

Lecture 14

Computational Barriers to Manipulation

Last Time

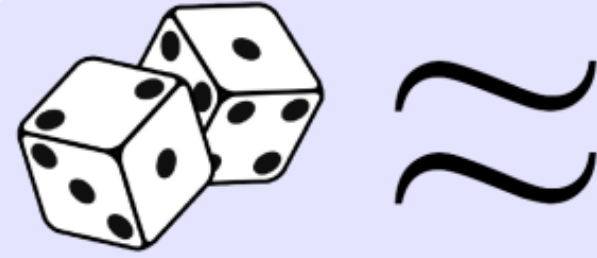
[Gibbard'73; Satterthwaite'75]

Any **onto** and **non-dictatorial** voting rule
must be **manipulable**.

Circumventing GS



Circumventing GS

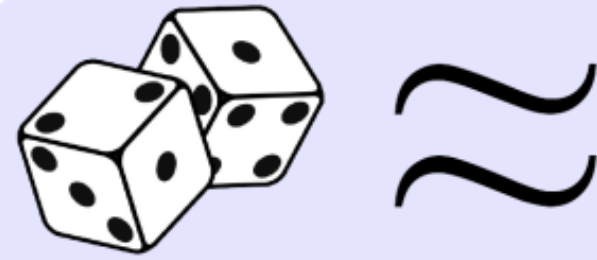


Circumventing GS

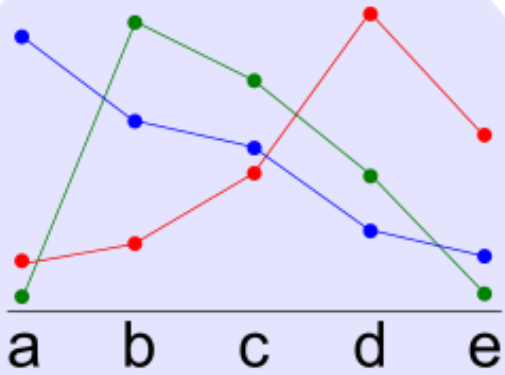


Circumventing GS





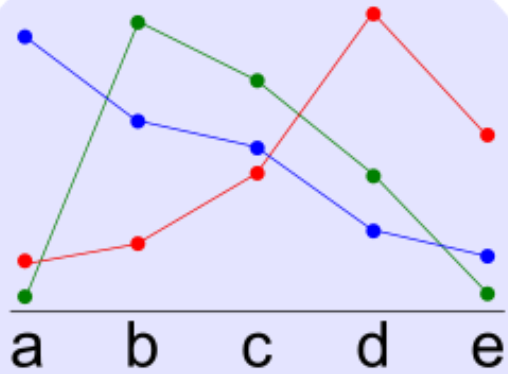
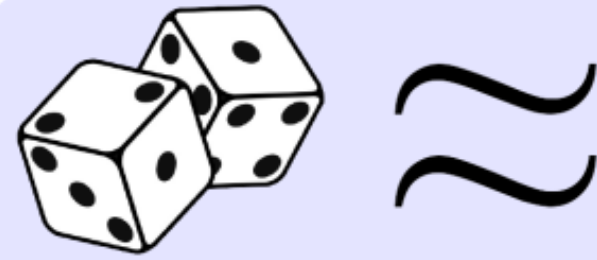
Circumventing GS



Next lecture



Circumventing GS



Next lecture

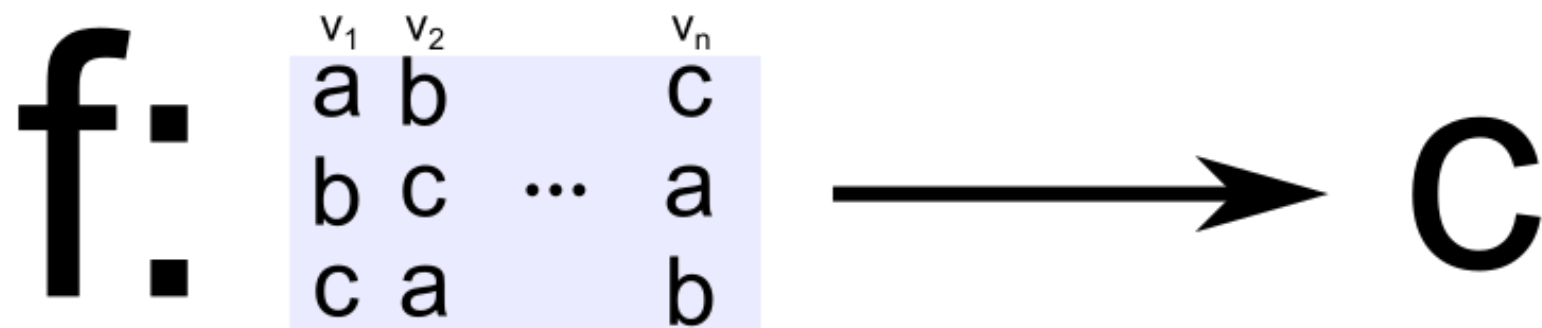


Today



VOTING RULE

A mapping from preference profiles to candidates.



f-Manipulation

f-Manipulation

Input:

- A set of candidates and a set of voters v_1, v_2, \dots, v_n

f-Manipulation

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- A set of candidates and a set of voters v_1, v_2, \dots, v_n
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f-Manipulation

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- A set of candidates and a set of voters v_1, v_2, \dots, v_n
- Votes P_2, \dots, P_n of all non-manipulating voters v_2, \dots, v_n
- Manipulator v_1 's favorite candidate c

f-Manipulation

Input:

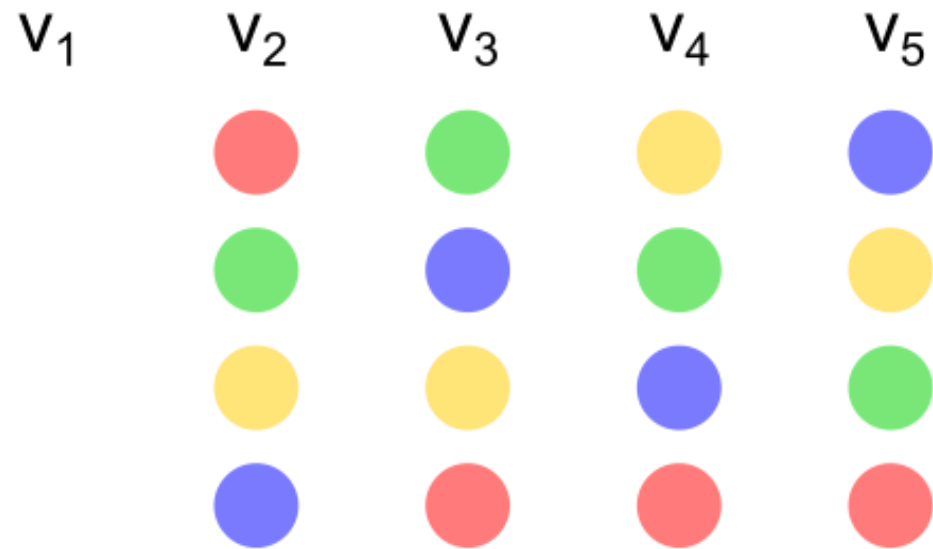
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- Manipulator v_1 's favorite candidate c

Question:

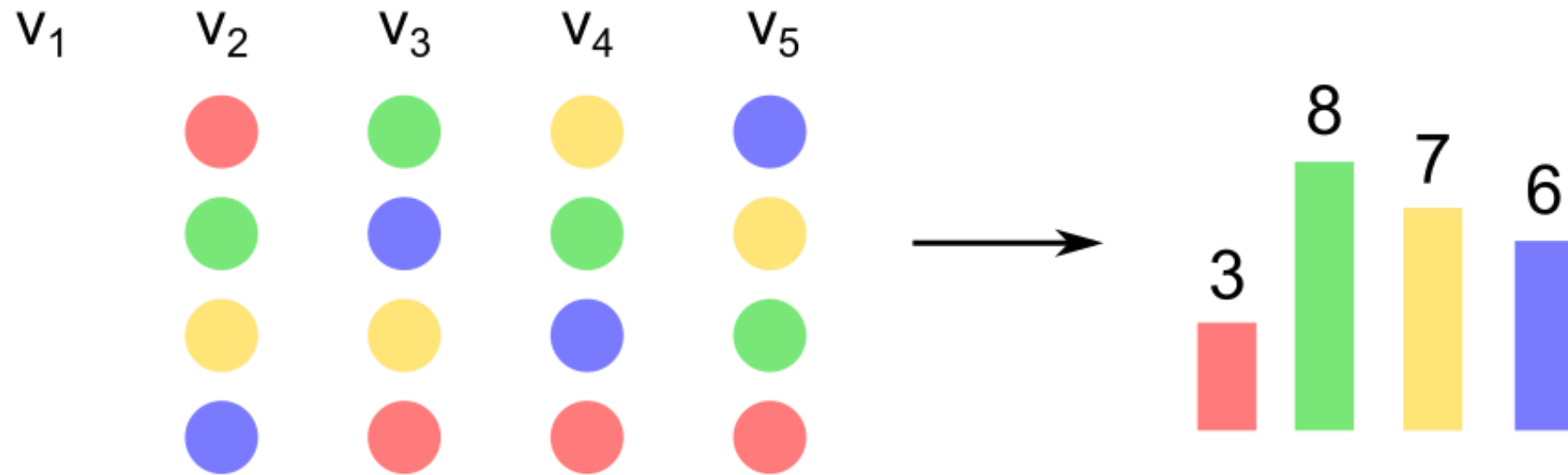
Does there exist a vote P_1 of the manipulator v_1 such that

$$f(P_1, P_2, \dots, P_n) = c?$$

Manipulation under Borda Count

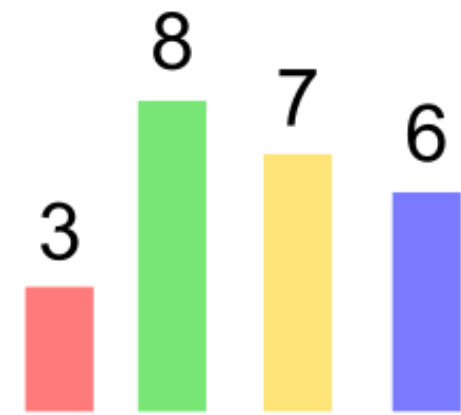
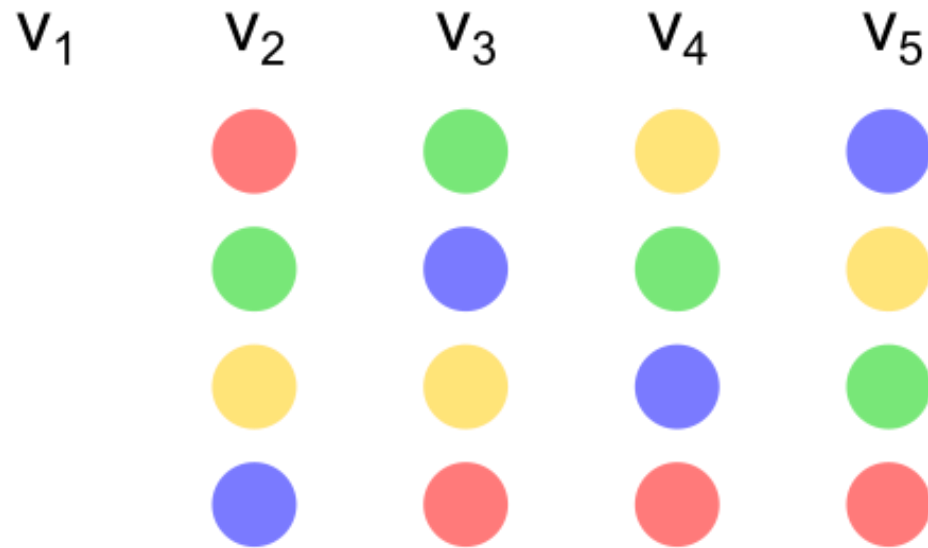


Manipulation under Borda Count



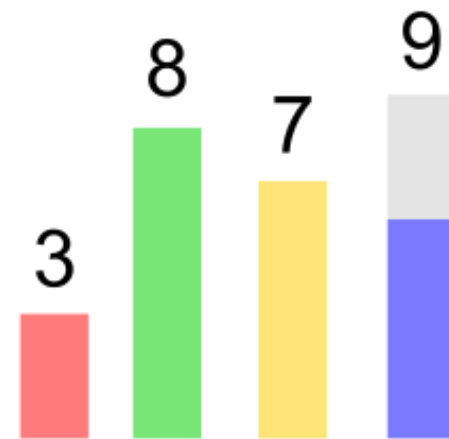
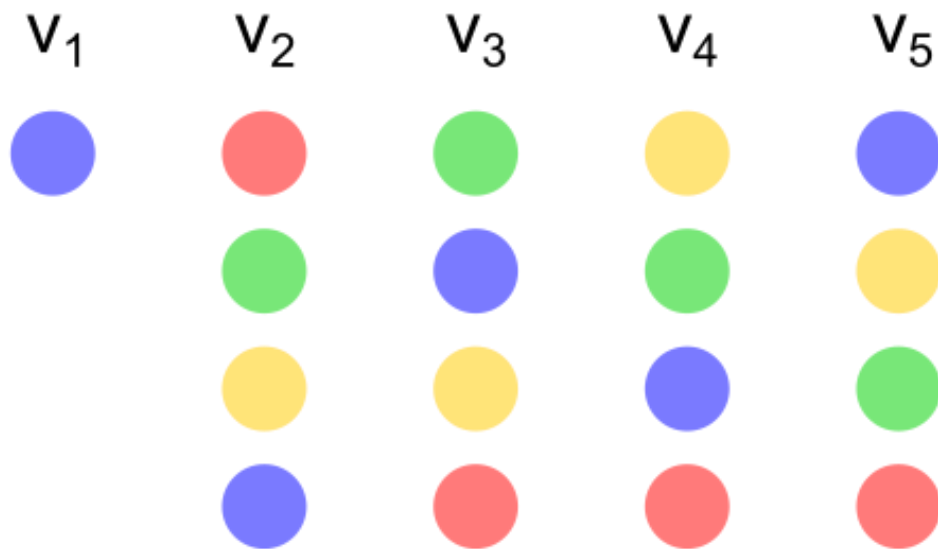
Manipulation under Borda Count

Can I make ● win?



