

COL866: Special Topics in Algorithms

Lecture 13

Manipulation in Voting

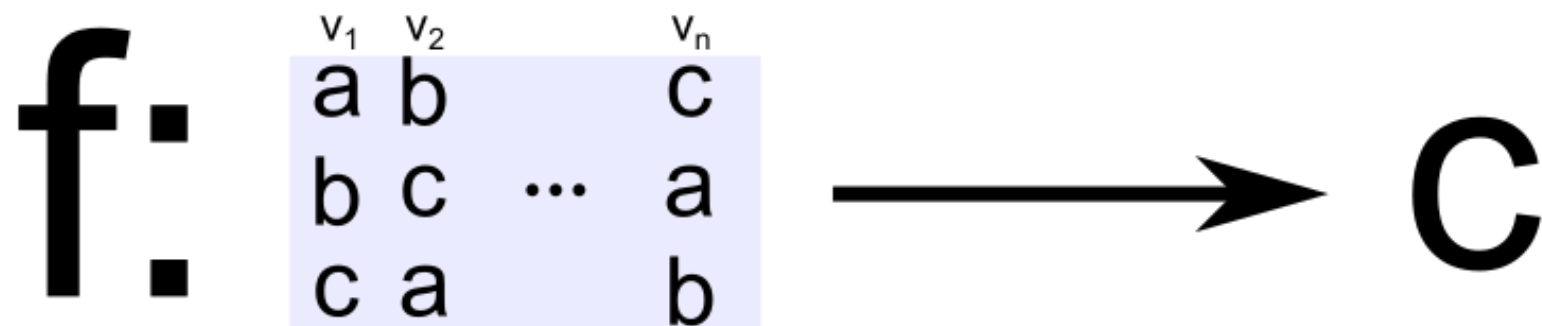
Oct 07, 2022

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Rohit Vaish

VOTING RULE

A mapping from preference profiles to candidates.



(also known as a *social choice function*)

Score-based

Plurality

Borda Count

Runoff-based

Plurality with Runoff

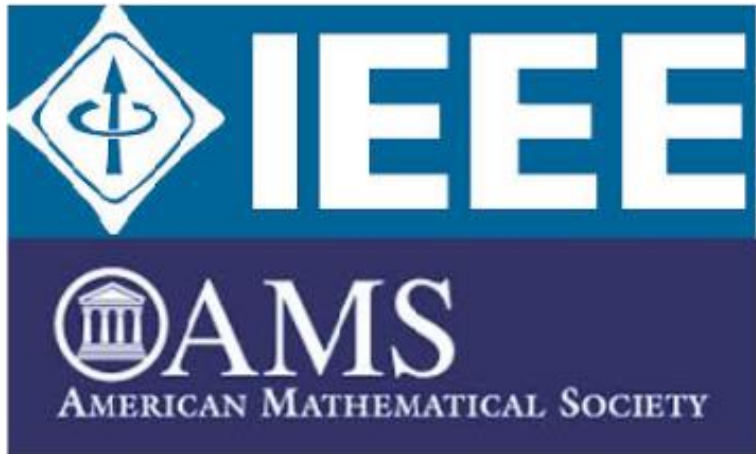
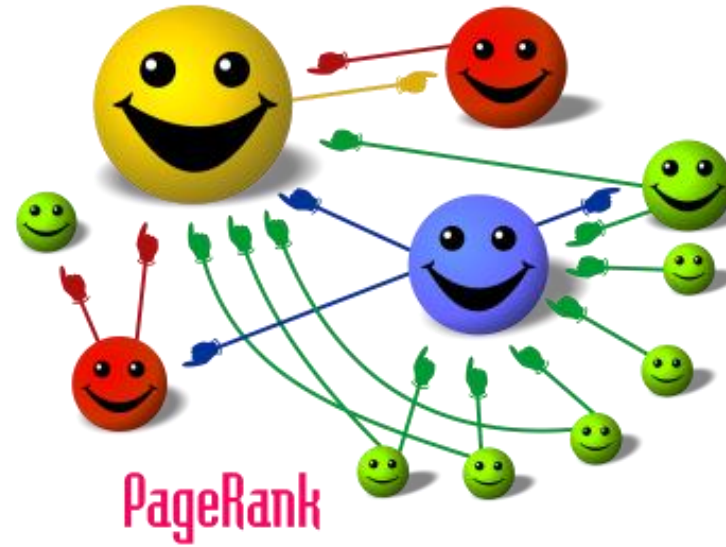
Single Transferable
Vote

Head-to-head
election based

Copeland

Schulze

Voting Rules are Everywhere!



A word cloud of various voting systems and methods. The words are arranged in a roughly triangular shape, pointing downwards. The colors of the words include yellow, orange, blue, white, and brown. The words are: Bucklin, Count, Voting, Random, Plurality, Kemeny-Young, Minimax, Ranked, Copeland, Winner, Pairs, Borda, Majority-Judgement, Range, Schulze, and Approval.

Bucklin
Count Voting
Random Plurality
Kemeny-Young Minimax
Ranked Copeland Winner
Pairs Borda Majority-Judgement
Range Schulze Approval

One Rule to Rule Them All?

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Axiomatic Approach

- Formulate a set of "reasonable" properties
- Check if there is a voting rule that satisfies them

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Expectation

Unique voting rule with
all the desirable properties



One Rule to Rule Them All?

Axiomatic Approach

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- Check if there is a voting rule that satisfies them

Expectation

Unique voting rule with all the desirable properties



Reality

No such voting rule





ONTO

For any candidate "a", there exists a profile where "a" wins.

$$f\left(\begin{array}{ccc} v_1 & v_2 & v_n \\ \text{[Redacted Profile]} \end{array}\right) = a$$

STRATEGYPROOF

No voter can improve by misreporting its preferences.

For any profile , any voter v_i , and any misreport , it must be that

$$f\left(\begin{array}{ccc} v_1 & v_i & v_n \\ \text{light blue box} \end{array}\right) \succeq_i f\left(\begin{array}{ccc} v_1 & \text{light red box} & v_n \\ \text{light blue box} \end{array}\right)$$

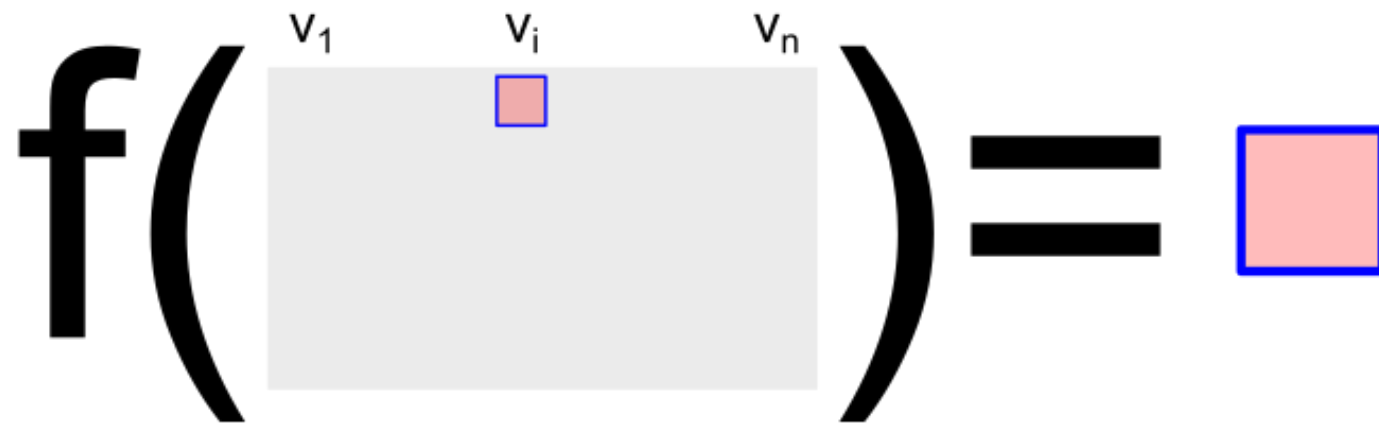


An Obviously Bad Voting Rule

A voting rule is called a **dictatorship** if there exists a voter v_i such that for any preferences of the other voters, the voting outcome is the favorite candidate of voter v_i .

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$$f\left(\begin{array}{ccc} v_1 & v_i & v_n \\ \text{[gray box]} & \text{[red box]} & \text{[gray box]} \end{array}\right) = \text{[red box]}$$
The diagram shows a function f in large black font. To its right is a large opening parenthesis. Inside the parenthesis is a light gray rectangular box representing a profile of preferences. Above the gray box, three labels are positioned: v_1 above the left side, v_i above the center, and v_n above the right side. Within the gray box, centered under the v_i label, is a small red square with a blue border. To the right of the gray box is a double equals sign. To the right of the equals sign is a larger red square with a blue border, representing the outcome of the function.

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A voting rule is called a **dictatorship** if there exists a voter v_i such that for any preferences of the other voters, the voting outcome is the favorite candidate of voter v_i .

$$f\left(\begin{array}{ccc} v_1 & v_i & v_n \\ \text{[gray box]} & \text{[red box]} & \text{[gray box]} \end{array}\right) = \text{[red box]}$$

A dictatorship is onto and strategyproof.



[Gibbard'73; Satterthwaite'75]

With three or more candidates,
a voting rule is **onto** and **strategyproof**
if and only if it is a **dictatorship**.



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Onto + strategyproof but two candidates:



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Majority



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≥ 3 candidates + strategyproof but not onto:



[Gibbard'73; Satterthwaite'75]

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≥ 3 candidates + strategyproof but not onto: Constant/Restricted Majority



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Onto + strategyproof but two candidates:

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≥ 3 candidates + strategyproof but not onto: Constant/Restricted Majority

≥ 3 candidates + onto but not strategyproof:

Plurality/Borda/...

With three or more candidates,
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With three or more candidates,
a voting rule is **unanimous** and **monotone**
if and only if it is a **dictatorship**.

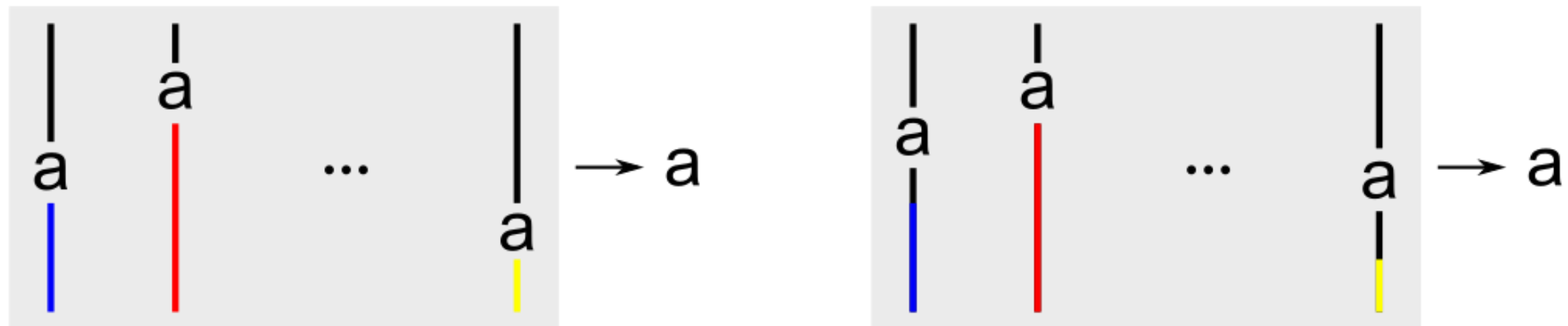
UNANIMOUS

If all voters agree on their top choice, then the voting rule picks it.

$$f\left(\begin{array}{c} v_1 \quad v_2 \quad \dots \quad v_n \\ a \quad a \quad \dots \quad a \\ \cdot \quad \cdot \quad \dots \quad \cdot \\ \cdot \quad \cdot \quad \dots \quad \cdot \\ \cdot \quad \cdot \quad \dots \quad \cdot \end{array}\right) = a$$

MONOTONE

Suppose "a" is the current winner, and for each voter, any candidate that was ranked below "a" in its old vote is ranked below "a" in its new vote, then "a" continues to be the winner.



support of "a"

Strategyproofness = Monotonicity

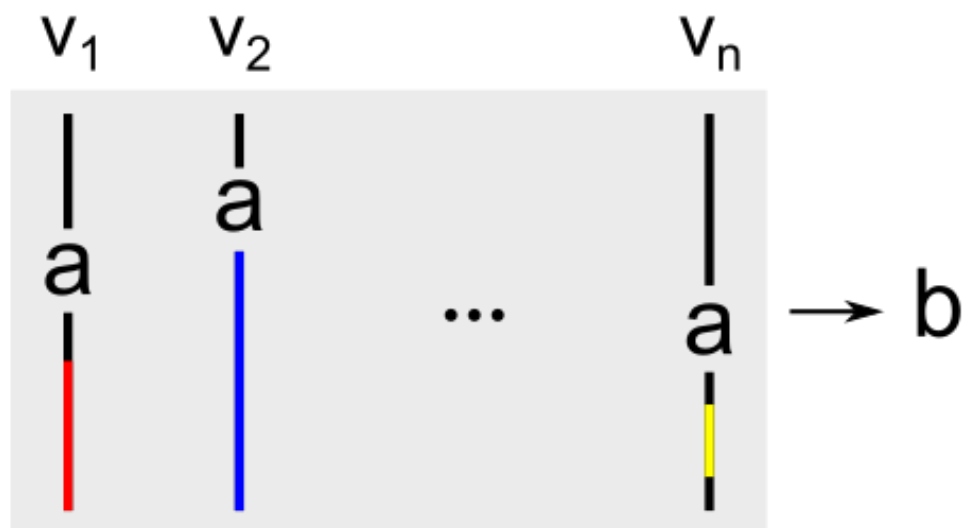
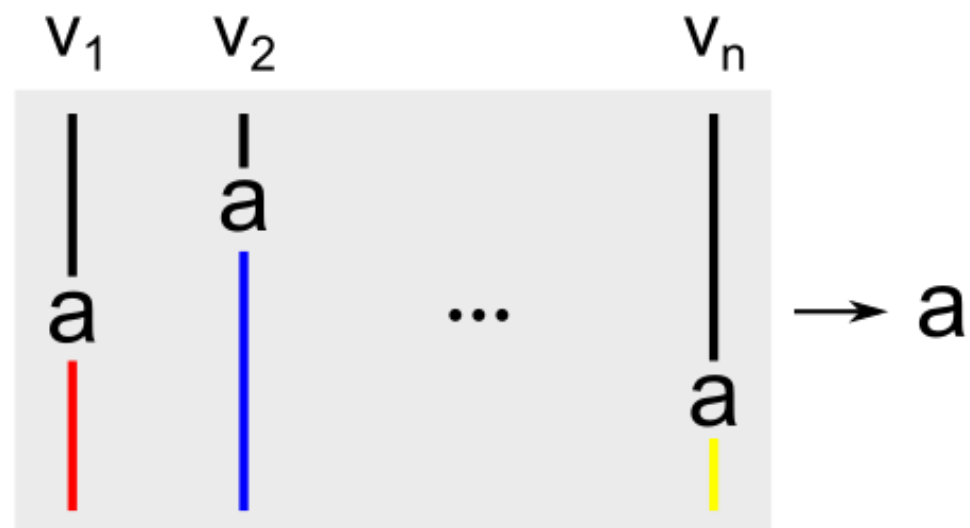
STRATEGYPROOF \Rightarrow MON

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Suppose SP but not MON.

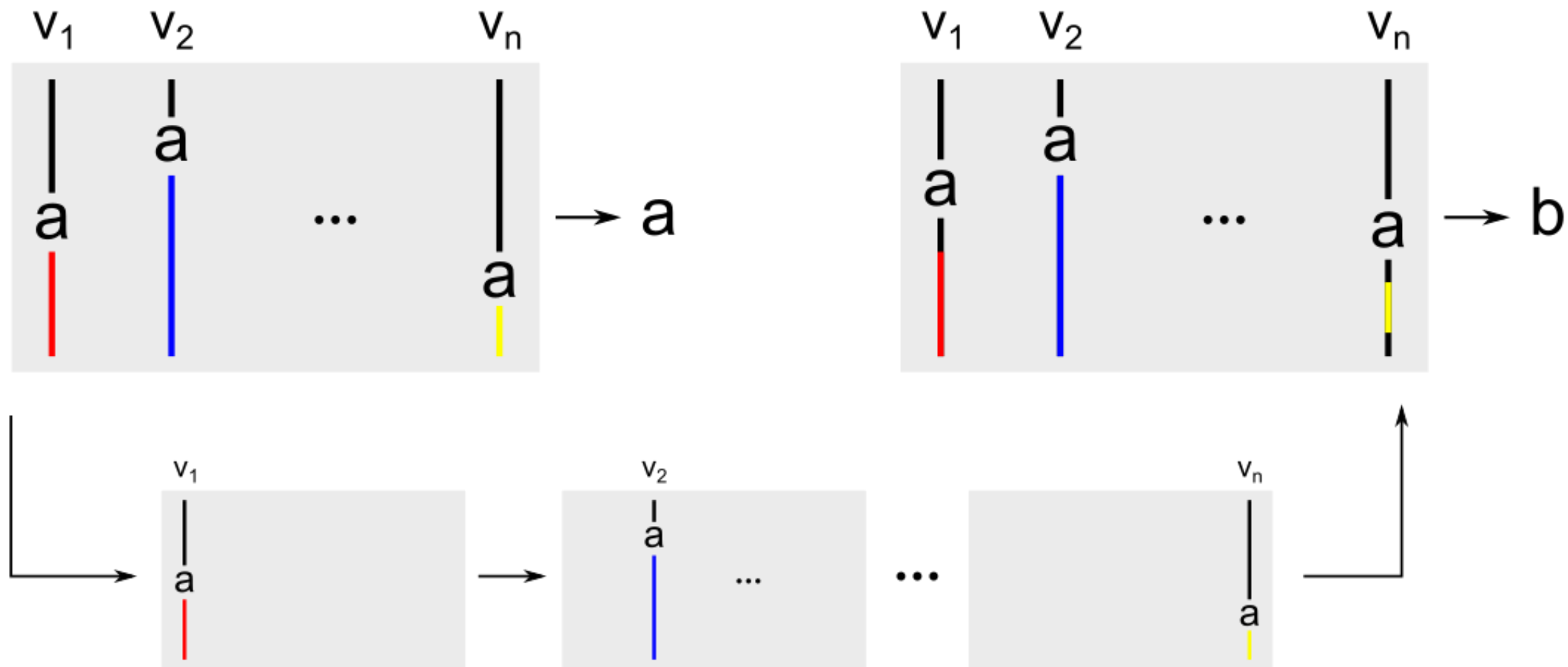
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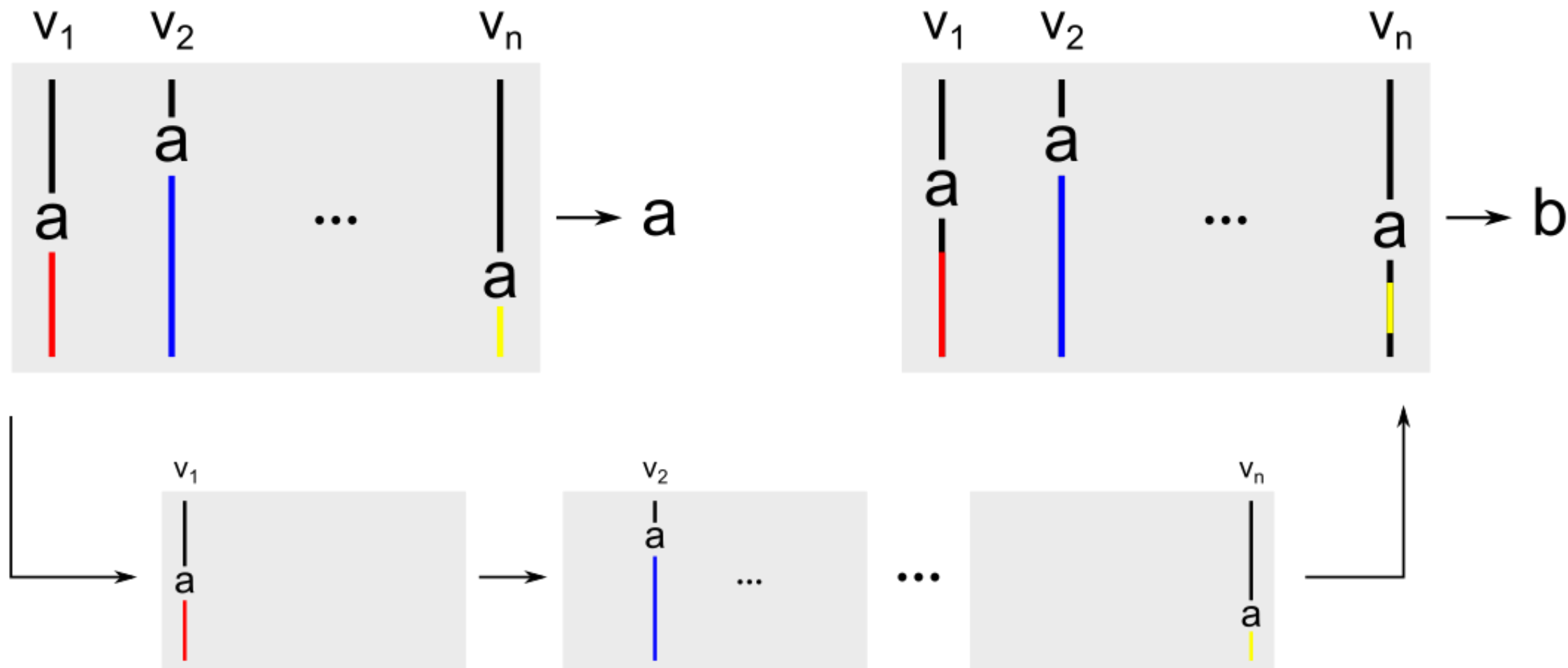
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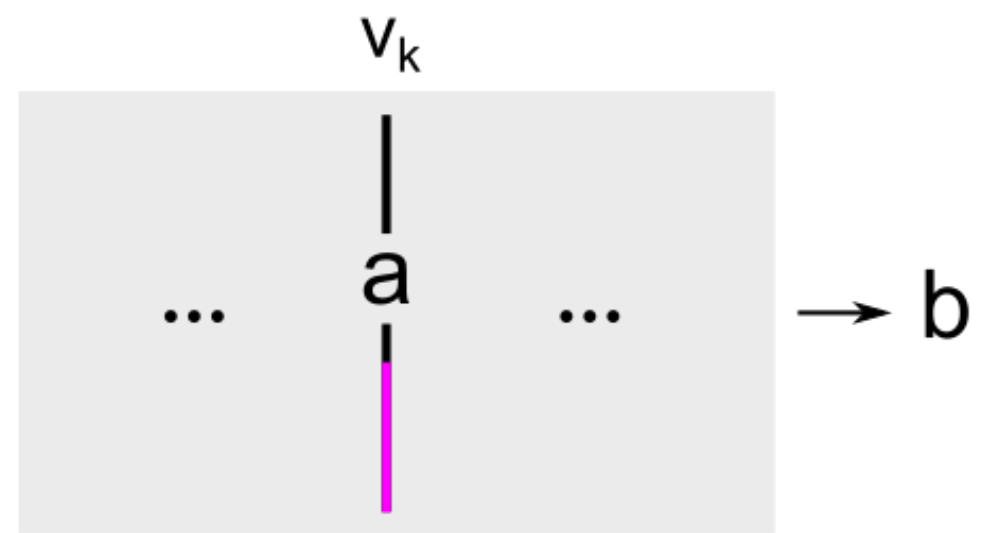
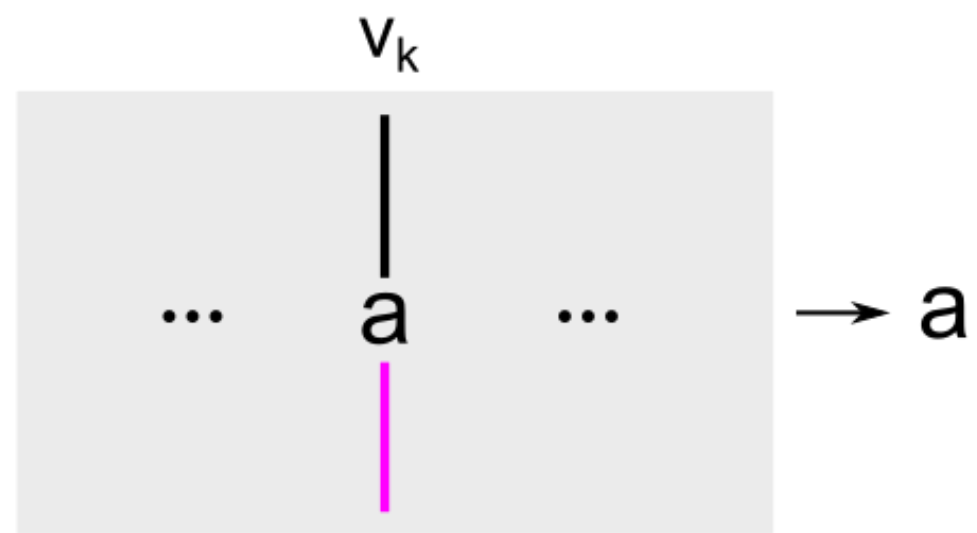
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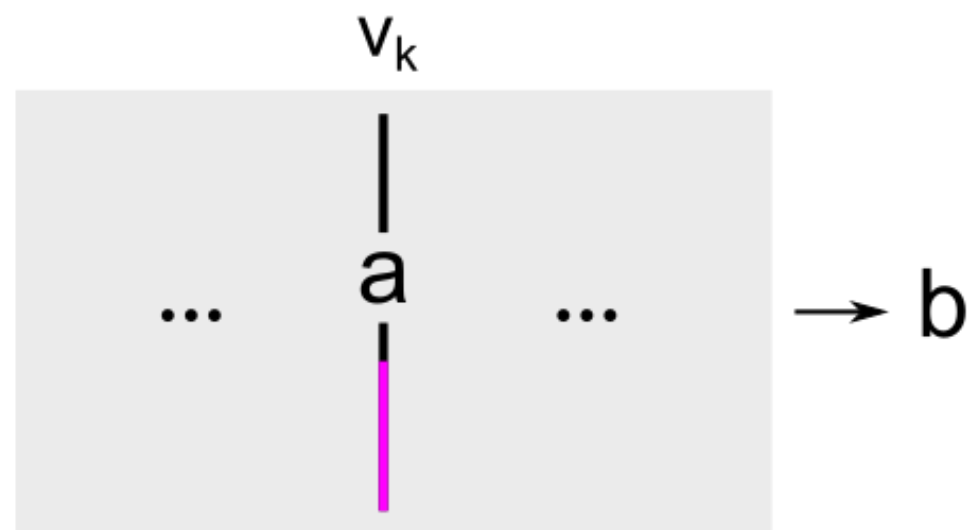
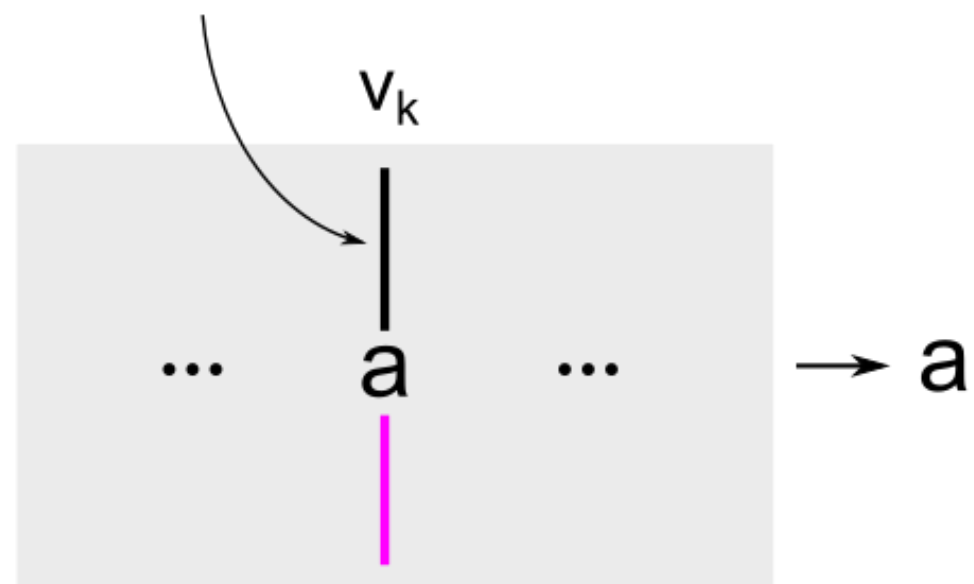
Suppose v_k is the first voter for which the outcome changes.

