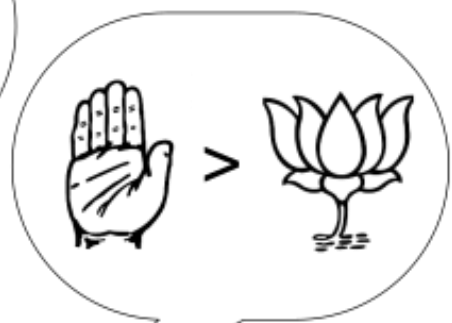
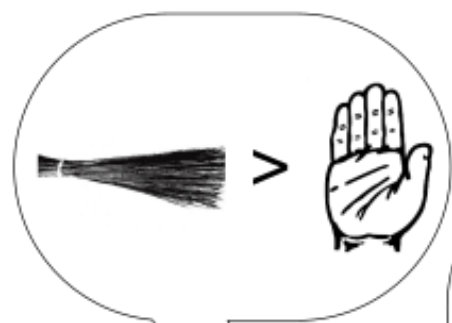


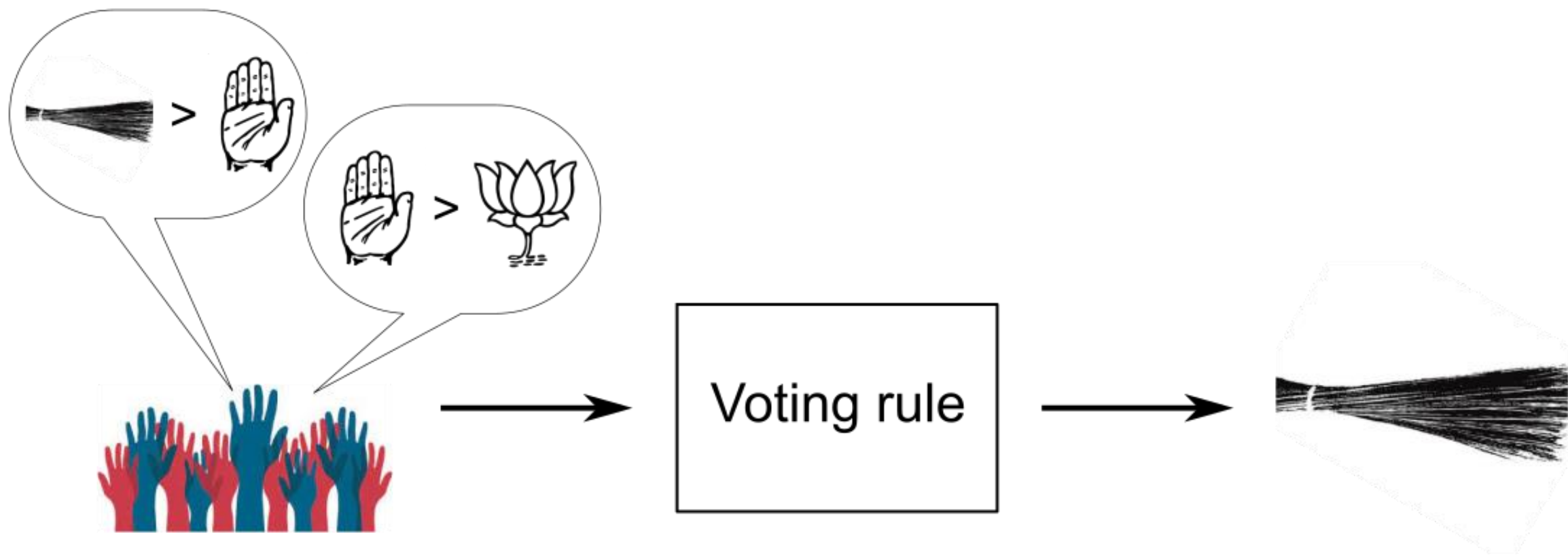
# Lecture 12

## Voting Rules



Voting rule



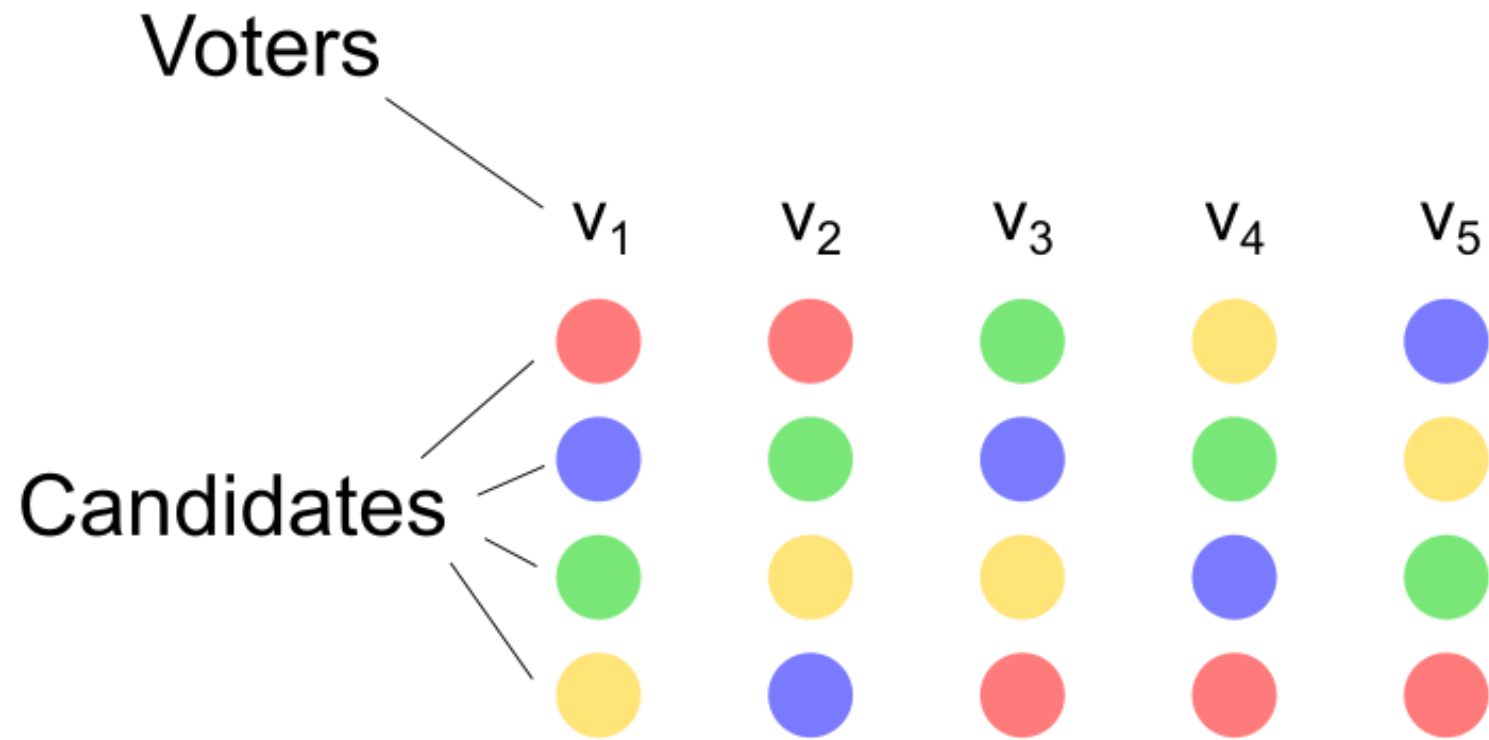


amazon

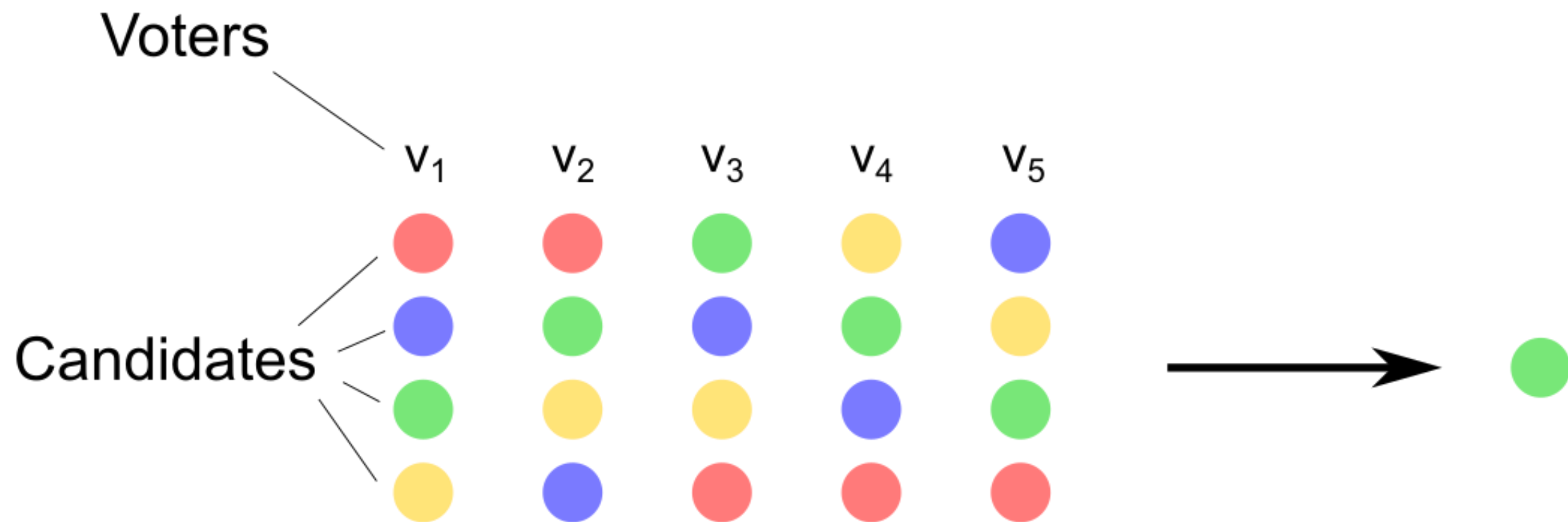


# Voting Setup

# Voting Setup

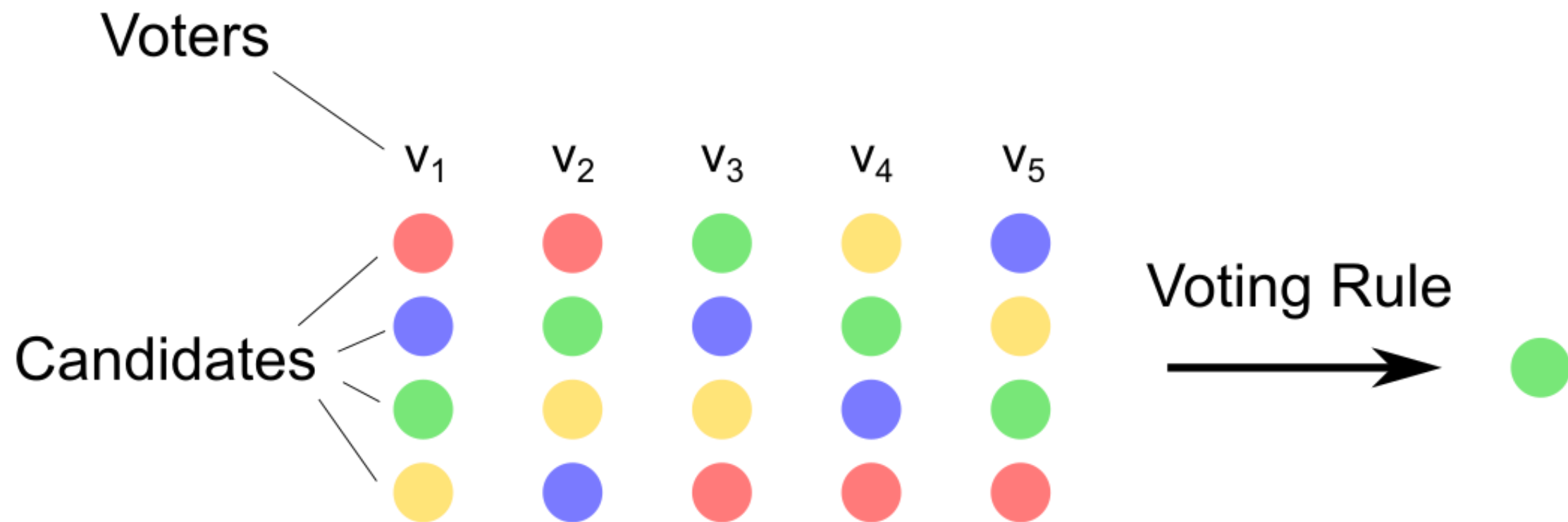


# Voting Setup



**Goal:** Pick exactly one winning candidate.

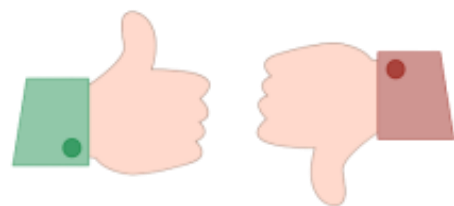
# Voting Setup



**Goal:** Pick exactly one winning candidate.



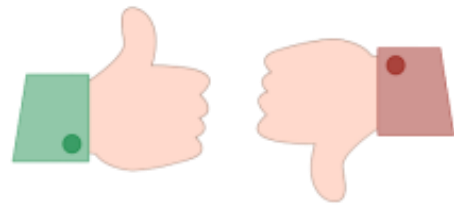
Voting Rule







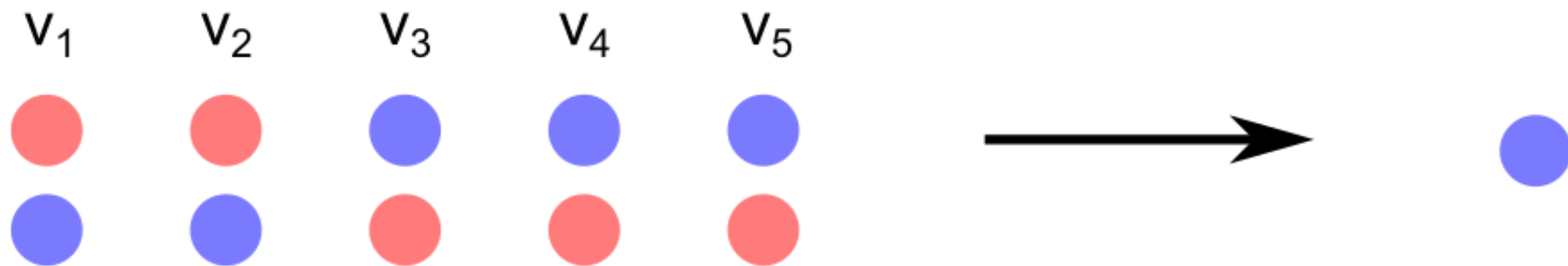
Voting Rule



# Voting with Two Candidates

# Voting with Two Candidates

Majority!



# Voting with More Than Two Candidates

# Voting with More Than Two Candidates

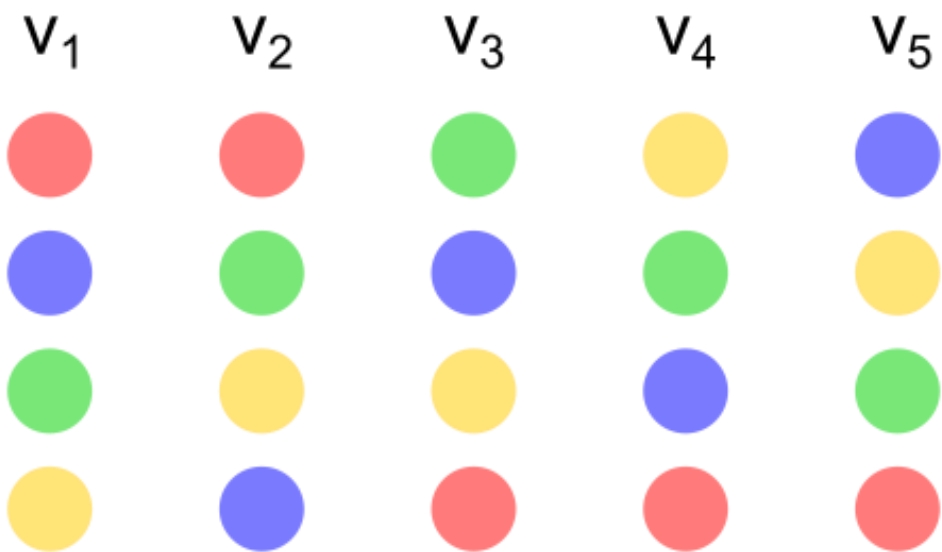
No candidate may have a majority...

1

Plurality

# 1

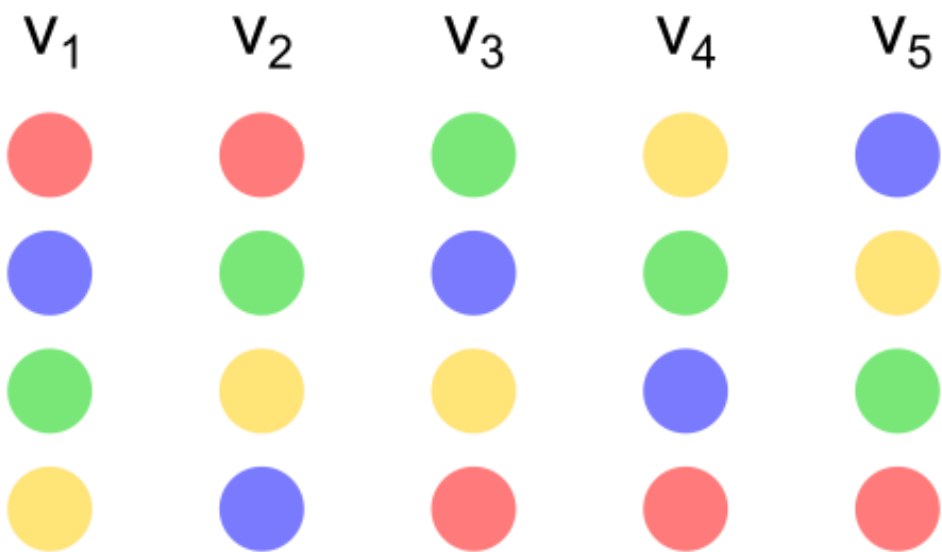
Plurality



# 1

Candidate with the most first-place votes wins

Plurality

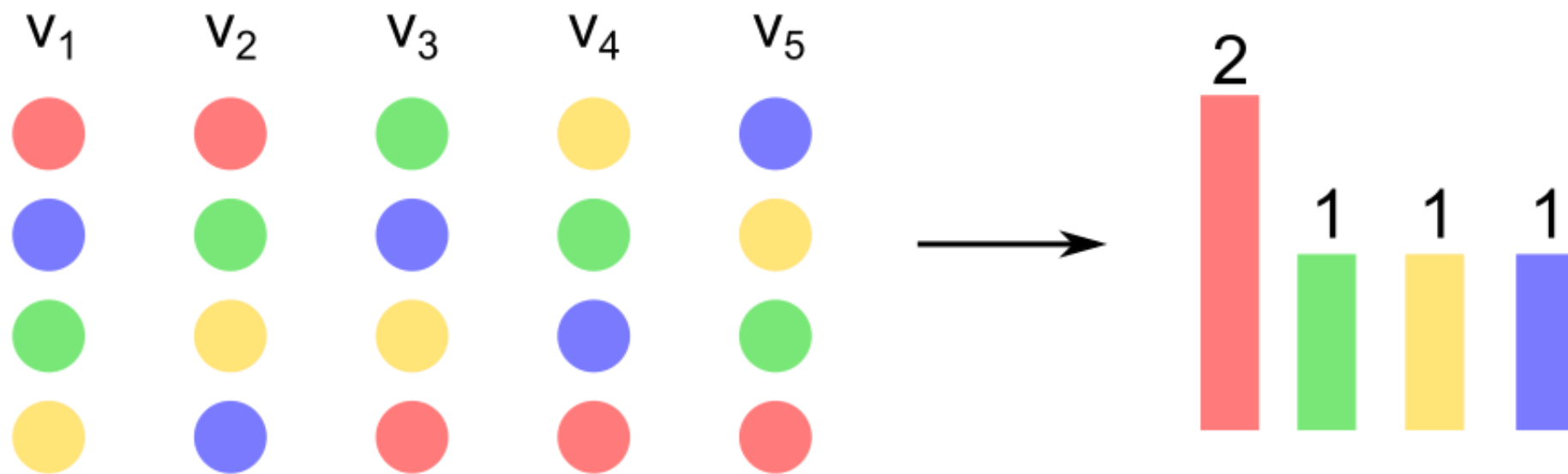




# 1

Candidate with the most first-place votes wins

Plurality



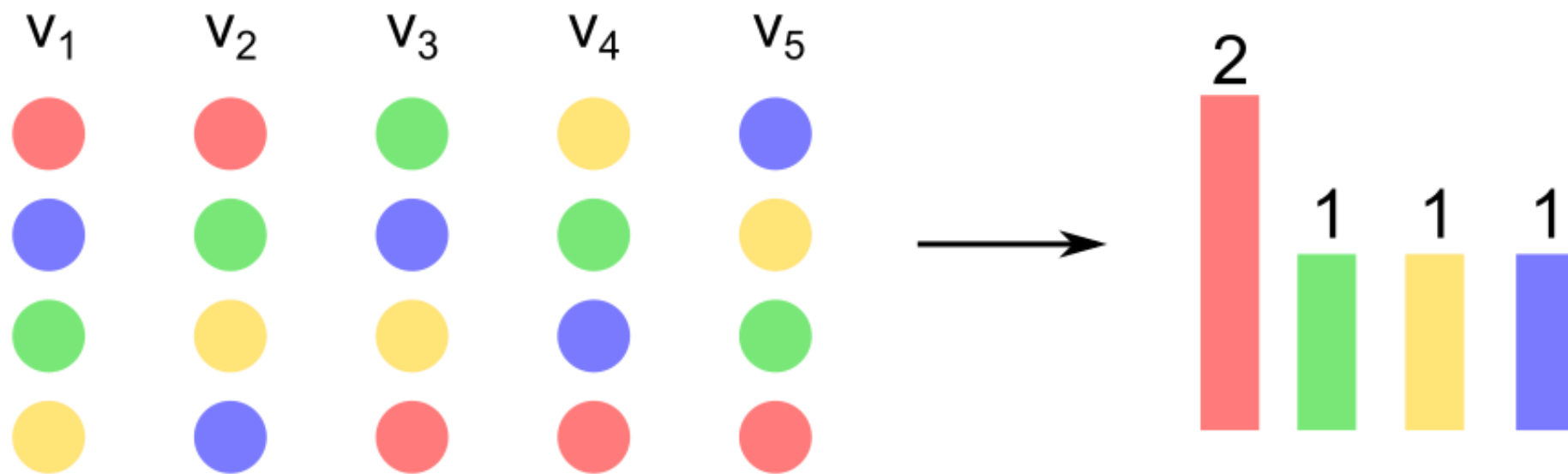
Plurality winner: ●

# 1

Candidate with the most first-place votes wins\*

\*subject to tie-breaking: lexicographic, random, ...