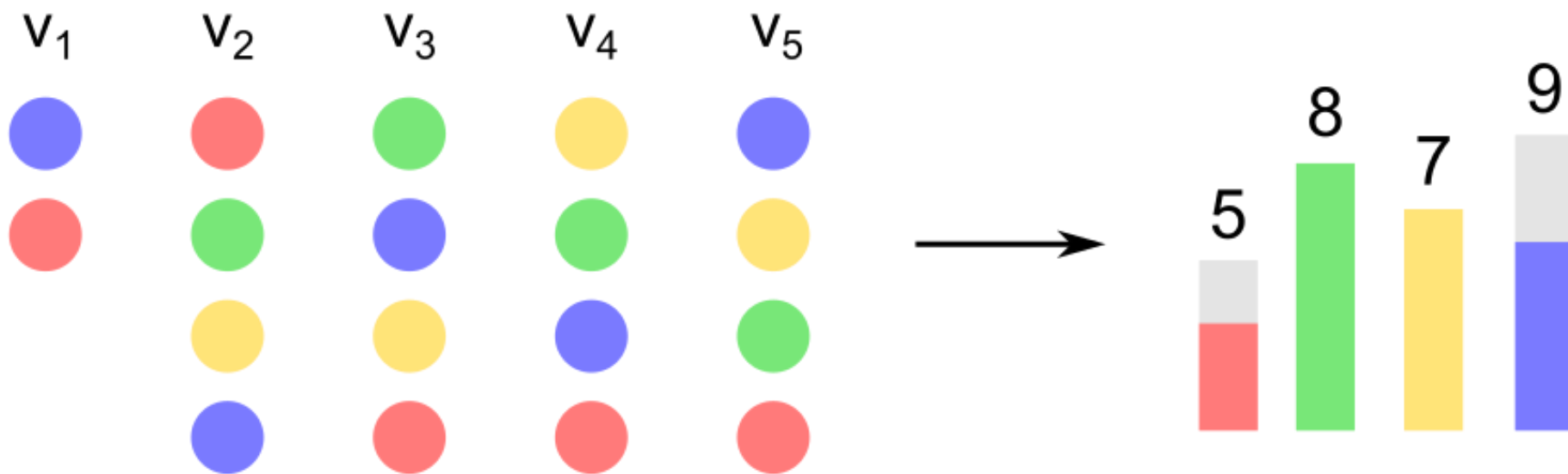
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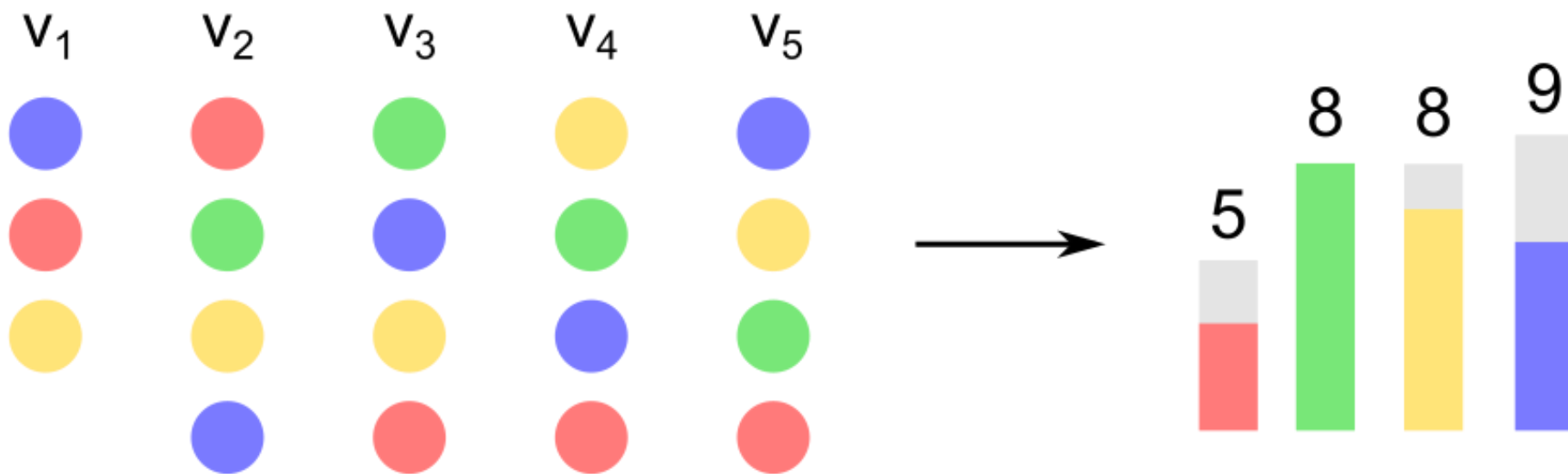
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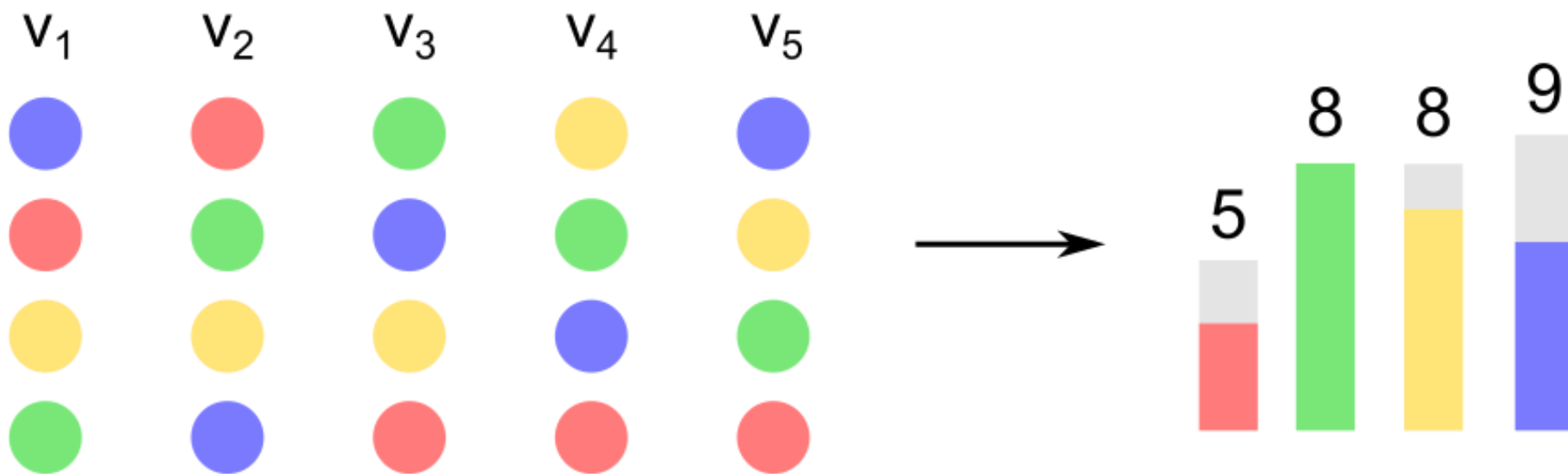
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Manipulation under Borda Count

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A Greedy Strategy

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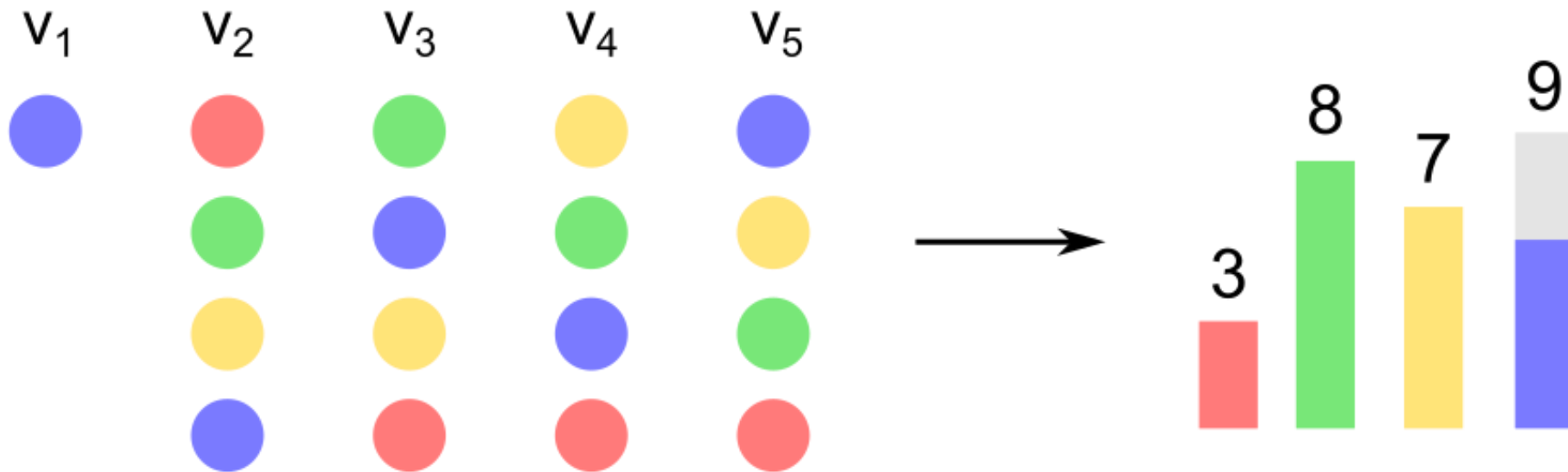
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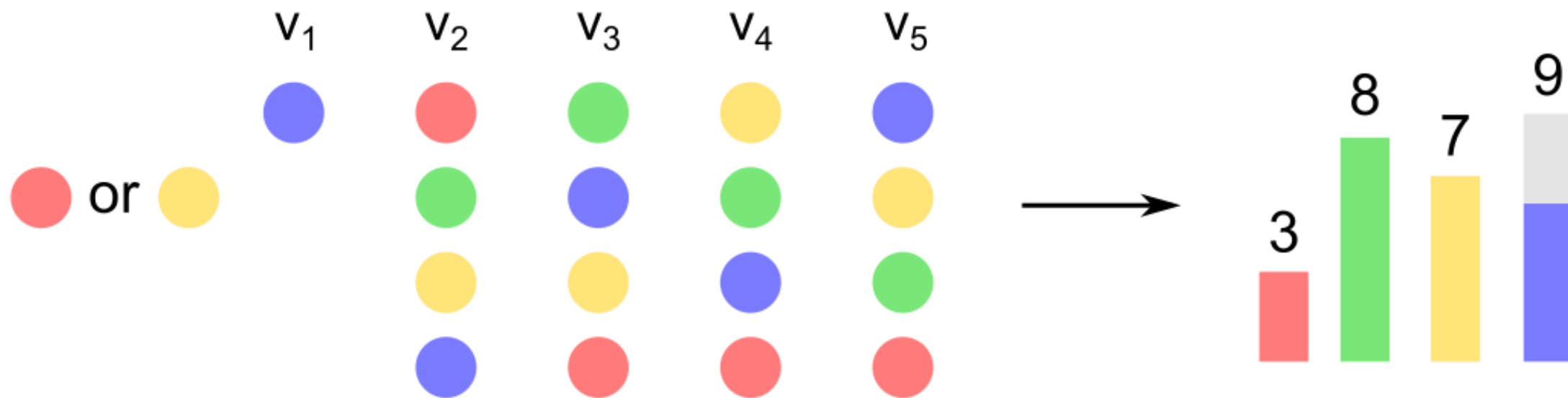
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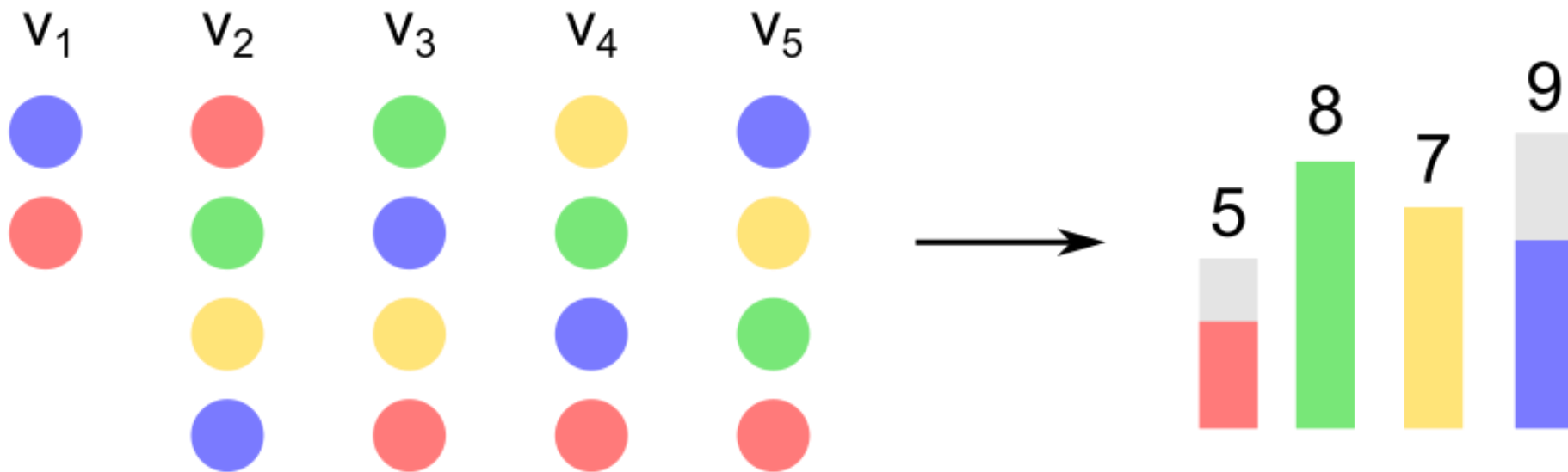
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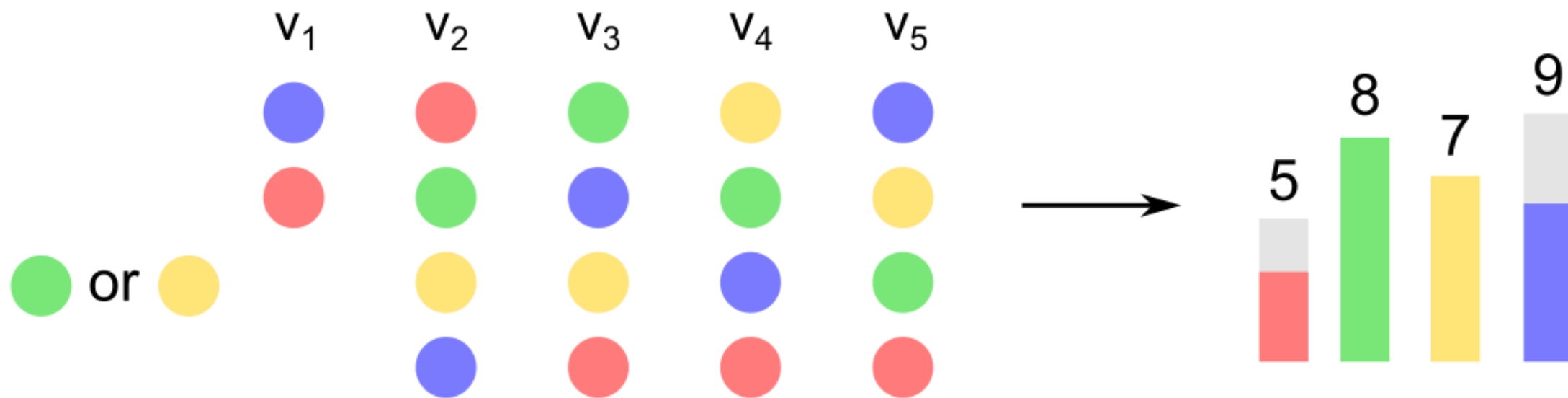
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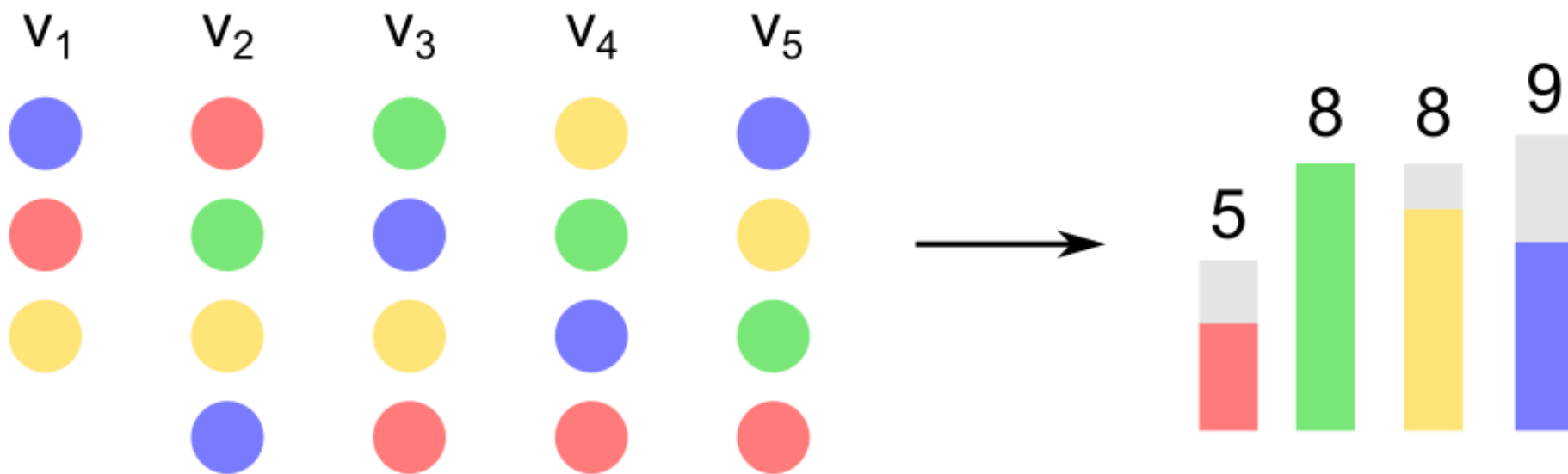
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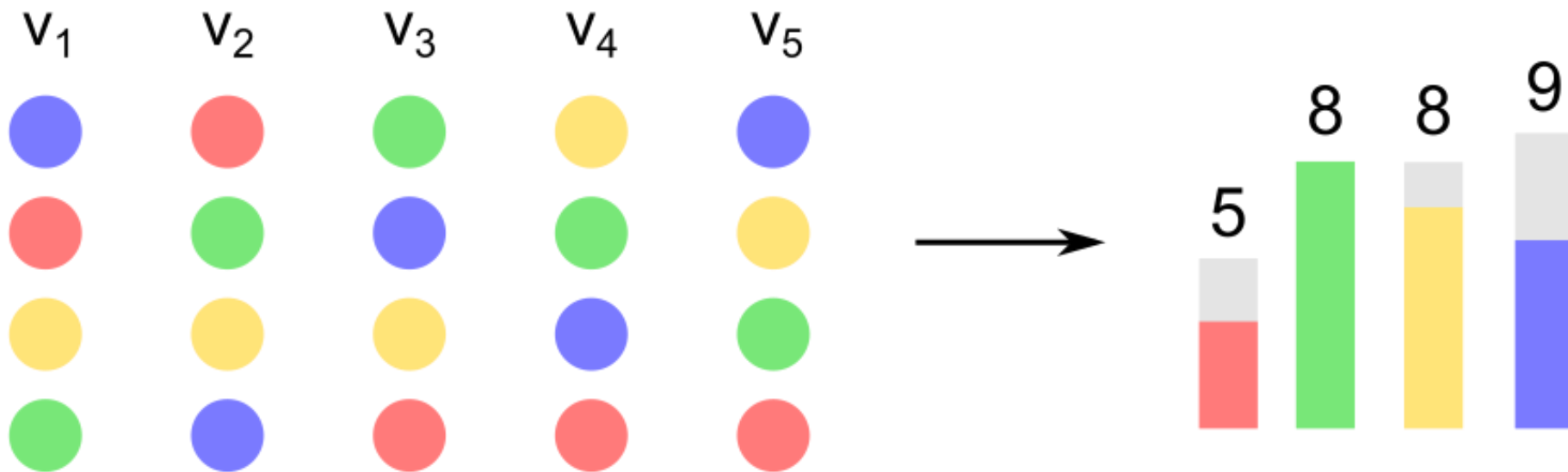
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Manipulation under Borda Count

