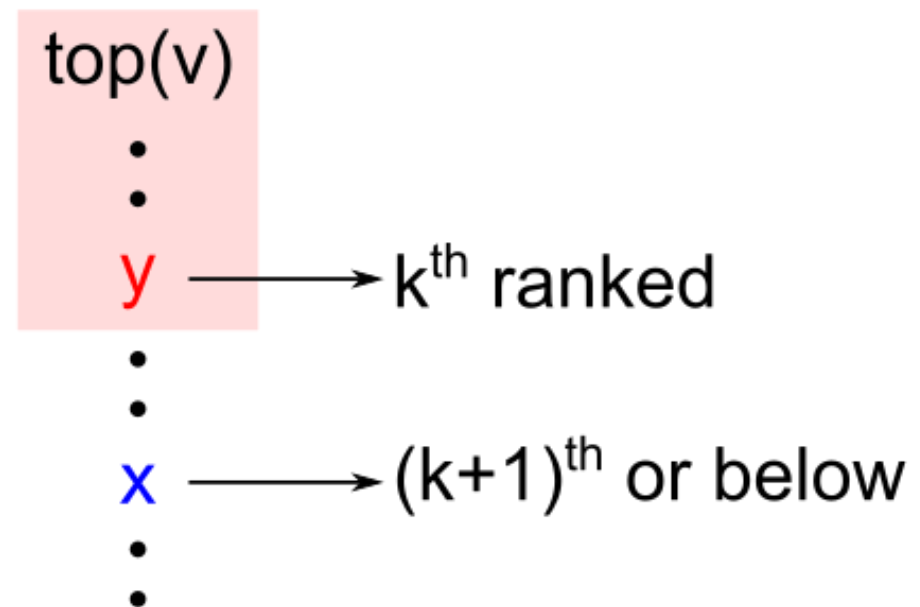
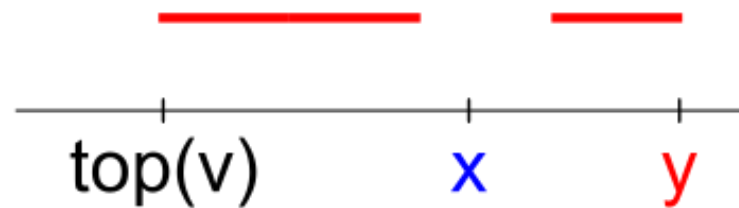
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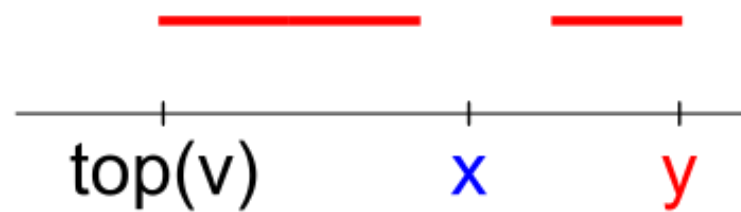
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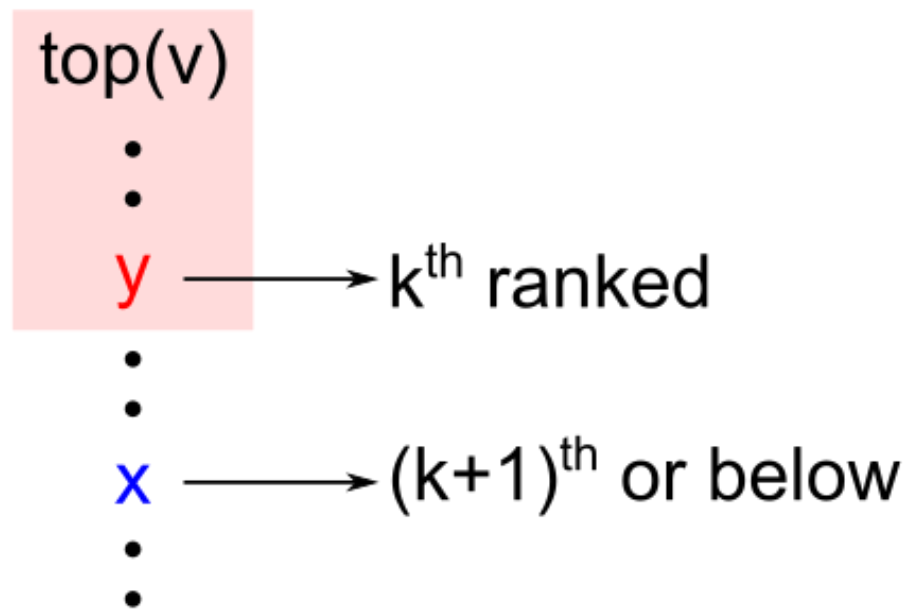
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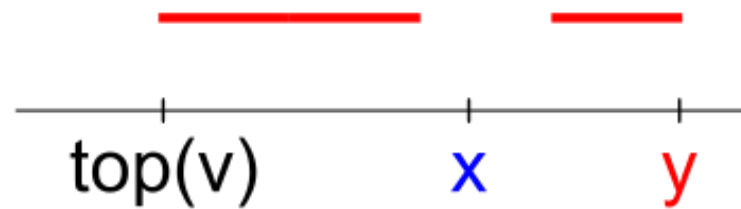
x disconnects the top- k segment