Plurality

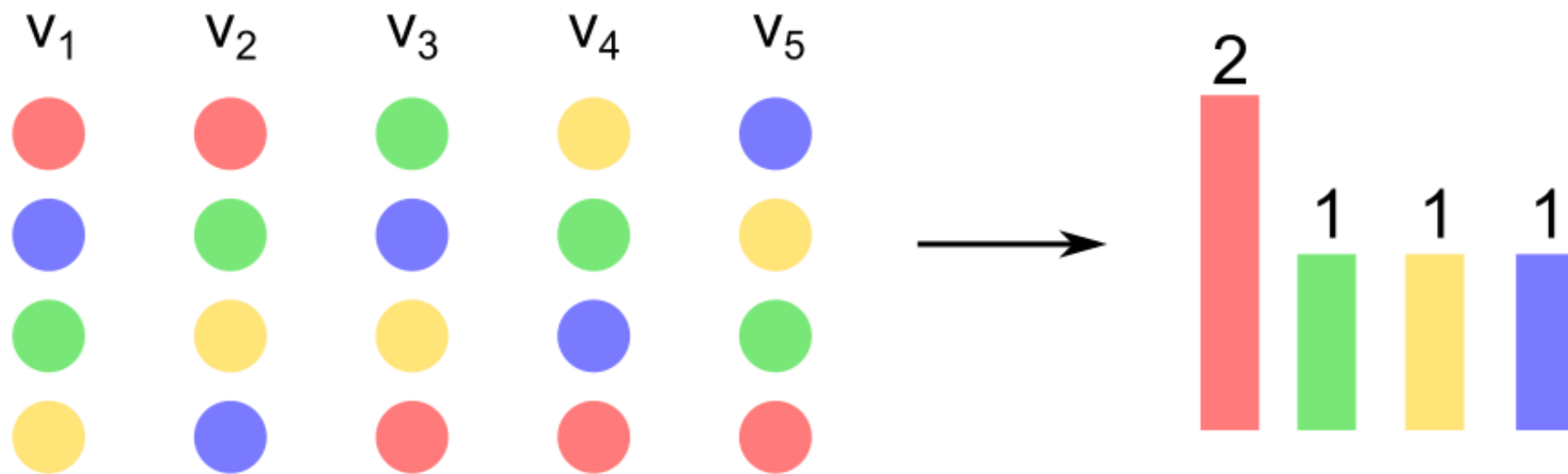


Plurality winner: ●

# 1

**Problem:** A majority prefers ● over the Plurality winner.

Plurality

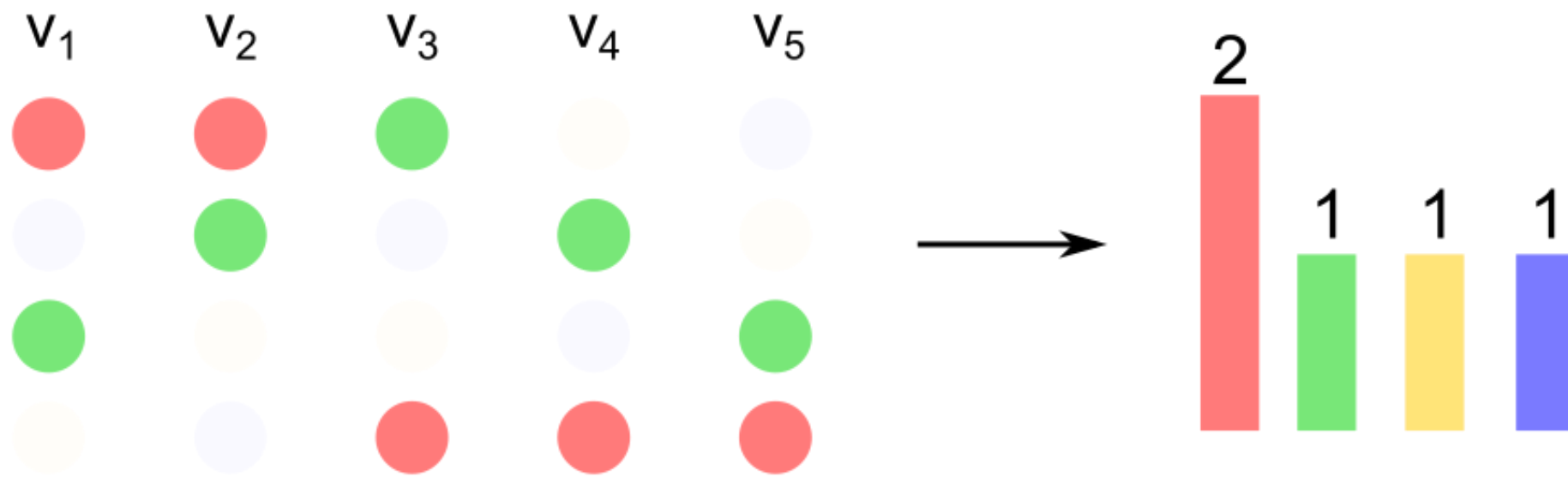


Plurality winner: ●

# 1

**Problem:** A majority prefers ● over the Plurality winner.

Plurality



Plurality winner: ●

# 1

Plurality

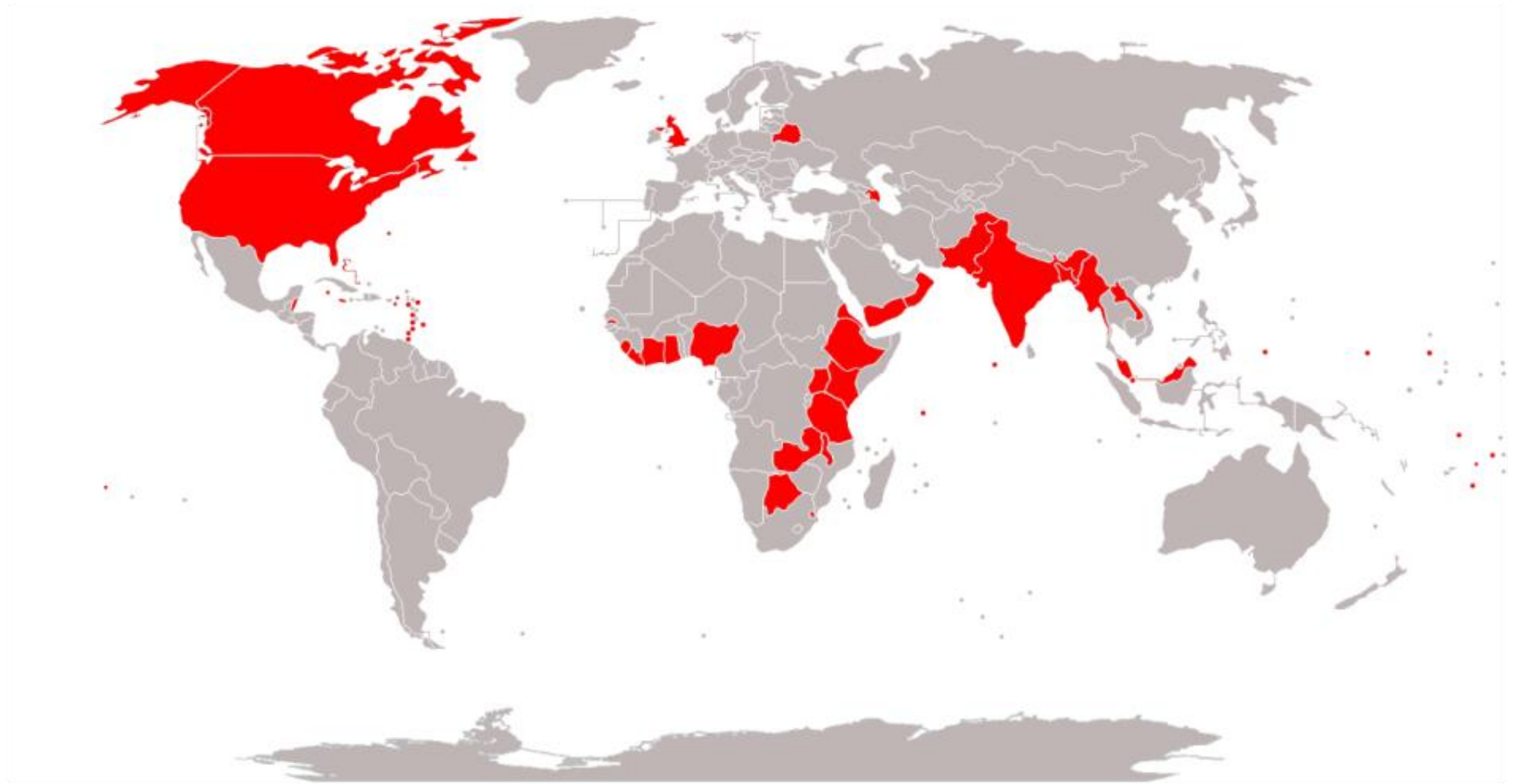


Image Source: Wikipedia article on "Electoral system" (Jan 2022)

2

Borda  
Count

# 2

Borda  
Count



Jean-Charles de Borda  
(1733-1799)

# 2

## Borda Count



Jean-Charles de Borda  
(1733-1799)

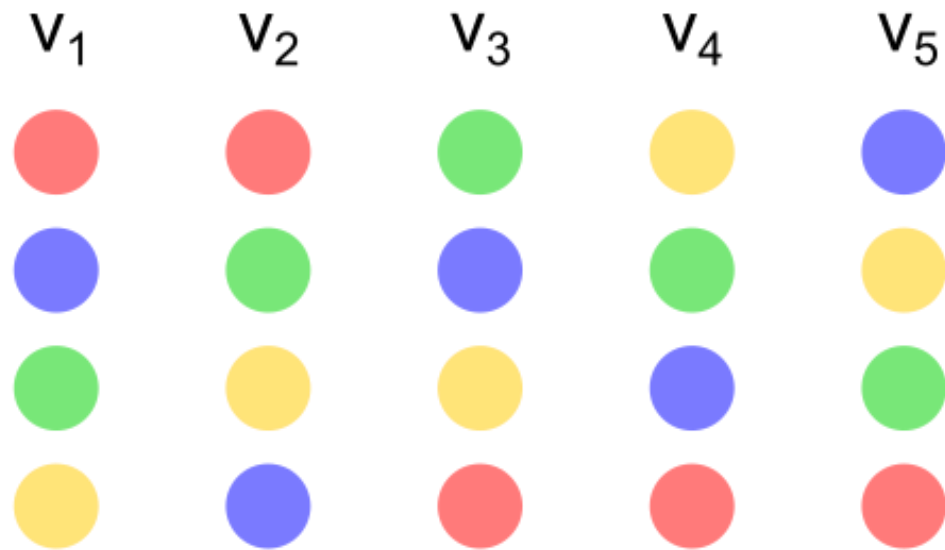




# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

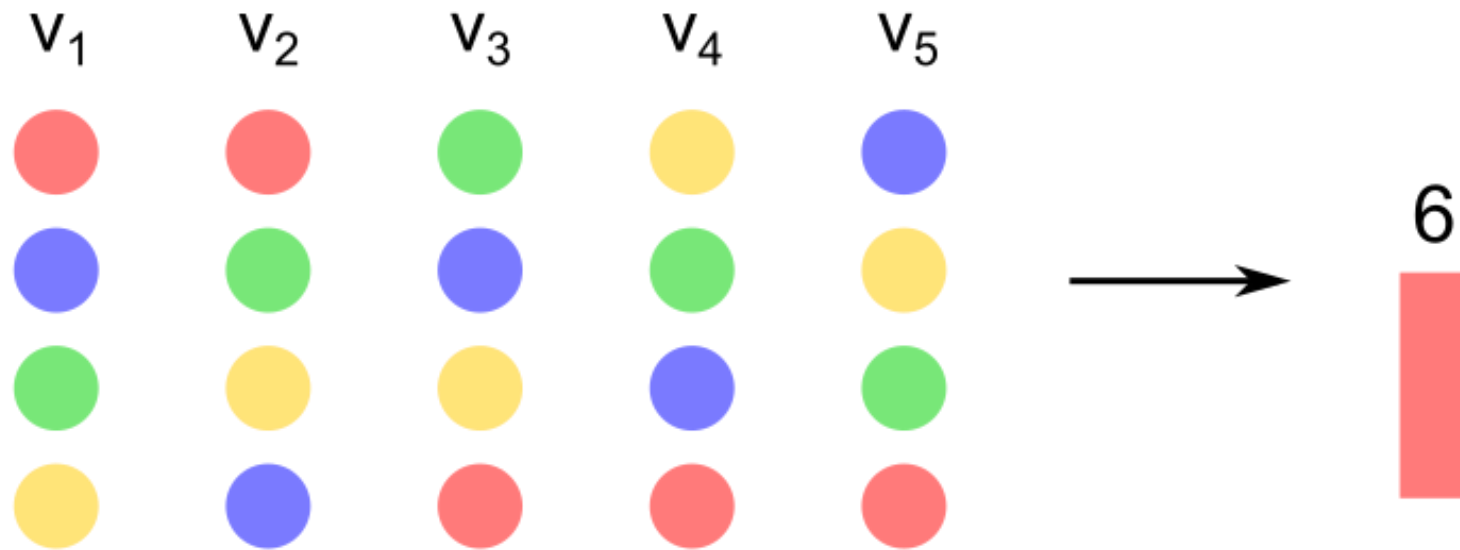
Borda  
Count



# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

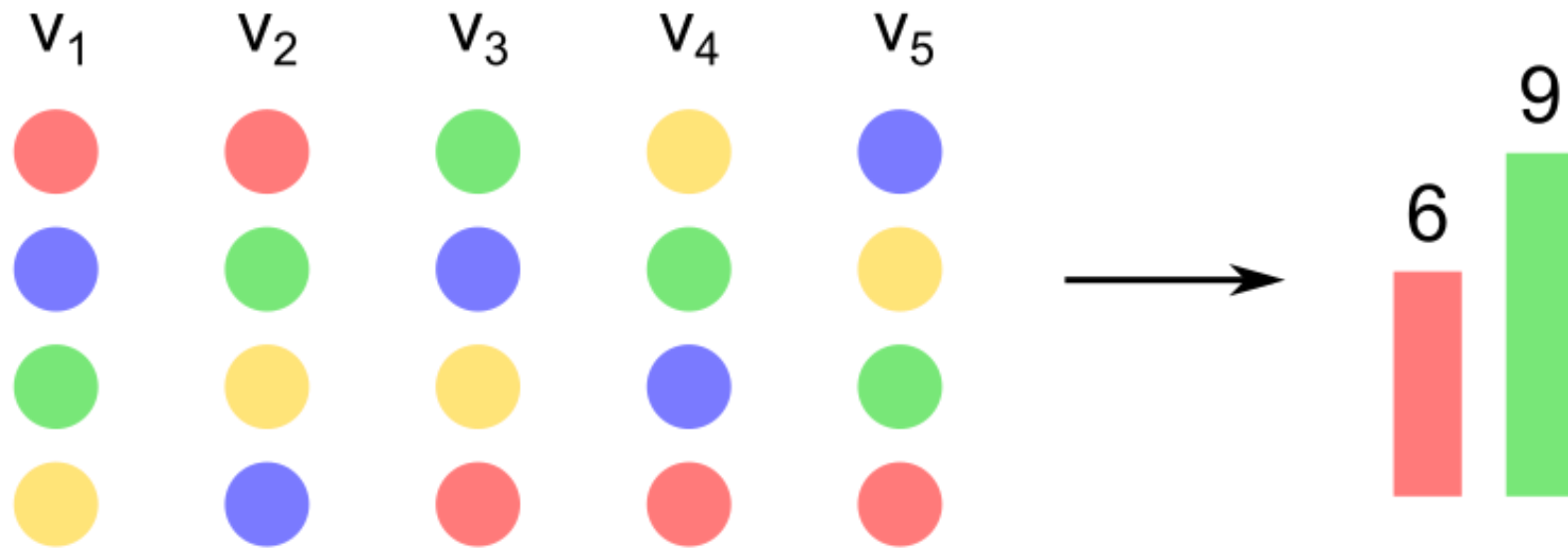
Borda  
Count



# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

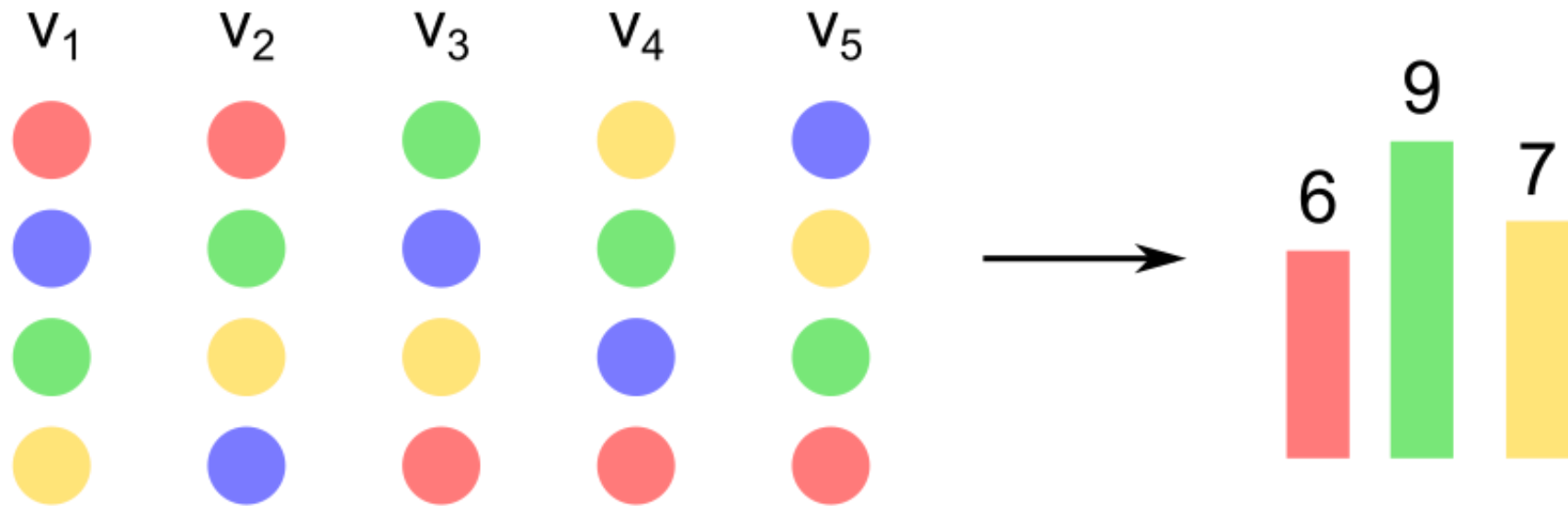
Borda  
Count



# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

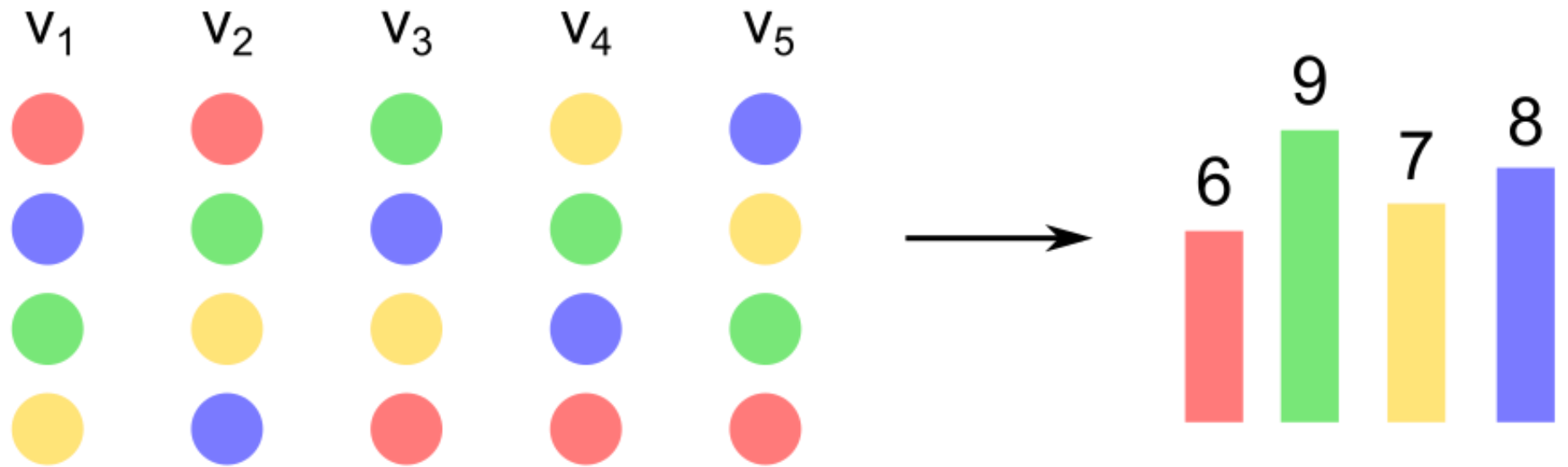
Borda  
Count



# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

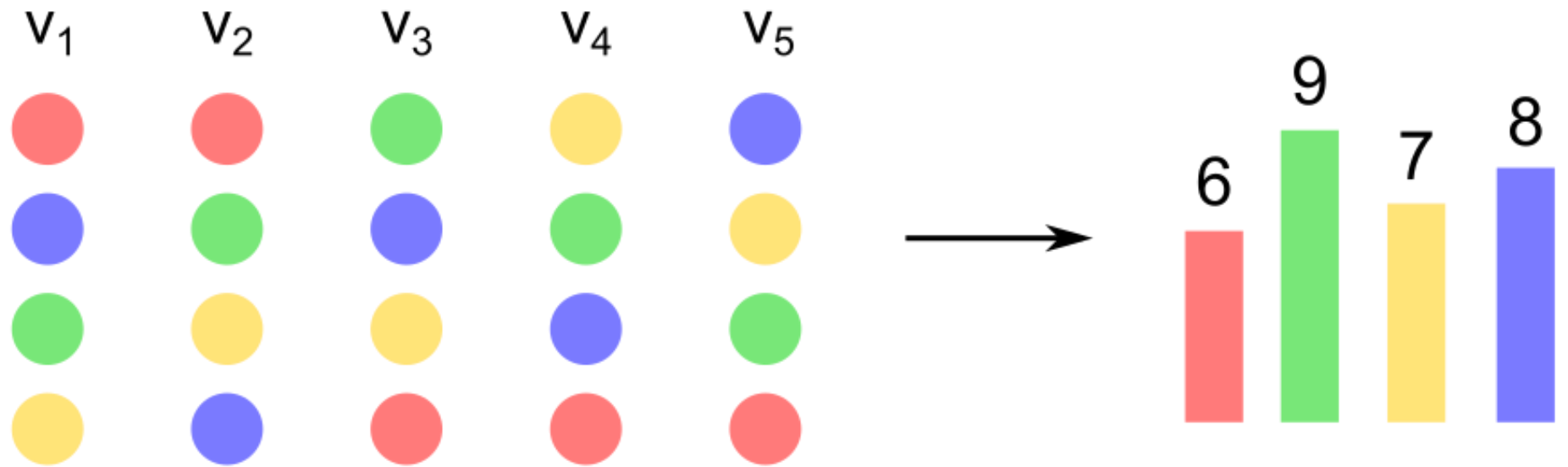
Borda  
Count



# 2

Each voter gives its  $k^{\text{th}}$  ranked candidate  $m-k$  points, where  $m$  is the number of candidates.