STRATEGYPROOF \Rightarrow MON



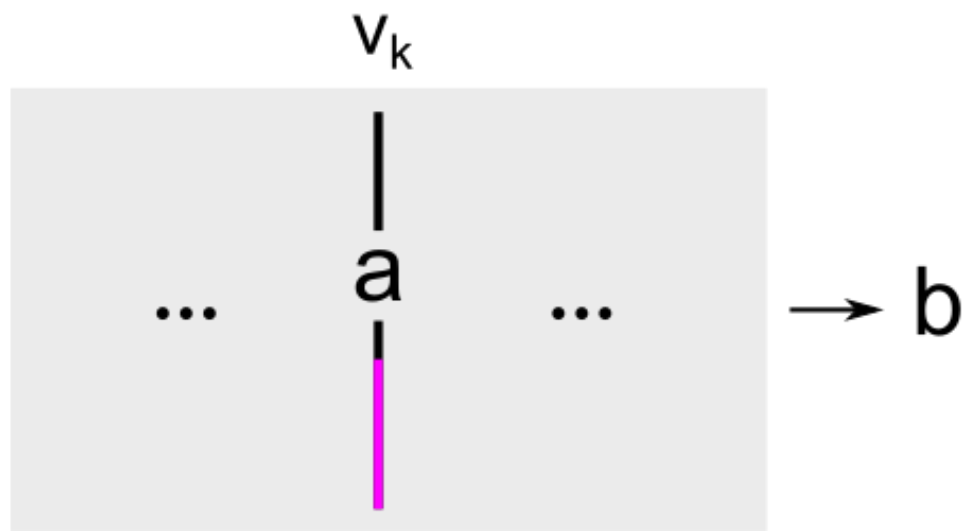
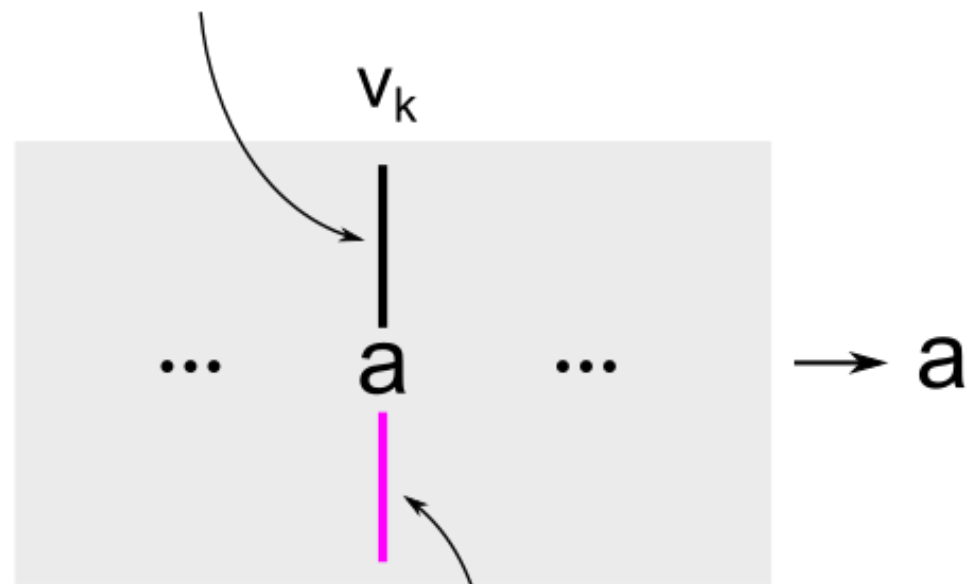
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b can't be here (SP)



STRATEGYPROOF \Rightarrow MON

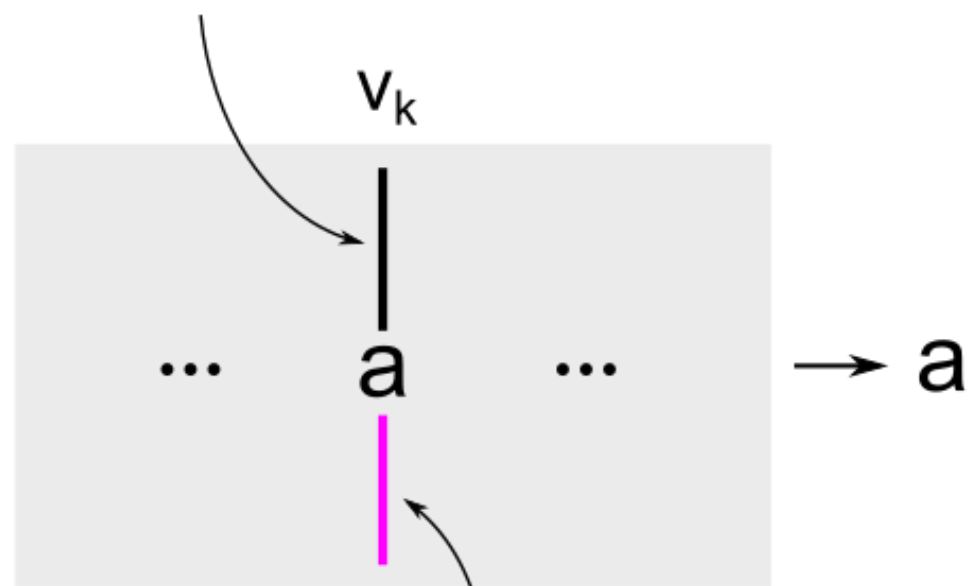
b can't be here (SP)



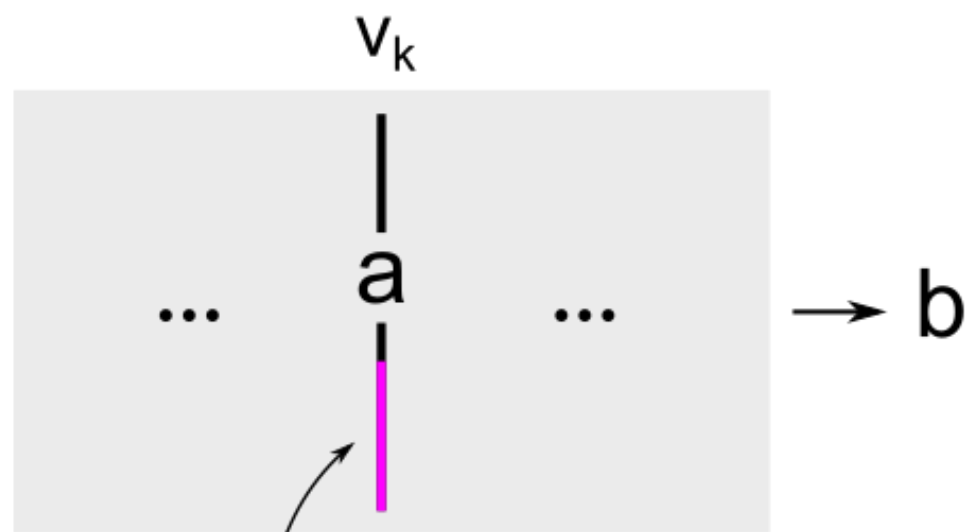
so it must be here

STRATEGYPROOF \Rightarrow MON

b can't be here (SP)



so it must be here

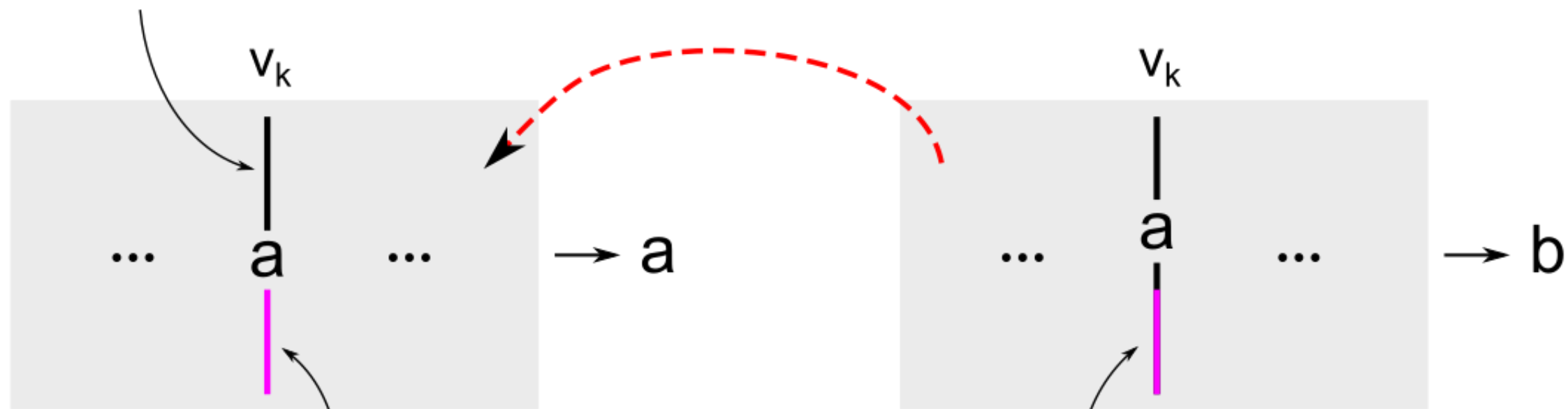


Then, it must be here

STRATEGYPROOF \Rightarrow MON

b can't be here (SP)

But then there is a **reverse manipulation**

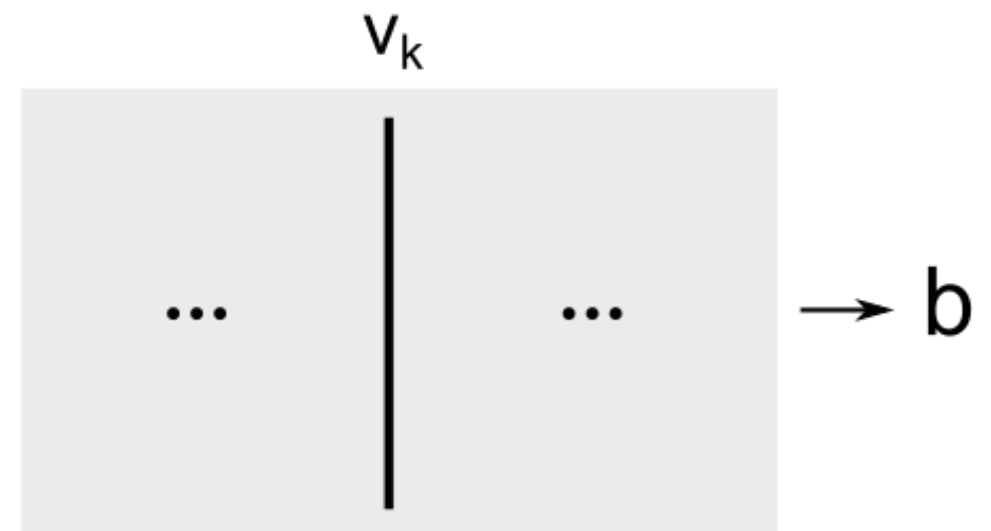
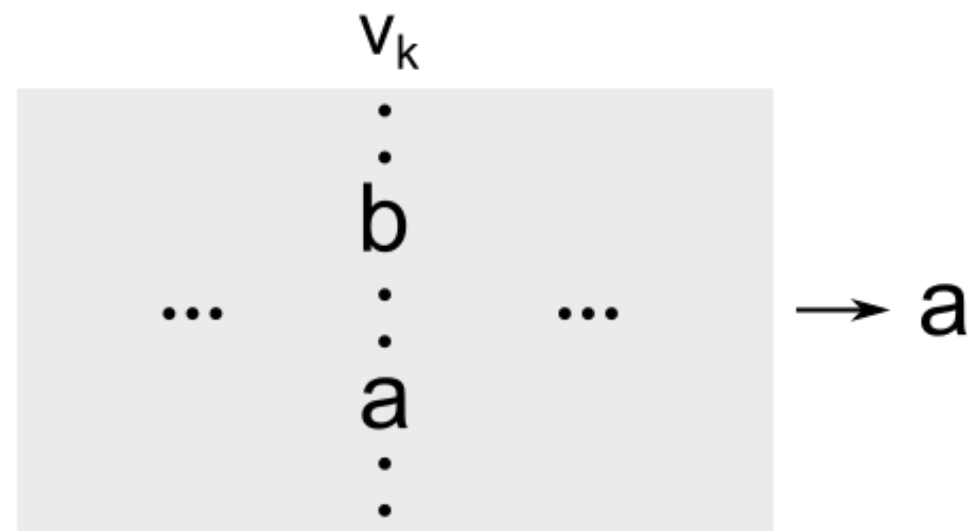


so it must be here

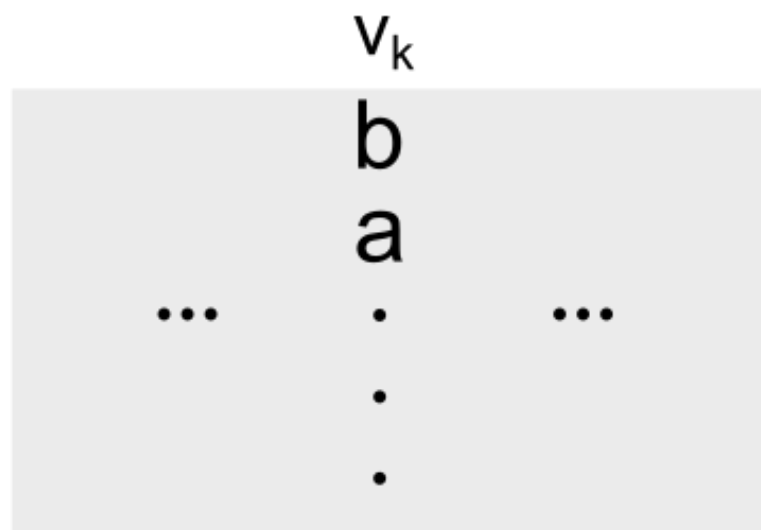
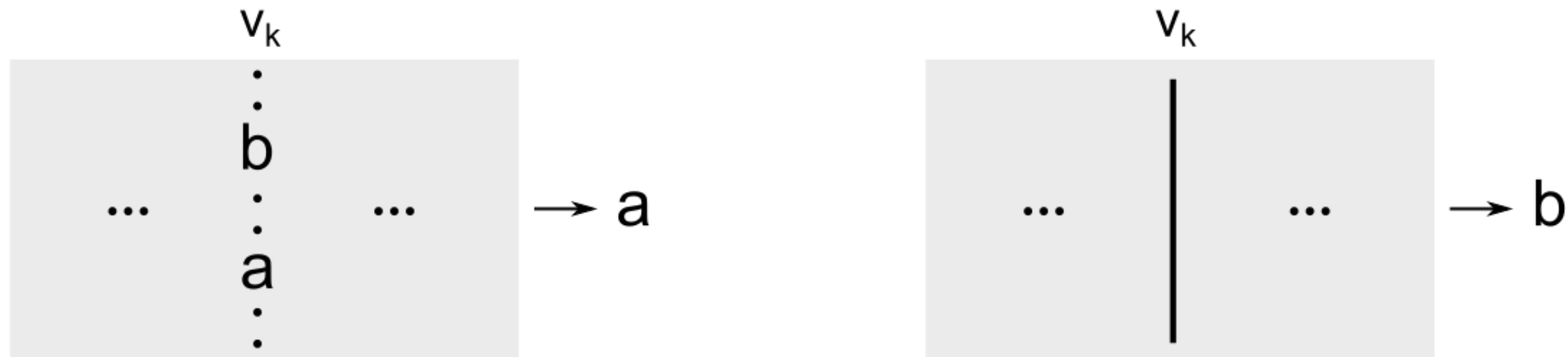
Then, it must be here