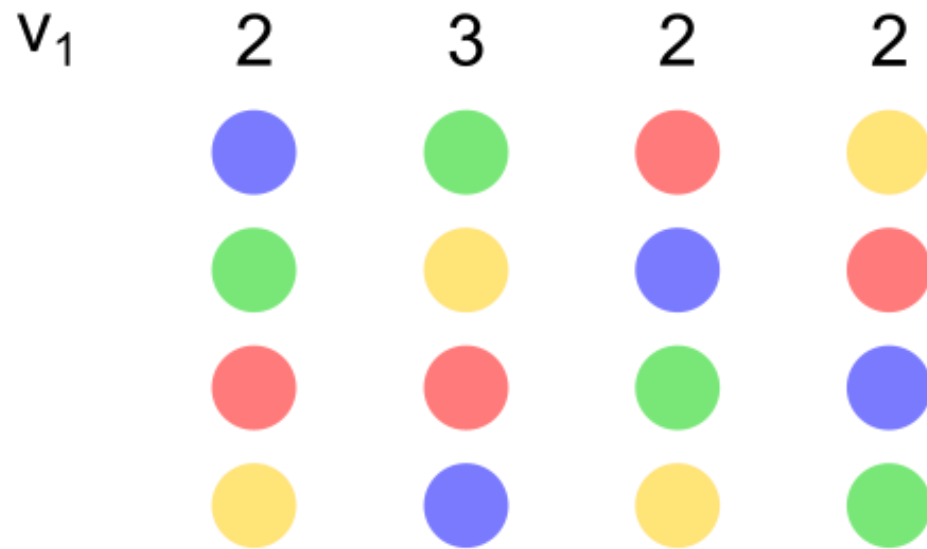
Can I make ● win?





The greedy strategy does not always work.

Manipulation under STV



Manipulation under STV

Can I make ● win?

v_1

2

3

2

2



Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

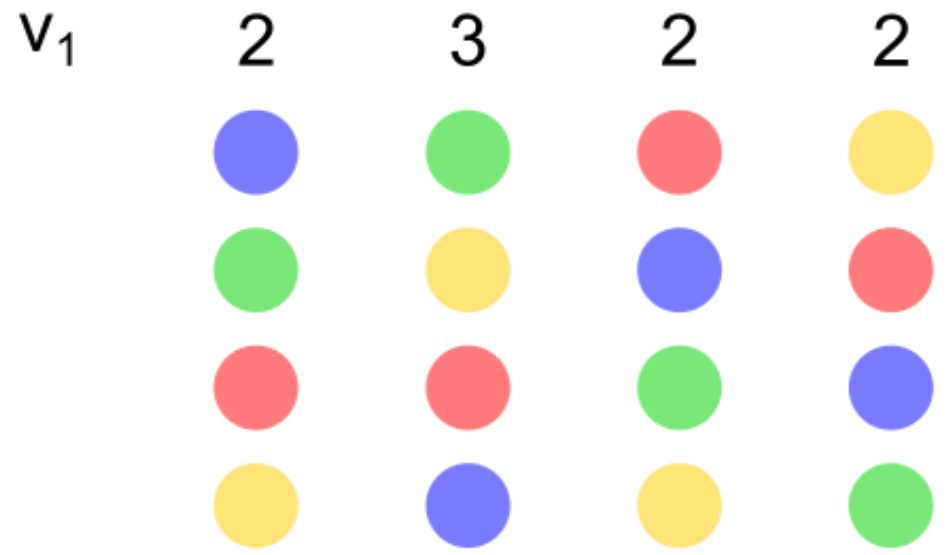
2



Manipulation under STV

Can I make  win?

Tie-breaking rule
 >  >  > 

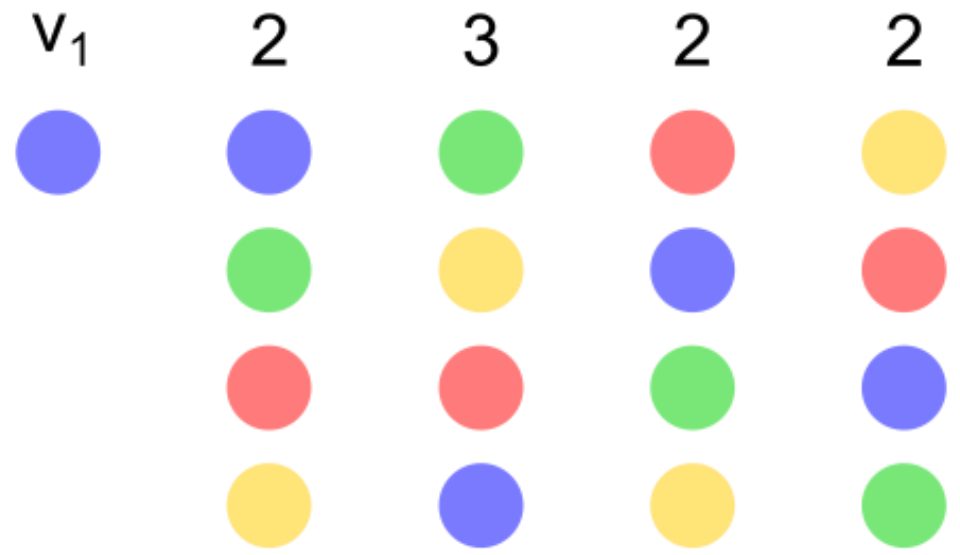


Let's follow the greedy strategy and put  at the top.

Manipulation under STV

Can I make  win?

Tie-breaking rule
 >  >  > 

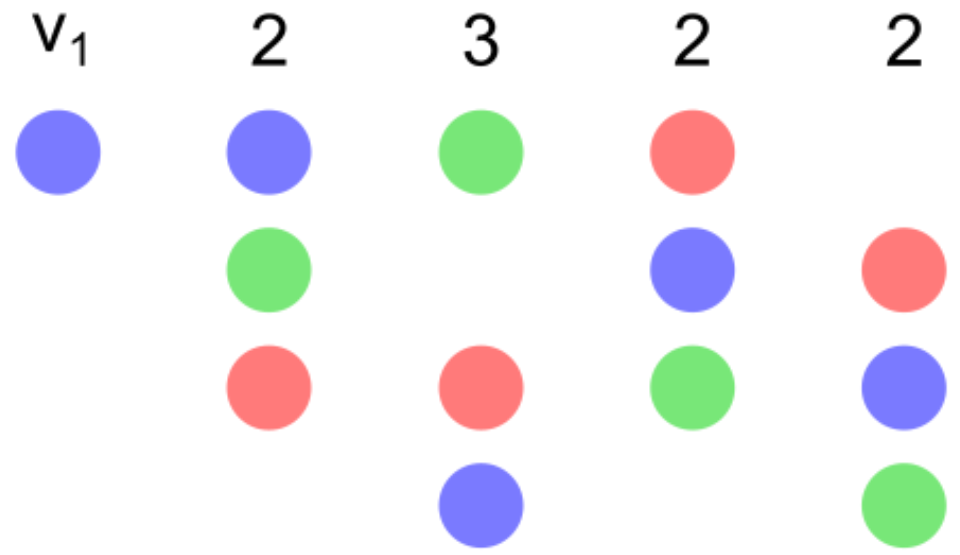


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Manipulation under STV

Can I make  win?

Tie-breaking rule
 >  >  > 



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Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

2



Let's follow the greedy strategy and put  at the top.

 is eliminated in the next round (due to tie-breaking rule).

Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

2



Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

2



Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

2



Manipulation under STV

Can I make  win?

Tie-breaking rule

 >  >  > 

v_1

2

3

2

2



Manipulation under STV

Can I make  win?