Borda  
Count

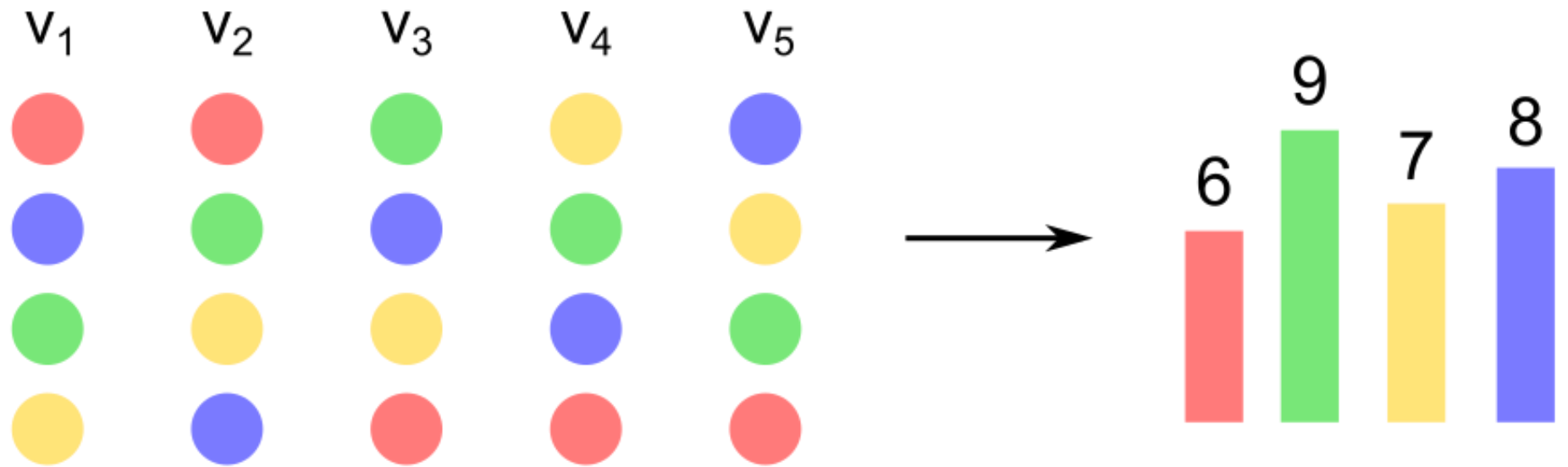


Borda winner: ●

# 2

**Problem:** Susceptible to strategic voting (manipulation).

Borda  
Count

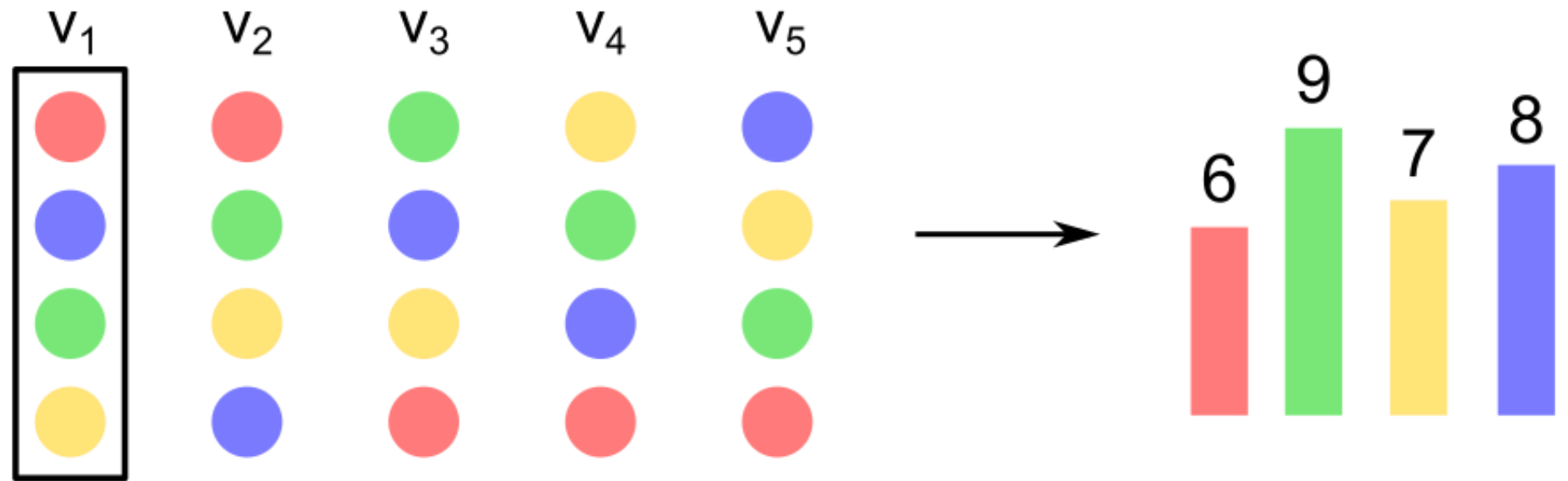


Borda winner: ●

# 2

**Problem:** Susceptible to strategic voting (manipulation).

Borda  
Count



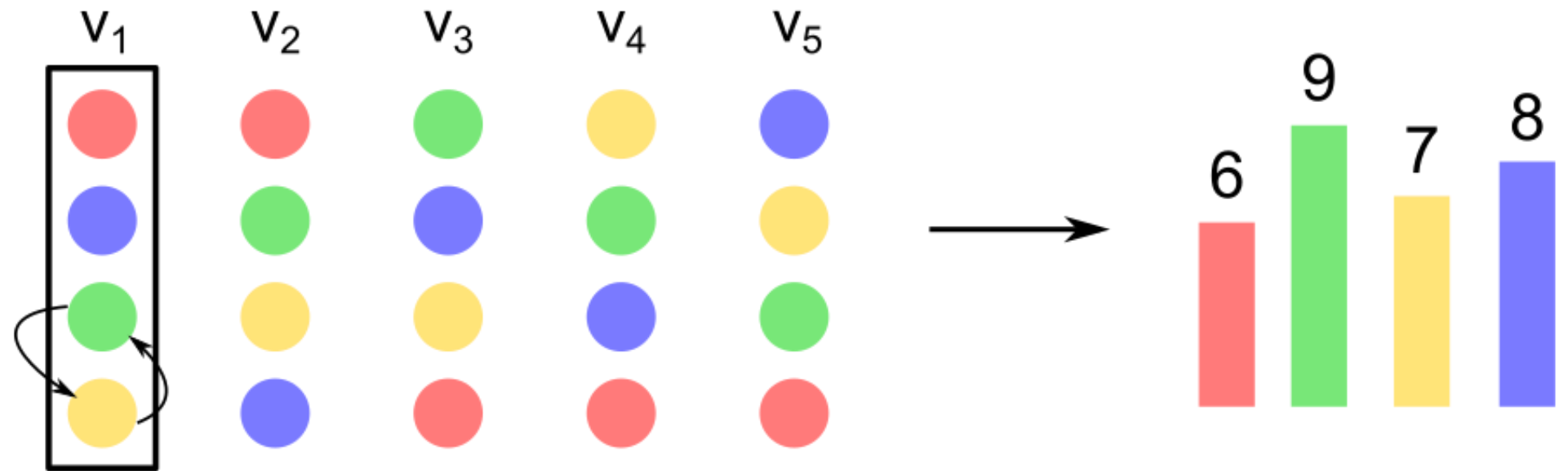
Borda winner: ●



# 2

**Problem:** Susceptible to strategic voting (manipulation).

Borda  
Count

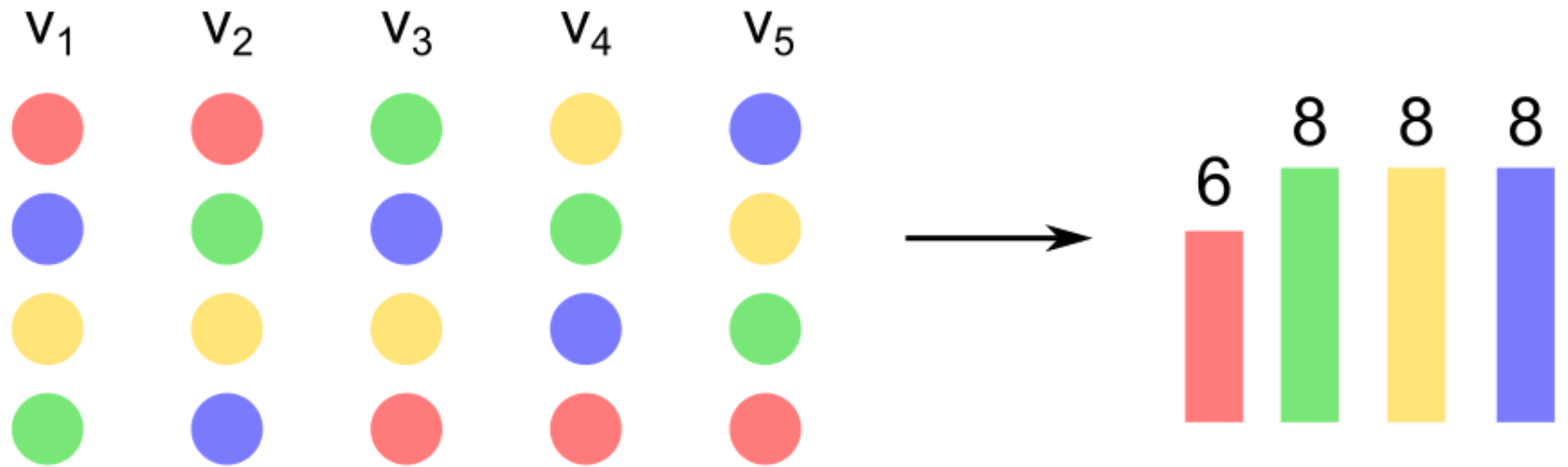


Borda winner: ●

# 2

**Problem:** Susceptible to strategic voting (manipulation).

Borda  
Count



Borda winner: ●

# 2

**Problem:** Susceptible to strategic voting (manipulation).

"My scheme is intended for only honest men."



Borda  
Count

# 2

Borda  
Count



# 3

Plurality  
With  
Runoff

# 3

## Plurality With Runoff



# 3

Plurality  
With  
Runoff

# 3

The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

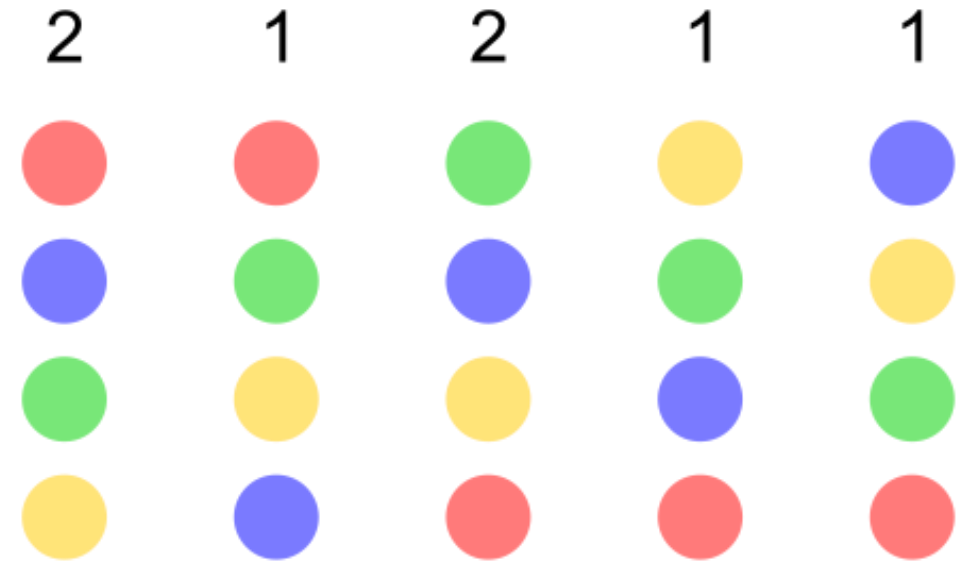
Plurality  
With  
Runoff



# 3

The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

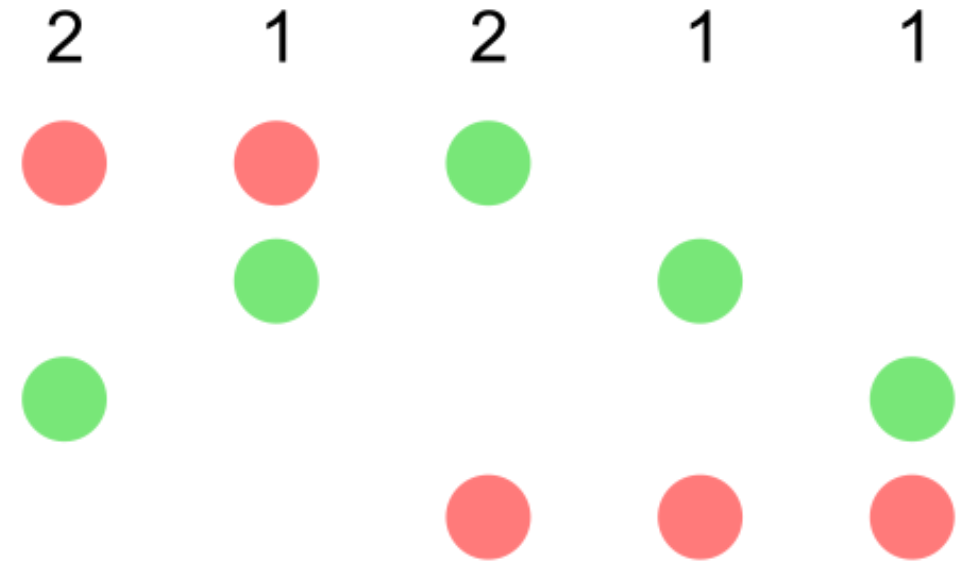
Plurality  
With  
Runoff



# 3

The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

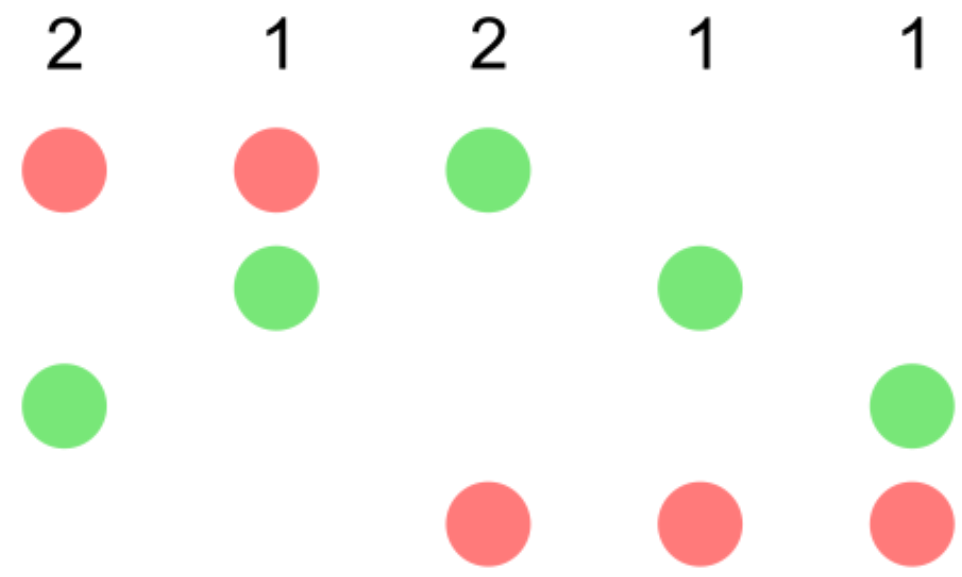
Plurality  
With  
Runoff



# 3

The two candidates with the highest Plurality scores in the first round go head-to-head in the next round

Plurality  
With  
Runoff



Plurality with runoff winner: ●

4

Single  
Transferable  
Vote

(Instant-Runoff)

# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

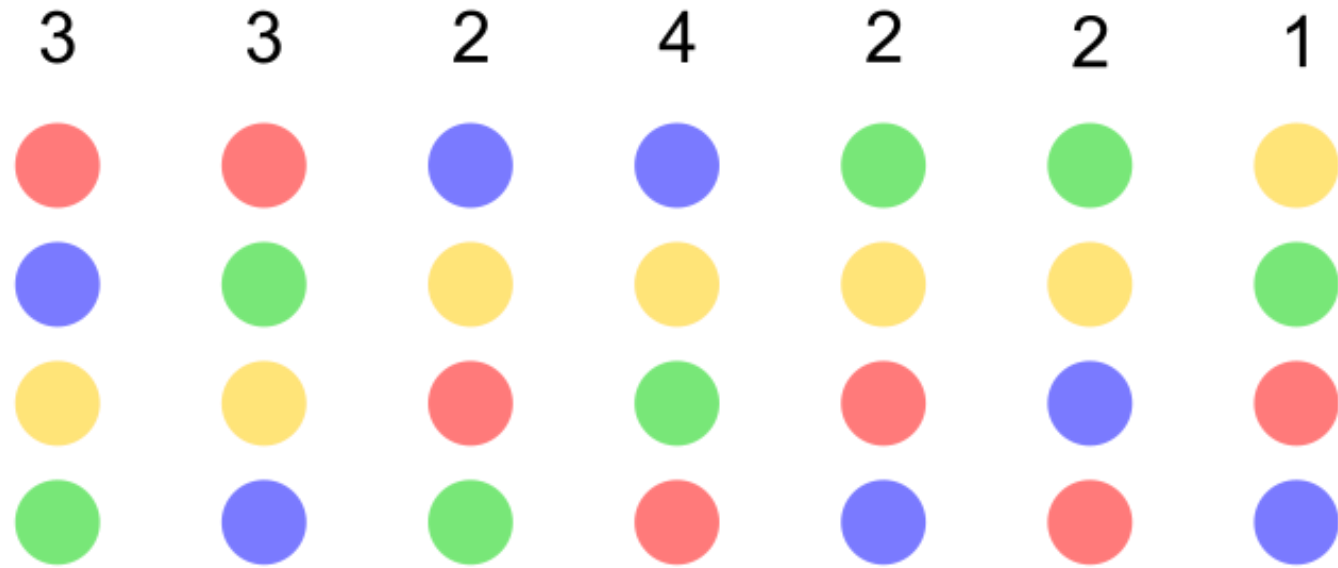
Single  
Transferable  
Vote

(Instant-Runoff)

# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

Single  
Transferable  
Vote  
(Instant-Runoff)

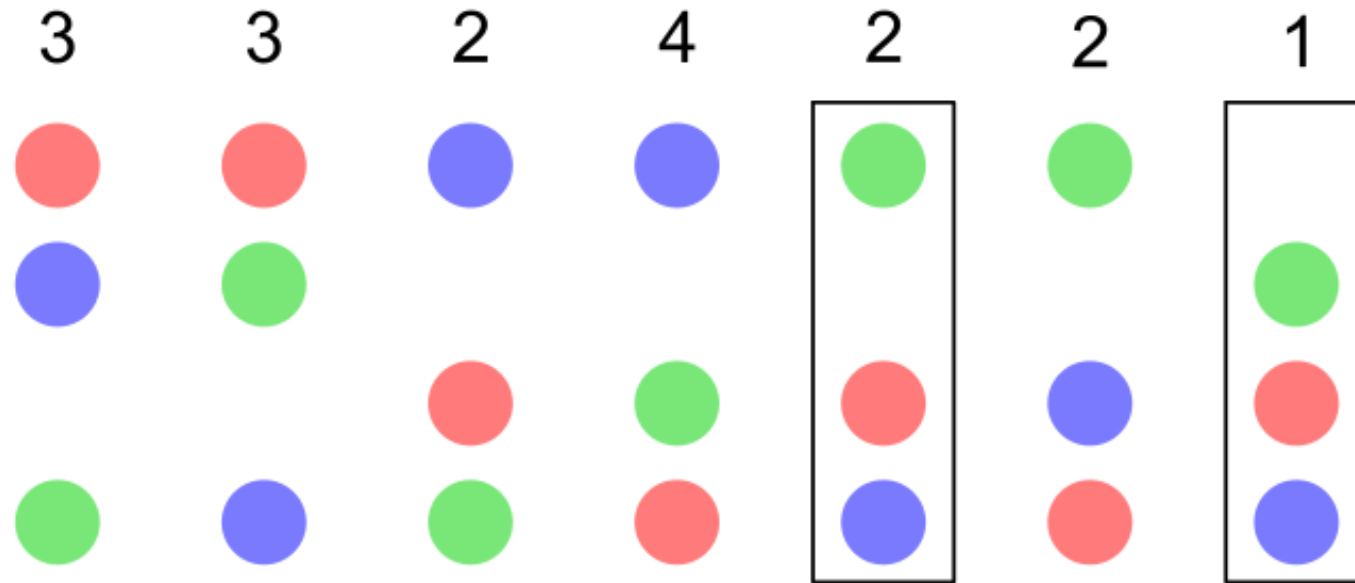




# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

Single  
Transferable  
Vote  
(Instant-Runoff)

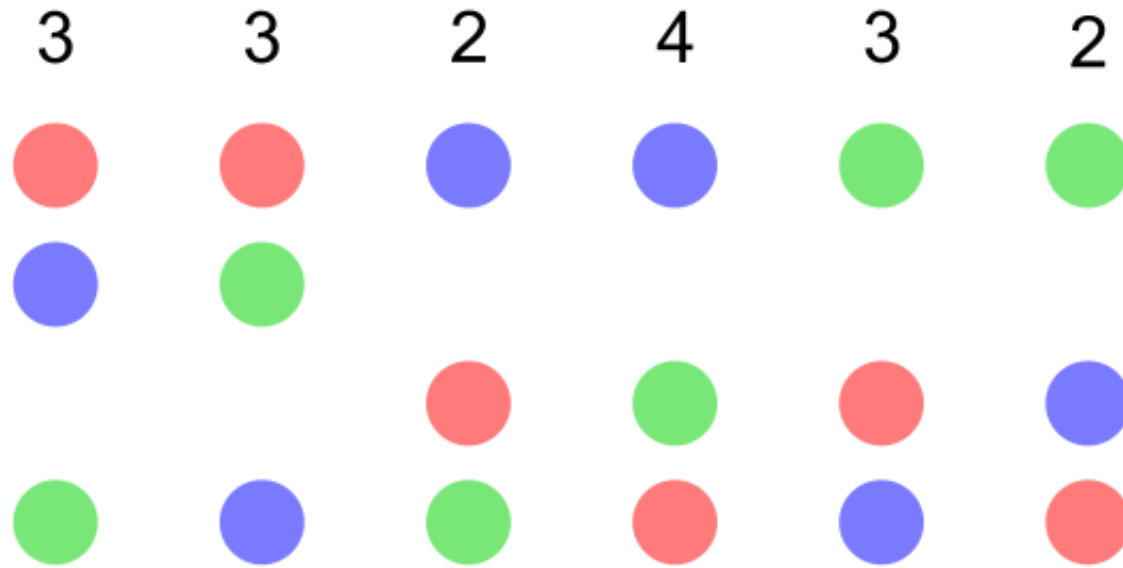




# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

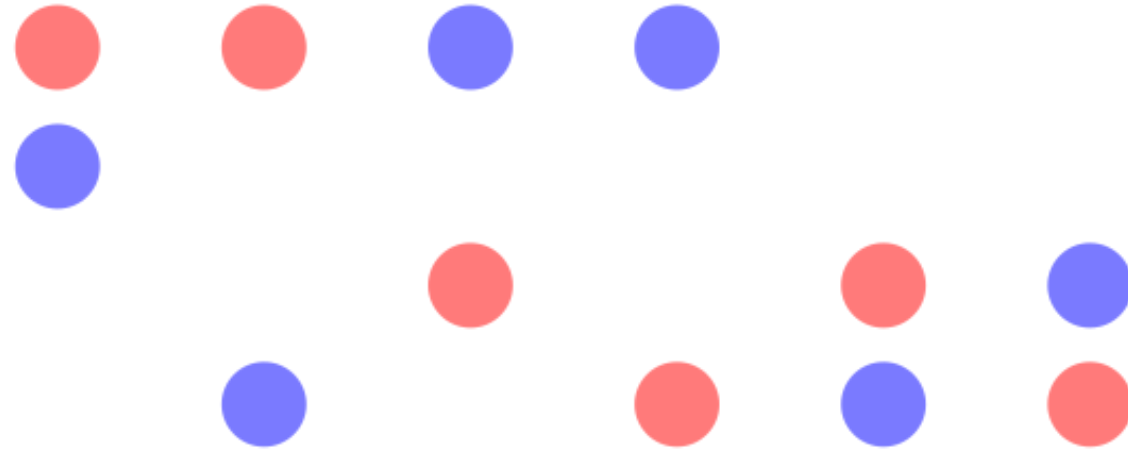
Single  
Transferable  
Vote  
(Instant-Runoff)



# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

3      3      2      4      3      2



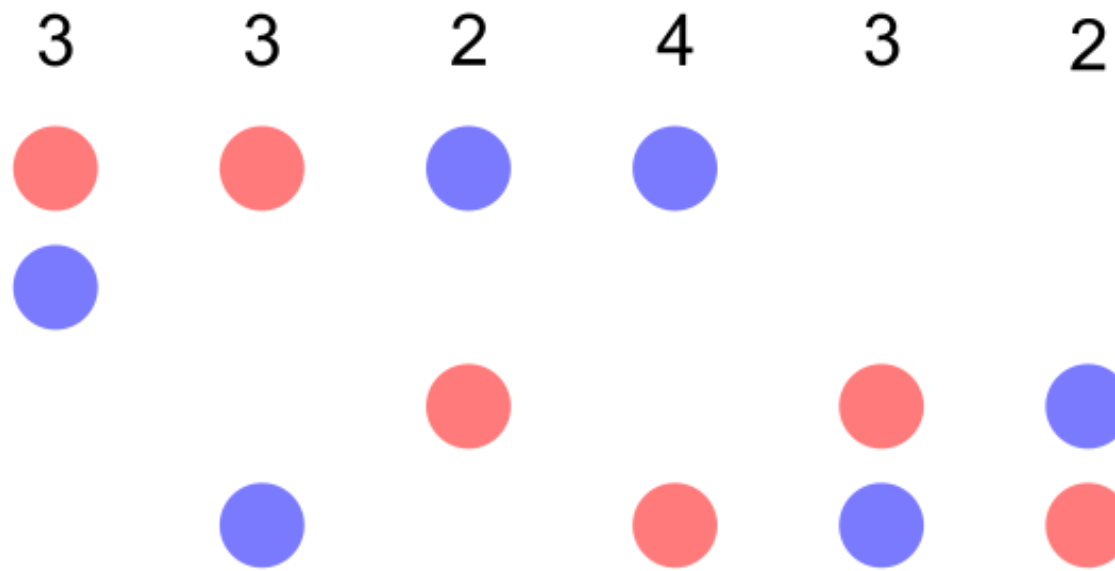
Single  
Transferable  
Vote

(Instant-Runoff)

# 4

In each round, eliminate the candidate with the lowest Plurality score, and transfer its supporters' votes

Single  
Transferable  
Vote  
(Instant-Runoff)