MON \Rightarrow STRATEGYPROOF

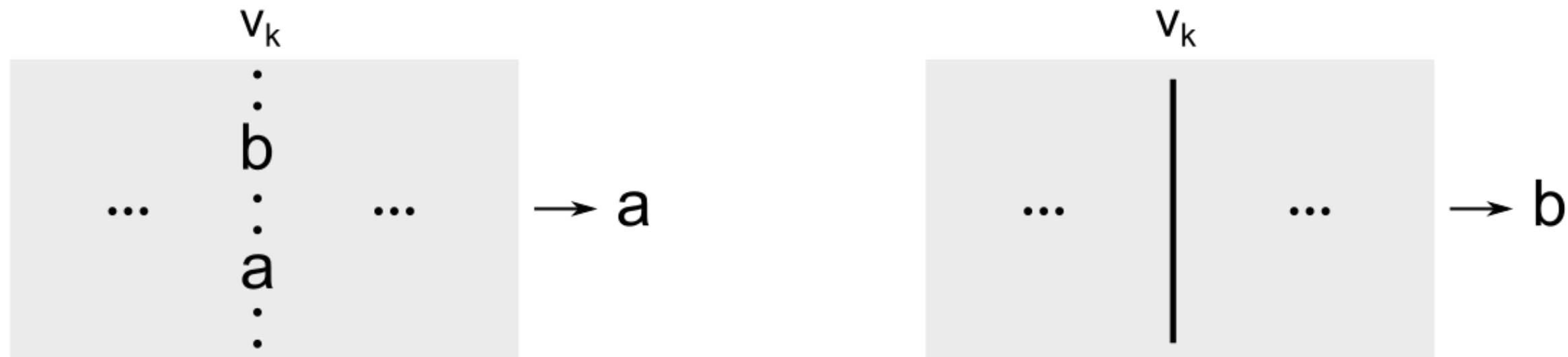
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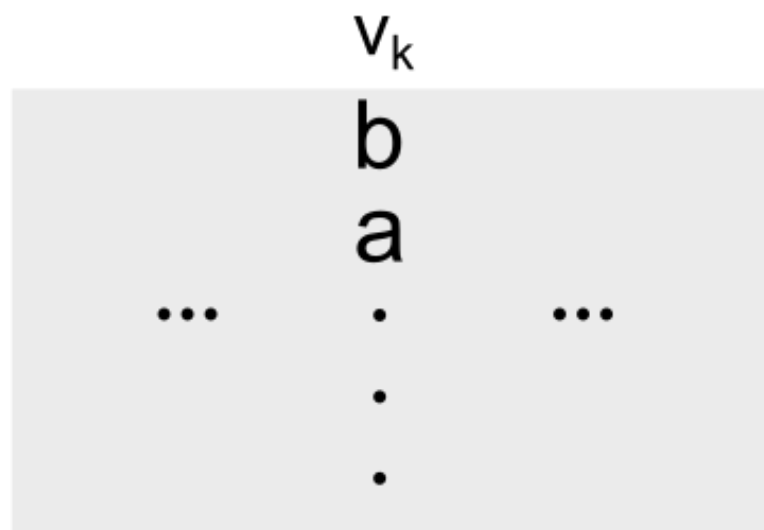
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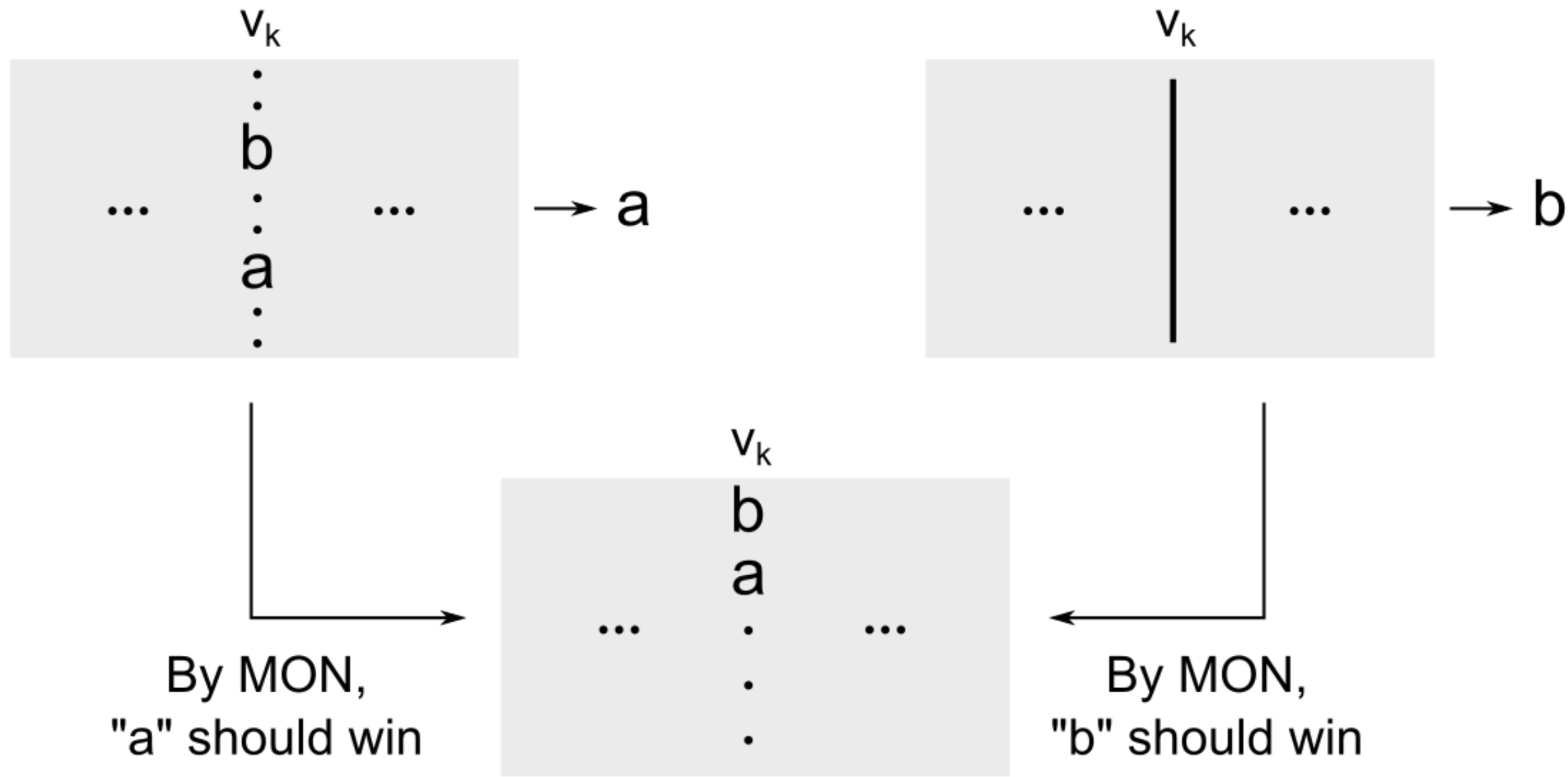
MON \Rightarrow STRATEGYPROOF



By MON,
"a" should win



MON \Rightarrow STRATEGYPROOF



With three or more candidates,
a voting rule is **onto** and **strategyproof**
if and only if it is a **dictatorship**.

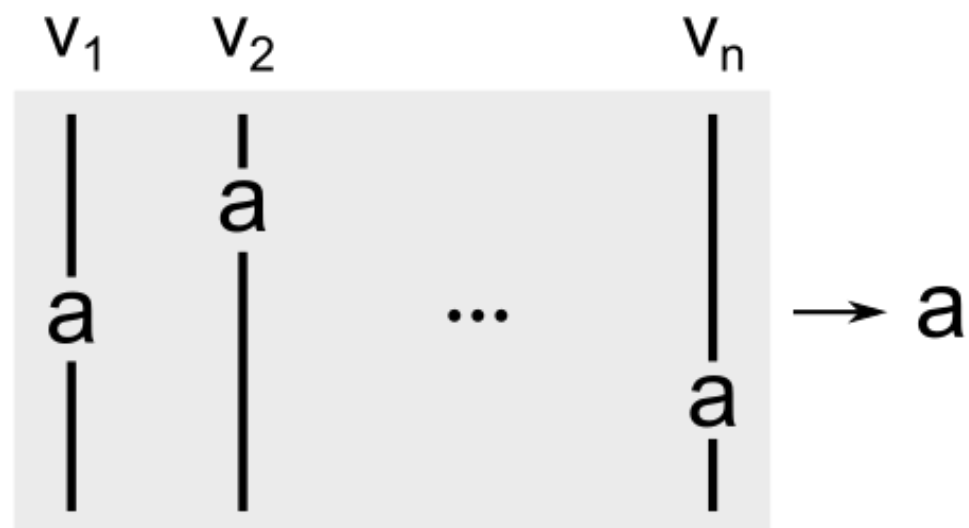


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MON/SP + ONTO \Rightarrow UNAN

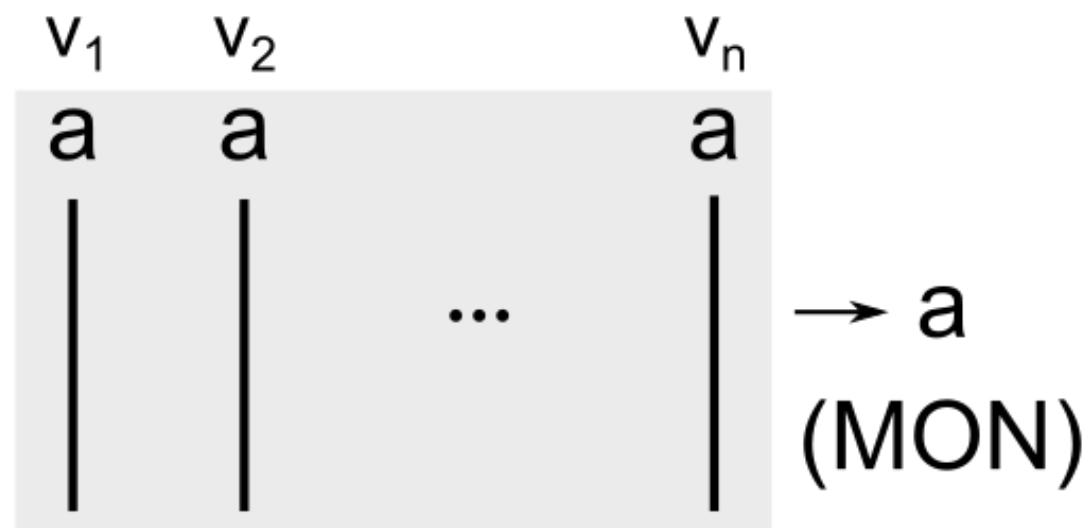
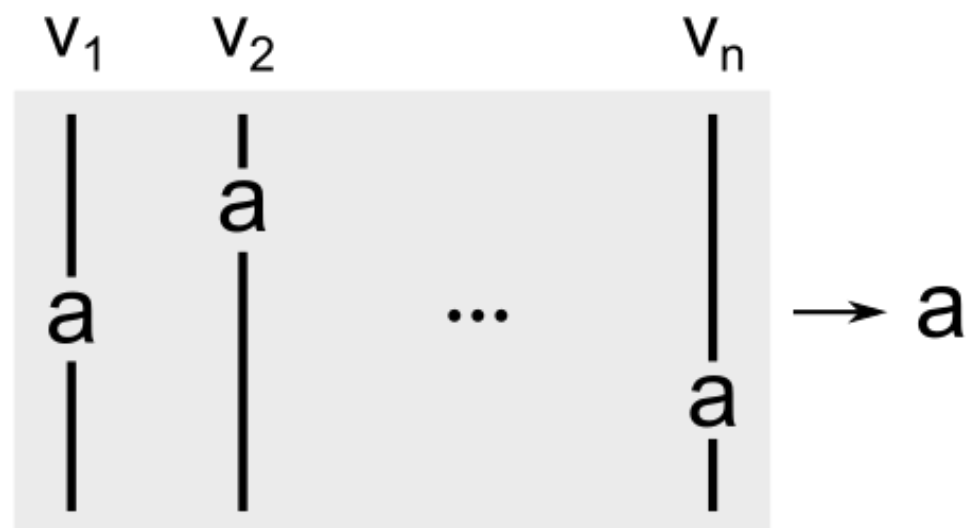
MON/SP + ONTO \Rightarrow UNAN

For any candidate "a", by ONTO, there is a profile where "a" wins



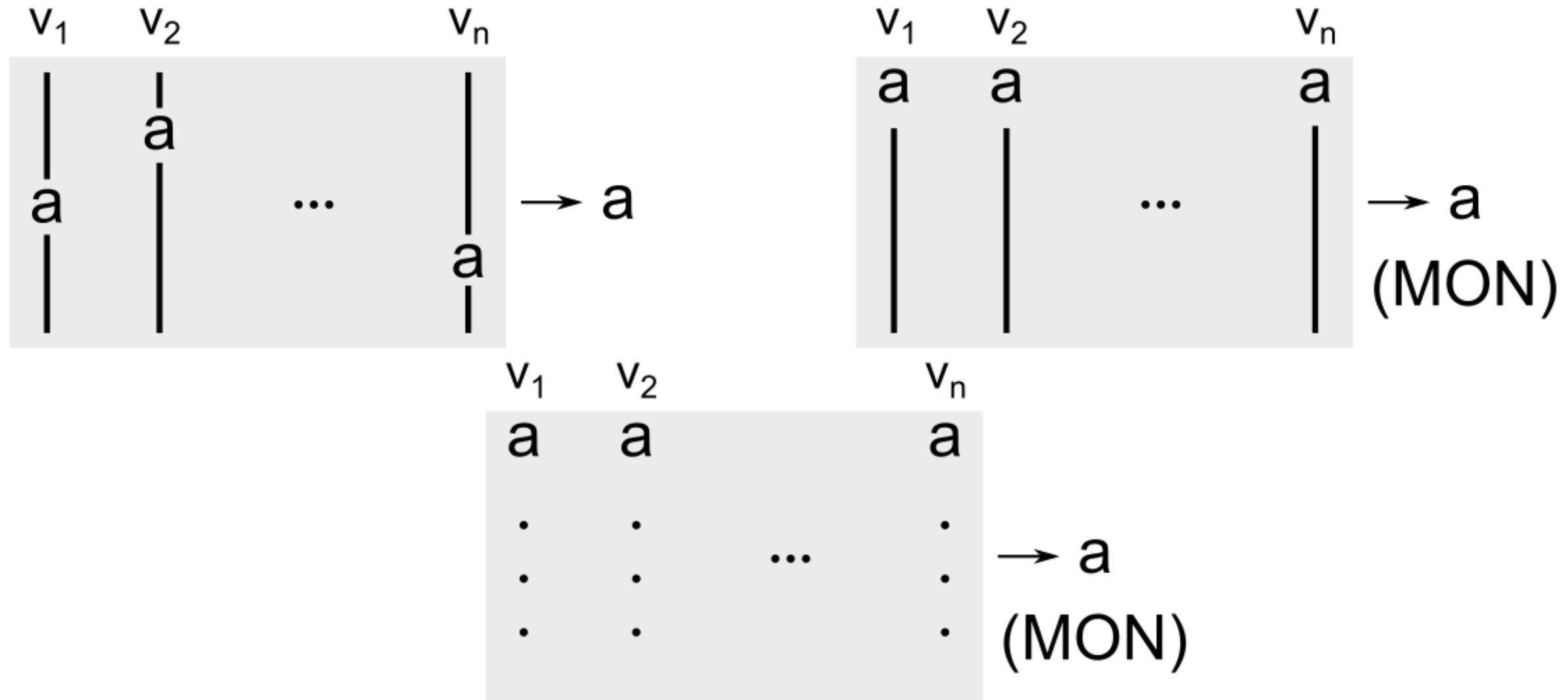
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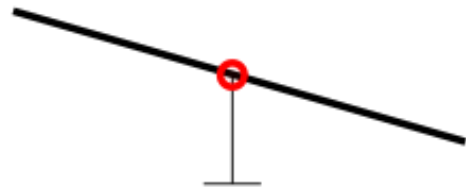
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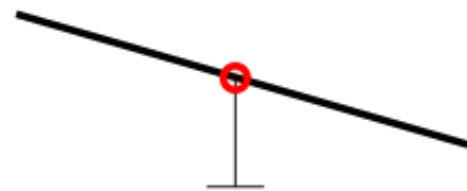
UNAN + MON \Rightarrow DICTATOR

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Identify a
"pivotal voter" v_p
for the pair $\{a,b\}$

UNAN + MON \Rightarrow DICTATOR

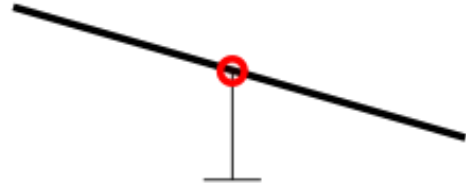


.	.	a	.	.
.
.
a	a	.	a	a

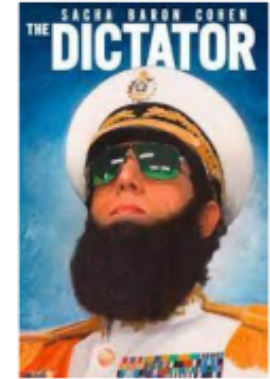
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Show that "a" wins
if v_p likes it
even if everyone
else ranks it last

UNAN + MON \Rightarrow DICTATOR



.	.	a	.	.
.
.
a	a	.	a	a

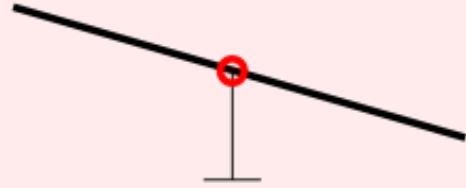


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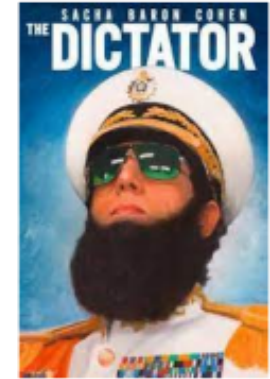
v_p is the
dictator for "a"
(and every other
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.	.	a	.	.
.
.
a	a	.	a	a



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v_p is the
dictator for "a"
(and every other
candidate)

v_1	v_2		v_n
a	a		a
·	·		·
·	·	...	·
·	·		·
b	b		b

v_1	v_2		v_n
a	a		a
·	·		·
·	·	...	·
·	·		·
b	b		b

 → a (UNAN)

v_1	v_2		v_n	
a	a		a	
·	·		·	
·	·	...	·	→ a (UNAN)
·	·		·	
b	b		b	

Gradually promote "b" in each voter's list until it wins

v_1	v_2		v_n
a	a		a
·	·		·
·	·	...	·
·	·		·
b	b		b

→ a (UNAN)

Gradually promote "b" in each voter's list until it wins

v_1	v_2		v_n
a	a		a
b	·		·
·	·	...	·
·	·		·
·	b		b

v_1	v_2		v_n	
a	a		a	→ a (UNAN)
·	·		·	
·	·	...	·	
·	·		·	
b	b		b	

Gradually promote "b" in each voter's list until it wins

v_1	v_2		v_n	
a	a		a	→ a (MON)
b	·		·	
·	·	...	·	
·	·		·	
·	b		b	

v_1	v_2		v_n	
a	a		a	→ a (UNAN)
·	·		·	
·	·	...	·	
·	·		·	
b	b		b	

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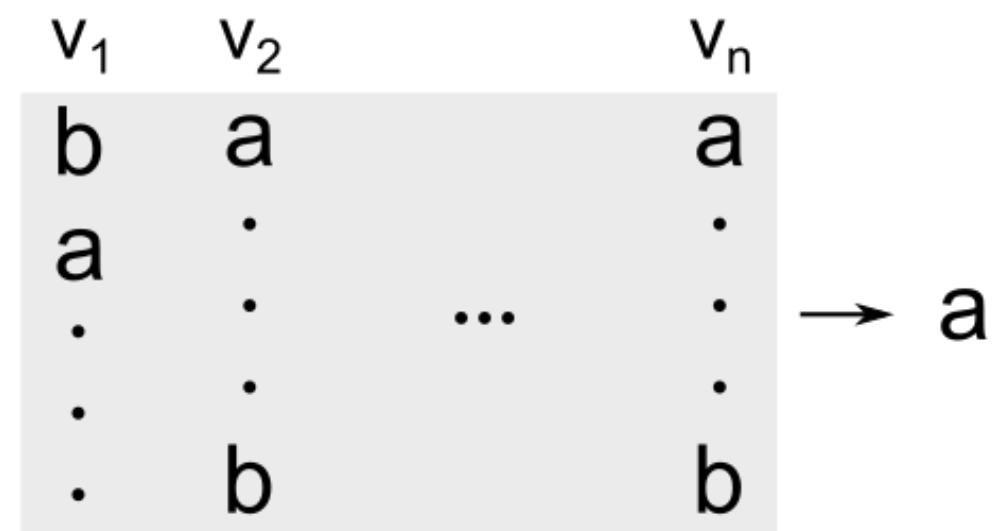
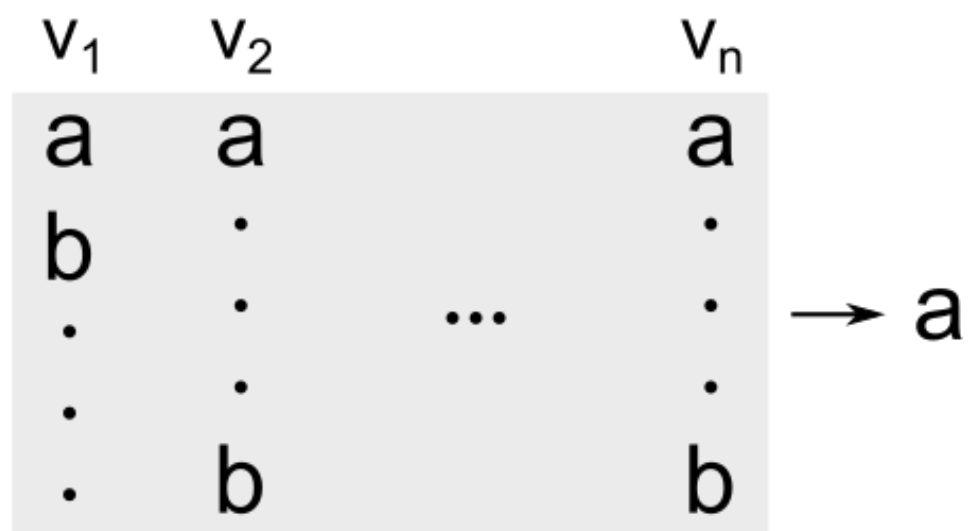
v_1	v_2		v_n		v_1	v_2		v_n	
a	a		a	→ a (MON)	b	a		a	
b	·		·		a	·		·	
·	·	...	·		·	·	...	·	
·	·		·		·	·		·	
·	b		b		·	b		b	

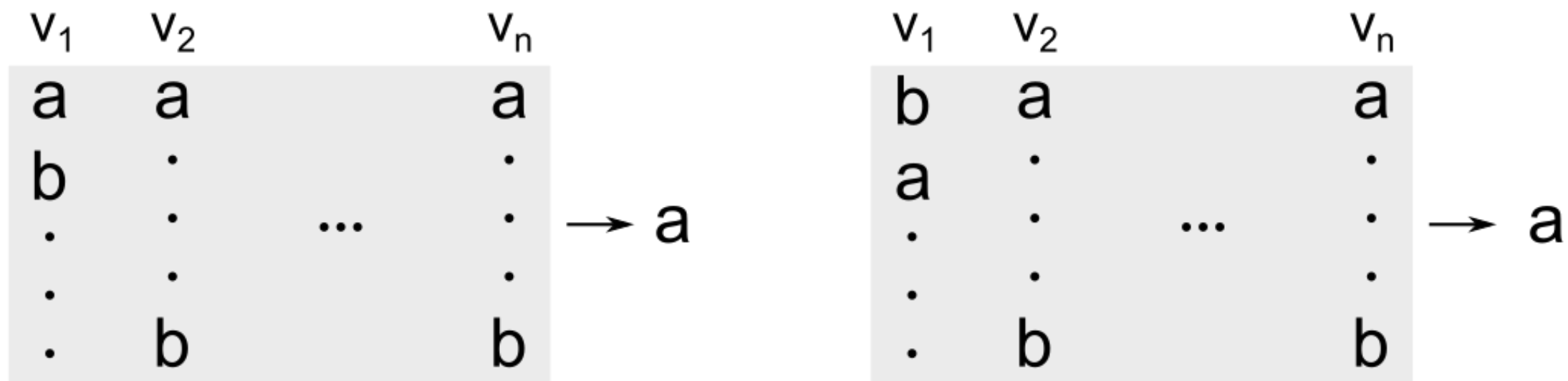
v_1	v_2		v_n	
a	a		a	→ a (UNAN)
·	·		·	
·	·	...	·	
·	·		·	
b	b		b	