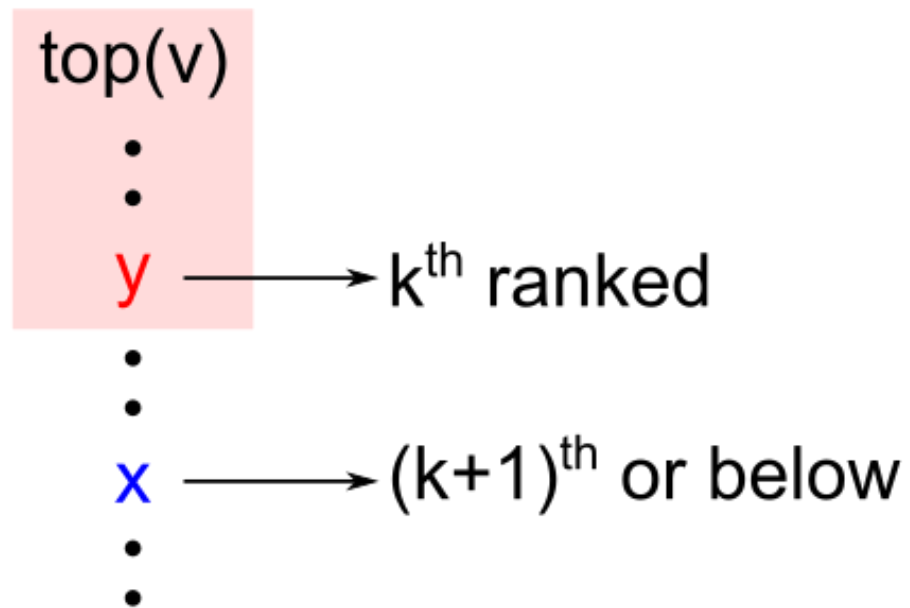
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Single-Peaked \Rightarrow Contiguous Segments

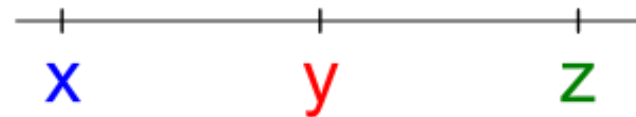
Suppose single-peaked property holds w.r.t. the axis $<$.