STV winner: ●

# 4

Single  
Transferable  
Vote

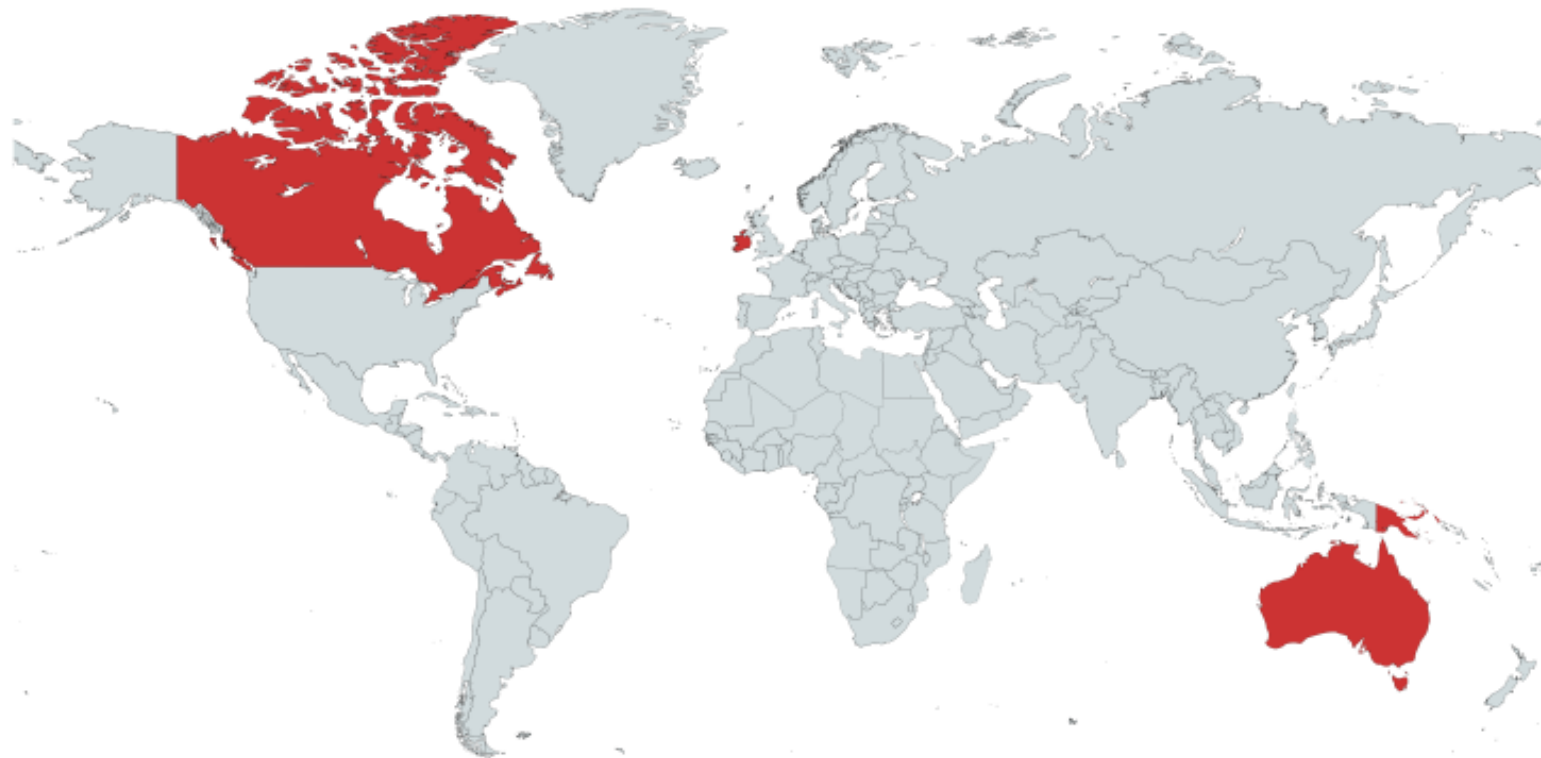
(Instant-Runoff)



4

Single  
Transferable  
Vote

(Instant-Runoff)



4

Single  
Transferable  
Vote

(Instant-Runoff)



# 4

Single  
Transferable  
Vote

(Instant-Runoff)

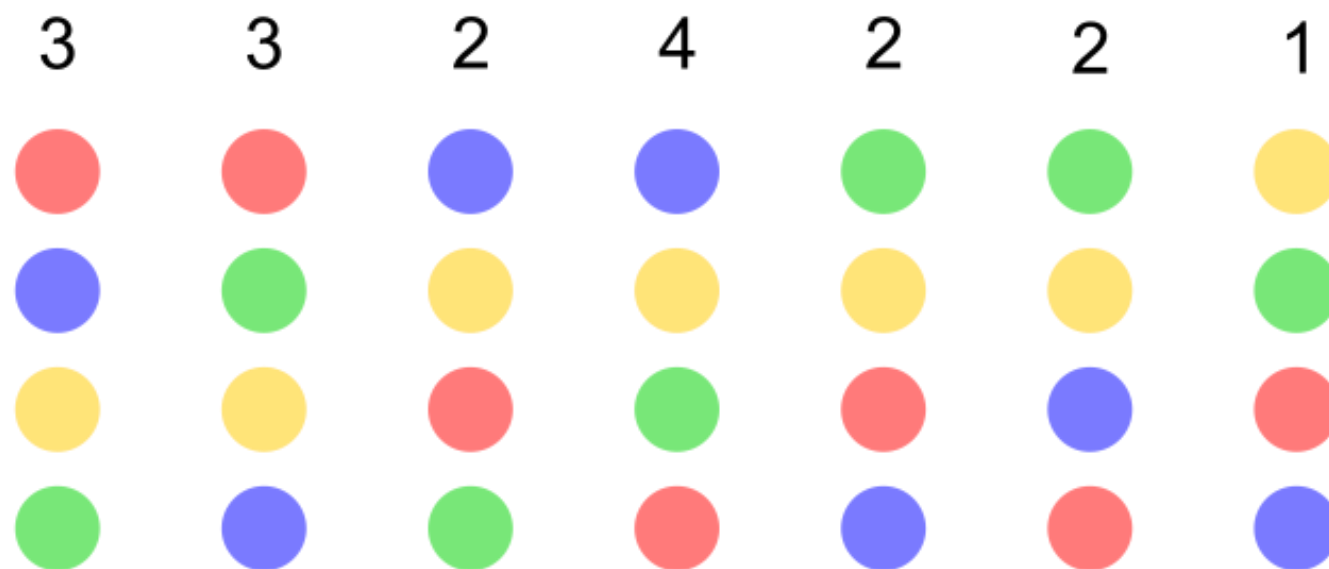
**Problem:** Failure of *monotonicity*  
(improving a candidate's support could make it worse off)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)



Recall that STV winner was 

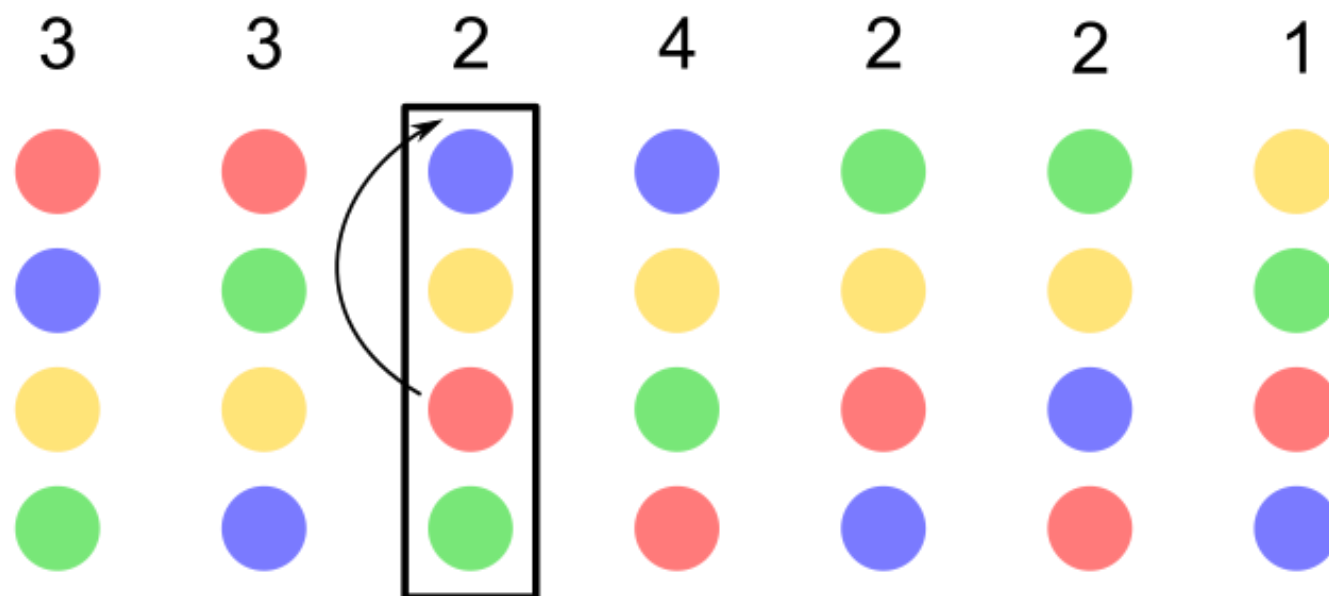


# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)

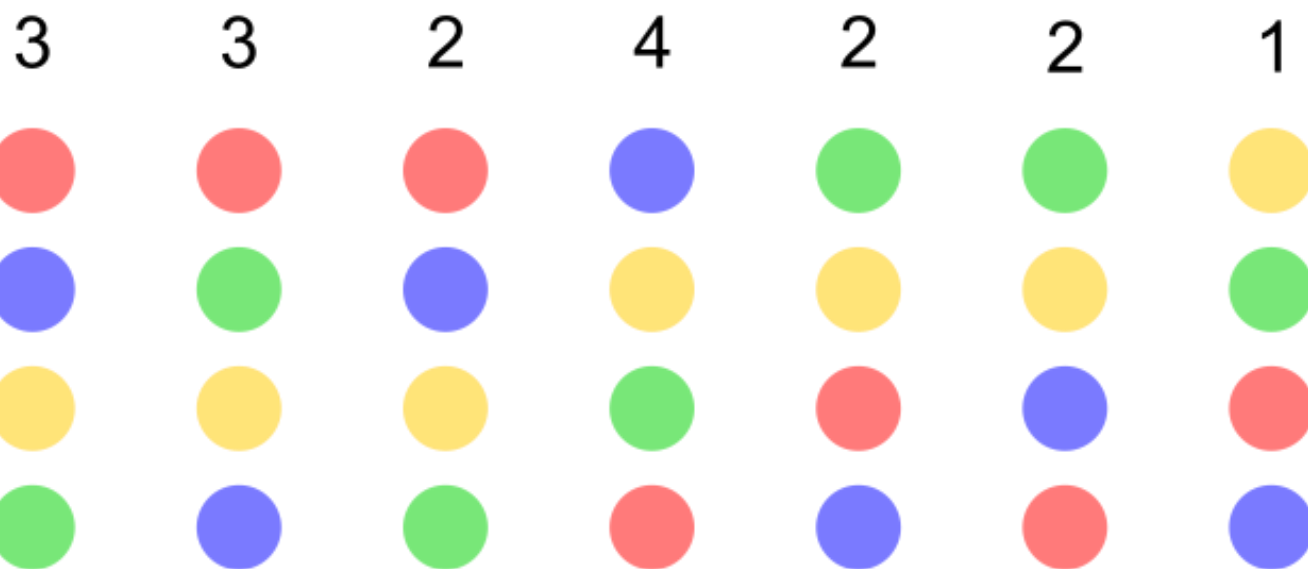


Recall that STV winner was ●

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was 

Single  
Transferable  
Vote

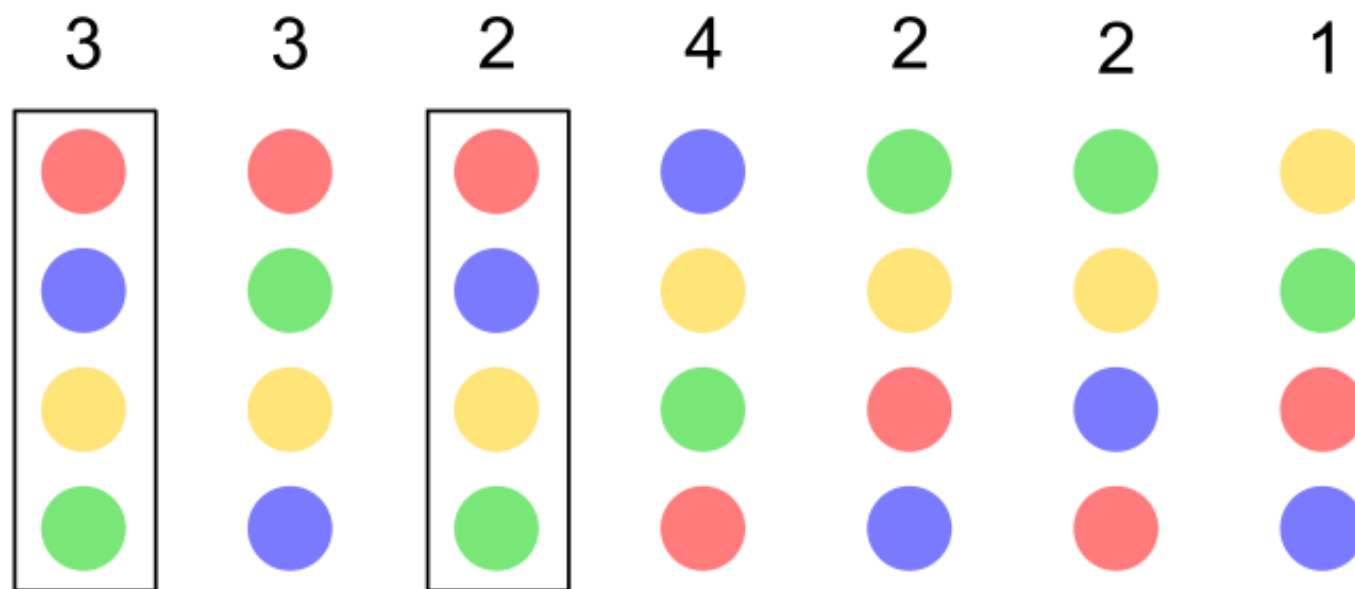
(Instant-Runoff)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)

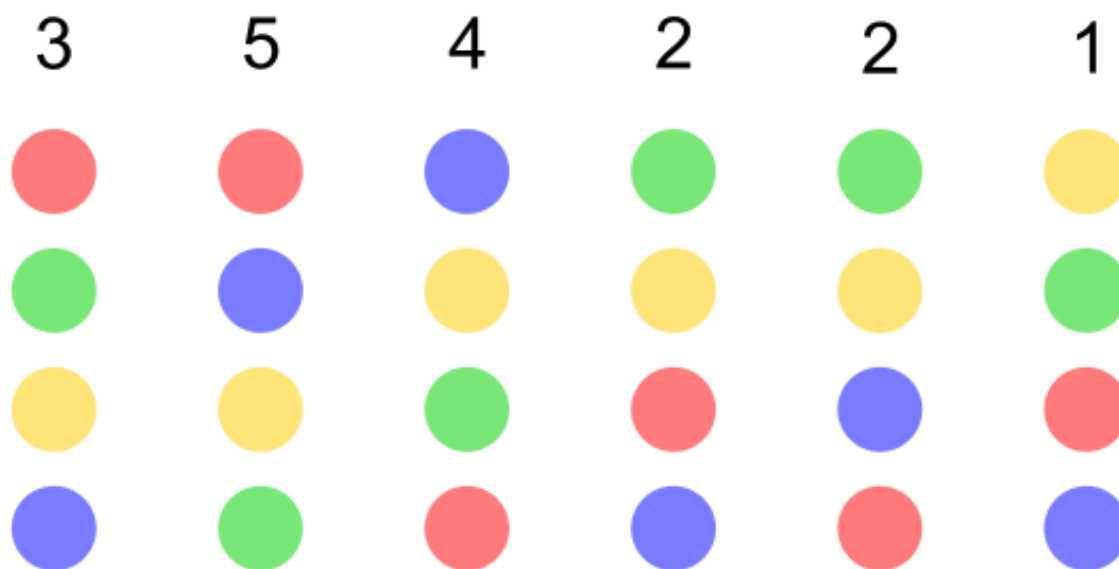


Recall that STV winner was 

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was 

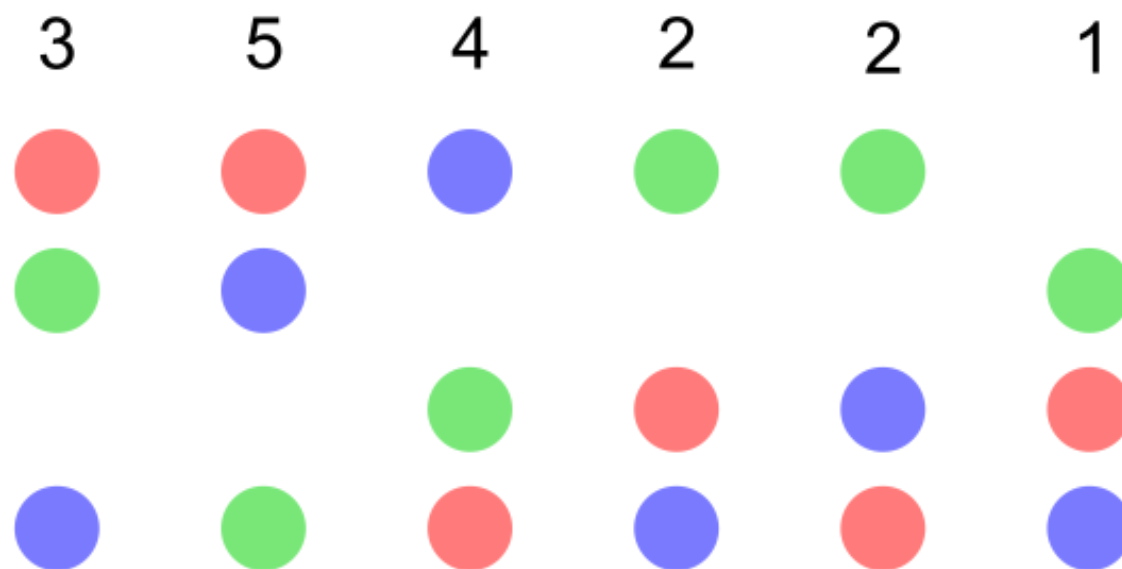
Single  
Transferable  
Vote  
(Instant-Runoff)

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)

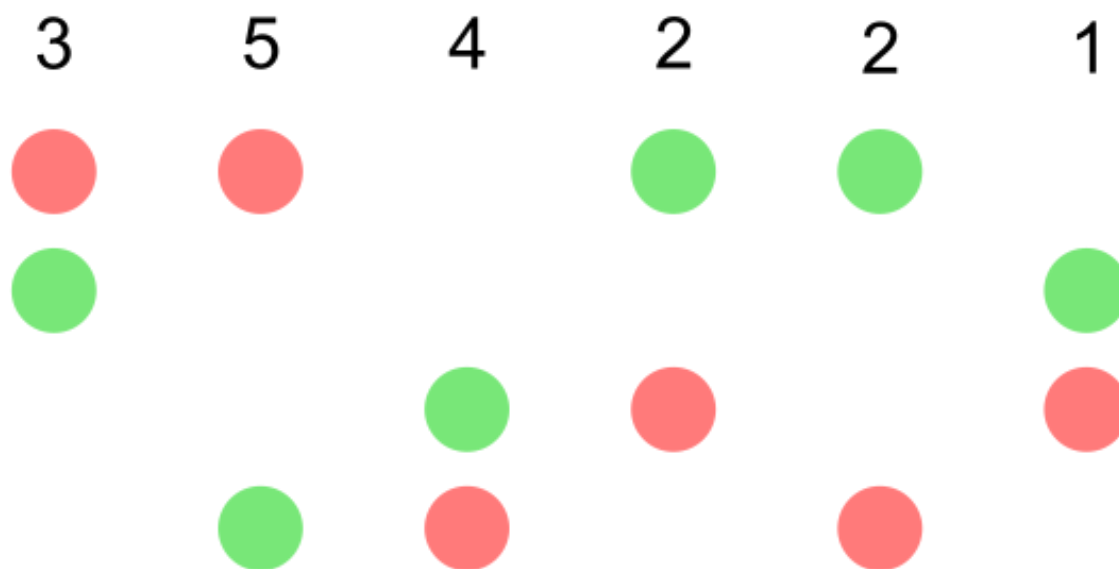


Recall that STV winner was 

# 4

## Problem: Failure of *monotonicity*

(improving a candidate's support could make it worse off)



Recall that STV winner was ●

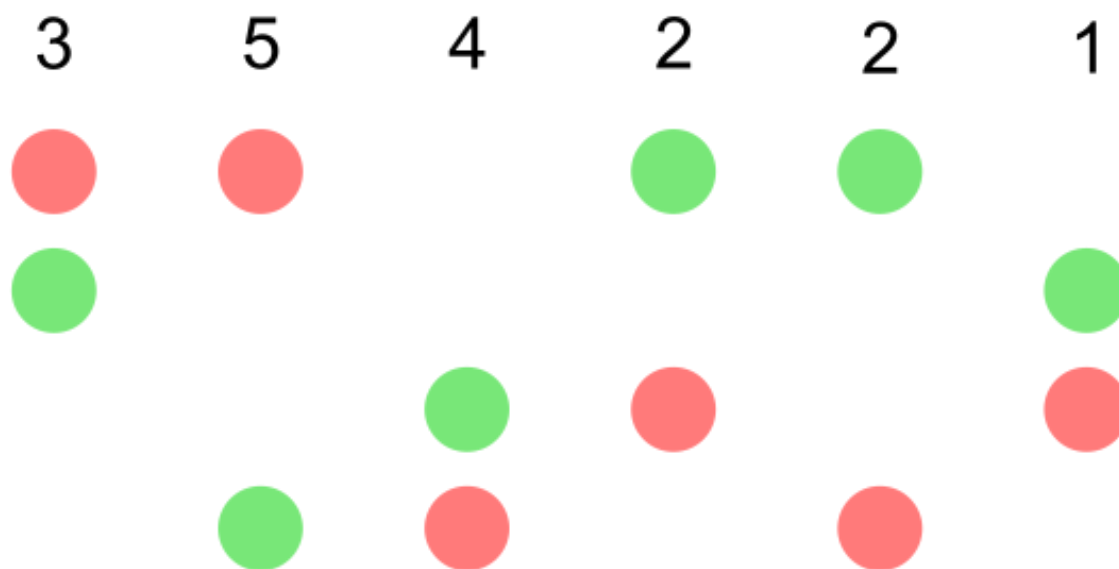
Single  
Transferable  
Vote

(Instant-Runoff)

# 4

**Problem:** Failure of *monotonicity*  
(improving a candidate's support could make it worse off)

Single  
Transferable  
Vote  
(Instant-Runoff)



Recall that STV winner was ●  
but now it's ●

# 4

Single  
Transferable  
Vote

(Instant-Runoff)

## 2016 host city election ballots results [\[ edit \]](#)

City	NOC	Round 1	Round 2	Round 3
<a href="#">Rio de Janeiro</a>	 <a href="#">Brazil</a> (COB)	26	<b>46</b>	<b>66</b>
<a href="#">Madrid</a>	 <a href="#">Spain</a> (COE)	<b>28</b>	29	32
<a href="#">Tokyo</a>	 <a href="#">Japan</a> (JOC)	22	20	—
<a href="#">Chicago</a>	 <a href="#">United States</a> (USOC)	18	—	—

Venue
<a href="#">Bella Center</a> <b>121st IOC Session</b> October 2, 2009  <a href="#">Copenhagen</a>

Vote details	Round 1	Round 2	Round 3
Eligible members	95	97	99
Participants	94	96	98
Abstentions	0	1	0
Valid ballots	94	95	98



# 4

## 2016 host city election ballots results [\[ edit \]](#)

City	NOC	Round 1	Round 2	Round 3
Rio de Janeiro	 Brazil (COB)	26	46	66
Madrid	 Spain (COE)	28	29	32
Tokyo	 Japan (JOC)	22	20	—
Chicago	 United States (USOC)	18	—	—

Venue
Bella Center <b>121st IOC Session</b> October 2, 2009  Copenhagen

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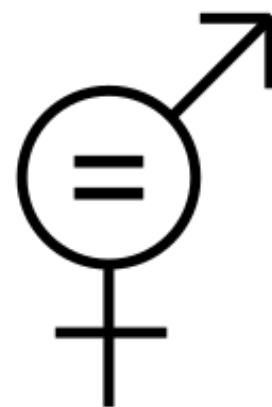
Single  
Transferable  
Vote  
  
(Instant-Runoff)



Nicolas de Condorcet  
(1743-1794)



Nicolas de Condorcet  
(1743-1794)



If a candidate beats every other candidate  
in a head-to-head election, select it!

If a candidate beats every other candidate  
in a head-to-head election, select it!