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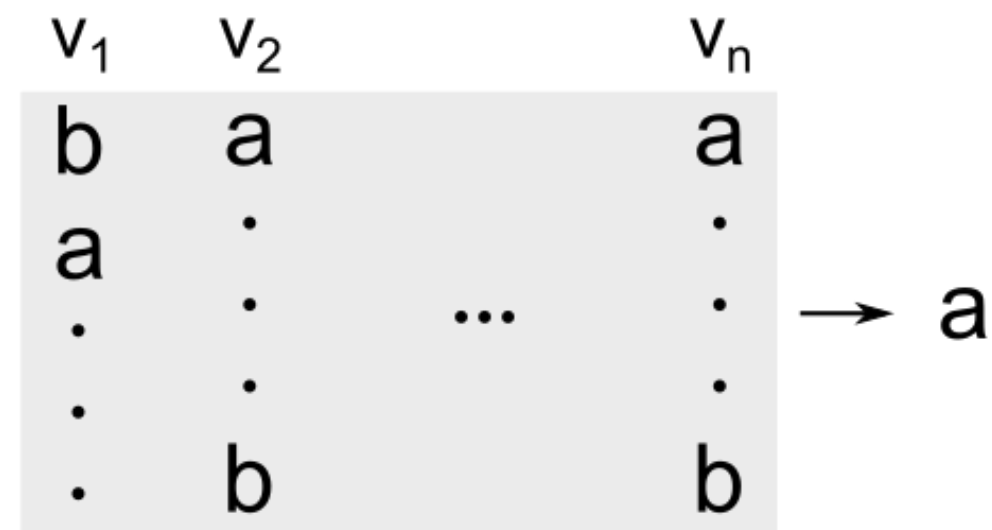
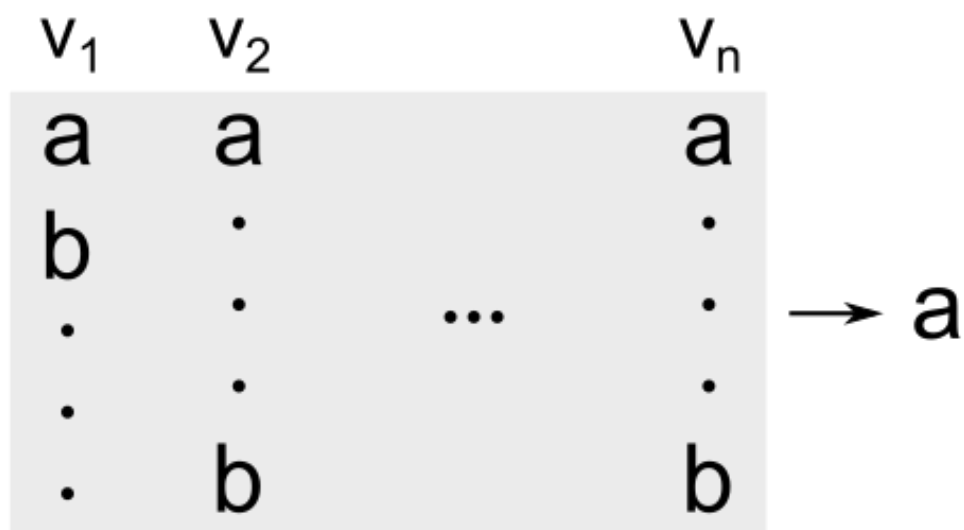
v_1	v_2		v_n	
a	a		a	→ a (MON)
b	·		·	
·	·	...	·	
·	·		·	
·	b		b	

v_1	v_2		v_n	
b	a		a	→ {a,b} (MON)
a	·		·	
·	·	...	·	
·	·		·	
·	b		b	

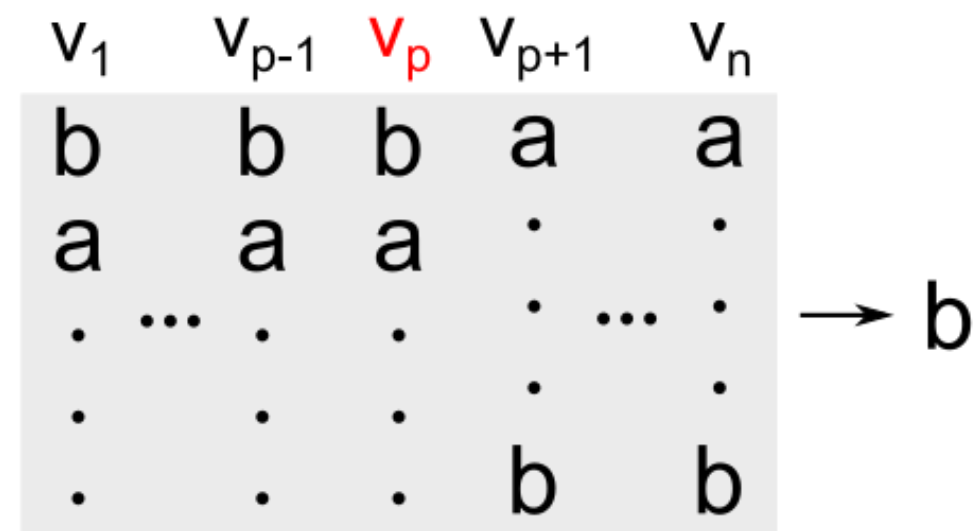
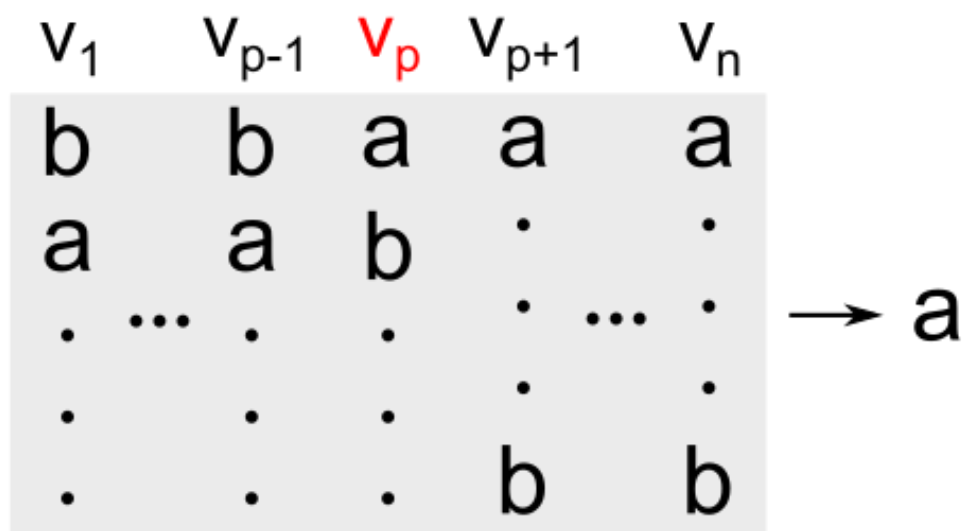




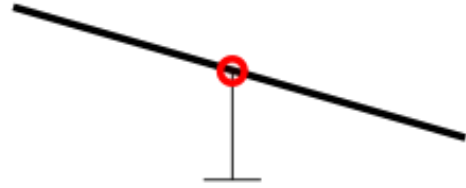
"b" must win *eventually* due to UNAN



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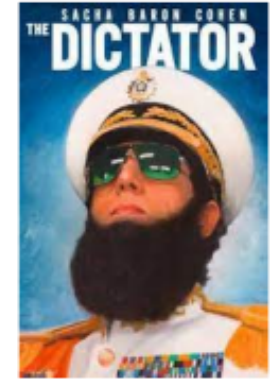
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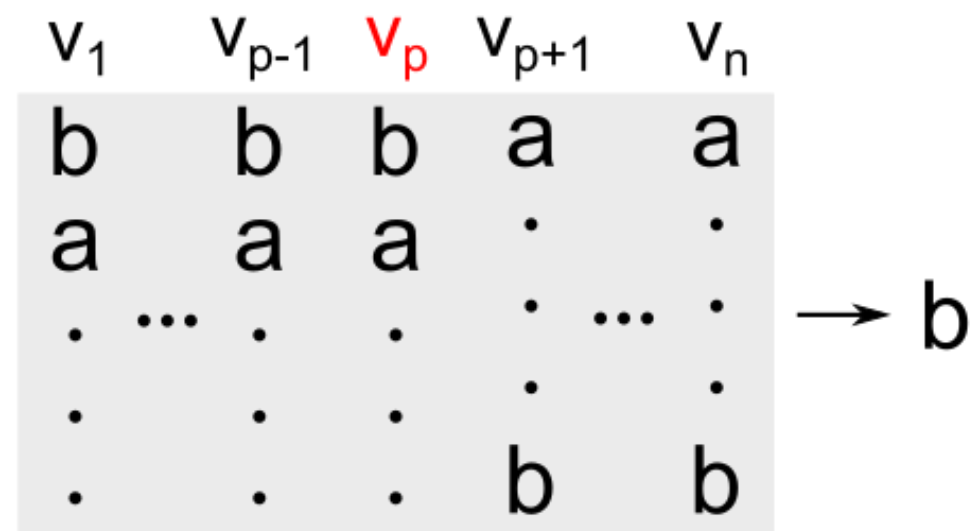
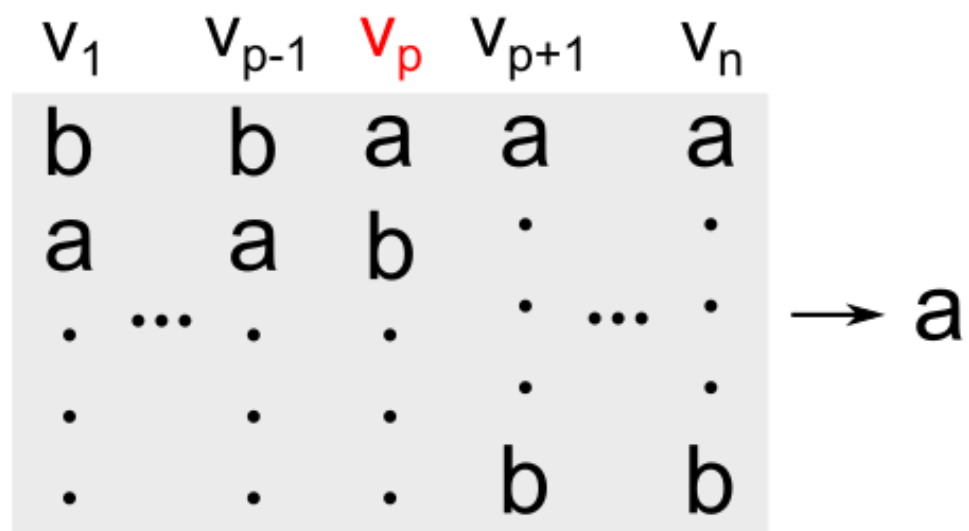
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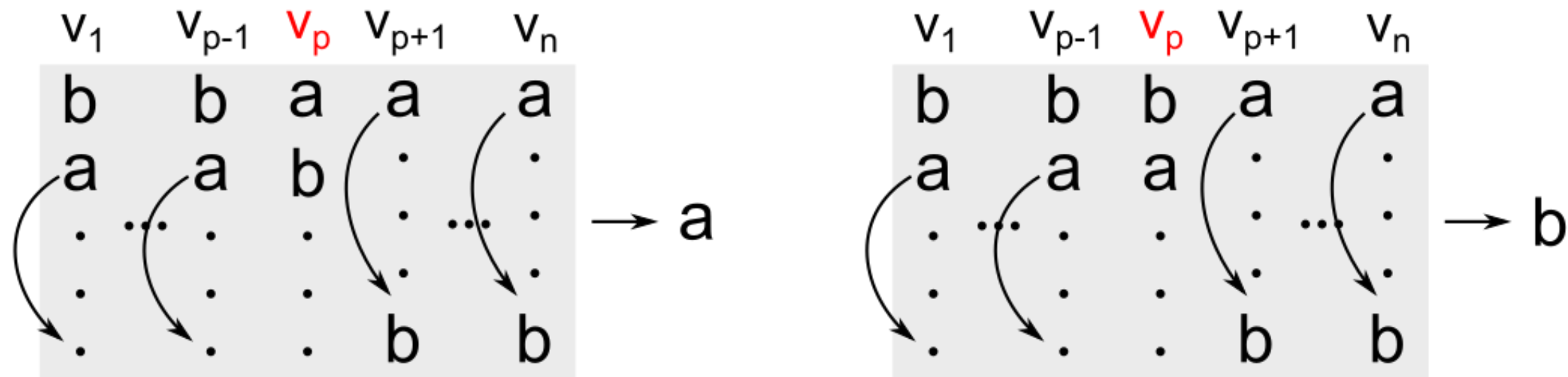
.	.	a	.	.
.
.
a	a	.	a	a

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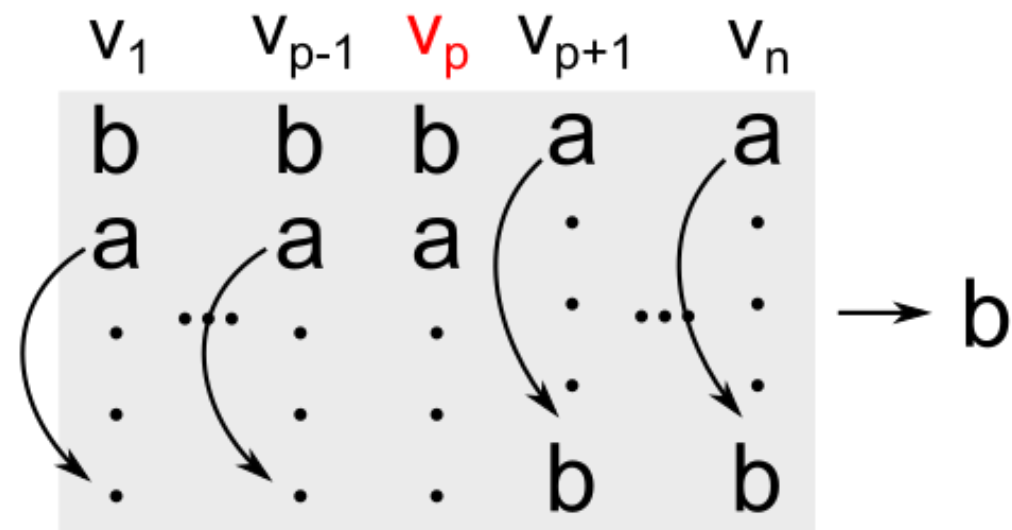
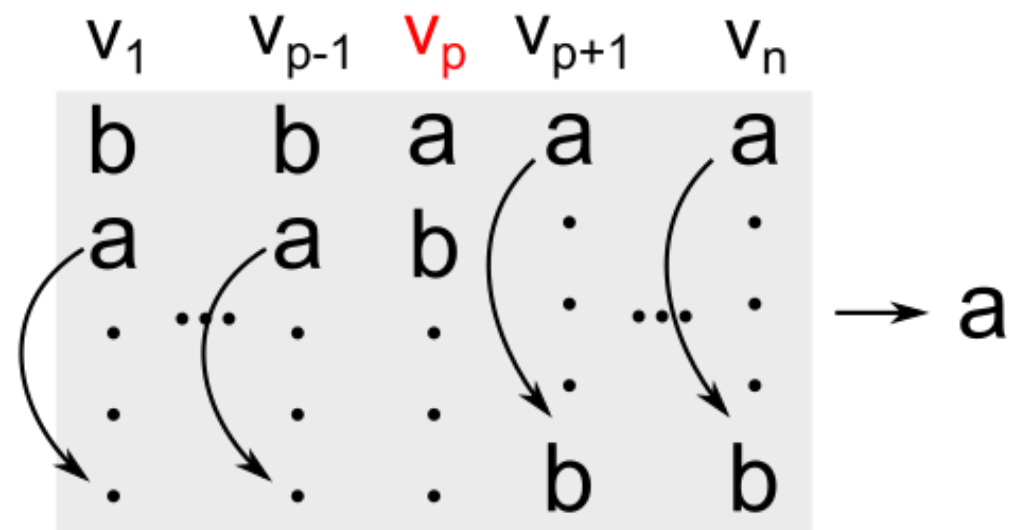


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candidate)

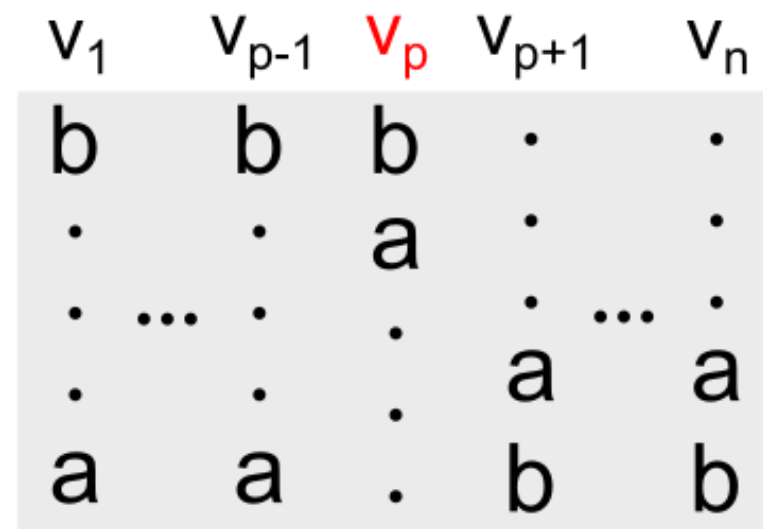
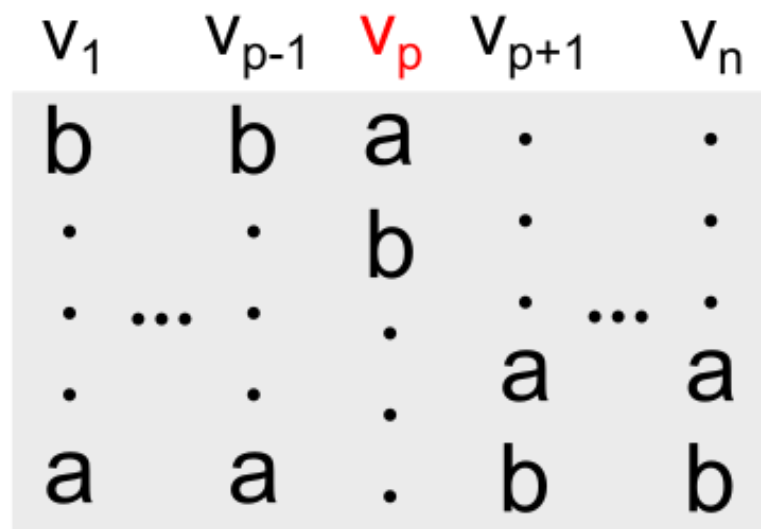


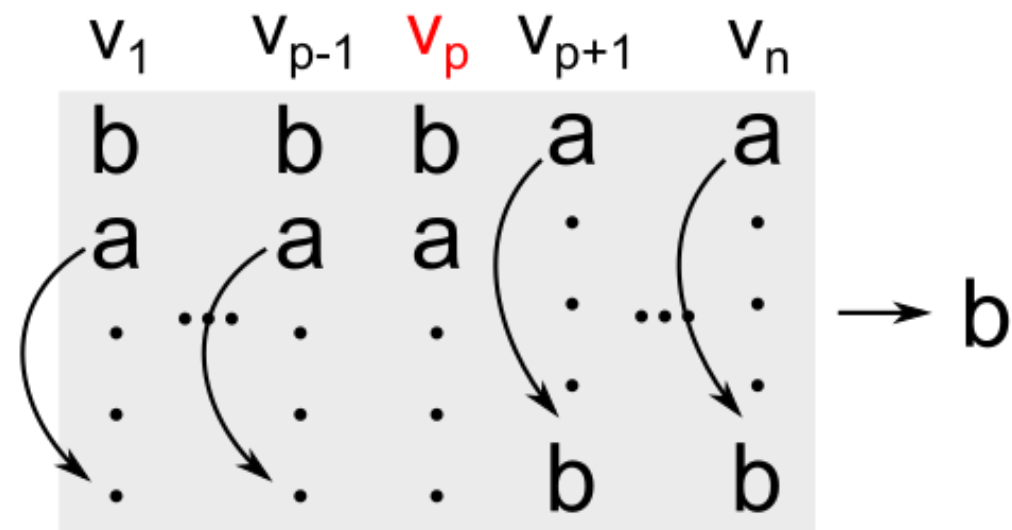
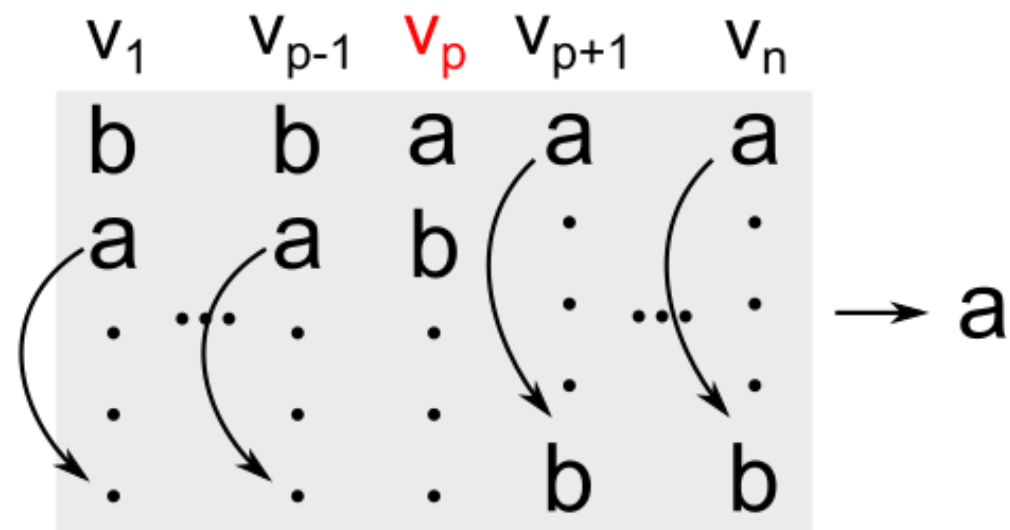