Then, for any $x < y < z$, a voter v will never rank y below x and z .
("no valleys" property)

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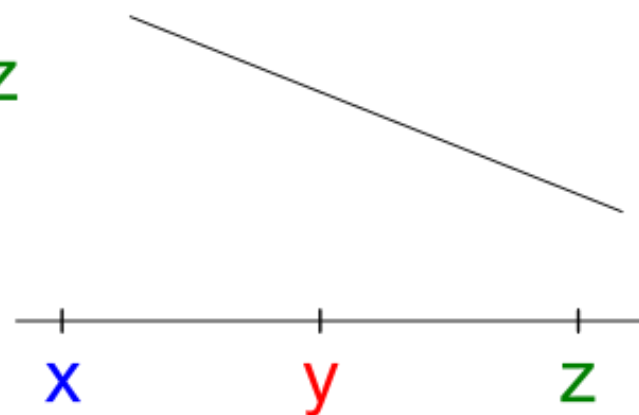


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if $\text{top}(v)$ is to the left of y
then y is preferred over z



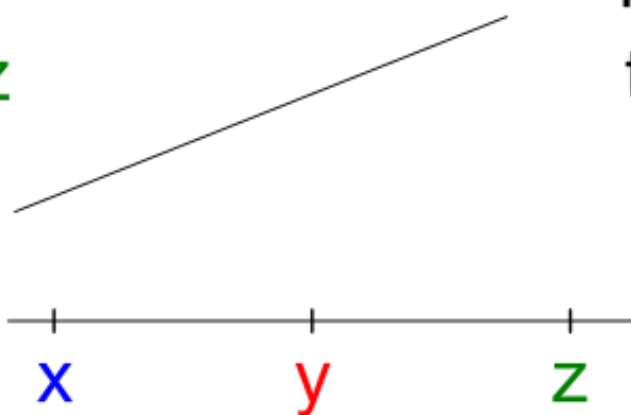
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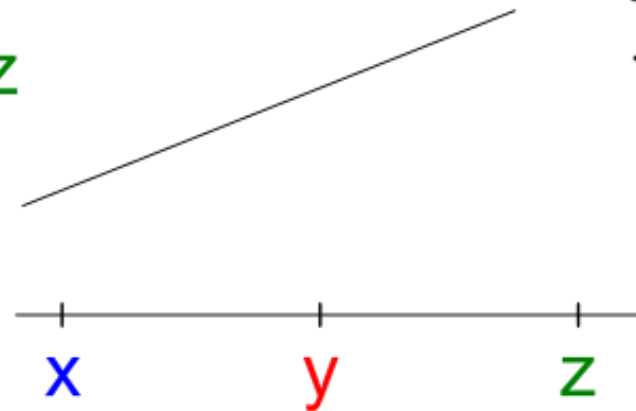
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Thus, single peaked (w.r.t. $<$) \Rightarrow no valleys (w.r.t. $<$).

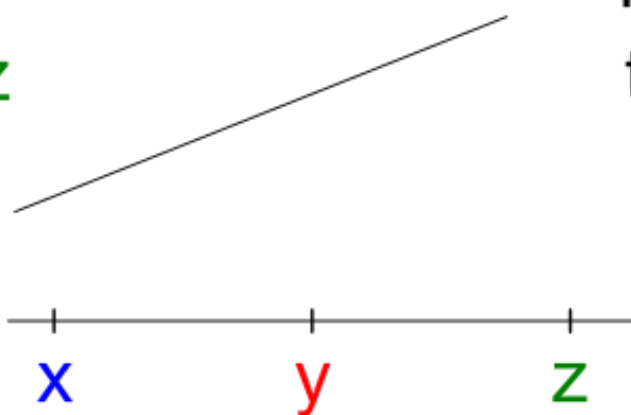
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Thus, single peaked (w.r.t. $<$) \Rightarrow no valleys (w.r.t. $<$).

Let us now show that no valleys (w.r.t. $<$) \Rightarrow contiguous segments (w.r.t. $<$).

Single-Peaked \Rightarrow Contiguous Segments

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Suppose, for contradiction, that for some voter v , the set of top k candidates is not contiguous (w.r.t. $<$).

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Pick the smallest such k .