Tie-breaking rule

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v_1

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3

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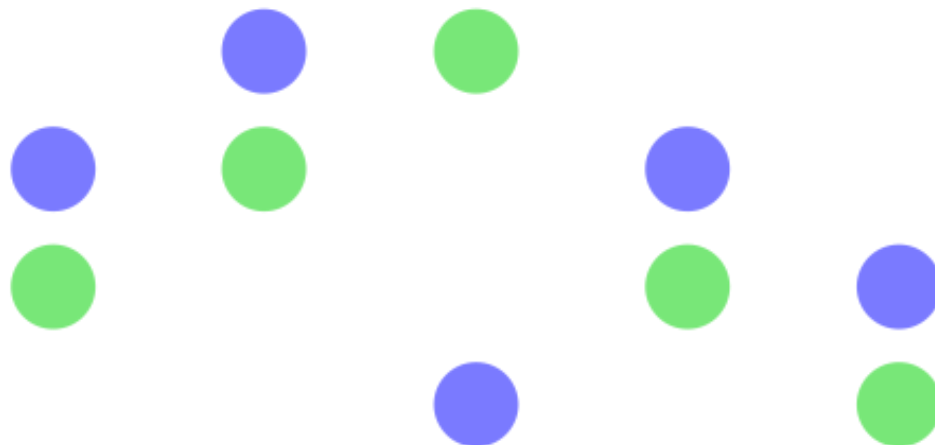
v_1

2

3

2

2



STV winner: 



So, *when* does the greedy strategy work?

[Bartholdi, Tovey and Trick, SCW 1989]

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- **Score-based**: There exists a *scoring function* $s: (P_1, x) \rightarrow \mathbb{R}$ such that for any vote P_1 of v_1 , the f -winner is the candidate maximizing $s(P_1, x)$.

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- **Efficiency**: The voting rule f can be evaluated in polynomial time.

In particular, for $f \in \{\text{Plurality, Borda, Copeland}\}$.

Voting rule

Scoring function

Voting rule

Scoring function

Plurality

p_x = Plurality score of x from P_2, \dots, P_n

$$s(P_1, x) = \begin{cases} 1 + p_x & \text{if } x \text{ is top-ranked in } P_1 \\ p_x & \text{otherwise} \end{cases}$$

Voting rule

Scoring function

Plurality

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Borda

$$b_x = \text{Borda score of } x \text{ from } P_2, \dots, P_n$$
$$s(P_1, x) = b_x + \# \text{candidates below } x \text{ in } P_1$$

Voting rule

Scoring function

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$$s(P_1, x) = b_x + \# \text{candidates below } x \text{ in } P_1$$

Copeland

$$s(P_1, x) = \# \text{candidates } x \text{ beats in a head-to-head} + 0.5 \cdot \# \text{candidates that } x \text{ ties with in a head-to-head}$$

(based on all votes P_1, P_2, \dots, P_n)

Correctness of Greedy Strategy

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Need to show:

If there is a winning vote for c , then the greedy strategy must also find one.

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Suppose, for contradiction, that there exists a winning vote W but the greedy strategy returns 'No'.

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W

x

k

c

d

•

•

s

b

q

Correctness of Greedy Strategy

Suppose, for contradiction, that there exists a winning vote W but the greedy strategy returns 'No'.

Let P be the partial list constructed by greedy before termination.

W

x

k

c

d

•

•

s

b

q

Correctness of Greedy Strategy

Suppose, for contradiction, that there exists a winning vote **W** but the greedy strategy returns 'No'.

Let **P** be the partial list constructed by greedy before termination.

W

x

k

c

d

•

•

s

b

q

P

c

x

q

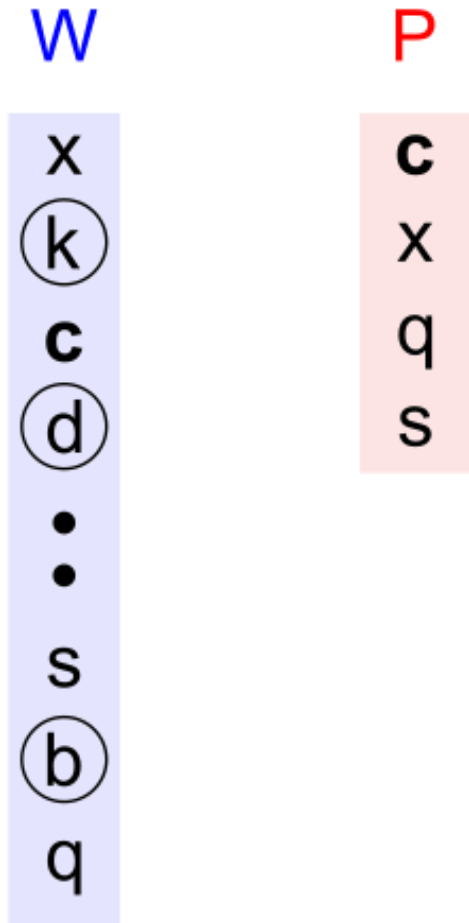
s

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Let P be the partial list constructed by greedy before termination.

Consider the set of candidates that were not ranked by P . Among them, let k be ranked highest in W .

Extend P by placing k in the next available position and arbitrarily ranking the remaining candidates.

W

x

(k)

c

(d)

•

•

s

(b)

q

P

c

x

q

s

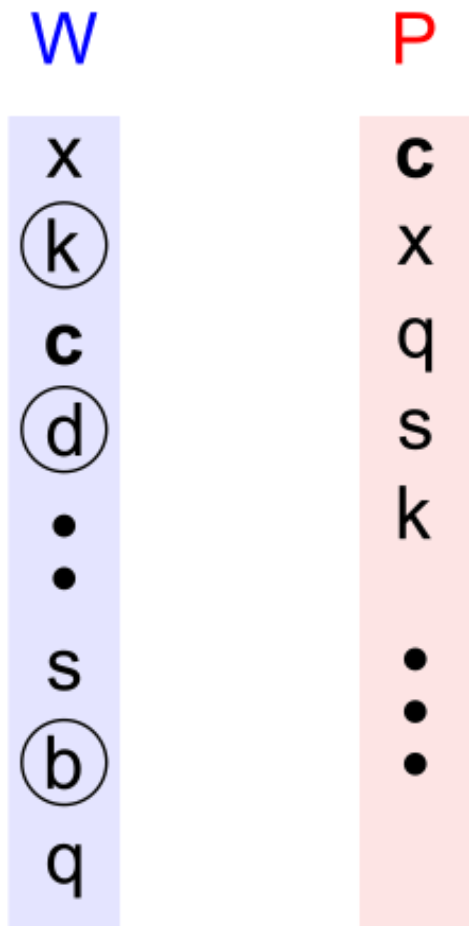
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Correctness of Greedy Strategy

$s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{c})$ by monotonicity of s

W

x
Ⓚ
c
ⓓ
•
•
s
Ⓟ
q

P

c
x
q
s
k
•
•
•

Correctness of Greedy Strategy

$$s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{c})$$

by monotonicity of s

$$s(\mathbf{W}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{k})$$

since c wins under \mathbf{W}

\mathbf{W}

x
k
c
d
•
•
s
b
q

\mathbf{P}

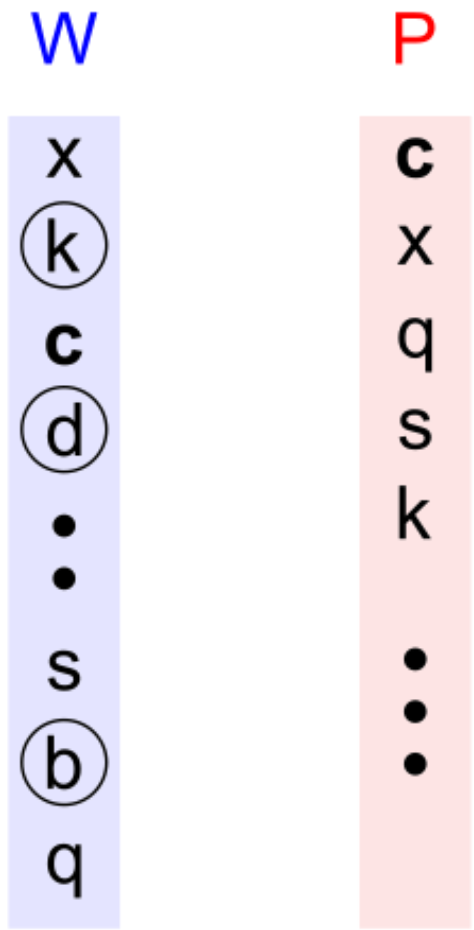
c
x
q
s
k
•
•
•

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$s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{c})$ by monotonicity of s

$s(\mathbf{W}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{k})$ since c wins under \mathbf{W}

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$s(\mathbf{W}, \mathbf{k}) \geq s(\mathbf{P}, \mathbf{k})$ by monotonicity of s

Overall, $s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{P}, \mathbf{k})$.

\mathbf{W}

x

(k)

c

(d)

•

•

s

(b)

q

\mathbf{P}

c

x

q

s

k

•

•

•

Correctness of Greedy Strategy

$s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{W}, \mathbf{c})$ by monotonicity of s

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$s(\mathbf{W}, \mathbf{k}) \geq s(\mathbf{P}, \mathbf{k})$ by monotonicity of s

Overall, $s(\mathbf{P}, \mathbf{c}) \geq s(\mathbf{P}, \mathbf{k})$.

Thus, \mathbf{k} could not have prevented \mathbf{c} from winning,
and therefore greedy should have continued.

\mathbf{W}

x

k

c

d

•

•

s

b

q

\mathbf{P}

c

x

q

s

k

•

•

•

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\mathbf{W}

x

k

c

d

•

•

s

b

q

\mathbf{P}

c

x

q

s

k

•

•

•





Is manipulation *always* easy?

The Computational Difficulty of Manipulating an Election*

J. J. Bartholdi III, C. A. Tovey, and M. A. Trick**

School of Industrial and Systems Engineering, Georgia Institute of Technology,
Atlanta, GA 30332, USA

Received June 9, 1987 / Accepted July 29, 1988

Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.

For many voting rules, f-Manipulation is **NP-hard**.

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[Bartholdi, Tovey, and Trick, SCW 1989]

Copeland with second-order tie-breaking

In case of a tie, winner is the candidate whose defeated competitors have the highest sum of Copeland scores.

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[Bartholdi, Tovey, and Trick, SCW 1989]

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Single Transferable Vote (STV)

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Single Transferable Vote (STV)

[Xia, Zuckerman, Procaccia, Conitzer, Rosenschein, IJCAI 2009]

Ranked Pairs

Consider candidate pairs according to the margin of head-to-head victories, and create a ranking based on it while avoiding cycles.

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NP-hardness is **good news!**

No general-purpose efficient algorithm that correctly works on all preference profiles (unless $P=NP$).

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NP-hardness is **good news!**

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Using **worst-case** computational hardness as a barrier to manipulation.

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NP-hardness is **good news!**

No general-purpose efficient algorithm that correctly works on all preference profiles (unless $P=NP$).

Using **worst-case** computational hardness as a barrier to manipulation.

Note: NP-hard *even with* full information.

Remember this?

Method	Criterion	Sort:																				
		Majority	Maj. loser	Mutual maj.	Condorcet	Cond. loser	Smith/ISDA	LIIA	IIA	Cloneproof	Monotone	Consistency	Participation	Reversal symmetry	Polytime/resolvable	Summable	Later-no-		No favorite betrayal	Ballot type	Ranks	
		Harm	Help	=	>2																	
Approval	Rated ^[a]	No	No	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes ^[c]	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes ^[f]	Yes	Approvals	Yes	No
Borda count	No	Yes	No	No ^[b]	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	Yes	Yes
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No ^[b]	Teams, crowds	Yes	No ^[b]	No ^[b]	Yes	O(N ²)	No	O(N ²)	No ^[b]	No	No ^[b]	Ranking	Yes	Yes
IRV (AV)	Yes	Yes	Yes	No ^[b]	Yes	No ^[b]	No	No	Yes	No	No	No	No	O(N ²)	Yes ^[g]	O(N) ^[h]	Yes	Yes	No	Ranking	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No ^[b]	Spoilers	Yes	No ^[b] _[i]	No ^[b]	Yes	O(N!)	Yes	O(N ²) ^[j]	No ^[b]	No	No ^[b]	Ranking	Yes	Yes
Highest median/Majority judgment ^[k]	Rated ^[l]	Yes ^[m]	No ^[n]	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes	Yes	No ^[o]	No ^[p]	Depends ^[q]	O(N)	Yes	O(N) ^[r]	No ^[s]	Yes	Yes	Scores ^[t]	Yes	Yes
Minimax	Yes	No	No	Yes ^[u]	No	No	No	No ^[b]	Spoilers	Yes	No ^[b]	No ^[b]	No	O(N ²)	Yes	O(N ²)	No ^{[b][v]}	No	No ^[b]	Ranking	Yes	Yes
Plurality/FPTP	Yes	No	No	No ^[b]	No	No ^[b]	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A ^[v]	N/A ^[v]	No	Single mark	N/A	No
Score voting	No	No	No	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No ^[b]	Yes	Yes	No ^[b]	No ^{[p][b]}	Yes	O(N ³)	Yes	O(N ²)	No ^[b]	No	No ^{[p][b]}	Ranking	Yes	Yes
Runoff voting	Yes	Yes	No	No ^[b]	Yes	No ^[b]	No	No	Spoilers	No	No	No	No	O(N) ^[w]	Yes	O(N) ^[w]	Yes	Yes ^[x]	No	Single mark	N/A	No ^[y]
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No ^[b]	Yes	Yes	No ^[b]	No ^{[p][b]}	Yes	O(N ³)	Yes	O(N ²)	No ^[b]	No	No ^{[p][b]}	Ranking	Yes	Yes
STAR voting	No ^[z]	Yes	No ^[aa]	No ^{[b][c]}	Yes	No ^[b]	No	No	No	Yes	No	No	Depends ^[ab]	O(N)	Yes	O(N ²)	No	No	No ^[ac]	Scores	Yes	Yes
Sortition, arbitrary winner ^[ad]	No	No	No	No ^[b]	No	No ^[b]	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A
Random ballot ^[ae]	No	No	No	No ^[b]	No	No ^[b]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No

Single manipulator

Plurality

P

[Bartholdi, Tovey and Trick, SCW 1989]

Borda

P

[Bartholdi, Tovey and Trick, SCW 1989]

Copeland^α

(friendly tie-breaking)

P

[Bartholdi, Tovey and Trick, SCW 1989]