Push "a" to last for v_1, \dots, v_{p-1} and second last for v_{p+1}, \dots, v_n

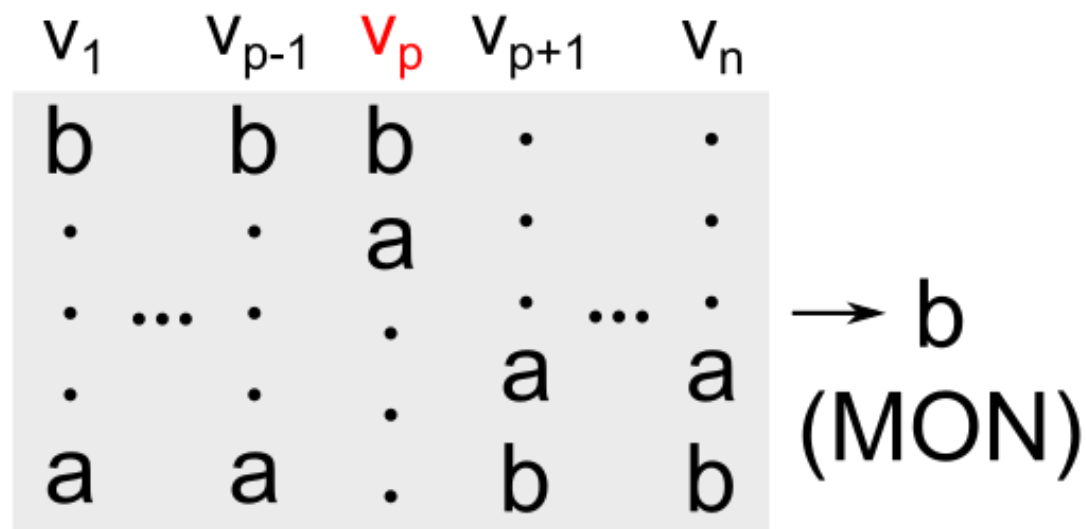
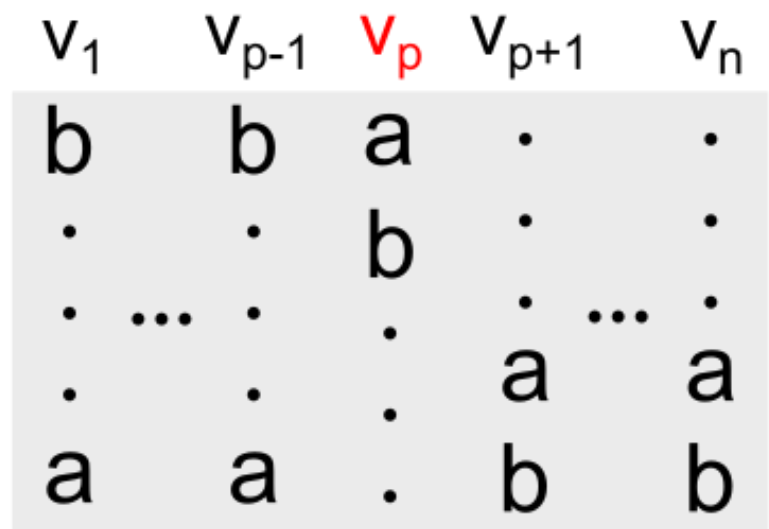


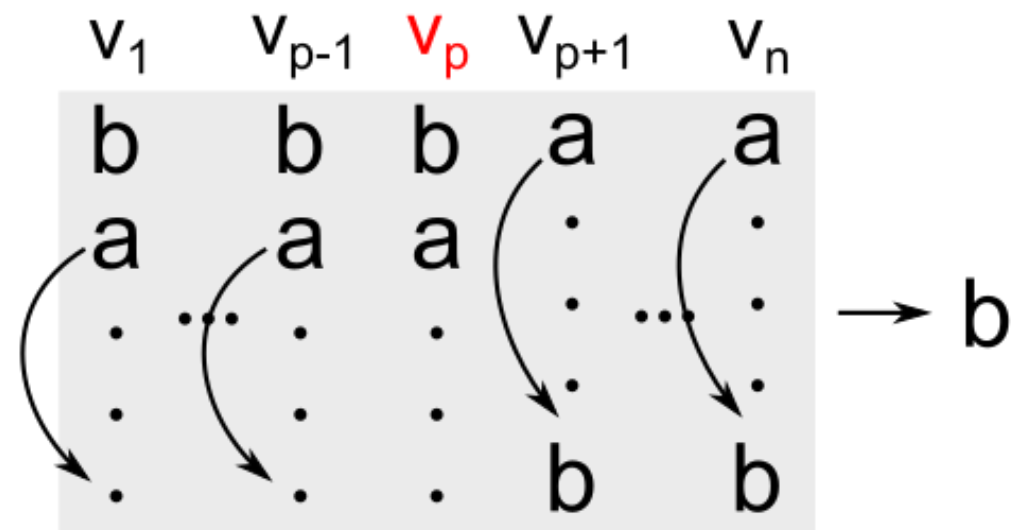
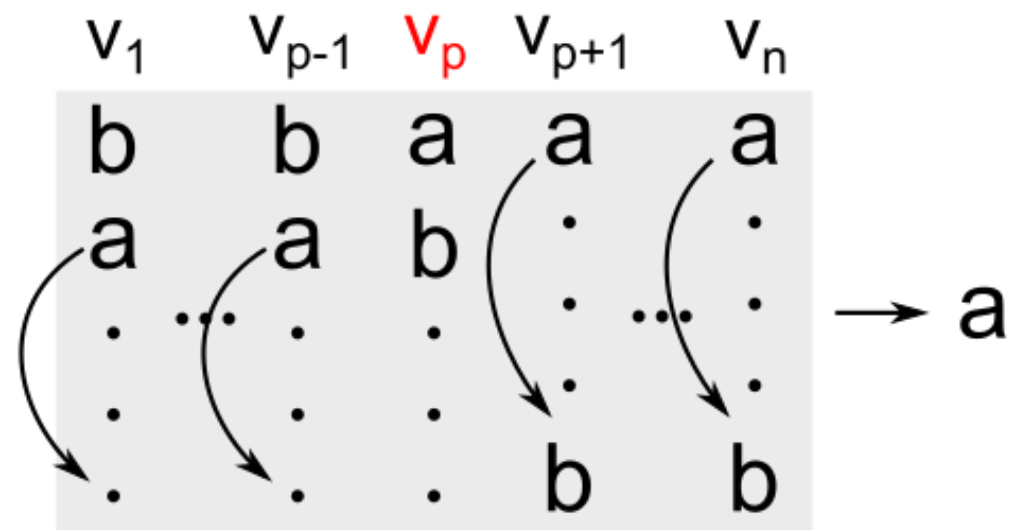
Push "a" to last for v_1, \dots, v_{p-1} and second last for v_{p+1}, \dots, v_n



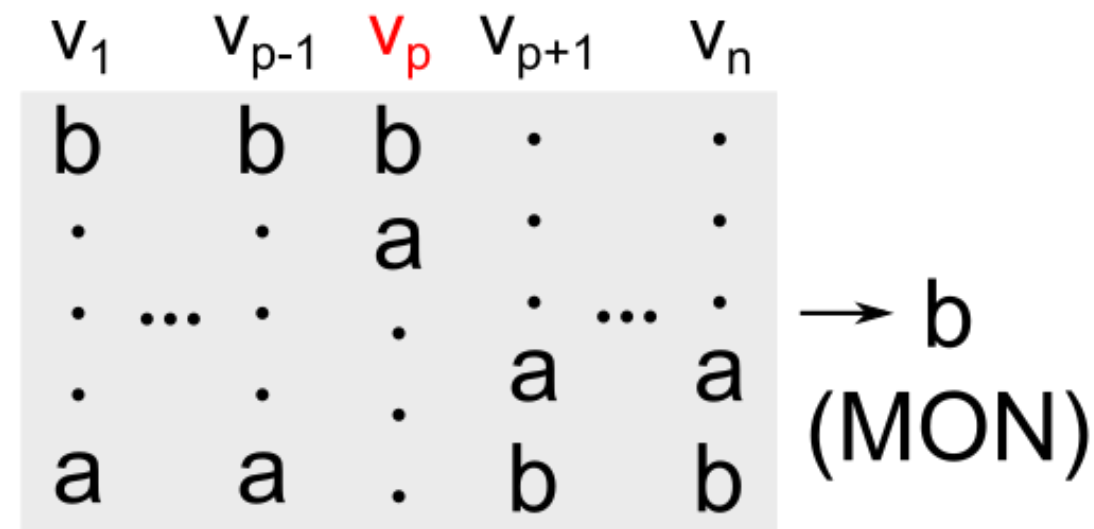
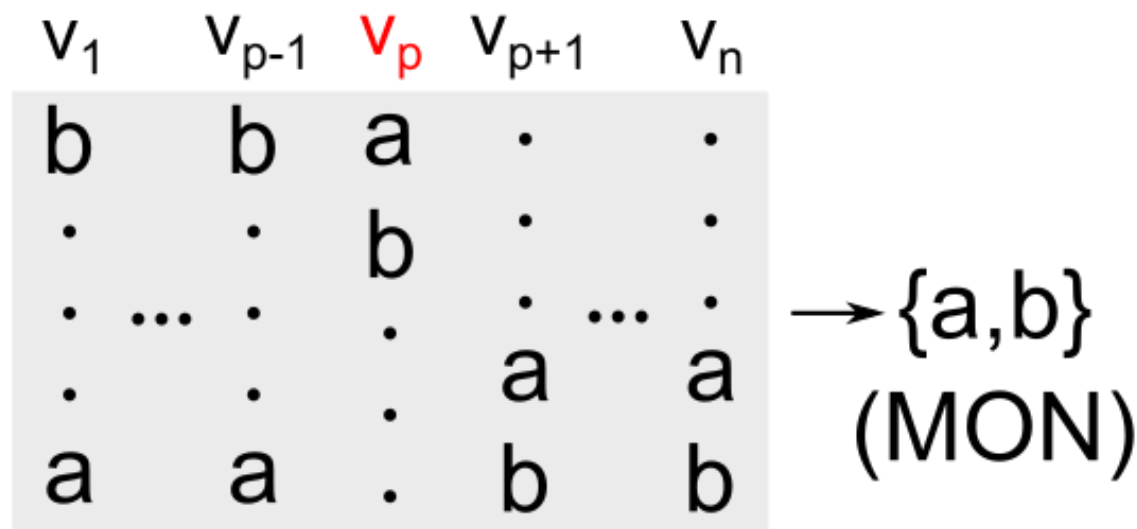


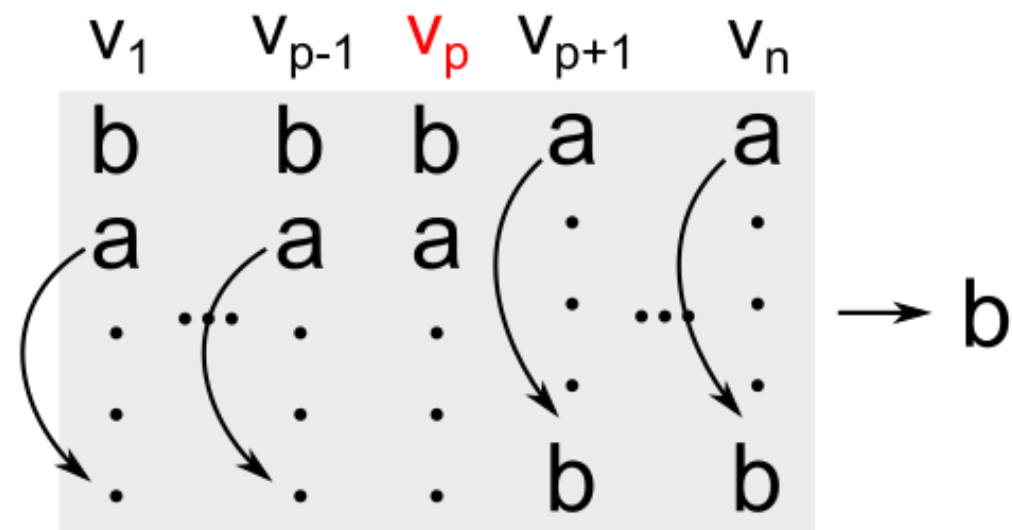
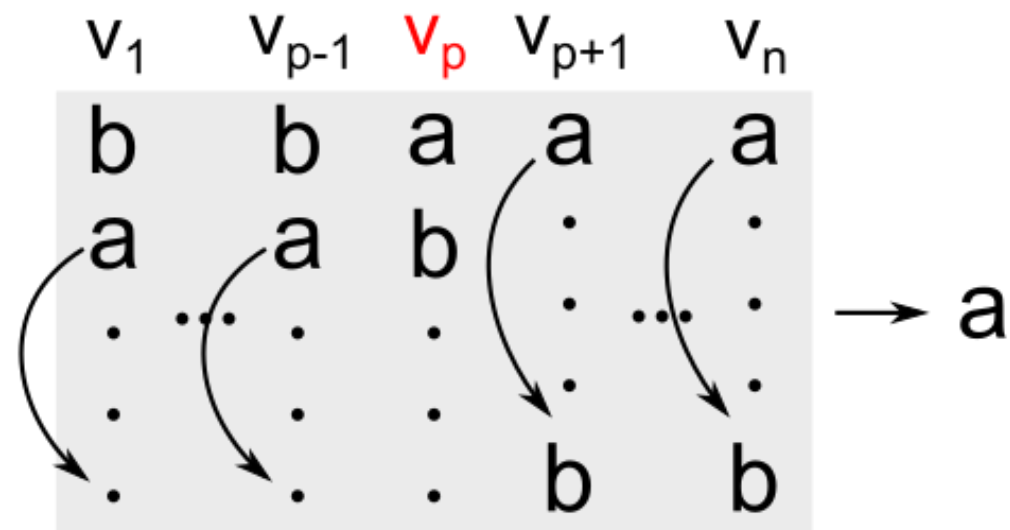
Push "a" to last for v_1, \dots, v_{p-1} and second last for v_{p+1}, \dots, v_n



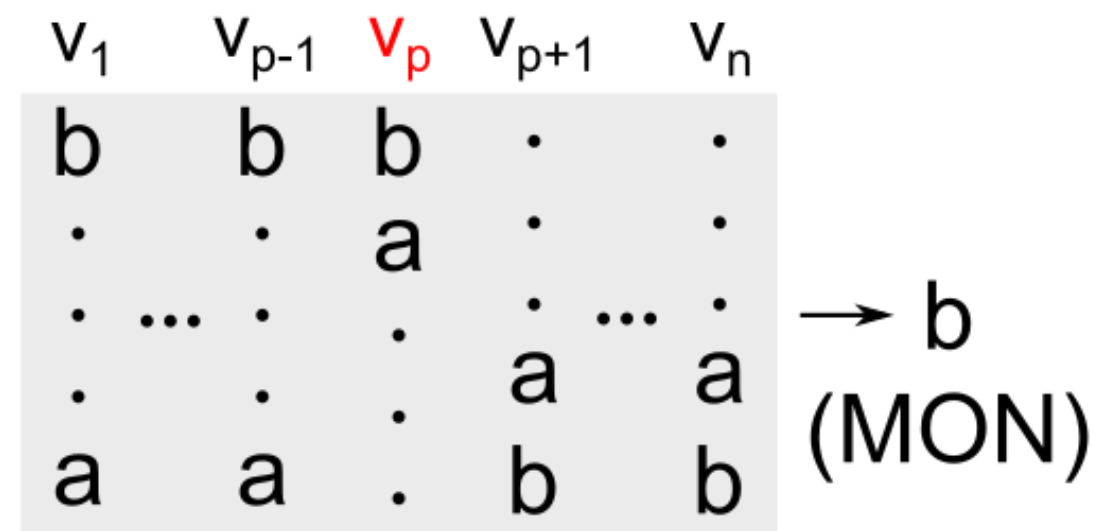
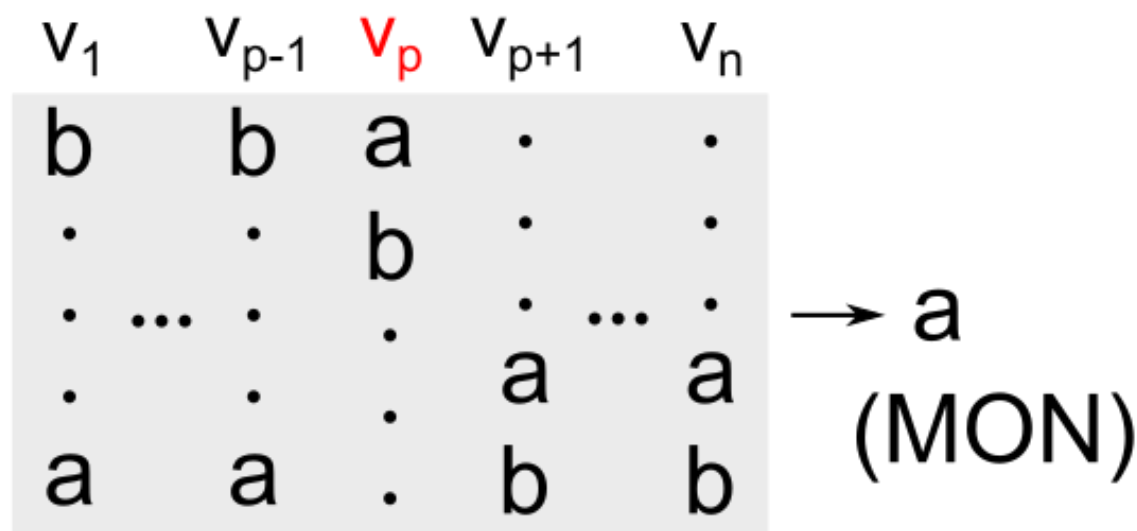


Push "a" to last for v_1, \dots, v_{p-1} and second last for v_{p+1}, \dots, v_n





Push "a" to last for v_1, \dots, v_{p-1} and second last for v_{p+1}, \dots, v_n



v_1	v_{p-1}	v_p	v_{p+1}	v_n
b	b	a	·	·
·	·	b	·	·
·	...	·	·	...
·	·	·	a	a
a	a	·	b	b

→ a
(MON)

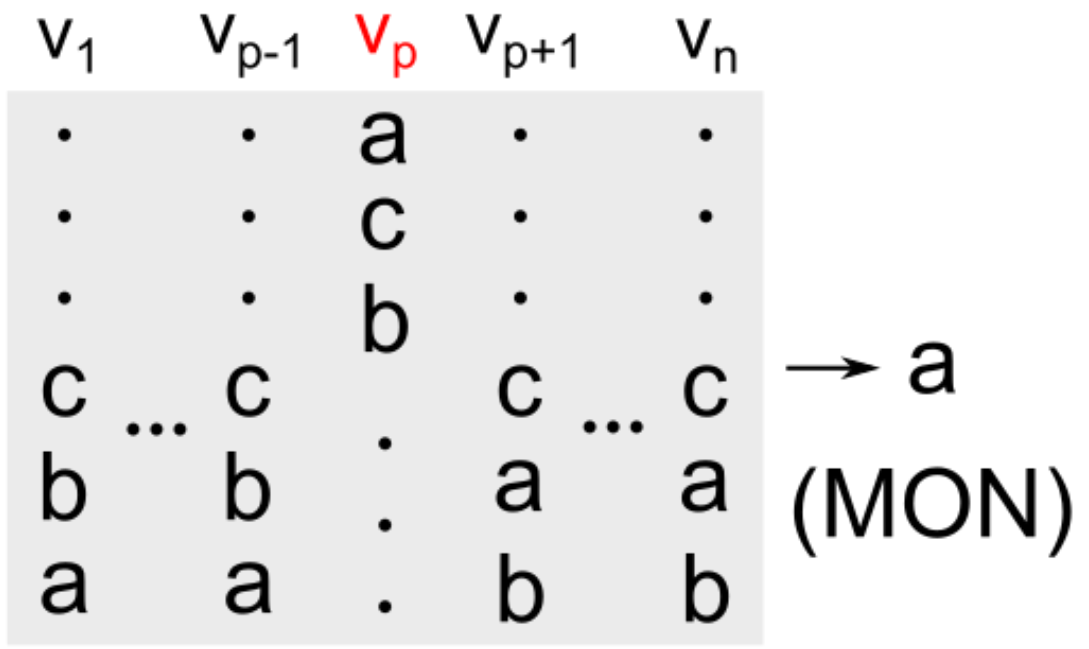
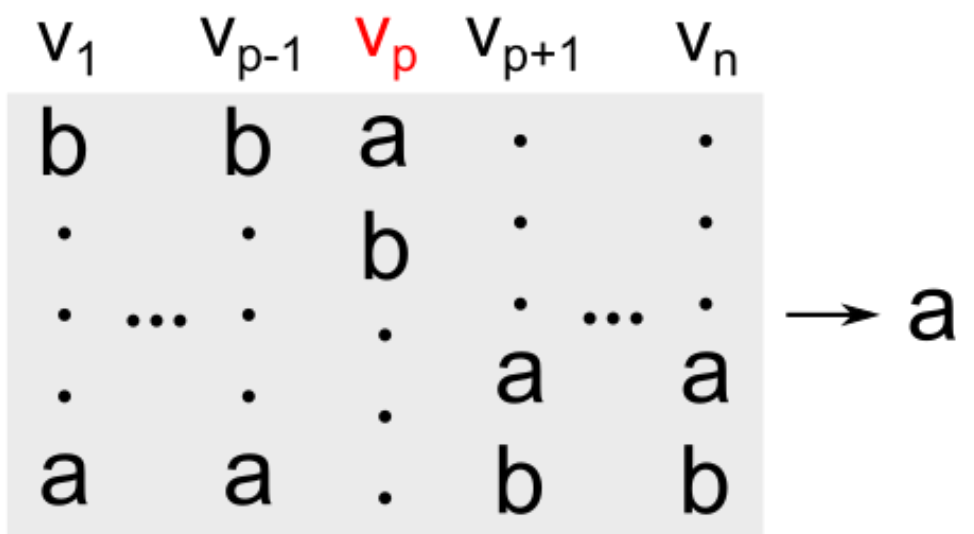
v_1	v_{p-1}	v_p	v_{p+1}	v_n
b	b	a	.	.
.	.	b	.	.
.
.	.	.	a	a
a	a	.	b	b