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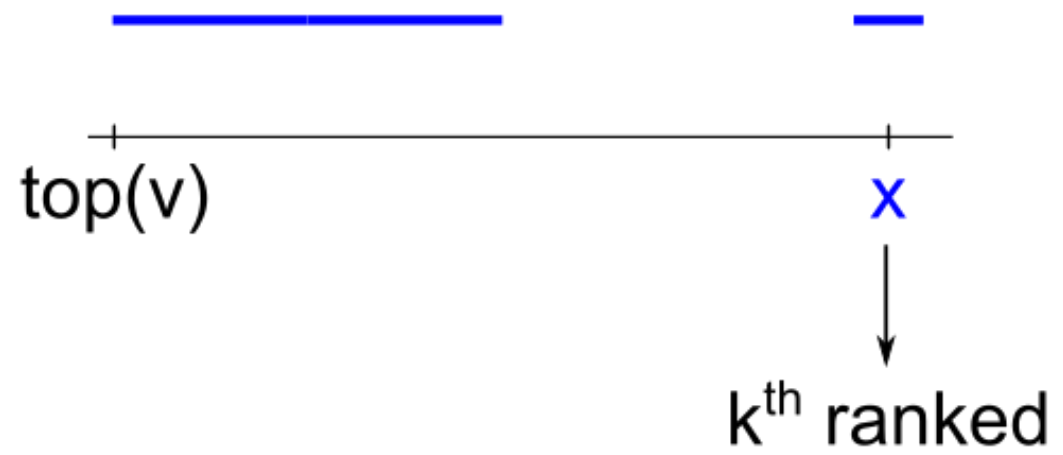
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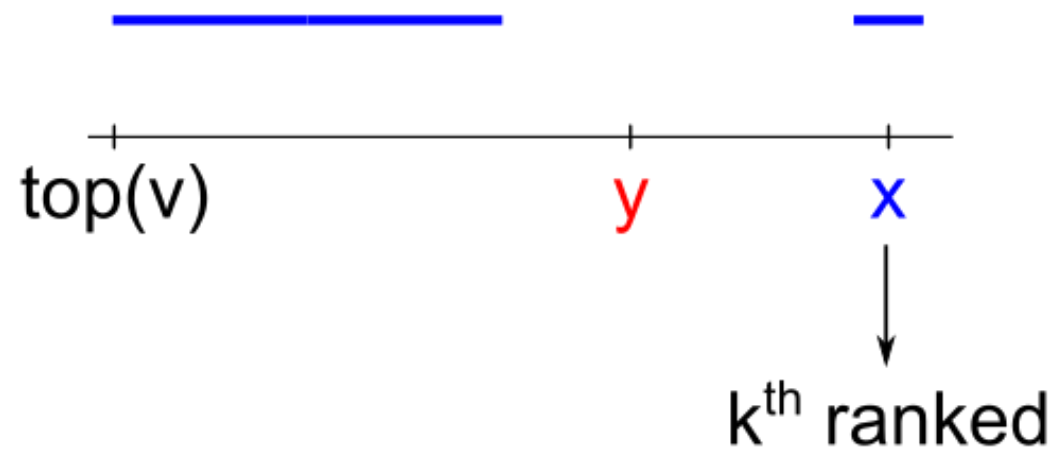
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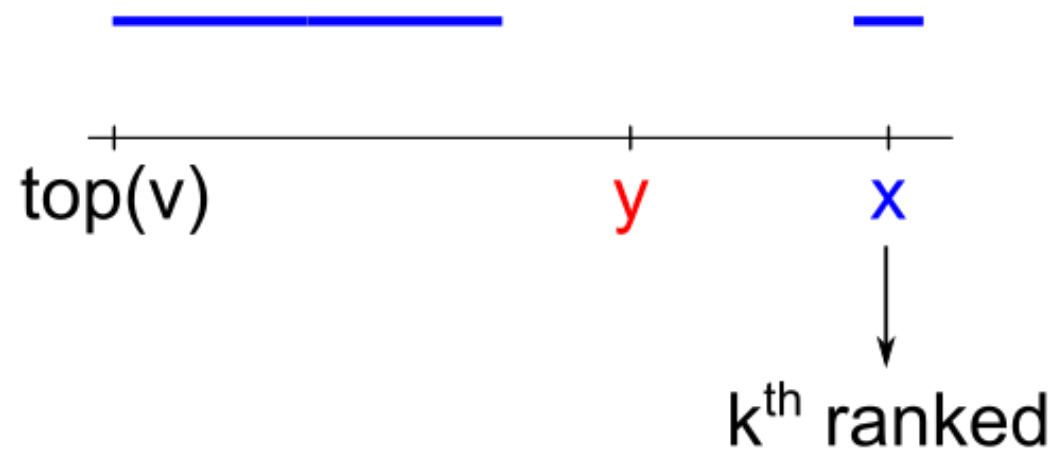