Ranked pairs

NP-hard

[Xia, Zuckerman, Procaccia, Conitzer,
and Rosenschein, IJCAI 2009]

Schulze

P

[Parkes and Xia, AAI 2012]

Single manipulator

Two manipulators

Plurality

P

[Bartholdi, Tovey and Trick, SCW 1989]

P

Borda

P

[Bartholdi, Tovey and Trick, SCW 1989]

NP-hard

[Betzler, Niedermeier and Woeginger, IJCAI 2011;
Davies, Katsirelos, Narodytska and Walsh, AAI 2011]

Copeland^α

(friendly tie-breaking)

P

[Bartholdi, Tovey and Trick, SCW 1989]

NP-hard

[Faliszewski, Hemaspaandra and Schnoor,
AAMAS 2008]

Ranked pairs

NP-hard

[Xia, Zuckerman, Procaccia, Conitzer,
and Rosenschein, IJCAI 2009]

NP-hard

[Xia, Zuckerman, Procaccia, Conitzer,
and Rosenschein, IJCAI 2009]

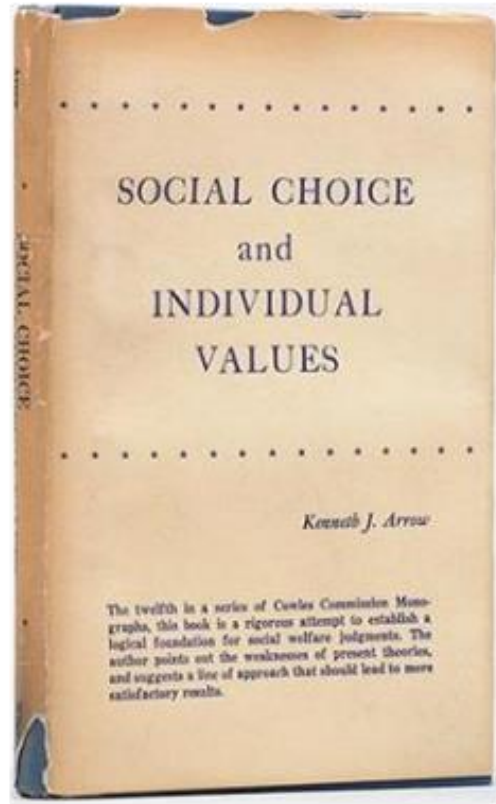
Schulze

P

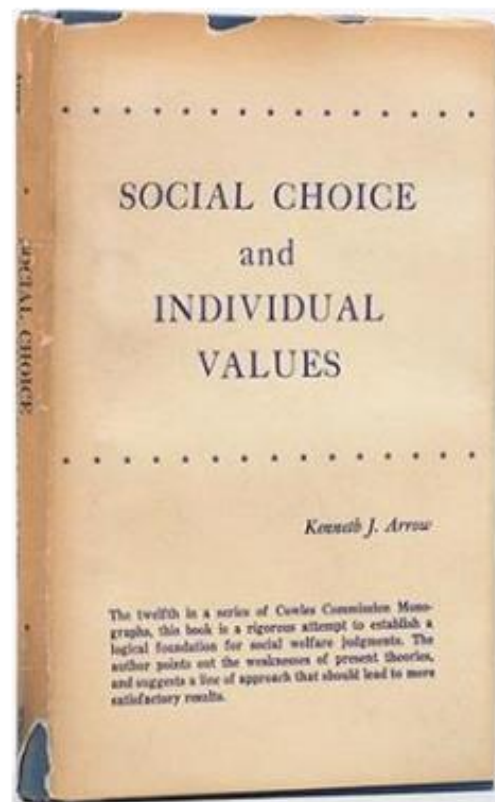
[Parkes and Xia, AAI 2012]

P

[Gaspers, Kalinowski, Narodytska and Walsh,
AAMAS 2013]



Social Choice Theory



Social Choice Theory

Soc Choice Welfare (1989) 6:227-241

**Social Choice
and Welfare**

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The Computational Difficulty of Manipulating an Election*

J. J. Bartholdi III, C. A. Tovey, and M. A. Trick**

School of Industrial and Systems Engineering, Georgia Institute of Technology,
Atlanta, GA 30332, USA

Received June 9, 1987 / Accepted July 29, 1988

Abstract. We show how computational complexity might protect the integrity of social choice. We exhibit a voting rule that efficiently computes winners but is computationally resistant to strategic manipulation. It is *NP*-complete for a manipulative voter to determine how to exploit knowledge of the preferences of others. In contrast, many standard voting schemes can be manipulated with only polynomial computational effort.



Computational Social Choice

Enough about voting. Let's talk sports!

ELIMINATION IN SPORTS

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Imagine we are at the halfway point of a sports tournament.

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Imagine we are at the halfway point of a sports tournament.

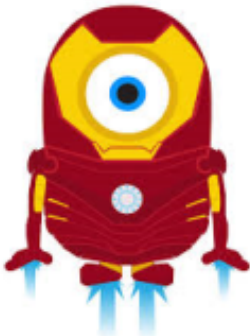
Some games have been played, others are still to go.

ELIMINATION IN SPORTS

Imagine we are at the halfway point of a sports tournament.

Some games have been played, others are still to go.

Q: Does my favorite team still have a chance of winning?



6



8



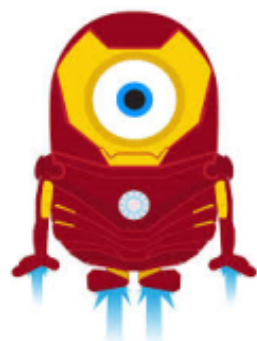
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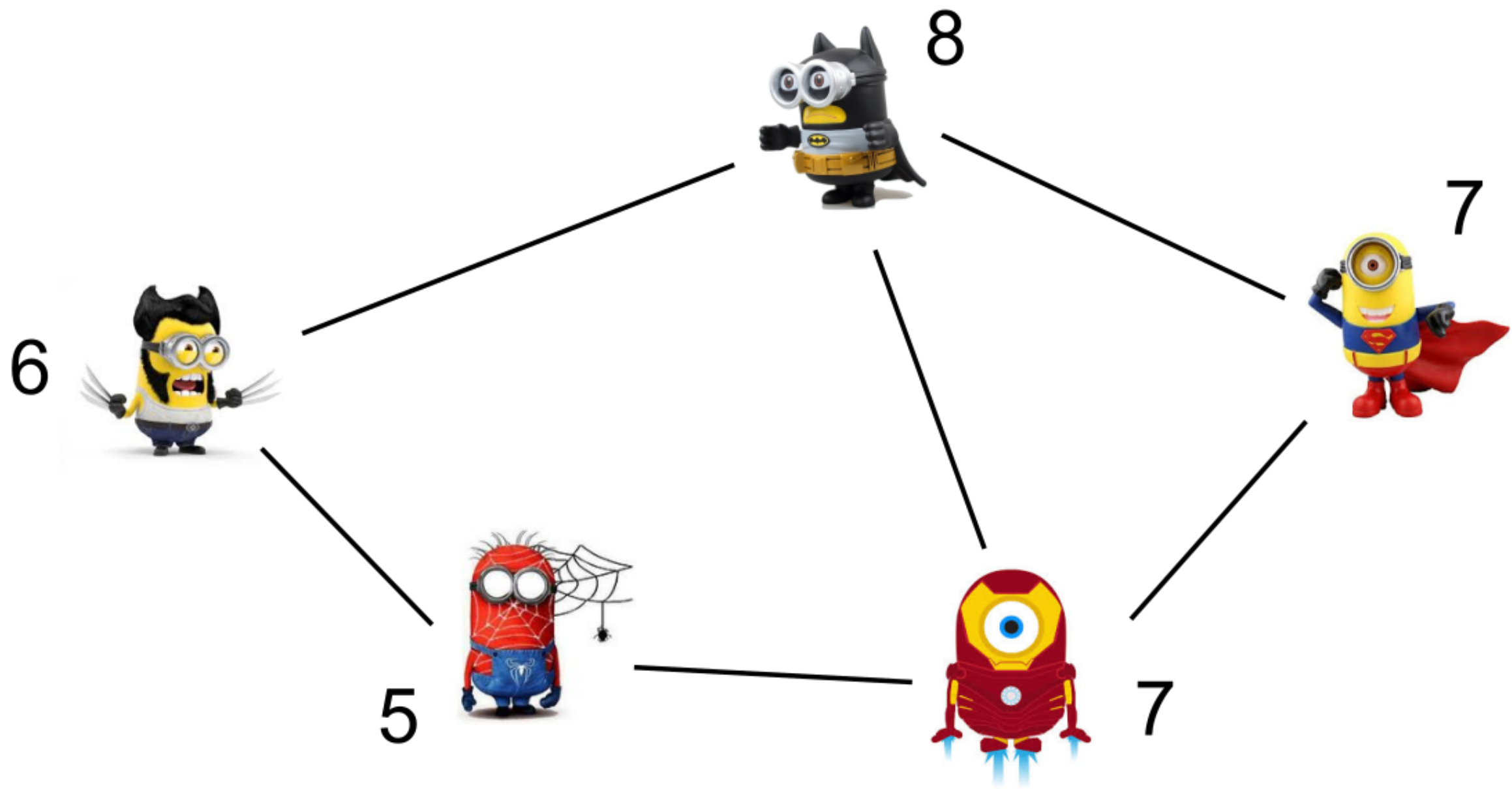


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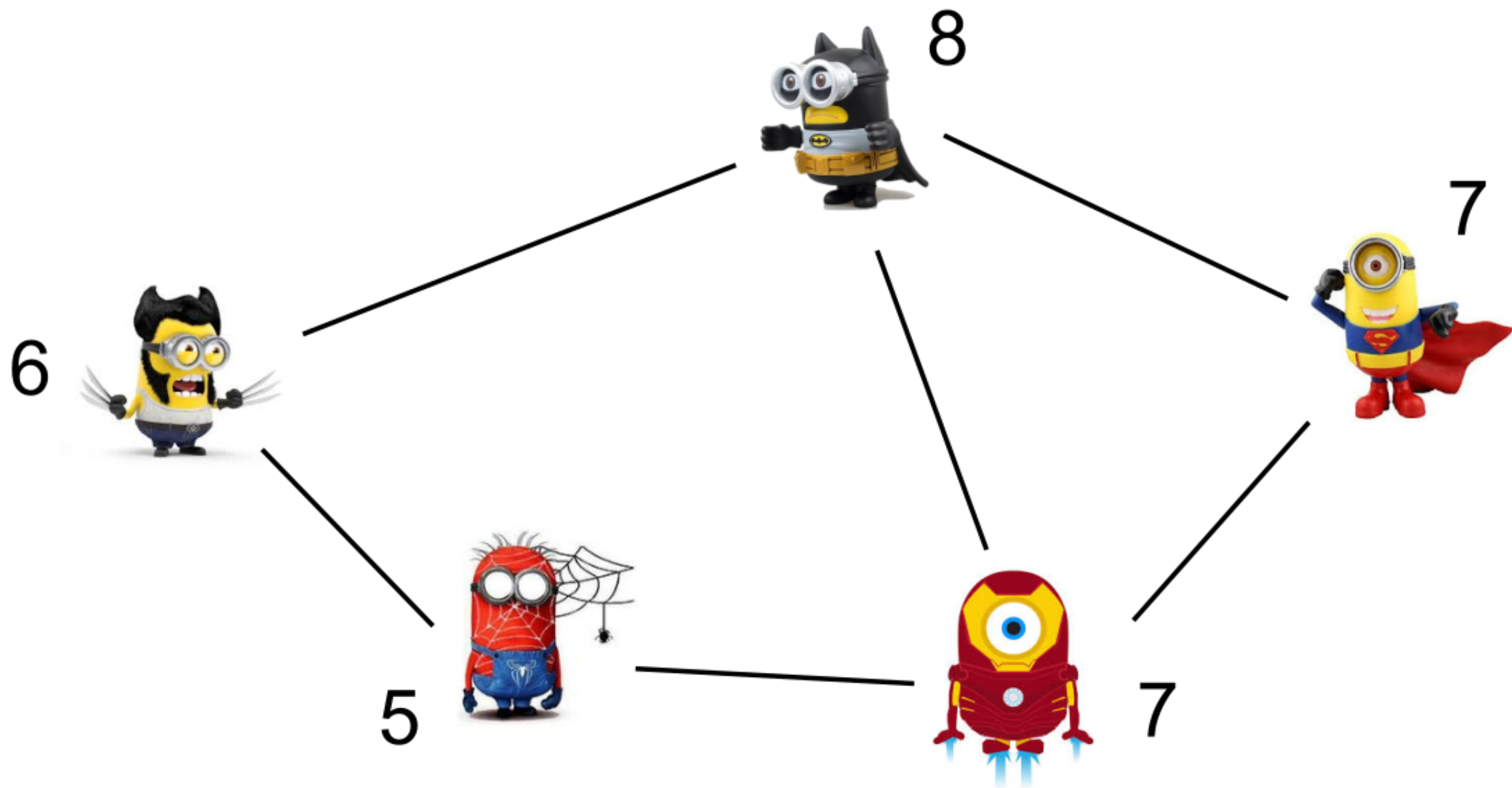