3



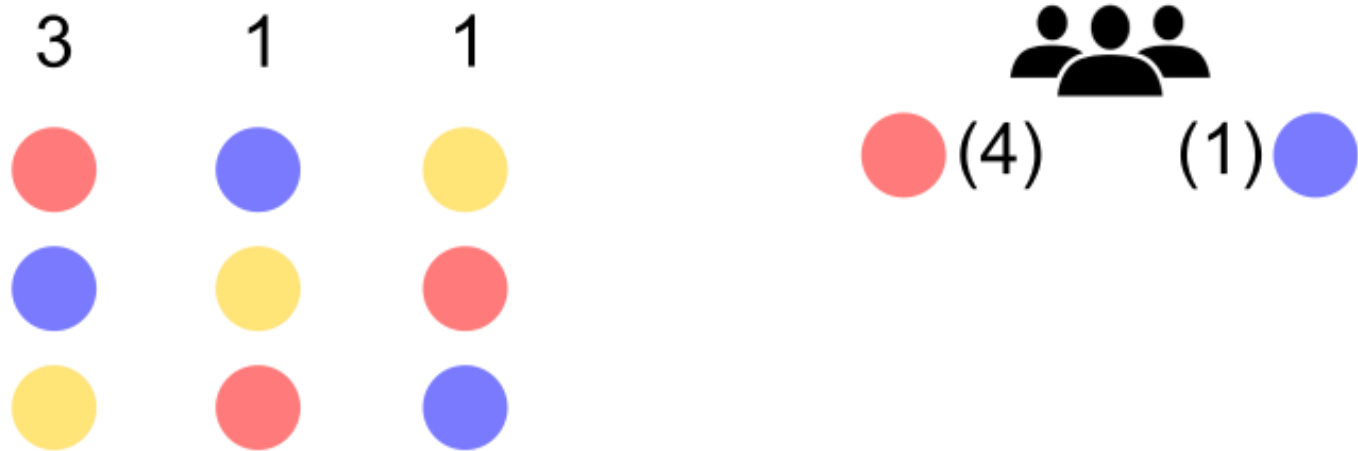
1



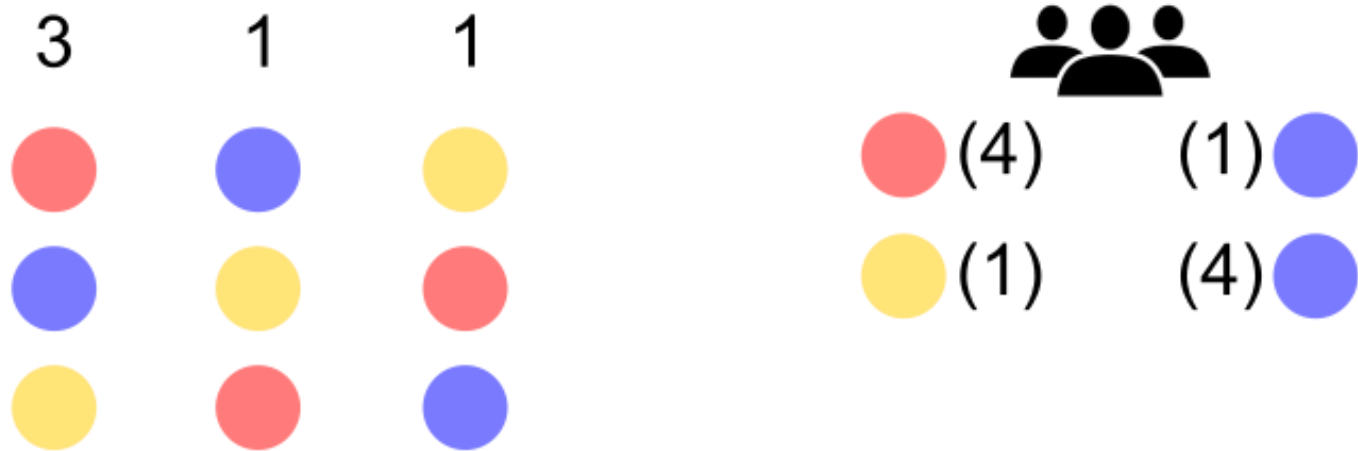
1



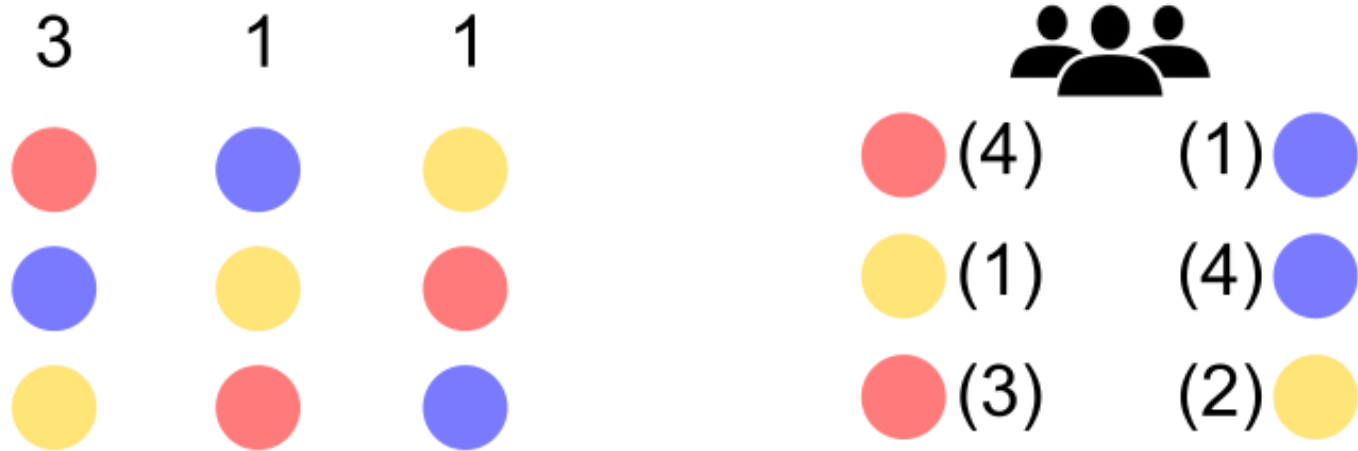
If a candidate beats every other candidate  
in a head-to-head election, select it!



If a candidate beats every other candidate  
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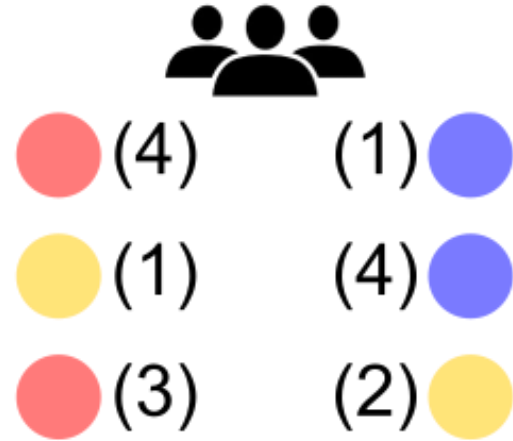
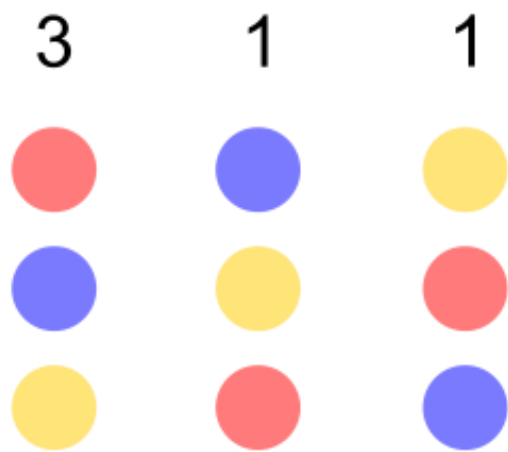


If a candidate beats every other candidate  
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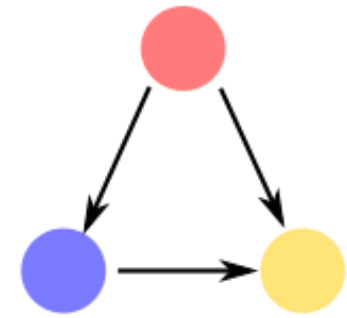




If a candidate beats every other candidate  
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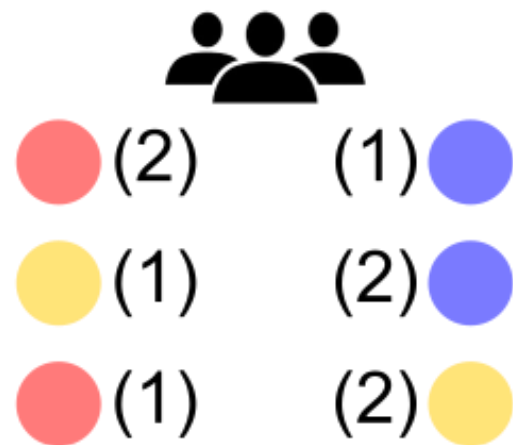
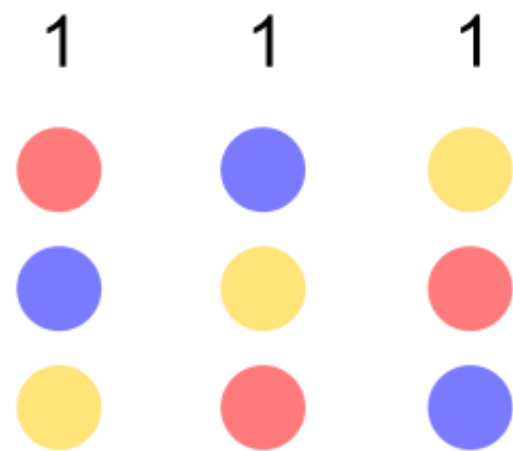
Condorcet winner



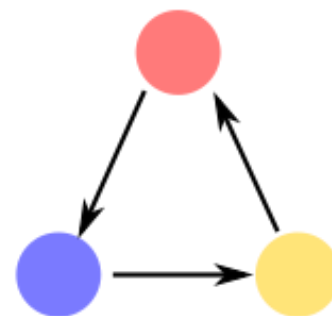
# Condorcet Criterion

**Problem:** A Condorcet winner may not exist

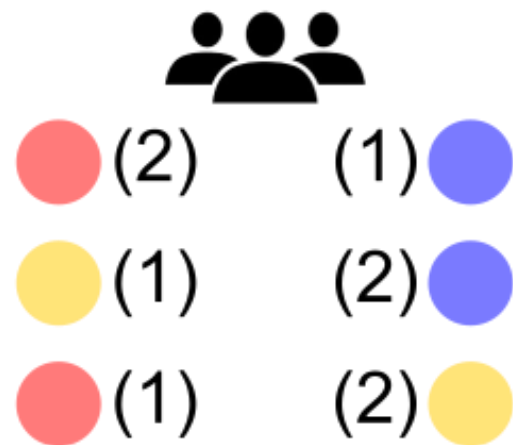
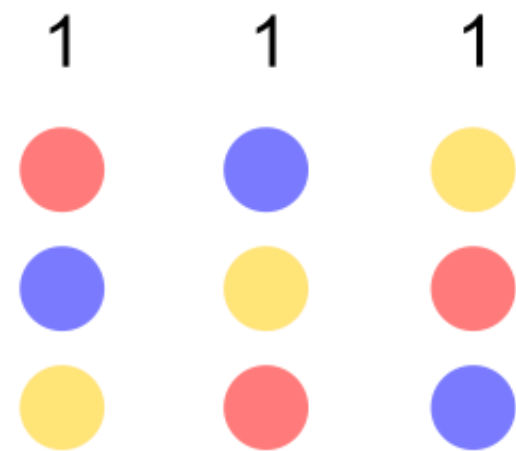
**Problem:** A Condorcet winner may not exist



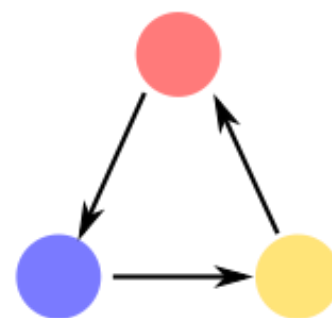
Condorcet cycle



**Problem:** A Condorcet winner may not exist



Condorcet cycle



**Condorcet paradox**

Transitivity of individual preferences  $\not\Rightarrow$  Transitivity of societal preferences

5

Copeland

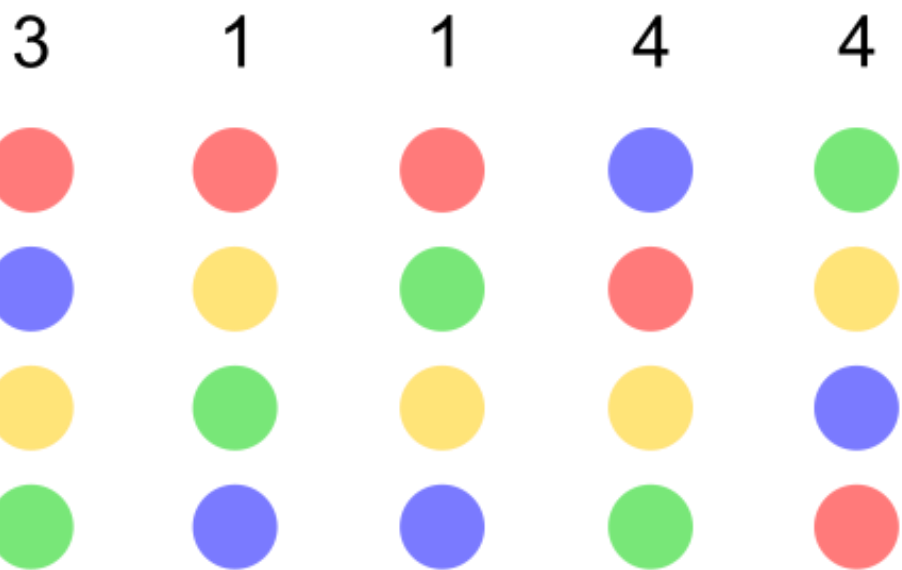
# 5

For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie

Copeland

# 5

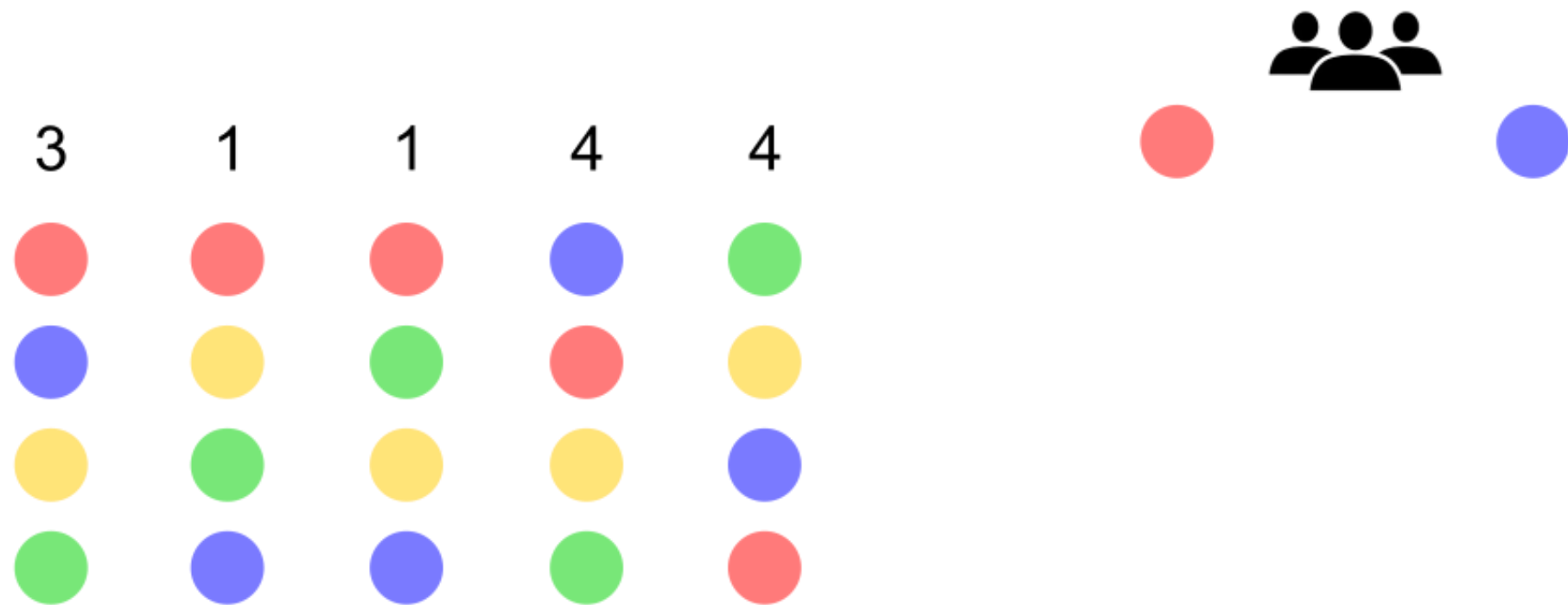
For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

# 5

For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie

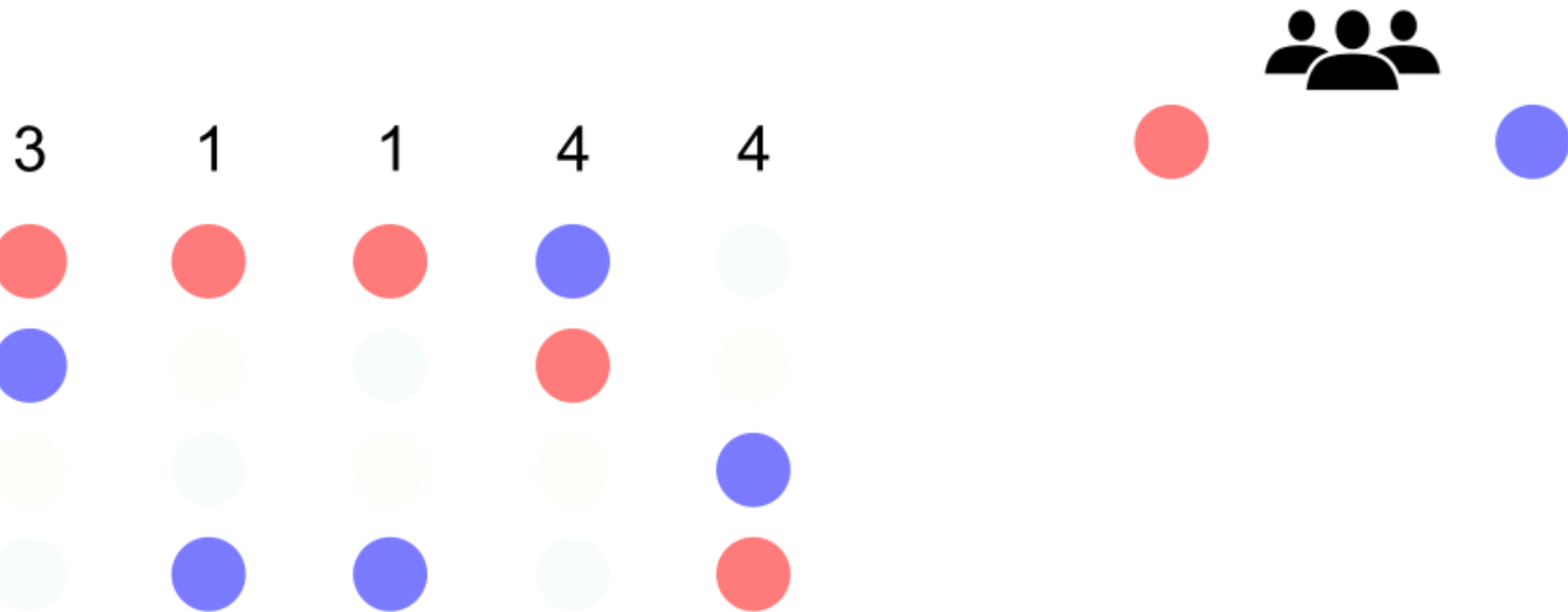


Copeland



# 5

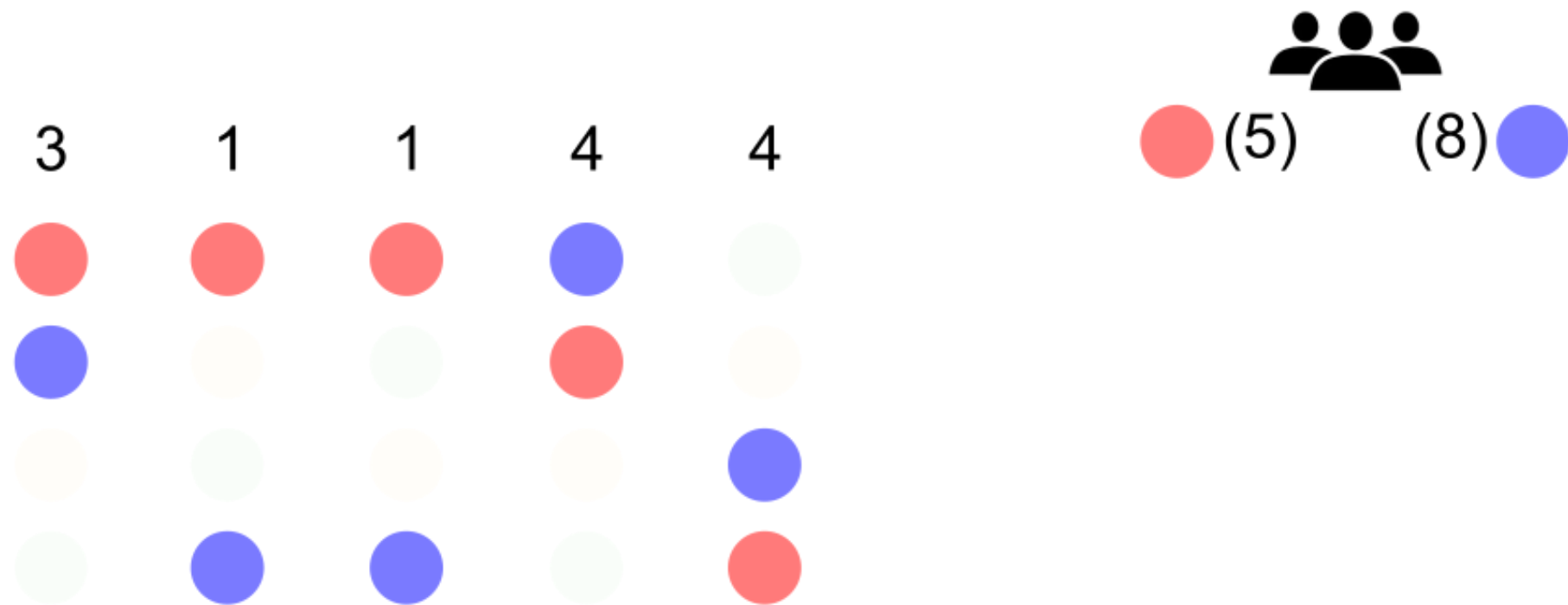
For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

# 5

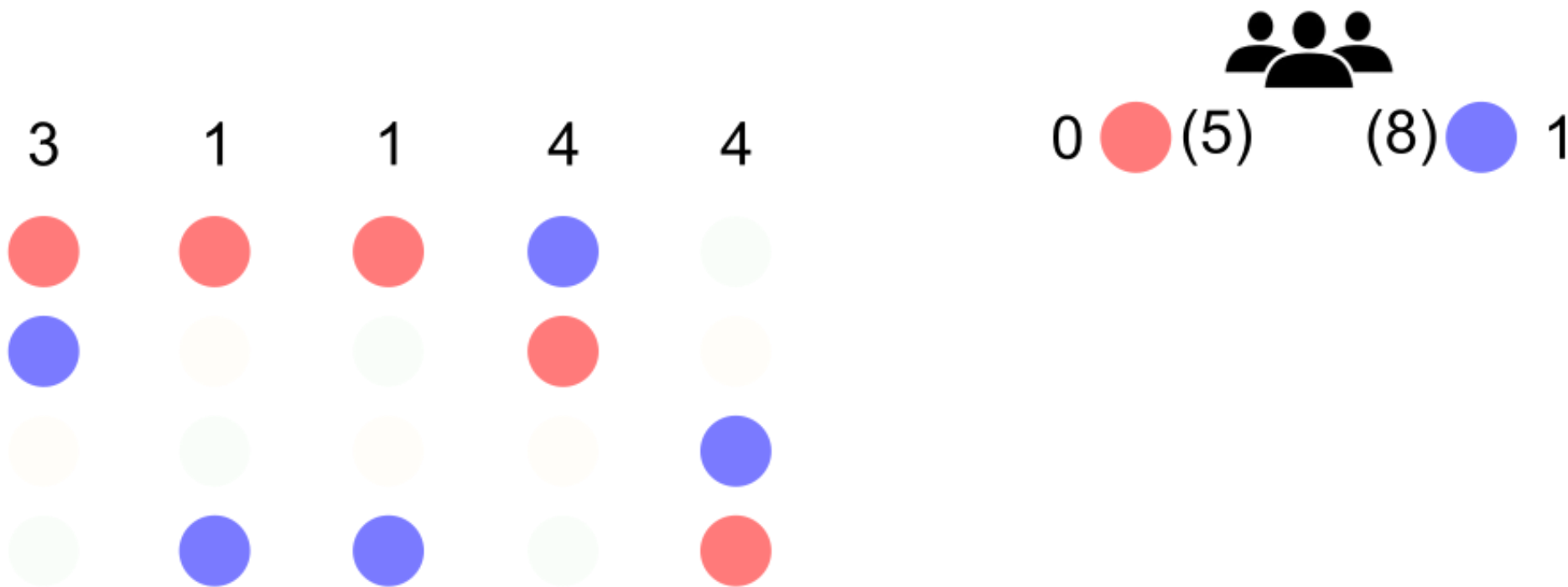
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Copeland