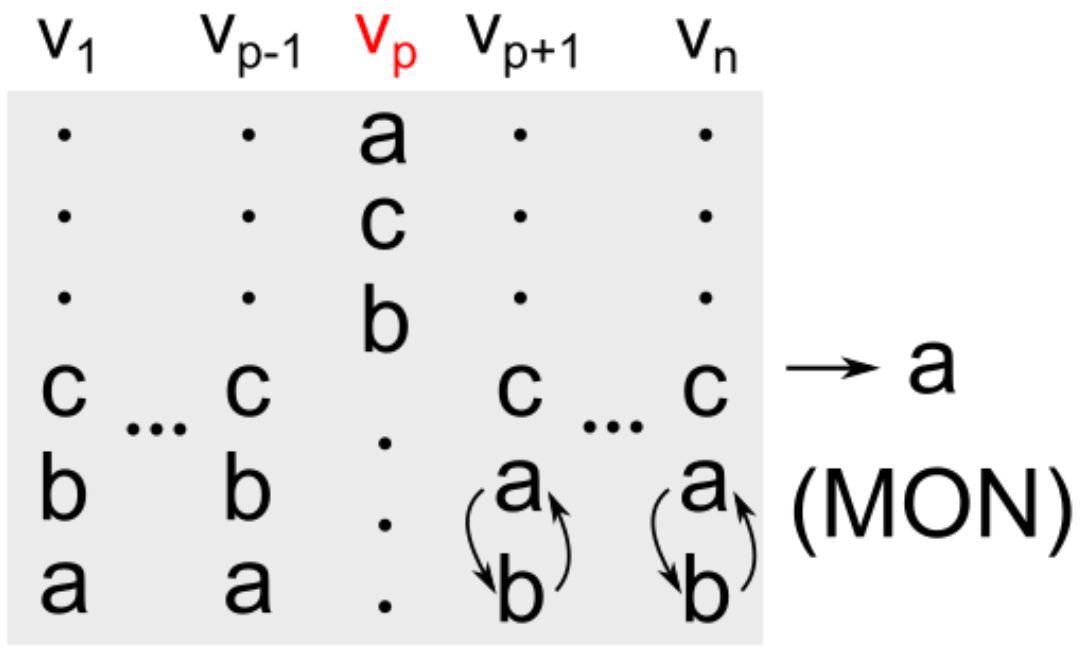
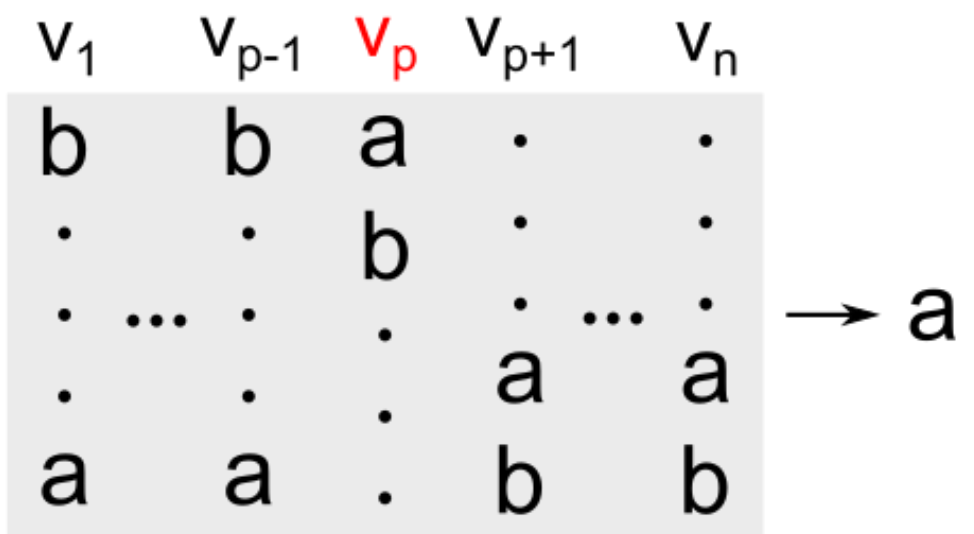
→ a

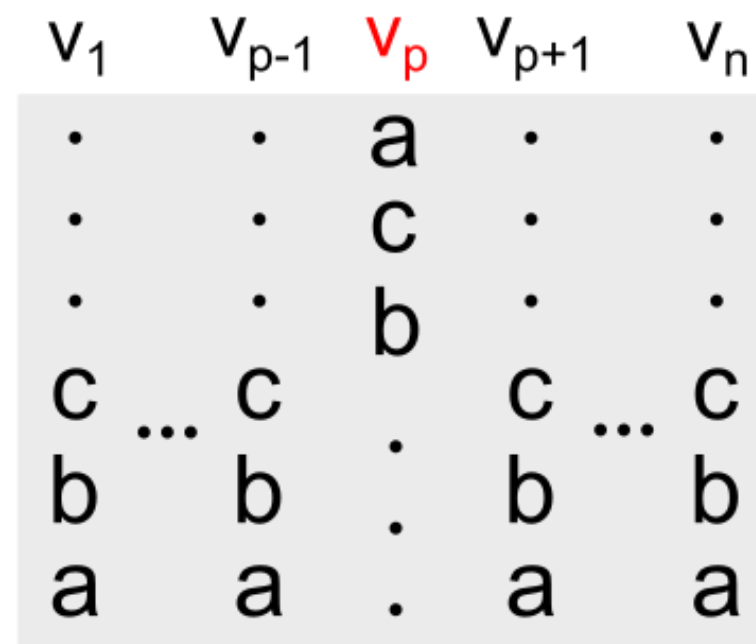
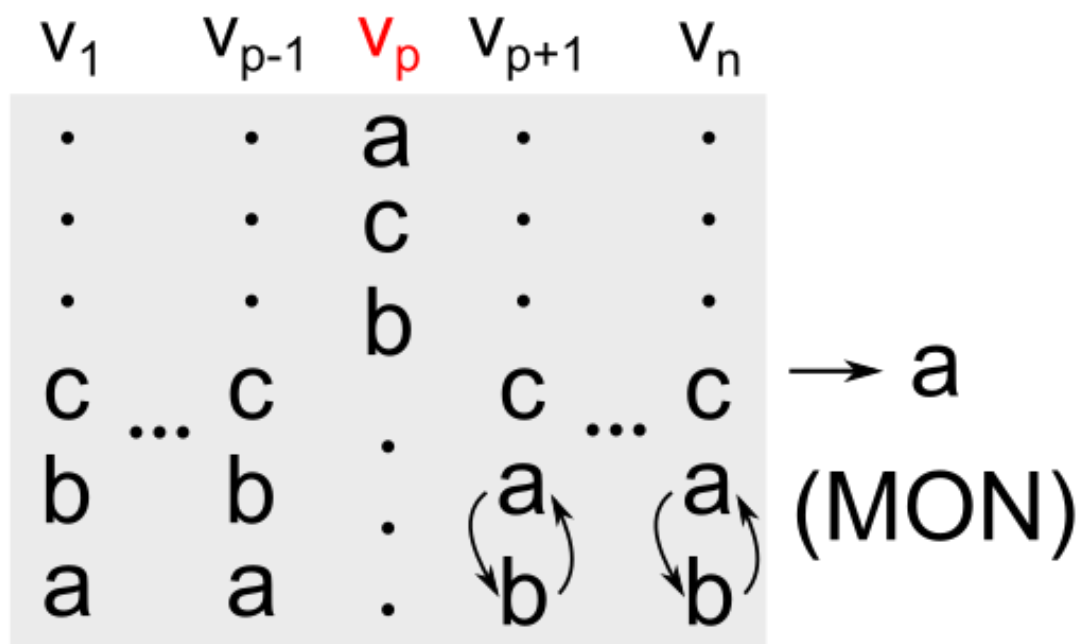
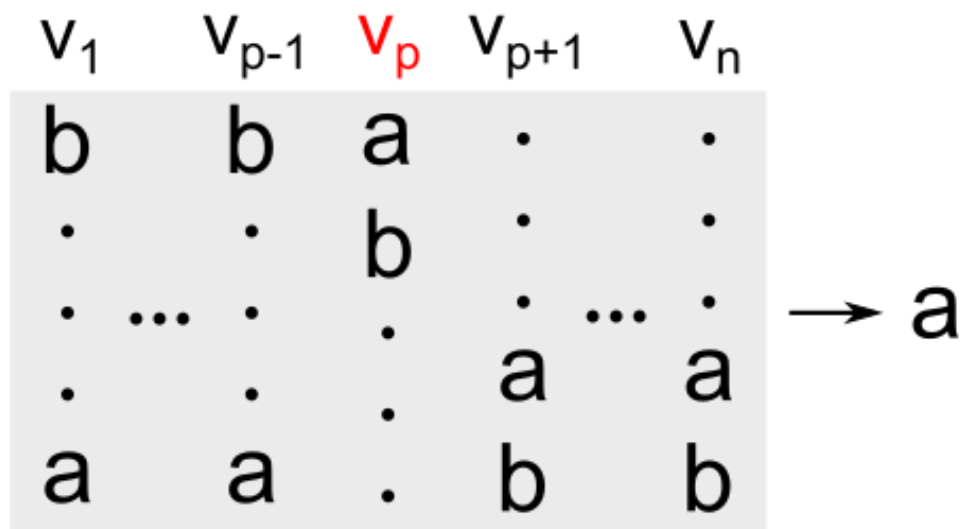
v_1	v_{p-1}	v_p	v_{p+1}	v_n
b	b	a	·	·
·	·	b	·	·
·	...	·	·	...
·	·	·	a	a
a	a	·	b	b

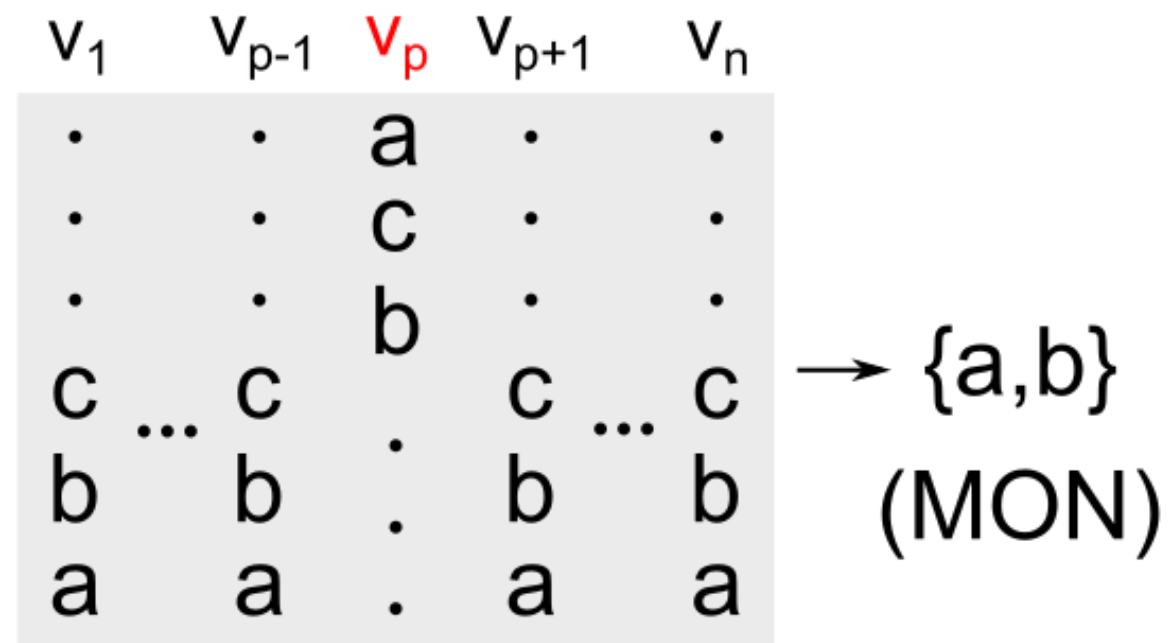
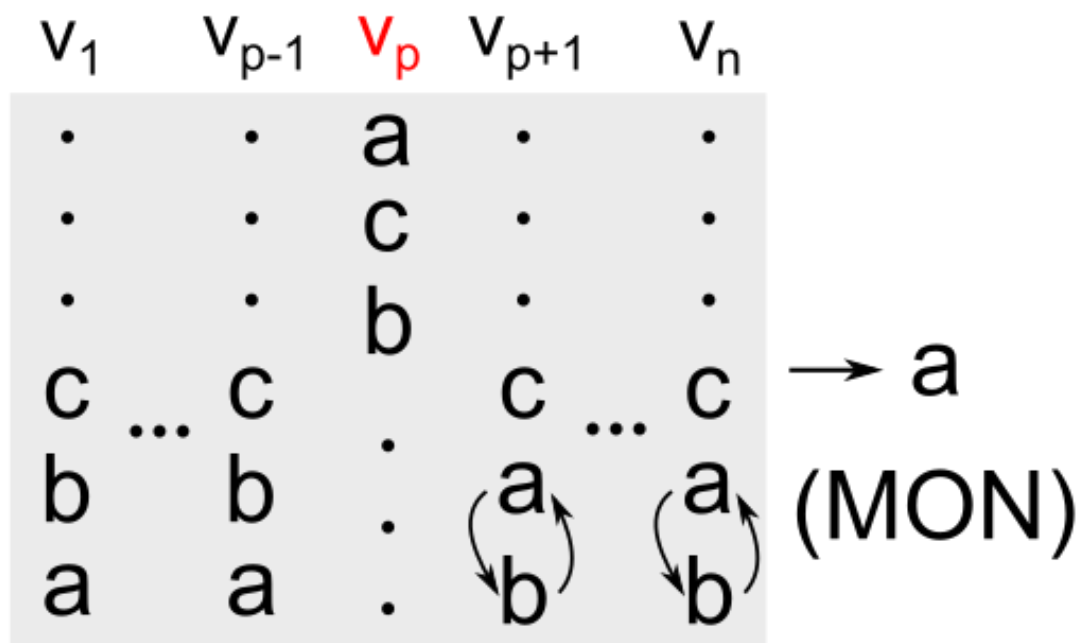
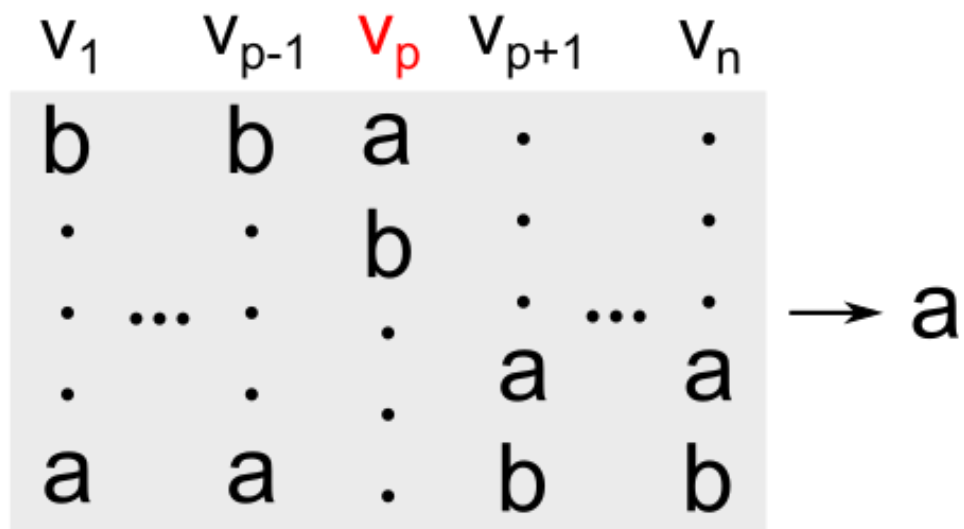
→ a

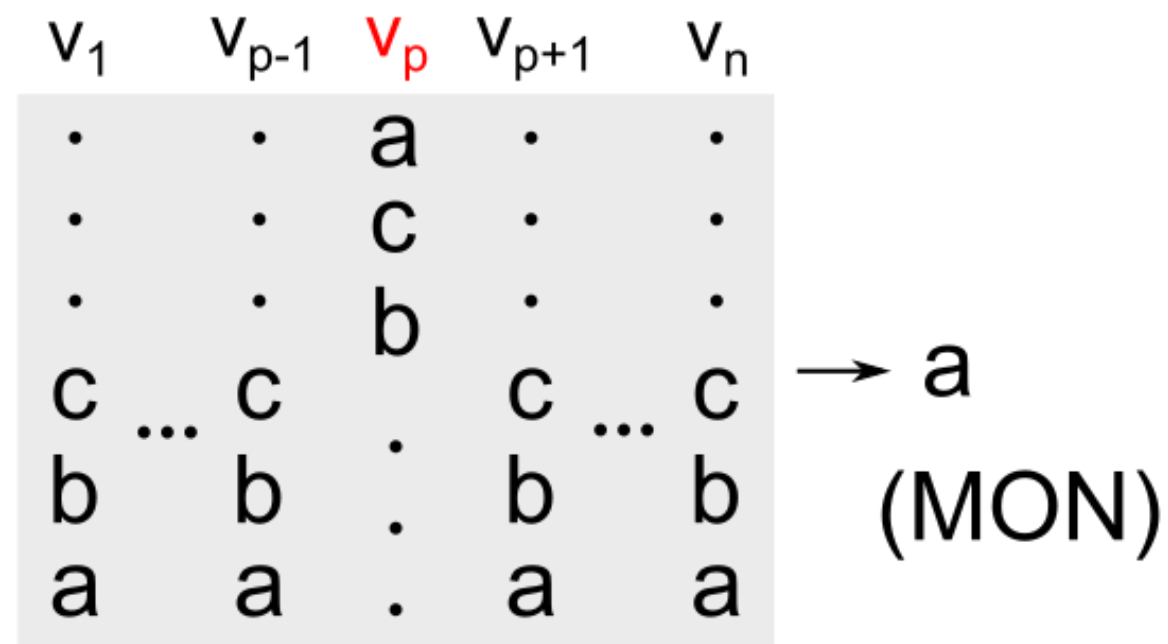
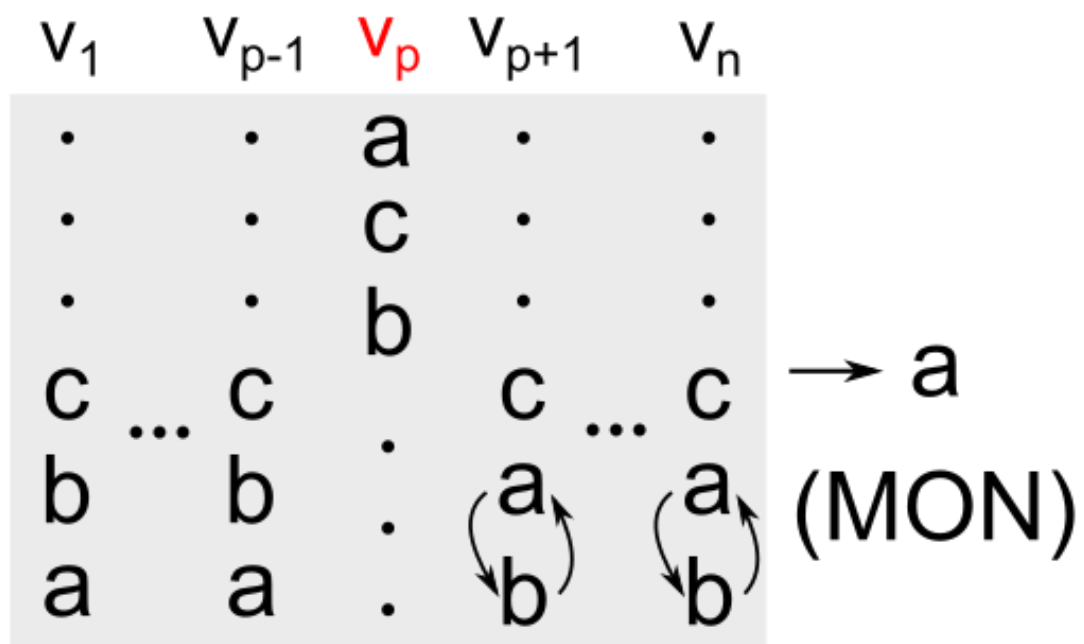
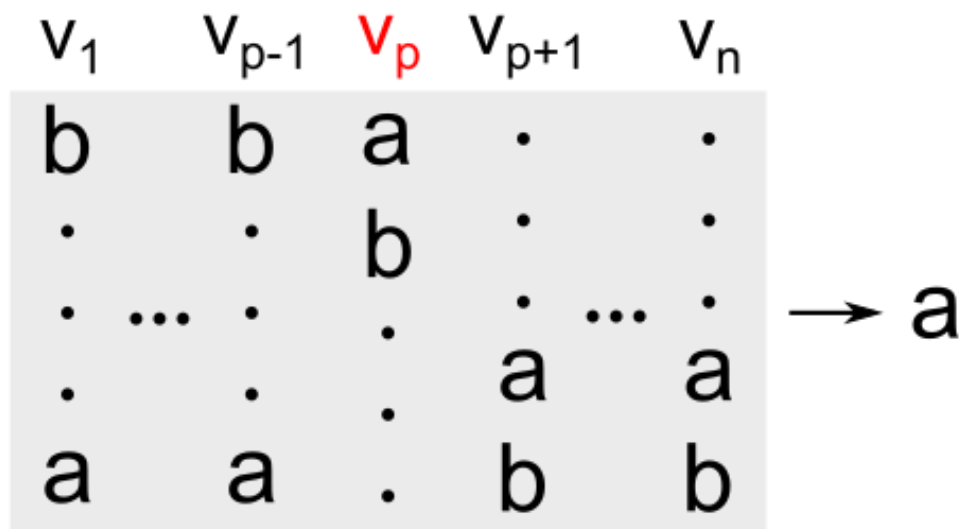
v_1	v_{p-1}	v_p	v_{p+1}	v_n
·	·	a	·	·
·	·	c	·	·
·	·	b	·	·
c	...	c	c	...
b	b	·	a	a
a	a	·	b	b