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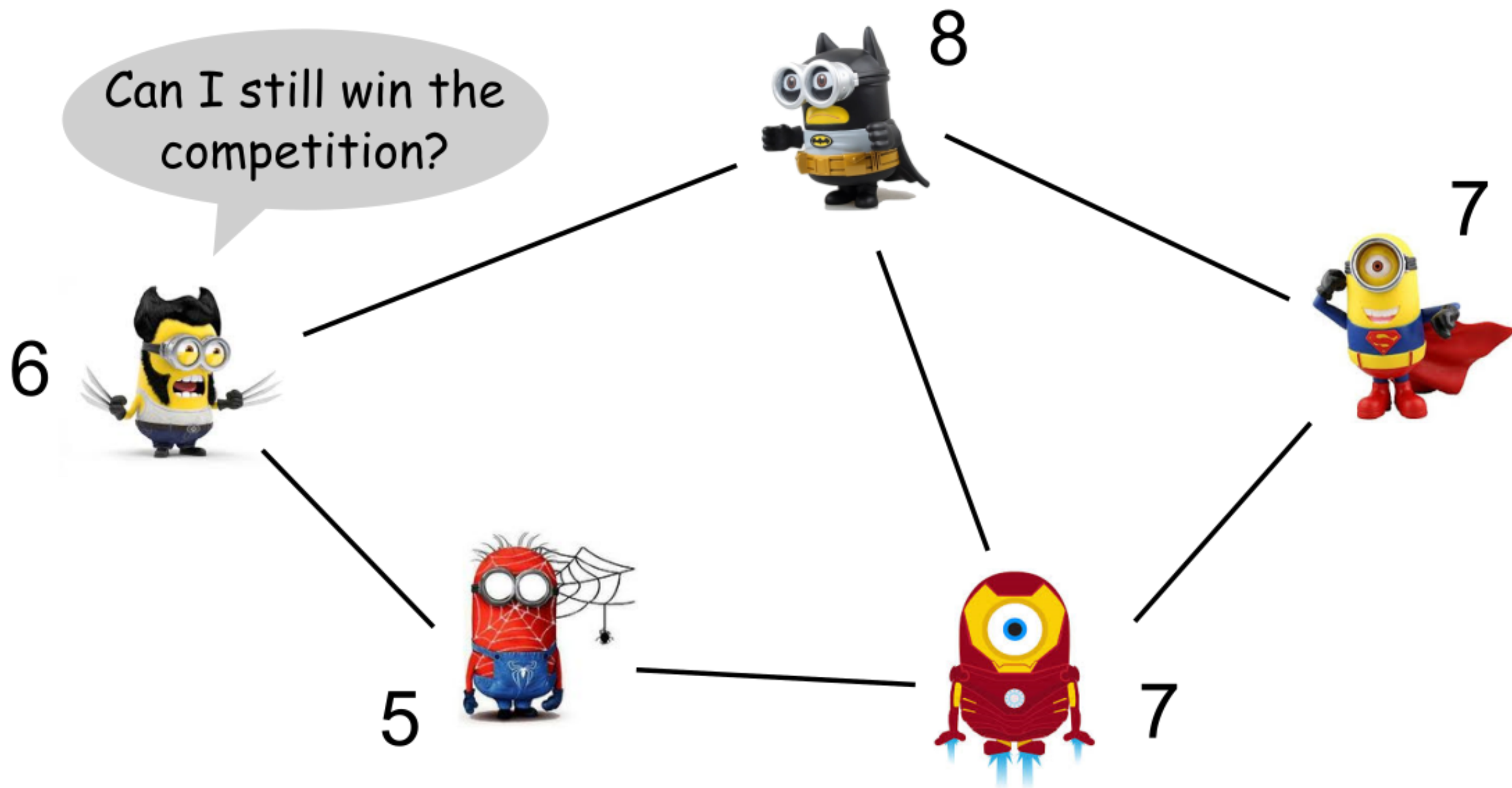




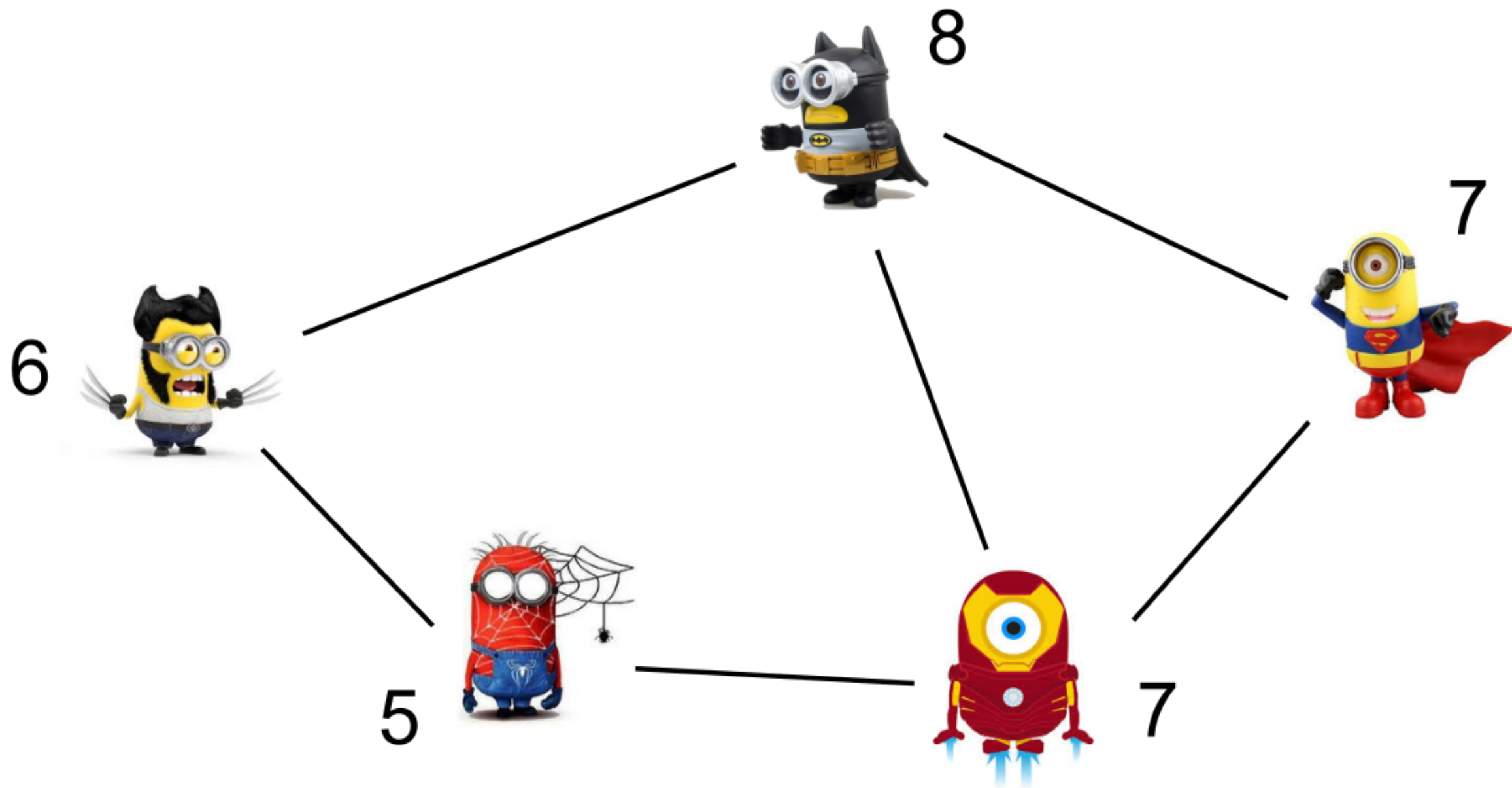
After each game, winner gets 1 point, loser get 0. No ties.



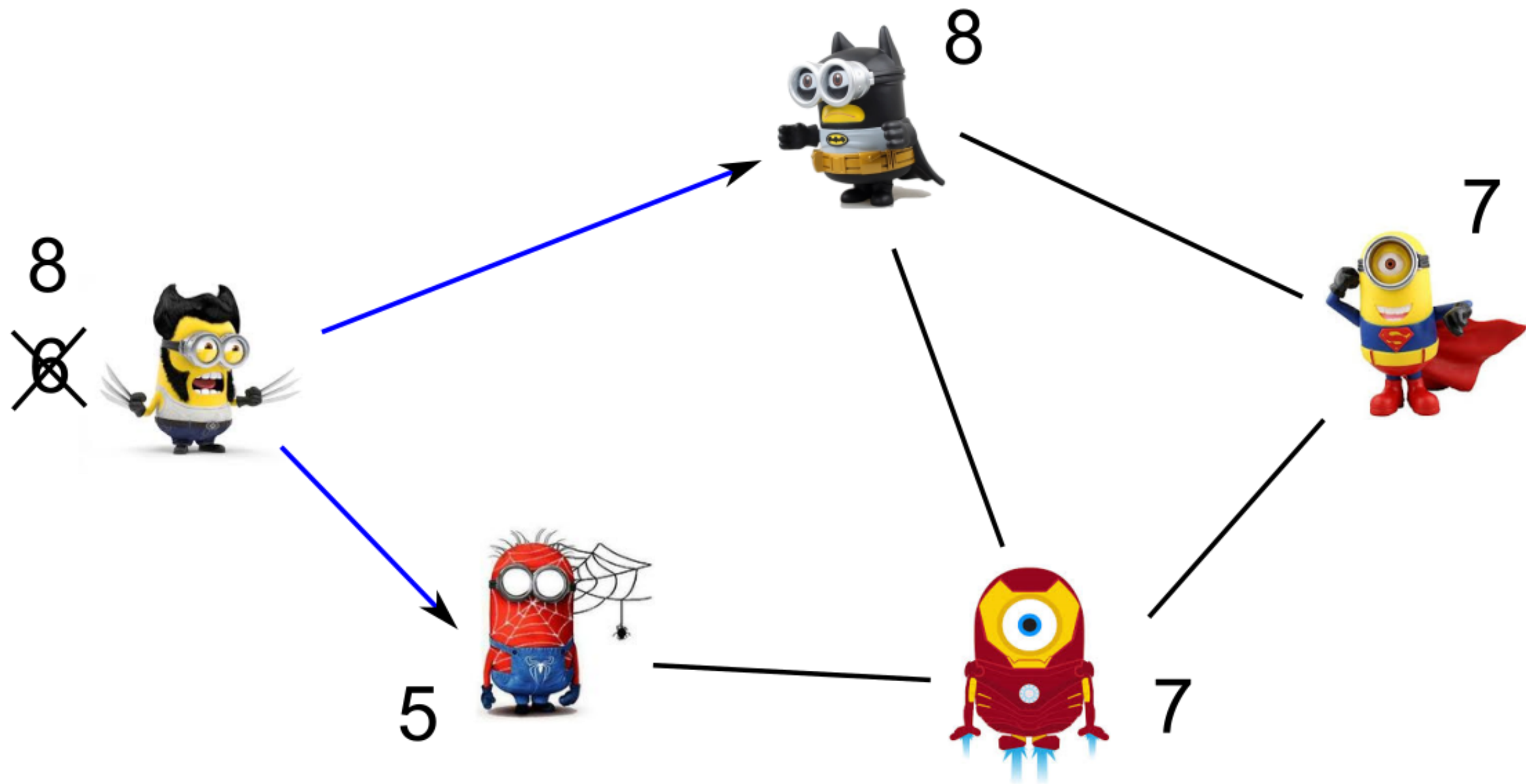
After each game, winner gets 1 point, loser get 0. No ties.



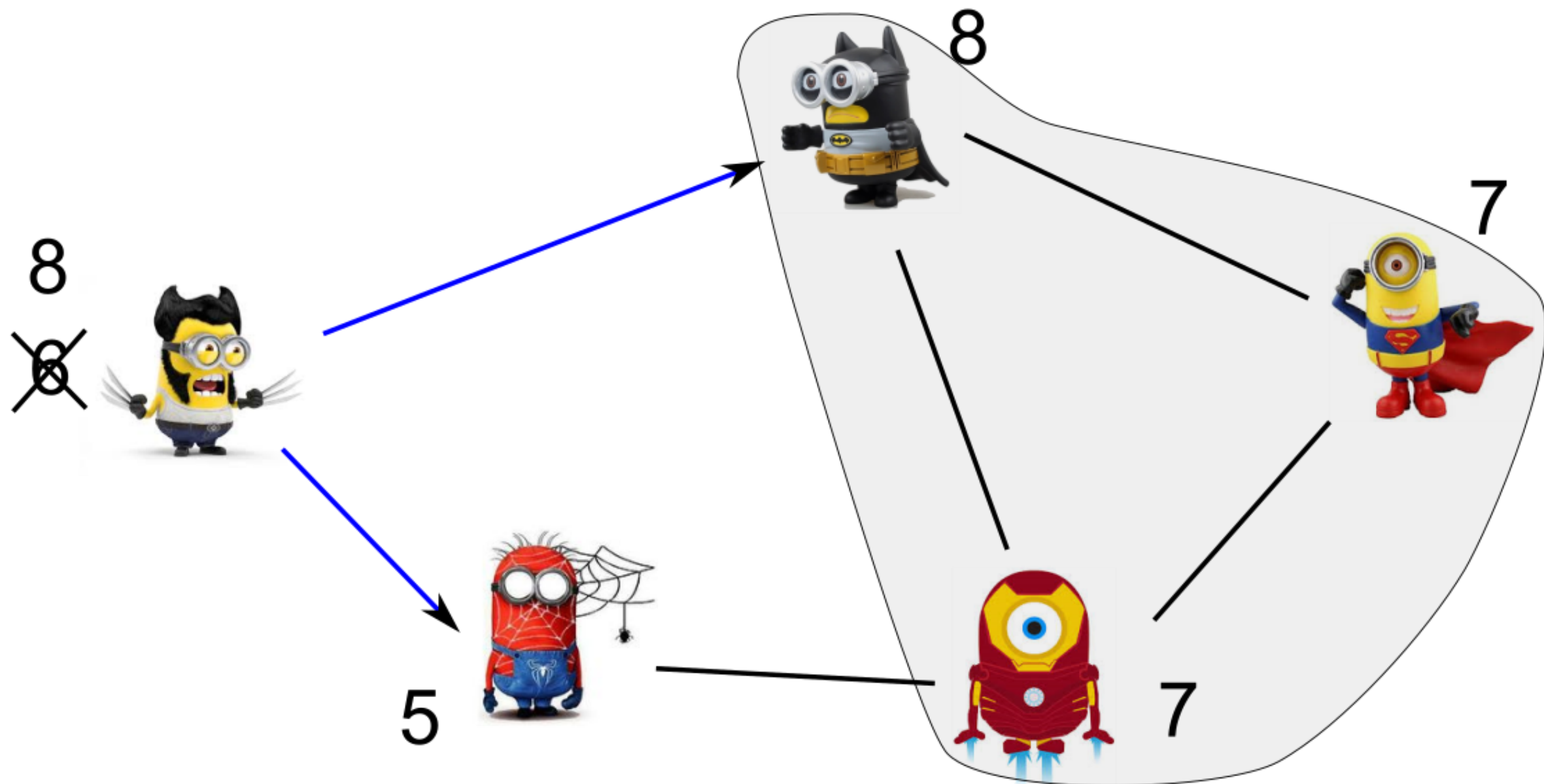
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One of these three will end up with at least 9 points

~~8~~



5



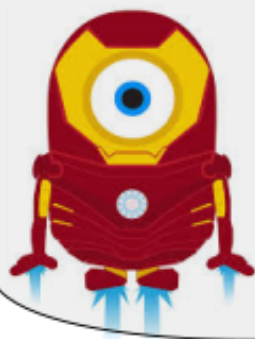
8



7



7




We will solve this problem using **max flow**.