Let y be a candidate that separates x from the top $(k-1)$ candidates

Single-Peaked \Rightarrow Contiguous Segments

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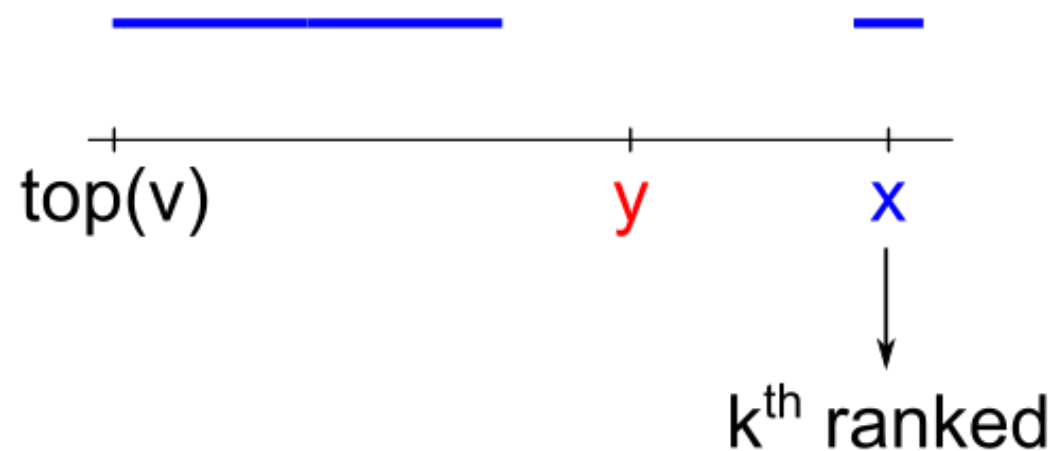
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Then, $\text{top}(v)$, y and x constitute a valley.

Single-Peaked \Rightarrow Contiguous Segments

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Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

We will use the contiguous segments property
to design an algorithm
for recognizing single-peaked preferences.

Recognizing Single-Peaked Prefs w.r.t. some axis

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But, before that, another digression.

Consecutive 1's Problem

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Given a 0-1 matrix, is there a permutation of columns such that all 1's in each row appear consecutively?

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1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0

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1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0



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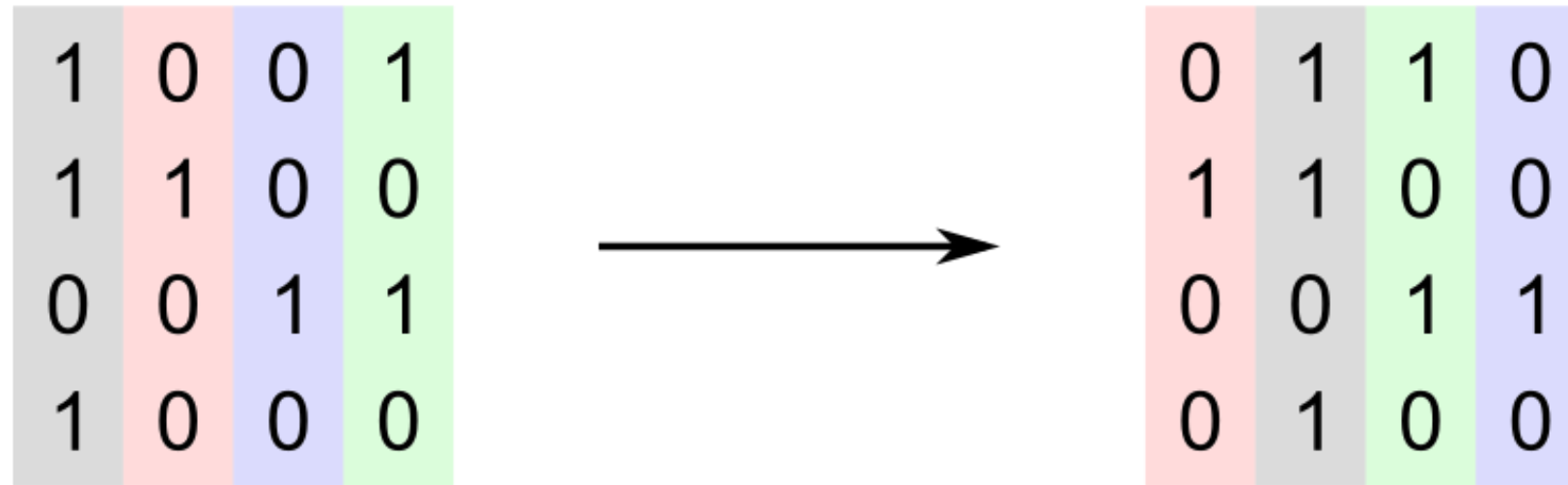
1	0	0	1
1	1	0	0
0	0	1	1
1	0	0	0