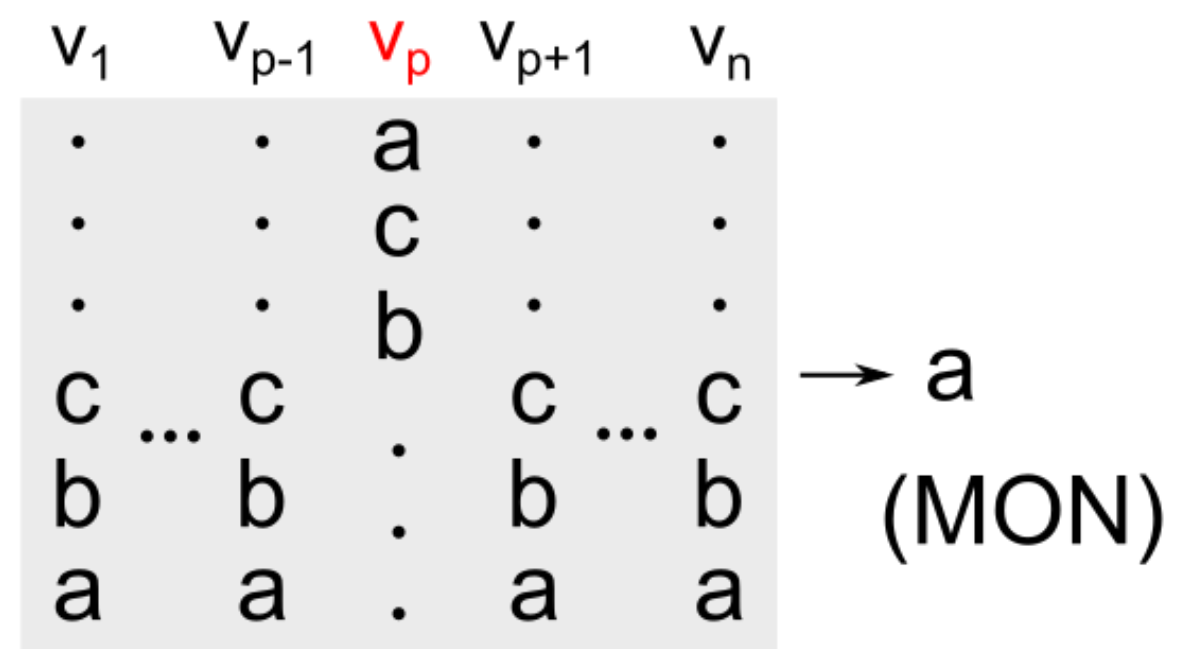


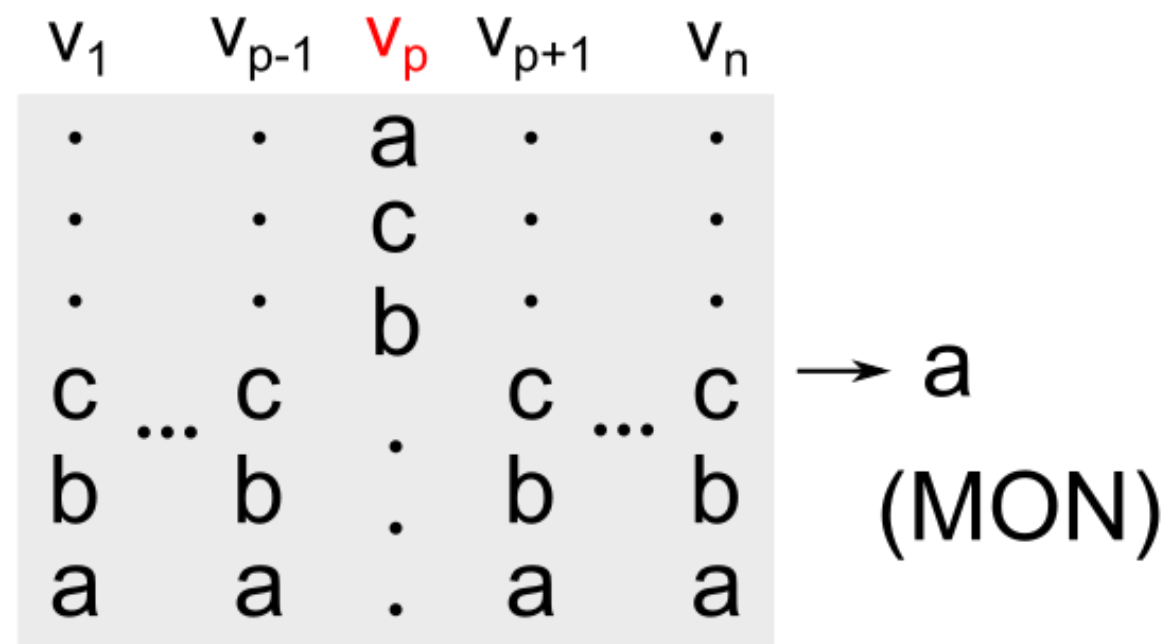
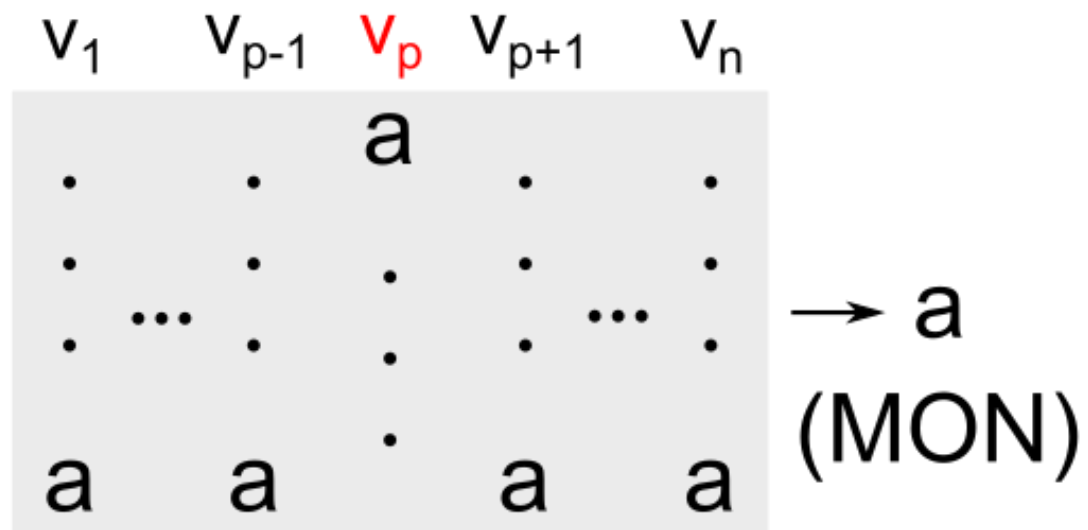






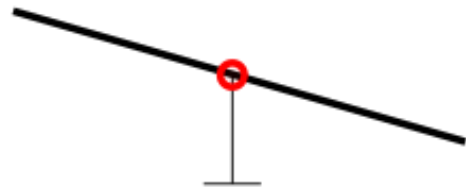






v_1	v_{p-1}	v_p	v_{p+1}	v_n	
·	·	a	·	·	
·	·	·	·	·	
·	...	·	·	...	→ a
a	a	·	a	a	(MON)

UNAN + MON \Rightarrow DICTATOR



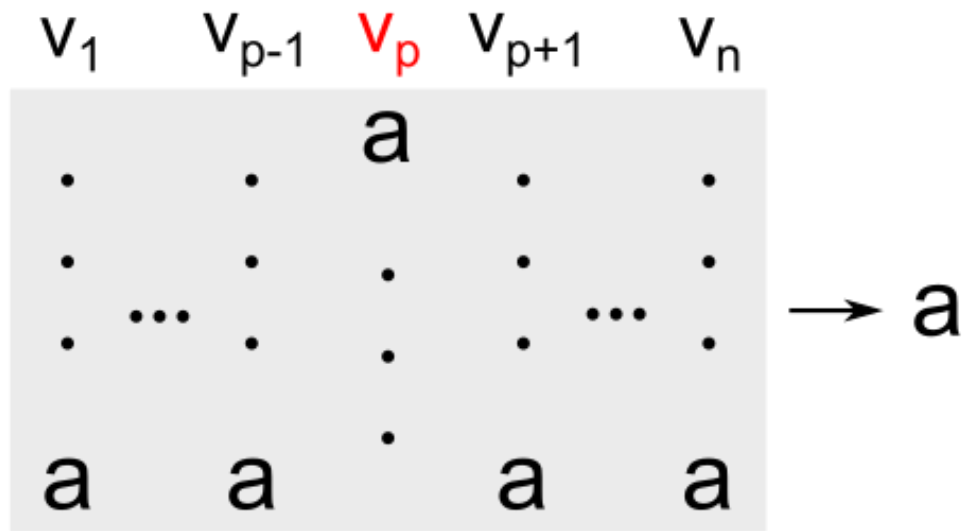
Identify a
"pivotal voter" v_p
for the pair $\{a, b\}$

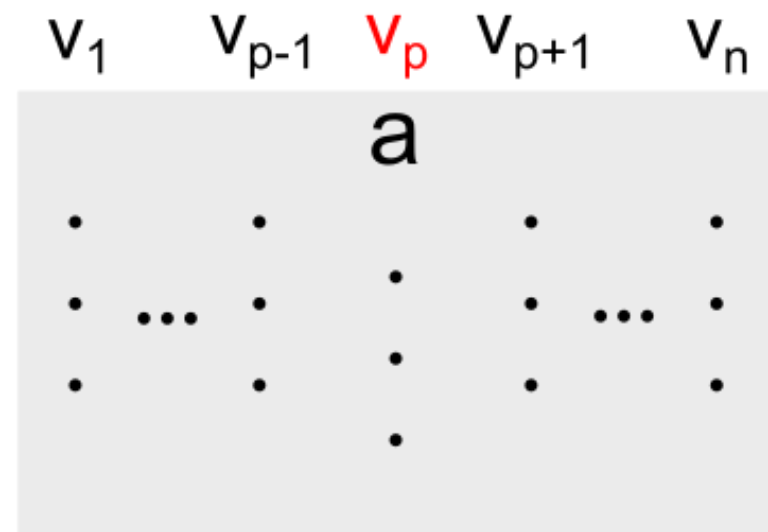
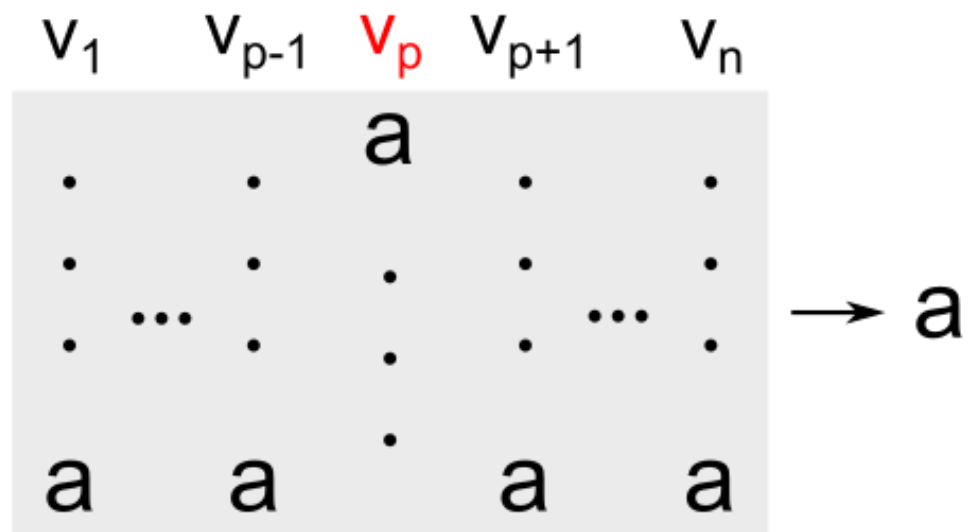
.	.	a	.	.
.
.
a	a	.	a	a

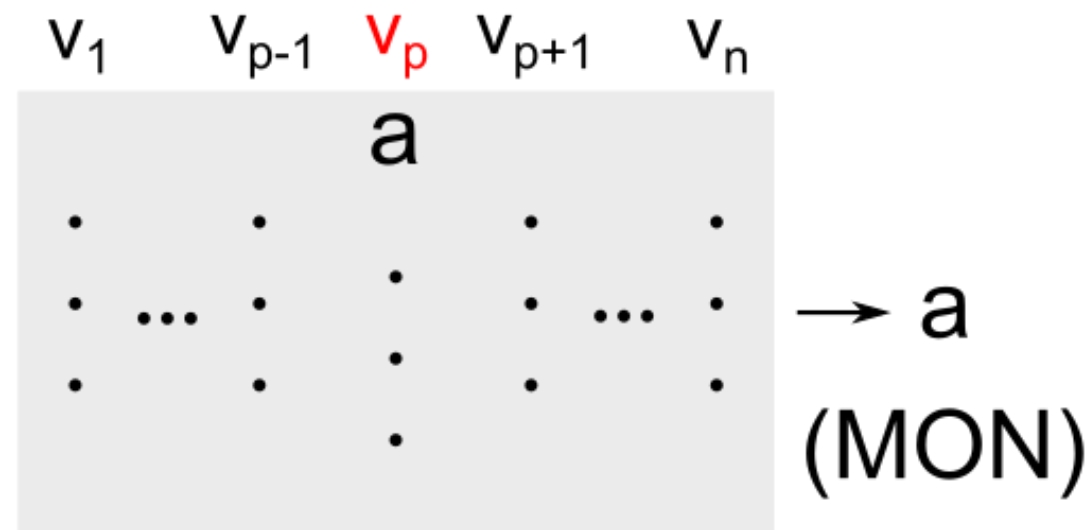
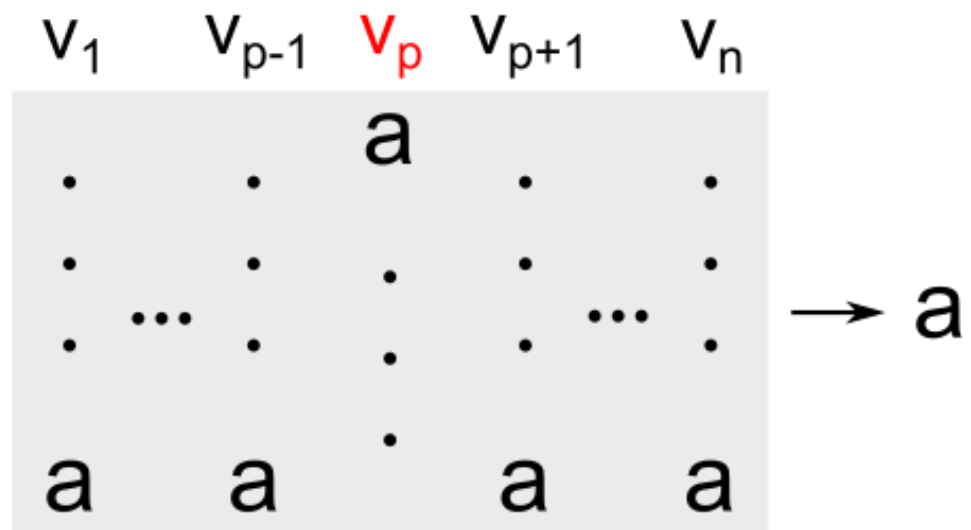
Show that "a" wins
if v_p likes it
even if everyone
else ranks it last

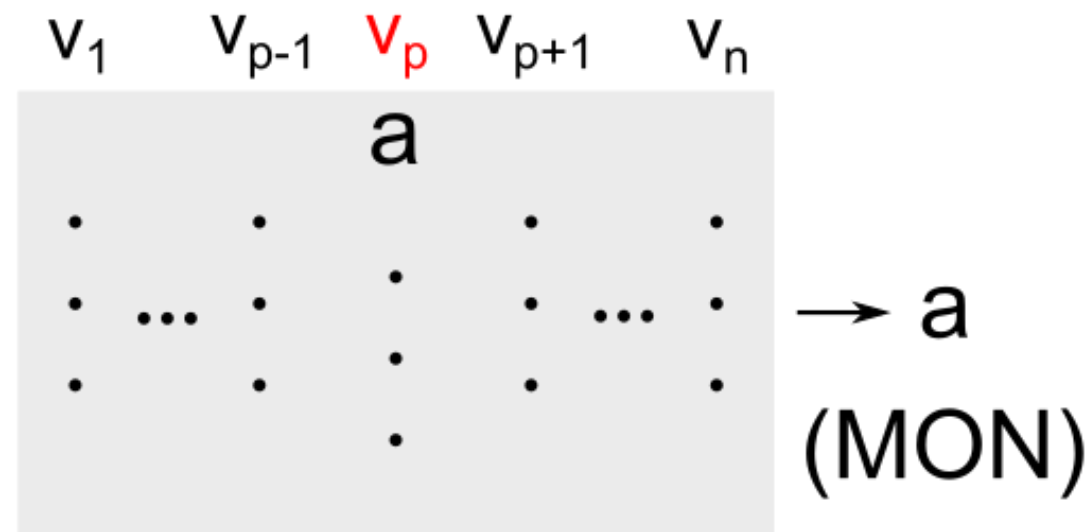
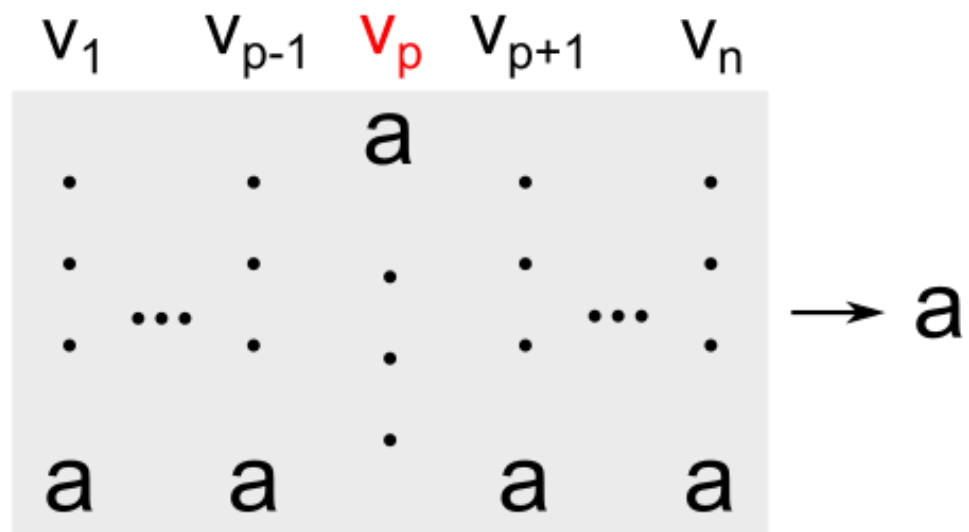


v_p is the
dictator for "a"
(and every other
candidate)

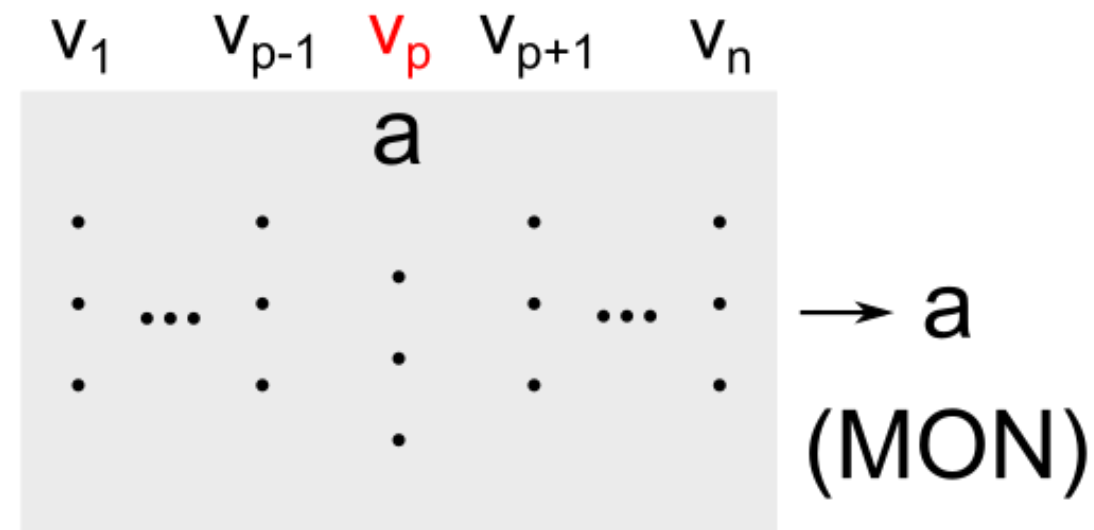
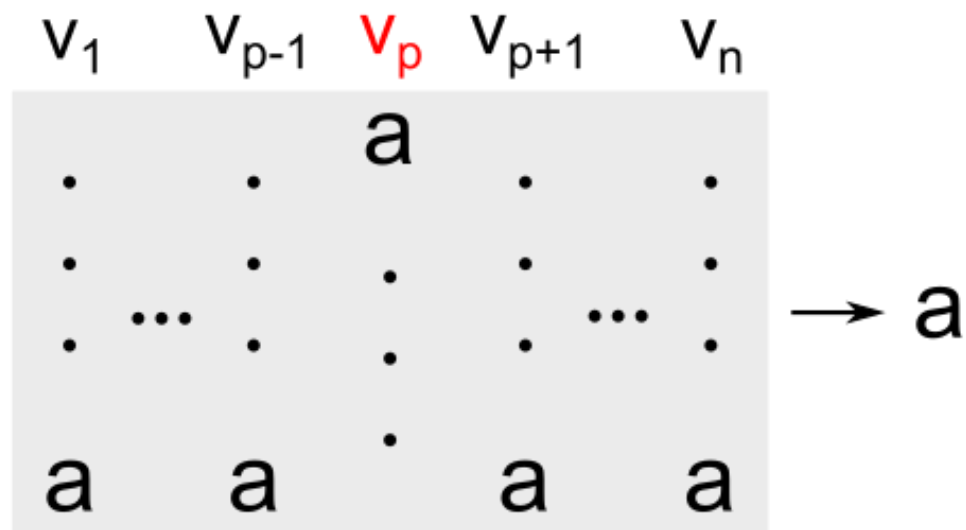






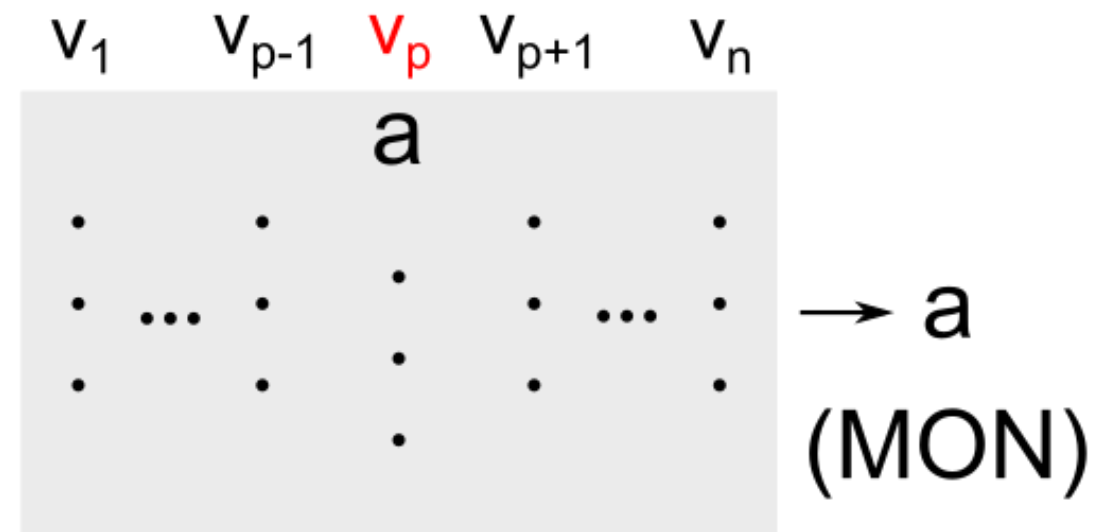
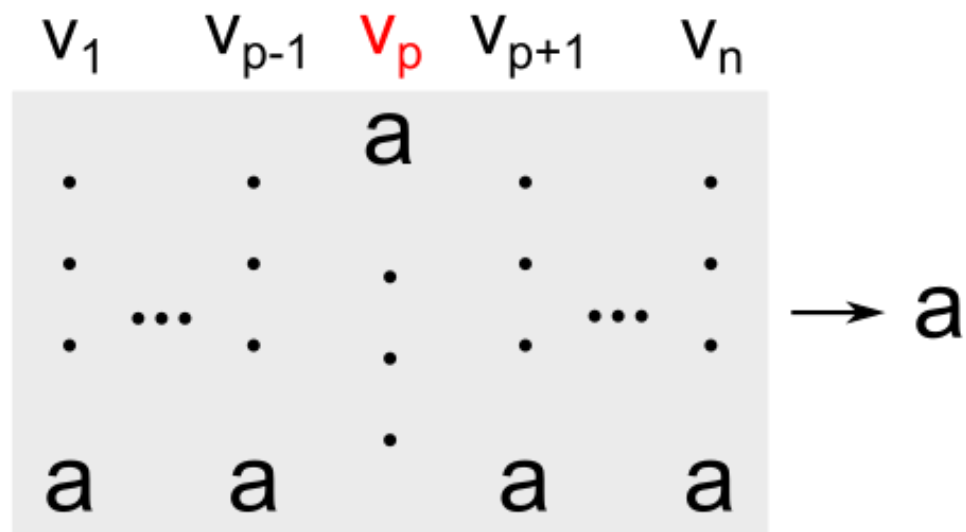


v_p is the dictator for "a"