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Step 1: Imagine  wins all its remaining games.


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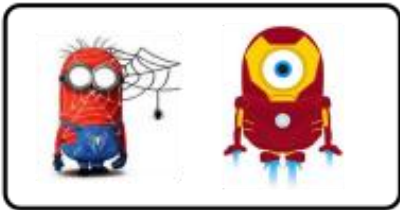
Doing so freezes the score of 

We will solve this problem using **max flow**.

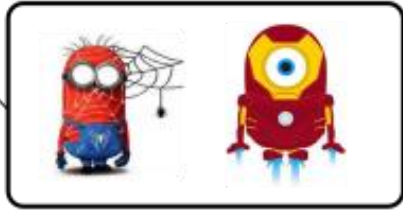
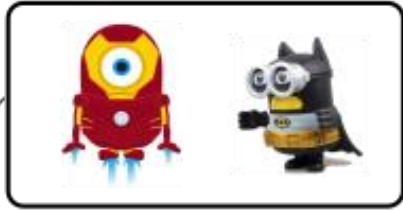
Step 1: Imagine  wins all its remaining games.

Doing so freezes the score of 

Step 2: Set up a flow network to check for a winning schedule.

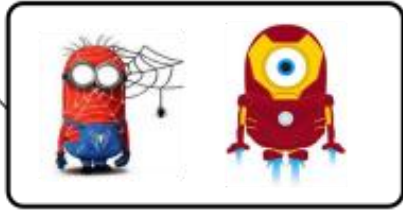
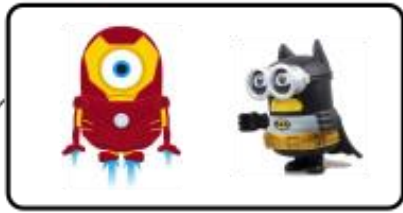


S



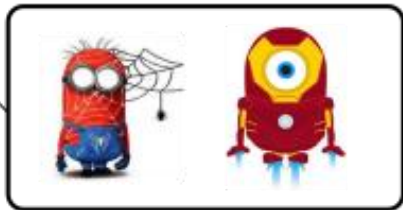
Cap = # points at stake
in each game

S



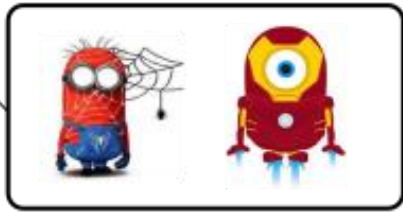
Cap = # points at stake
in each game

S



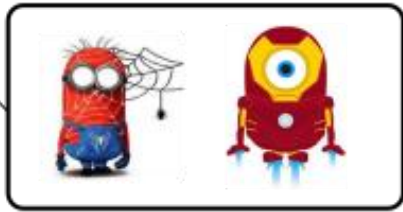
Cap = # points at stake
in each game

S



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in each game

S

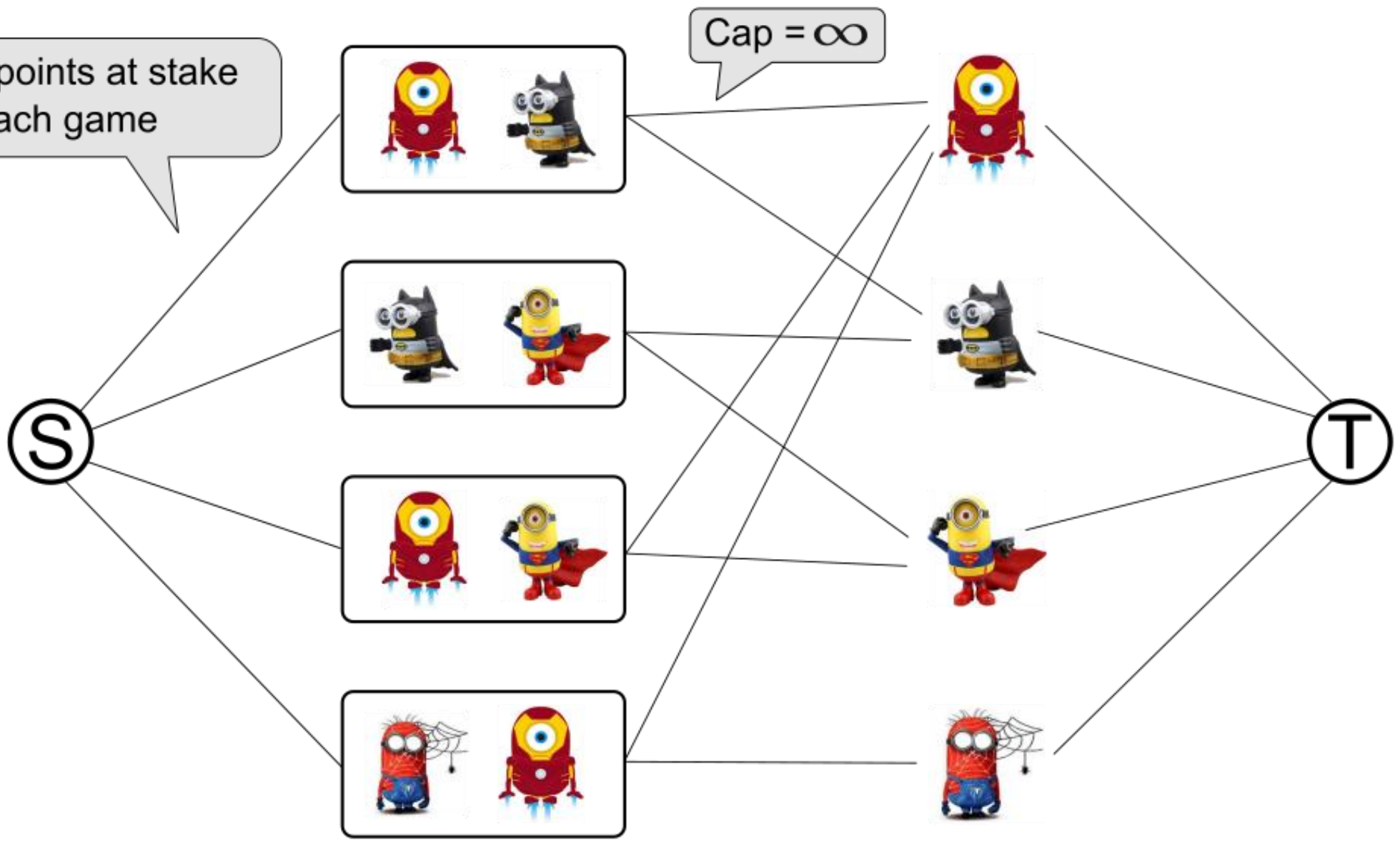


Cap = ∞



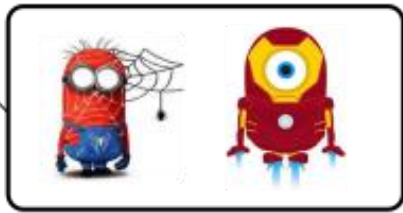
Cap = # points at stake in each game

Cap = ∞



Cap = # points at stake in each game

S



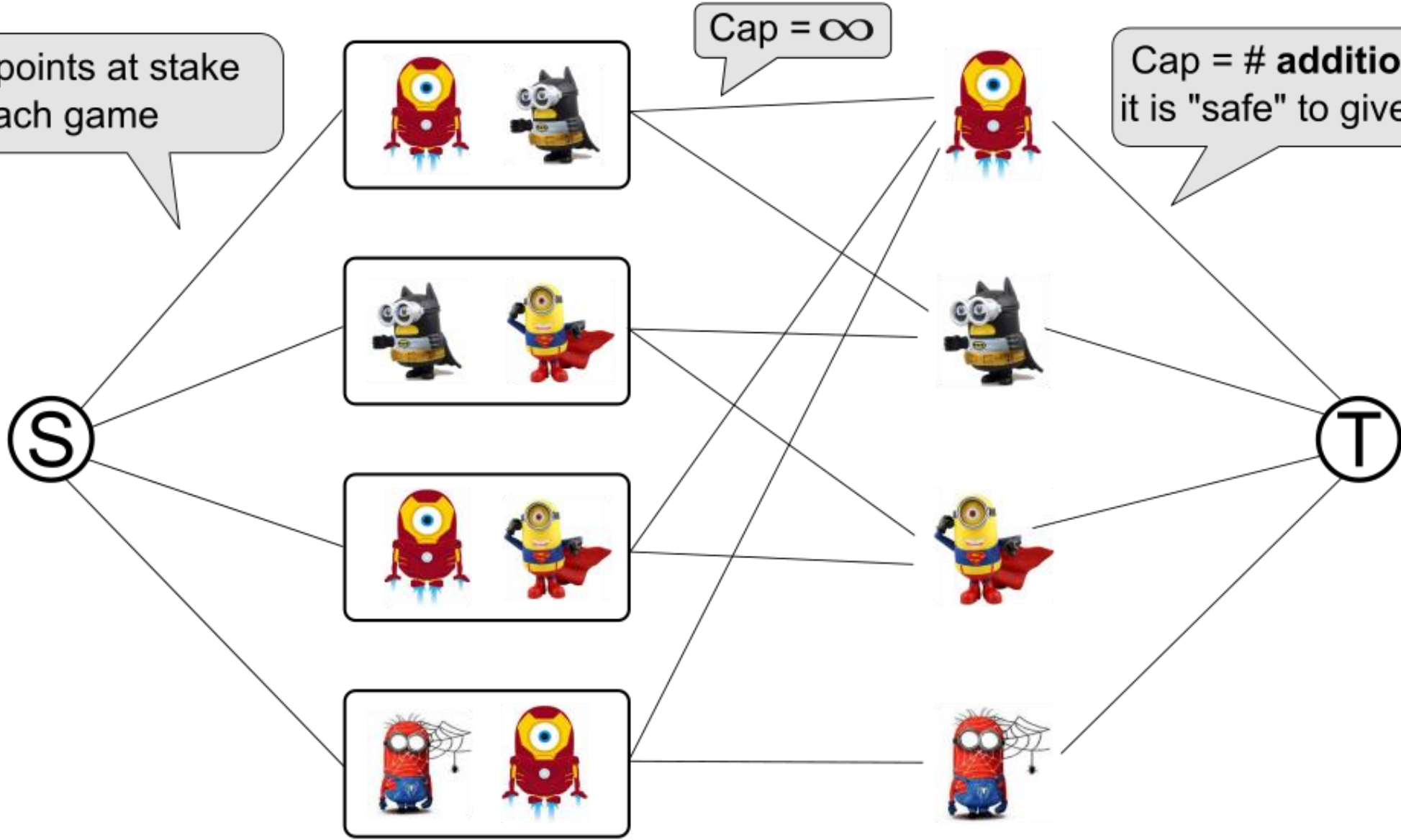
Cap = ∞



Cap = # **additional** points it is "safe" to give to a team

T

Cap = # points at stake in each game



Cap = # **additional** points it is "safe" to give to a team

There is a max flow that saturates the edges of $S \leftrightarrow T$ \Leftrightarrow There is a winning schedule.

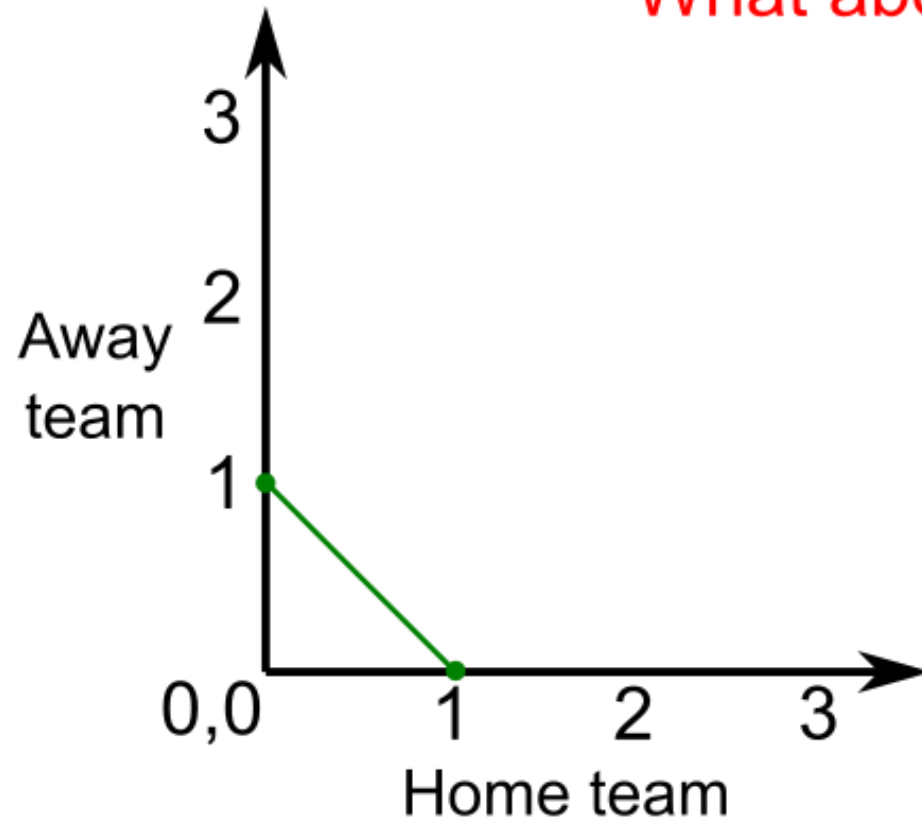
The minion championship used a $\{(0,1),(1,0)\}$ point system.

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What about other point systems?