# 5

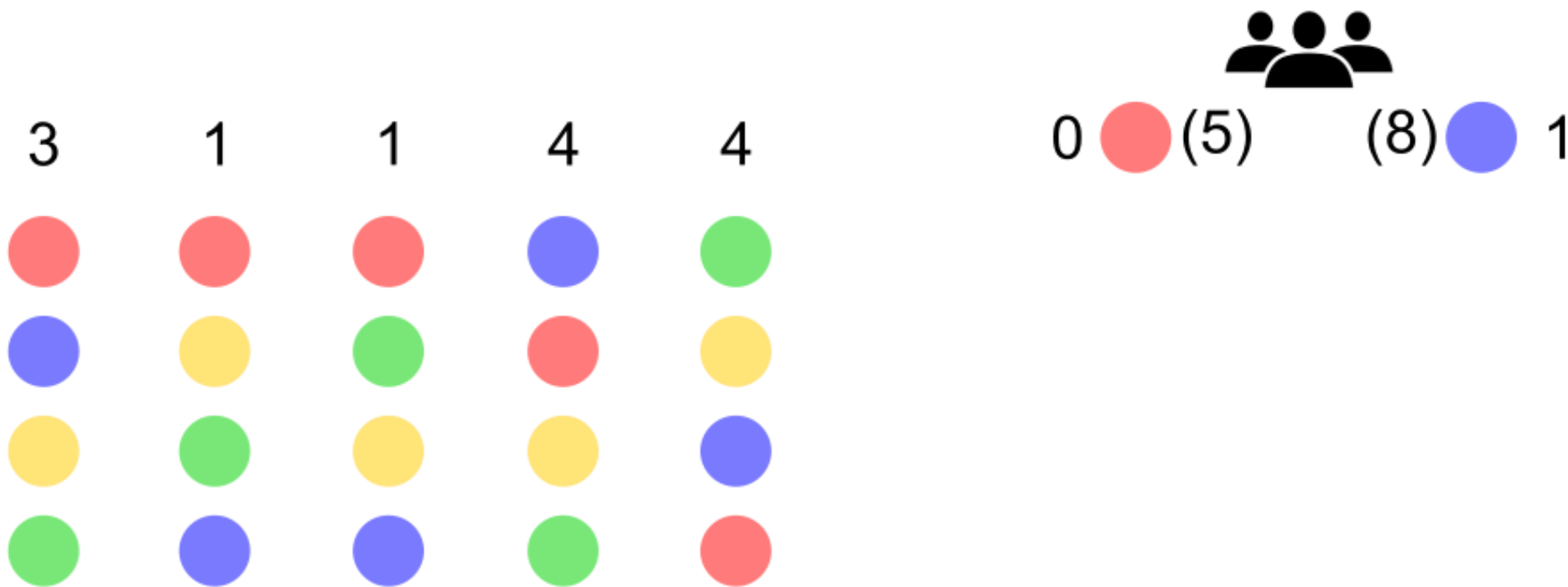
For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

# 5

For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

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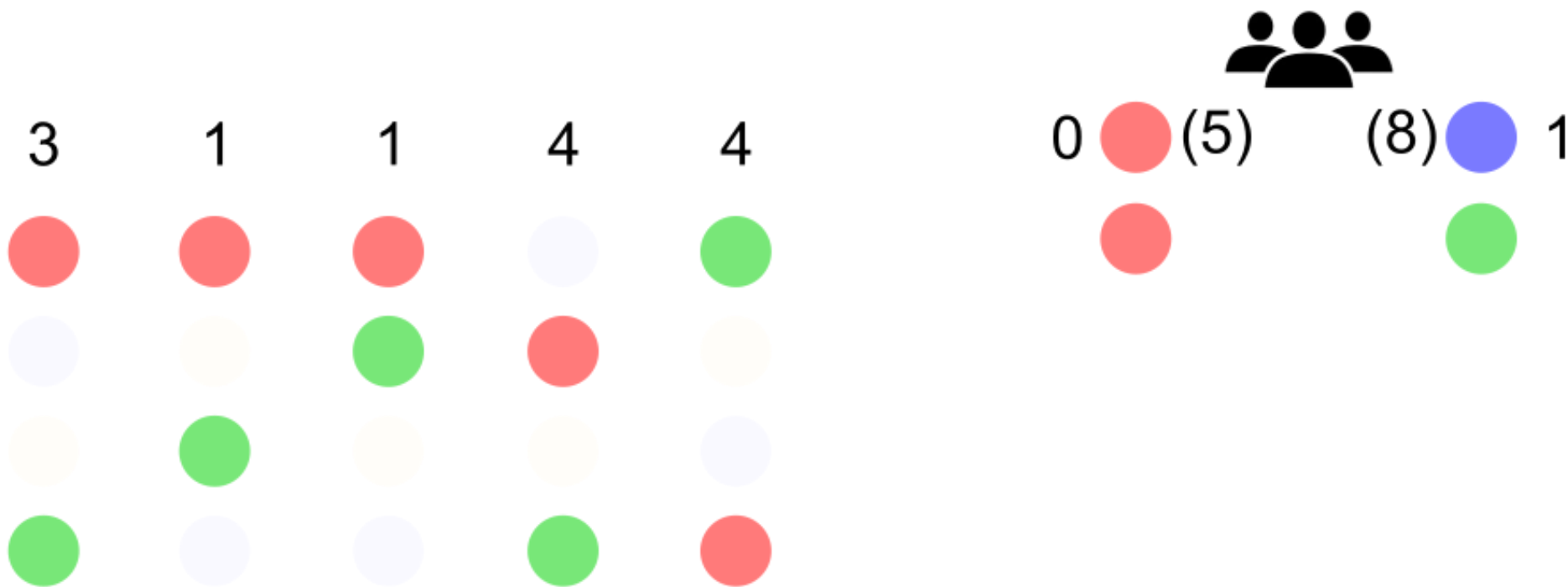
For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

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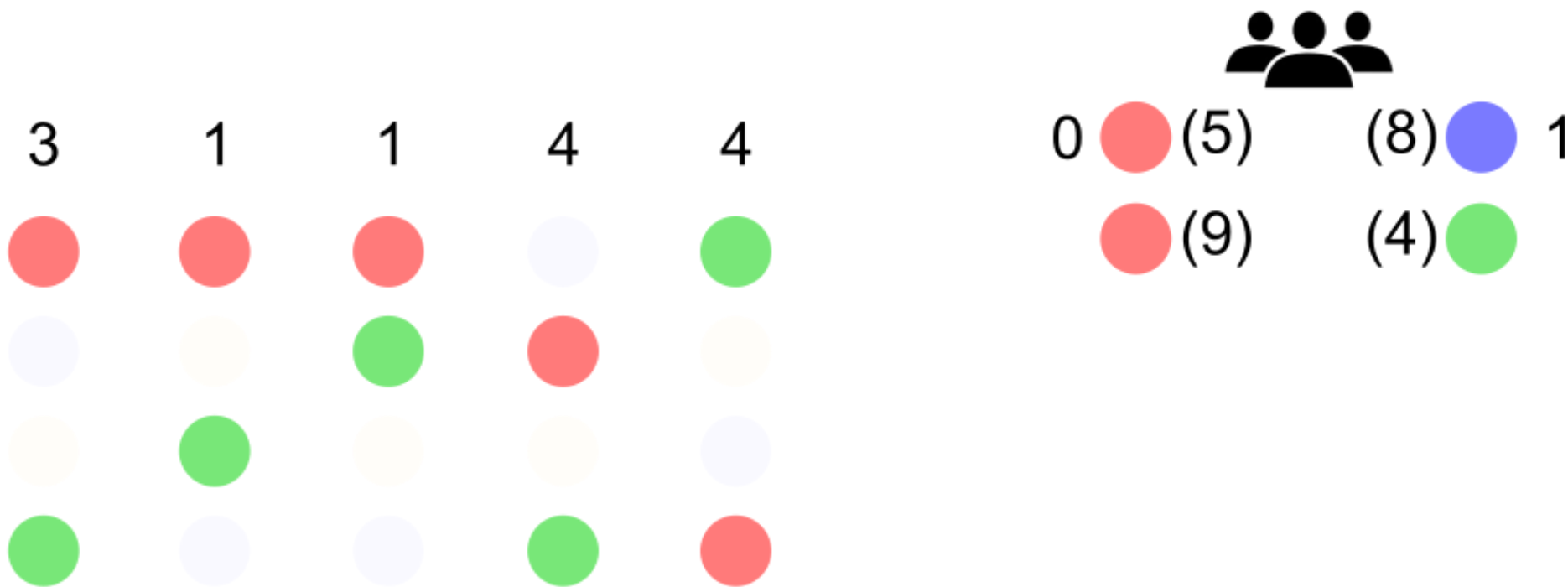
For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

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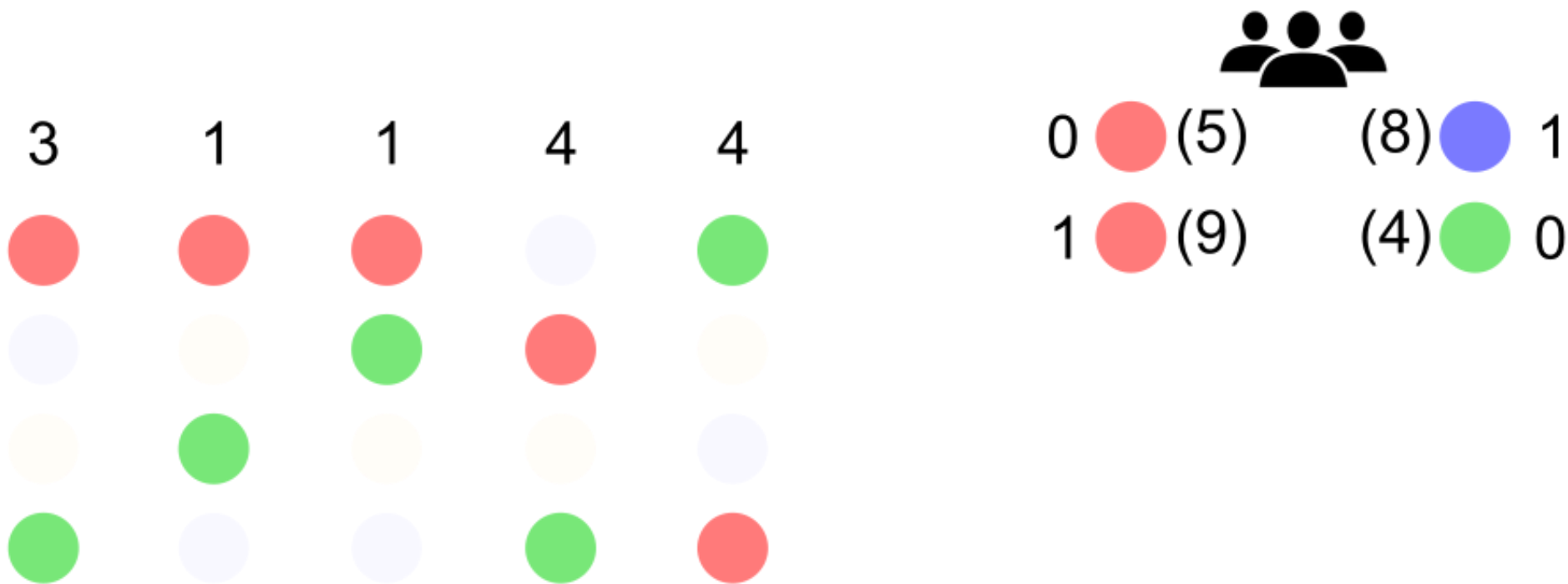
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Copeland

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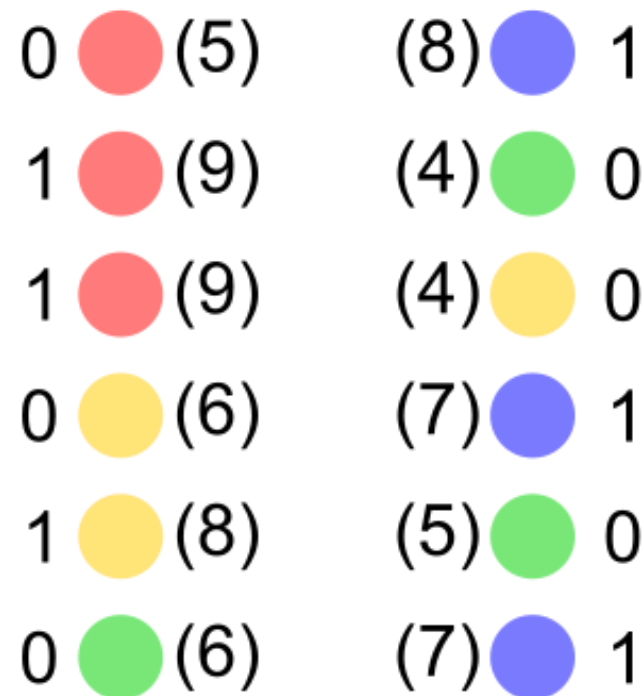
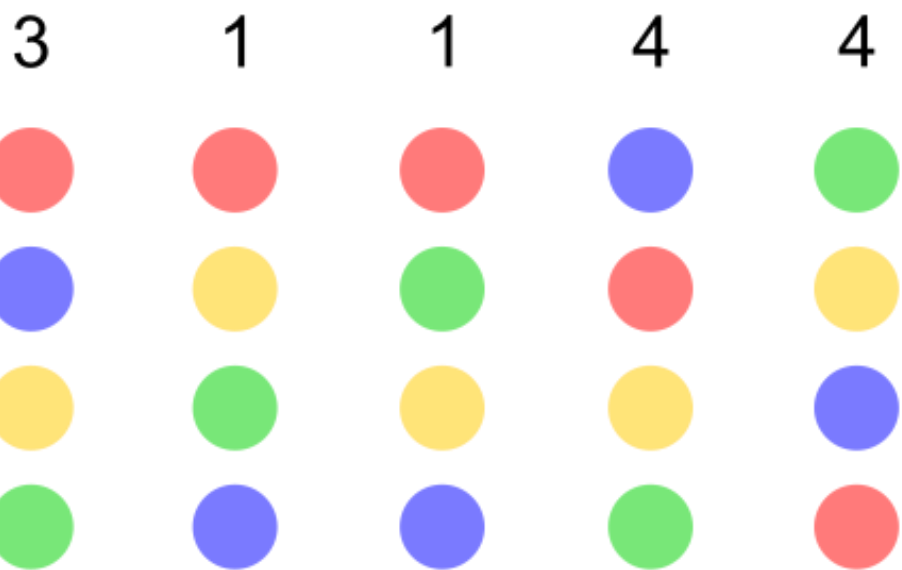


Copeland



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For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland

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For each head-to-head election, a candidate gets 1 point for winning, 0 for losing, and 0.5 for a tie



Copeland winner: 

Copeland

# 5



**BUNDESLIGA**



**Premier  
League**



**ICC WOMEN'S  
CRICKET WORLD CUP  
NEW ZEALAND 2022**



**INDIAN  
PREMIER  
LEAGUE**

Copeland

# 5

**Problem:** Voters are sometimes better off not voting

Copeland

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: 

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: ●

# 5

**Problem:** Voters are sometimes better off not voting

Copeland



Copeland winner: ●

6

Schulze



# 6

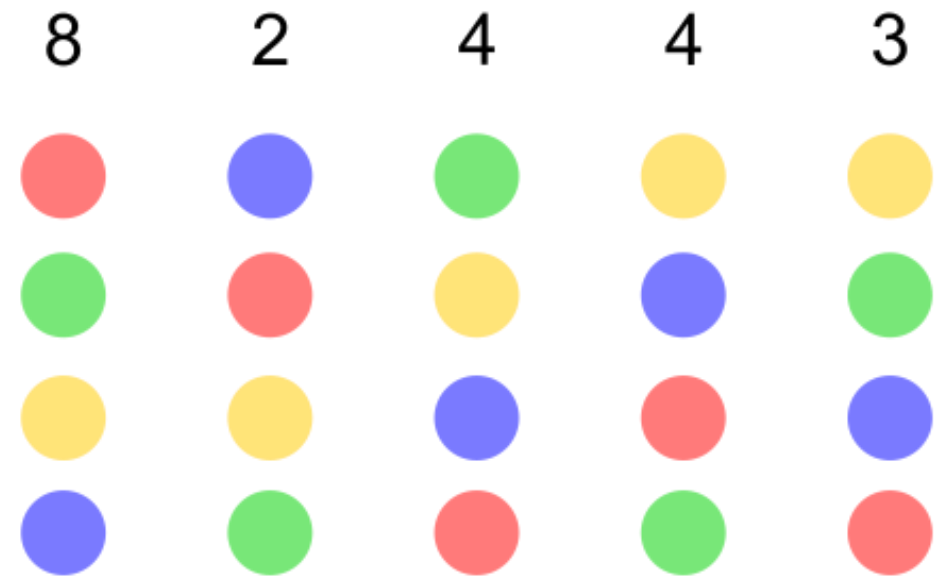
Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

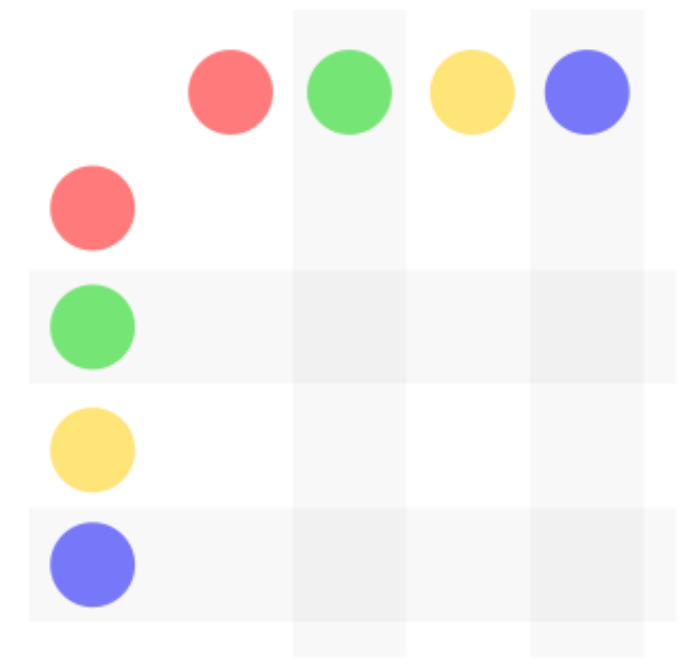
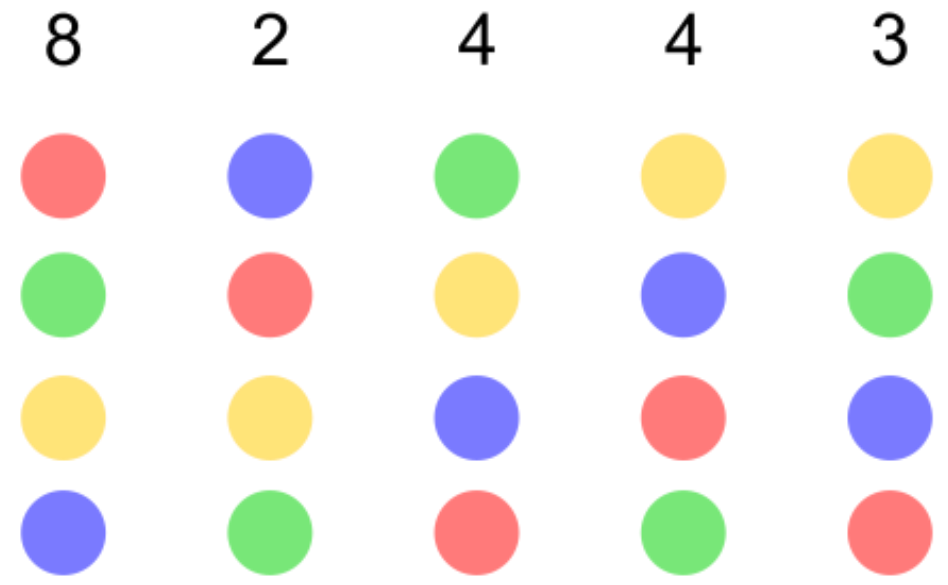
Schulze



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

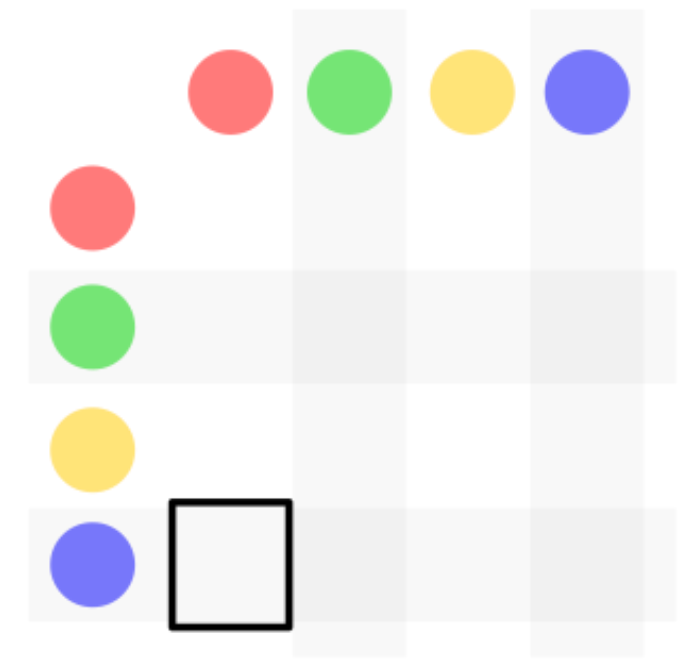
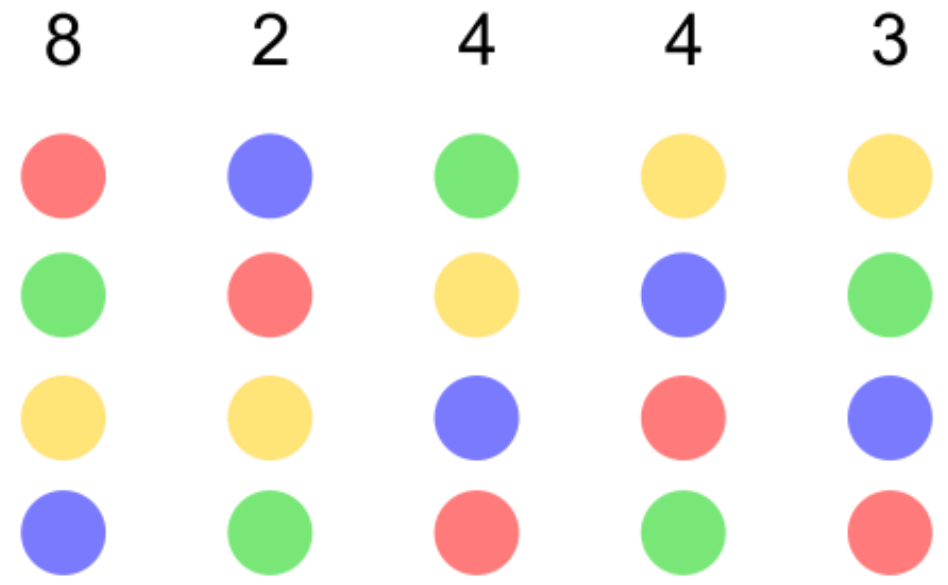
Schulze