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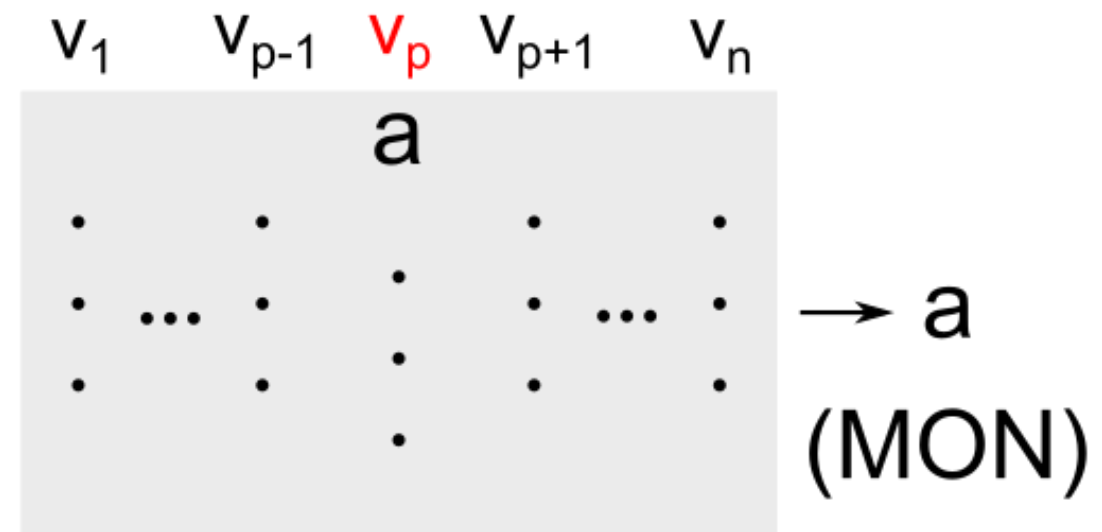
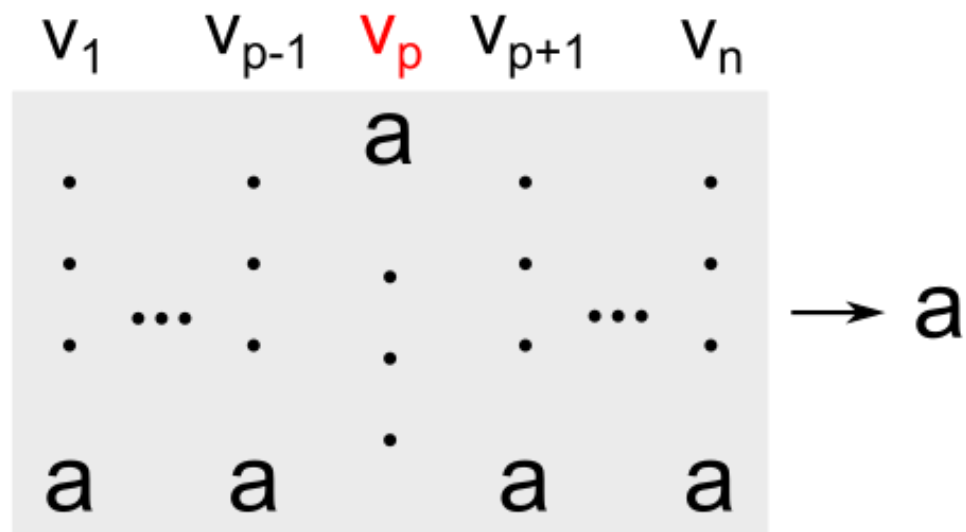
Since the choice of "a" was arbitrary, *every* candidate must have a dictatorial voter associated with it.



v_p is the dictator for "a"

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But there can't be distinct dictators for different candidates.

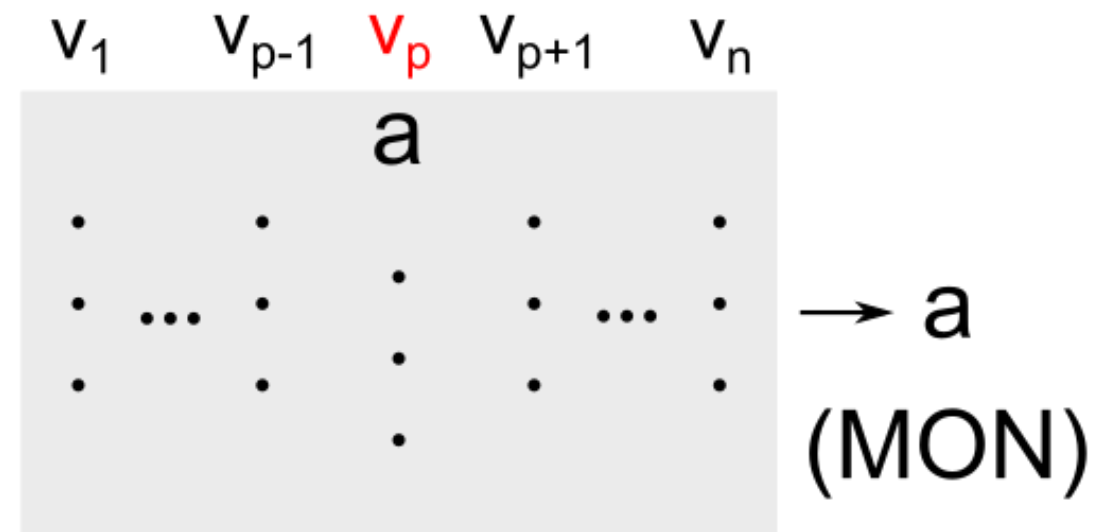
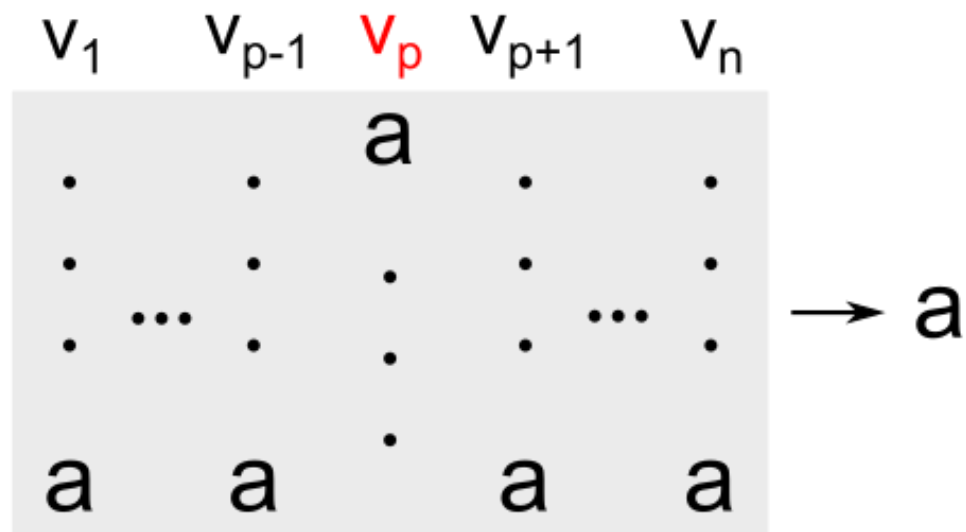


v_p is the dictator for "a"

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So, v_p is the dictator for all candidates.



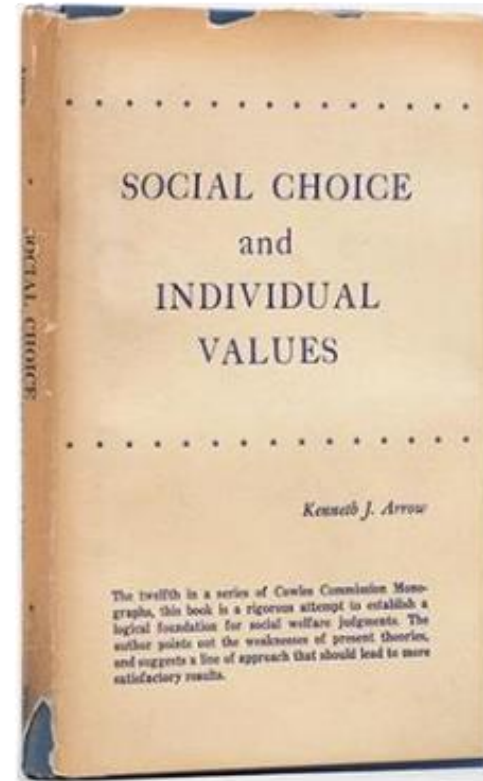
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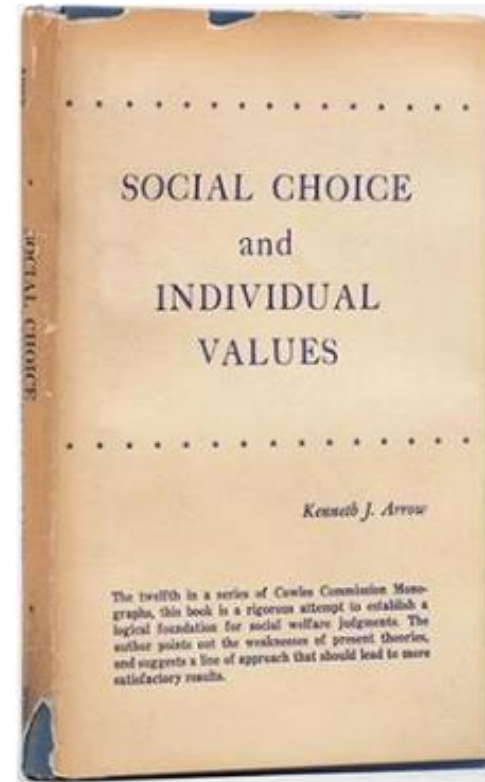
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The use of axiomatic approach in voting goes back much further.

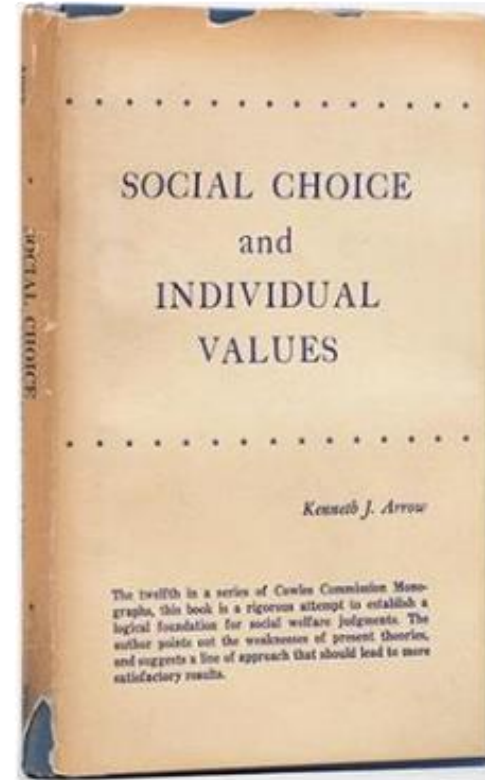


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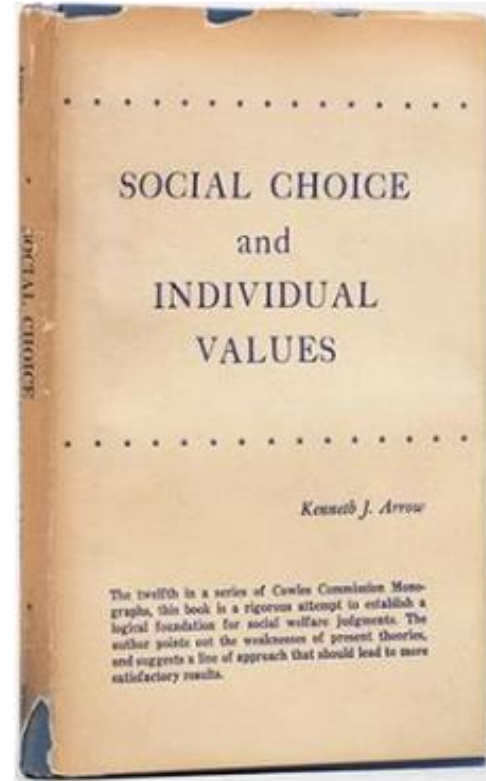
1951 PhD thesis of Kenneth Arrow

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1951 PhD thesis of Kenneth Arrow

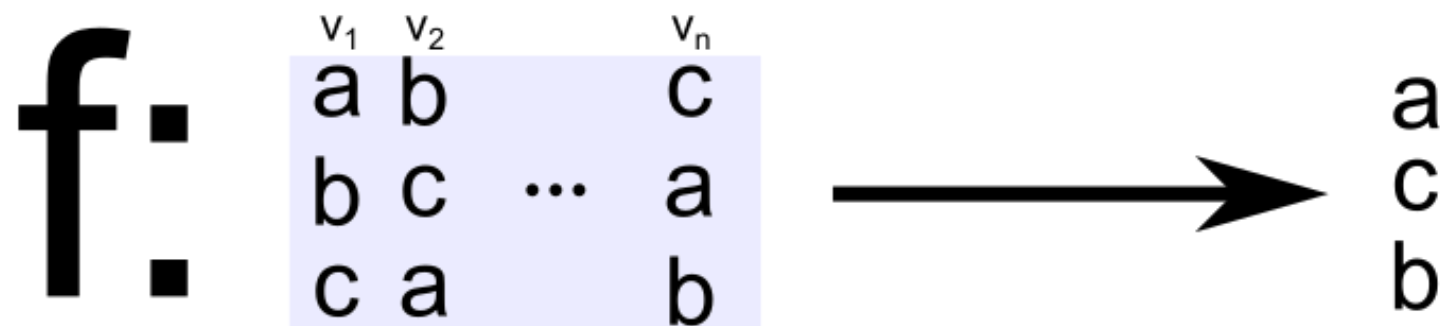
The use of axiomatic approach in voting goes back much further.



1951 PhD thesis of Kenneth Arrow

Voting Rules that Output Rankings

A mapping from preference profiles to rankings over candidates.



(also known as a *social welfare function* or SWF)

UNANIMOUS

If all voters prefer "a" over "b", then so does the SWF.

$$f\left(\begin{array}{c|c|c|c} v_1 & v_2 & & v_n \\ \hline \cdot & a & & \cdot \\ a & b & \dots & a \\ \cdot & \cdot & & b \\ \cdot & \cdot & & \cdot \\ b & \cdot & & \cdot \end{array}\right) = \begin{array}{c} \cdot \\ a \\ \cdot \\ \cdot \\ b \\ \cdot \\ \cdot \end{array}$$

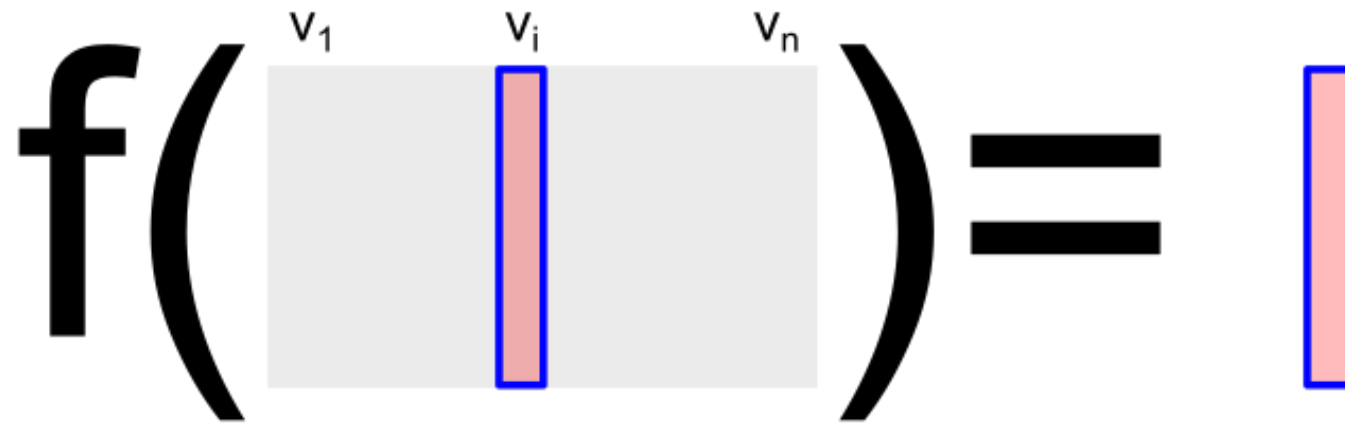
INDEPENDENCE OF IRRELEVANT ALTERNATIVES

If the relative ranking of "a" and "b" in each vote is unchanged, then their relative ranking in the SWF outcome is also unchanged.

$$f\left(\begin{array}{c|c|c|c} v_1 & v_2 & \dots & v_n \\ \hline \circ & b & & \\ a & \circ & \dots & a \\ b & a & & b \\ & & & \circ \end{array}\right) = \begin{array}{c} \cdot \\ a \\ \cdot \\ \cdot \\ b \\ \cdot \\ \cdot \end{array}$$

DICTATORSHIP

An SWF that mimics the preferences of a fixed voter on all inputs.

$$f\left(\begin{array}{ccc} v_1 & v_i & v_n \\ \text{[gray box]} & \text{[red box]} & \text{[gray box]} \end{array}\right) = \text{[red box]}$$
The diagram shows a function f in large black font. To its right is a large opening parenthesis. Inside the parenthesis is a gray rectangular area representing a vector of preferences. Above the gray area are three labels: v_1 , v_i , and v_n . A vertical red rectangle with a blue border is positioned in the center of the gray area, directly under the v_i label. To the right of the parenthesis is an equals sign, followed by another vertical red rectangle with a blue border, identical in size and position to the one inside the parenthesis.

DICTATORSHIP

An SWF that mimics the preferences of a fixed voter on all inputs.

$$f\left(\begin{array}{ccc} v_1 & v_i & v_n \\ \text{[gray box]} & \text{[red box]} & \text{[gray box]} \end{array}\right) = \text{[red box]}$$

A dictatorship is unanimous and IIA.



[Arrow'51]

With three or more candidates,
an SWF is **unanimous** and **IIA**
if and only if it is a **dictatorship**.



[Arrow'51]

With three or more candidates,
an SWF is **unanimous** and **IIA**
if and only if it is a **dictatorship**.

Proof almost identical to the one we saw for Gibb-Satt thm.

Sort: ↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
Criterion	Majority	Maj. loser	Mutual maj.	Condorcet	Cond. loser	Smith/ISDA	LIIA	IIA	Cloneproof	Monotone	Consistency	Participation	Reversal symmetry	Polytime/resolvable		Summable	Later-no-		No favorite betrayal	Ballot type	Ranks	
														Harm	Help		=	>2				
Method																						
Approval	Rated ^[a]	No	No	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes ^[e]	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes ^[f]	Yes	Approvals	Yes	No
Borda count	No	Yes	No	No ^[b]	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	Yes	Yes
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No ^[b]	Teams, crowds	Yes	No ^[b]	No ^[b]	Yes	O(N ²)	No	O(N ²)	No ^[b]	No	No ^[b]	Ranking	Yes	Yes
IRV (AV)	Yes	Yes	Yes	No ^[b]	Yes	No ^[b]	No	No	Yes	No	No	No	No	O(N ²)	Yes ^[g]	O(N) ^[h]	Yes	Yes	No	Ranking	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No ^[b]	Spoilers	Yes	No ^[b] _[i]	No ^[b]	Yes	O(N [!])	Yes	O(N ²) ^[j]	No ^[b]	No	No ^[b]	Ranking	Yes	Yes
Highest median/Majority judgment^[k]	Rated ^[l]	Yes ^[m]	No ^[n]	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes	Yes	No ^[o]	No ^[p]	Depends ^[q]	O(N)	Yes	O(N) ^[r]	No ^[s]	Yes	Yes	Scores ^[t]	Yes	Yes
Minimax	Yes	No	No	Yes ^[u]	No	No	No	No ^[b]	Spoilers	Yes	No ^[b]	No ^[b]	No	O(N ²)	Yes	O(N ²)	No ^{[b][u]}	No	No ^[b]	Ranking	Yes	Yes
Plurality/FPTP	Yes	No	No	No ^[b]	No	No ^[b]	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A ^[v]	N/A ^[v]	No	Single mark	N/A	No
Score voting	No	No	No	No ^{[b][c]}	No	No ^[b]	Yes	Yes ^[d]	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No ^[b]	Yes	Yes	No ^[b]	No ^{[p][b]}	Yes	O(N ³)	Yes	O(N ²)	No ^[b]	No	No ^{[p][b]}	Ranking	Yes	Yes
Runoff voting	Yes	Yes	No	No ^[b]	Yes	No ^[b]	No	No	Spoilers	No	No	No	No	O(N) ^[w]	Yes	O(N) ^[w]	Yes	Yes ^[x]	No	Single mark	N/A	No ^[y]
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No ^[b]	Yes	Yes	No ^[b]	No ^{[p][b]}	Yes	O(N ³)	Yes	O(N ²)	No ^[b]	No	No ^{[p][b]}	Ranking	Yes	Yes
STAR voting	No ^[z]	Yes	No ^[aa]	No ^{[b][c]}	Yes	No ^[b]	No	No	No	Yes	No	No	Depends ^[ab]	O(N)	Yes	O(N ²)	No	No	No ^[ac]	Scores	Yes	Yes
Sortition, arbitrary winner^[ad]	No	No	No	No ^[b]	No	No ^[b]	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A
Random ballot^[ae]	No	No	No	No ^[b]	No	No ^[b]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No

*"Most systems are not going to work badly all of the time.
All I proved is that all can work badly at times."*



Next Time

Algorithms for Manipulating Voting Rules

Oct 8 (Sat)

References

- A shared proof of Arrow's and Gibbard-Satterthwaite theorems: "Arrow's Theorem and the Gibbard-Satterthwaite Theorem: A Unified Approach" by Philip J. Reny.
<https://www.sciencedirect.com/science/article/pii/S0165176500003323>
- Another nice proof of Arrow's theorem:
https://www.youtube.com/watch?v=QLi_5LCwJ20
- The "big table" of voting rules is from the Wikipedia article "Comparison of Electoral Systems".
- A tribute to Arrow (see the end of the article for a nice anecdote):
<https://www.nytimes.com/2017/02/21/business/economy/kenneth-arrow-dead-nobel-laureate-in-economics.html>