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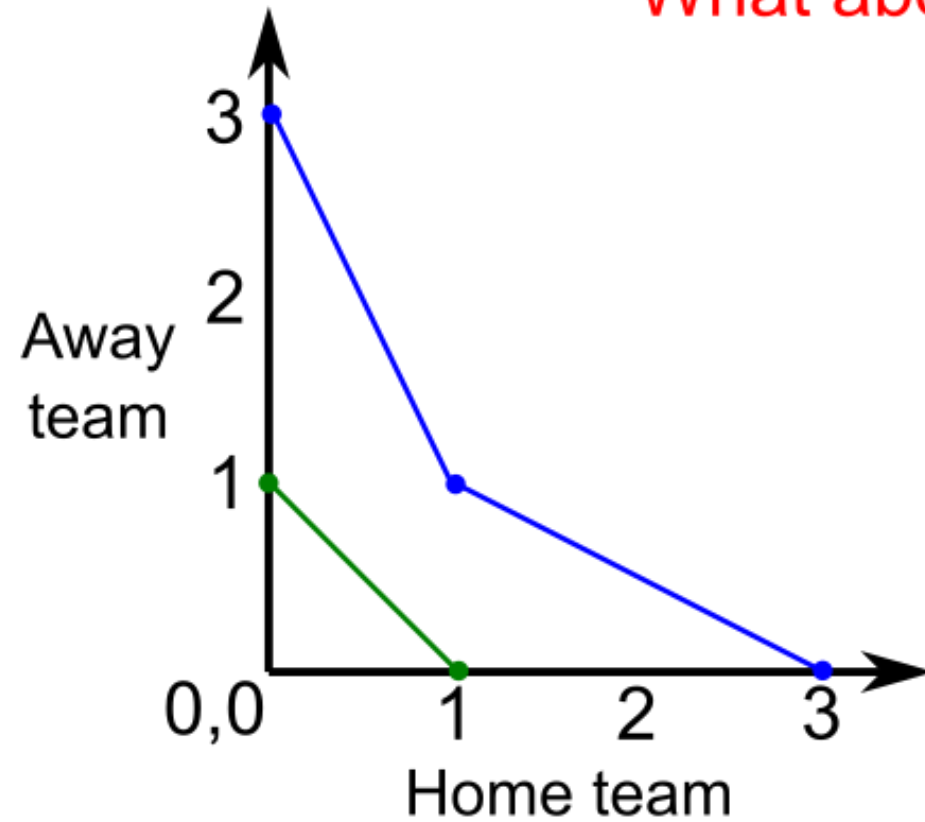
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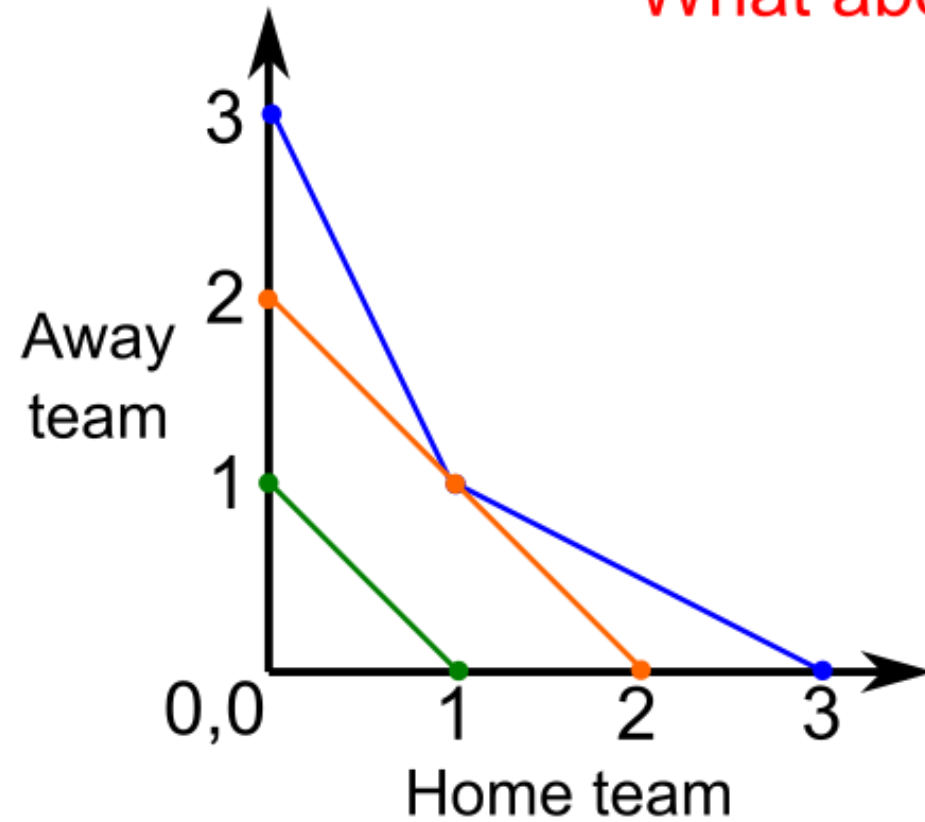
$\{(0,1),(1,0)\}$



$\{(0,3),(1,1),(3,0)\}$

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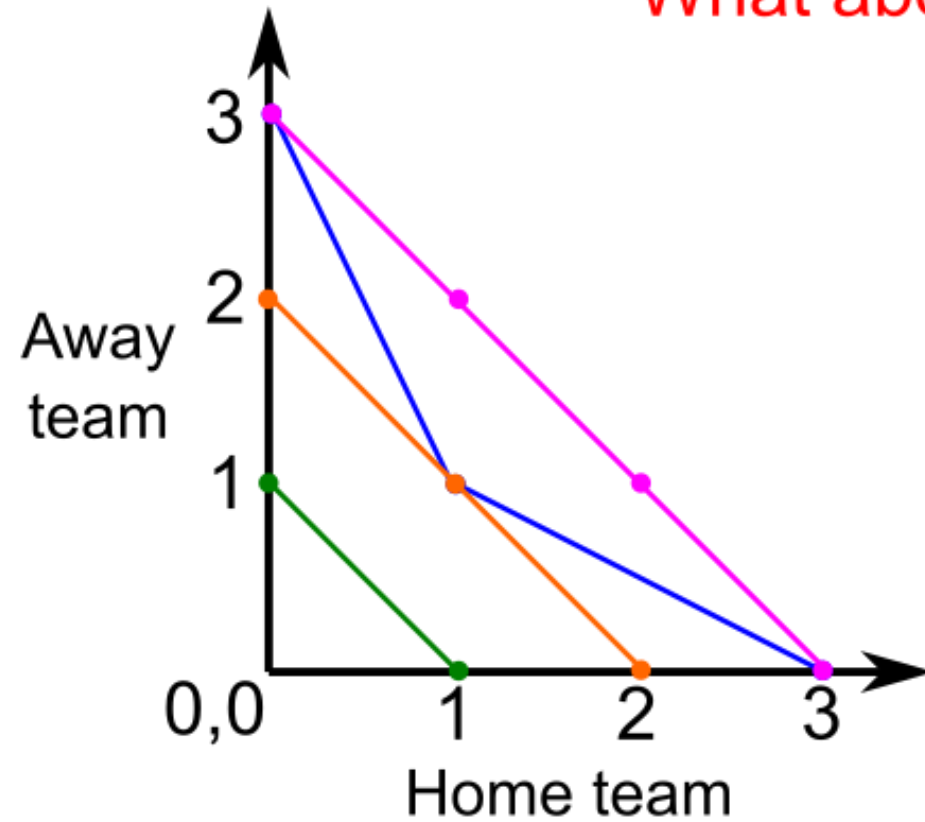
$\{(0,3),(1,1),(3,0)\}$



$\{(0,2),(1,1),(2,0)\}$

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What about other point systems?



$\{(0,1),(1,0)\}$



$\{(0,3),(1,1),(3,0)\}$



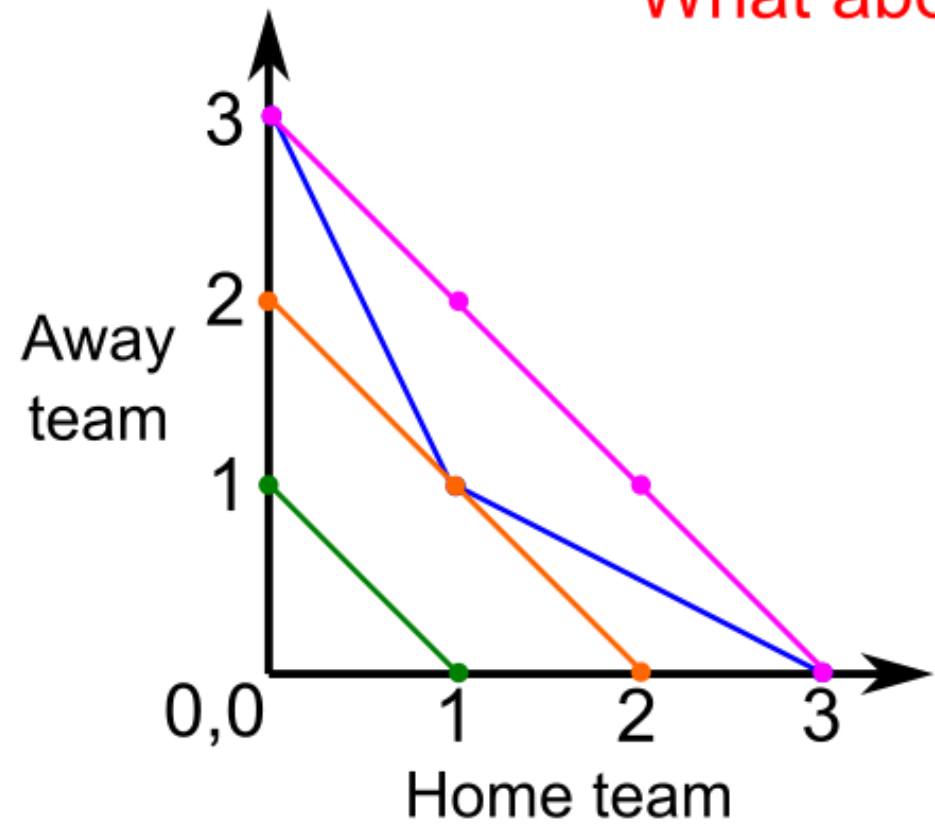
$\{(0,2),(1,1),(2,0)\}$



$\{(0,3),(1,2),(2,1),(3,0)\}$

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$\{(0,3),(1,1),(3,0)\}$



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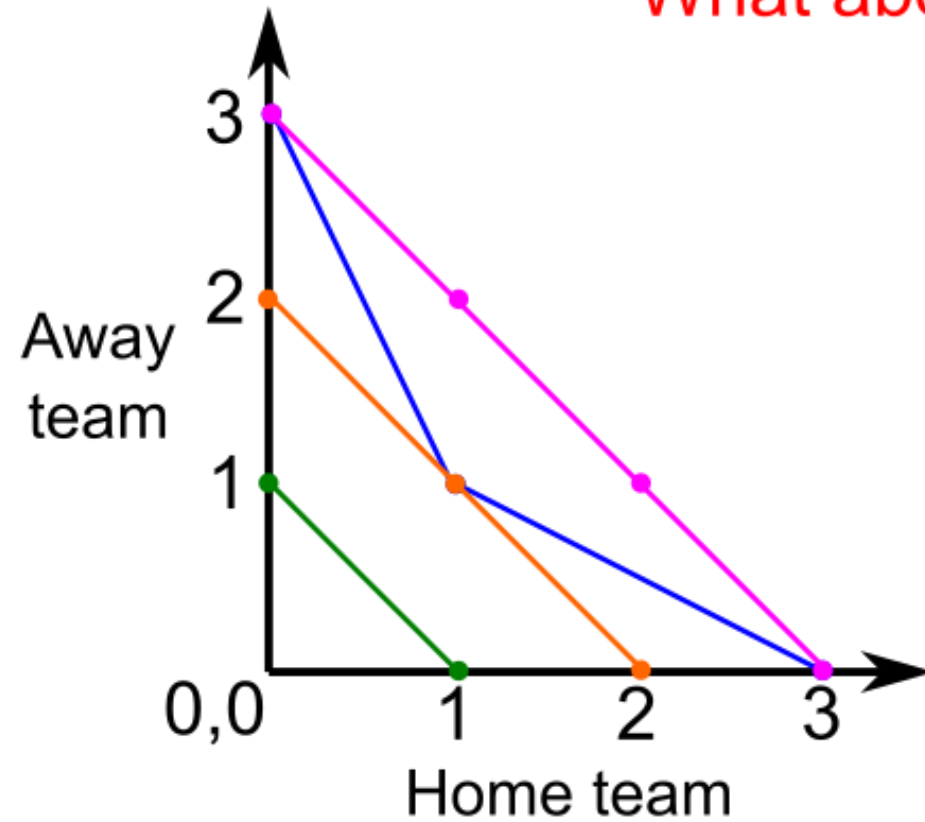


$\{(0,3),(1,2),(2,1),(3,0)\}$

[Kern and Paulusma, Disc. Opt. 2004]
Elimination problem is **NP-complete** for all point systems except for those that "line up nicely".

The minion championship used a $\{(0,1),(1,0)\}$ point system.

What about other point systems?



$\{(0,1),(1,0)\}$



$\{(0,3),(1,1),(3,0)\}$



$\{(0,2),(1,1),(2,0)\}$

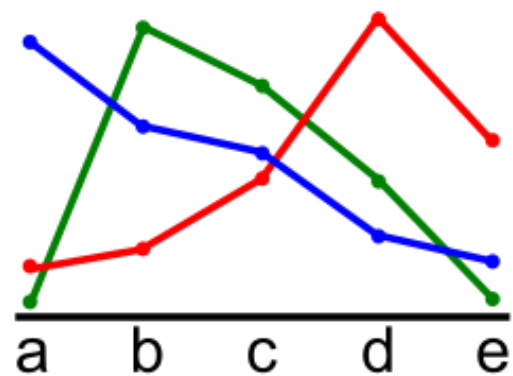


$\{(0,3),(1,2),(2,1),(3,0)\}$

Football is computationally harder than chess and ice hockey.

Next Time

Circumventing negative results
with structured preferences



References

- “Sports elimination via max flow” with IPL teams:
<https://www.youtube.com/watch?v=XK6qZjHWo9A>
- When it’s easy to recognize the *existence* of a beneficial manipulation but hard to *find* a manipulative vote.

“Search versus Decision for Election Manipulation Problems”

Hemaspaandra, Hemaspaandra, and Menton

<https://dl.acm.org/doi/10.1145/3369937>