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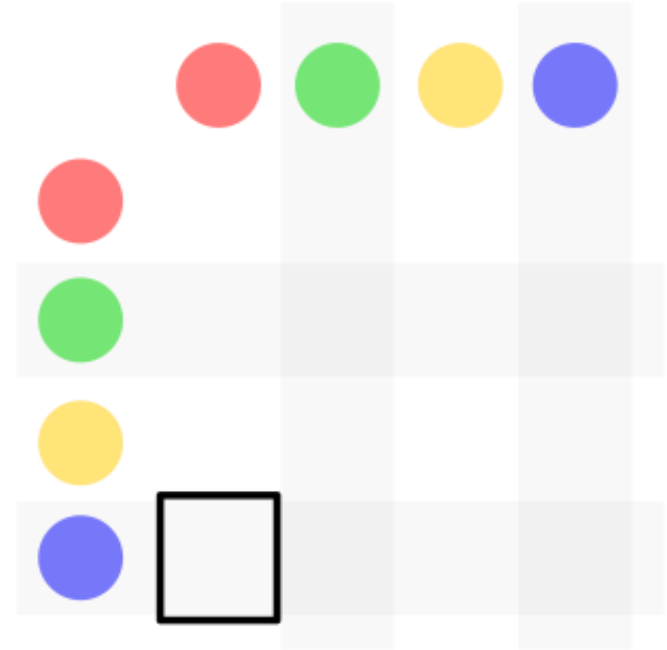
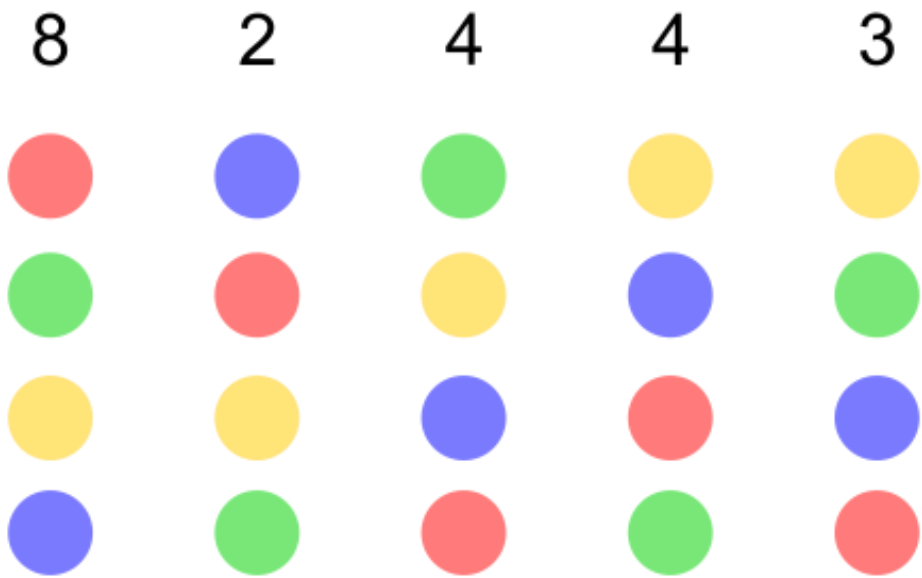
Schulze



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

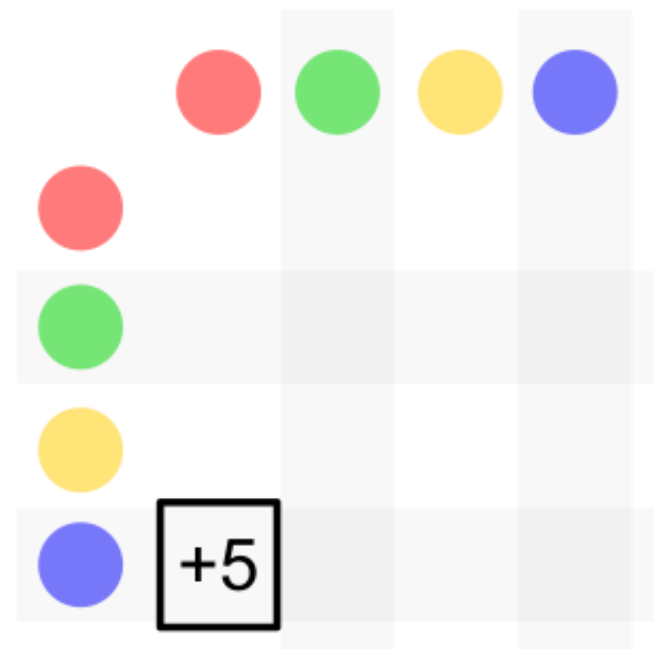
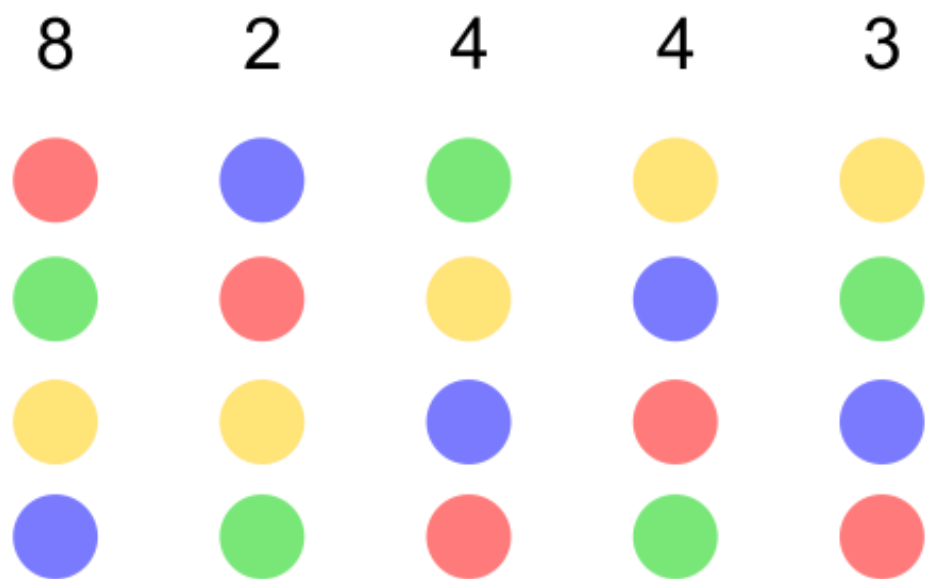


#voters who think  $\text{Blue} > \text{Red}$   $(2+4+4+3) - 8$   $\text{Red} > \text{Blue}$

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze



#voters who think ● > ●

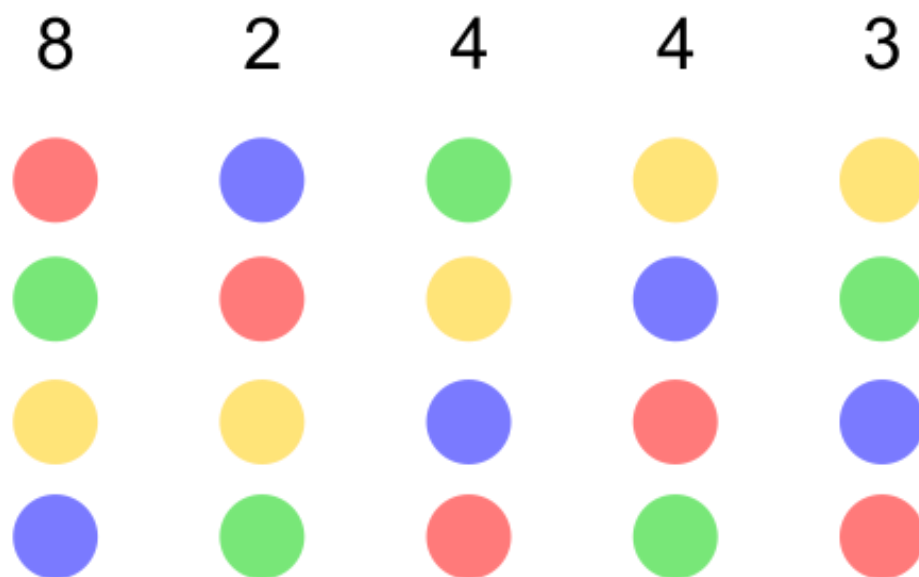
$$(2+4+4+3) - 8$$

#voters who think ● > ●



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



Schulze



	Red	Green	Yellow	Blue
Red	0	+7	-1	-5
Green	-7	0	+3	+9
Yellow	+1	-3	0	+17
Blue	<b>+5</b>	-9	-17	0

#voters who think  > 

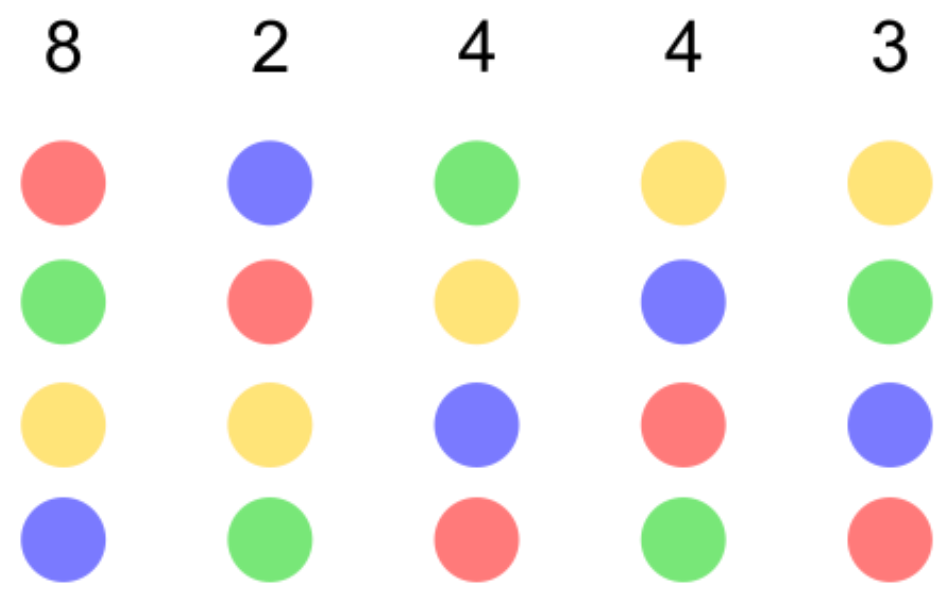
$$(2+4+4+3) - 8$$

#voters who think  > 

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze



	Red	Green	Yellow	Blue
Red	0	+7	-1	-5
Green	-7	0	+3	+9
Yellow	+1	-3	0	+17
Blue	+5	-9	-17	0



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

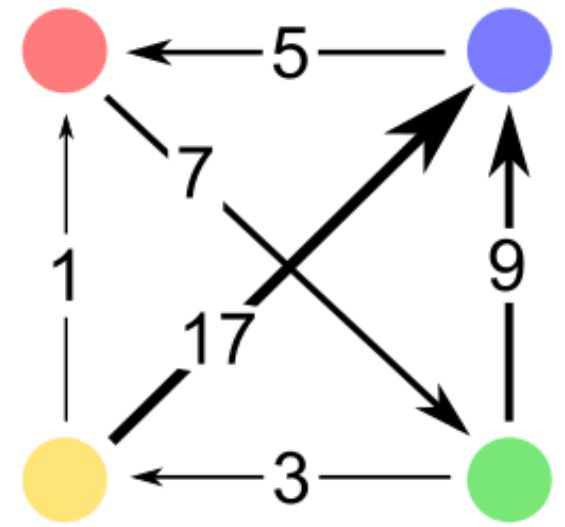
Schulze

				
	0	+7	-1	-5
	-7	0	+3	+9
	+1	-3	0	+17
	+5	-9	-17	0

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Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

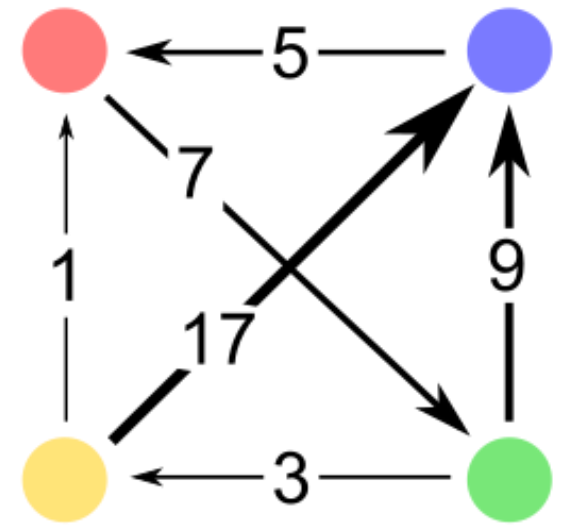


	●	●	●	●
●	0	+7	-1	-5
●	-7	0	+3	+9
●	+1	-3	0	+17
●	+5	-9	-17	0

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

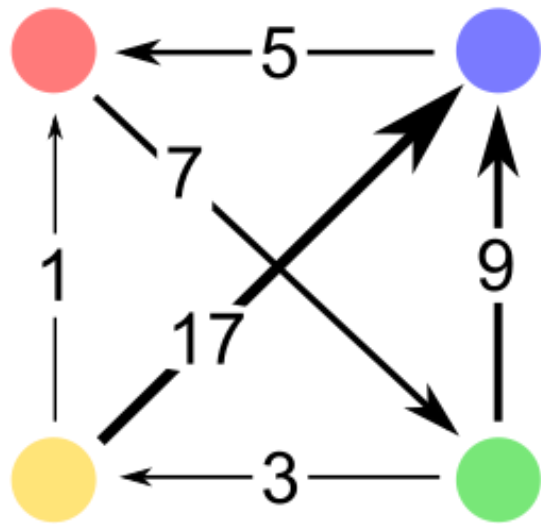
Schulze



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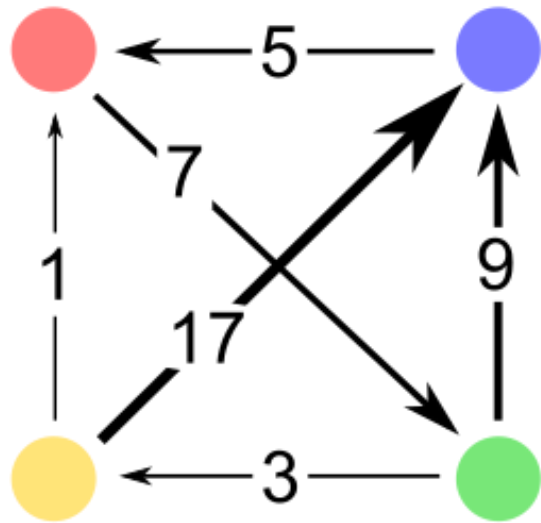
The **strength** of a path/chain is the weight of the **weakest** link in it



Schulze

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



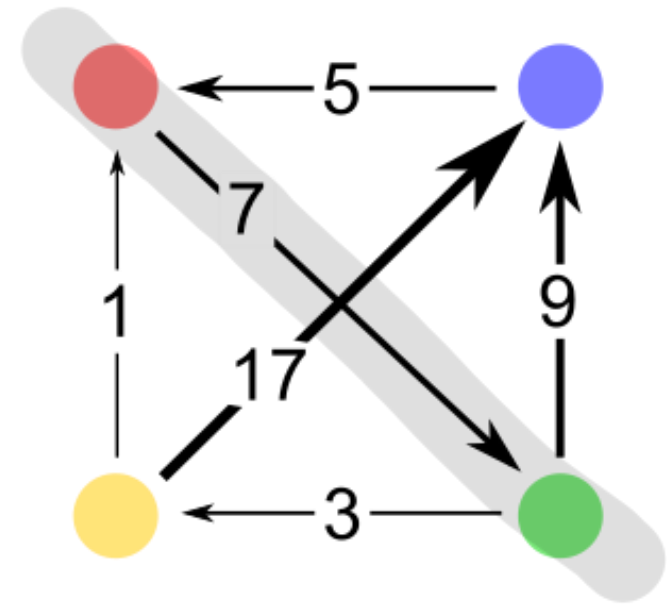
The **strength** of a path/chain is the weight of the **weakest** link in it

What's the strongest path from ● to ●?

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

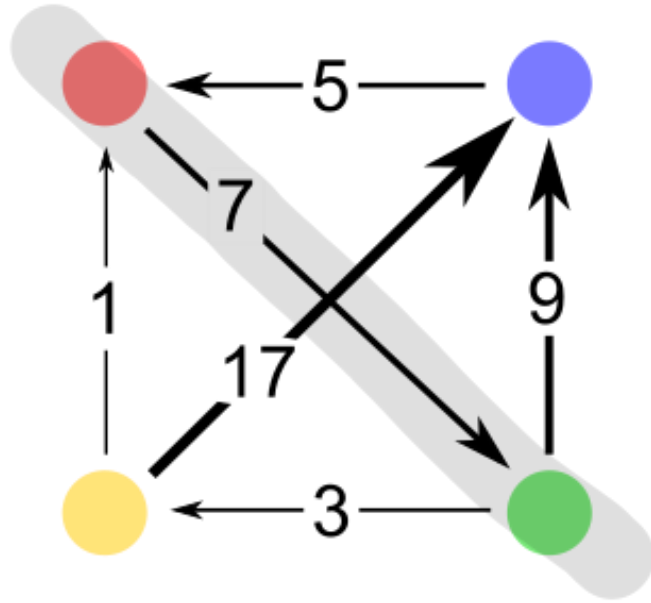


The **strength** of a path/chain is the weight of the **weakest** link in it

What's the strongest path from ● to ●?

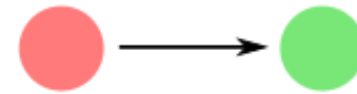
# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



The **strength** of a path/chain is the weight of the **weakest** link in it

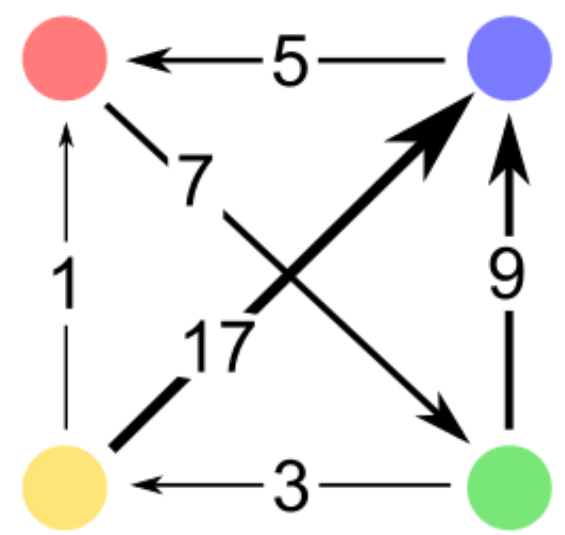
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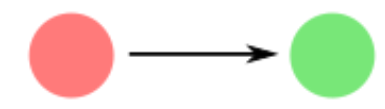
Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



The **strength** of a path/chain is the weight of the **weakest** link in it

What's the strongest path from red to green?

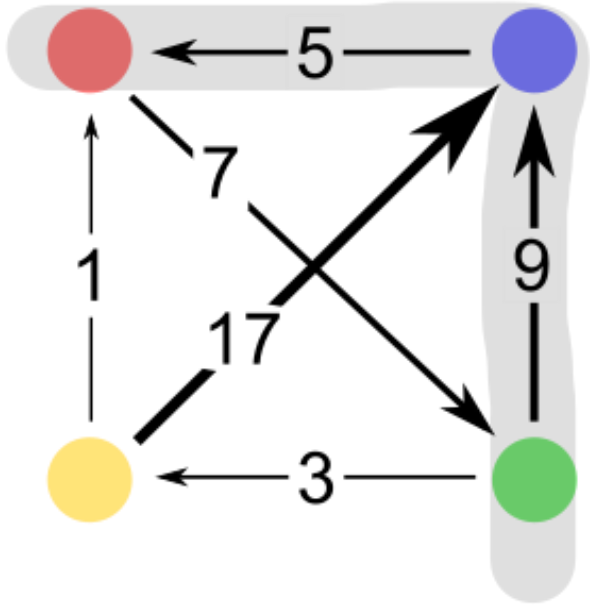


What's the strongest path from green to red?



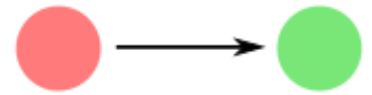
# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



The **strength** of a path/chain is the weight of the **weakest** link in it

What's the strongest path from red to green?



What's the strongest path from green to red?

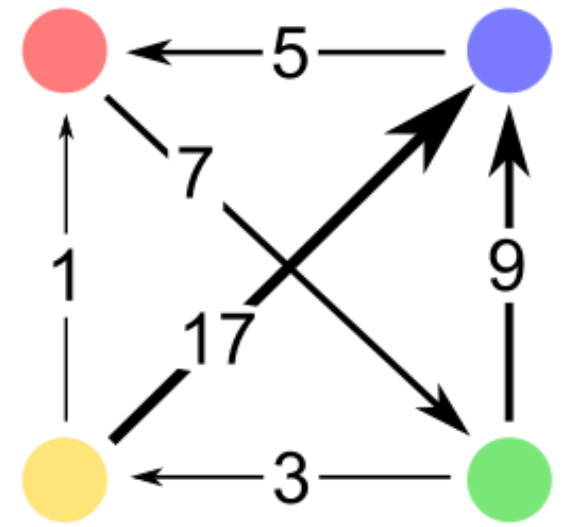


Schulze

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

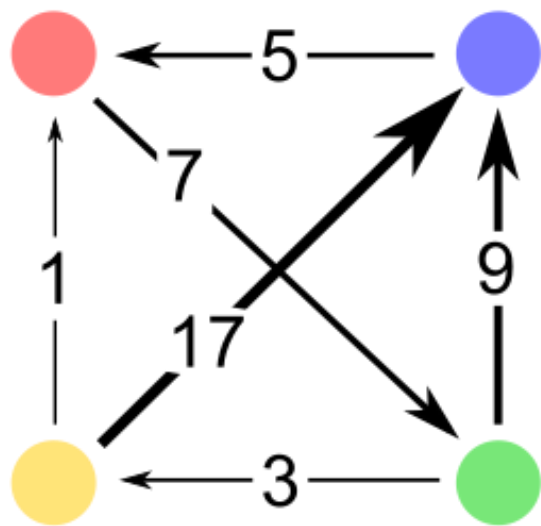
Schulze



# 6

Schulze

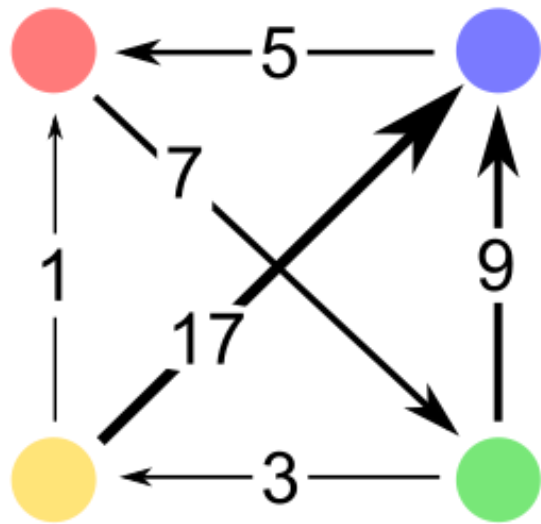
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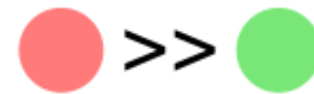
$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$

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Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

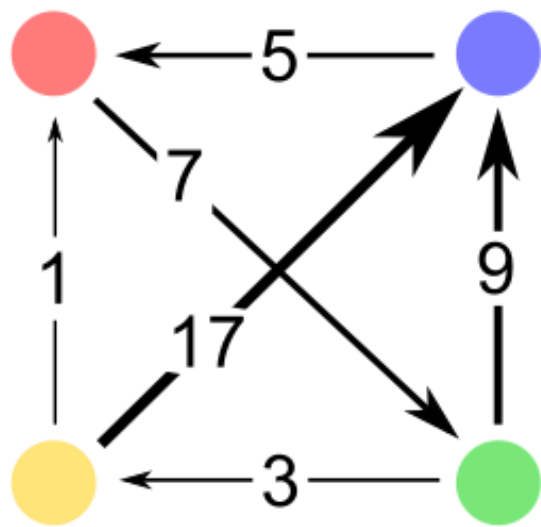


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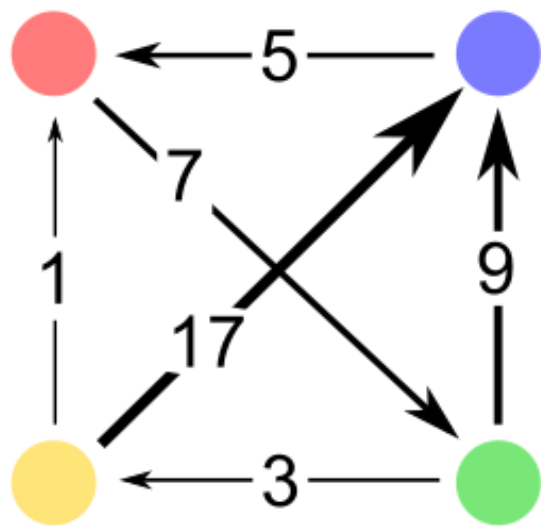
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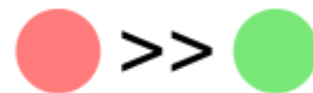
the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{green}}$  has strength 7