0	1	1	0
1	1	0	0
0	0	1	1
0	1	0	0

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[Booth and Leuker, JCSS 1976]

The consecutive 1's problem can be solved in polynomial time.

Recognizing Single-Peaked Prefs w.r.t. some axis

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e



Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

a	b	c	d	e
1	0	1	0	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

a	b	c	d	e
1	0	1	0	0
1	0	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1
	0	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1
	0	0	1	1	1
	1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1
	0	0	1	1	1
	1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1
	0	0	1	1	1
	1	0	1	1	1

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
1	0	1	0	0	
1	0	1	1	0	
1	1	1	1	0	
0	0	0	1	1	
0	0	1	1	1	
1	0	1	1	1	
1	1	0	0	0	

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
1	0	1	0	0	
1	0	1	1	0	
1	1	1	1	0	
0	0	0	1	1	
0	0	1	1	1	
1	0	1	1	1	
1	1	0	0	0	
1	1	1	0	0	

Recognizing Single-Peaked Prefs w.r.t. some axis

a
c
d
b
e

d
e
c
a
b

b
a
c
d
e

	a	b	c	d	e
1	0	1	0	0	
1	0	1	1	0	
1	1	1	1	0	
0	0	0	1	1	
0	0	1	1	1	
1	0	1	1	1	
1	1	0	0	0	
1	1	1	0	0	
1	1	1	1	0	

Recognizing Single-Peaked Prefs w.r.t. some axis

a c d b
c e a
d c c
b a d
e b e

	a	b	c	d	e
1	1	0	1	0	0
2	1	0	1	1	0
3	1	1	1	1	0
4	0	0	0	1	1
5	0	0	1	1	1
6	1	0	1	1	1
7	1	1	0	0	0
8	1	1	1	0	0
9	1	1	1	1	0

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

	a	b	c	d	e
1	0	1	0	0	
1	0	1	1	0	
1	1	1	1	0	
0	0	0	1	1	
0	0	1	1	1	
1	0	1	1	1	
1	1	0	0	0	
1	1	1	0	0	
1	1	1	1	0	

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if

its prefix matrix satisfies consecutive 1's property.

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

	a	b	c	d	e
	1	0	1	0	0
	1	0	1	1	0
	1	1	1	1	0
	0	0	0	1	1
	0	0	1	1	1
	1	0	1	1	1
	1	1	0	0	0
	1	1	1	0	0
	1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if

its prefix matrix satisfies consecutive 1's property.

Recognizing Single-Peaked Prefs w.r.t. some axis

a	d	b
c	e	a
d	c	c
b	a	d
e	b	e

b	a	c	d	e
0	1	1	0	0
0	1	1	1	0
1	1	1	1	0
0	0	0	1	1
0	0	1	1	1
0	1	1	1	1
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0

[Bartholdi and Trick, ORL 1986]

A preference profile is single-peaked
if and only if

its prefix matrix satisfies consecutive 1's property.