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$



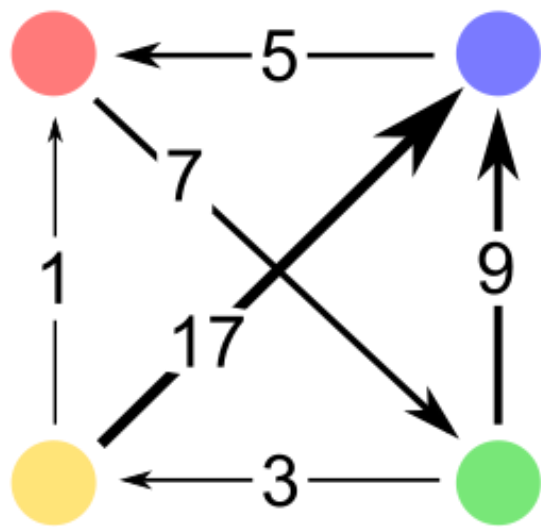
the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{green}}$  has strength 7

the strongest path from  $\textcircled{\text{green}}$  to  $\textcircled{\text{red}}$  has strength 5

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

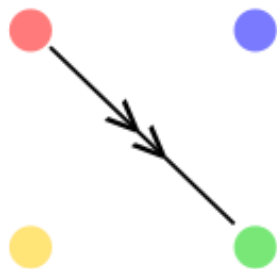


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the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{green}}$  has strength 7

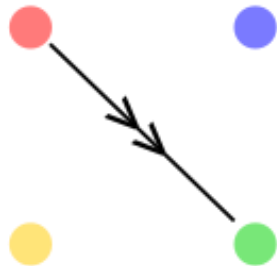
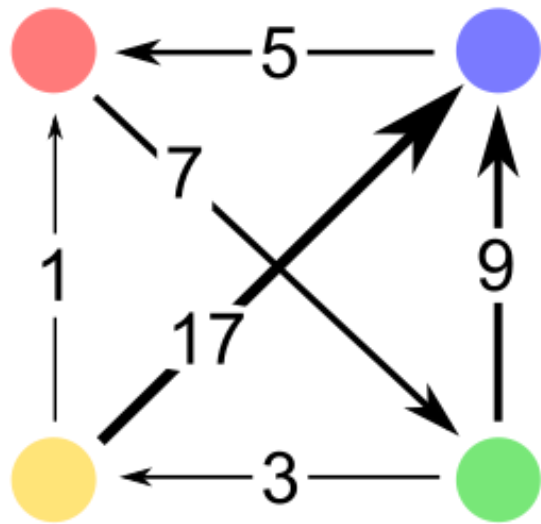
the strongest path from  $\textcircled{\text{green}}$  to  $\textcircled{\text{red}}$  has strength 5



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze



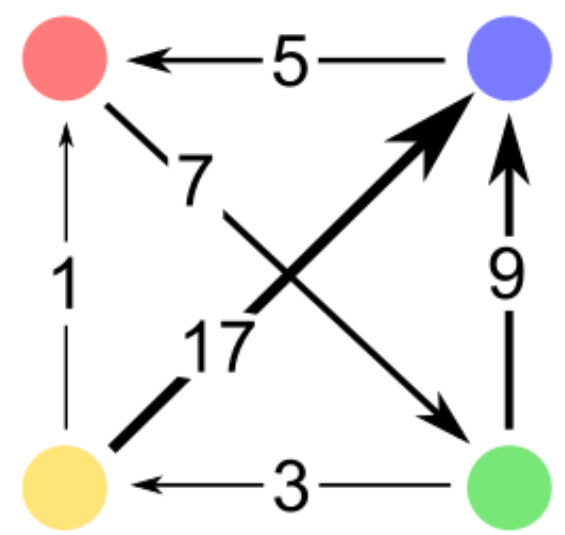
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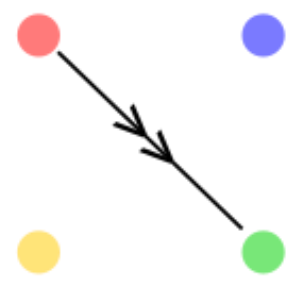
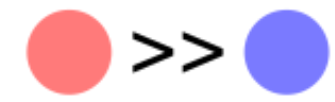
# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



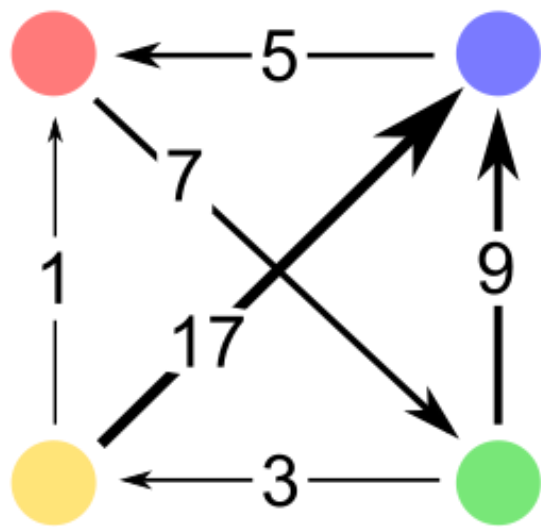
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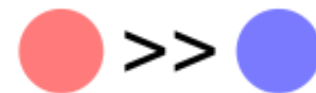
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Schulze

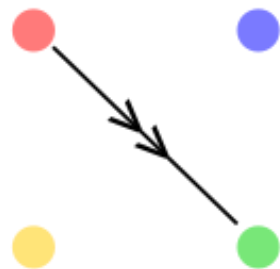
Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



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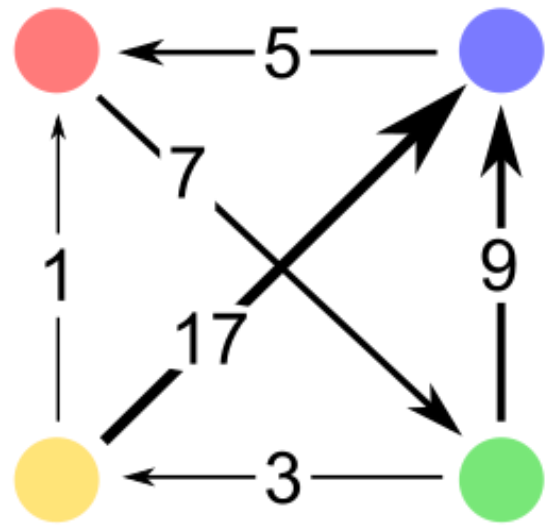
the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{blue}}$  has strength 7



# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

Schulze

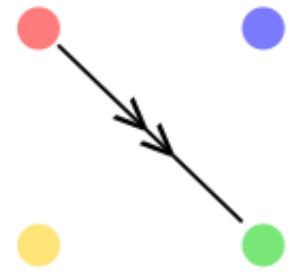


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the strongest path from red to blue has strength 7

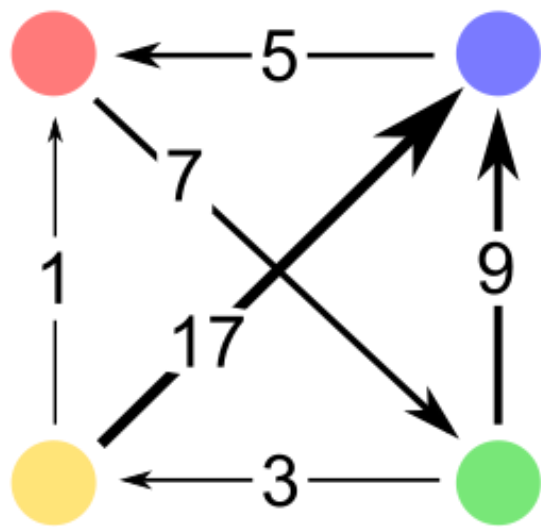
the strongest path from blue to red has strength 5



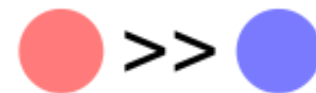
# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

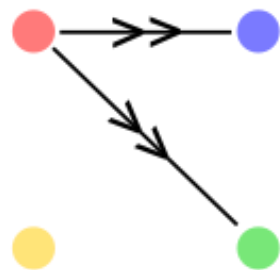


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the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{blue}}$  has strength 7

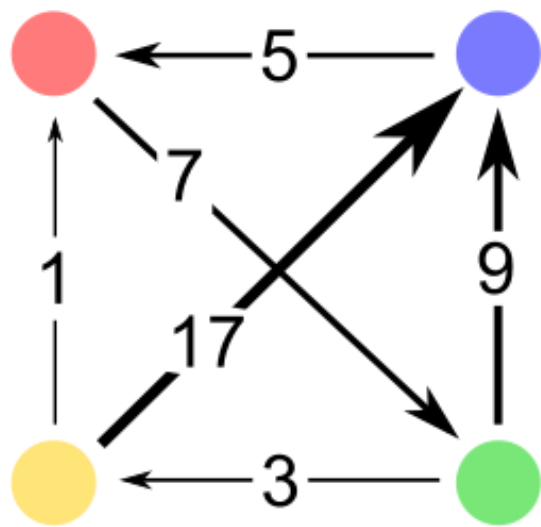
the strongest path from  $\textcircled{\text{blue}}$  to  $\textcircled{\text{red}}$  has strength 5



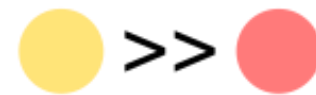
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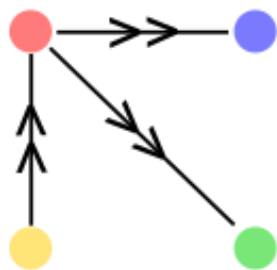


$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$



the strongest path from  $\textcircled{\text{yellow}}$  to  $\textcircled{\text{red}}$  has strength 5

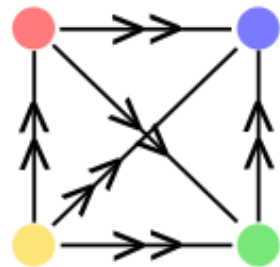
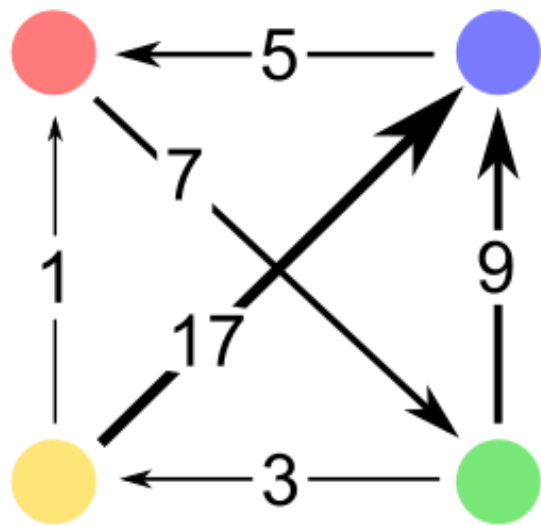
the strongest path from  $\textcircled{\text{red}}$  to  $\textcircled{\text{yellow}}$  has strength 3



# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

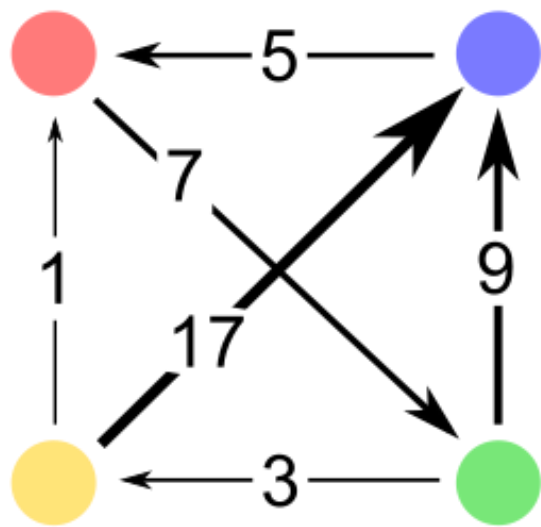


$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$

$\textcircled{y} \gg$  all others

# 6

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph

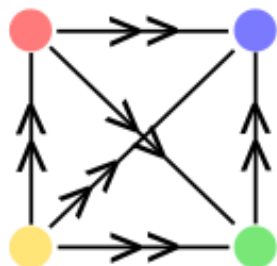


$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$

●  $\gg$  all others

Schulze winner: ●

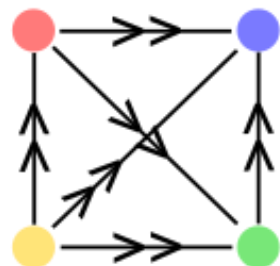
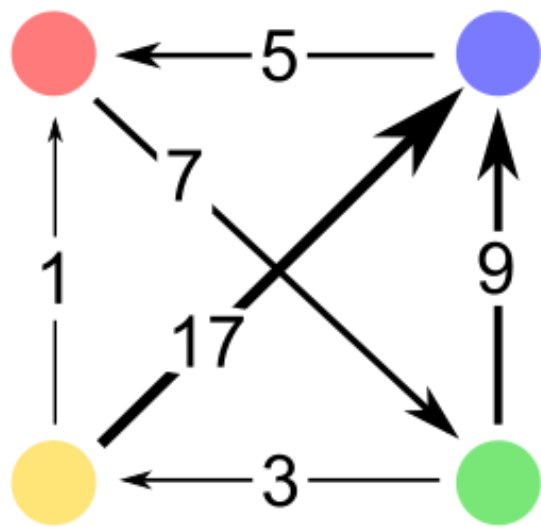
Schulze



# 6

Schulze

Winner is the candidate who "chain beats" every other candidate in the pairwise comparison graph



$\textcircled{a} \gg \textcircled{b}$  ( $\textcircled{a}$  "chain beats"  $\textcircled{b}$ ) if the *strongest* path from  $\textcircled{a}$  to  $\textcircled{b}$  is *stronger* than the *strongest* path from  $\textcircled{b}$  to  $\textcircled{a}$

A Schulze winner always exists!

The "chain beats" relation is transitive

$\textcircled{a} \gg \textcircled{b}$  and  $\textcircled{b} \gg \textcircled{c}$ , then  $\textcircled{a} \gg \textcircled{c}$



6



Schulze

Which Voting Rule is the Best?

# Which Voting Rule is the Best?

## **Voting Power in Practice Summer Workshop**

### **Assessing Alternative Voting Procedures**

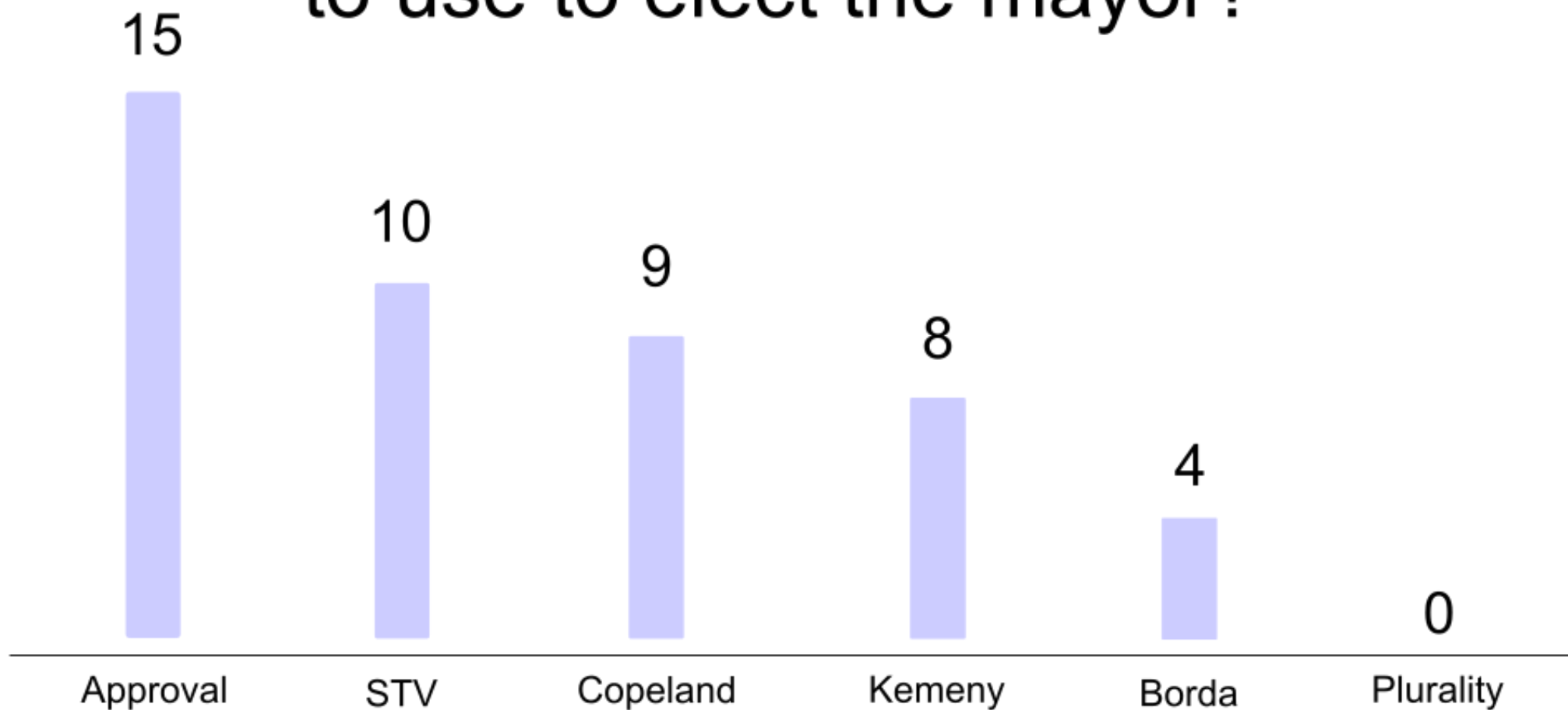
**Sponsored by The Leverhulme Trust**

Chateau du Baffy, Normandy, France  
30 July – 2 August 2010

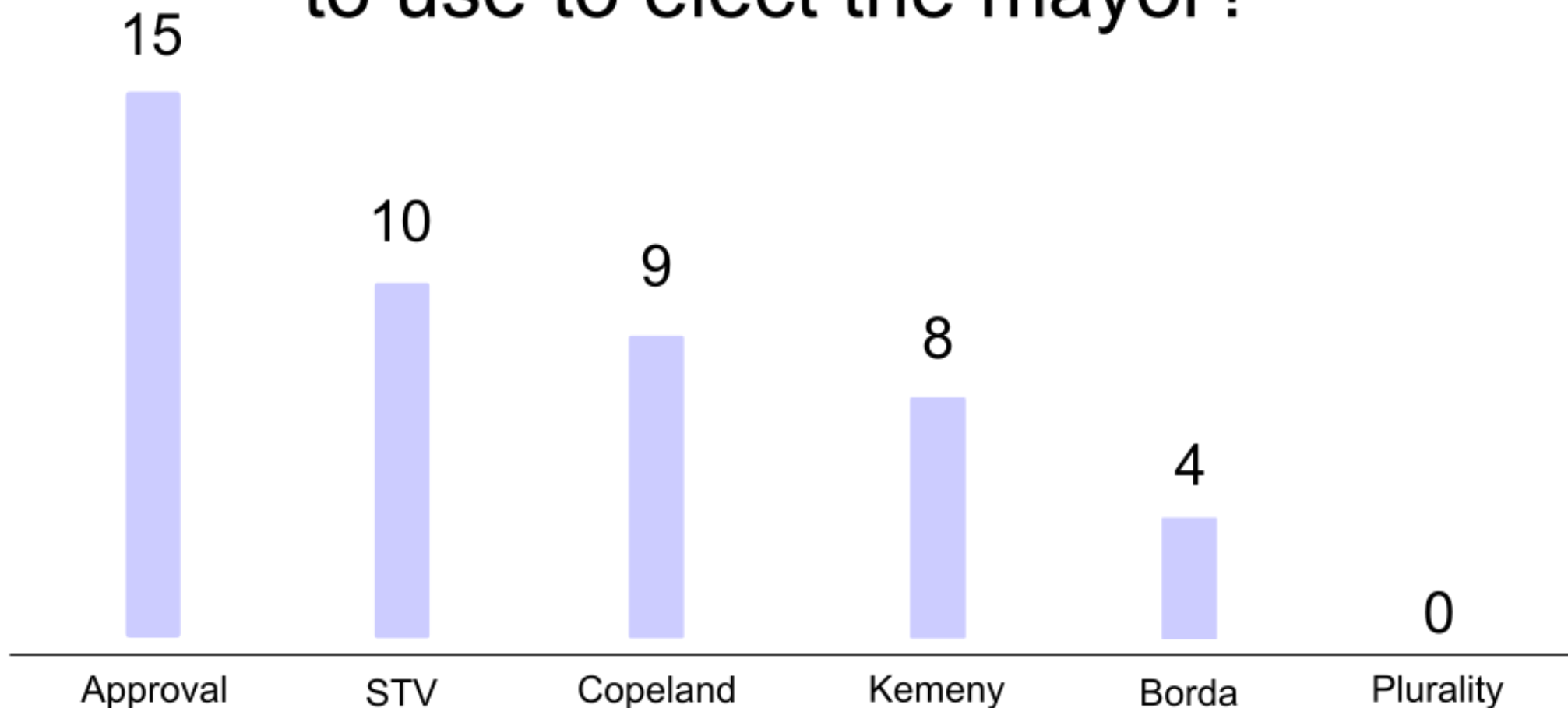
22 voting theorists

"What is the best voting rule for your town to use to elect the mayor?"

"What is the best voting rule for your town to use to elect the mayor?"



"What is the best voting rule for your town to use to elect the mayor?"



Votes were counted via approval voting.

18

12

10

9

4

2

Plurality



Plurality  
with runoff



Borda



Condorcet



(Copeland, Schulze)

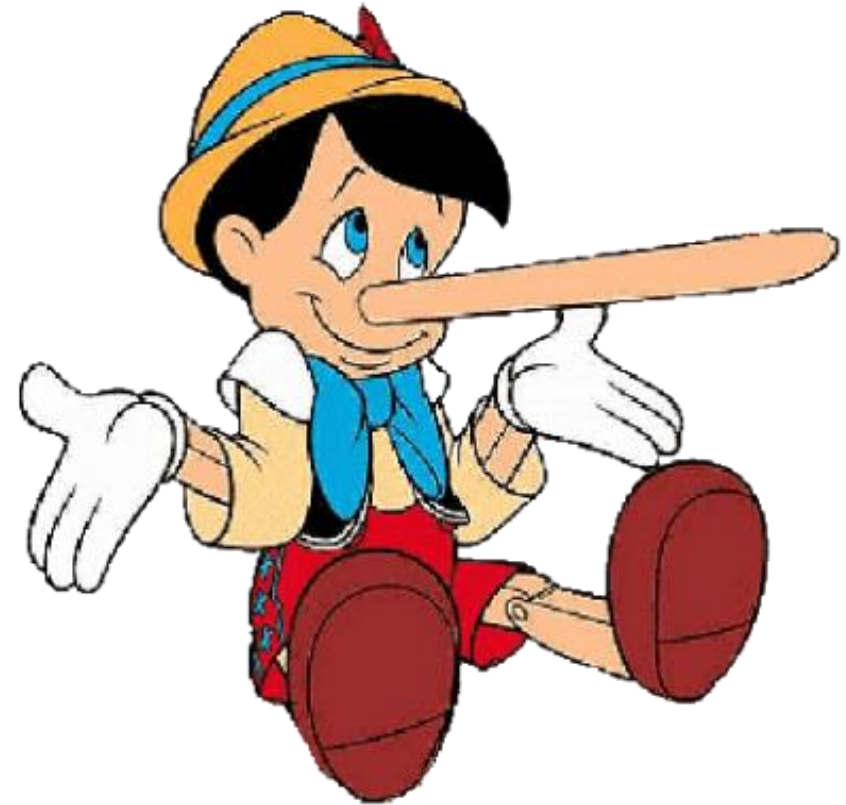


STV



# Next Time

Manipulation in voting  
Oct 7 (Fri)





# References

- “Disagreement between voting rules” example:  
<http://www.ams.org/publicoutreach/feature-column/fcarc-voting-decision>
- The “chain beats” terminology in the description of Schulze rule was borrowed from Hubert Bray’s explanatory video:  
[https://www.youtube.com/watch?v=\\_HVeN0GnnuA](https://www.youtube.com/watch?v=_HVeN0GnnuA)
- The Schulze rule example is from the paper “The Schulze Method of Voting” by Markus Schulze:  
<https://arxiv.org/abs/1804.02973>
- (Possibly) strategic voting in the selection of Olympics host:  
[https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic\\_2\\_Plurality\\_Runoff.pdf](https://www3.nd.edu/~apilking/math10170/Information/Lectures%202015/Topic_2_Plurality_Runoff.pdf)

# References

- “Which voting rule is the best” poll  
<https://hal.archives-ouvertes.fr/hal-00609810/document>