Do single-peaked preferences occur in real world?

PrefLib: A Library for Preferences

Data ▾

Elections

Matchings

Ratings

Search

About

PrefLib

PrefLib is a reference library of preference data and links assembled by *Nicholas Mattei*, *Toby Walsh* and lately *Simon Rey*. This site and library is proudly supported by the *Algorithmic Decision Theory group* at *Data61* and the *The COMSOC Group at the University of Amsterdam*.

We want to provide a comprehensive resource for the multiple research communities that deal with preferences, including computational social choice, recommender systems, data mining, machine learning, and combinatorial optimization, to name just a few.

For more information on PrefLib and some helpful tips on using it, please see Nick's Tutorial *Slides and Code* from *EXPLORE 2014*. Check out the *data type* page to learn more about the kind of data we provide.

Please see the *about* page for information about the site, contacting us, and our citation policy. We rely on the support of the community in order to grow the usefulness of this site. To contribute, please contact *Nicholas Mattei* at: [nsmattei{at}gmail](mailto:nsmattei@gmail.com) or *Simon Rey* at: [s.j.rey{at}uva{dot}nl](mailto:s.j.rey@uva.nl).

In Brief

We currently host:

- 11 types of data
- 38 datasets
- 3668 data files
- More than 3.37 Gb. of data

Other Links

Here are some links that you might find relevant as well.

- *DEMOCRATIX: A Declarative Approach to Winner Determination*

CHAPTER 15

A PREFLIB.ORG Retrospective: Lessons Learned and New Directions

Nicholas Mattei and Toby Walsh

Trends in Computational Social Choice

Realism. Perhaps the key motivating factor behind assembling PREFLIB was a desire to have realistic data. Many of the models studied in classical social choice seem to be chosen because they *seem* reasonable or were explicitly chosen for mathematical expediency. Perhaps nothing is more of an exemplar here than the fact that out of over 300 profiles containing strict, complete preference relations, absolutely none are single-peaked, a common profile restriction that has been called “natural” or “well-motivated” numerous times since its introduction by Black (1948). Collecting data has helped us to quantify what is reasonable. Now we have to start using the data.

Reminders

Project Update Meetings (Oct 11 and 12)

Mid-Semester Feedback (due Oct 13)

Next Time

Rank Aggregation

Oct 18 (Tue)

References

- Single-peaked preferences in theory:

Duncan Black

“On the Rationale of Group Decision-Making”

Journal of Political Economy, Feb 1948, 56(1), pg 23-34

<https://www.journals.uchicago.edu/doi/10.1086/256633>

- Single-peaked preferences in the real-world:

Nicholas Mattei and Toby Walsh

“A PREFLIB.ORG Retrospective: Lessons Learned and New Directions”

Chapter 15 in Trends in Computational Social Choice

<https://research.illc.uva.nl/COST-IC1205/BookDocs/TrendsCOMSOC.pdf>

References

- Lecture by Edith Elkind on restricted preference domains:
<https://www.youtube.com/watch?v=q8vc8Znoev0>
https://www.youtube.com/watch?v=vL_U-5tlQu4

- Strategyproof voting rules using “phantom” voters:

Hervé Moulin

“On Strategyproofness and Single-Peakedness”

Public Choice, 35(4), 1980, pp. 437-455

<https://www.jstor.org/stable/30023824>

References

- Recognizing single-peaked preferences:

John Bartholdi III and Michael A. Trick

“Stable Matching with Preferences Derived from a Psychological Model”

Operations Research Letters, 5(4), 1986, pp. 165-169

<https://www.sciencedirect.com/science/article/pii/0167637786900726>

- Consecutive 1's Problem:

Kellogg S. Booth and George S. Lueker

“Testing for the Consecutive One Property, Interval Graphs, and Graph Planarity using PQ Tree Algorithms”

JCSS, 13(3), 1976, pp. 335-379

<https://www.sciencedirect.com/science/article/pii/S0022000076800451>

